Three-body resonances from Lattice QCD

ω-meson, PRL 133 (2024) 21, arXiv:2407.16659, $\pi(1300)$: arXiv:2510.09476

Carsten Urbach

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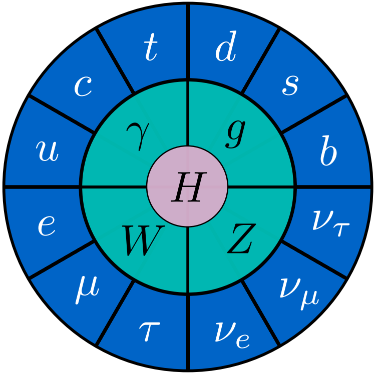
Standard Model of Particle Physics



- standard model highly successful

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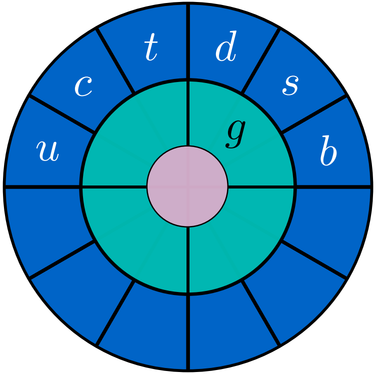


- standard model highly successful
- strong interaction part

$\Rightarrow$ quantum chromodynamics (QCD)

- gives rise to **hadron spectrum**
  mesons and baryons
- still new hadrons are being found
  X, Y, Z states
- **most hadrons are resonances**

The ω-meson

- experimentally discovered in 1961

  [Maglich et al., PRL 7 (1961)]

- theoretically predicted by Nambu (1957) to explain the electromagnetic form factors of the nucleon

  [Nambu, Phys. Rev. 106, 1366 (1957)]

- also expected in the vector meson dominance (VMD) picture

  [Sakurai, Ann. Phys. 11, 1 (1960)]

- dominates the isoscalar response in VMD

1961 press conference on ω discovery

[NARA & DVIDS Public Domain Archive, photographer: D. Cooksey]

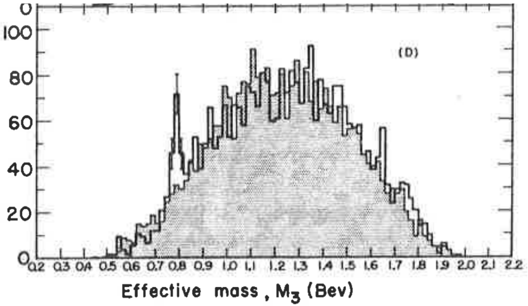
The ω-meson

- triplets identified in

$$\bar{p} + p \to \pi^+ + \pi^+ + \pi^- + \pi^- + \pi^0$$

  decays

- $I^G(J^{PC}) = 0^-(1^{--})$
- PDG: $M_\omega = 782$ MeV, width 8.5 MeV
- $\approx 90\%$ decays to $\pi^+\pi^-\pi^0$
- about 1.5% $\to \pi^+\pi^-$



[B. Maglich et al, PRL 7 (1961)]

(BeV is one Bevatron $\equiv$ GeV)

What Do We Know Theoretically?

- in 2022, we reviewed the theory status of hadron resonances

  [Mai, Meißner, Urbach, Phys.Rept. 1001 (2023) 1-66]

- focus on calculations from first principles

What Do We Know Theoretically?

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- focus on calculations from first principles

given the huge number of experimentally known states:

surprisingly little is know theoretically!

almost exclusively $H \quad \rightarrow \quad XY$

The $\pi(1300)$ Excited Pion

- first excited pion state
- first observed in 1981

  [Ananeva et al., JETP Lett. 34, 488 (1981)]
- mass of 1300 ± 100 MeV

  $200 - 600$ MeV width
- even existence debated

  [P. d'Argent et al., JHEP 05, 143]

Citation: R.L. Workman *et al.* (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022)

π(1300) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\rho\pi$	seen
Γ_2	$\pi(\pi\pi)_{S\text{-wave}}$	seen
Γ_3	$\gamma\gamma$	

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- almost degenerate to the $\eta(1295)$ and $K(1460)$
- interesting insights into emergence of hadrons

$\Rightarrow$ even for excited pions room for significant contributions!

Quantum Chromodynamics

$$S[A_\mu, \bar\psi, \psi] = \int d^4x\ \left\{\frac{1}{4}G_{\mu\nu}^2 + \bar\psi_q\left(\mathrm{i}\gamma_\mu D_\mu + m_q\right)\psi_q\right\}$$

- astonishingly simple action, intriguingly complex dynamics
- running coupling: QCD is non-perturbative at low energies

$\Rightarrow$ hadron spectrum requires non-perturbative methods

Lattice QCD Regularisation

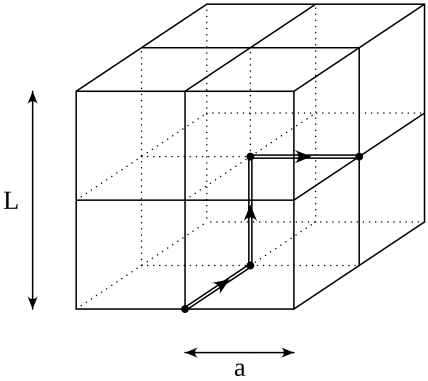
- quantum field theory requires regularisation

- lattice regularisation:

$\Rightarrow$ discretise space-time

  - hyper-cubic $L^3 \times T$-lattice with lattice spacing a

  $\Rightarrow$ momentum cut-off: $k_{\mathrm{max}} \propto 1/a$
  - derivatives $\Rightarrow$ finite differences
  - integrals $\Rightarrow$ sums
  - gauge potentials A_μ in $G_{\mu\nu}$ $\Rightarrow$ link matrices U_μ ('')



- work in Euclidean space-time $\Rightarrow$ **use Monte Carlo**

Lattice QCD Regularisation

- Monte Carlo: access to equilibrium, vacuum properties
- fundamental observables:
  Euclidean correlation functions

$$\langle \mathcal{O}_i^\dagger(p,t)\,\mathcal{O}_j(p,t')\rangle \;\propto\; \sum_n c_{i,n}c_{j,n}e^{-E_n t}$$

- with interpolating operators $\mathcal{O}_i$ with certain quantum numbers
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- **continuum limit**:

$$\lim_{a\to 0}$$

(i.e. at least 3 lattice spacing values)

- **infinite volume limit**:

$$\lim_{L\to\infty}$$

- **physical mass limit**:

$$\lim_{m_\ell\to m_\ell^{\mathrm{phys}}} \quad \text{or} \quad M_\pi^2 \to (M_\pi^{\mathrm{phys}})^2$$

And then: Compute the Spectrum?

- lattice stochastic methods:
  work in finite volume / Euclidean space-time

$\Rightarrow$ real valued, quantised eigenvalues of the lattice Hamiltonian
no continuum of states

- Maiani and Testa:
  interactions properties cannot be studied directly

  [Maiani and Testa, (1990)]

$\Rightarrow$ there is no one-to-one correspondence of an energy level to a resonance state

- the connection is only provided by the Lüscher method!

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yes and no!

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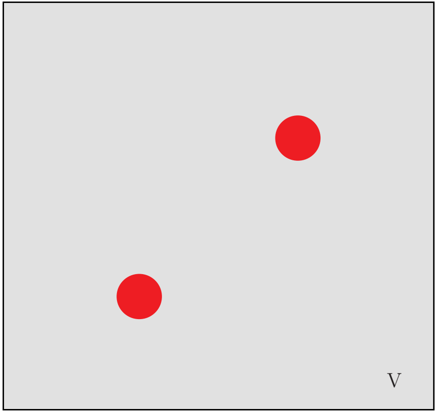
Lüscher Method

finite volume: boon and bane!



- for $V \to \infty$:

$\Rightarrow$ interaction probability very low

$\Rightarrow$ $E_{2p}(p = 0) = 2E_{1p}$

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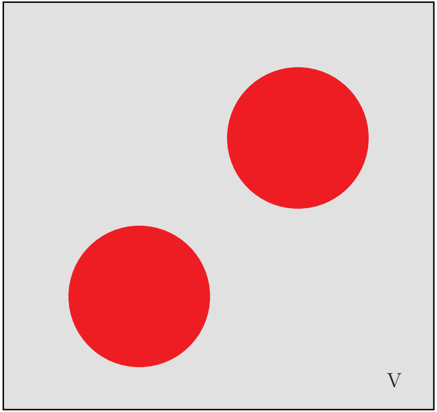

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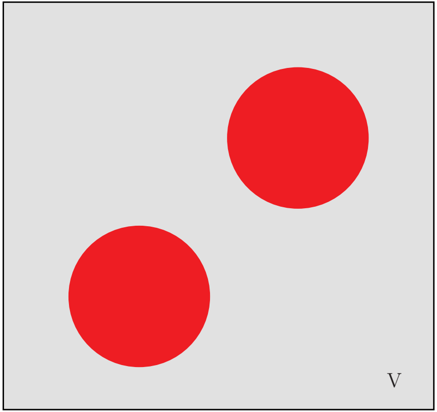

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- Lüscher: correction in $1/V$ related to scattering properties!

[Lüscher, 1986]

The $1+1$-dimensional Analog

- plane wave acquires phase shift $\delta(k)$
- finite extend L, periodic BC

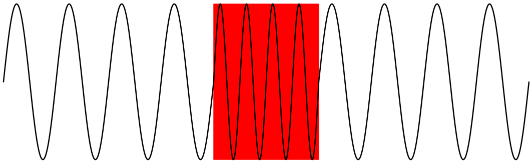
$$e^{\mathrm{i}kL+2\mathrm{i}\delta(k)} = e^{ik0} = 1$$

- quantisation condition

$$k_n L + 2\delta(k_n) = 2n\pi$$

- momenta k_n from dispersion relation

$$W_n = 2\sqrt{m^2 + k_n^2}$$

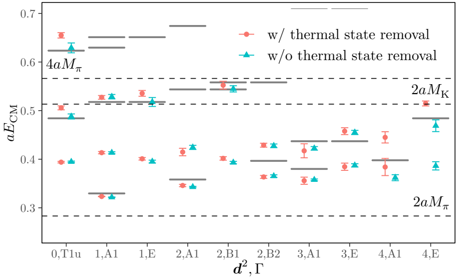


Procedure

1. determine non-interacting m
2. determine energies W_n
3. $W_n \quad \to \quad k_n$
4. $k_n \quad \to \quad \delta(k_n)$

The General Case

Lattice Energy Levels E



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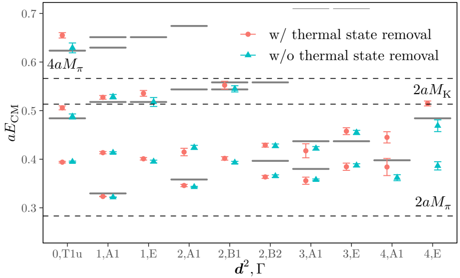

Determinant Equation

$$\det\left[\mathcal{M}^{\Gamma,\mathbf{d}}(E) - \cot(\delta)\right]^{\Lambda} = 0$$

($\mathcal{M}$ Lüscher function)

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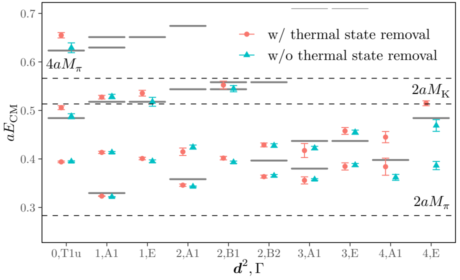

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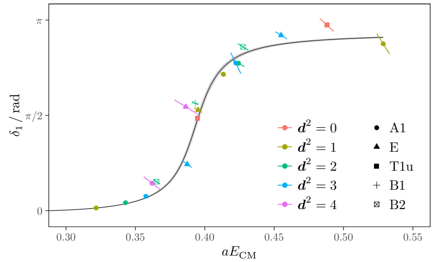

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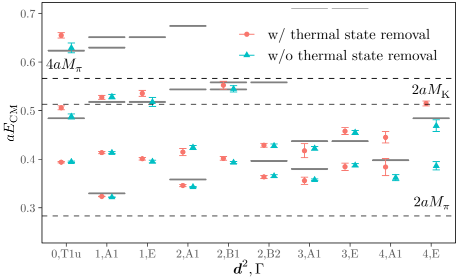

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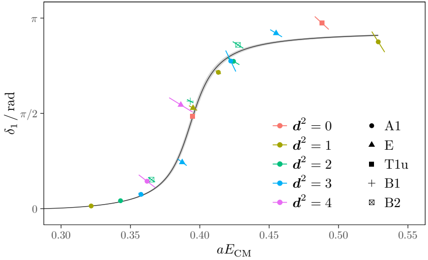
Parametrise Energy Dependence

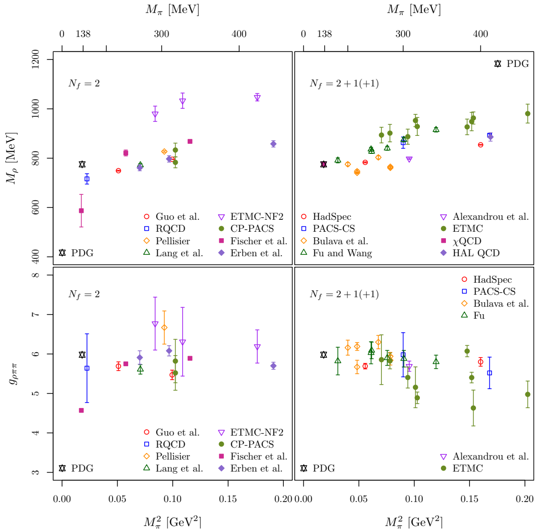
e.g. Breit-Wigner or better



Example: The $\rho(770)$ Meson

- ρ -resonance a poster Breit-Wigner resonance
- ρ decays to $\pi\pi$ in a P-wave
- best studied resonance from Lattice QCD
- summary of 16 Lattice studies

  [Mai, Meißner, Urbach, Phys. Rept. 1001 (2023) 1-66]
- bare lattice results for $N_f = 2$ and $N_f = 2 + 1(+1)$
- systematics clearly visible



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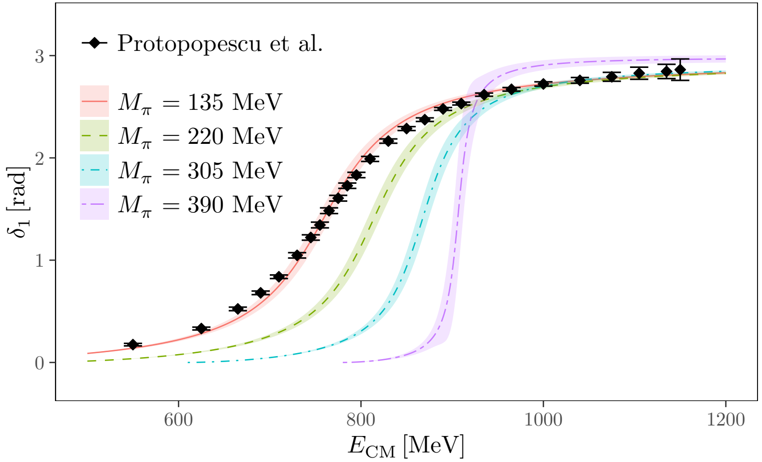
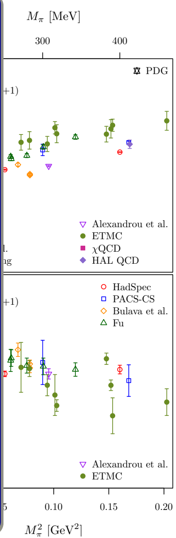


[Werner et al., Phys.Lett.B 819 (2021) 136449]



A Third Particle Enters the Game

Lattice Energy Levels E
Finite Volume, discrete, real

- Three particle decays highly relevant
- Three-pion decays of
  $K, \eta, \omega, a_1(1260), a_1(1420), \pi(1300)$
- Decays of exotica, e.g.:
  $X(3872) \to \bar{D}^* D \to \bar{D}D\pi$,
  $Y(4260) \to J/\psi\,\pi\pi$
- Roper resonance $\to \pi N$ and $\pi\pi N$

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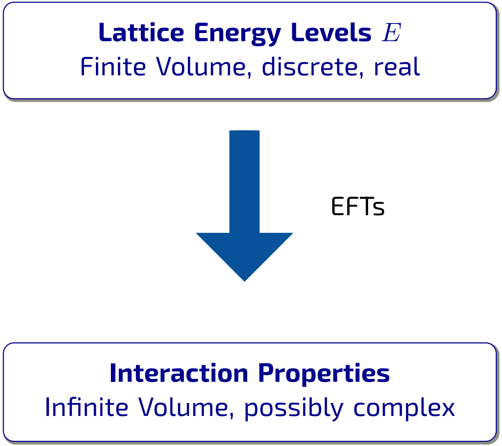

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Three equivalent EFTs

- RFT (Relativistic Field Theory)
  [Hansen, Sharpe, 2014]
- NREFT (Non-relativistic EFT)
  [Hammer, Pang, Rusetsky, 2017]
- FVU (Finite Volume Unitarity)
  [Mai, Döring, 2017]

Here: Finite Volume Unitarity (FVU)

builds on

- unitarity ($\rightarrow$ probability conservation)
- analyticity ($\rightarrow$ causality)
- crossing symmetry

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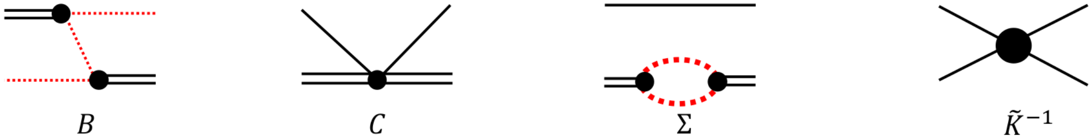
Quantisation Condition

$$\det\left[2L^3 E_{\mathbf{p}}(\tilde{K}^{-1} - \Sigma^L) - B - C\right]^{\Lambda} = 0$$

[Mai & Döring, EPJA 53 (2017)]



Lattice Details

- based on CLQCD ensembles

  [CLQCD, Hu et al, PRD 109 (2024)]

- tadpole tree-level Symanzik improved gauge
- $N_f = 2 + 1$ dynamical quark flavours
- tadpole improved clover fermions
- one level of stout smearing with $\rho = 0.125$

  [Morningstar and Peardon, PRD 69 (2004)]

- possible $O(a\alpha_s)$ lattice artefacts

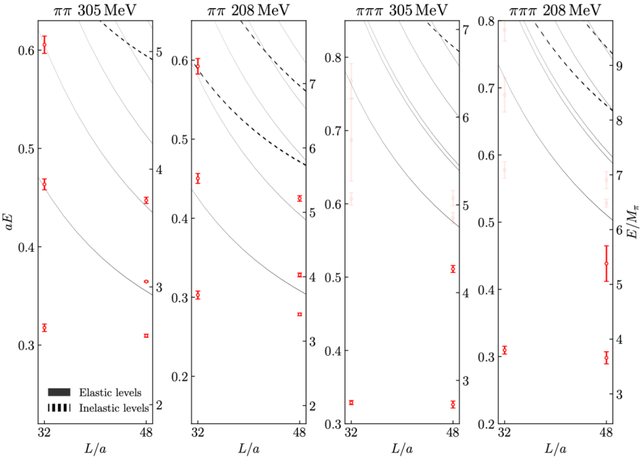
Ensemble	Volume	M_π/MeV	N_{confs}
F32P21	$32^3 \times 64$	$206.8(2.1)$	459
F48P21	$48^3 \times 96$	$207.58(76)$	221
F32P30	$32^3 \times 96$	$303.61(71)$	777
F48P30	$48^3 \times 96$	$304.95(49)$	201

- two pion mass values
- with two volumes each
- $a = 0.07746(18)$ fm

Finite Volume Spectrum: Parametrizations

First look at energy levels:

- red: interacting energy levels
- see attractive interaction for $\pi\pi$ and $\pi\pi\pi$
- $M_\pi = 305$ MeV: ω bound
- $M_\pi = 208$ MeV: ω decays
- need to parametrise



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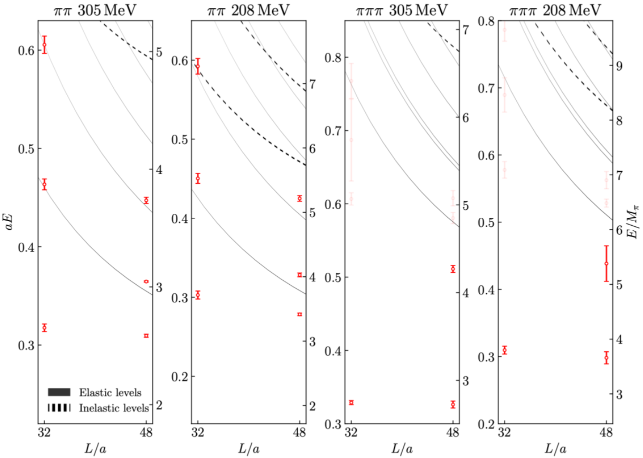

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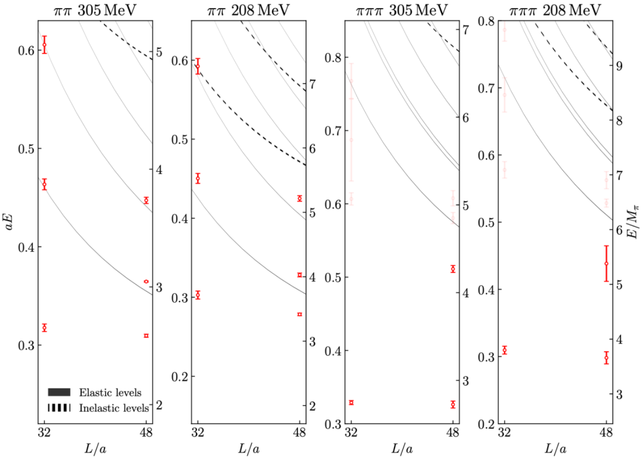
Generic Form

$$K^{-1} \propto a_0 + a_1 \sigma_p(s)$$

$$c_{11} = \frac{c_0}{s + M_\omega^2} + c_1$$

σ_p: two-body invariant mass



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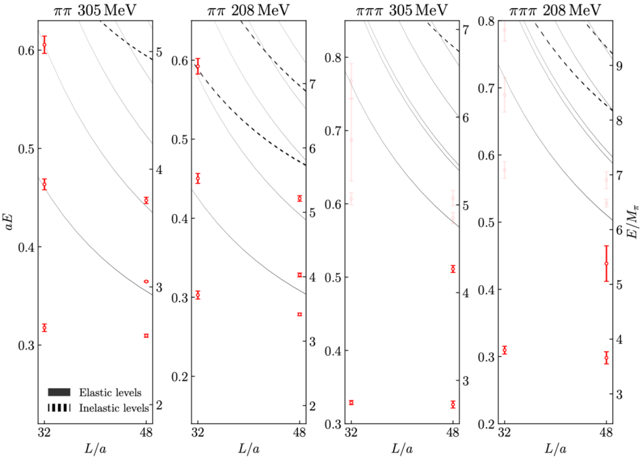
EFT Form

- EFT with vector mesons

  [for a review: Meißner, Phys.Rept. 161 (1988)]

- re-express in $M_{\pi,\rho\omega}$, f_π and g

$$g_{\rho\pi\pi} = g_{\omega\rho\pi} = g$$



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- $f_\pi(M_\pi^2)$ from chiral PT

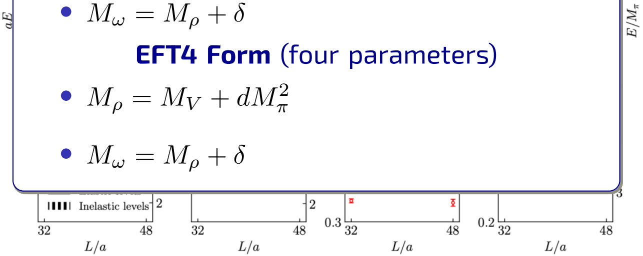
  [Gasser,Leutwyler, Annals Phys. 158 (1984)]

EFT2 Form (two parameters)

- $M_\rho = \sqrt{2} g f_\pi$ (KFSR)
- $M_\omega = M_\rho + \delta$

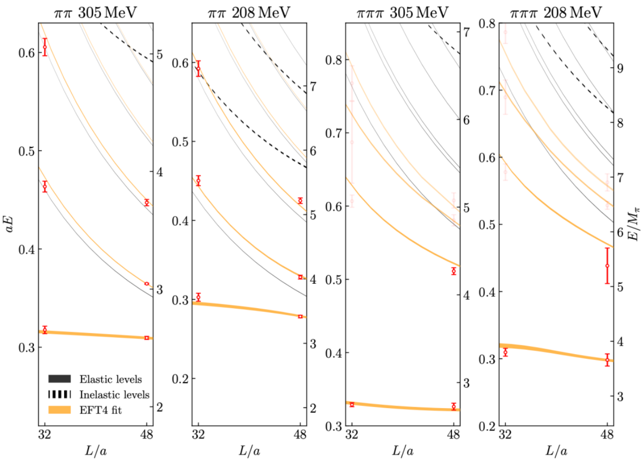
EFT4 Form (four parameters)

- $M_\rho = M_V + d M_\pi^2$
- $M_\omega = M_\rho + \delta$



Finite Volume Spectrum: EFT4 Fit

- parameters g, M_V, d, δ
- data described reasonably well
- even not included energy levels described
- removing ensemble with smallest $M_\pi \cdot L$ doesn't change results significantly
- $\chi^2/\text{dof} = 2.3$



Final Step: Pole Positions (ω meson)

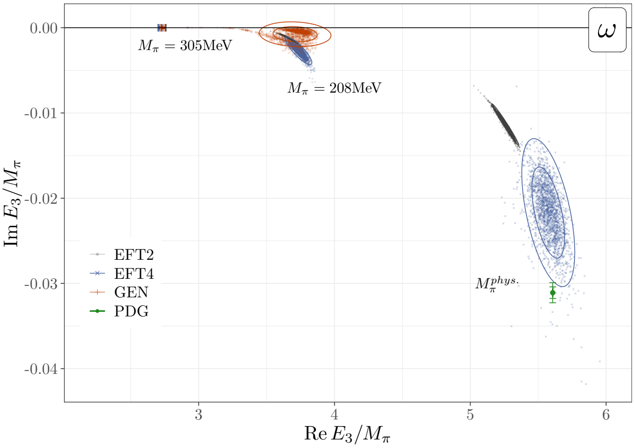
- with $C, \tilde{K}$ as input
- solve an integral equation

  [Mai, Döring, EPJA 53 (2017)]
- analytically continue to 2nd Riemann Sheet

$\Rightarrow$ Pole positions of ω

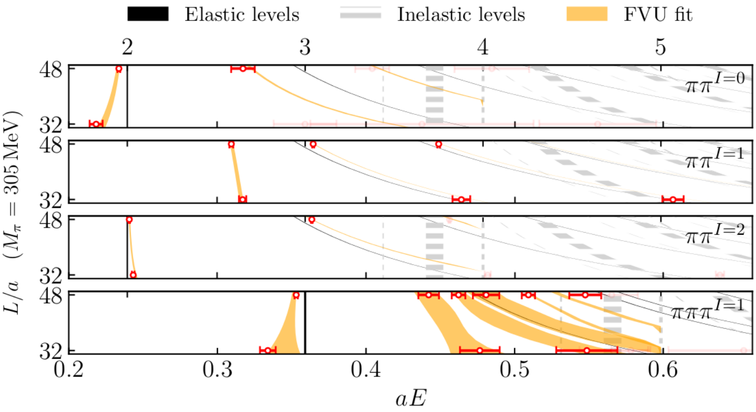
$778.0(11.2) - i3.0(5)$ MeV

[Yan et int. CU, PRL 133 (2024) 21]



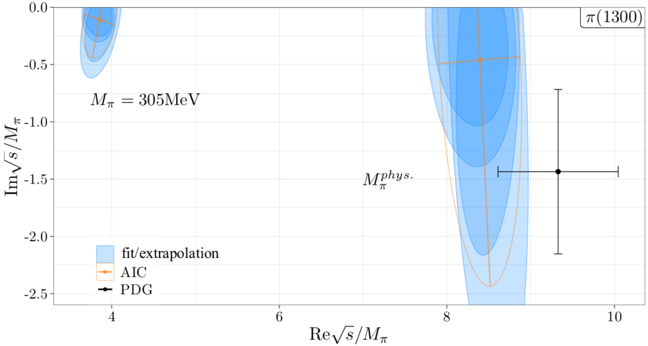
$\pi(1300)$: The Same Procedure

- more difficult due to large expected resonance mass
- three $\pi\pi$ isospin channels relevant
- ground state is a single pion
- for $M_\pi = 305$ MeV sufficient no of states below $6M_\pi$
- at $M_\pi = 208$ MeV too few states



$\pi(1300)$ Pole Position

- rely on $M_\pi = 305$ MeV ensembles only
- perform a model averaging procedure
- inverse amplitude method for extrapolation in M_π^2
- with large uncertainties we find evidence for a pole



The (notorious) Challenge: Extracting Energy Levels

- Euclidean Correlation Functions

$$C(t) = \sum_{l=0}^{N_s} A_l \, e^{-E_l t}$$

$E_0 < E_1 < \ldots$

- obtained from stochastic (MC) simulations
- want to estimate energy levels E_l
- and amplitudes A_l (matrix elements)

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Challenge

- signal-to-noise problem

$$\text{StN} \propto \exp(-\Delta E t)$$

with in general $\Delta E > 0$

[Lepage (1989)]

- signal deteriorates exponentially
- increasing severity with l

Example: The Effective Mass of the Nucleon

- effective mass

$$M_{\text{eff}}(t) = -\frac{1}{\delta t}\log\left(\frac{C(t+\delta t)}{C(t)}\right)$$

- since $E_0 < E_{l\neq 0}$

$$\lim_{t\to\infty} M_{\text{eff}}(t) = E_0$$

- all other contributions to C exponentially suppressed
- but t finite
  $\Rightarrow$ excited state contaminations

Example: The Effective Mass of the Nucleon

- effective mass

$$M_{\text{eff}}(t) = -\frac{1}{\delta t}\log\left(\frac{C(t+\delta t)}{C(t)}\right)$$

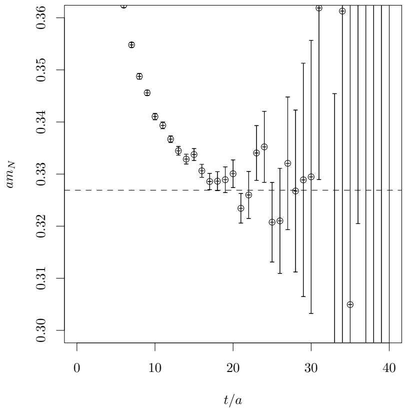
- since $E_0 < E_{l\neq 0}$

$$\lim_{t\to\infty} M_{\text{eff}}(t) = E_0$$

- all other contributions to C exponentially suppressed

- but t finite
  $\Rightarrow$ excited state contaminations

Example Nucleon M_{eff}



[ETMC (2021)]

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Goal

Find a method that either works at smaller t-values or reduces the noise at large t-values

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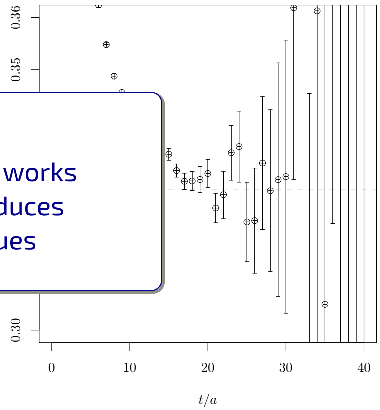

[ETMC (2021)]

Generalised Effective Mass

- build correlator matrix

$$C_{\alpha\beta}(t) = \langle O_\alpha^\dagger(0) O_\beta(t) \rangle$$

- O_α with appropriate quantum numbers
- cast as eigenvalue problem

$$C(t_0)^{-1} \cdot C(t)\, v_n(t, t_0) = \lambda_n(t, t_0)\, v_n(t, t_0)$$

- with gen. effective mass

$$\lambda_n(t, t_0) \propto e^{-E_n(t-t_0)}$$

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- but: no. of operators limited...

- create more operators for free!

$$O_{\Delta t,\alpha}(t) = O_\alpha(t+\Delta t) = T(\Delta t)\, O_\alpha(t)$$

Generalised Pencil of Function method

[Schiel, PRD92, 034512 (2015)]

- leads to the square Hankel matrix

$$H^{\alpha\beta,m}_{ij}(t) \;=\; C_{\alpha\beta}(t+i+j)\,, \quad i+j < 2m$$

- and the eigenvalue problem

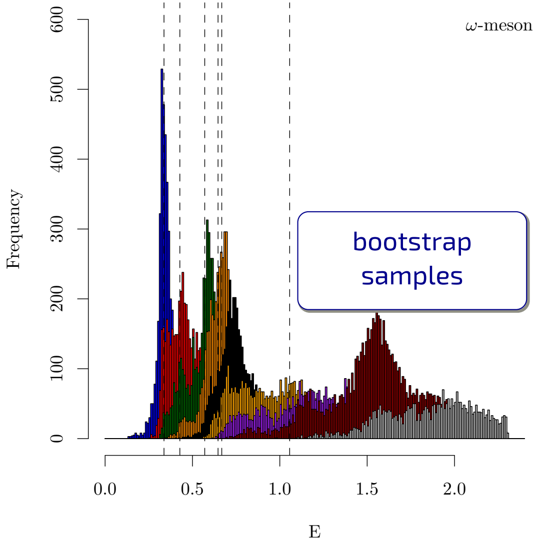
$$H^m(t_0)^{-1} \cdot H^m(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, v_n(t,t_0)$$

And Here Comes the Noise...

- the method enforces an effective noise model

$$e^{\mathrm{Re}(E)t}e^{\mathrm{i}\,\mathrm{Im}(E)t}$$

- the larger the matrix, the more spurious modes are found
- and they mix with the physical ones



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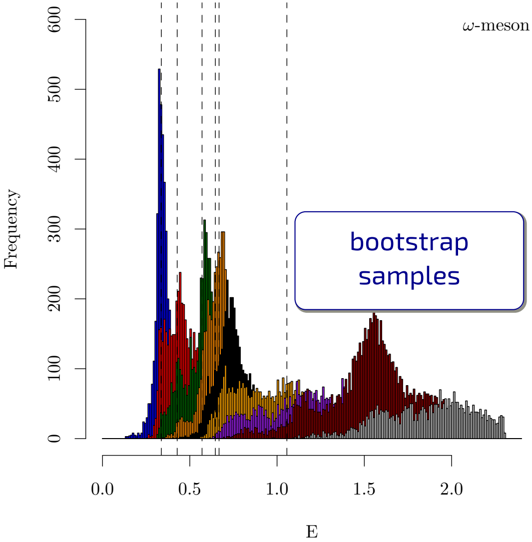
find/invent criteria for physical states

or reformulate the problem!



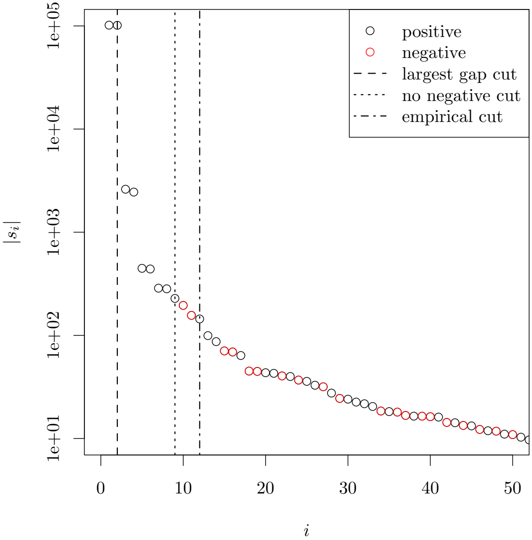
The Reformulated Problem

What is the best approximation to the correlator with k terms in a Frobenius norm?

- solution is known!
- diagonalise H (or SVD)

$$H = U^\dagger S U$$

- set all but the k largest modulus eigenvalues $|s_{i>k}|$ to zero
- reconstruct $\tilde{H} = U_k^\dagger S_k U_k$ and solve for eigenvalues

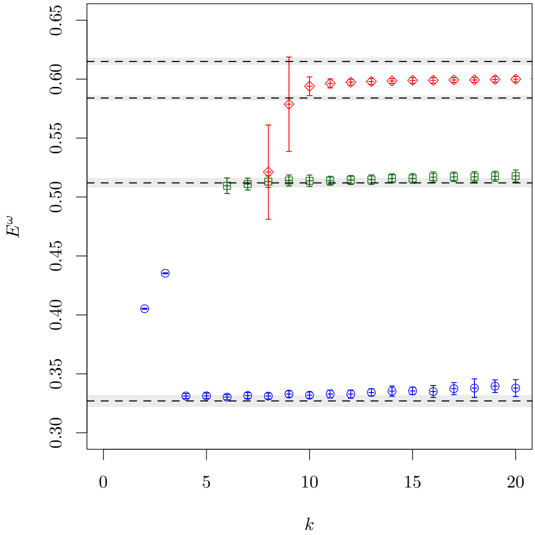


THC Results

- truncated Hankel correlator (THC) method

  [Ostmeyer, Urbach, arXiv:2510.15500]

- single parameter k
- all data included
- stable spectrum for $k \geq k_{\text{cut}}$
- directly applicable to **imaginary time evolution** in quantum computing



What can be improved...

- ω and $\pi(1300)$ are clearly exploratory studies!
- more ensembles, more M_π-values, larger volumes
- more energy levels by including more frames and irreps
- include also operators with strange quarks
- investigate the φ and the ω–φ mixing
- take the continuum limit
- include other partial waves

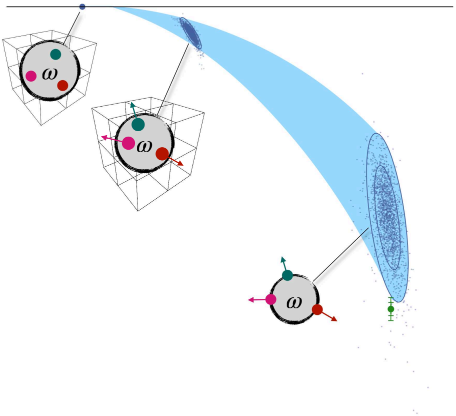
Summary

- resonances in LQCD challenging problem
- presented a first calculation of the ω and $\pi(1300)$ resonances
- using EFTs, obtained results at physical point
- ω pole position

$$(778(11)-i3.0(5))\ \text{MeV}$$

- $\pi(1300)$ pole position

$$(1169(46)-i62(170))\ \text{MeV}$$



[Yan, Mai, Garofalo, Meißner, Liu, Liu, Urbach, arXiv:2407.16659]

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