Precision calculations at colliders

Giulia Zanderighi

IFT Christmas Meeting, Madrid, 10th November 2025

Why is high energy interesting

Higher energy allows to access heavier particles: $E = mc^2$

Particle	Collider / Experiment	Energy (GeV)	Year	Mass (GeV/c²)
J/ψ	SPEAR	3.1	1974	3.097
Tau (τ)	SLAC/LBL (SPEAR)	4	1975	1.777
Bottomonium (Υ)	Fermilab/Cornell	9.46	1977	9.460
W boson	UA1 / CERN SPS	540	1983	80.379
Z boson	UA1 / CERN SPS	540	1983	91.1876
Top quark	Tevatron	1800	1995	173.0
Higgs boson	LHC (ATLAS/CMS)	7000	2012	125.1

In the last 50 years, every time a new energy frontier was explored, a new particle was found

… but times have changed

With the discovery of the Higgs boson the Standard Model of particle physics is complete
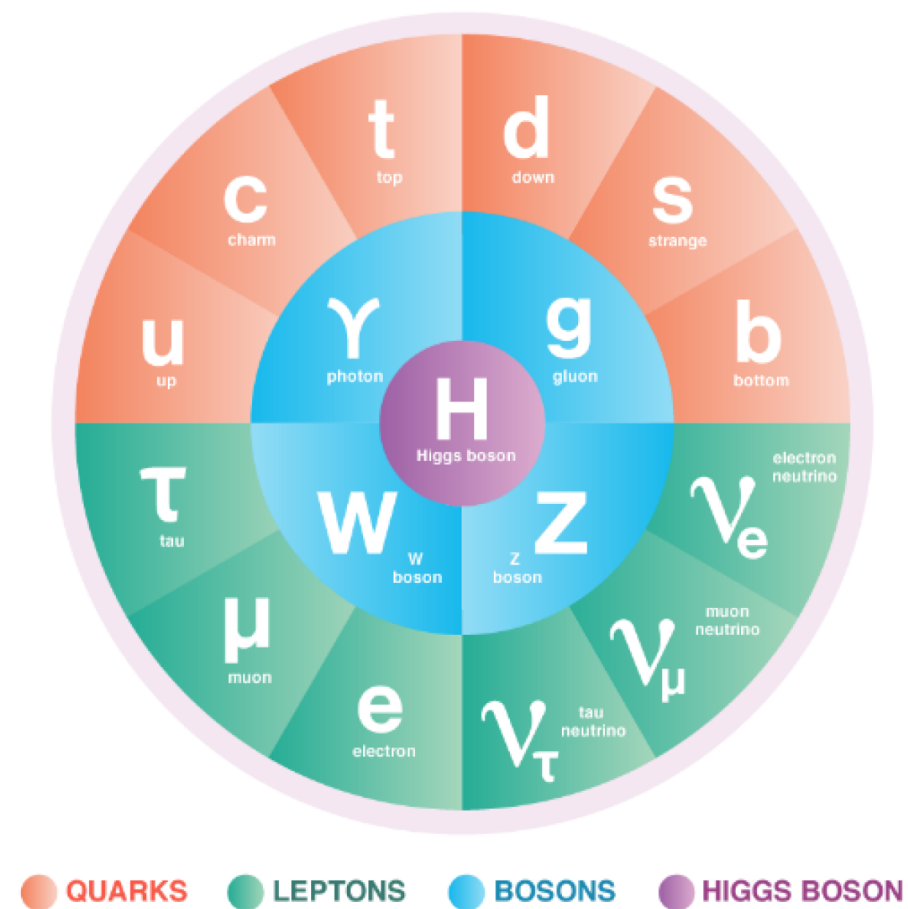
No new discovery can be easily anticipated

TeV-scale new physics?

(As far as I know) The only hint of New Physics at the TeV energy scale comes from the hierarchy problem in the Higgs sector

The argument: **if** there are new particles, that interact in any way with SM particles, they would give rise to corrections to the Higgs mass that depend quadratically on the New Physics scale

$$\text{BSM}$$

$$h \text{ ---} \bigcirc \text{--- } h \qquad \Delta m_h^2 \propto \frac{g_h^2}{16\pi^2} M_{\text{BSM}}^2$$

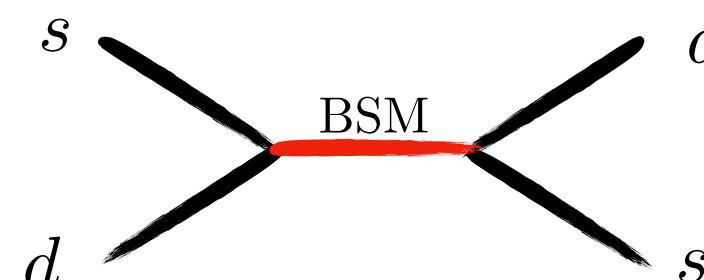*⇒ Since the Higgs is light, new physics must be around the corner …*

The Higgs & flavour tension

Tension between hierarchy problem in the Higgs section and absence of New Physics in the flavour sector is a major puzzle

$$\Delta m_h^2 \propto \frac{g_h^2}{16\pi^2} M_{\text{BSM}}^2$$

☞ Light Higgs suggests *light* physics Beyond Standard Model (BSM)

$$\Delta m_K \propto \frac{g_{sd}^2}{M_{\text{BSM}}^2}$$

☞ No effects in flavour physics suggests BSM must be *heavy*
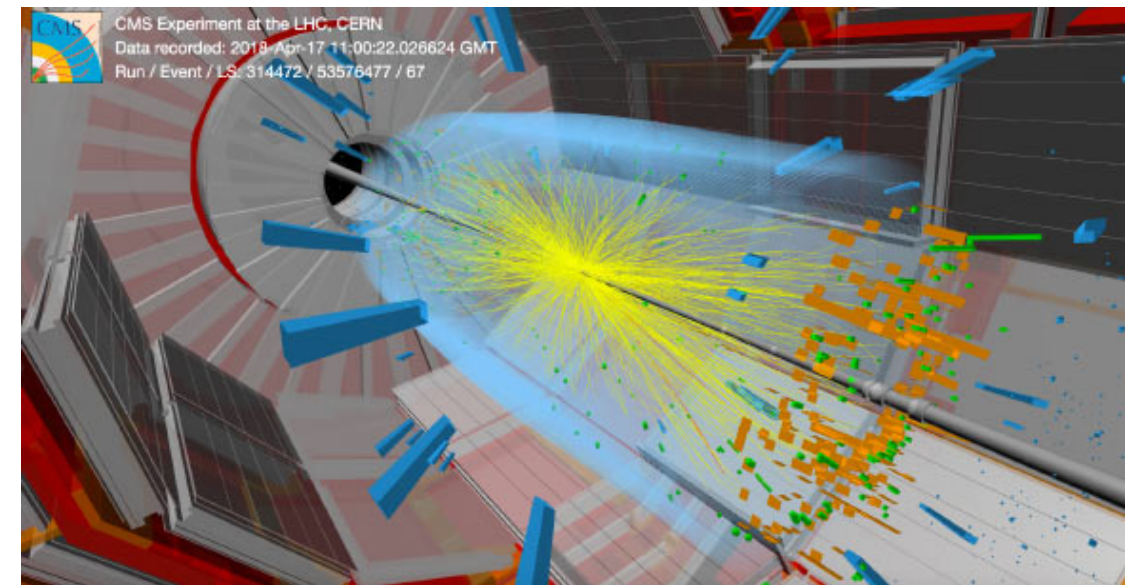
- With this in mind, exploring the energy frontier is our best opportunity to understand Nature from first principles

- At the same time, exploring the Standard Model as a fundamental theory in its own right is equally compelling

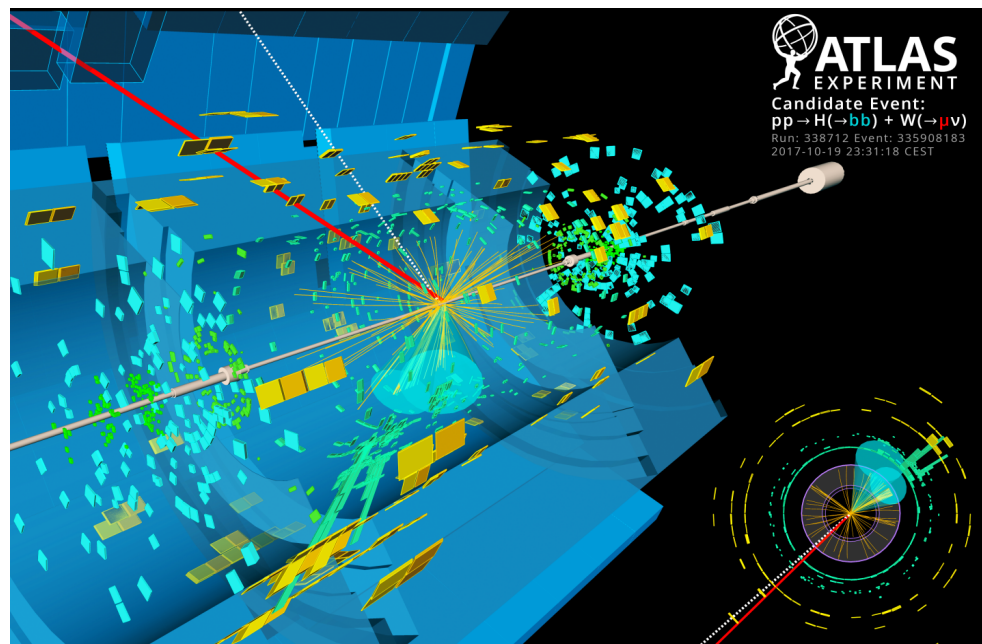# QCD ubiquitous at the LHC

LHC events typically involve jets, bunches of collimated hadrons
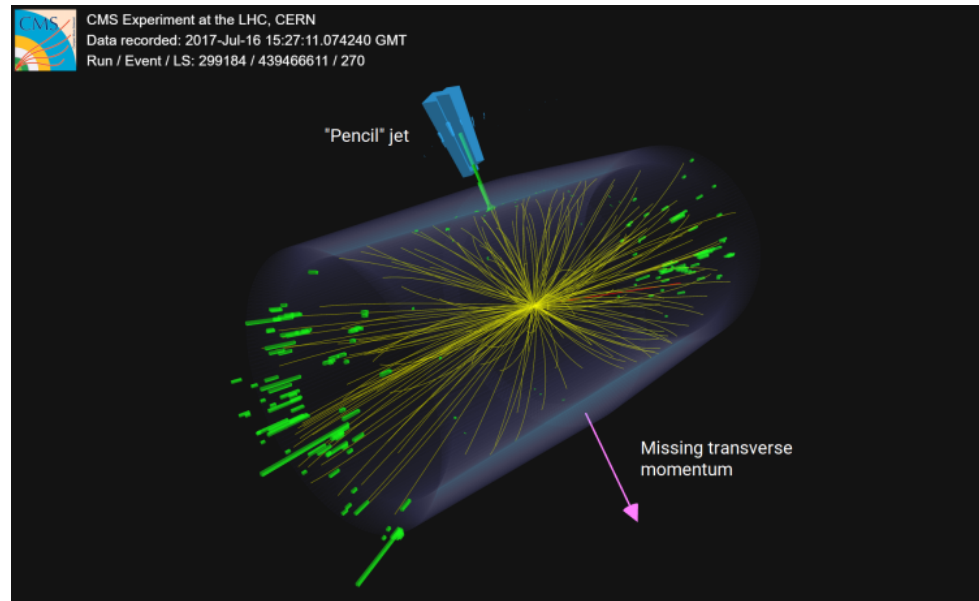
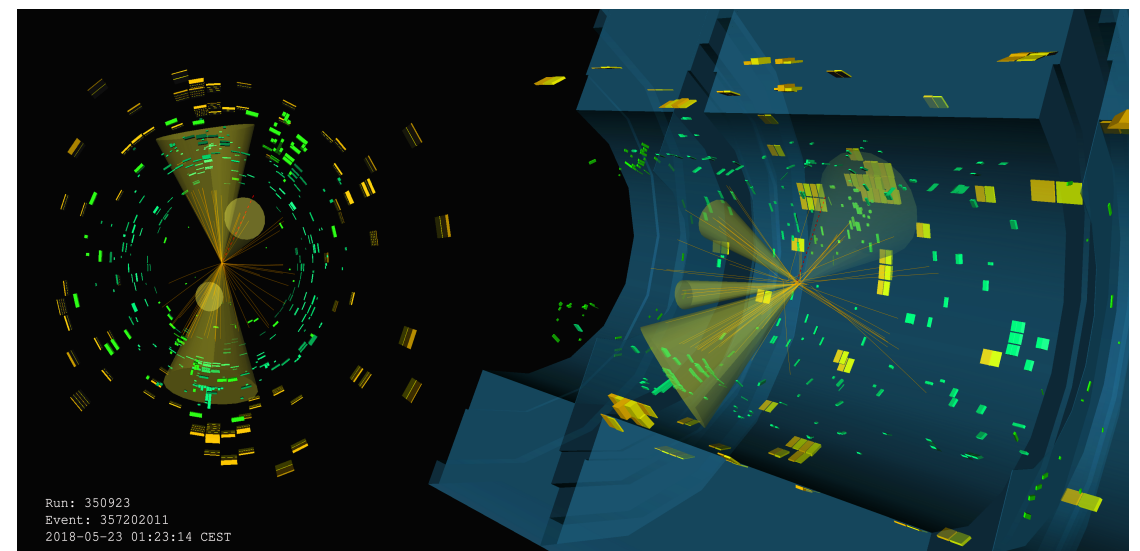

Search for Dark Matter in mono-jet events

Search for mini-blackholes

Precision Higgs

SUSY Searches

Key to best interpret LHC collisions is to control QCD radiation

Electroweak and QCD interactions

The Standard Model: a gauge theory with gauge group

$$SU(3) \times SU(2) \times U(1)$$

$$\alpha_s(M_Z) \sim 0.118 \qquad \alpha_{\text{weak}}(M_Z) \sim 0.01 \qquad \alpha_{\text{em}}(M_Z) \sim \frac{1}{128}$$
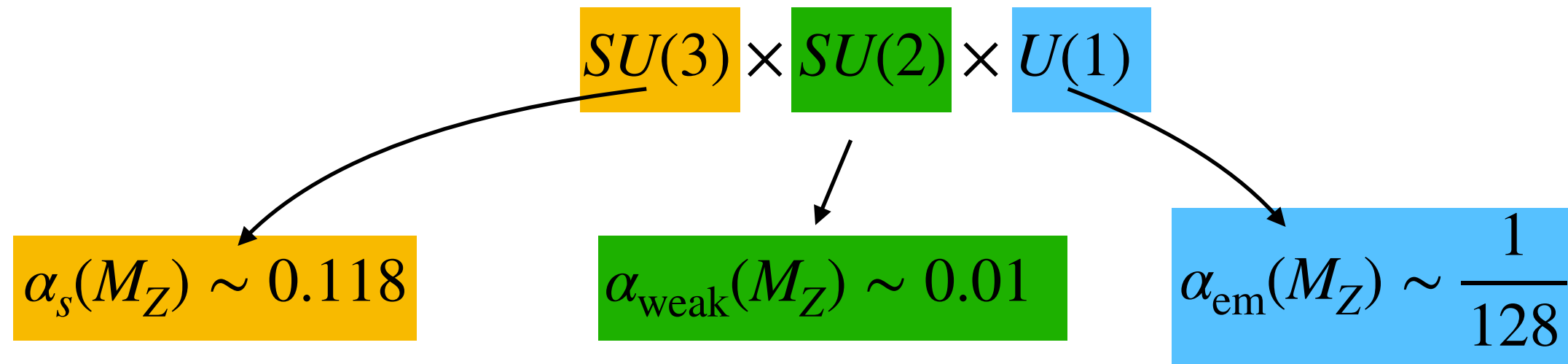
QCD interaction dominate. Rule of thumb: $\alpha_s^2 \sim \alpha_{\text{em}} \sim \alpha_{\text{ew}}$

Interactions at collider energies are perturbative, so one can compute cross sections as expansions in the couplings:

$$\sigma = \sigma_0 + \alpha \, \sigma_1 + \alpha \, \sigma_2 + \ldots$$

LO   NLO   NNLO

Role of precision

- test the Standard Model (SM) with highest precision and controlled theory uncertainties

- constrain fundamental parameters and parton distribution functions (PDFs)

- probe New Physics via small deviations from SM predictions

- optimise experimental analyses through design of better observables/selection criteria suitable for precision calculations

- guide future collider designs and priorities

Potential of LHC and future machines, in particular e⁺e⁻ colliders, cannot be exploited without precision theory predictions

Precision a reality

Z-boson transverse momentum used recently to extract strong coupling with high precision

$$\alpha_s(m_Z) = 0.11828^{+0.00084}_{-0.00088}$$

ATLAS, 2309.12986

Precision a reality

Z-boson kinematics to below a percent
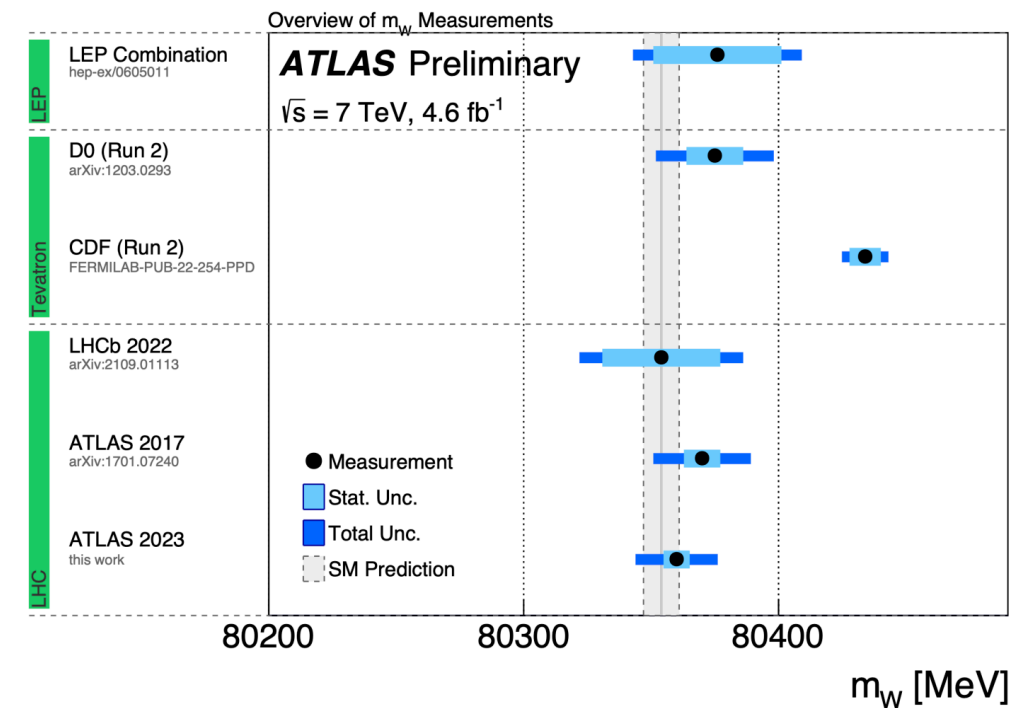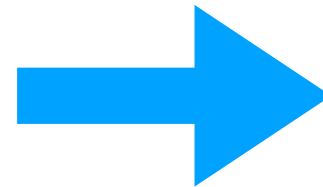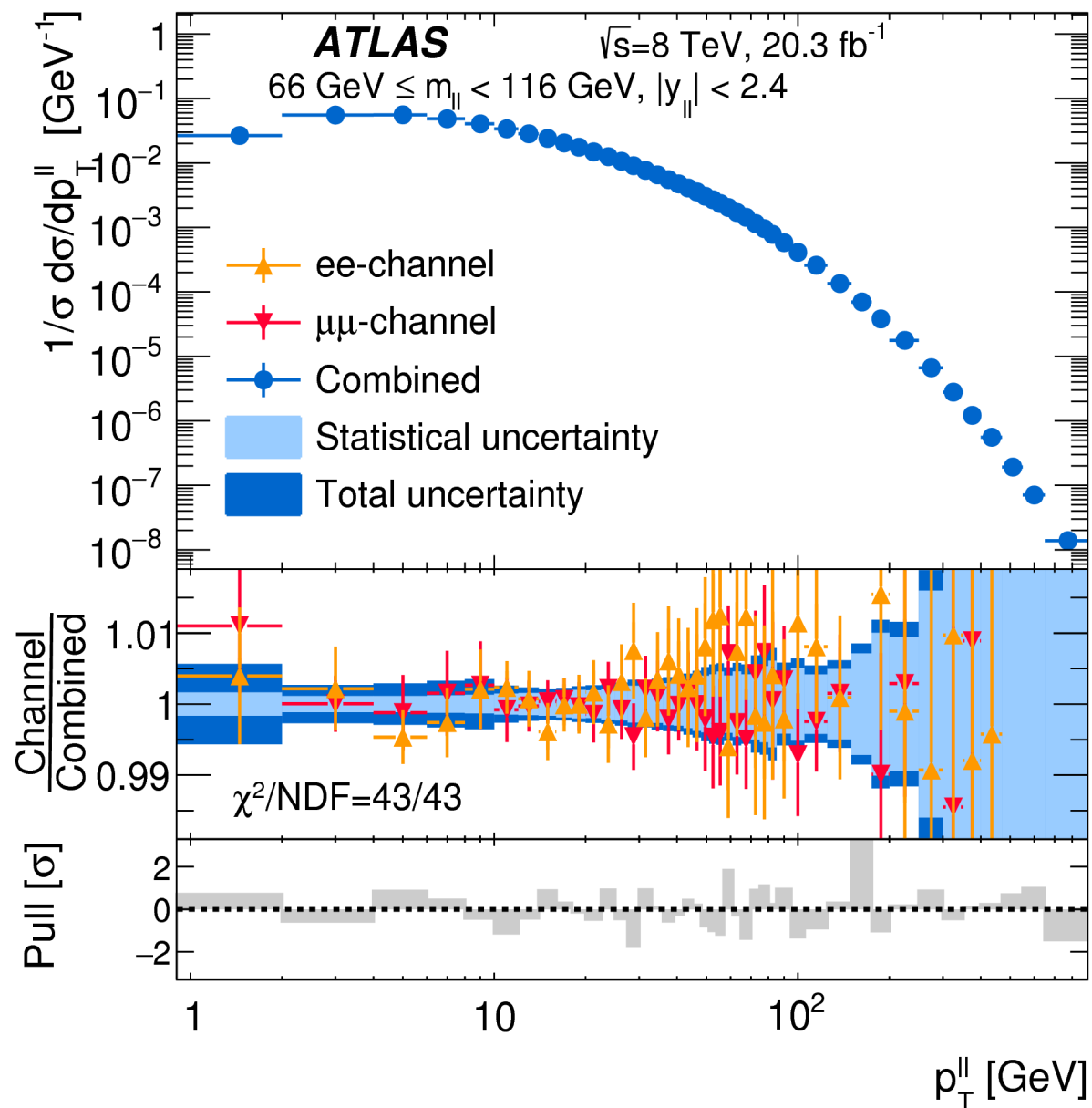
Crucial input for W-boson mass extraction

Collider events: real world

Theorist point of view

time/energy scale

block 2: evolution

$\mathrm{d}\hat{\sigma}_0$

block 1: hard process

MPI   MPI

block 3: hadronization

from MLHAD

○ hard interaction
● resonance decays
■ matching/merging
■ FSR
■ ISR
■ QED
■ weak showers
■ hard onium
○ multiparton interactions
■ beam remnants
■ strings
■ primary hadrons
■ secondary hadrons
■ hadronic rescattering

● meson
▲ baryon
▼ antibaryon
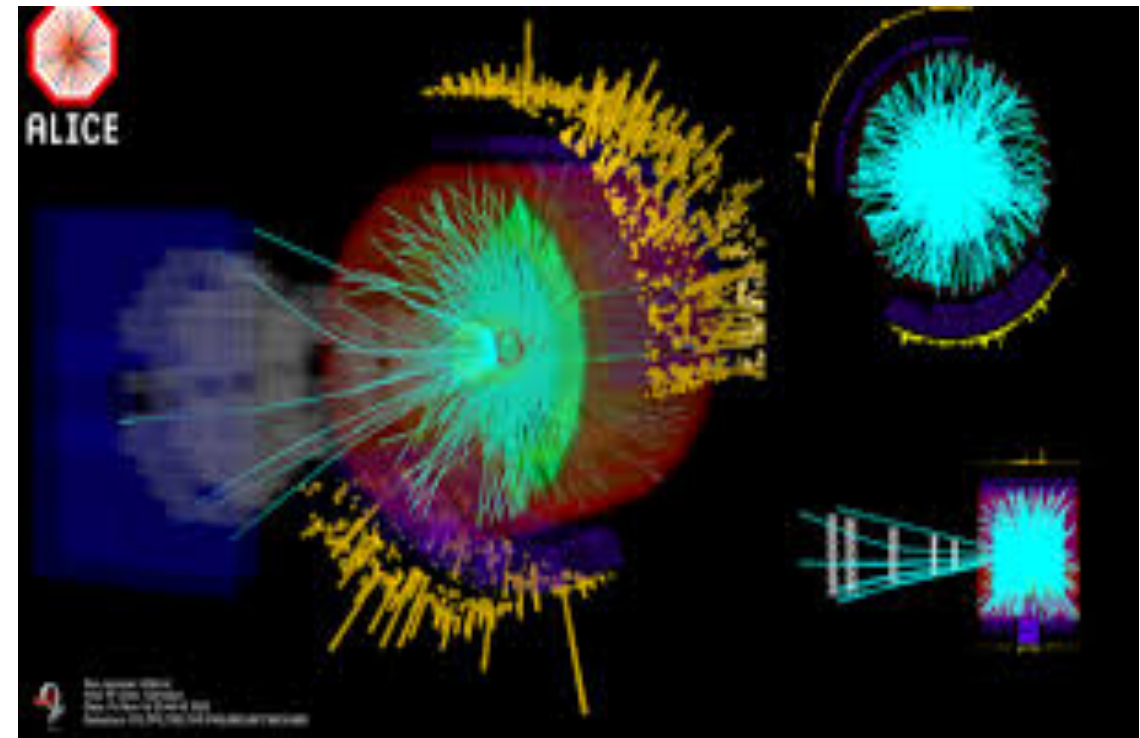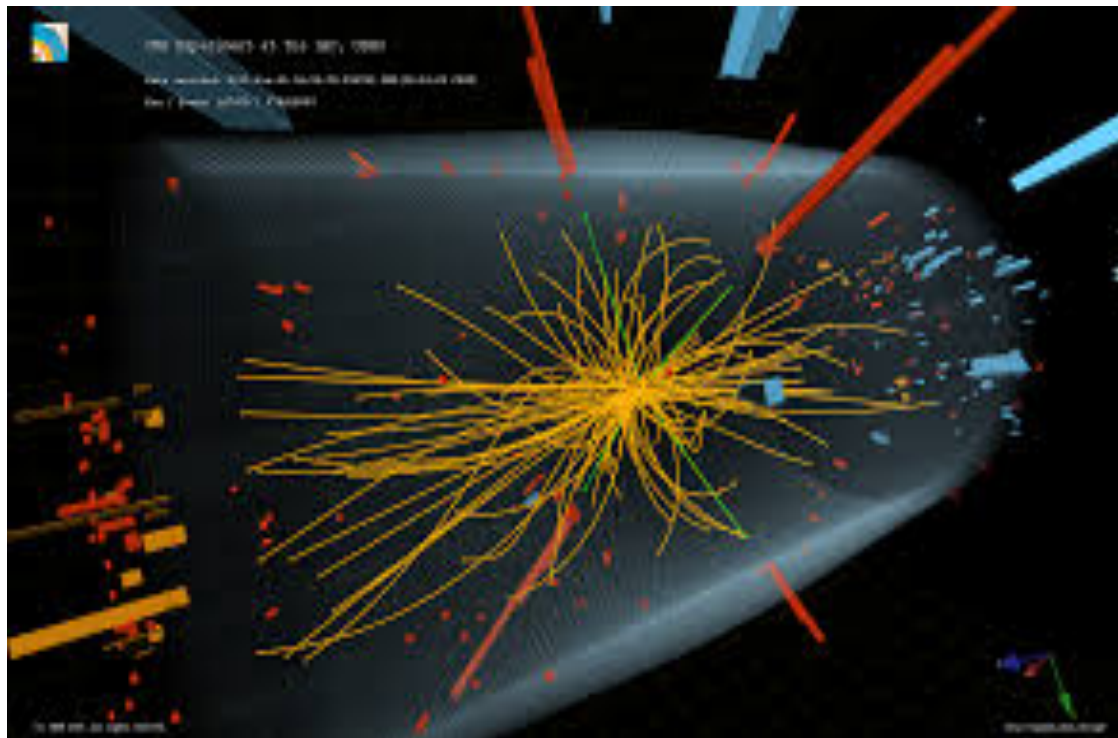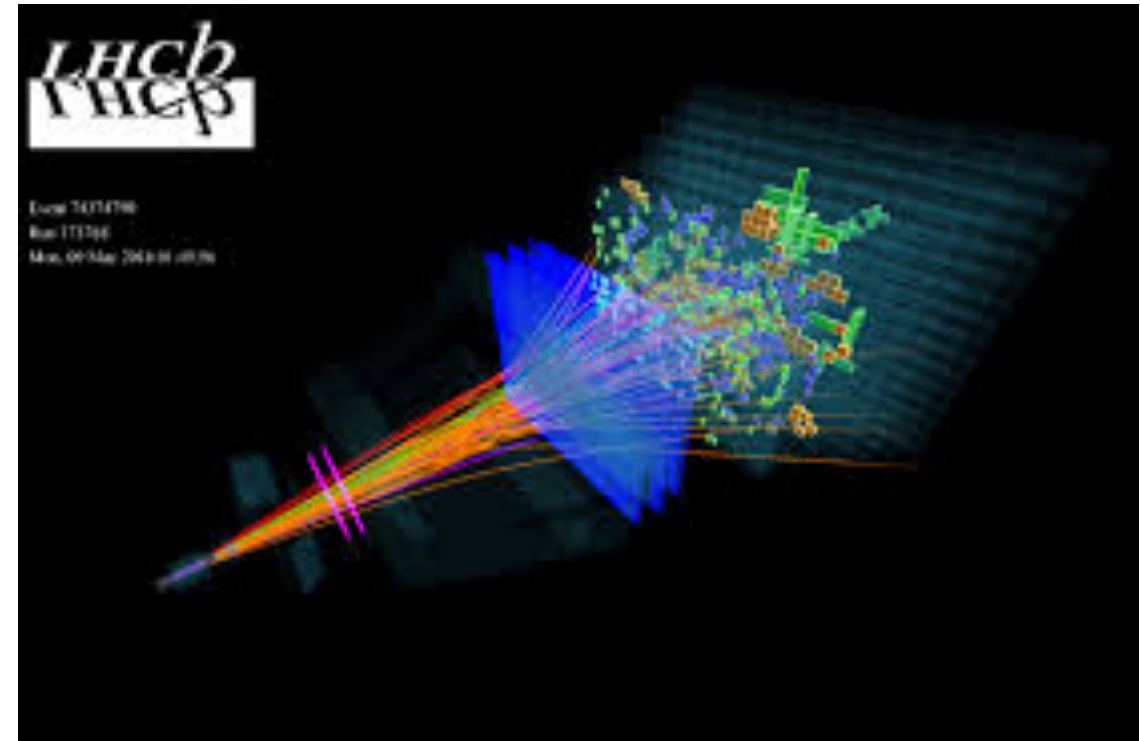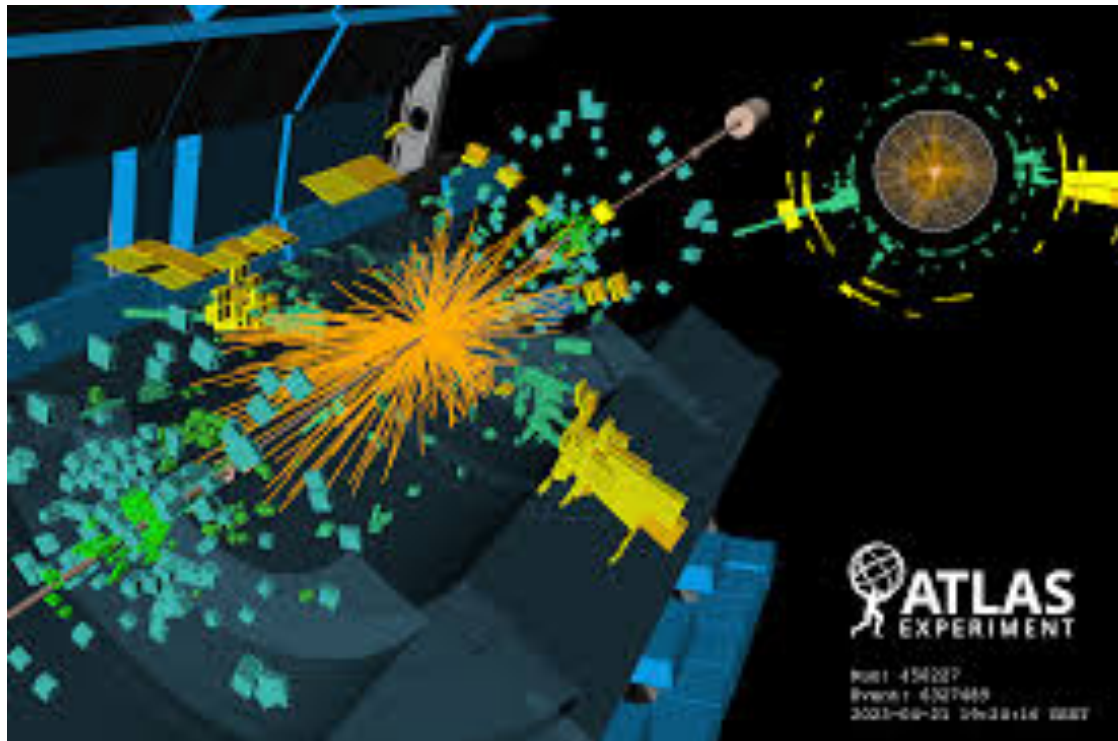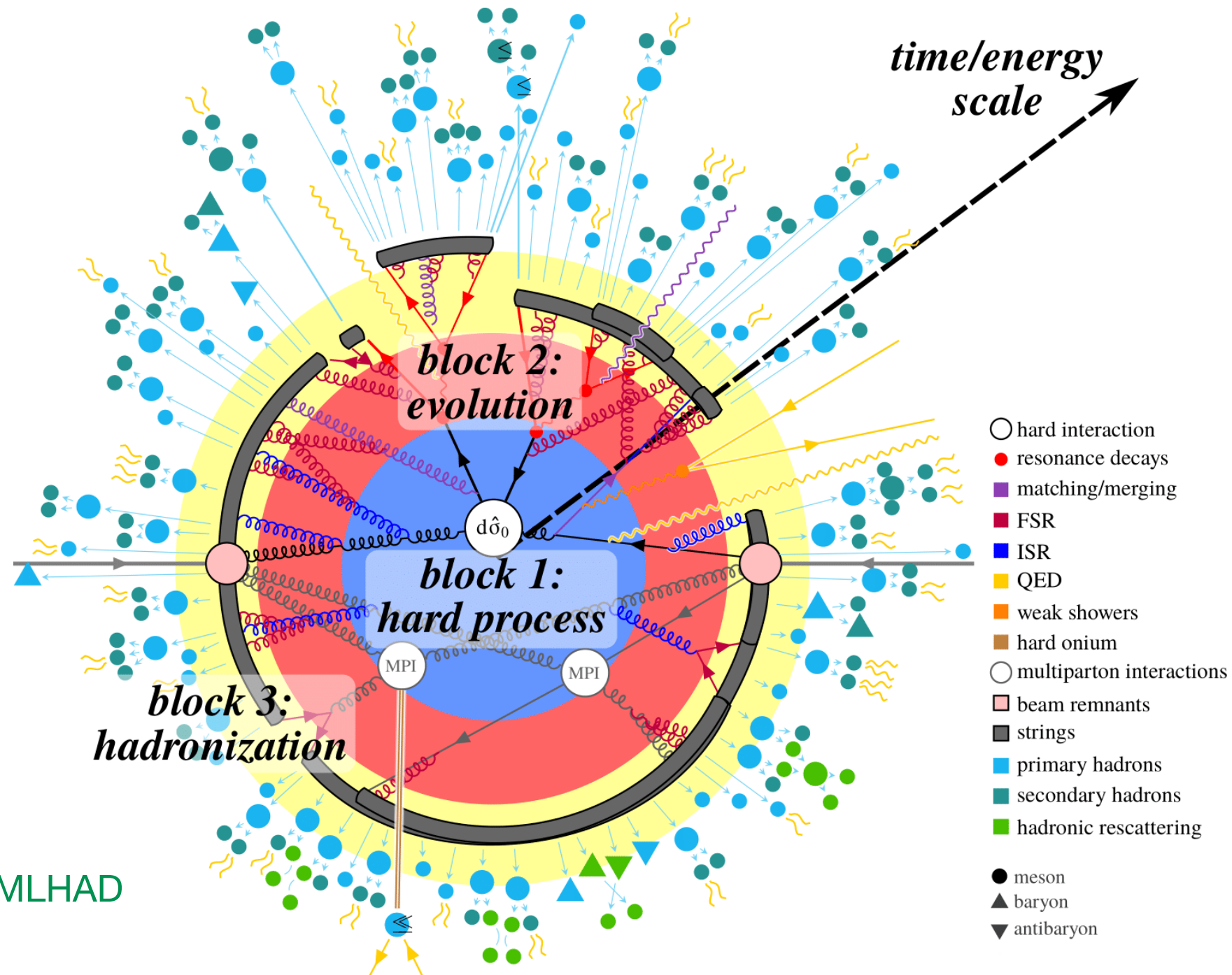
Clearly, a multilateral challenge

NLO: one-loop amplitudes

Main **one-loop amplitude providers**:

- ▶ **BlackHat(https://blackhat.hepforge.org/)**

- ▶ **Collier (https://collier.hepforge.org)**

- ▶ **GoSam (https://gosam.hepforge.org)**

- ▶ **Golem95 (https://golem.hepforge.org/)**

- ▶ **Helac-NLO/Helac-1Loop (https://helac-phegas.web.cern.ch)**

- ▶ **Ninja (https://ninja.hepforge.org/)**

- ▶ **Njet (https://www.physik.hu-berlin.de/de/pep/tools/njet)**

- ▶ **NLOX (http://www.hep.fsu.edu/~nlox/)**

- ▶ **OpenLoops (https://openloops.hepforge.org)**

- ▶ **Recola/Recola2 (https://recola.hepforge.org)**

✓ Fast, automated generation and numerical evaluation of one-loop amplitudes

✓ Easy interface with Sherpa, Herwig, POWHEG, and others

✓ QCD only, or full SM (QCD+EW)

NLO: general purpose tools

1. MadGraph5_aMC@NLO (https://launchpad.net/mg5amcnlo)

- Full automation of NLO QCD and EW

- Process generation via FeynRules/UFO. Supports parton showers via aMC@NLO

2. Sherpa+OpenLoops (https://sherpa.hepforge.org, https://openloops.hepforge.org)

- SHERPA handles phase space, subtraction, matching, and showering

- OpenLoops provides fast NLO matrix elements. Efficient for multi-leg processes

3. Herwig+Matchbox (https://herwig.hepforge.org)

- Herwig's Matchbox module enables automated NLO QCD corrections and matching

- Works with external amplitude providers (OpenLoops, MadGraph, etc.)

4. POWHEG-BOX (http://powhegbox.mib.infn.it)

- NLO with matching to parton showers (POWHEG method)

- Semi-automated; requires user input for new processes

5. MCFM (https://mcfm.fnal.gov)

- Parton-level code for NLO calculations (less automated)

- Mostly SM processes. Mostly based on analytic calculations, very stable and fast

...

NLO automation

NLO QCD calculations are now fully automated, thanks to:

✓ the ability to reduce one-loop amplitudes to master integrals

✓ analytic knowledge of all relevant master integrals

✓ a systematic understanding of how to handle intermediate divergences (e.g. subtraction/slicing methods)

✓ the availability of tools for computing tree-level real radiation
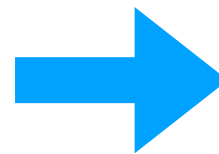
Open challenges at NLO

- numerical stability and efficiency for high multiplicity processes
- NLO corrections to loop-induced processes

NNLO: status

2 → 1

DY, van Neerven et al.

W/Z total, H total, Harlander, Kilgore

H total, Anastasiou, Melnikov

H total, Ravindran, Smith, van Neerven

WH total, Brein, Djouadi, Harlander

H diff., Anastasiou, Melnikov, Petriello

H diff., Anastasiou, Melnikov, Petriello

W diff., Melnikov, Petriello

W/Z diff., Melnikov, Petriello

H diff., Catani Grazzini

W/Z diff., Catani, LC, de Florian, Ferrera, Grazzini

VBF total, Bolzoni, Maltoni, Moch, Zaro

WH diff., Ferrera, Grazzini, Tramontano

γγ, Catani, LC, de Florian, Ferrera, Grazzini

2 → 2

Hj (partial), Boughezal et al.

ttbar total, Czakon, Fiedler, Mitov

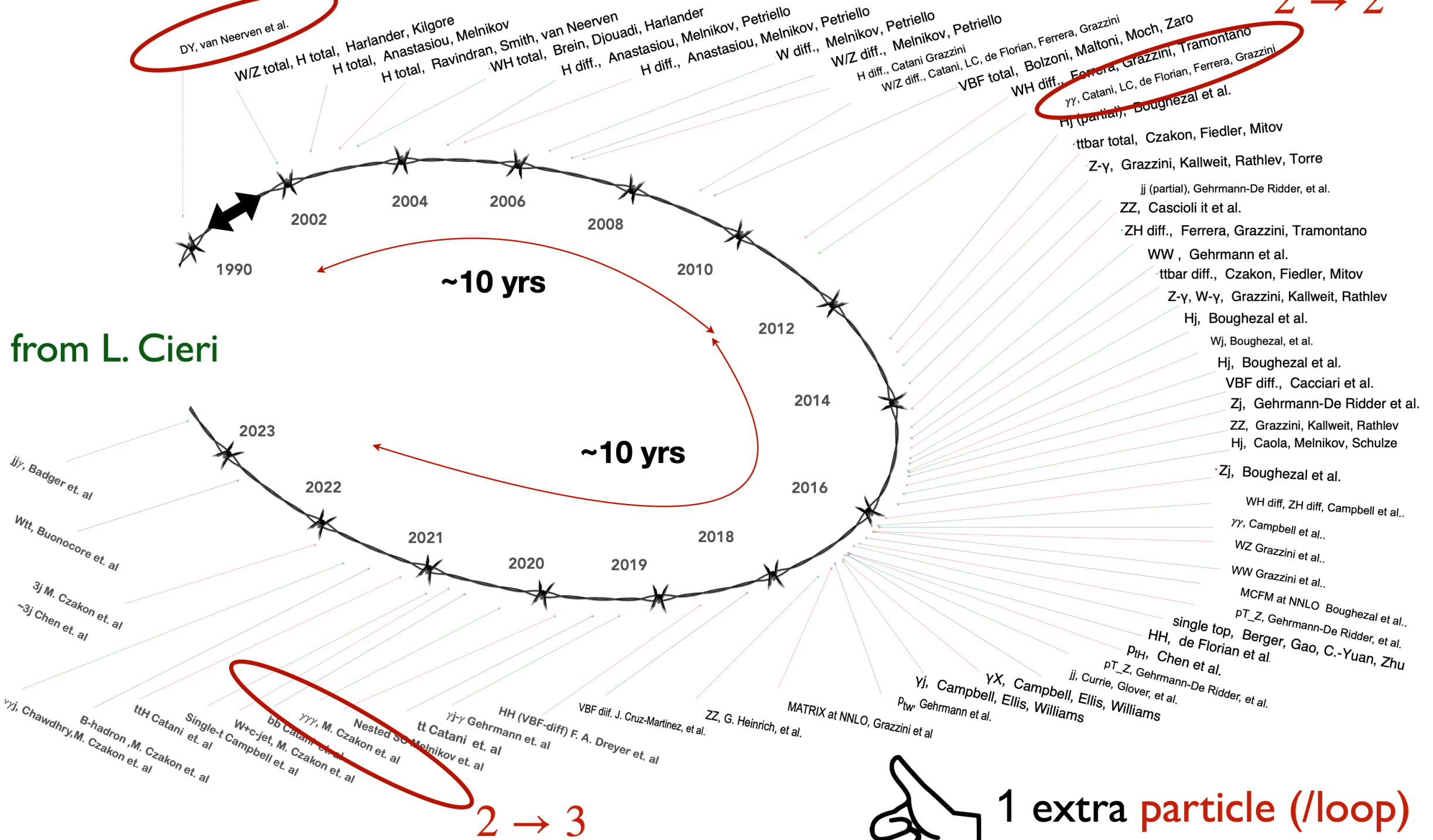Z-γ, Grazzini, Kallweit, Rathlev, Torre

jj (partial), Gehrmann-De Ridder, et al.

ZZ, Cascioli it et al.

ZH diff., Ferrera, Grazzini, Tramontano

WW, Gehrmann et al.

ttbar diff., Czakon, Fiedler, Mitov

Z-γ, W-γ, Grazzini, Kallweit, Rathlev

Hj, Boughezal et al.

Wj, Boughezal, et al.

Hj, Boughezal et al.

VBF diff., Cacciari et al.

Zj, Gehrmann-De Ridder et al.

ZZ, Grazzini, Kallweit, Rathlev

Hj, Caola, Melnikov, Schulze

Zj, Boughezal et al.

WH diff, ZH diff, Campbell et al..

γγ, Campbell et al..

WZ Grazzini et al..

WW Grazzini et al..

MCFM at NNLO Boughezal et al..

pT_Z, Gehrmann-De Ridder, et al.

single top, Berger, Gao, C.-Yuan, Zhu

HH, de Florian et al.

pTH, Chen et al.

ji, Currie, Glover, et al.

pT_Z, Gehrmann-De Ridder, et al.

γX, Campbell, Ellis, Williams

γj, Campbell, Ellis, Williams

ptw, Gehrmann et al.

MATRIX at NNLO, Grazzini et al

ZZ, G. Heinrich, et al.

VBF diff. J. Cruz-Martinez, et al.

HH (VBF-diff) F. A. Dreyer, et al.

γj-γ Gehrmann et. al

tt Catani et. al

Nested SC Melnikov et. al

γγγ, M. Czakon et. al

bb Catani et. al

W+c-jet, M. Czakon et. al

Single-t Campbell et. al

ttH Catani et. al

B-hadron, M. Czakon et. al

γγj, Chawdhry, M. Czakon et. al

~3j Chen et. al

3j M. Czakon et. al

Wtt, Buonocore et. al

jjγ, Badger et. al

from L. Cieri

~10 yrs

~10 yrs

2 → 3

1 extra **particle** (/loop) every 10 years

⇒ *no full automation yet*

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NNLO: ingredients

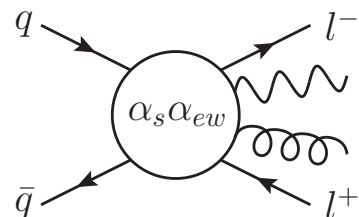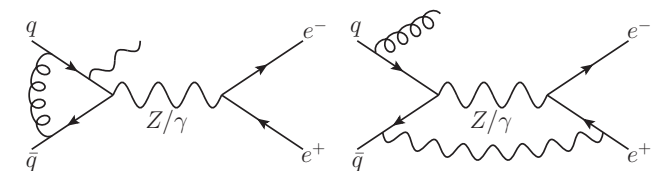Three main ingredients at NNLO:
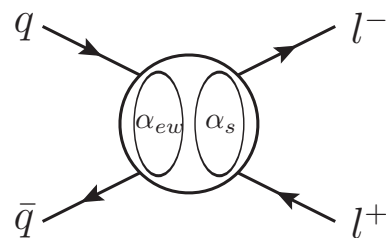
1. two-loop virtual + one-loop squared

2. one-loop with one extra radiation

[issue:numerical stability in unresolved regions]

3. Tree-level double real

diagrams from C. Signorile-Signorile

Sample diagrams shown for mixed QCD-EW corrections to Drell-Yan production
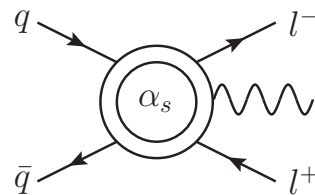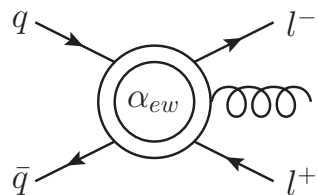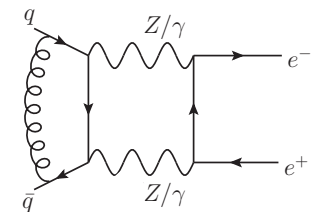
NNLO: ingredients

Three main ingredients at NNLO:
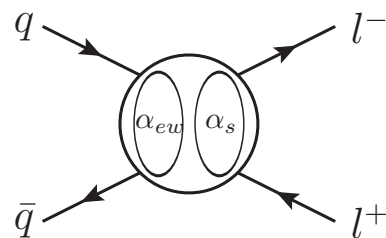
1. two-loop virtual + one-loop squared
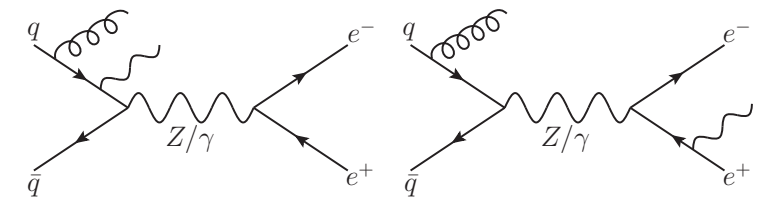
The hardest challenge ✗

2. one-loop with one extra radiation ✓

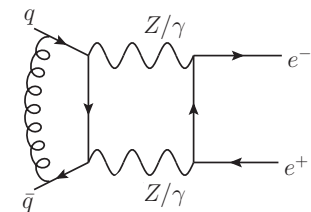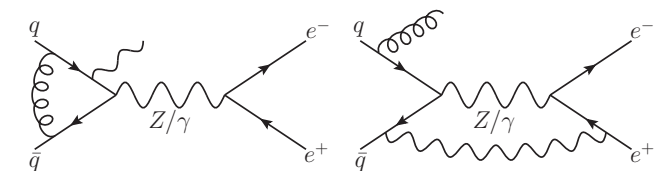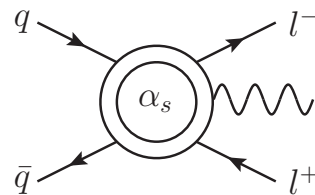*[issue:numerical stability in unresolved regions]*

3. Tree-level double real ✓

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Sample diagrams shown for mixed QCD-EW corrections to Drell-Yan production

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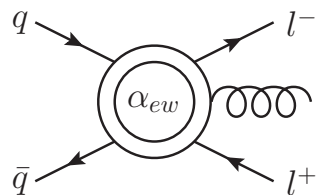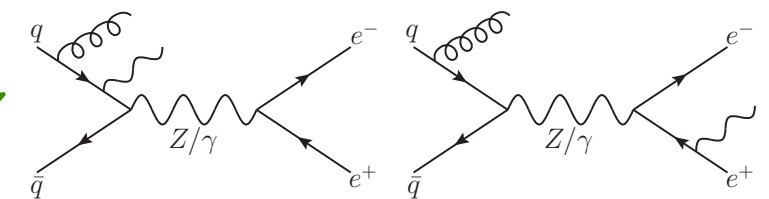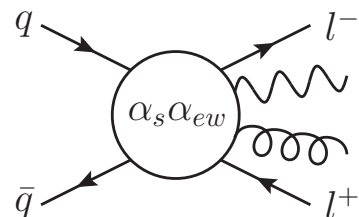
2-loop calculations
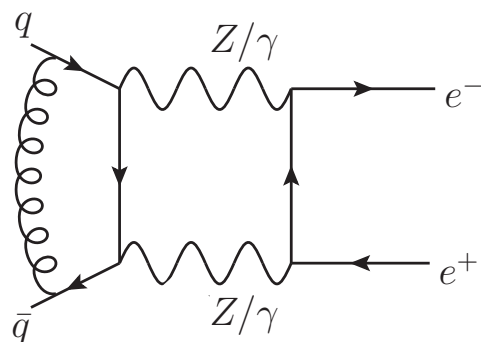
Feynman rules

$$C_{\mu_1...\mu_n\nu_1...\nu_n}(\{p_{\text{ext.}}\}) \int \frac{d^D l_1}{(2\pi)^4} \frac{d^D l_2}{(2\pi)^4} \frac{l_1^{\mu_1} \ldots l_1^{\mu_n} l_2^{\nu_1} \ldots l_2^{\nu_m}}{D_1 \ldots D_N}$$

$$D_i = q_i^2 - m_i^2$$

- The goal is to express a large number of tensor integrals appearing in Feynman diagrams as linear combinations of a small set of master integrals.

- Well established reduction-techniques, e.g.

  ‣ projection on form factors

  ‣ integration-by-parts (IBP) identities

  ‣ Lorentz invariance identities

  ‣ dimensional shift relations

  ‣ ...

2-loop challenge #1: reduction

While the flowchart is rather straightforward:

- define integral family
- generate IBP identities
- solve Laporta algorithm

The challenges involved are substantial and complex:

- explosion in number of terms and algebraic complexity
- finding minimal, linearly independent topologies not trivial
- finding and exploiting symmetries

Determine spanning cuts

Generate IBPs on $\mathcal{C}_1$-cut

Generate IBPs on $\mathcal{C}_2$-cut

...

Consistency Check

Shorten IBPs (optional)

Reduce IBPs on $\mathcal{C}_1$-cut

Reduce IBPs on $\mathcal{C}_2$-cut

...

Merge reduced IBPs on all cuts
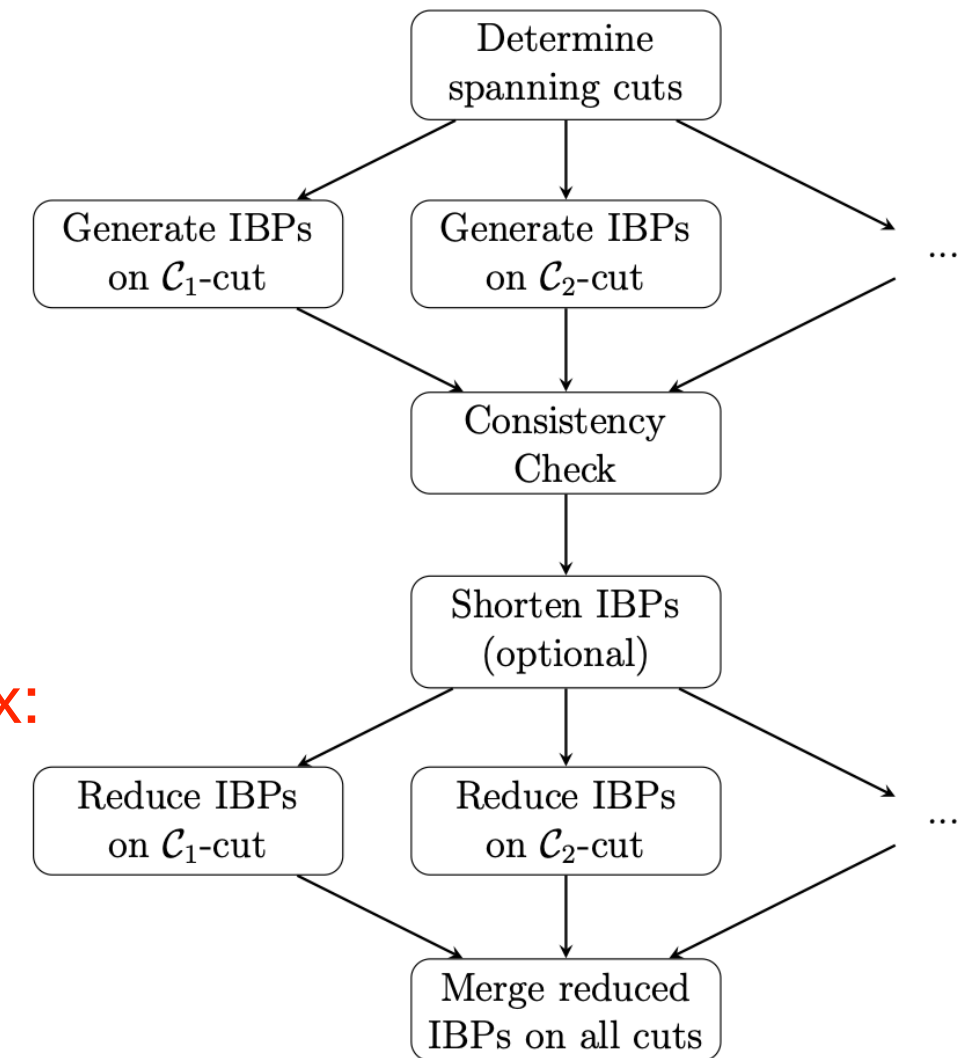
from 2502.20778
[NeatIBP 1.1+Kira]

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2-loop challenge #1: reduction

Main public tools for IBP reduction of two-loop Integrals:

1. **AIR** (https://people.phys.ethz.ch/~pheno/air/)

   ○ Automates IBP reduction for scalar and tensor integrals to master integrals.

2. **BLADE** (https://gitee.com/multiloop-pku/blade)

   ○ Fast IBP reduction in block triangular form

2. **FiniteFlow** (https://github.com/peraro/finiteflow-mathtools/)

   ○ multivariate functional reconstruction using finite fields and dataflow graphs.

3. **Fire** (https://gitlab.com/feynmanintegrals/fire)

   ○ Uses the Laporta algorithm for efficient IBP reduction of multi-loop integrals.

4. **Kira** (https://gitlab.com/kira-pyred/kira)

   ○ High-performance tool for IBP reduction, particularly for multi-loop integrals.

5. **LiteRed** (https://github.com/rnlg/LiteRed2)

   ○ A C++ program for IBP reduction, optimized for speed and simplicity.

6. **NeatIBP** (https://github.com/yzhphy/NeatIBP)

   ○ small-size integration-by-parts relations for Feynman integrals.

7. **Reduze** (https://reduze.hepforge.org/)

   ○ Automates IBP reduction for scalar and tensor integrals, support both one- and two-loop level.

Steady progress allows reduction of increasingly complicated processes

2-loop challenge #2: masters

At NLO: the full set of master integral for any process is fully known analytically

At NNLO: the complete set of master integrals is not known. Often non-trivial to verify linear independence

Techniques for evaluation:

▸ difference and differential equations (DE), including auxiliary mass method

▸ Mellin–Barnes representations

▸ sector decomposition

▸ expansion in limits

▸ iterated integrals

▸ …

▸ ongoing development of **new analytic and numerical methods**

2-loop challenge #2: masters

Most used public codes:

1. pySecDec (https://github.com/gudrunhe/secdeco)

- Numerical evaluation of master integrals using sector decomposition

2. FIESTA (https://gitlab.com/feynmanintegrals/fiesta)

- Numerical evaluation of master integrals using sector decomposition

3. AMFLOW (https://gitlab.com/multiloop-pku/amflow)

- Numerical evolution of master integrals using finite flow method

▸ powerful checks of analytic calculations

▸ used directly in numerical/phenomenological implementations

▸ if the evaluation is very slow, used to construct integration grids to be just interpolated on-the-fly

2-loop challenge #2: masters

Open challenges:

- Elliptic and beyond:
  Master integrals with elliptic or more general functions (e.g. modular forms)
  are not expressible in terms of multiple polylogarithms; the theory of these
  functions is still being developed.

- Numerical evaluation stability:
  Some integrals can only be evaluated numerically, but the precision and
  speed required for collider applications are often challenging/prohibitive.

- Automation bottlenecks:
  While IBP and DE methods are largely automated, choosing good bases and
  simplifying expressions still requires expert intervention.

**Frontier: 2 to 2 process with full off-shell legs, 2-loop integrals with
3-4 scales, multi-leg QCD-EW mixed contributions …**

NNLO challenge #3: singularities

While full results are finite, double-virtual, real-virtual and double-real amplitudes involve intermediate singularities. Since they live in different phase-spaces (Φ_n, Φ_{n+1}, and Φ_{n+2}) the cancellation of singularities must occur *before numerical evaluation*.
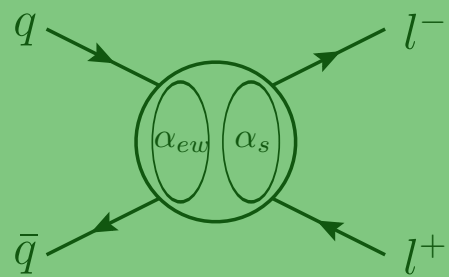
Two-loop virtual Φ_n
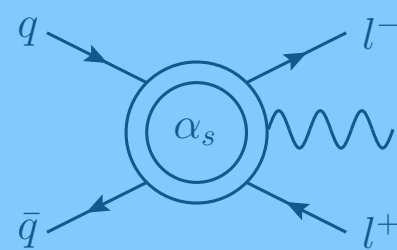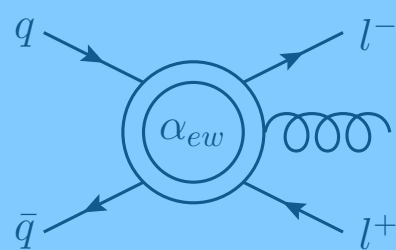
Real-virtual Φ_{n+1}
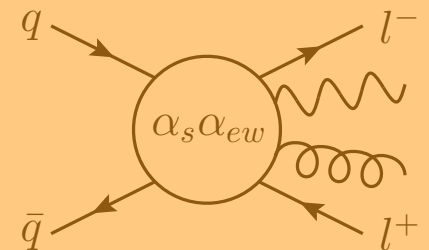
Double real-Φ_{n+2}

NNLO challenge #3: singularities

Two core ideas

Slicing

finite ⇒ numerical evaluation

$$\sigma(X) = \int_0^{\tau_{\text{cut}}} \frac{d\sigma_{\text{approx}}(X)}{d\tau} + \int_{\tau_{\text{cut}}} \frac{d\sigma(X)}{d\tau}$$

approximate evaluation close to soft-collinear limits

- Non-local in the phase space
- Dependence on the slicing parameter

Subtraction

finite ⇒ numerical evaluation

$$\int d\Phi_D |\mathcal{M}|^2 F_{\text{J}} = \int d\Phi_4 \left(|\mathcal{M}|^2 F_{\text{J}} - K \right) + \int d\Phi_D K$$

soft-collinear counterterms ⇒ poles in 1/ε

- Local in the phase space
- Construction and integration of counterterms hard/process specific

Many practical realisations of these ideas at two-loops and beyond

NNLO challenge #3: singularities

| Method	Local?	Analytic?	Examples	Public tool
Antenna subtraction [Gehrmann-De Ridder et al. '05]	✓	✓	$e^+e^- \to$ 3 jets, DIS, Z/H+jet, dijets, diphoton, VBF Higgs …	NNLOJet
q_T subtraction [Catani, Grazzini '07]	✗	✓	Colour singlet processes (2 to 2) , tt, bb, ttH, ttW	HNNLO, DYNNLO, MCFM
N-jettiness subtraction [Boughezal et al. '15; Gaunt et al. '15]	✗	✓	V+jet, H+jet, top decay, single top	Geneva, MCFM
Sector improved residue subtraction [Czakon '10; Czakon, Heymes '14]	✓	✗	tt, Higgs+jet, inclusive jet, W+jet, 3γ, 2γ +jet, γ + 2 jet,3 jets, B-hadrons	STRIPPER (private)
Local analytic subtraction [Magnea, Maina, Pelliccioli, Signorile-S., Torielli, Ucciratial. '18-'20]	✓	✓	$e^+e^- \to$ 2 jets	Private code
Nested soft–collinear subtraction [Caola, Melnikov, Rontsch '17]	✓	✓	VH, VBF QCD-EW Drell Yan	Private code
ColourFull subtraction [Del Duca, Duhr, Somogyi, Trocsanyi '05]	✓	✓	$e^+e^- \to$ 3 jets, H → bb, H	NNLOCAL
Projection to Born [Cacciari, Dreyer, Karlberg, Salam, GZ '15]	✓	✓	VFB H, DIS, H → bb, HH, t-channel single top [also jet in DIS, H → bb, gg → @ N3LO)	Private codes
Loop Treel duality [Bierenbaum, Catani, Draggiotis, Rodrigo '10, Capatti, HIrschi, Kermanschah, Pelloni, Ruijl '19]	✓	✗	$\gamma^* \to$ 2 jets, $\gamma^* \to$ tt	Private code
Locally finite two-loops [Anastasiosu, Sterman '18]	✓	✗	Multi-photon in e+e-, multi-Higgs and EW bosons finite finite top-mass, electroweak production through gluon fusion	Private code

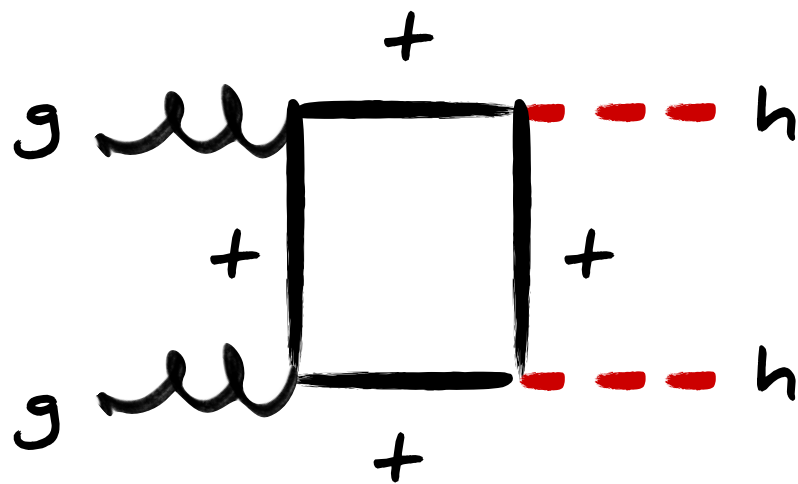# Example: gg → HH
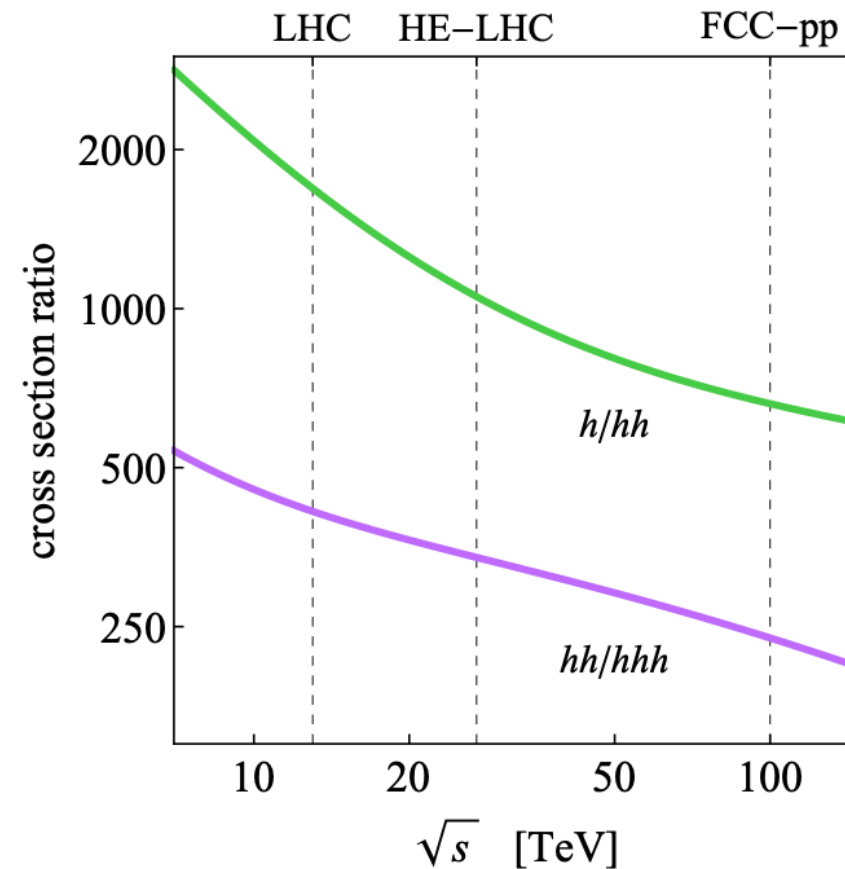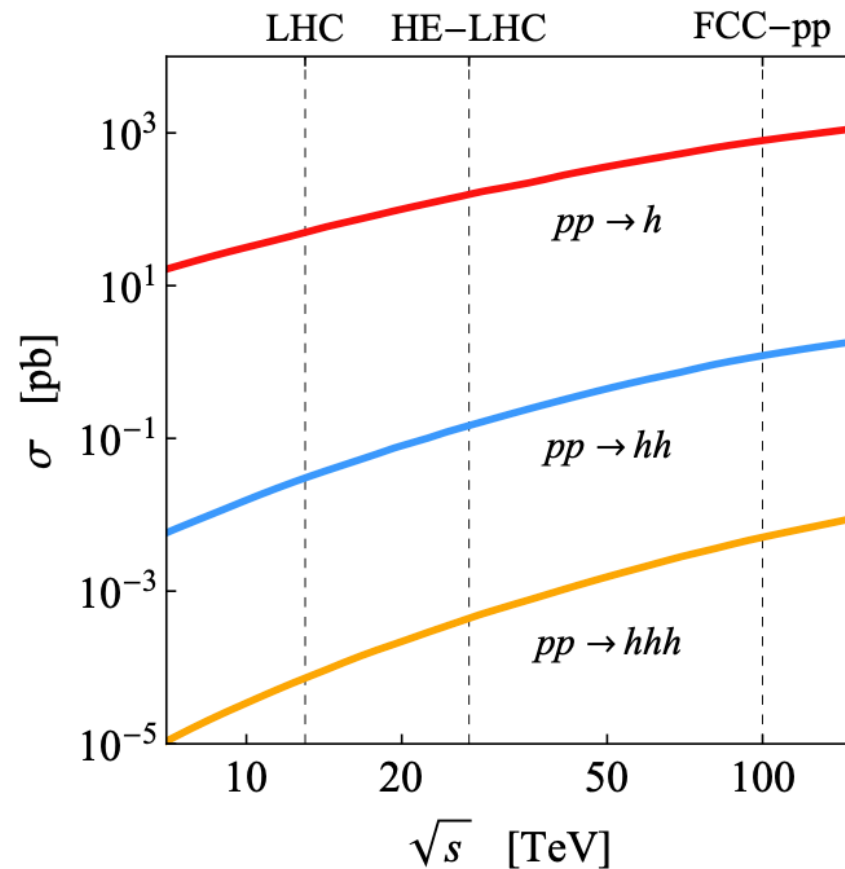
- Double-Higgs production golden channel for the direct measurement of the Higgs self-coupling, the footprint of the SM Higgs boson and of Higgs potential

- Small cross section because of accidental cancellations and phase space effect

- Interesting also in the context of possible new resonances (2HDM, MSSM, RS, …)

gg → HH: LO
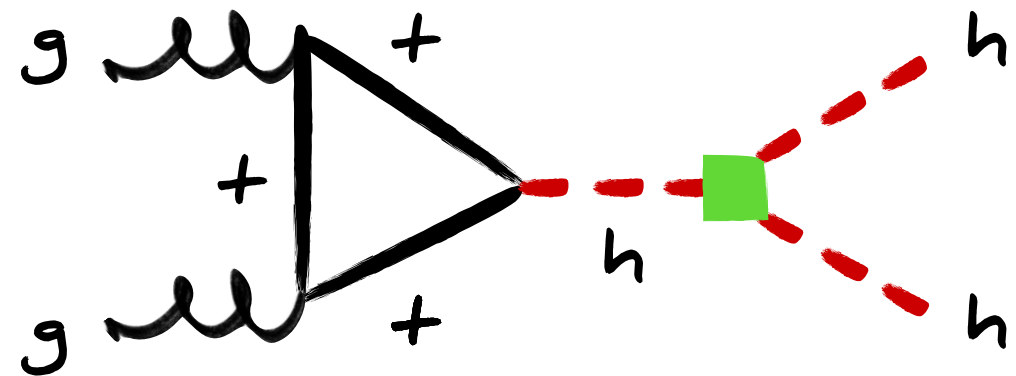
LO amplitude ✓

Eboli, Marques, Novaes, Natale '87
Glover, van der Bij '88

Going beyond LO highly non-trivial

Figure 1. Left: Total production cross section for $pp \to h$
as a function of $\sqrt{s}$. Right: Dependence of the cross
$\sigma(pp \to hh)/\sigma(pp \to hhh)$ (purple) on the collider C
state-of-the-art SM calculations of single-Higgs [2–4],

obvious way to get access to the cubic and quartic i
production. Unfortunately, all multi-Higgs produc
seen from Figure 1, making already LHC measur

Figure 1. Left: Total production cross section for $pp \to h$ (red), $pp \to hh$ (blue) and $pp \to hhh$ (yellow) produ
as a function of $\sqrt{s}$. Right: Dependence of the cross section ratio $\sigma(pp \to h)/\sigma(pp \to hh)$ (green) and
$\sigma(pp \to hh)/\sigma(pp \to hhh)$ (purple) on the collider CM energy. The shown predictions are based on the

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gg → HH: beyond LO

NLO QCD:

- large-m_t [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]
- numeric   [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16]
             [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
- large-m_t + threshold exp. Padé   [Gröber, Maier, Rauh '17]
- high-energy expansion   [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion   [Bonciani, Degrassi, Giardino, Gröber '18]
       + high-energy expansion   [Bagnaschi, Degrassi, Gröber '23]

NNLO QCD:

- large-m_t virtuals   [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15, Davies; Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals   [Davies, Herren, Mishima, Steinhauser '19 '21]
- light fermion corrections at $p_T = 0$ [Davies, Schönwald, Steinhauser '23]

N3LO QCD:

- Wilson coefficient C_{HH} [Spira '16; Gerlach, Herren, Steinhauser '18]
- HTL   [Chen, Li, Shao, Wang '19]

from Kay Schönwald

gg → HH: beyond LO
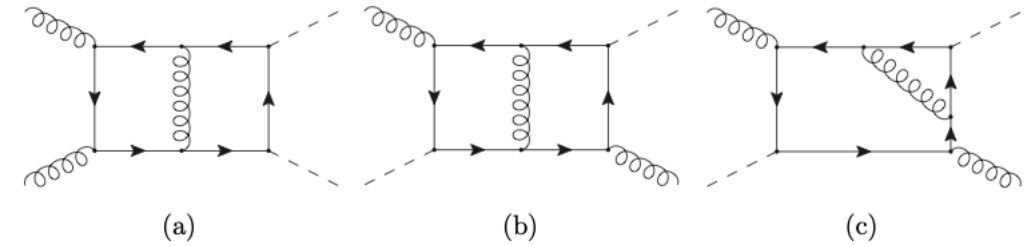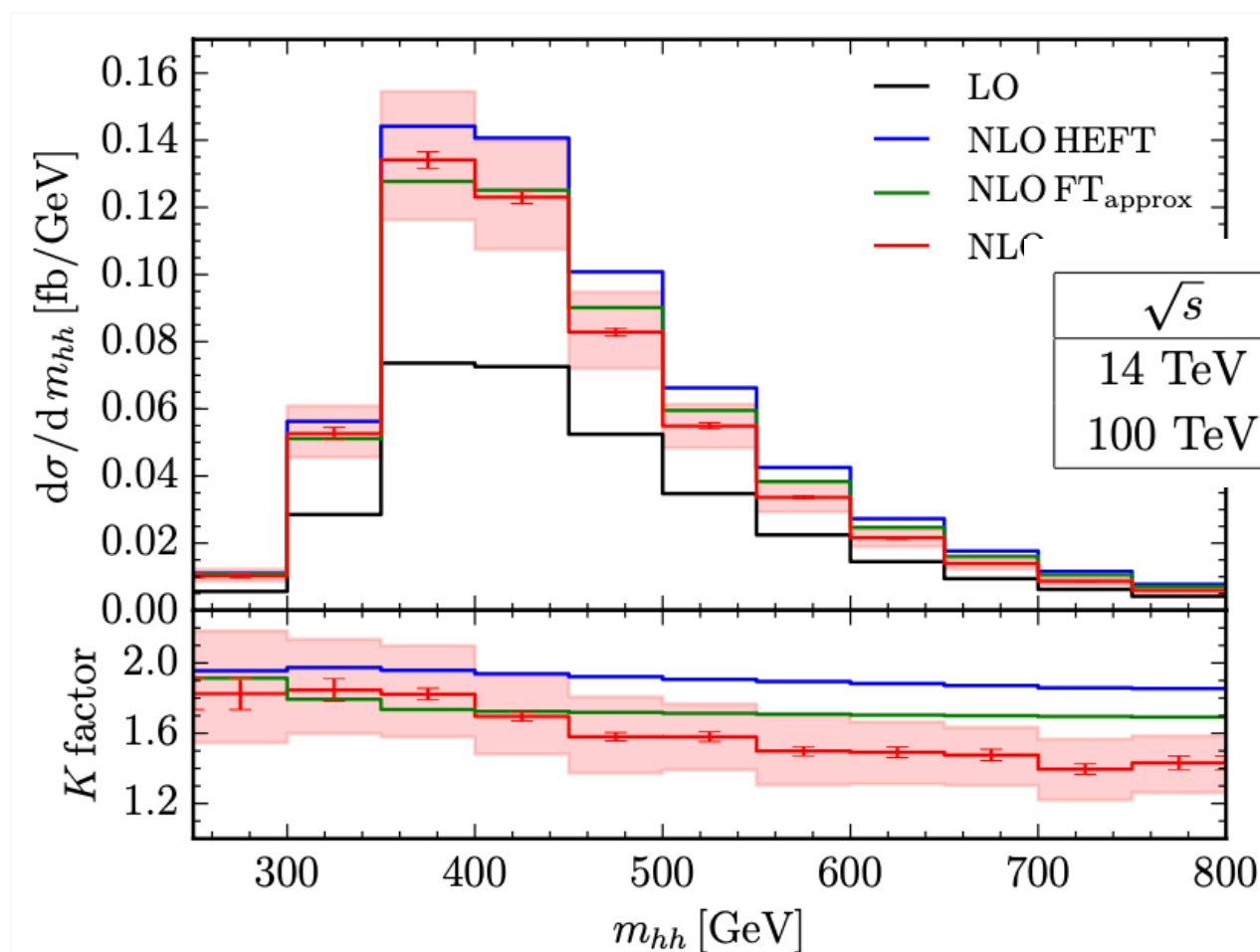
Borowka et al. '16

As for single Higgs production very large NLO corrections and large residual uncertainties (m_t scheme choice?) ⇒ strong motivation to improve precision

gg → HH: approx NNLO

Since a full NNLO (3-loop) is out of reach, clever approximations developed:

‣ High energy expansion $s, |t|, \gg m_t^2, m_H^2$

‣ Small-t expansion $s, m_t^2, \gg |t|, m_H^2$

Idea is to fully cover the physically relevant phase space with the two approximations (reminiscent, and exploits, the strategy of regions, according to which full results are recovered by summing over all relevant regions at integrand level)

Robust validation using overlap regions, lower orders and numerical results

from Kay Schönwald
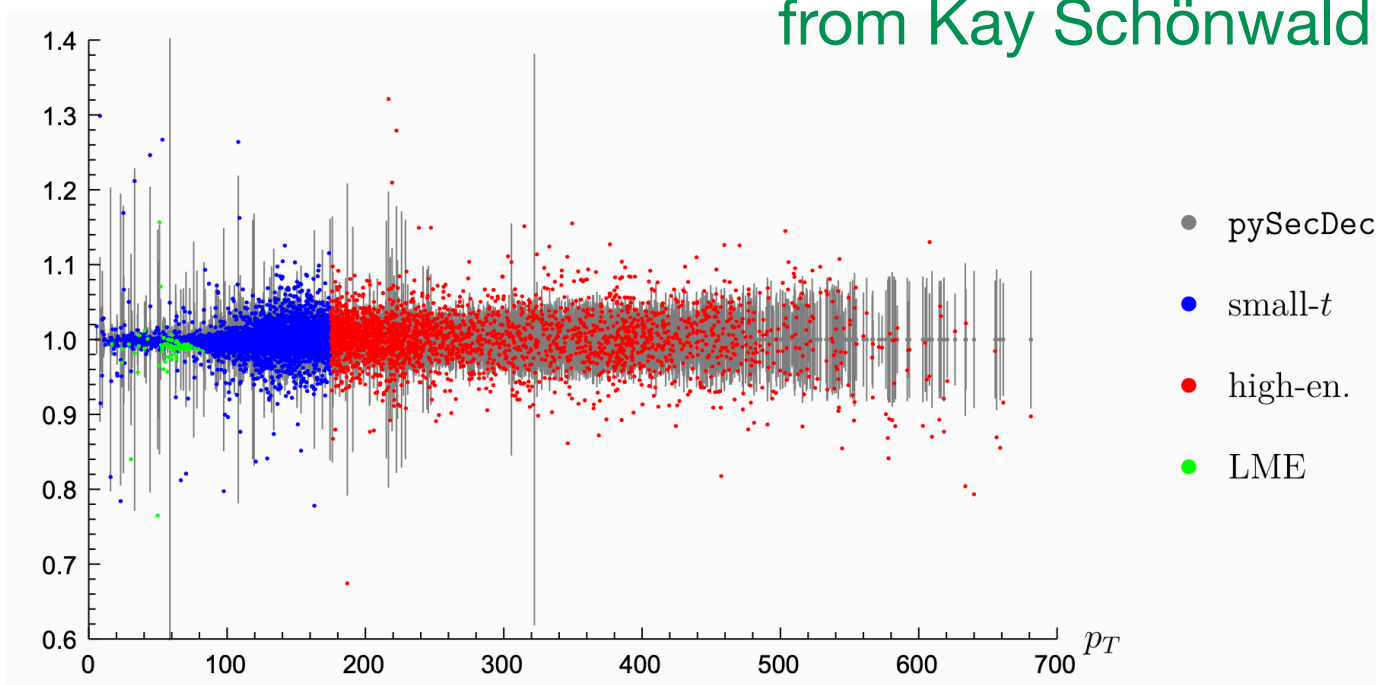
These and similar approximations (soft Higgs, massification, threshold exp., large N_c, …) are expected to become increasingly relevant as we aim to **halve theoretical uncertainties** — a common target in future projections.

gg → HH: electroweak?

Electroweak effects of order a few percent for inclusive cross-sections, but enhanced in tails of distributions, where New Physics effects might appear, because of large Sudakov logarithms.

Scales involved:

$$s, t, M_H^2, M_W^2, m_t^2, M_Z^2, m_b^2, \dots$$
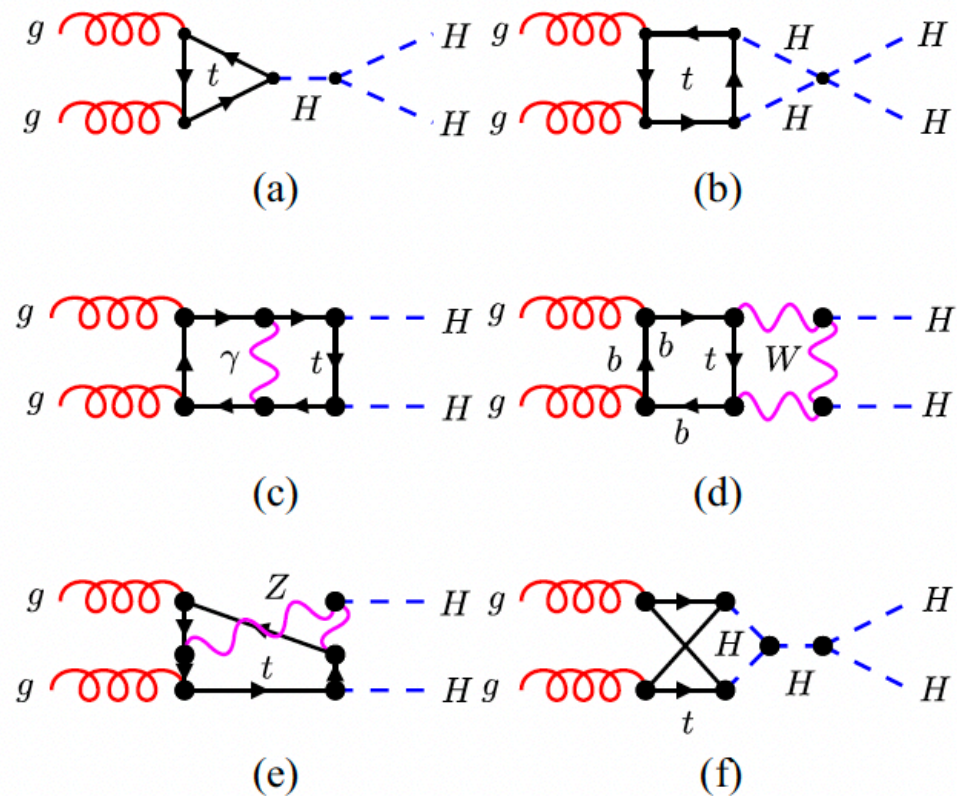
Given the # of scales involved, even the calculation of the leading electroweak effects is analytically extremely challenging. As for QCD corrections, expand in regions to obtain approximate results. Alternatively numerical results available.

gg → HH: electroweak?

NLO EW effects are known fully numerically:

Bi, Huang, Huang, Ma, Yu, 2311.16963

TABLE I. LO and NLO EW corrected integrated cross sections (in fb) with $\sqrt{s} = 14$ TeV based on 1.8×10^4 reweighted events. The uncertainties arise from statistical errors in phase space integration.

μ	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$ factor	0.958(1)	0.957(1)	0.954(1)

4-5% on inclusive cross section

10-15% on distributions
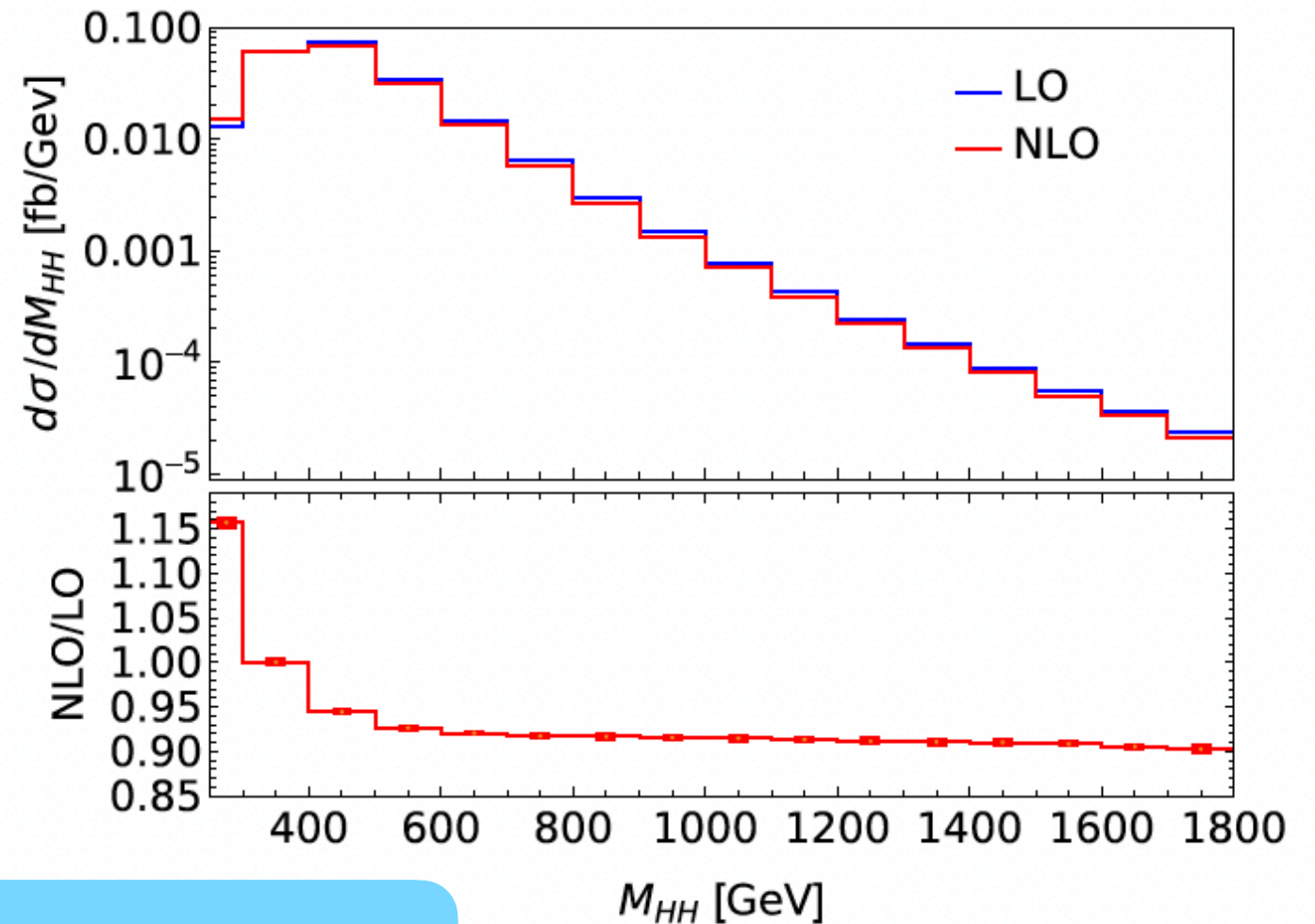
When targeting % precision, EW or mixed QCD-EW effects must be fully under control.

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Analytic versus numerical?

Rapid progress is happening on both the analytical and numerical fronts. Each has clear strengths and limitations — but their synergy is driving remarkable advances in precision calculations.

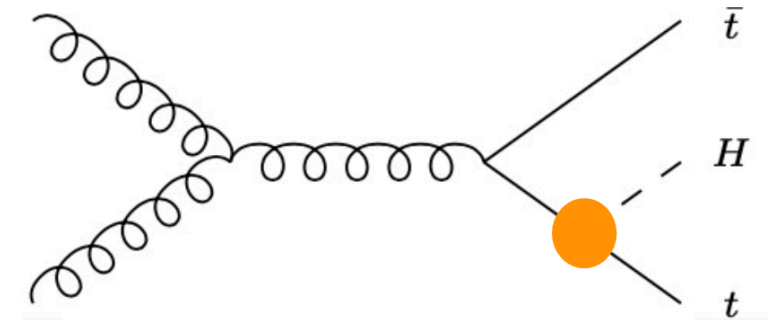| Analytic	Numerical
Fast, efficient	Slow, less efficient
Arbitrary precision	Limited precision (less so recently)
Complexity increases dramatically with the number of scales involved	More suitable for multi-loop and multi-scale problems
Analytic control of function (logs etc), allows insight into the result and control	Hard to extract analytic structure (limits, etc.)

Approximate ttH at NNLO

Catani et al., 2210.04846

Two-loop pp → ttH amplitudes still missing.

Idea: approximate with amplitudes with a soft Higgs emitted off heavy quarks

σ [fb]	$\sqrt{s} = 13\,\text{TeV}$		$\sqrt{s} = 100\,\text{TeV}$	
	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}}\big|_{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO,H}}\big|_{\text{soft}}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

Test the procedure at NLO

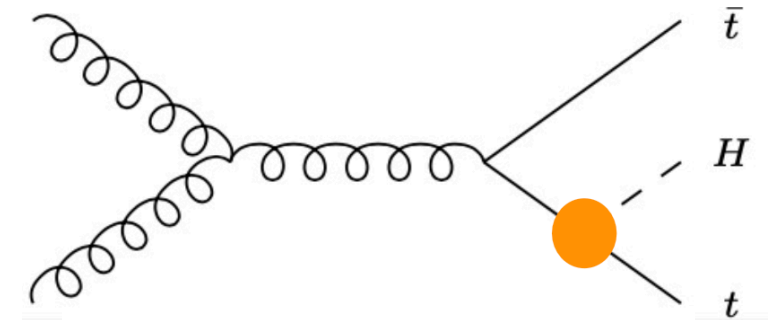▸approximation not that great and works better for qq then gg channel

Approximate ttH at NNLO

Catani et al., 2210.04846

Two-loop pp → ttH amplitudes still missing.

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$\Delta\sigma_{\mathrm{NNLO,H}}\big|_{\mathrm{soft}}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

Size of approx. NNLO

▸ but two-loop corrections are very small (below a %)

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Approximate ttH at NNLO

Catani et al., 2210.04846
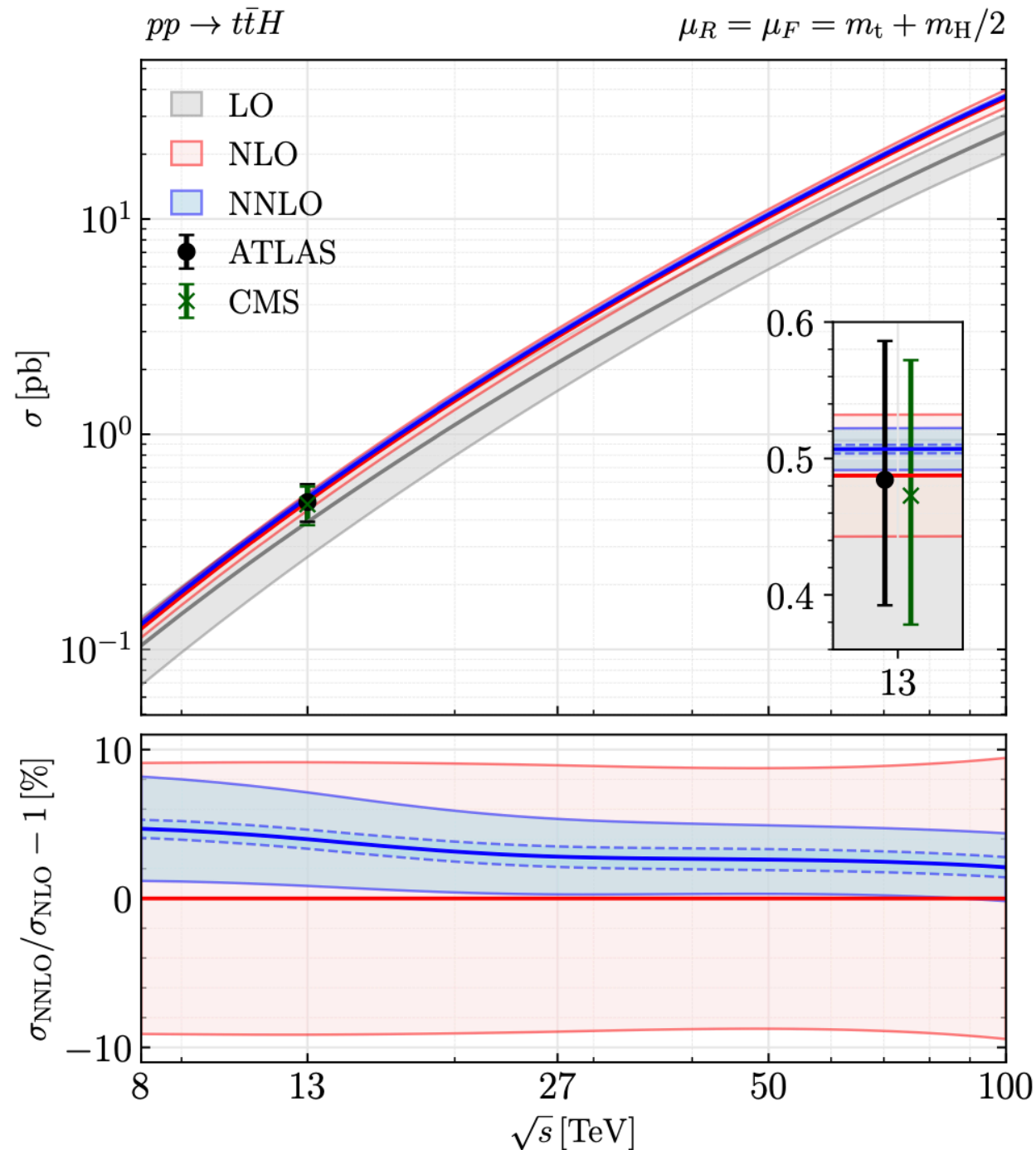
⇒ estimated uncertainty on the total cross section at the few percent level

Interesting to validate this once full NNLO is available

Approximate ttW at NNLO

Buonocore et al., 2306.16311

Two approximations ("soft W" and "massification") estimate missing two-loop ttW corrections, yielding 10% accuracy at NLO and similar results at NNLO, even outside their ideal validity.
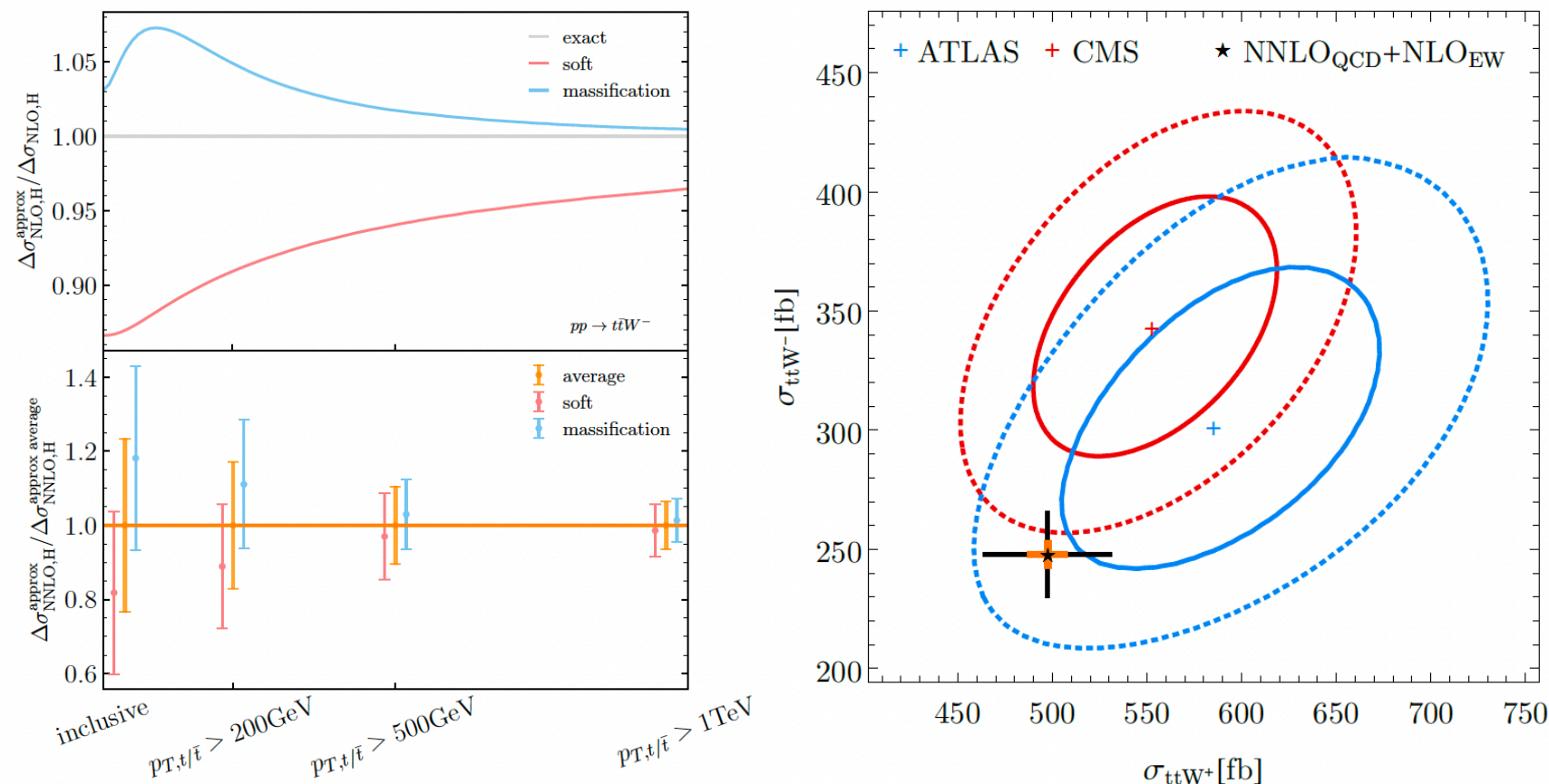
Clever approximations enable precision for key processes (ttH, ttW, ttZ, ...) crucial to constrain SM parameters (y_t, g_{ttZ}, ...).

N³LO status

DY-like (no color in final state)
$2 \rightarrow 1$

Higgs (TH, app.) C. Anastasiou, C. Duhr, F. Dulat, F. Herzog and B. Mistlberger

Higgs (VBF) F. A. Dreyer and A. Karlberg

Higgs (Diff in TH app.) F. Dulat, B. Mistlberger and A. Pelloni
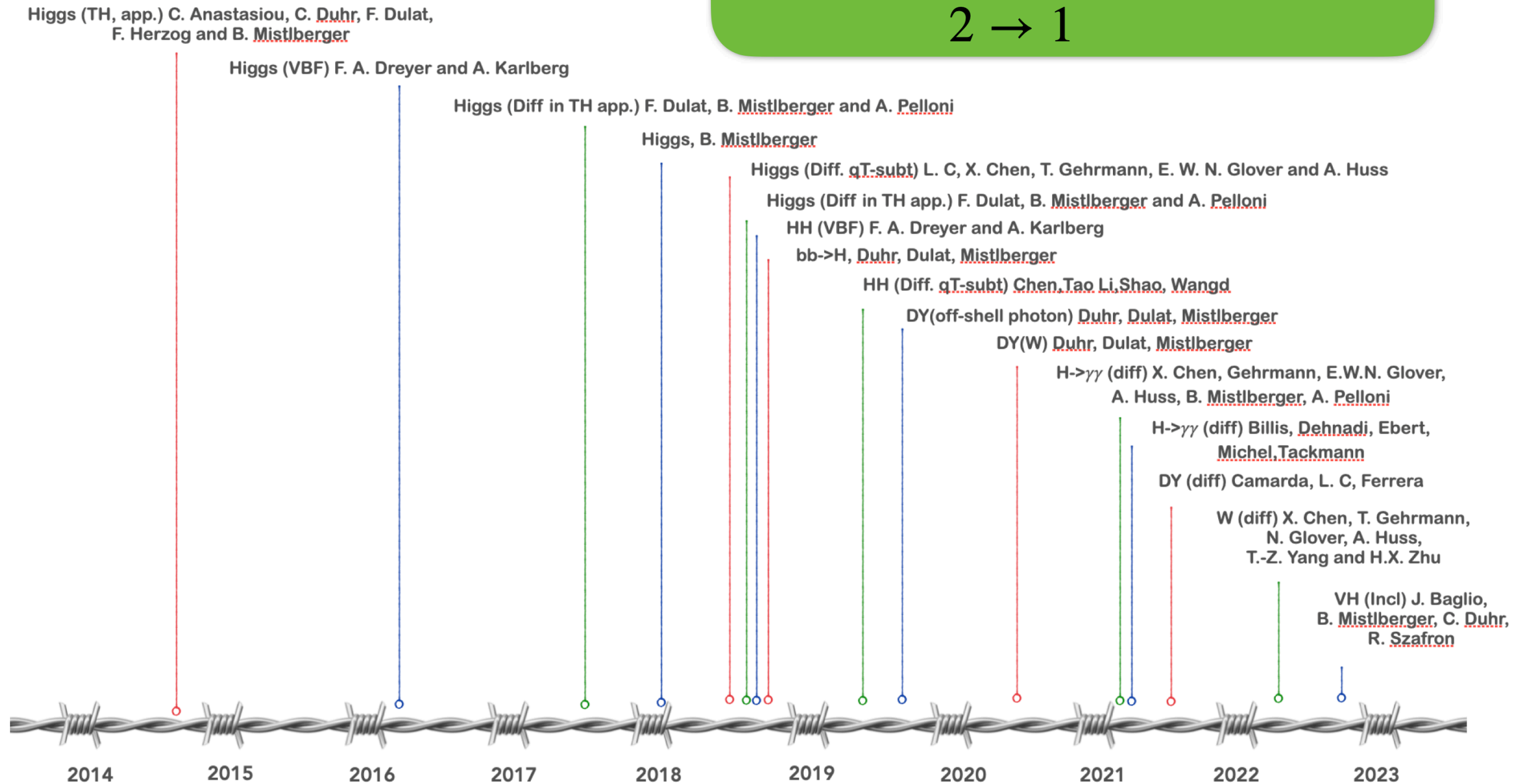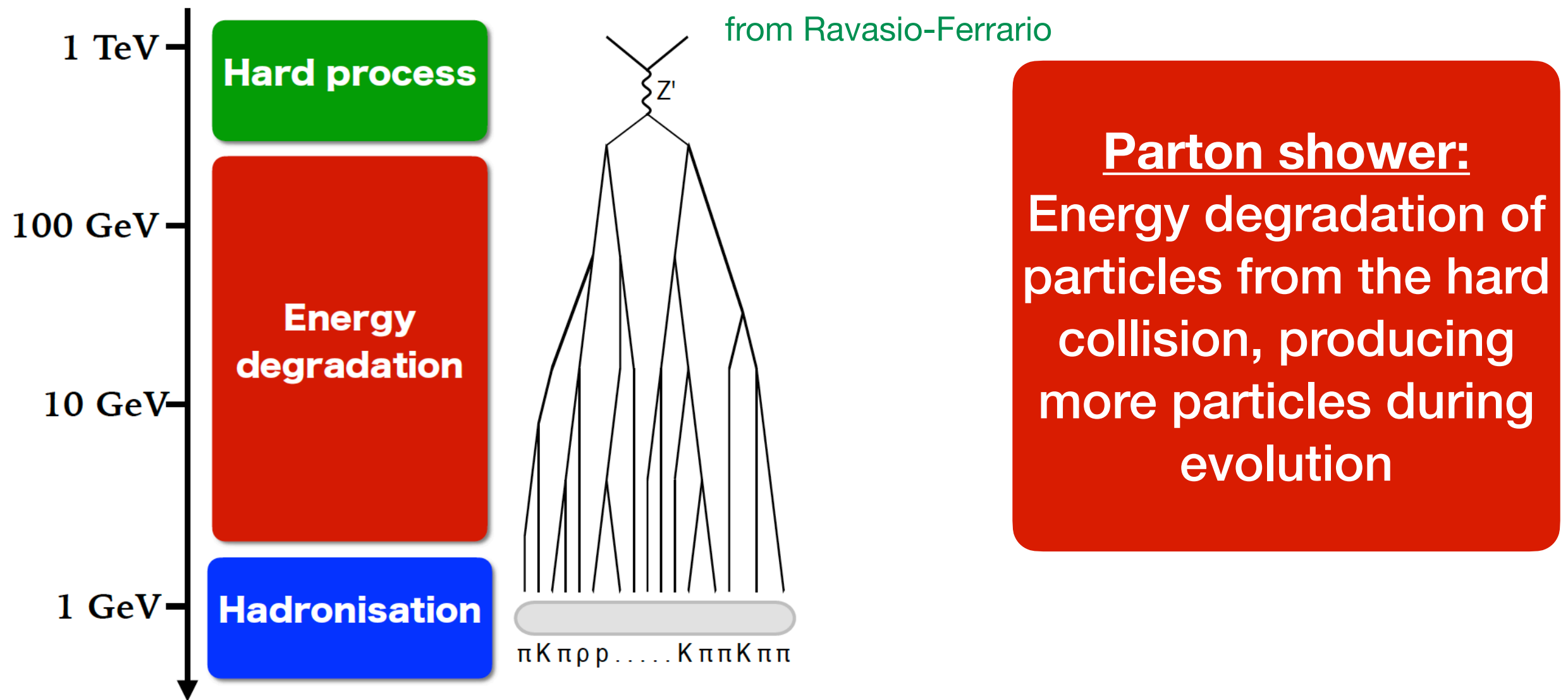
Higgs, B. Mistlberger

Higgs (Diff. qT-subt) L. C, X. Chen, T. Gehrmann, E. W. N. Glover and A. Huss

Higgs (Diff in TH app.) F. Dulat, B. Mistlberger and A. Pelloni

HH (VBF) F. A. Dreyer and A. Karlberg

bb->H, Duhr, Dulat, Mistlberger

HH (Diff. qT-subt) Chen,Tao Li,Shao, Wangd

DY(off-shell photon) Duhr, Dulat, Mistlberger

DY(W) Duhr, Dulat, Mistlberger

H->$\gamma\gamma$ (diff) X. Chen, Gehrmann, E.W.N. Glover, A. Huss, B. Mistlberger, A. Pelloni

H->$\gamma\gamma$ (diff) Billis, Dehnadi, Ebert, Michel,Tackmann

DY (diff) Camarda, L. C, Ferrera

W (diff) X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang and H.X. Zhu

VH (Incl) J. Baglio, B. Mistlberger, C. Duhr, R. Szafron

2014   2015   2016   2017   2018   2019   2020   2021   2022   2023

from L. Cieri

Parton shower

from Ravasio-Ferrario

1 TeV — **Hard process**

100 GeV —

Energy degradation

10 GeV —

1 GeV — **Hadronisation**

π K π ρ ρ . . . . . K π π K π π

Parton shower:
Energy degradation of particles from the hard collision, producing more particles during evolution
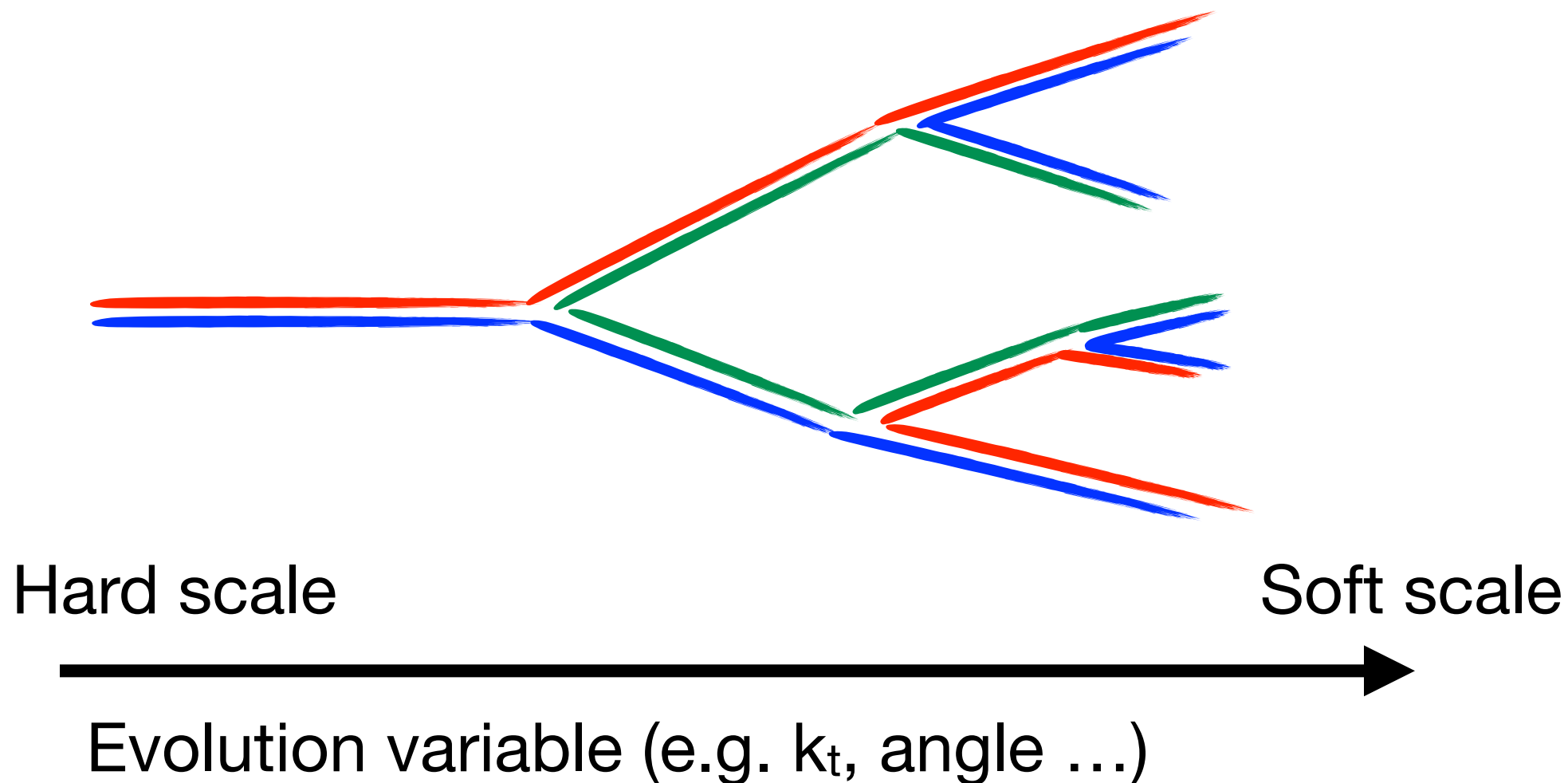
Parton showers are ubiquitous at the LHC

Modelling QCD processes, event simulation, background estimates, unfolding, detector simulation, …

Parton showers

Single algorithm for arbitrary observables: using recursively $1 \to 2$ splittings, parton showers describe highly complex final states

Hard scale   Soft scale

Evolution variable (e.g. k_t, angle …)

First parton showers in the '80s. Yet, for a very long time accuracy of parton showers not clear: uncertainty assessed by using different tools.

Marchesini, Webber '87; Bengtsson, Sjostrand '86

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Key elements of showers

Ingredients/ambiguities:

• Splitting kernels / matrix elements

• Ordering variable

• Recoil

• Vetoed region

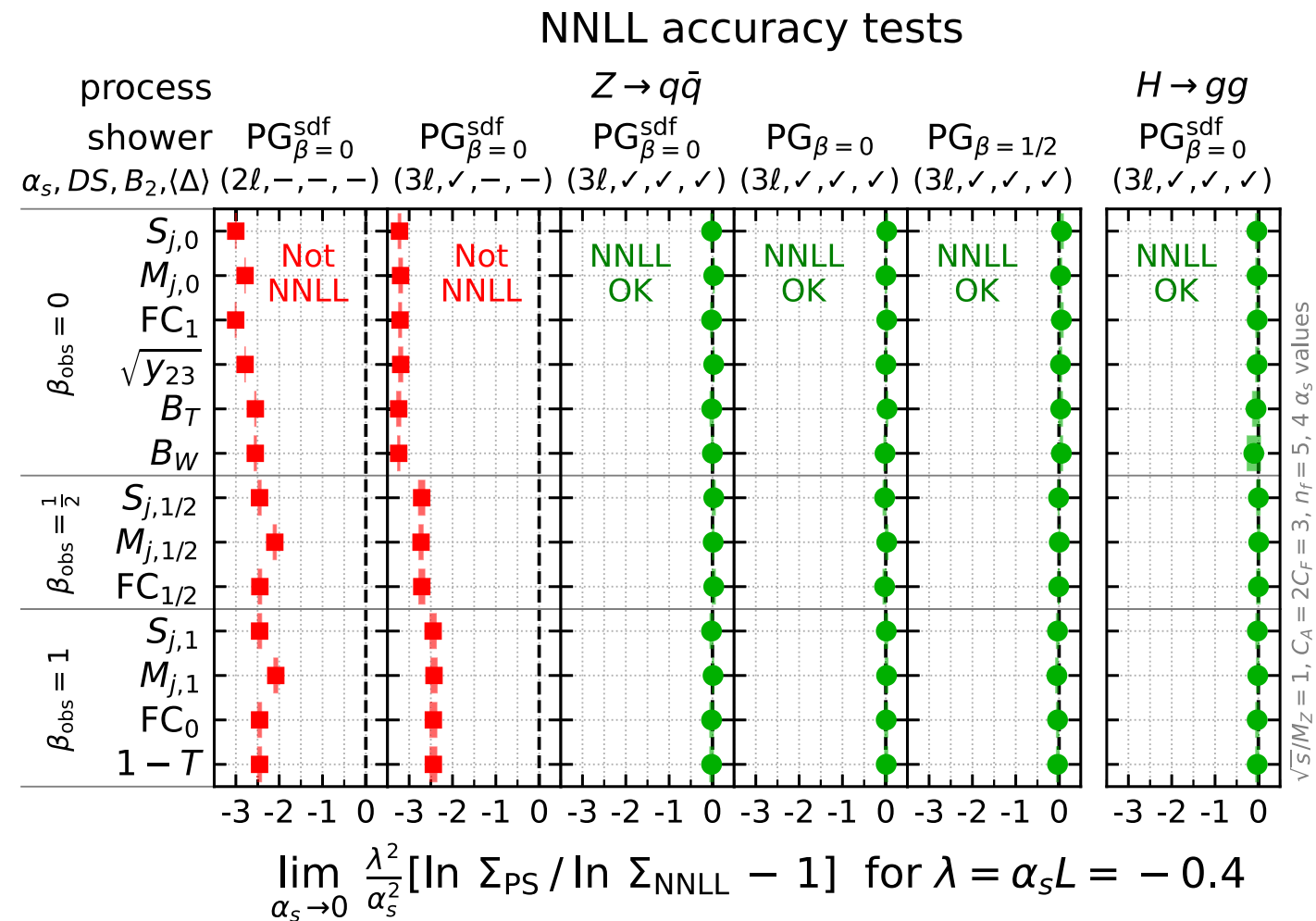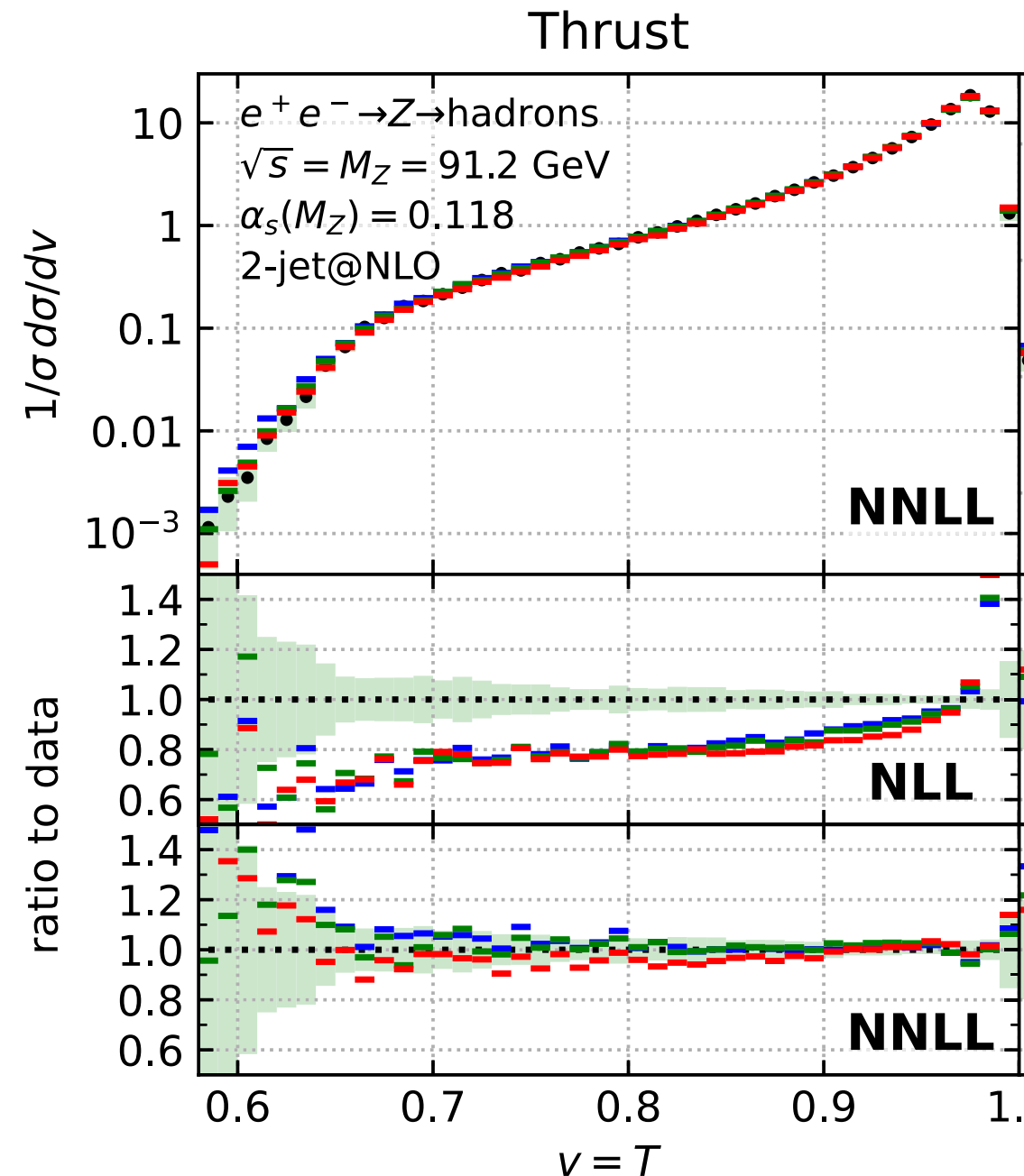• Colour/spin treatments

• ...

1

2
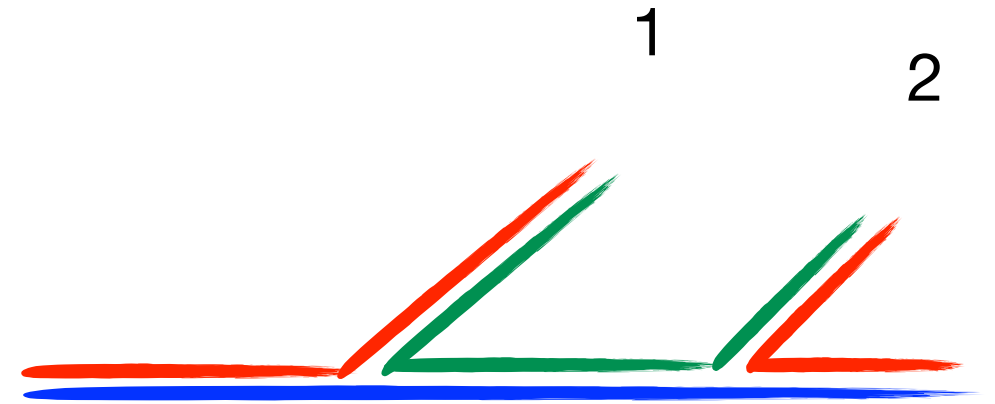
Dipole recoil in k_t-ordered shower: emission 2 takes recoil from 1. This is wrong in a large phase space region

Design principle for modern parton showers:

All elements of the shower should respect the absence of crosstalk between disparate scales, i.e. it should **respect QCD factorisation**

Modern parton showers

Thrust

$e^+e^- \to Z \to$ hadrons
$\sqrt{s} = M_Z = 91.2$ GeV
$\alpha_s(M_Z) = 0.118$
2-jet@NLO

NNLL accuracy tests

$$\lim_{\alpha_s \to 0} \frac{\lambda^2}{\alpha_s^2}[\ln \Sigma_{PS} / \ln \Sigma_{NNLL} - 1] \text{ for } \lambda = \alpha_s L = -0.4$$

Novel parton showers reaching NLL and even NNLL accuracy

Knowledge from resummations of event shapes crucial to achieve these goals and validate them

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Modern parton showers

The development of parton shower has seen a dramatic change in recent years. Key new elements of modern parton showers include

- Improvements in the accuracy of the parton shower

- Numerical procedure to validate the accuracy

- Understanding that some parton showers have lower accuracy $\Rightarrow$ can be disregarded when assessing theory uncertainties

ALARIC, DEDUCTOR, PANSCALES, HERWIG7, ...

A revolution in parton shower developments is ongoing. Will be crucial for Run 3, HL-LHC and FCC.

Parton shower matching

Different methods developed. NNLOPS with leading logarithmic accuracy in the shower well understood
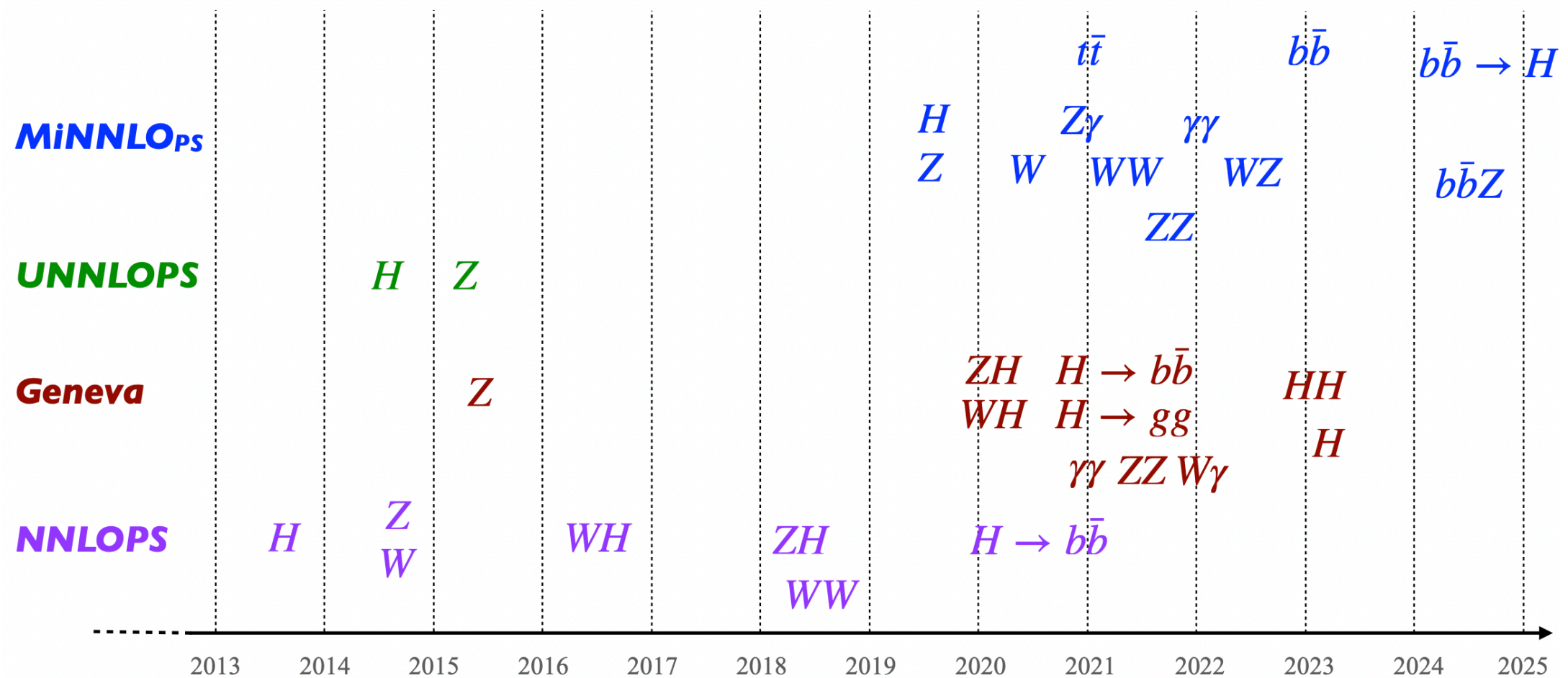
Not yet clear how to preserve accuracy of more accurate showers in the matching
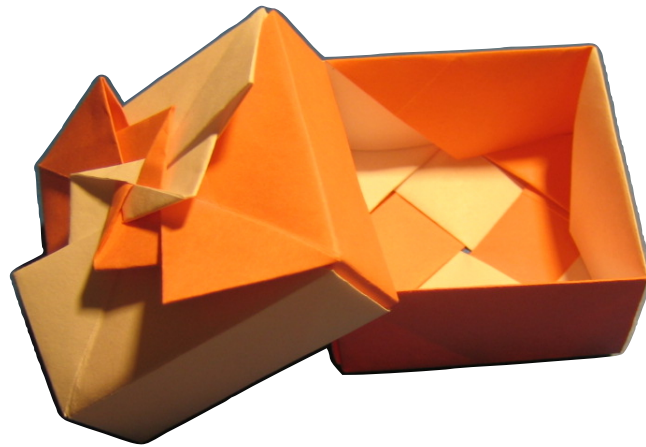
Art versus automation

Automation is key to making precision scalable and accessible, but it can foster the misconception that precision is merely about brute force. Brute force is insufficient for LHC precision. Deep understanding & breakthroughs prerequisite for practical automation.

PYTHIA

The POWHEG BOX

QCDLoop

Caravel

SHERPA

H7

MADGRAPH5 aMC@NLO

Conclusion

Progress beyond expectations $\Rightarrow$ remarkable success of theorists

- Progress not due to cranking old machinery but driven by new ideas and developments of new calculational methods

- Strong complementary between numerical and analytical methods

Many calculations eagerly awaited and in sight in the next five years

[in the meantime, clever and robust approximations reduce theory uncertainties]

Perturbative QCD well on track to keep up with experimental precision