Bridges between Quantum information and Many-body physics

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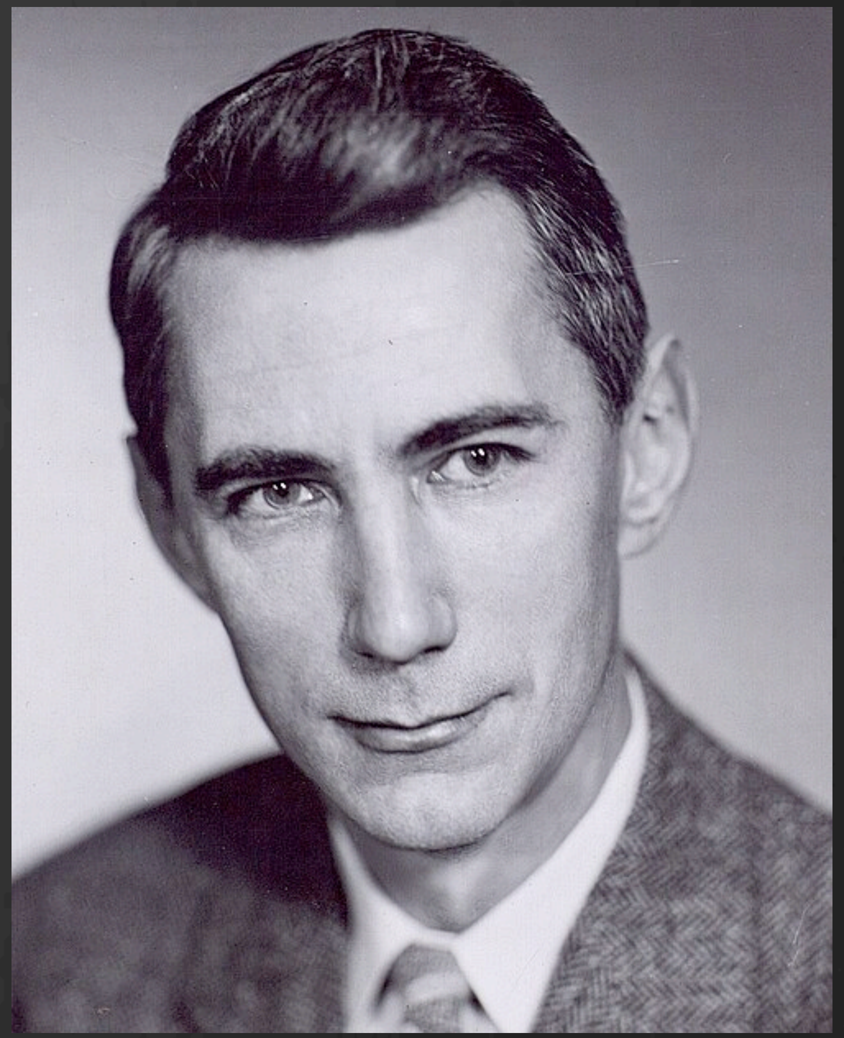
11 Dec 2025

1. Entropy
in information theory and physics

My greatest concern was what to call it. I thought of calling it 'information,' but the word was overly used, so I decided to call it 'uncertainty.' When I discussed it with John von Neumann, he had a better idea. Von Neumann told me, 'You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, no one really knows what entropy really is, so in a debate you will always have the advantage.'



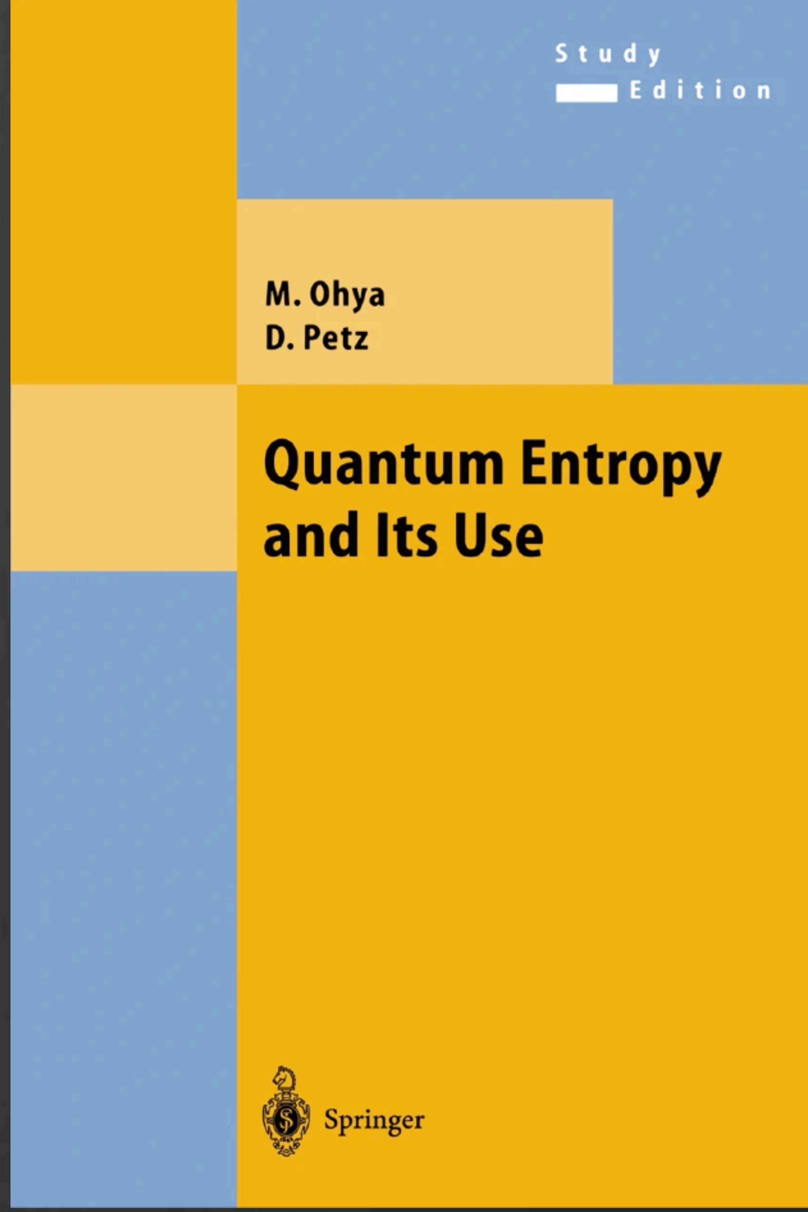
Claude Shannon

Information theory
content of a message

Statistical Physics
phase-space volume

Bridge
compression

Free energy and relative entropy



$F(\rho) = \mathrm{tr}[\rho H] - T \cdot S(\rho)$

$D(\rho \| \sigma) = \mathrm{tr}[\rho(\log \rho - \log \sigma)]$

<u>minimized by Gibbs state</u>

$$\sigma_T := \frac{e^{-H/T}}{Z}$$

<u>Bridge via Jaynes' principle:</u>

$$D(\rho \| \sigma_T) = F(\rho) - F(\sigma_T)$$

<u>Implications</u>

- $D(\rho \| \sigma_T) \geq 0$ means σ_T minimizes free energy.
- $D(\rho \| \sigma_T) \geq \frac{1}{2} \|\rho - \sigma_T\|_1^2$ makes this robust.

Means that $F(\rho) \approx F(\sigma_T)$ implies $\rho \approx \sigma_T$.

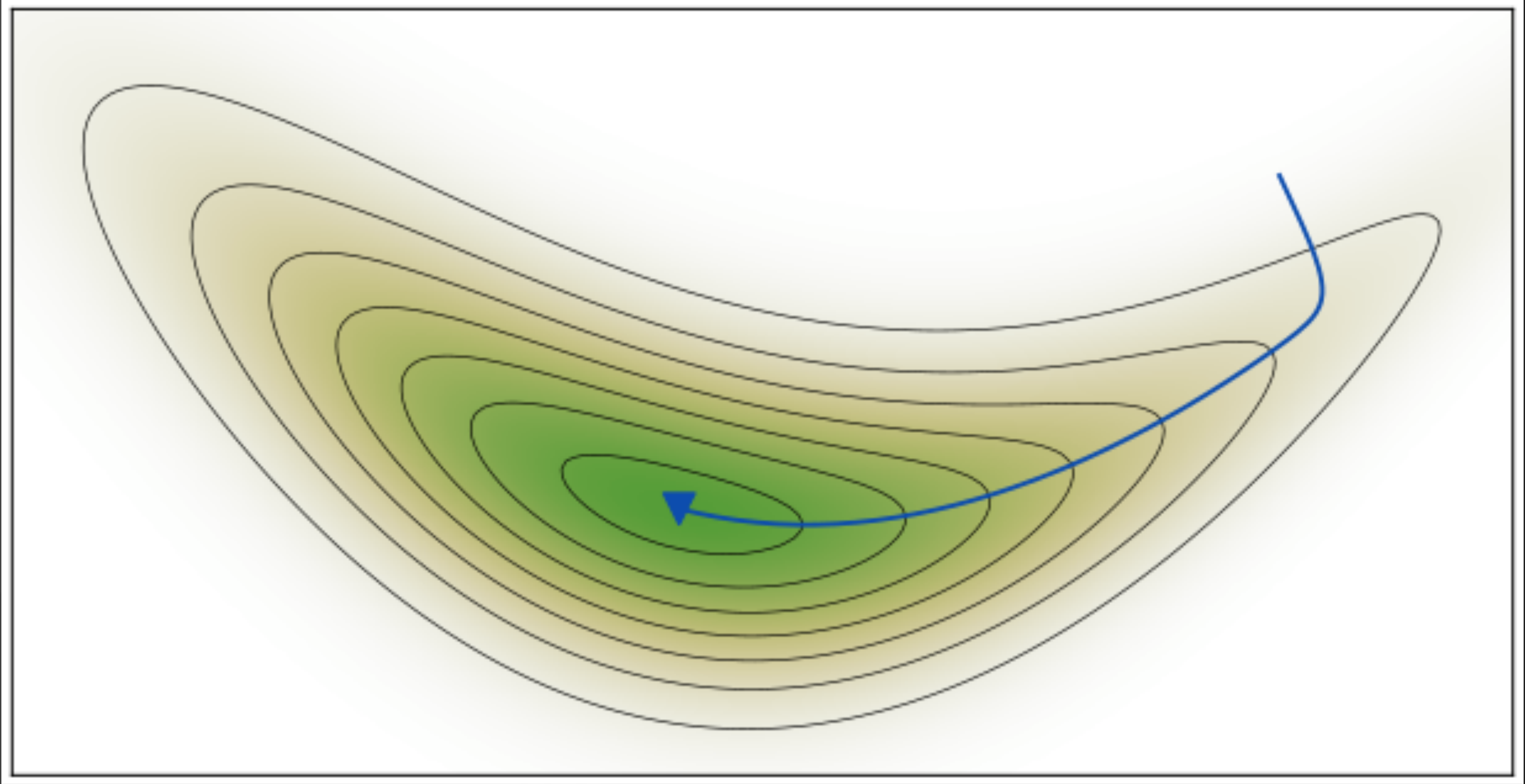
Information Theory and Statistical Mechanics

E. T. JAYNES
Department of Physics, Stanford University, Stanford, California
(Received September 4, 1956; revised manuscript received March 4, 1957)

Information Theory and Statistical Mechanics. II

E. T. JAYNES
Department of Physics, Stanford University, California
(Received March 15, 1957)

Entropic approach to gradient descent



gradient descent

$$x_{t+1} = \arg\min \eta \langle x_{t+1}, \nabla f(x_t) \rangle + \frac{1}{2} \| x_{t+1} - x_t \|^2$$

move downhill   not too fast

$$\Rightarrow \quad x_{t+1} = x_t - \eta \nabla f(x_t)$$

converges in O(dim) steps

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mirror descent

$$\rho_{t+1} = \arg\min \eta \,\mathrm{tr}[\rho_{t+1} \nabla f(\rho_t)] + D(\rho_{t+1} \| \rho_t)$$

move downhill   relative entropy

$$\Rightarrow \quad \log \rho_{t+1} = \log \rho_t - \eta \nabla f(\rho_t) + cI$$

matrix multiplicative weights

converges in O(log dim) steps

Entropic approach to thermalization in open systems

$$\dot{\rho} = \mathscr{L}(\rho)$$

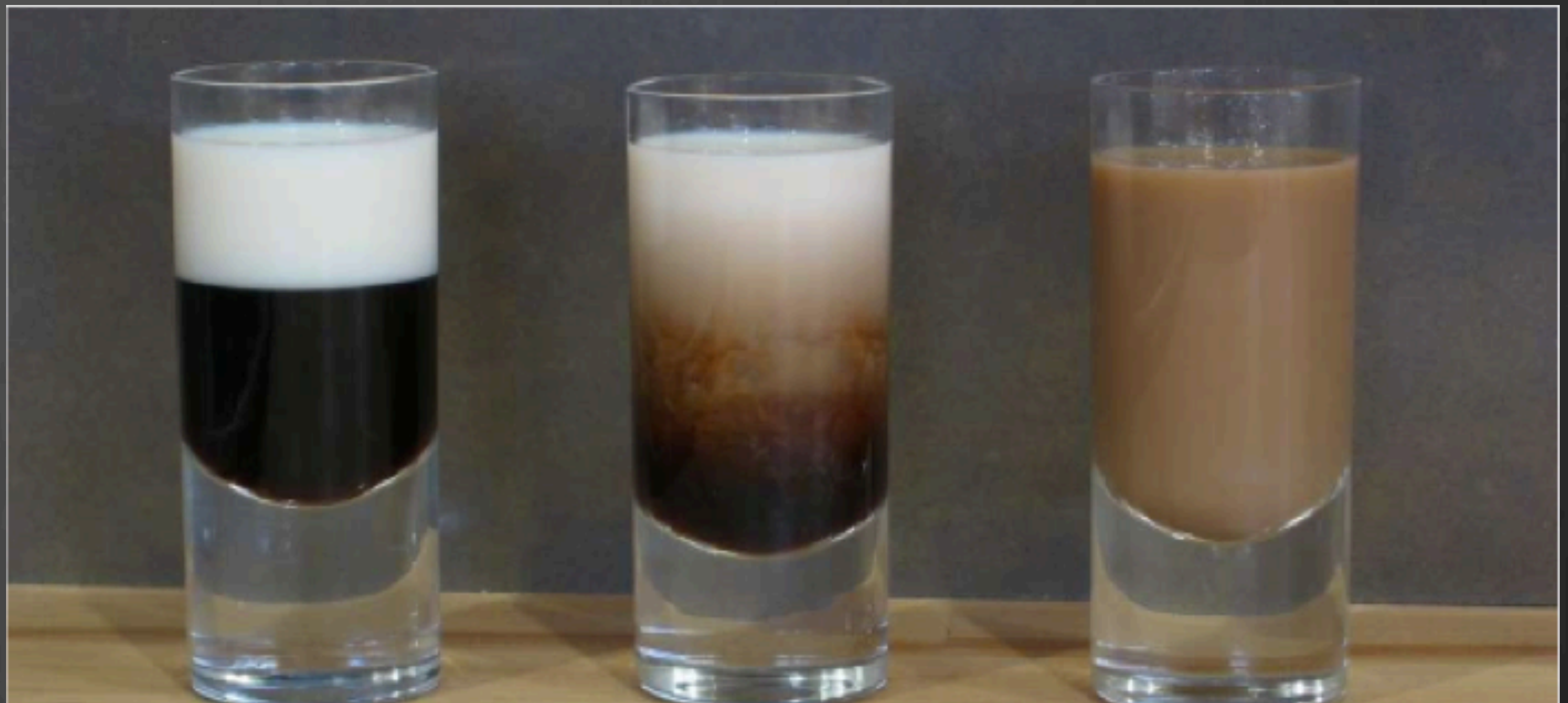


Photo credit: Sean Carroll

converges to stationary state σ
satisfying $\mathscr{L}(\sigma) = 0$

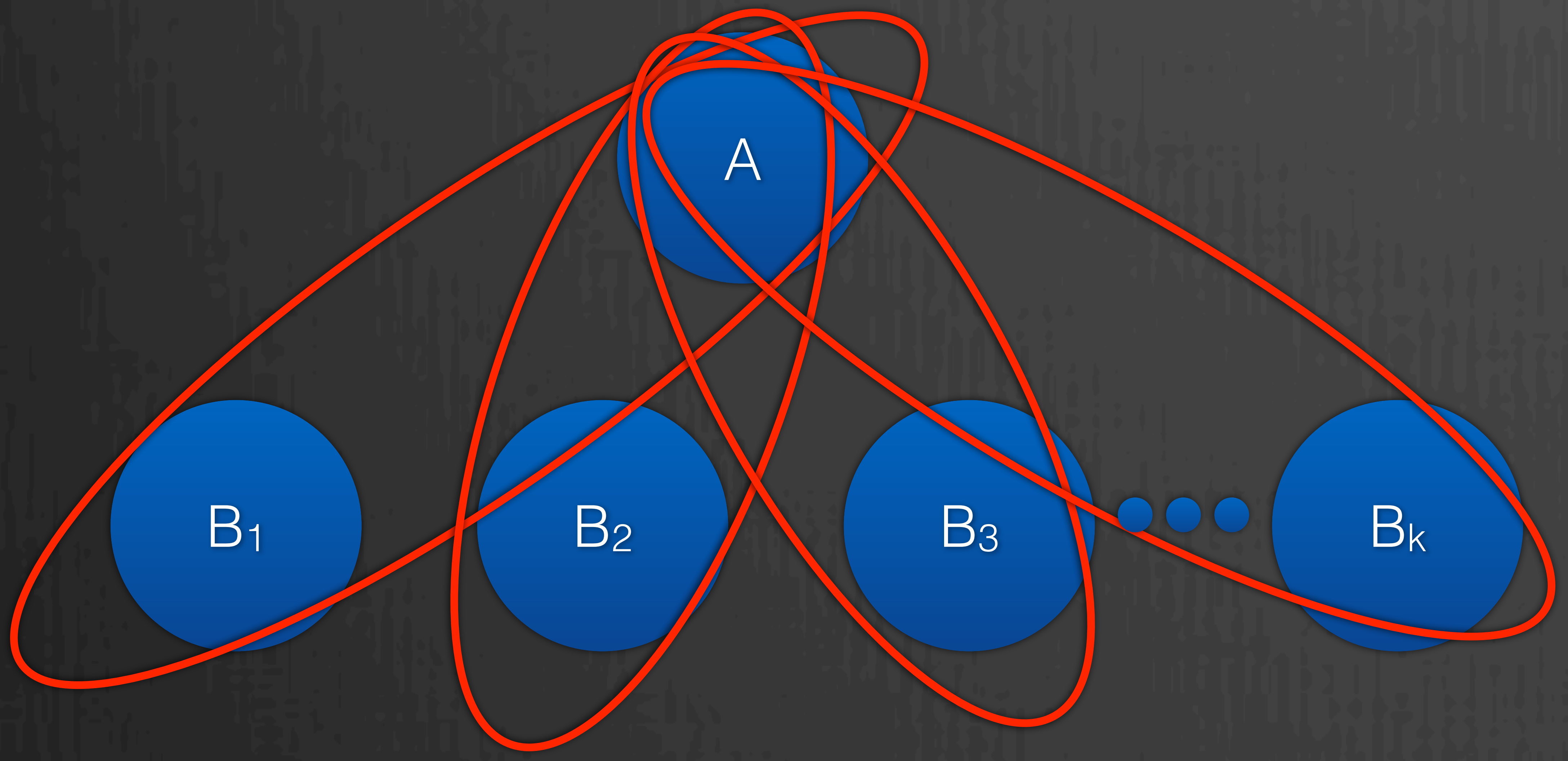
spectral gap: $\|\rho(t) - \sigma\|_1 \leq \|\sigma^{-1}\|^{-1/2} e^{-\text{gap}(\mathscr{L})t}$

MLSI: $D(\rho(t)\|\sigma) \leq D(\rho(0)\|\sigma)e^{-\alpha(\mathscr{L})t}$

Entropic approach to mean-field theory

[Brandão, H. 1310.0017]

$$\rho_{AB_1B_2\cdots B_k}$$



Monogamy of entanglement

ρ_{AB_i} ≈ unentangled for most i in {1, ... k}
if $k \gg \log(\dim A)$

- Previous methods needed $k \sim \dim A$.
- Quantifies Zurek's decoherence model.

arXiv:1210.6367 and arXiv:1205.4484

Algorithm for unique games

Estimating max {tr[Hρ] : ρ separable}
in time $d^{O(\log d)}$ where d = dim ρ.

Previous methods needed time $d^{O(d)}$.

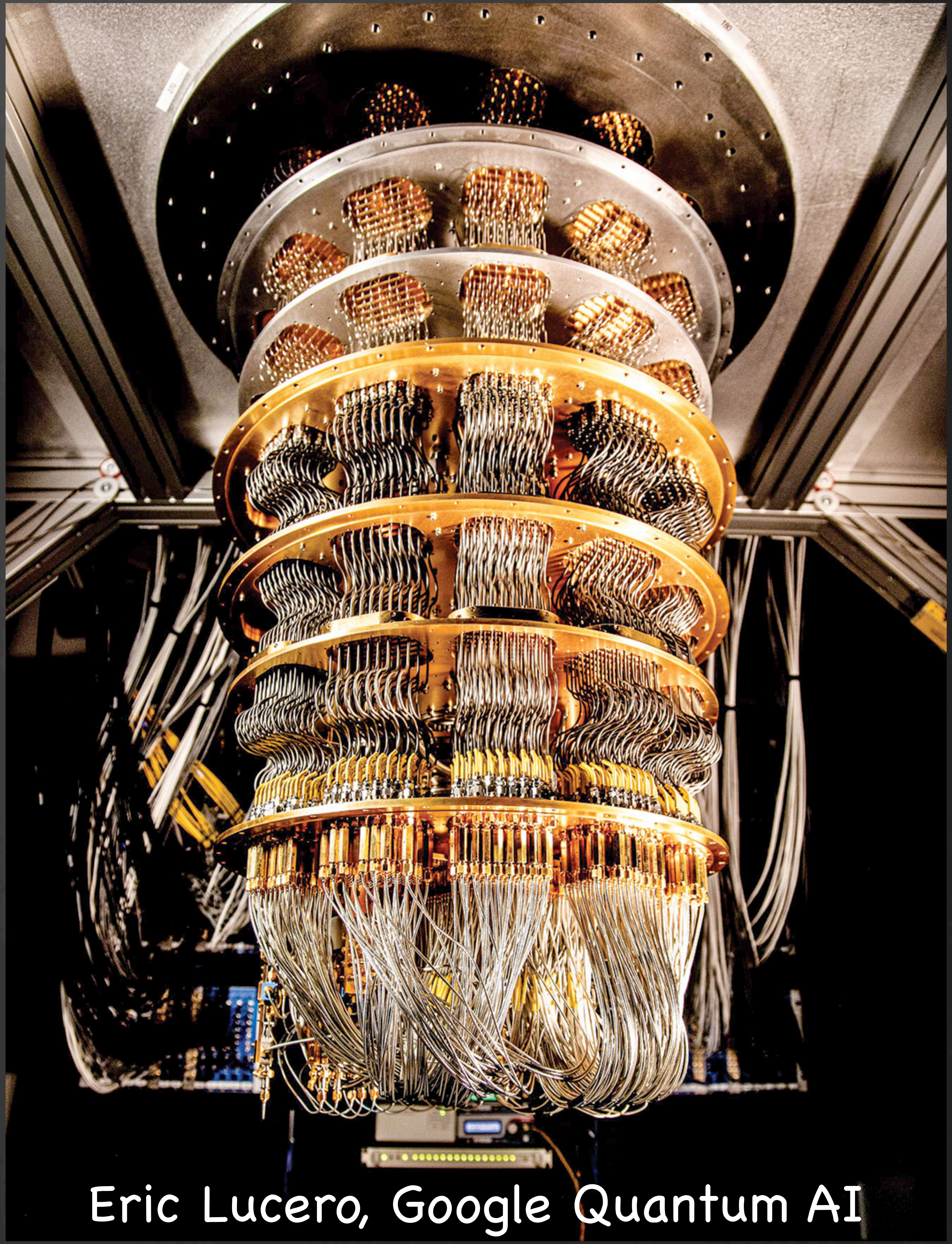
Builds on quantum de Finetti theorems from
[Caves, Fuchs, Schack. quant-ph/0104088]
[Koenig, Renner. quant-ph/0410229]
[Christandl, Toner. 0712.0916]

2. Why are there quantum speedups?

- Speedups for quantum simulation, cryptanalysis and optimization.

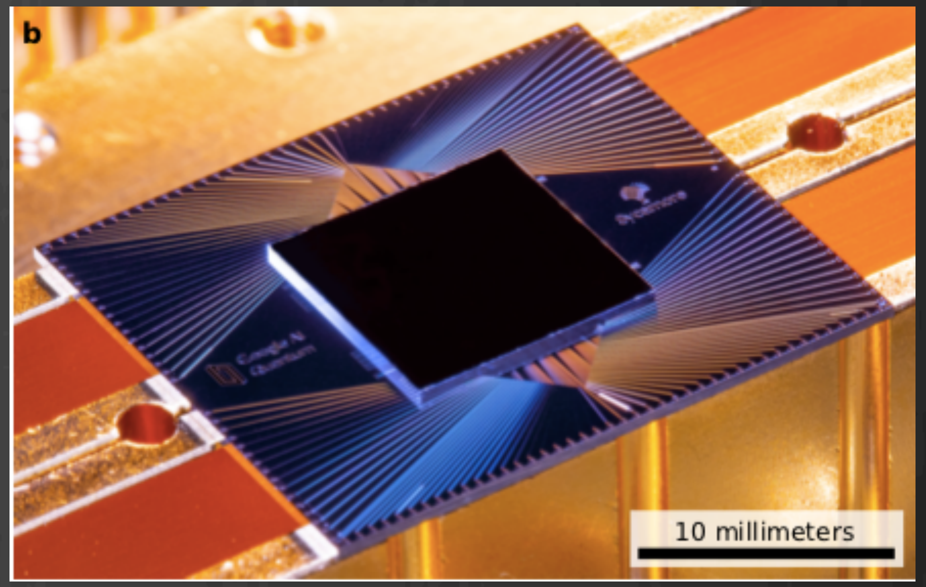
- Easier to find when there is no speedup

  Limited entanglement –>
  efficient classical simulation –>
  limited speedup.



Eric Lucero, Google Quantum AI

2-D circuits



Quantum supremacy using a programmable superconducting processor

Google AI Quantum and collaborators†

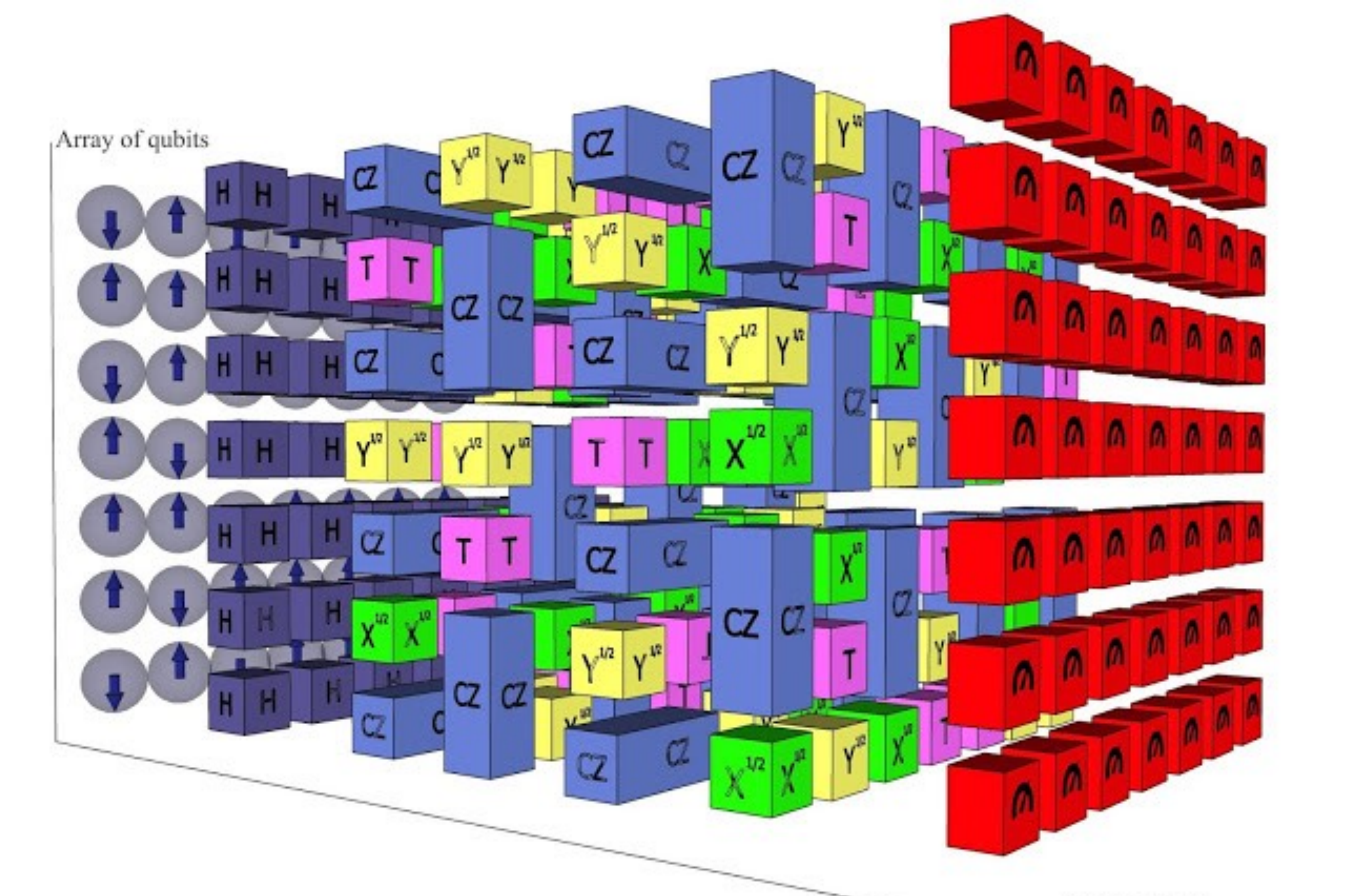


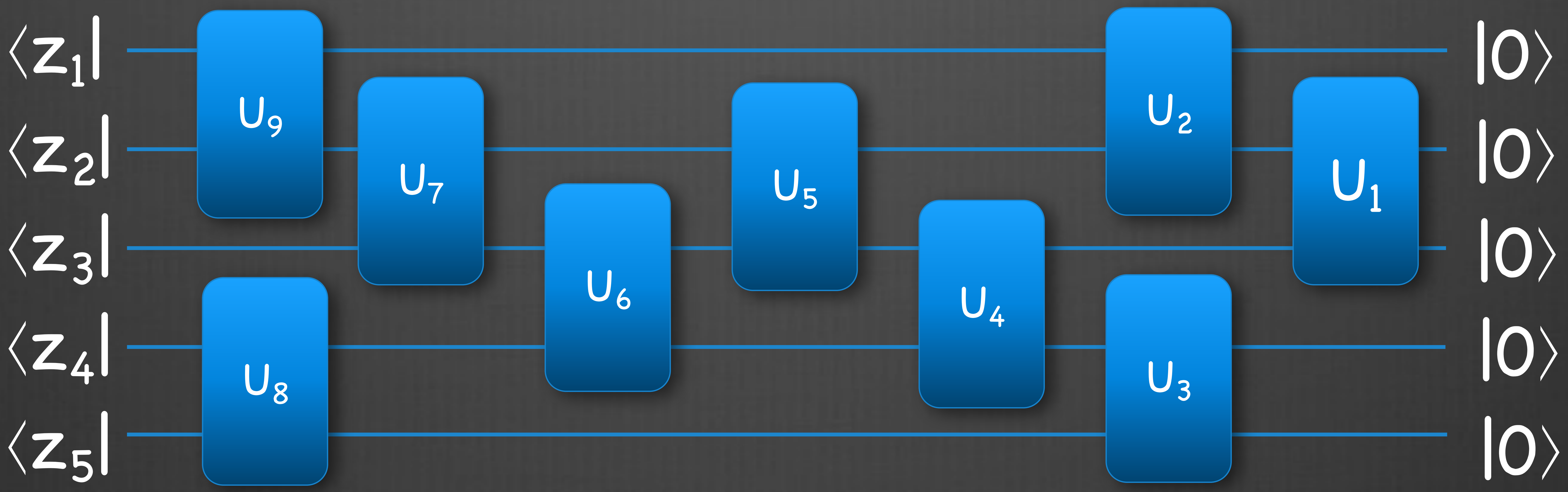
Photo credit: Google Quantum AI Lab

- 1-D shallow circuits are easy regardless of the choice of gates.
- Are shallow 2-D circuits hard to simulate?
- Are random circuits a good benchmark task?

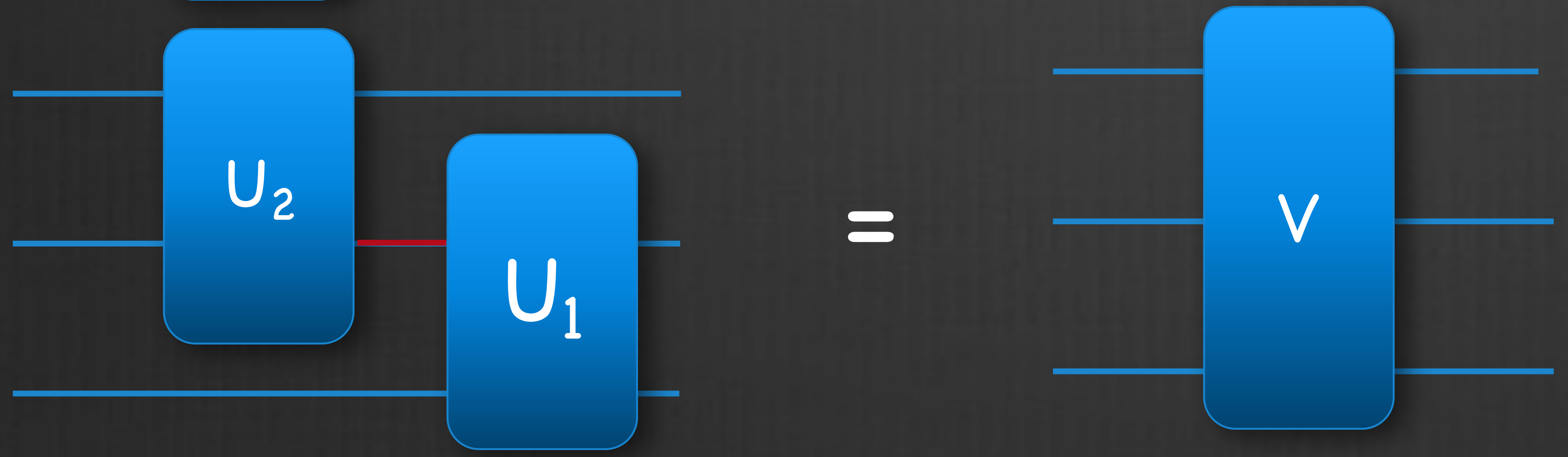
53 qubits with depth 20 in 2019
simulated with 2^{59} FLOPS in 2025

tensor contraction for time evolution

$\langle z_1 z_2 z_3 z_4 z_5 |\, U_9 U_8 U_7 U_6 U_5 U_4 U_3 U_2 U_1 \,|00000\rangle =$



= tensor with 4 indices, each dim 2

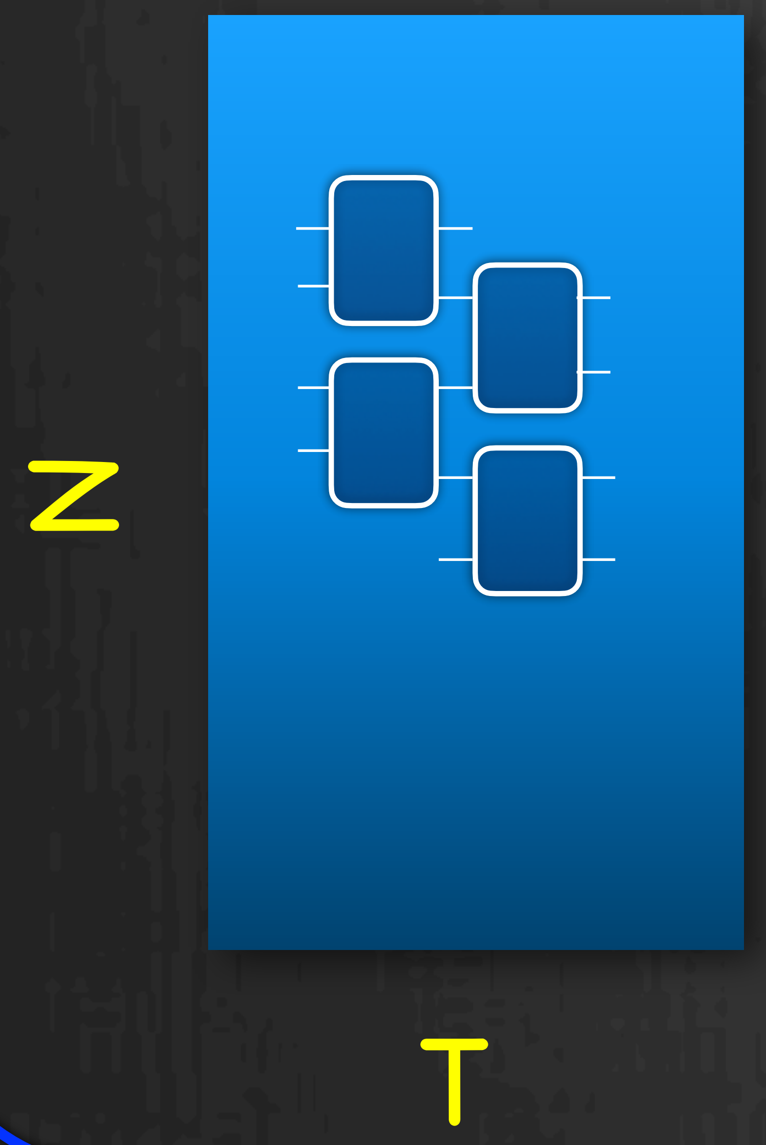


tensor contraction:
sum over

Tensor-network simulations of shallow 2-D circuits

Brandão, Dalzell, H, LaPlaca, Napp. PRX 2022

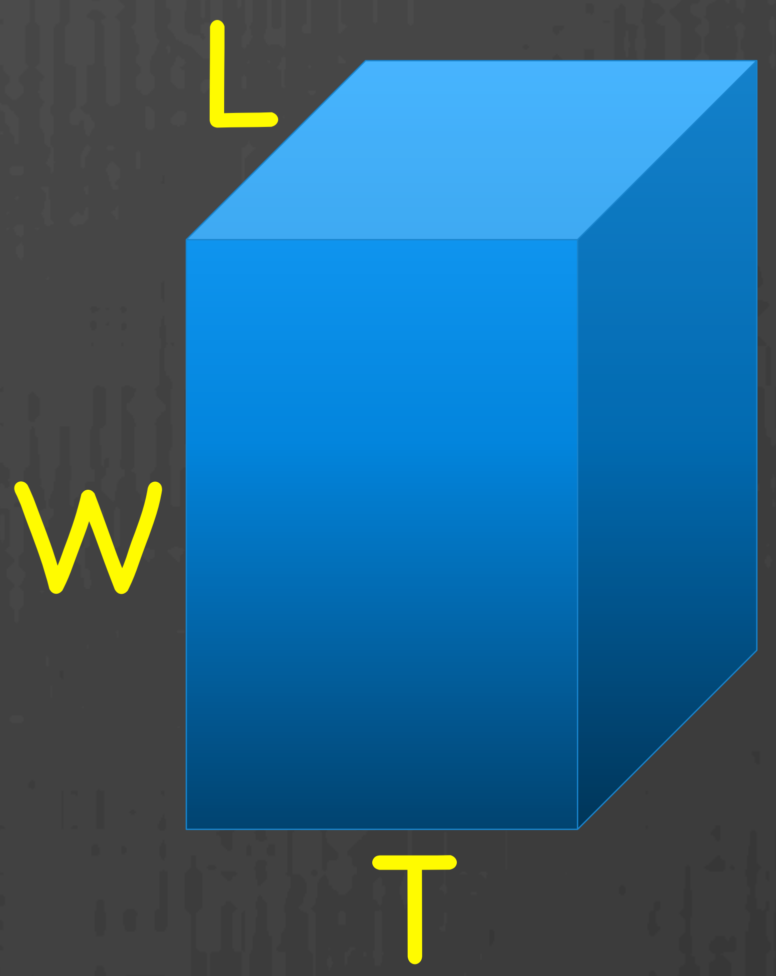
1D circuits



tensor-network simulation in time $T\cdot 2^N$ or $N\cdot 2^T$.

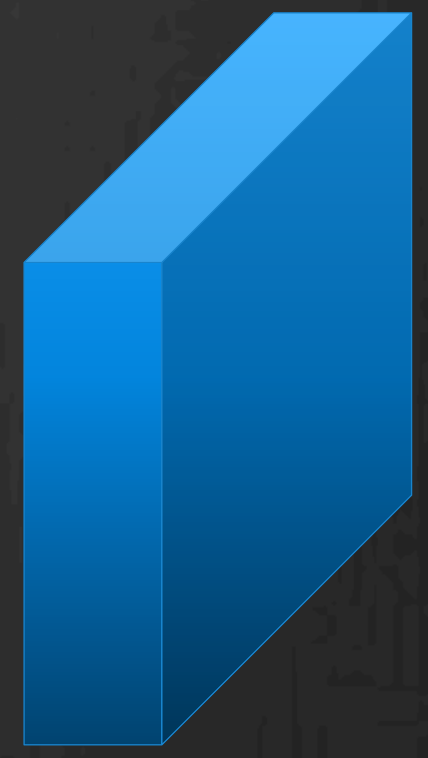
Fast if $T = O(1)$.

2D circuits



can be simulated in time 2^{LW} or 2^{LT} or 2^{WT}

Depth $T=O(1)$ circuit on $\sqrt{N} \times \sqrt{N}$ grid

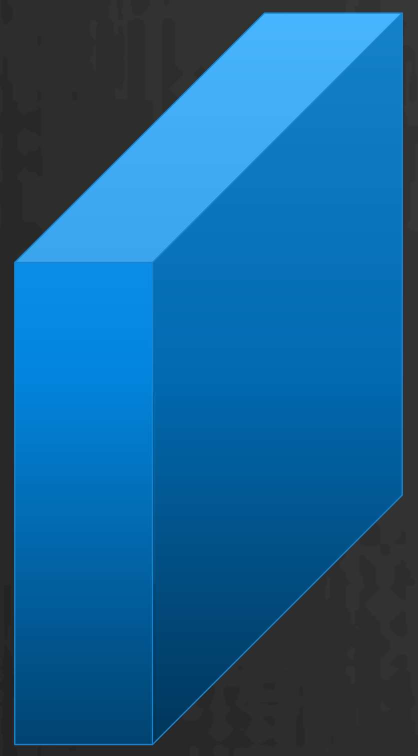


Worst-case hard when $T \geq 3$ from cluster states.

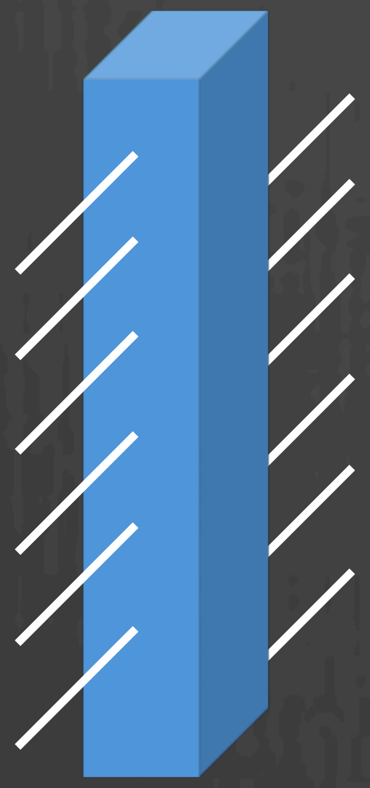
Tensor-network simulations of random shallow 2D circuits

Brandão, Dalzell, H, LaPlaca, Napp. PRX 2022

Depth T=O(1) circuit on $\sqrt{N} \times \sqrt{N}$ grid

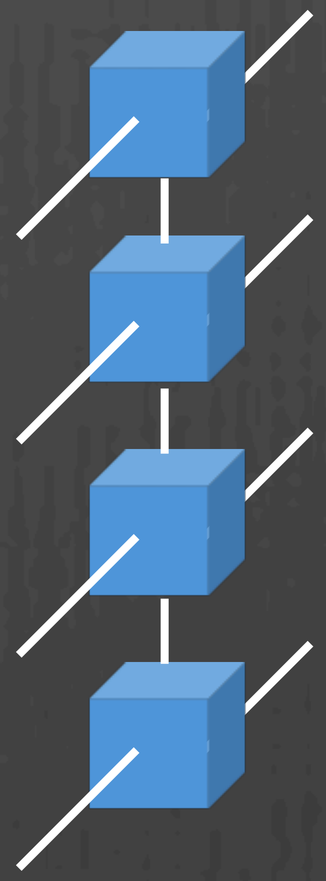


Intermediate tensors



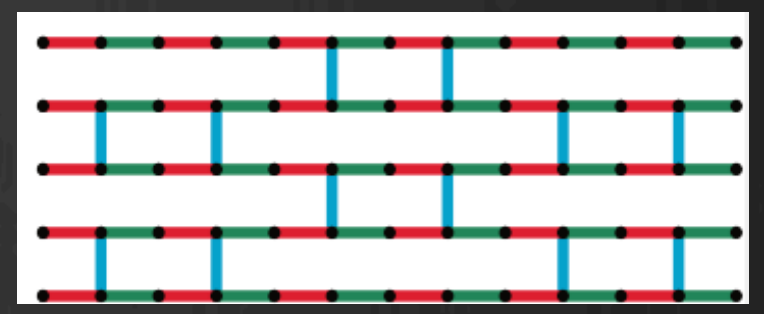
Contraction like $\sqrt{N}$ qubits evolving non-unitarily for time $\sqrt{N}$.

Express as matrix-product state

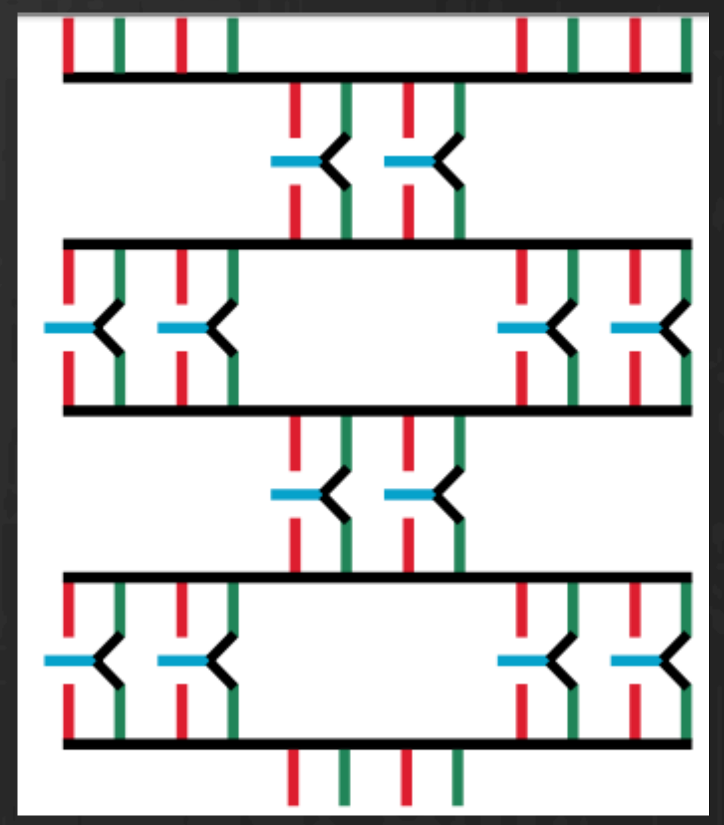


Cost depends on entanglement.

Entanglement related to Ising model surface tension

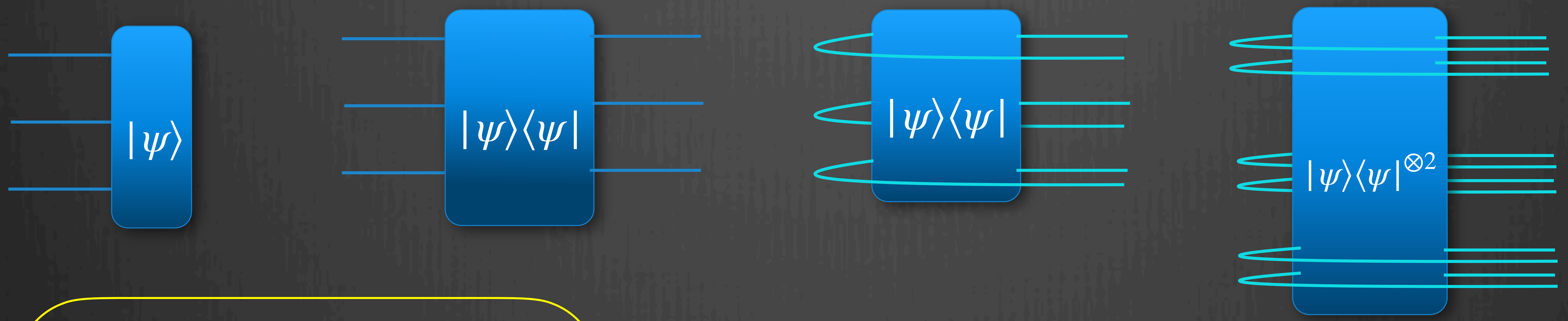


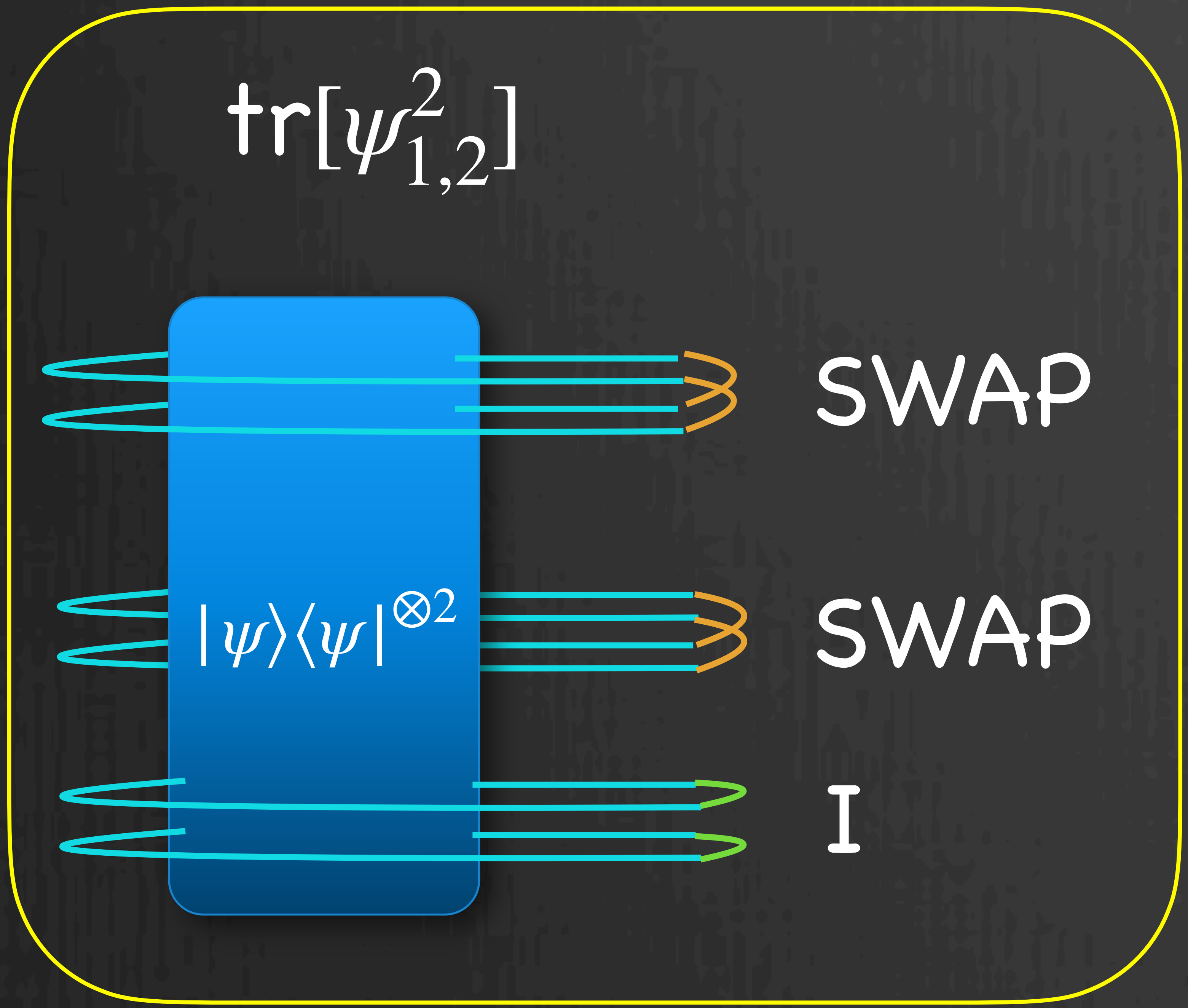
circuit

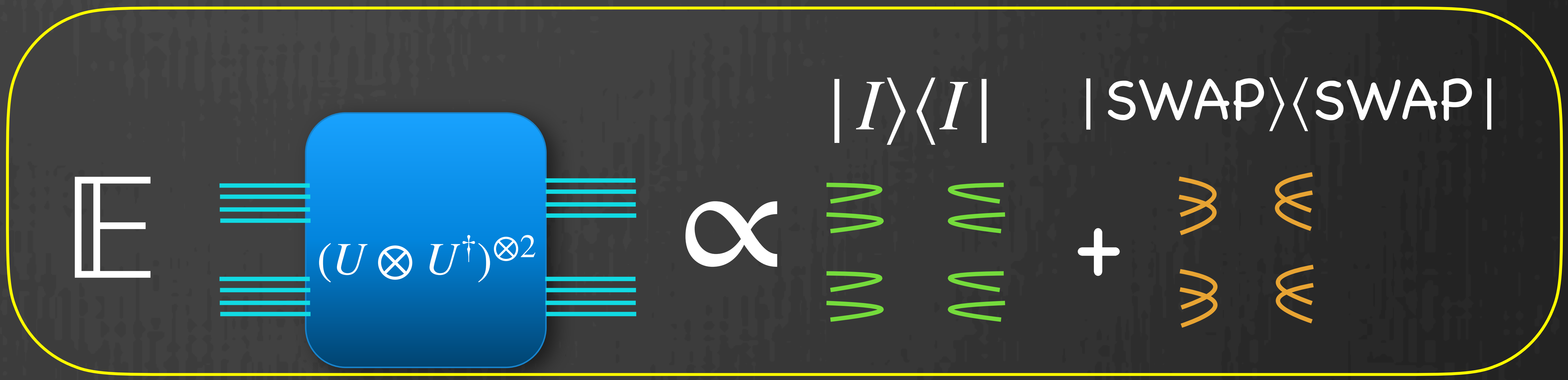


anisotropic Ising

The stat-mech method for $\mathrm{tr}[\psi_A^2]$







$\mathrm{tr}[\psi_A^3], \mathrm{tr}[\psi_A^4], \ldots, -\mathrm{tr}[\psi_A \log \psi_A]$

are more challenging

3. Closed-system thermalization from entanglement

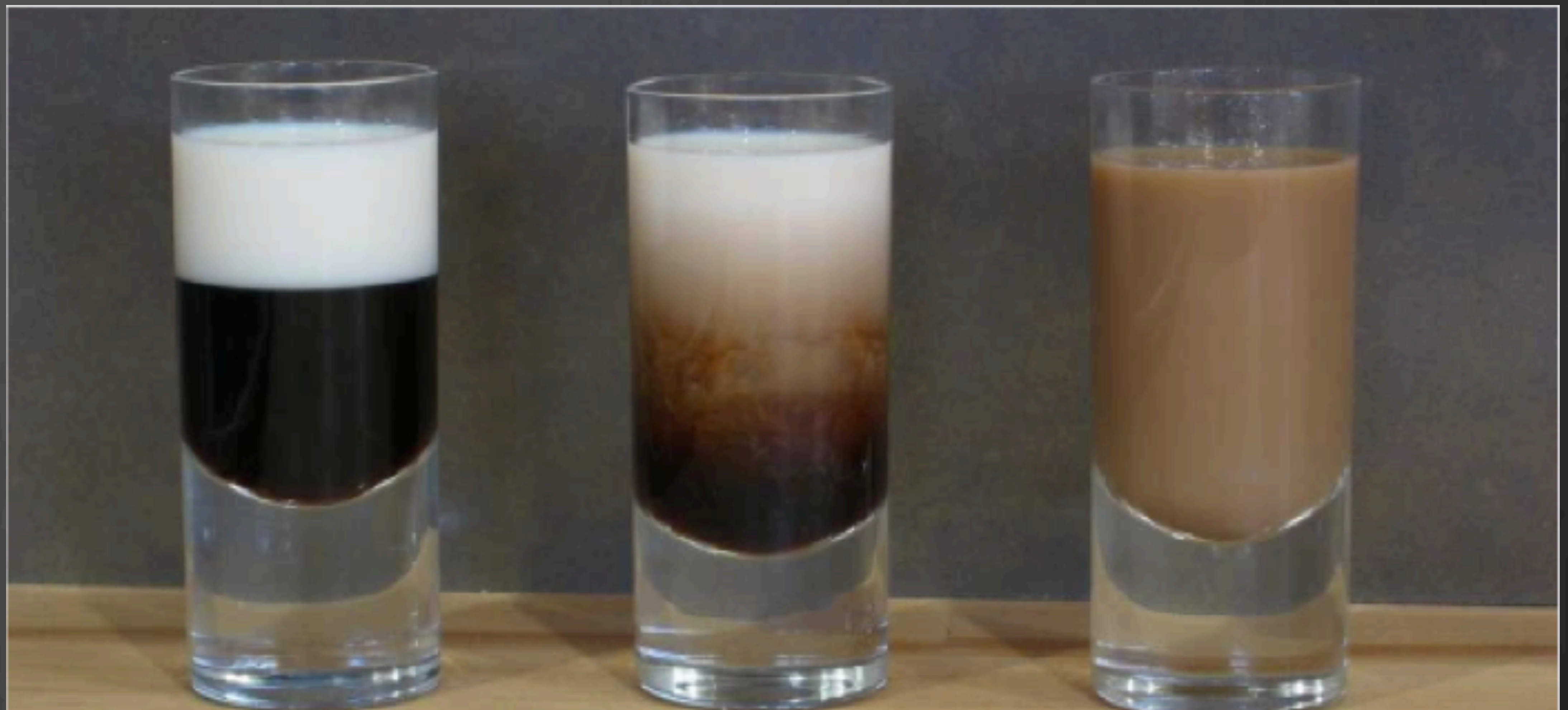


Photo credit: Sean Carroll

<u>Gibbs distribution</u>

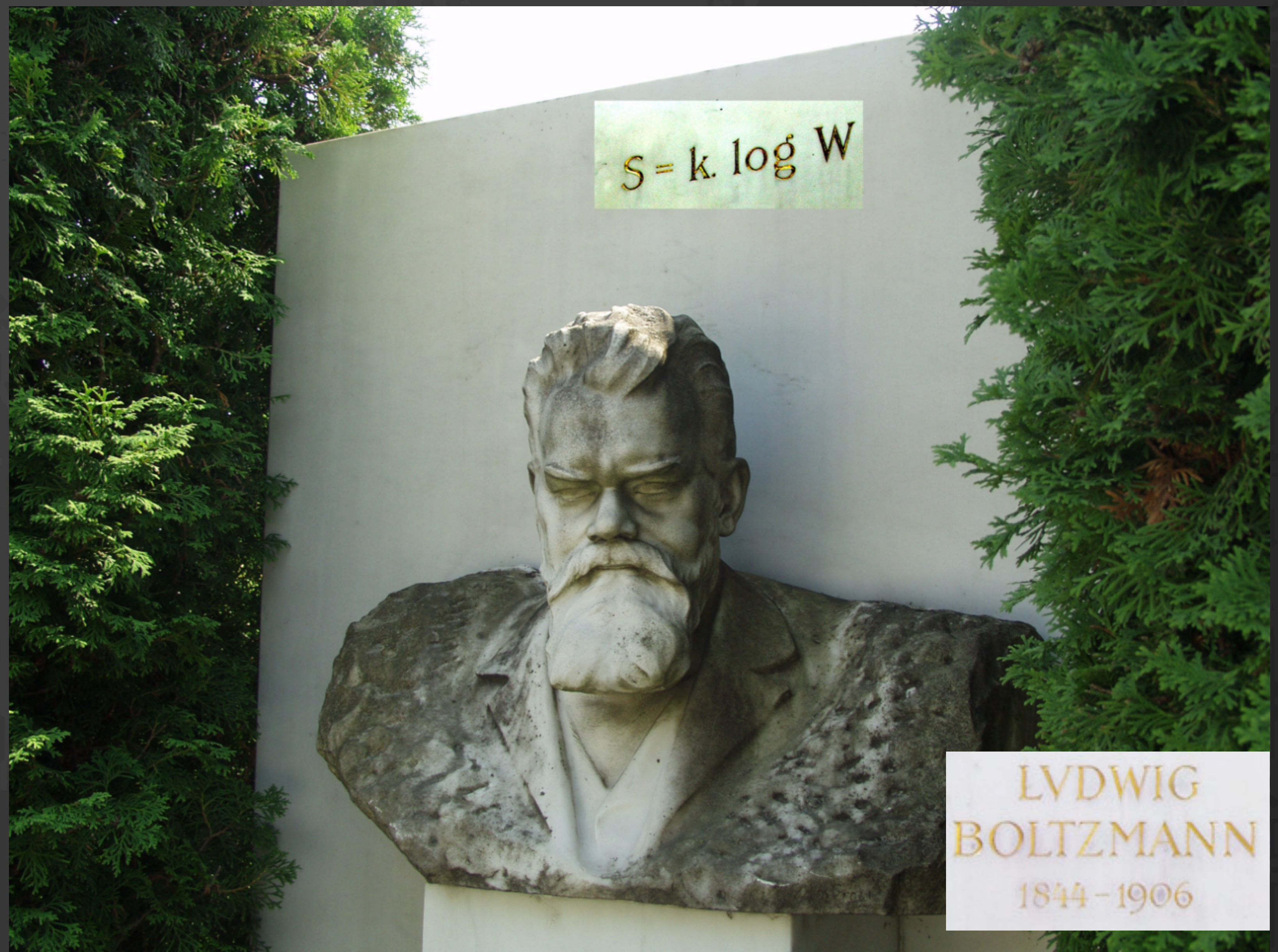
$$\Pr[x] = \frac{e^{-E(x)/T}}{Z}$$

<u>Gibbs state</u>

$$\sigma_T = \frac{e^{-\frac{H}{T}}}{Z}$$

<u>Why random?</u>

- Maximum entropy principle?
- Ergodic hypothesis?
- Entanglement?



Thermalization hypotheses


Photo credit: Sean Carroll

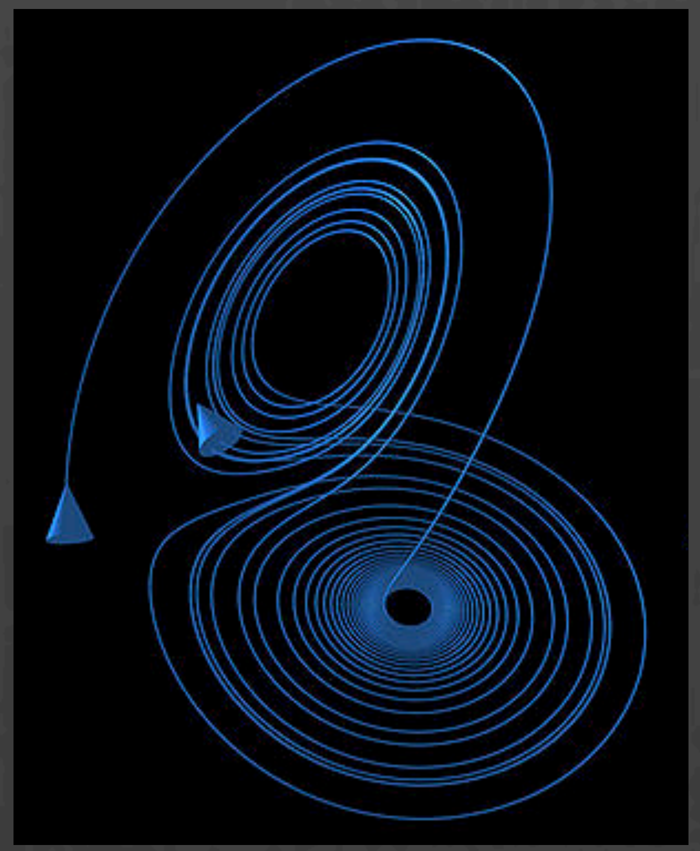
Maximum entropy principle

max $S(\rho)$ subject to constraint on $\langle H\rangle = \mathrm{tr}[\rho H]$

- e^{-iHt} doesn't change entropy
- ignorance is observer-dependent

Ergodic hypothesis

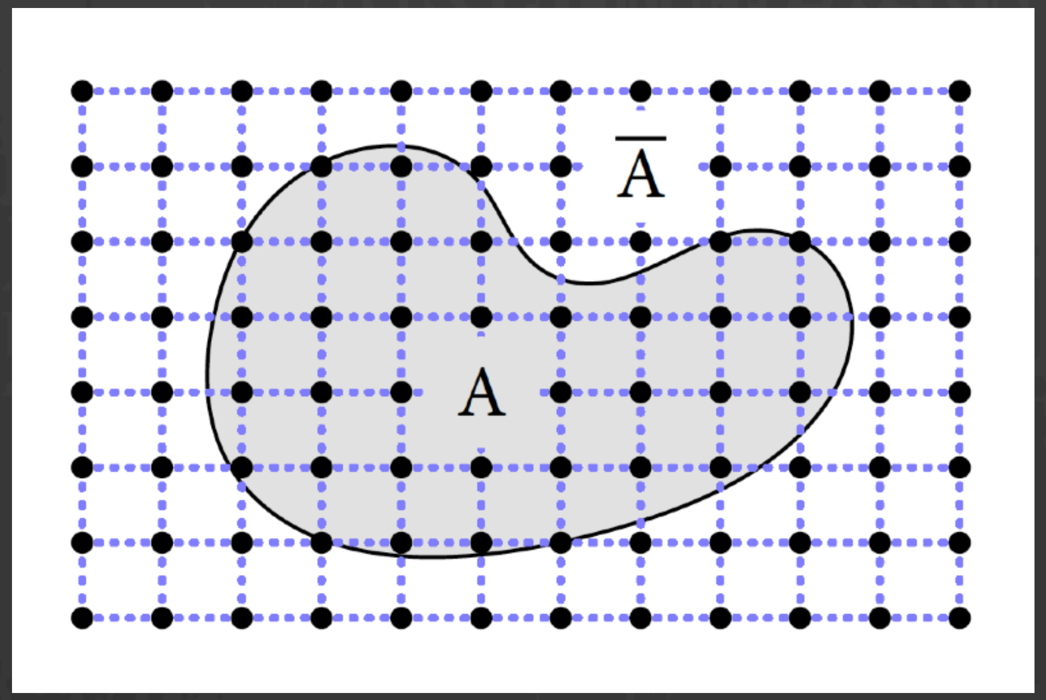
Replace ρ with $\lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{d}t\, e^{-iHt}\rho e^{iHt}$



- Doesn't work for all states (e.g. energy eigenstates)
- Requires time average.

Entanglement

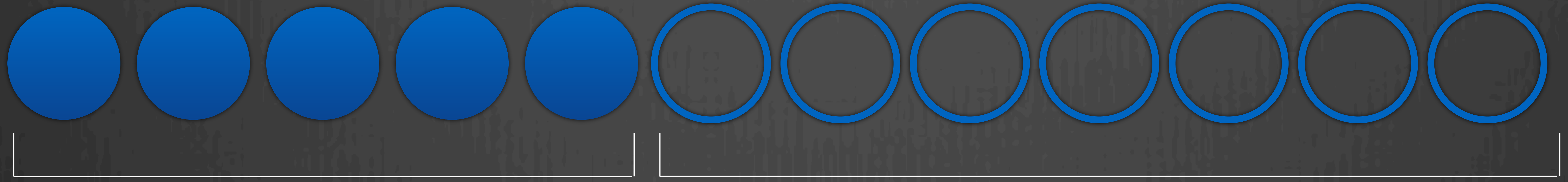
View $\bar{A}$ as a bath for A.



- Most pure states of fixed total energy have $\rho_A \approx e^{-H/T}/Z$
- Most states are unphysical.

Thermalization from entanglement

N fermion modes

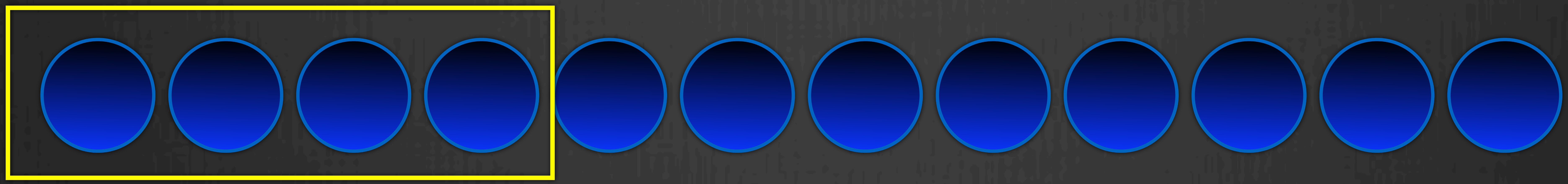


n occupied   N-n empty

evolve under

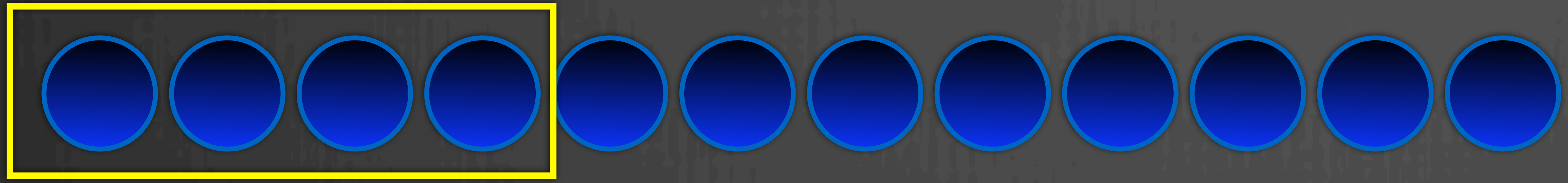
$$H = \mu \sum_{1\le i\le N} c_i^\dagger c_i + \sum_{1\le i,j\le N} h_{ij} c_i^\dagger c_j + \epsilon \sum_{1\le i,j,k,l\le N} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

$$\mu \gg 1 \gg \epsilon$$



subsystem of L modes

Thermalization from entanglement

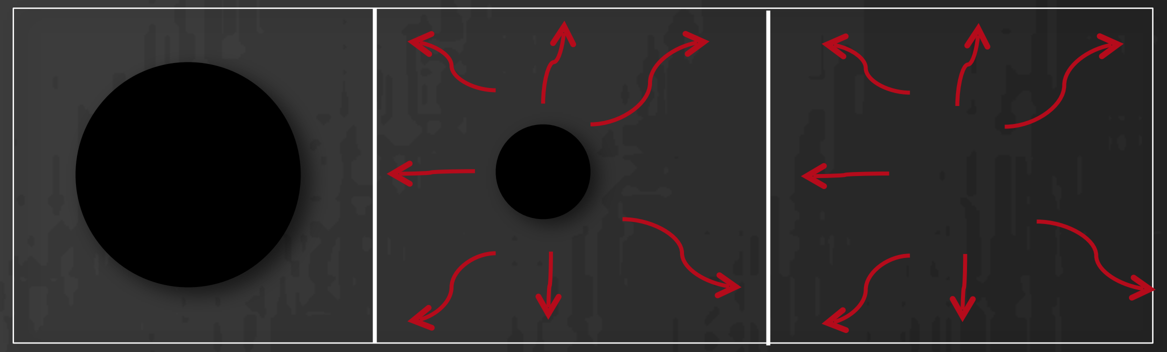


subsystem of L modes

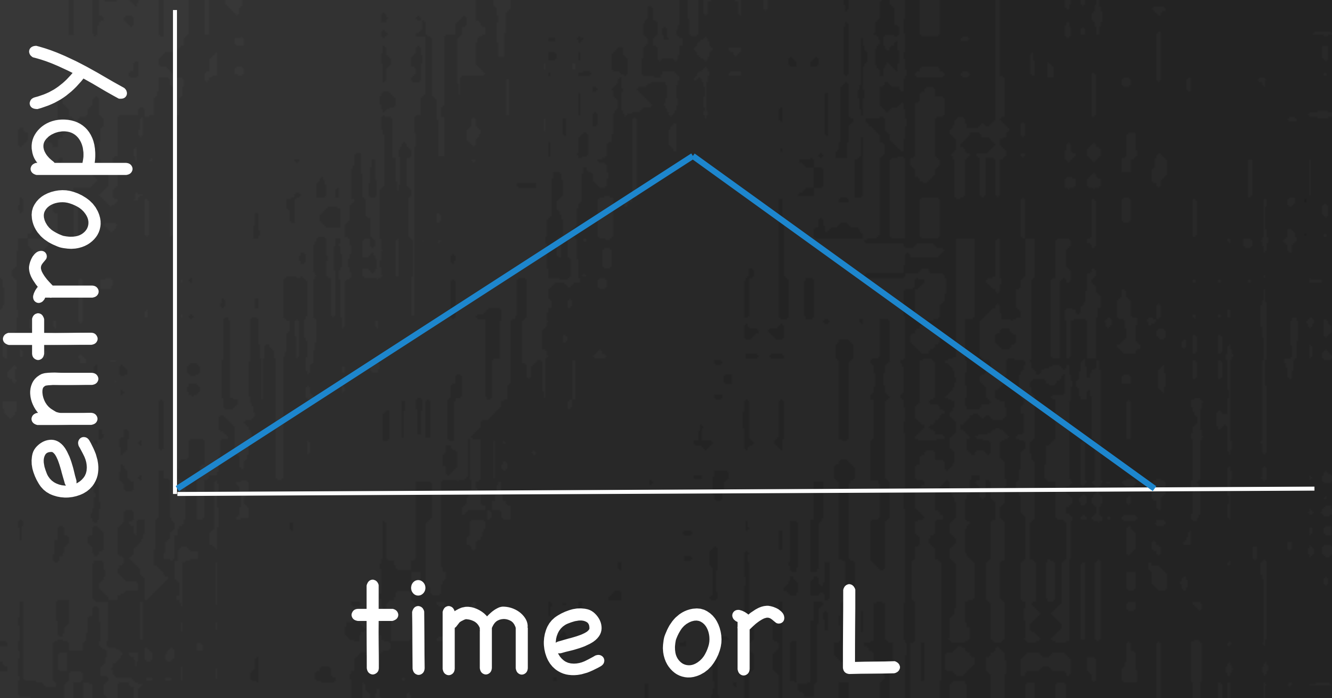
with Yichen Huang
2209.09826
2302.10165

- ≈ thermal if $L \ll \dfrac{N}{\log N}$
- ETH (Eigenstate Thermalization Hypothesis) only for L ≤ $\sqrt{N}$
- entanglement entropy ≈ thermal entropy for all $L \leq \dfrac{N}{2}$
- significance?
  model system for thermalization, analogous to
  TFIM for quantum phase transitions or
  toric code for topological order.
- general h_{ij}? ETH ~ RIP (Restricted Isometry Property)

Cartoon history of black holes



"Page curve"



Thanks!

Entropic approach to mean-field and optimization: 1210.6367, 1205.4484, 1301.0017

Tensor network simulation of short-time evolutions: 2001.00021

Thermalization in closed systems: 2209.09826, 2302.10165