# Single pion production from interactions with dark matter

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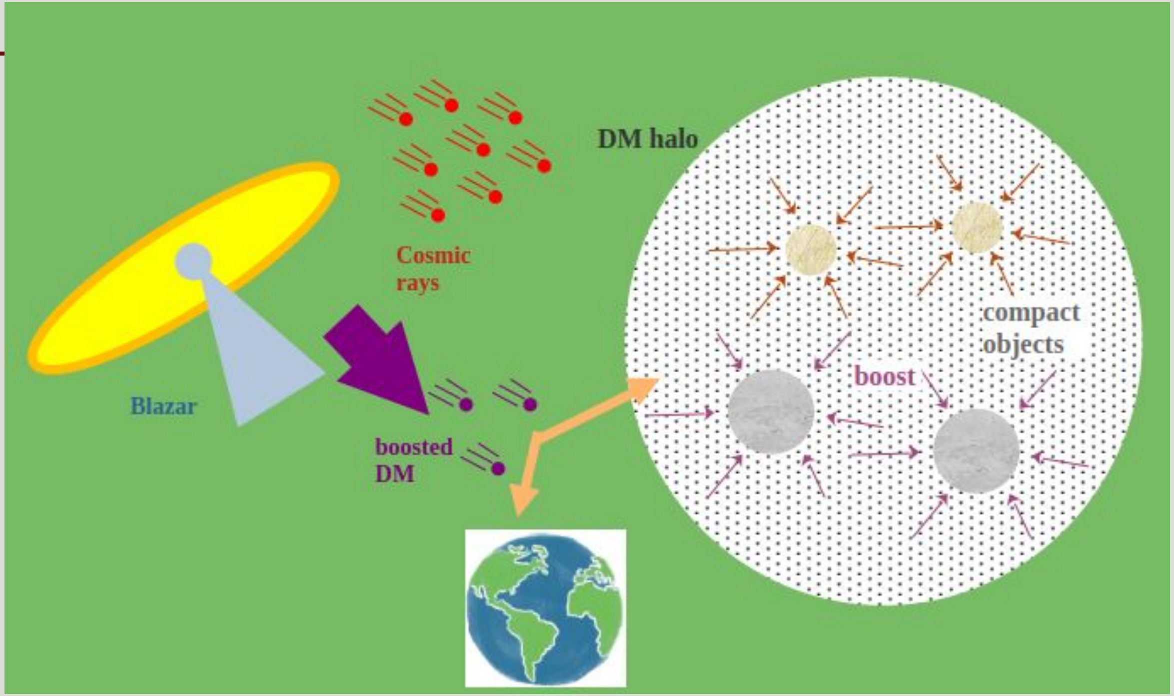
Based on PRD 111 (2025) 11, 115013 and JHEP 05 (2025) 160,
with Maura Ramírez-Quezada and Shihwen Hor

# Agenda

# Motivation

# Motivation

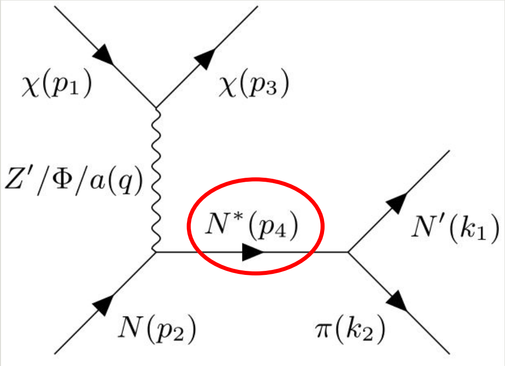
Astrophysical processes with DM can be highly energetic

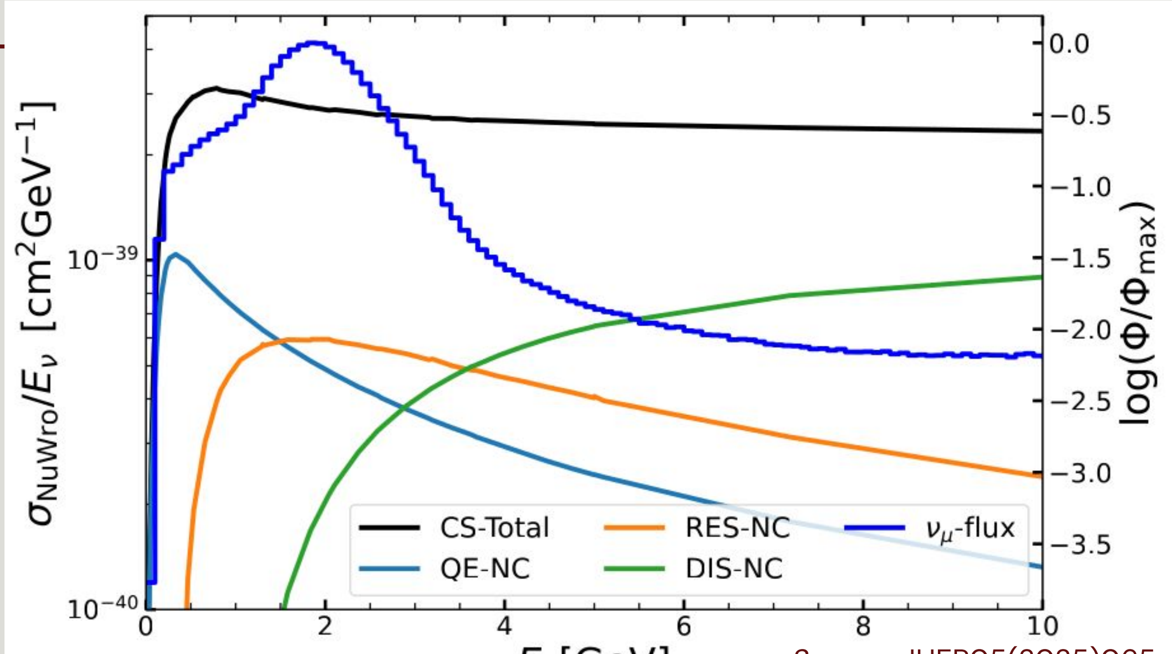


# Motivation

## Similar case in SM: ν NC cross sections

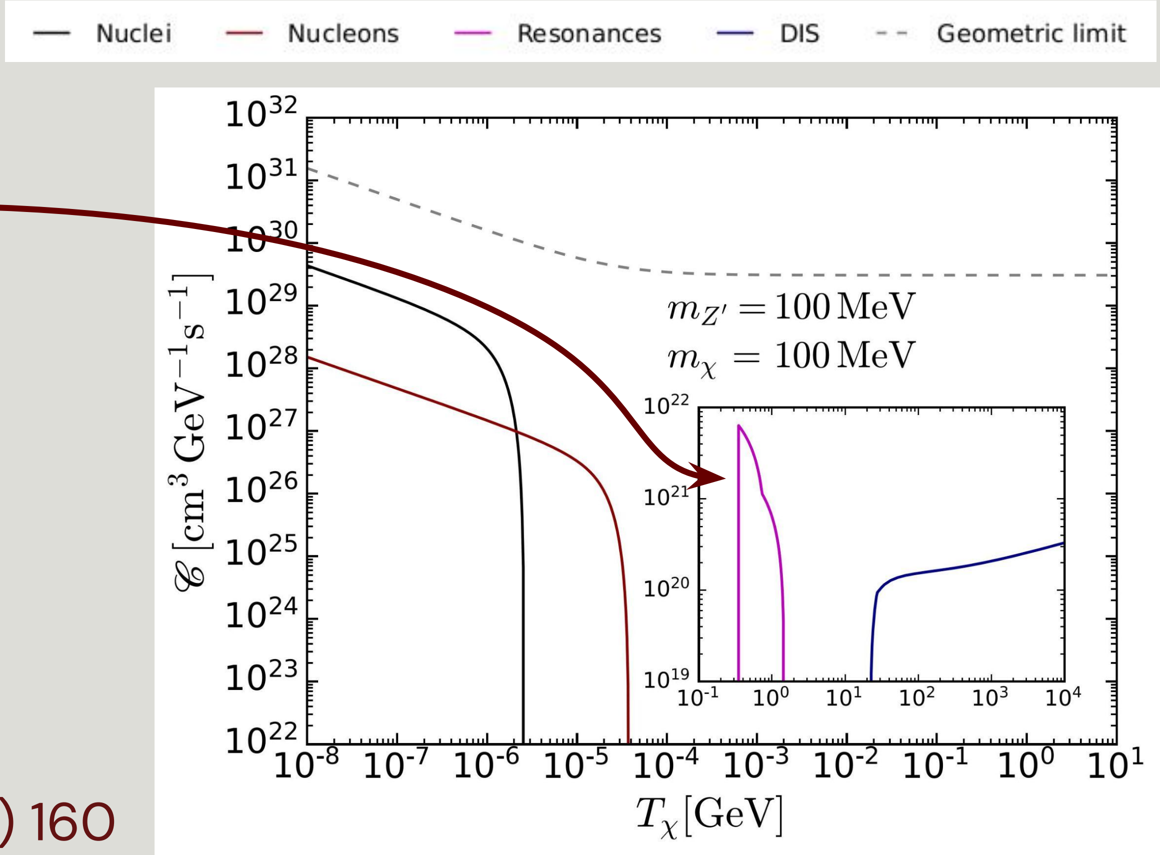
χ N → χ N* → χ N' π




*Source*: JHEP05(2025)065

RES




Capture rates
JHEP 05 (2025) 160

# Model: FKR & RS

# Brief historical account



**Current Matrix Elements from a Relativistic Quark Model***

R. P. Feynman, M. Kislinger, and F. Ravndal
*Lauritsen Laboratory of Physics, California Institute of Technology, Pasadena, California 91109*

(Received 17 December 1970)

A relativistic equation to represent the symmetric quark model of hadrons with harmonic interaction is used to define and calculate matrix elements of vector and axial-vector currents. Elements between states with large mass differences are too big compared to experiment, so a factor whose functional form involves one arbitrary constant is introduced to compensate this. The vector elements are compared with experiments on photoelectric meson production, $K_{l3}$ decay, and $\omega \to \pi\gamma$. Pseudoscalar-meson decay widths of hadrons are calculated supposing the amplitude is proportional (with one new scale constant) to the divergence of the axial-vector current matrix elements. Starting only from these two constants, the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{3}{4}$ agree with the experimental values within 40%. The problems of extending this calculational scheme to a viable physical theory are discussed.

**Feynman, Kislinger, Ravndal**. PRD, vol.3, n. 11, (1971)

IL NUOVO CIMENTO    VOL. 18 A, N. 3    1 Dicembre 1973

**Weak Production of Nuclear Resonances in a Relativistic Quark Model** (*).

F. RAVNDAL
*California Institute of Technology - Pasadena, Cal.*

(ricevuto il 29 Maggio 1973)

**Ravndal**. Il nuovo cimento, 18 A, 3 (1973)

ANNALS OF PHYSICS **133**, 79–153 (1981)

**Neutrino-Excitation of Baryon Resonances and Single Pion Production**

DIETER REIN AND LALIT M. SEHGAL
*III Physikalisches Institut, Technische Hochschule, Aachen, West Germany*

Received October 31, 1980

This is an attempt to describe all existing data on neutrino production of single pions in the resonance region up to $W = 2$ GeV in terms of the relativistic quark model of

**Rein, Sehgal**. Annals of Physics, 133, 79–153 (1981)

Modern Physics Letters A
Vol. 19, No. 38 (2004) 2815–2829
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**LEPTON POLARIZATION IN NEUTRINO–NUCLEON INTERACTIONS**

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Received 30 August 2004

We derive generic formulas for the polarization density matrix of leptons produced in

**Kuzmin, Lyubushkin, Naumov**. MPL A, 19, 38, 2815–2829 (2004)

PHYSICAL REVIEW D **76**, 113004 (2007)

**Lepton mass effects in single pion production by neutrinos**

Ch. Berger*
*I. Physikalisches Institut der RWTH, Aachen, Germany*

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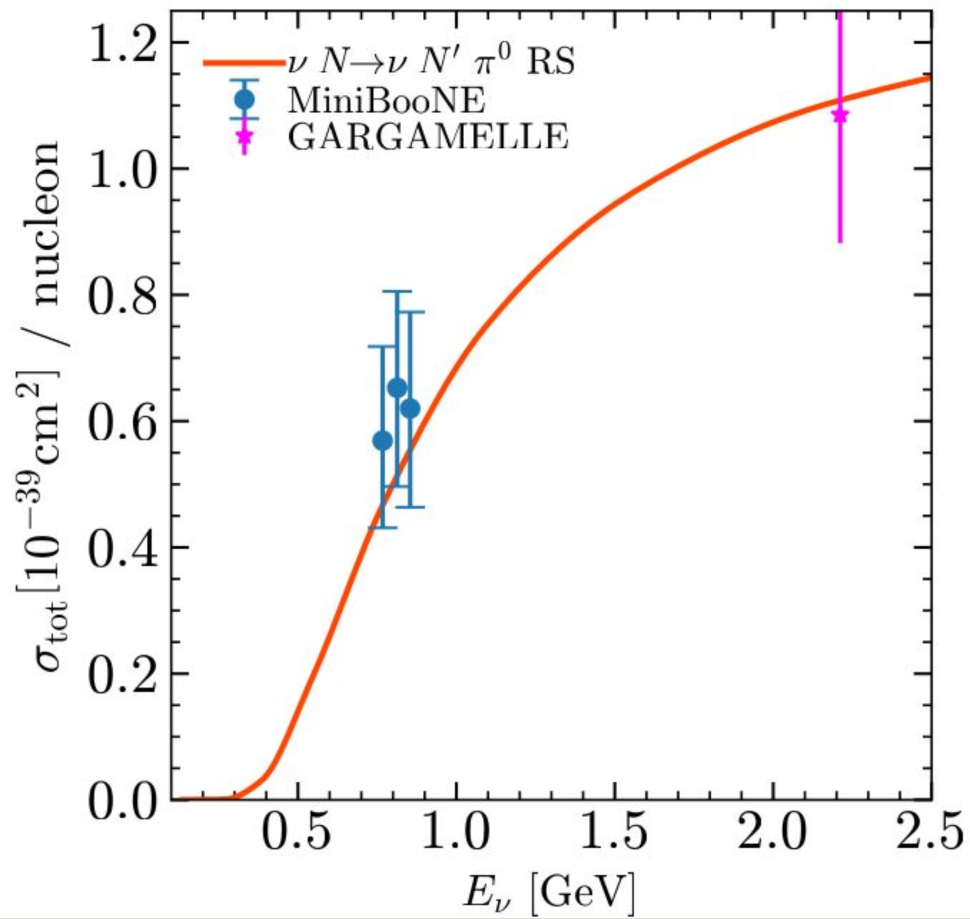
(Received 28 September 2007; published 27 December 2007)

We reconsider the Feynman-Kislinger-Ravndal model applied to neutrino-excitation of baryon resonances. The effects of lepton mass are included, using the formalism of Kuzmin, Lyubushkin, and Naumov. In addition we take account of the pion-pole contribution to the hadronic axial vector current. Application of this new formalism to the reaction $\nu_\mu + p \to \mu^- + \Delta^{++}$ at $E_\nu \sim 1$ GeV gives a suppressed cross section at small angles, in agreement with the screening correction in Adler's forward-scattering theorem. Application to the process $\nu_\tau + p \to \tau^- + \Delta^{++}$ at $E_\nu \sim 7$ GeV leads to the prediction of right-handed $\tau^-$ polarization for forward-going leptons, in line with a calculation based on an isobar model. Our formalism represents an improved version of the Rein-Sehgal model, incorporating lepton mass effects in a manner consistent with partially conserved axial-vector current.

DOI: 10.1103/PhysRevD.76.113004    PACS numbers: 13.15.+g, 11.40.Ha, 12.39.Ki

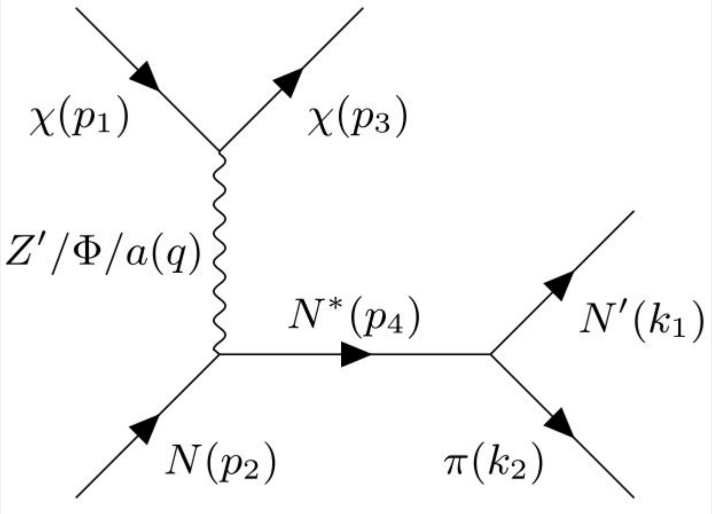
**Berger, Sehgal**. PRD, 76, 113004 (2007)

## Validity for neutrinos




## Let's use it for DM






**Channels**:

1. $\chi p \to \chi n \pi^+$
2. $\chi p \to \chi p \pi^0$
3. $\chi n \to \chi n \pi^0$
4. $\chi n \to \chi p \pi^-$

# Different scenarios

### Vector - axial

$$\mathcal{L}_I^{\mathrm{V-A}} = \sum_{f \in SM} g_{fZ'} \, \overline{f} \, \gamma^\mu \left( c_V^f - c_A^f \, \gamma^5 \right) f \, Z'_\mu$$
$$+ \, g_D \, \overline{\chi} \, \gamma^\mu \left( c_V^\chi - c_A^\chi \, \gamma^5 \right) \chi \, Z'_\mu \, ,$$

### Scalar

$$\mathcal{L}_I^{\mathrm{S}} = \sum_{f \in SM} g_{f\Phi} \, \overline{f} \, f \, \Phi \; + \; g_D \, \overline{\chi} \, \chi \, \Phi$$

### Pseudoscalar

$$\mathcal{L}_I^{\mathrm{P}} = - \, i \sum_{f \in SM} g_{fa} \overline{f} \gamma^5 f a - i g_D \overline{\chi} \gamma^5 \chi a$$

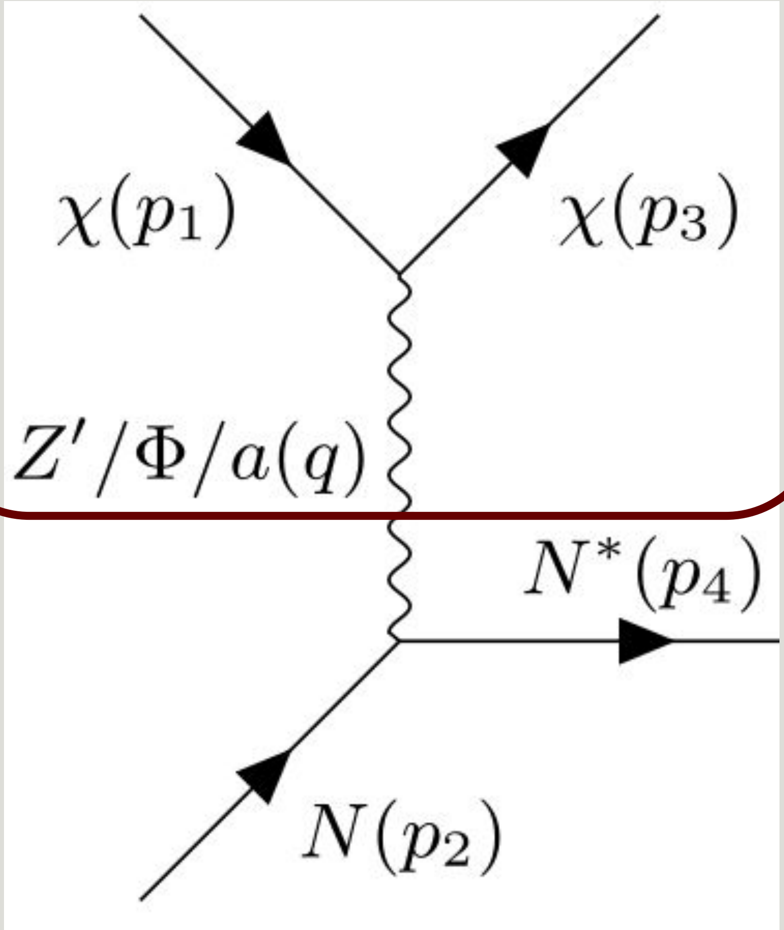
MEM = $V^{\mu}$

Matrix Element

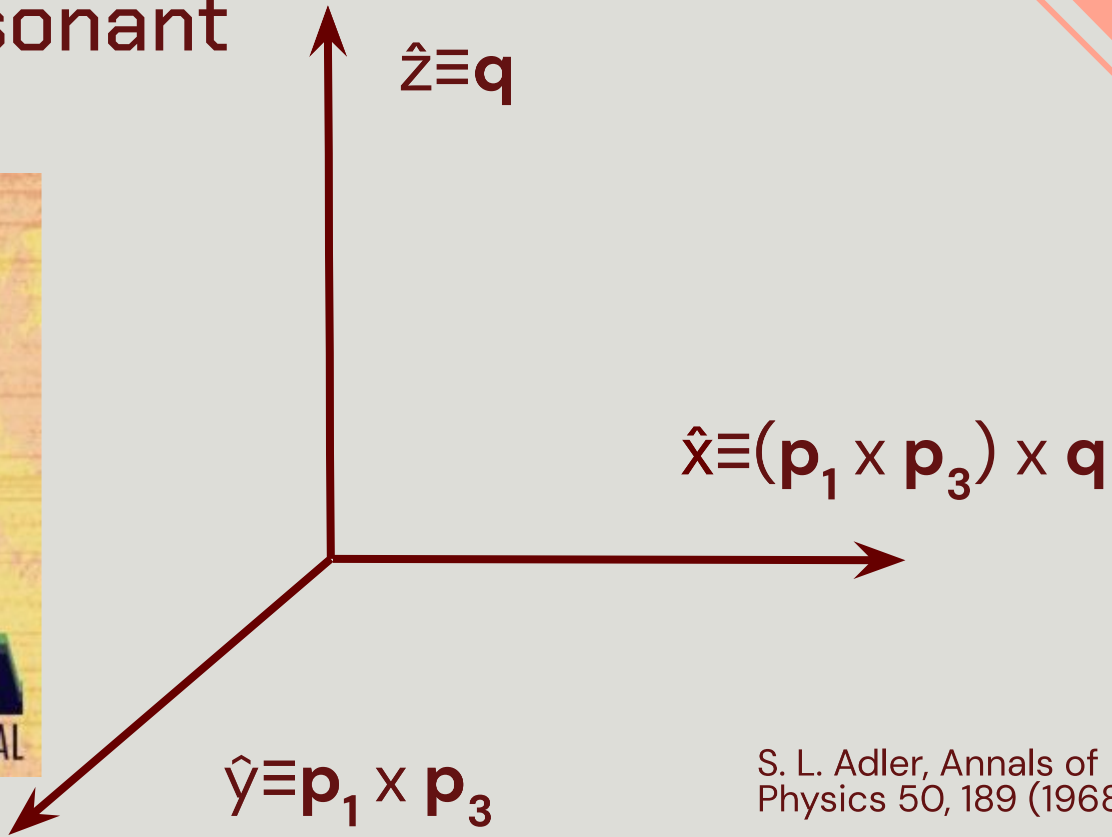
Mediator

$$V^{\nu}_{\lambda_1\lambda_2} \equiv \left[\bar{u}_{\chi,\lambda_2}\gamma_\mu\left(c_V^\chi - \gamma^5 c_A^\chi\right) u_{\chi,\lambda_1}\right]\left(g^{\mu\nu} - \frac{q^{\mathrm{IB}\mu}q^{\mathrm{IB}\nu}}{M_{Z'}^2}\right)$$

$$\mathcal{M}(\chi(p_1,\lambda_1)N(p_2) \rightarrow \chi(p_3,\lambda_2)N^*(p_4))$$
$$= \frac{g_D g_{NZ'}}{q^2 - m_{Z'}^2}\left[\overline{u}_{p_3\lambda_2}\gamma_\mu\left(c_V^\chi - c_A^\chi\gamma^5\right)u_{p_1\lambda_1}\right] \times$$
$$\times \left(g^{\mu\nu} - q^{\mathrm{IB}\mu}q^{\mathrm{IB}\nu}/m_{Z'}^2\right)\langle N^*|\, J_\nu^{V+}(0) - J_\nu^{A+}(0)\,|N\rangle$$
$$= 2M\frac{g_D g_{NZ'}}{q^2 - m_{Z'}^2}V^{\mu}_{\lambda_1\lambda_2}(\langle N^*|F_\mu^V|N\rangle - \langle N^*|F_\mu^A|N\rangle)$$



# Isobaric / Resonant frame


S. L. Adler, Annals of Physics 50, 189 (1968)

# Work separately for each polarization

$$e_L^\mu = \frac{1}{\sqrt{2}}(0, 1, -i, 0)\,,$$

$$e_R^\mu = \frac{1}{\sqrt{2}}(0, -1, -i, 0)\,,$$

$$e_s^\mu(\lambda_1\lambda_2) = \frac{(V^0_{\lambda_1\lambda_2}, 0, 0, V^3_{\lambda_1\lambda_2})}{\sqrt{\left|(V^0_{\lambda_1\lambda_2})^2 - (V^3_{\lambda_1\lambda_2})^2\right|}}$$

We will expand the MEM

$$V^\mu_{\lambda_1\lambda_2} = C_L^{\lambda_1\lambda_2} e_L^\mu$$

$$+ C_R^{\lambda_1\lambda_2} e_R^\mu$$

$$+ C_s^{\lambda_1\lambda_2} e_s^\mu(\lambda_1\lambda_2)$$

# Amplitude

$$\mathcal{M}(\chi(p_1,\lambda_1)N(p_2) \to \chi(p_3,\lambda_2)N^*(p_4)) =$$

$$2W\frac{g_D g_{NZ'}}{q^2 - m_{Z'}^2}\Big(\langle N^*|\, C_L F_-^V + C_R F_+^V + C_{\lambda_1\lambda_2} F_0^V \,|N\rangle$$

$$- \langle N^*|\, F_-^A C_L + F_+^A C_R + F_0^A C_{\lambda_1\lambda_2} \,|N\rangle\Big)$$

$$F_0^{\mathrm{RS}} \equiv F_0 \times \sqrt{\frac{-q^2}{|\vec{q}^{\,\mathrm{IB}}|^2}}$$

# Matrix Element Mediator

$$V^0_{\lambda_1\lambda_2} = \left(\left(1 - 2\delta^-_{\lambda_1}\delta^-_{\lambda_2}\right) c^\chi_V \Delta_{+,(\lambda_1\lambda_2)} - \sigma^{\delta^-_{\lambda_1\lambda_2}} c^\chi_A \Delta_{-,(\lambda_1\lambda_2)}\right) \sqrt{1 + \lambda_1\lambda_2\cos\delta},$$

$$V^1_{\lambda_1\lambda_2} = \left(1 - 2\delta^-_{\lambda_1}\delta^-_{\lambda_2}\right)\left(\sigma^{\delta^-_{\lambda_1\lambda_2}} c^\chi_V \Delta_{-,(\lambda_1\lambda_2)} - \left(1 - 2\delta^-_{\lambda_1}\delta^+_{\lambda_2}\right) c^\chi_A \Delta_{+,(\lambda_1\lambda_2)}\right) \frac{\left||\vec{p}_1^{\,\mathrm{IB}}| + \lambda_1\lambda_2|\vec{p}_3^{\,\mathrm{IB}}|\right|}{|\vec{q}^{\,\mathrm{IB}}|} \sqrt{1 - \lambda_1\lambda_2\cos\delta},$$

$$V^2_{\lambda_1\lambda_2} = i\left(\left(1 - 2\delta^+_{\lambda_1}\delta^-_{\lambda_2}\right) \sigma^{\delta^-_{\lambda_1\lambda_2}} c^\chi_V \Delta_{-,(\lambda_1\lambda_2)} - c^\chi_A \Delta_{+,(\lambda_1\lambda_2)}\right) \sqrt{1 - \lambda_1\lambda_2\cos\delta},$$

$$V^3_{\lambda_1\lambda_2} = \left(1 - 2\delta^+_{\lambda_1}\delta^+_{\lambda_2}\right)\left(\sigma^{\delta^-_{\lambda_1\lambda_2}} c^\chi_V \Delta_{-,(\lambda_1\lambda_2)} - \left(1 - 2\delta^-_{\lambda_1}\delta^+_{\lambda_2}\right) c^\chi_A \Delta_{+,(\lambda_1\lambda_2)}\right) \frac{\left||\vec{p}_1^{\,\mathrm{IB}}| - \lambda_1\lambda_2|\vec{p}_3^{\,\mathrm{IB}}|\right|}{|\vec{q}^{\,\mathrm{IB}}|} \sqrt{1 + \lambda_1\lambda_2\cos\delta},$$

$$\Delta_{\alpha,\beta} \equiv \sqrt{E_1^{\mathrm{IB}} E_3^{\mathrm{IB}} + \alpha\, m_\chi^2 + \beta\, |\vec{p}_1^{\,\mathrm{IB}}|\, |\vec{p}_3^{\,\mathrm{IB}}|}$$

$$\sigma \equiv \mathrm{sgn}(q^2 + m_N q_0)$$

# Differential cross section

$$\frac{d^2\sigma}{dq^2 dW} = \frac{g_D^2 g_{NZ'}^2}{4\pi^2 \left(q^2 - m_{Z'}^2\right)^2} \frac{W}{m_N} \sum_{\lambda_1\lambda_2} \left( \left|C_L^{\lambda_1\lambda_2}\right|^2 \sigma_L^{\lambda_1\lambda_2} + \left|C_R^{\lambda_1\lambda_2}\right|^2 \sigma_R^{\lambda_1\lambda_2} + \left|C_s^{\lambda_1\lambda_2}\right|^2 \sigma_s^{\lambda_1\lambda_2} \right) \delta(W - M)$$

$$\sigma_R^{\lambda_1\lambda_2} = K \sum_{j_z} \left| \sum_{k=V,A} s_k \langle N, j_z + 1 | F_+^k | N^*, j_z \rangle \right|^2 ,$$

$$\sigma_L^{\lambda_1\lambda_2} = K \sum_{j_z} \left| \sum_{k=V,A} s_k \langle N, j_z - 1 | F_-^k | N^*, j_z \rangle \right|^2 ,$$

$$\sigma_s^{\lambda_1\lambda_2} = K \sum_{j_z} \left| \sum_{k=V,A} s_k \langle N, j_z | F_{0\pm\lambda_1\lambda_2}^k | N^*, j_z \rangle \right|^2 ,$$

$$K \equiv \frac{\pi}{16} \frac{W}{m_N |\vec{p}_{\mathrm{IB},1}|^2}$$

$$s_V = 1, \; s_A = -1$$

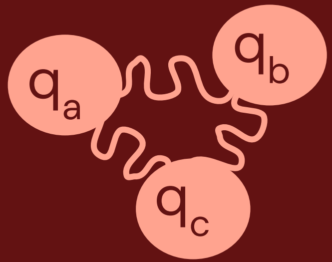
$$f_{\pm|2j_z|}^{V(A)} \equiv \langle N, j_z \pm 1 | F_\pm^{V(A)} | N^*, j_z \rangle ,$$

$$f_{0\pm\lambda_1\lambda_2}^{V(A)} \equiv \langle N, \pm 1/2 | F_{0\pm\lambda_1\lambda_2}^{V(A)} | N^*, \pm 1/2 \rangle .$$

# FKR model

# Model for baryon dynamics

Harmonic oscillators




$$r_a, r_b, r_c \to R_{CM}, \mathbf{x}, \mathbf{y}$$

$$p^2 \equiv \not{p}\not{p}$$

$$H = \frac{1}{2m} P^2 + \tfrac{1}{2} m \omega_0{}^2 X^2 .$$

Multiply by $2m$ and set $m^2 \omega_0{}^2 = \Omega^2$, to get

$$2mH = P^2 + \Omega^2 X^2 ,$$

$$\mathfrak{N} = -\Omega(a_\mu^* a_\mu + b_\mu^* b_\mu) + C$$

from Regge trajectories

$$\not{p}_\alpha \to \not{p}_\alpha - e_\alpha \not{A}(u_\alpha) \longrightarrow j_\mu^V = 3 \sum_\alpha e_\alpha (\not{p}_\alpha \gamma_\mu e^{i q \cdot u_\alpha} + \gamma_\mu e^{i q \cdot u_\alpha} \not{p}_\alpha)$$

$$\not{p}_\alpha \to \not{p}_\alpha - e'_\alpha \gamma_5 \not{B}(u_\alpha) \longrightarrow j_\mu^A = 3i \sum_\alpha e'_\alpha (\not{p}_\alpha \gamma_5 \gamma_\mu e^{i q \cdot u_\alpha} + \gamma_5 \gamma_\mu e^{i q \cdot u_\alpha} \not{p}_\alpha)$$

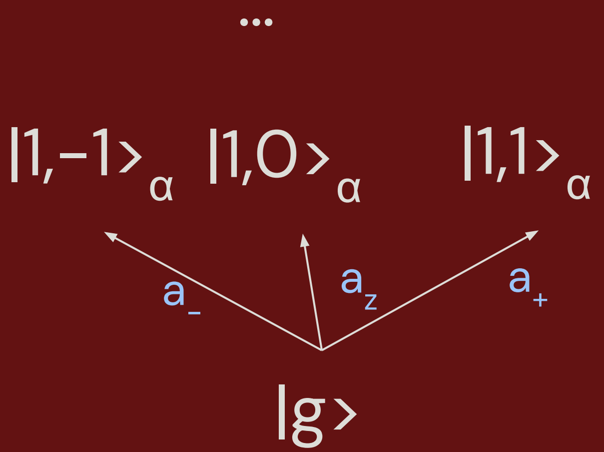
# SU(6) representations

$$^{2s}(G_{SU(3)})_J\,[G_{SU(6)}, L^P]_N$$

$$6\otimes6\otimes6 = 56 \oplus 70_\alpha \oplus 70_\beta \oplus 20$$

**([u,d,s] x [½,-½])**

- **56**: $^2(8)$, $^4(10)$
- **70**: $^2(1)$, $^2(8)$, $^4(8)$, $^2(10)$
- **20**: $^4(1)$, $^2(8)$



# Operators

$$F_{\pm}^{V} = -9e_a\left(R^V\sigma_{\pm} + T^V a_{\mp}\right)e^{-\lambda a_z},$$
$$F_{0\pm}^{V,\lambda_1\lambda_2} = 9e_a S_{\lambda_1\lambda_2} e^{-\lambda a_z},$$

$$\lambda \equiv \sqrt{\frac{2}{\Omega}\frac{m_N}{W}}|\vec{q}|,$$
$$R^V \equiv \frac{\sqrt{2}m_N}{W}\frac{(W+m_N)|\vec{q}|}{(W+m_N)^2-q^2}F^V(q^2),$$
$$T^V \equiv \frac{\sqrt{\Omega}}{3\sqrt{2}W}F^V(q^2),$$
$$S_{\lambda_1\lambda_2} \equiv \frac{m_N}{W|\vec{q}|}\frac{V^3_{\lambda_1\lambda_2}q^0_{\mathrm{IB}} - V^0_{\lambda_1\lambda_2}|\vec{q}_{\mathrm{IB}}|}{C_s^{\lambda_1\lambda_2}} \times\left(\frac{1}{6}-\frac{q^2}{6m_N^2}-\frac{W}{2m_N}\right)F^V(q^2).$$

$F^V$, $F^A$
$\sim \exp(q^2/\Omega)$

$$F_{\pm}^{A} = -9e_a\left(R^A\sigma_{\pm} + T^A a_{\mp}\right)e^{-\lambda a_z},$$
$$F_{0\pm}^{A,\lambda_1\lambda_2} = -9e_a\left[C_{\lambda_1\lambda_2}\sigma_z + B_{\lambda_1\lambda_2}\vec{\sigma}\cdot\vec{a}\right]e^{-\lambda a_z},$$

$$R^A \equiv \frac{\sqrt{2}Z}{6W}\left(W+m_N+\frac{2N\Omega W}{(W+m_N)^2-q^2}\right)F^A(q^2),$$
$$T^A \equiv \frac{\sqrt{2\Omega}Z}{3}\frac{m_N|\vec{q}|}{W((W+m_N)^2-q^2)}F^A(q^2),$$
$$B_{\lambda_1\lambda_2} \equiv \frac{1}{C_s^{\lambda_1\lambda_2}}\sqrt{\frac{\Omega}{2}}\left(V^0_{\lambda_1\lambda_2}+V^3_{\lambda_1\lambda_2}\frac{|\vec{q}^{\,\mathrm{IB}}|}{am_N}\right)Z\frac{F^A(q^2)}{3W},$$
$$C_{\lambda_1\lambda_2} \equiv \frac{1}{C_s^{\lambda_1\lambda_2}}\left[(V^0_{\lambda_1\lambda_2}|\vec{q}^{\,\mathrm{IB}}|-V^3_{\lambda_1\lambda_2}q_0^{\mathrm{IB}})\left(\frac{1}{3}+\frac{q_0^{\mathrm{IB}}}{am_N}\right) +V^3_{\lambda_1\lambda_2}\left(\frac{2}{3}W+\frac{q^2}{am_N}+\frac{N\Omega}{3am_N}\right)\right]\frac{Z}{2W}F^A(q^2),$$
$$a \equiv 1+\frac{W^2+m_N^2-q^2}{2m_N W}.$$

| Resonance | $f_{2s}$ | Vector | Axial | Scalar | Pseudoscalar |
|---|---|---|---|---|---|
| $P_{33}(1232)^4(10)_{3/2}[56,0^+]_0$ | $f_{-3}$ | $\sqrt{6}g_1^V R^V$ | $\sqrt{6}g_1^A R^A$ | | |
| | $f_{-1}$ | $\sqrt{2}g_1^V R^V$ | $\sqrt{2}g_1^A R^A$ | | |
| | $f_{+1}$ | $-\sqrt{2}g_1^V R^V$ | $\sqrt{2}g_1^A R^A$ | | |
| | $f_{+3}$ | $-\sqrt{6}g_1^V R^V$ | $\sqrt{6}g_1^A R^A$ | | |
| | $f_{0^+}$ | 0 | $-2\sqrt{2}g_1^A C$ | 0 | $\frac{2\sqrt{2}}{9}g_1^P S^P$ |
| | $f_{0^-}$ | 0 | $-2\sqrt{2}g_1^A C$ | 0 | $\frac{2\sqrt{2}}{9}g_1^P S^P$ |
| $P_{11}(1440)^2(8)_{1/2}[56,0^+]_2$ | $f_{-1}$ | $-\frac{1}{2\sqrt{3}}g_4^V\lambda^2 R^V$ | $-\frac{1}{2\sqrt{3}}g_4^A\lambda^2 R^A$ | | |
| | $f_{+1}$ | $-\frac{1}{2\sqrt{3}}g_4^V\lambda^2 R^V$ | $\frac{1}{2\sqrt{3}}g_4^A\lambda^2 R^A$ | | |
| | $f_{0^+}$ | $-\frac{\sqrt{3}}{2}g_5^V\lambda^2 S$ | $\frac{1}{2\sqrt{3}}g_4^A\lambda(\lambda C-2B)$ | $-\frac{1}{6\sqrt{3}}g_5^S\lambda^2 S^S$ | $-\frac{1}{18\sqrt{3}}g_4^P\lambda S^P$ |
| | $f_{0^-}$ | $-\frac{\sqrt{3}}{2}g_5^V\lambda^2 S$ | $-\frac{1}{2\sqrt{3}}g_4^A\lambda(\lambda C-2B)$ | $-\frac{1}{6\sqrt{3}}g_5^S\lambda^2 S^S$ | $\frac{1}{18\sqrt{3}}g_4^P\lambda S^P$ |
| $D_{13}(1520)^2(8)_{3/2}[70,1^-]_1$ | $f_{-3}$ | $-\frac{3}{\sqrt{2}}g_1^V T^V$ | $-\frac{3}{\sqrt{2}}g_1^A T^A$ | | |
| | $f_{-1}$ | $-\sqrt{\frac{3}{2}}g_1^V T^V+\frac{1}{\sqrt{3}}g_2^V\lambda R^V$ | $-\sqrt{\frac{3}{2}}g_1^A T^A+\frac{1}{\sqrt{3}}g_2^A\lambda R^A$ | | |
| | $f_{+1}$ | $-\sqrt{\frac{3}{2}}g_1^V T^V+\frac{1}{\sqrt{3}}g_2^V\lambda R^V$ | $\sqrt{\frac{3}{2}}g_1^A T^A-\frac{1}{\sqrt{3}}g_2^A\lambda R^A$ | | |
| | $f_{+3}$ | $-\frac{3}{\sqrt{2}}g_1^V T^V$ | $\frac{3}{\sqrt{2}}g_1^A T^A$ | | |

And so on... 18 in total

# Decay to N’π

## Nearly on-shell resonances

# Full transformation

$$\mathcal{M} \to a_{\mathrm{CG}} \times \mathcal{M} \times \eta_r(W) \times \sqrt{x_E^r} \times \mathrm{sgn}(N_r^*)$$

Clebsch-Gordan coefficient for decay channel

Breit-Wigner factor

Branching ratio for decay into Nπ

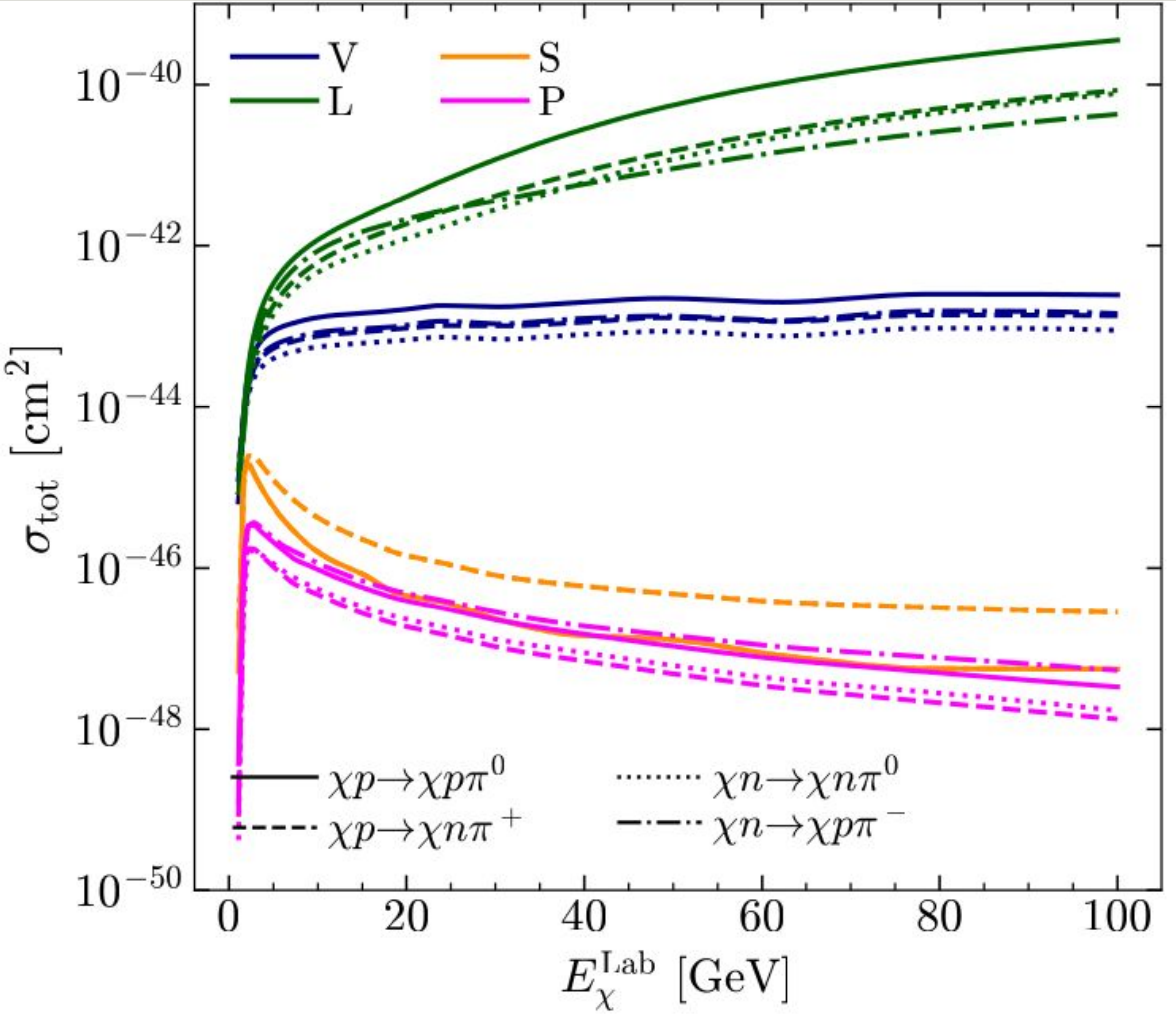
sign of decay (relevant for interferences)

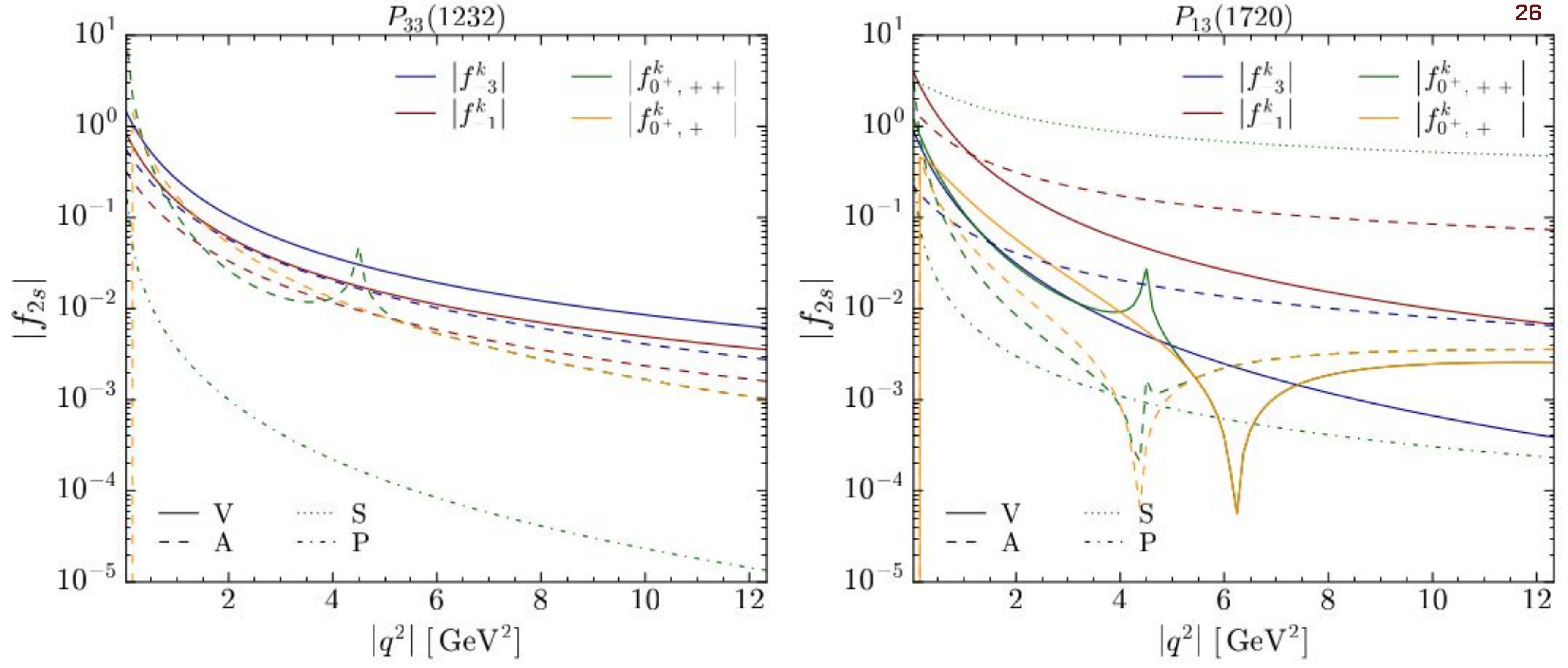
# Some plots

$m_{Z'/\varphi/a} = 1$ GeV
$m_\chi = 0.5$ GeV
$g_D = 0.1$
$g_{NZ'/\varphi} = 10^{-6}$
$g_{Na} = 10^{-5}$

**Vector**
$g_u^V = 2g_d^V = 2$

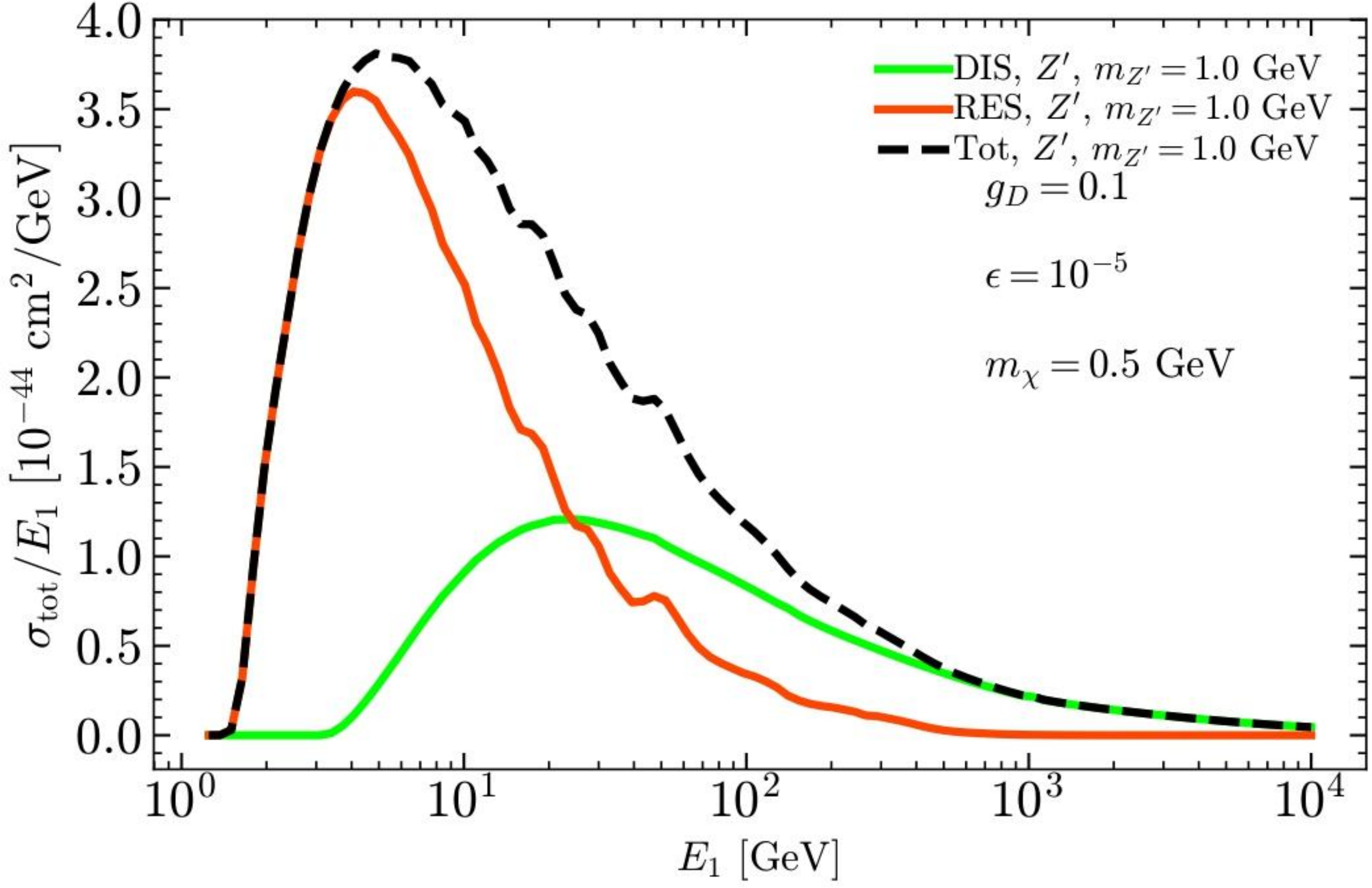
**Left**
$g_u^V = 2g_d^V = 1$






$E_\chi$ = 10 GeV

W = $(W_{min} + W_{max})/2 \sim 2.5$ GeV



DIS, $Z'$, $m_{Z'} = 1.0$ GeV
RES, $Z'$, $m_{Z'} = 1.0$ GeV
Tot, $Z'$, $m_{Z'} = 1.0$ GeV
$g_D = 0.1$
$\epsilon = 10^{-5}$
$m_\chi = 0.5$ GeV
$\sigma_{\rm tot}/E_1$ [$10^{-44}$ cm$^2$/GeV]
$E_1$ [GeV]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$

# Conclusions

# Conclusions

| | |
|---|---|
| R&S model (and FKR) gives us an interesting and productive framework to compute single pion resonant production via DM interactions with ordinary matter. | There are approaches that take into account a better structured group of form factors and treat the 3/2 fermions in a more proper way, using the Rarita-Schwinger formalism. |
| Although the model is based on a simplified effective approach, it adapts well to BSM scenarios at (any?) energy range. | Other more modern approaches sometimes rely more strongly on experimental results (fits) and are suitable for some energy ranges (tested, for example, for neutrinos). |
| With the FKR model, we can model other kinds of interactions, with different Lorentz structures. | The scalar and pseudoscalar cases are built from Ansätze. |
| The model is especially valid for interactions in astrophysical scenarios, where uncertainties are not small. | Terrestrial probes of the model might be possible, but must be taken as mere probes. |

# Extra material

“A truly relativistic quantum-mechanical theory today seems available only in the complexities of field theory with its many virtual states involving, for example, pairs, etc. It is so complex that no particular dynamic regularities among the resonances are expected of it, other than those resulting from symmetries of the original Hamiltonian. We have gone in a different direction, sacrificing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and in which we can calculate as many things as possible —not expecting the results to agree exactly with experiment, but to see how closely our "shadow of the truth" equation gives a partial reflection of reality.” (FKR, 2706)

$$p_1 = (E_1, 0, 0, |\vec{p}_1|)\,,$$
$$p_2 = (m_N, 0, 0, 0)\,,$$
$$p_3 = (E_3, |\vec{p}_3| \sin\theta, 0, |\vec{p}_3| \cos\theta)\,,$$
$$p_4 = p_1 + p_2 - p_3,$$
$$q = p_1 - p_2\,.$$

$$\Gamma = R^{xy}(\theta_2)\, B^{z}(\chi)\, R^{xz}(\theta_1)$$

$$p_1^{\rm IB} = \left(\frac{(E_1 - E_3)^2 + 2m_N E_1 - Q^2}{2W}, A_{13}, 0, B_{13}^-\right),$$
$$p_2^{\rm IB} = \left(\frac{m_N(E_1 - E_3 + m_N)}{W}, 0, 0, -\frac{m_N Q}{W}\right),$$
$$p_3^{\rm IB} = \left(-\frac{(E_1 - E_3)^2 - 2m_N E_3 - Q^2}{2W}, A_{13}, 0, B_{13}^+\right),$$
$$p_4^{\rm IB} = (W, 0, 0, 0)\,,$$
$$A_{13} = \frac{\sqrt{(|\vec{p}_1| + |\vec{p}_3| - Q)\,(|\vec{p}_1| - |\vec{p}_3| + Q)\,(-|\vec{p}_1| + |\vec{p}_3| + Q)\,(|\vec{p}_1| + |\vec{p}_3| + Q))}}{2Q},$$
$$B_{13}^{\pm} = \frac{(E_1^2 - E_3^2)(E_1 - E_3 + m_N) - (E_1 + E_3 \pm m_N)Q^2}{2QW},$$

$$Q^* = \frac{m_N |\vec{q}|}{W}$$

$$\nu^* = \frac{(E_1 - E_3)(E_1 - E_3 + m_N) - |\vec{q}|^2}{W}$$

$$q^{\mathrm{IB}} = (\nu^*, 0, 0, Q^*)$$

$$p_1^{\mathrm{IB}} = \left(\frac{(E_1 - E_3)^2 + 2m_N E_1 - Q^2}{2W}, A_{13}, 0, B_{13}^-\right),$$

$$p_2^{\mathrm{IB}} = \left(\frac{m_N(E_1 - E_3 + m_N)}{W}, 0, 0, -\frac{m_N Q}{W}\right),$$

$$p_3^{\mathrm{IB}} = \left(-\frac{(E_1 - E_3)^2 - 2m_N E_3 - Q^2}{2W}, A_{13}, 0, B_{13}^+\right),$$

$$p_4^{\mathrm{IB}} = (W, 0, 0, 0),$$

$$A_{13} = \frac{\sqrt{(|\vec{p}_1| + |\vec{p}_3| - Q)(|\vec{p}_1| - |\vec{p}_3| + Q)(-|\vec{p}_1| + |\vec{p}_3| + Q)(|\vec{p}_1| + |\vec{p}_3| + Q)}}{2Q},$$

$$B_{13}^{\pm} = \frac{(E_1^2 - E_3^2)(E_1 - E_3 + m_N) - (E_1 + E_3 \pm m_N)Q^2}{2QW},$$

Q*

W–ν*

# Limits of integration

$$m_N + m_\pi \leq W \leq \sqrt{s} - m_\chi,$$

$$2m_\chi^2 - \frac{A_q B_q}{s} - \frac{\sqrt{A_q^2 - s m_\chi^2}\sqrt{B_q^2 - 4 s m_\chi^2}}{s} \leq |q^2| \leq 2m_\chi^2 - \frac{A_q B_q}{s} + \frac{\sqrt{A_q^2 - s m_\chi^2}\sqrt{B_q^2 - 4 s m_\chi^2}}{s}$$

where $A_q \equiv \left(E_1 m_N + m_\chi^2\right)$, $B_q \equiv \left(m_\chi^2 + s - W^2\right)$, and $s$ is the Mandelstam variable.

# Axial correction

PCAC

$$\mathcal{L}_q = i \sum_i \overline{q}_i \gamma_\mu q_i$$

Free quark Lagrangian, massless limit

Symmetries: $SU(n)_L$ x $SU(n)_R$

$$q_i \rightarrow \exp -i\theta^A T_{Aij} q_j$$

$$q_i \rightarrow \exp -i\theta^A T_A \gamma_{5ij} q_j$$

Noether currents

$$V_\mu^A = \overline{q}_i \gamma_\mu T_{ij}^A q_j$$

$$A_\mu^A = \overline{q}_i \gamma_\mu \gamma_5 T_{ij}^A q_j$$

# Axial correction

The axial charge does not annihilate the vacuum -> G = $SU(n)_L \times SU(n)_R$ is broken by light quarks to H = $SU(n)_{L+R}$

We will have $n^2-1$ massless Goldstone bosons associated with the coset space G/H

$$< 0|\partial_\mu A^\mu(x)_{ij}|\pi >= \sqrt{2} f_\pi m_\pi^2 \pi_i \delta_{ij}$$

$$\partial_\mu V^\mu_{ij} = (m_i - m_j)\overline{q}_i(i)q_j$$

$$\partial_\mu A^\mu_{ij} = (m_i + m_j)\overline{q}_i(i\gamma_5)q_j$$

**PCAC**
(at q = 0)

$$-(m_u + m_d) < \overline{u}u + \overline{d}d >= 2m_\pi^2 f_\pi^2$$

# Axial correction

There is a relation between the coupling of the pion to baryonic currents, the axial form factor and the decay of the pion, $f_\pi$ ($\pi \to \mu\, \nu$): the Goldberger-Treiman relation:

$$f_\pi g_{\pi NN}(0) = m_N g_A(0)$$

$$\langle N(p_2)|A_\mu|N(p_1)\rangle = \bar{u}(p_2)[\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)]\gamma_5 u(p_1)$$

$$\mathcal{A} \equiv \langle N(p_2)|\partial^\mu A_\mu|N(p_1)\rangle = \bar{u}(p_2)[2M_N g_A(q^2) + q^2 g_P(q^2)](i\gamma_5)u(p_1)$$

$$g_P(q^2) = \frac{\sqrt{2} f_\pi}{m_\pi^2 - q^2}\sqrt{2} g_{\pi NN}$$

→ **a pion pole appears!**

# Axial correction

We need to modify our axial operator, such that it also exhibits the pion pole and respects the previous relations. This was first shown by Ravndal (Nuovo Cim.A 18 (1973) 385-415):

$$\overline{A}^j_\mu = A^j_\mu + q_\mu \frac{(q\cdot A^j)}{m_\pi^2 - q^2}$$

$$B_{\lambda_1\lambda_2} \to B_{\lambda_1\lambda_2} + \frac{1}{C_s^{\lambda_1\lambda_2}} \frac{Z}{2W} \left(V^0_{\lambda_1\lambda_2} q_0^{\mathrm{IB}} - V^3_{\lambda_1\lambda_2} |\vec{q}^{\,\mathrm{IB}}|\right) \times \frac{\frac{2}{3}\sqrt{\frac{\Omega}{2}}\left(q_0^{\mathrm{IB}} + \frac{|\vec{q}^{\,\mathrm{IB}}|^2}{am_N}\right)}{m_\pi^2 - q^2} F^A(q^2),$$

$$C_{\lambda_1\lambda_2} \to C_{\lambda_1\lambda_2} + \frac{1}{C_s^{\lambda_1\lambda_2}} \frac{Z}{2W} \left(V^0_{\lambda_1\lambda_2} q_0^{\mathrm{IB}} - V^3_{\lambda_1\lambda_2} |\vec{q}^{\,\mathrm{IB}}|\right) \times \frac{|\vec{q}^{\,\mathrm{IB}}|\left(\frac{2}{3}W + \frac{q^2}{am_N} + \frac{N\Omega}{3am_N}\right)}{m_\pi^2 - q^2} F^A(q^2).$$

# Form factors
## (Rein & Sehgal)

$g^3 \exp(q^2/\Omega)$

$$F^V(q^2) = \left(1 - \frac{q^2}{4m_N^2}\right)^{0.5-N} \left(1 - \frac{q^2}{m_V^2}\right)^{-2}$$

$m_V = 0.84$ GeV

$$F^A(q^2) = \left(1 - \frac{q^2}{4m_N^2}\right)^{0.5-N} \left(1 - \frac{q^2}{m_A^2}\right)^{-2}$$

$m_A = 0.95$ GeV

# Form factors (vector)
## (Graczyk & Sobczyk, PRD 77 (2008) 053001)

$$G_V^{RS,new}(W,Q^2) = \frac{1}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left(1+\frac{Q^2}{4W^2}\right)^{-\frac{N}{2}}\sqrt{3(G_3(W,Q^2))^2+(G_1(W,Q^2))^2} \tag{36}$$

$$G_V^{RS,new}(W,Q^2) = \frac{1}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left(1+\frac{Q^2}{4M^2}\right)^{-N}\sqrt{3(G_3(W,Q^2))^2+(G_1(W,Q^2))^2} \tag{37}$$

$$G_3(W,Q^2) \;=\; \frac{1}{2\sqrt{3}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}+\frac{C_3^V}{M}(W+M)\right], \tag{38}$$

$$G_1(W,Q^2) \;=\; -\frac{1}{2\sqrt{3}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}-C_3^V\frac{(M+W)M+Q^2}{MW}\right] \tag{39}$$

# Form factors (axial)

## (Graczyk & Sobczyk, PRD 77 (2008) 053001)

$$C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1+\frac{Q^2}{M_a^2}\right)^2}, \quad C_5^A(0) = 1.2, \quad M_a^2 \approx 0.54\,\mathrm{GeV}^2.$$

$$C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1+\frac{Q^2}{M_a^2}\right)^2\left(1+\frac{Q^2}{M_b^2}\right)}, \quad C_5^A(0) \approx 0.88, \quad M_a^2 \approx 9.71\,\mathrm{GeV}^2, \quad M_b^2 \approx 0.35\,\mathrm{GeV}^2$$

$$\widetilde{G}_A^{RS,new}(W,Q^2) = \frac{\sqrt{3}}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left(1+\frac{Q^2}{4M^2}\right)^{-N}\left[1-\frac{W^2-Q^2-M^2}{8M^2}\right]C_5^A(Q^2)$$

# Modeling resonances

FKR used a simple notation to account for the different symmetries in the states of the hadrons:

$$|S\rangle = |xyz\rangle_S = \frac{1}{\sqrt{6}}(|xyz\rangle + |xzy\rangle + |yxz\rangle + |yzx\rangle + |zxy\rangle + |zyx\rangle),$$

$$|\alpha\rangle = |xyz\rangle_\alpha = \frac{1}{2\sqrt{3}}(|xyz\rangle + |xzy\rangle + |yxz\rangle + |yzx\rangle - 2|zxy\rangle - 2|zyx\rangle),$$

$$|\beta\rangle = |xyz\rangle_\beta = \tfrac{1}{2}(|xyz\rangle - |xzy\rangle + |yxz\rangle - |yzx\rangle),$$

$$|A\rangle = |xyz\rangle_A = \frac{1}{\sqrt{6}}(-|xyz\rangle + |xzy\rangle - |yzx\rangle + |yxz\rangle - |zxy\rangle + |zyx\rangle),$$

$$|1\rangle_S|2\rangle_S = |\ \rangle_S, \quad |1\rangle_S|2\rangle_\alpha = |\ \rangle_\alpha,$$

$$|1\rangle_S|2\rangle_\beta = |\ \rangle_\beta, \quad |1\rangle_S|2\rangle_A = |\ \rangle_A,$$

$$|1\rangle_A|2\rangle_S = |\ \rangle_A, \quad |1\rangle_A|2\rangle_\alpha = |\ \rangle_\beta,$$

$$-|1\rangle_A|2\rangle_\beta = |\ \rangle_\alpha, \quad |1\rangle_A|2\rangle_A = |\ \rangle_S,$$

$$\frac{1}{\sqrt{2}}(+|1\rangle_\alpha|2\rangle_\alpha + |1\rangle_\beta|2\rangle_\beta) = |\ \rangle_S,$$

$$\frac{1}{\sqrt{2}}(-|1\rangle_\alpha|2\rangle_\alpha + |1\rangle_\beta|2\rangle_\beta) = |\ \rangle_\alpha,$$

$$\frac{1}{\sqrt{2}}(+|1\rangle_\alpha|2\rangle_\beta + |1\rangle_\beta|2\rangle_\alpha) = |\ \rangle_\beta,$$

$$\frac{1}{\sqrt{2}}(-|1\rangle_\alpha|2\rangle_\beta + |1\rangle_\beta|2\rangle_\alpha) = |\ \rangle_A.$$

# Modeling resonances

Spin states:

$$\left|\tfrac{3}{2}, +\tfrac{3}{2}\right\rangle_S = \left|+++\right\rangle_S ,$$
$$\left|\tfrac{3}{2}, +\tfrac{1}{2}\right\rangle_S = \left|++-\right\rangle_S ,$$
$$\left|\tfrac{3}{2}, -\tfrac{1}{2}\right\rangle_S = \left|+--\right\rangle_S ,$$
$$\left|\tfrac{3}{2}, -\tfrac{3}{2}\right\rangle_S = \left|---\right\rangle_S ,$$

$$\left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle_\alpha = +\left|++-\right\rangle_\alpha ,$$
$$\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle_\alpha = -\left|--+\right\rangle_\alpha ,$$

$$\left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle_\beta = +\left|++-\right\rangle_\beta ,$$
$$\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle_\beta = -\left|--+\right\rangle_\beta .$$

$$J_- \left| j, m \right> = \sqrt{(j+m)(j-m+1)} \left| j, m-1 \right>$$
$$J_-^{H \equiv G_1 \otimes G_2 \otimes G_3 \ldots} = \left(J_-^{G_1} \otimes 1_{G_2} \otimes 1_{G_3} \otimes \ldots\right) + \left(1_{G_1} \otimes J_-^{G_2} \otimes 1_{G_3} \otimes \ldots\right) + \left(1_{G_1} \otimes 1_{G_2} \otimes J_-^{G_3} \otimes \ldots\right) + \ldots$$

# Modeling resonances

Unitary spin ⊗ spin:

$$|\underline{70}\rangle_\beta: \quad {}^4(\underline{8})_\beta = |\tfrac{3}{2}\rangle_S\,|\underline{8}\rangle_\beta\,,$$

$$\quad {}^2(\underline{10})_\beta = |\tfrac{1}{2}\rangle_\beta\,|\underline{10}\rangle_S\,,$$

$$\quad {}^2(\underline{8})_\beta = \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2}\rangle_\alpha\,|\underline{8}\rangle_\beta + |\tfrac{1}{2}\rangle_\beta\,|\underline{8}\rangle_\alpha\right),$$

$$\quad {}^2(\underline{1})_\beta = |\tfrac{1}{2}\rangle_\alpha\,|\underline{1}\rangle_A\,;$$

$$|\underline{20}\rangle_A: \quad {}^4(\underline{1})_A = |\tfrac{3}{2}\rangle_S\,|\underline{1}\rangle_A\,,$$

$$\quad {}^2(\underline{8})_A = \frac{1}{\sqrt{2}}\left(-|\tfrac{1}{2}\rangle_\alpha\,|\underline{8}\rangle_\beta + |\tfrac{1}{2}\rangle_\beta\,|\underline{8}\rangle_\alpha\right).$$

$$|\underline{56}\rangle_S: \quad {}^4(\underline{10}) = |\tfrac{3}{2}\rangle_S\,|\underline{10}\rangle_S\,,$$

$$\quad {}^2(\underline{8}) = \frac{1}{\sqrt{2}}\left(|\tfrac{1}{2}\rangle_\alpha\,|\underline{8}\rangle_\alpha + |\tfrac{1}{2}\rangle_\beta\,|\underline{8}\rangle_\beta\right);$$

$$|\underline{70}\rangle_\alpha: \quad {}^4(\underline{8})_\alpha = |\tfrac{3}{2}\rangle_S\,|\underline{8}\rangle_\alpha\,,$$

$$\quad {}^2(\underline{10})_\alpha = |\tfrac{1}{2}\rangle_\alpha\,|\underline{10}\rangle_S\,,$$

$$\quad {}^2(\underline{8})_\alpha = \frac{1}{\sqrt{2}}\left(-|\tfrac{1}{2}\rangle_\alpha\,|\underline{8}\rangle_\alpha + |\tfrac{1}{2}\rangle_\beta\,|\underline{8}\rangle_\beta\right),$$

$$\quad {}^2(\underline{1})_\alpha = -|\tfrac{1}{2}\rangle_\beta\,|\underline{1}\rangle_A\,,$$

# Modeling resonances

## Space configuration (orbital momentum, L)

**N = 0**

$$|\underline{56},0\rangle = |\underline{56}\rangle_S\,|g\rangle$$

**N = 1**

$$|1,+1\rangle^1_\alpha = a^*_+|g\rangle\,,$$

$$|1,0\rangle^1_\alpha = a^*_z\,|g\rangle\,,$$

$$|1,-1\rangle^1_\alpha = a^*_-\,|g\rangle\,,$$

for example

$$|\underline{70},1\rangle = \frac{1}{\sqrt{2}}\left(|\underline{70}\rangle_\alpha\,|1\rangle^1_\alpha + |\underline{70}\rangle_\beta\,|1\rangle^1_\beta\right)$$

**N = 2**

$$|2\rangle^2_S,|0\rangle^2_S = \frac{1}{\sqrt{2}}\left(|1\rangle^1_\alpha|1\rangle^1_\alpha + |1\rangle^1_\beta|1\rangle^1_\beta\right),$$

$$|2\rangle^2_\alpha,|0\rangle^2_\alpha = \frac{1}{\sqrt{2}}\left(-|1\rangle^1_\alpha|1\rangle^1_\alpha + |1\rangle^1_\beta|1\rangle^1_\beta\right),$$

$$|2\rangle^2_\beta,|0\rangle^2_\beta = \frac{1}{\sqrt{2}}\left(|1\rangle^1_\alpha|1\rangle^1_\beta + |1\rangle^1_\beta|1\rangle^1_\alpha\right),$$

$$|1\rangle^2_A = \frac{1}{\sqrt{2}}\left(-|1\rangle^1_\alpha|1\rangle^1_\beta + |1\rangle^1_\beta|1\rangle^1_\alpha\right).$$

# Modeling resonances

Charge operator, $e_a$:

For an interaction with a proton, we have states of the form: $|uud\rangle_k$, where k = S, α, β. Action of $e_a$:

$$e_a|q_1q_2q_3\rangle = g_{q_1}|q_1q_2q_3\rangle$$

$$|(8)_\alpha\rangle = |uud\rangle_\alpha = \frac{1}{2\sqrt{3}}\left(\sqrt{2}\,|uud\rangle + \sqrt{2}\,|udu\rangle - 2\sqrt{2}\,|duu\rangle\right) = \frac{1}{\sqrt{6}}\left(|uud\rangle + |udu\rangle - 2\,|duu\rangle\right)$$

$$|(8)_\beta\rangle = |uud\rangle_\beta = \frac{1}{2}\left(\sqrt{2}\,|uud\rangle - \sqrt{2}\,|udu\rangle\right) = \frac{1}{\sqrt{2}}\left(|uud\rangle - |udu\rangle\right)$$

$$|(10)_S\rangle = |uud\rangle_S = \frac{1}{\sqrt{6}}\left(\sqrt{2}\,|uud\rangle + \sqrt{2}\,|udu\rangle + \sqrt{2}\,|duu\rangle\right) = \frac{1}{\sqrt{3}}\left(|uud\rangle + |udu\rangle + |duu\rangle\right)$$

$$\langle(\underline{8})_\alpha|e_a|(\underline{10})_S\rangle = \tfrac{1}{3}\sqrt{2}(e_u - e_d)\,,$$

$$\langle(\underline{8})_\alpha|e_a|(\underline{8})_\alpha\rangle = \tfrac{1}{3}(e_u + 2e_d)\,,$$

$$\langle(\underline{8})_\beta|e_a|(\underline{8})_\beta\rangle = e_u\,,$$

$$\langle(\underline{8})_\beta|e_a|(\underline{10})_S\rangle = 0\,,$$

$$\langle(\underline{8})_\alpha|e_a|(\underline{8})_\beta\rangle = 0\,.$$

$$\langle(8)_\alpha|\,e_a\,|(10)_S\rangle = \left(\frac{1}{\sqrt{6}}\left(\langle uud| + \langle udu| - 2\,\langle duu|\right)\right) e_a \left(\frac{1}{\sqrt{3}}\left(|uud\rangle + |udu\rangle + |duu\rangle\right)\right) =$$

$$\frac{1}{3\sqrt{2}}\left(\langle uud| + \langle udu| - 2\,\langle duu|\right)\left(g_u\,|uud\rangle + g_u\,|udu\rangle + g_d\,|duu\rangle\right) = \frac{1}{3\sqrt{2}}\left(g_u + g_u - 2\,g_d\right) = \frac{\sqrt{2}}{3}\left(g_u - g_d\right)$$

# Clebsch - Gordan coefficients



$$1.\ T\left(\chi p \rightarrow \chi \mathrm{p}\pi^0\right) = \sqrt{\tfrac{2}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*+}\right) - \sqrt{\tfrac{1}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*+}\right) = \sqrt{\tfrac{2}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*0}\right) - \sqrt{\tfrac{1}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*+}\right)$$

$$2.\ T\left(\chi p \rightarrow \chi \mathrm{n}\pi^+\right) = \sqrt{\tfrac{1}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*+}\right) + \sqrt{\tfrac{2}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*+}\right) = \sqrt{\tfrac{1}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*0}\right) + \sqrt{\tfrac{2}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*+}\right)$$

$$3.\ T\left(\chi n \rightarrow \chi \mathrm{n}\pi^0\right) = \sqrt{\tfrac{2}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*+}\right) + \sqrt{\tfrac{1}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*0}\right) = \sqrt{\tfrac{2}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*0}\right) + \sqrt{\tfrac{1}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*0}\right)$$

$$4.\ T\left(\chi n \rightarrow \chi \mathrm{p}\pi^-\right) = \sqrt{\tfrac{1}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*+}\right) - \sqrt{\tfrac{2}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*0}\right) = \sqrt{\tfrac{1}{3}}\,\Sigma_{I=3/2\,\mathrm{res}}\, a^{Z'}\left(N_3^{*0}\right) - \sqrt{\tfrac{2}{3}}\,\Sigma_{I=1/2\,\mathrm{res}}\, a^{Z'}\left(N_1^{*0}\right)$$

# From on-shell to nearly on-shell

We will transform the Dirac delta distribution into a non-relativistic Breit-Wigner distribution

$$\delta(W - M_r) \longrightarrow |\eta_r(W)|^2$$

$$\eta_r(W) = \sqrt{\frac{\Gamma_r}{2\pi N_r}} \frac{1}{W - M_r + i\Gamma_r/2}$$

# From on-shell to nearly on-shell

$$\eta_r(W) = \sqrt{\frac{\Gamma_r}{2\pi N_r}} \frac{1}{W - M_r + i\Gamma_r/2}$$

$$N_r = \int_{W_{\min}}^{\infty} dW \frac{\Gamma_r}{2\pi} \frac{1}{(W - M_r)^2 + \Gamma_r^2/4}$$

$$\Gamma_r = \Gamma_r^0 \left( \frac{q_\pi(W)}{q_\pi(M_r)} \right)^{2L+1}$$

$$q_\pi(x) = \frac{\sqrt{(x^2 - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2}}{2x}$$

# Interference rules



$L_{2I,2J}$

**1.** $\chi p \rightarrow \chi n \pi^{+}$

$$|T(\chi p \rightarrow \chi n\pi^{+})|^2 = \left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(S^{+}_{31}) + \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(S^{+}_{11})\right|^2 + \Sigma_{j=1,3}\left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(P^{+}_{3\,j}) + \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(P^{+}_{1\,j})\right|^2 + \Sigma_{j=3,5}\left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(D^{+}_{3\,j}) + \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(D^{+}_{1\,j})\right|^2 + \Sigma_{j=5,7}\left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(F^{+}_{3\,j}) + \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(F^{+}_{1\,j})\right|^2$$

**2.** $\chi p \rightarrow \chi p \pi^{0}$

$$|T(\chi p \rightarrow \chi p\pi^{0})|^2 = \left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(S^{+}_{31}) - \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(S^{+}_{11})\right|^2 + \Sigma_{j=1,3}\left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(P^{+}_{3\,j}) - \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(P^{+}_{1\,j})\right|^2 + \Sigma_{j=3,5}\left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(D^{+}_{3\,j}) - \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(D^{+}_{1\,j})\right|^2 + \Sigma_{j=5,7}\left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(F^{+}_{3\,j}) - \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(F^{+}_{1\,j})\right|^2$$

**3.** $\chi n \rightarrow \chi n \pi^{0}$

$$|T(\chi n \rightarrow \chi n\pi^{0})|^2 = \left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(S^{0}_{31}) + \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(S^{0}_{11})\right|^2 + \Sigma_{j=1,3}\left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(P^{0}_{3\,j}) + \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(P^{0}_{1\,j})\right|^2 + \Sigma_{j=3,5}\left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(D^{0}_{3\,j}) + \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(D^{0}_{1\,j})\right|^2 + \Sigma_{j=5,7}\left|\sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(F^{0}_{3\,j}) + \sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(F^{0}_{1\,j})\right|^2$$

**4.** $\chi n \rightarrow \chi p \pi^{-}$

$$|T(\chi n \rightarrow \chi p\pi^{-})|^2 = \left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(S^{0}_{31}) - \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(S^{0}_{11})\right|^2 + \Sigma_{j=1,3}\left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(P^{0}_{3\,j}) - \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(P^{0}_{1\,j})\right|^2 + \Sigma_{j=3,5}\left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(D^{0}_{3\,j}) - \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(D^{0}_{1\,j})\right|^2 + \Sigma_{j=5,7}\left|\sqrt{\tfrac{1}{3}}\,\Sigma a^{Z'}(F^{0}_{3\,j}) - \sqrt{\tfrac{2}{3}}\,\Sigma a^{Z'}(F^{0}_{1\,j})\right|^2$$

# Example

## Δ(1232)

$$P_{33}(1232)\ :\ {}^4(10)_{3/2}\,[56,0^+].$$

$$\text{State (Res)} = |\underline{10}_s\rangle\,|\tfrac{3}{2}\rangle_s\,|g\rangle$$

We'll compute: $f_{-3} \equiv \langle N, \tfrac{1}{2} | F | N^*, \tfrac{3}{2}\rangle$

$$\text{State (p)} = \frac{1}{\sqrt{2}}\left(\langle \underline{8}_\alpha|_\alpha\langle +| + \langle \underline{8}_\beta|_\beta\langle +|\right)$$

$$f_{-3} = -\frac{9}{\sqrt{2}}\left(\langle \underline{8}_\alpha|_\alpha\langle +| + \langle \underline{8}_\beta|_\beta\langle +|\right)\langle g|\, e_a\,(T a_+ + R\sigma_-)\, e^{-ka_z}$$

$$\times |\underline{10}_s\rangle\,|\tfrac{3}{2},\tfrac{3}{2}\rangle_s\,|g\rangle$$

$$= \left(-\frac{9}{\sqrt{2}}\right)\underbrace{\langle \underline{8}_\alpha | e_a | \underline{10}_s\rangle_\alpha}_{\frac{\sqrt{2}}{3}(g_u - g_d)}\ \underbrace{\langle + | R\sigma_- | \tfrac{3}{2},\tfrac{3}{2}\rangle_s}_{-\sqrt{\frac{2}{3}}R}\ \underbrace{\langle g|g\rangle}_{1}$$

$$= \boxed{\sqrt{6}\,(g_u - g_d)\,R}$$

# Scalar mediator

$$\mathcal{M}(\chi(p_1,\lambda_1)N(p_2) \to \chi(p_3,\lambda_2)N^*(p_4))$$
$$= \frac{g_D g_{N\Phi}}{q^2 - m_\Phi^2}[\bar{u}_{p_3\lambda_2} u_{p_1\lambda_1}]\langle N^*|J_S^+(0)|N\rangle,$$
$$= 2M \frac{g_D g_{N\Phi}}{q^2 - m_\Phi^2} V_S^{\lambda_1\lambda_2}\langle N^*|F_S|N\rangle,$$

$$\frac{d^2\sigma}{dq^2 dW} = \frac{1}{64\pi}\frac{g_D^2 g_{N\Phi}^2}{(q^2 - m_\Phi^2)^2}\frac{W^2}{m_N^2|\vec{p}_1|^2} \times \sum_{\lambda_1,\lambda_2}|V_S^{\lambda_1\lambda_2}|^2 \sum_{i=\pm}|\langle N^*|F_S^i|N\rangle|^2 \delta(W-M)$$

$$V_S^{\lambda_1\lambda_2} = (1 - 2\delta_{\lambda_1}^- \delta_{\lambda_2}^-)\frac{\Delta^S_{(-\lambda_1\lambda_2)}}{A_S}\sqrt{1 + \lambda_1\lambda_2 \cos\delta},$$

$$\Delta^S_\pm \equiv (E_1^{\mathrm{IB}} + m_\chi)(E_3^{\mathrm{IB}} + m_\chi) \pm |\vec{p}_1^{\,\mathrm{IB}}||\vec{p}_3^{\,\mathrm{IB}}|$$

$$A_S \equiv \sqrt{2(E_1^{\mathrm{IB}} + m_\chi)(E_3^{\mathrm{IB}} + m_\chi)}.$$

$$F^S_{0^\pm} = e_k S^S e^{-\lambda a_z},$$

$$S^S \equiv 3F^S(q^2).$$

$$F^S(q^2) = \frac{F^S(0)}{1 - \frac{q^2}{m_S^2}},$$

# Psuedoscalar mediator

$$\mathcal{M}(\chi(p_1,\lambda_1)N(p_2)\to\chi(p_3,\lambda_2)N^*(p_4))$$
$$=\frac{g_D g_{Na}}{q^2-m_a^2}[-i\bar{u}_{p_3\lambda_2}\gamma^5 u_{p_1\lambda_1}]\langle N^*|J_P^+(0)|N\rangle,$$
$$=-i2M\frac{g_D g_{Na}}{q^2-m_a^2}V_P^{\lambda_1\lambda_2}\langle N^*|F_P|N\rangle,$$

$$\frac{d^2\sigma}{dq^2dW}=\frac{1}{64\pi}\frac{g_D^2 g_{Na}^2}{(q^2-m_a^2)^2}\frac{W^2}{m_N^2|\vec{p}_1|^2}\sum_{\lambda_1,\lambda_2}|V_P^{\lambda_1\lambda_2}|^2 \times\sum_{i=\pm}|\langle N^*|F_P^i|N\rangle|^2\delta(W-M).$$

$$V_P^{\lambda_1\lambda_2}=(1-2\delta^-_{\lambda_1}\delta^-_{\lambda_2})\frac{\Delta^P_{(-\lambda_1\lambda_2)}}{A_S}\sqrt{1+\lambda_1\lambda_2\cos\delta}$$

$$\Delta^P_\pm\equiv(E_3^{\rm IB}+m_\chi)|\vec{p}_1^{\rm IB}|\pm(E_1^{\rm IB}+m_\chi)|\vec{p}_3^{\rm IB}|$$

$$F^P_{0^\pm}=e_kS^P\sigma_z e^{-\lambda a_z},$$

$$F^P(q^2)=\frac{F^P(0)}{1-\frac{q^2}{m_P^2}}$$

$$S^P\equiv\frac{6m_N|\vec{q}|}{(W+m_N)^2-q^2}F^P(q^2)$$

$$S^P\to S^P\left(\frac{m_\pi^2}{m_\pi^2-q^2}\right) =\frac{6m_N m_\pi^2|\vec{q}|}{[(W+m_N)^2-q^2](m_\pi^2-q^2)}F^P(q^2)$$