A new approach to studying neutrino dynamics in the primordial era

Based on [2411.00931], [2411.00892], [2409.15129], [2409.07378]

Vsevolod Syvolap, Maksym Ovchynnikov

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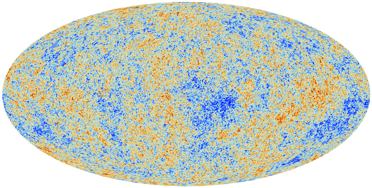
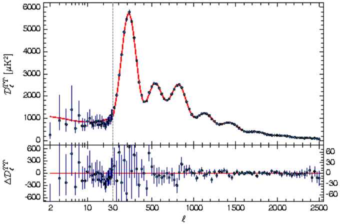
Cosmological neutrinos, N_{eff} and CMB

- Cosmic Microwave Background (CMB) is among the most precise probes of cosmology and new physics
- Produced at $T \approx 3000K$ when the Universe becomes transparent for photons, it carries information about cosmological history
- CMB power spectrum - shows the temperature fluctuation, it is affected by the Hubble rate





Cosmological neutrinos, N_{eff} and CMB

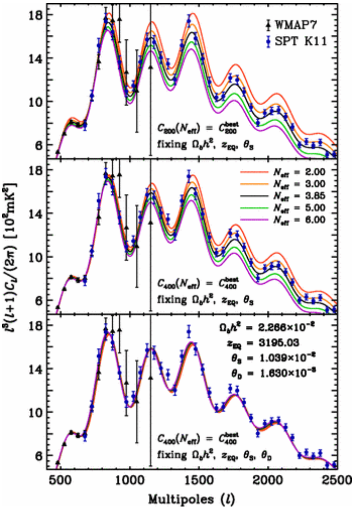
- Cosmic neutrinos (and hypothetical particles) make a contribution to the Hubble rate
- Quantified via effective number of neutrinos N_{eff}

$$N_{\text{eff}} = \frac{8}{7}\left(\frac{\rho_{\text{rad}}}{\rho_\gamma} - 1\right)\left(\frac{11}{4}\right)^{4/3}$$

- Current measurements combined with BAO give a limit for non-photon radiation density (68% CL)

$$N_{\text{eff}} = 2.99 \pm 0.17 - \text{ Planck}$$
$$N_{\text{eff}} = 2.89 \pm 0.11 - \text{ ACT}$$

- Simons Observatory aim to improve the precision to $\sigma(N_{\text{eff}}) < 0.07$



Cosmological neutrinos, $N_{\rm eff}$ and CMB

- In the Standard Model only relic neutrinos contribute to $N_{\rm eff}$, exact value depend on their history
- At $T \gg$ MeV neutrinos are kept in equilibrium

$$f_\nu = \frac{1}{e^{p/T}+1}$$

- At temperature $T \approx$ few$\times$ MeV neutrinos start to decouple from plasma keeping close to equilibrium spectra

$$\Gamma_{\rm weak} \approx G_F^2 T^5, \quad H \approx \frac{T^2}{M_{\rm pl}^*}, \quad \Gamma_{\rm weak}/H \sim \left(\frac{T}{1\ \rm MeV}\right)^3 \tag{1}$$

- Below MeV, most neutrinos are decoupled and free-streaming
- Accurate calculation within SM cosmology [2306.05460]

$$N_{\rm eff} = 3.043 \tag{2}$$

FIPs Effects in the BBN Era

- Feebly interacting particles (FIPs) or non-standard scenarios can spoil this picture or modify the results.
- Additional lepton asymmetry
- Late reheating
- Primordial black hole (PBH) evaporation
- FIPs ($\tau_{\text{FIP}} \gtrsim 10^{-2}$ s), produced in the early Universe (e.g., HNLs, dark photons, scalars), can affect cosmology in several ways:
  1. Contributing to the expansion rate
  2. Entropy injection (EM sector) at decays
  3. Injection of high-energy neutrinos $\Rightarrow$ spectral distortions
  4. Meson or unstable lepton decays $\Rightarrow$ entropy release $+$ secondary non-thermal neutrinos
- All of these processes can affect N_{eff} and the CMB.

Boltzmann Equation

- At MeV-scale the standard cosmological system consists of an equilibrium EM sector, neutrinos and tiny fraction of baryons
- Dynamics of neutrinos can be described via the Boltzmann equation:

$$\frac{\partial f_{\nu_\alpha}(E_\nu, t)}{\partial t} - E_\nu H \frac{\partial f_{\nu_\alpha}(E_\nu, t)}{\partial E_\nu} = \sum_\beta \langle P_{\beta\alpha} \rangle \mathcal{I}_{\text{coll},\nu_\beta}\left[E_\nu, f_{\nu_\alpha}, f_{\nu_\beta}, T\right] \quad (3)$$

$$\mathcal{I}_{\text{coll},\nu_\alpha}(p_1) = \frac{1}{2E_{\nu_\alpha}} \sum \int \prod_{i=2}^{m} \frac{d^3\mathbf{p}_i}{(2\pi)^3\, 2E_i} \prod_{f=1}^{n-m} \frac{d^3\mathbf{p}'_f}{(2\pi)^3\, 2E'_f} \times |\mathcal{M}|^2\, F[f]\, (2\pi)^4\, \delta^{(4)}\left(\sum_{i=1}^{m} p_i - \sum_{f=1}^{n-m} p'_f\right). \quad (4)$$

- And coupled equations describing the expansion of the Universe and thermodynamics
- Non-standard FIPs' scenarios might include the evolution of new particles

Solving the Boltzmann System

- Solving this system is necessary to obtain an accurate value of N_{eff}.
- Two main approaches exist for solving the Boltzmann equation:

Solution

- **Integrated approach** — assume neutrino distributions $f_\nu \equiv f_{\text{FD}}(T)$, which reduces the problem to a system of ODEs for T_γ and T_{ν_i}.
- **Discretized approach** — directly solve the Boltzmann equation numerically on a fixed energy grid for each f_{ν_i}.

Integrated Approach

Pros

- Simple and fast — useful for estimating ν evolution.
- Intuitively easier to interpret.
- Convenient for low-energy neutrino or EM injections.

Cons

- Breaks down if neutrinos are highly non-thermal ($E_\nu \gg T$).
- May yield *qualitatively* incorrect results in such regimes.
- Less accurate than the discretized approach.

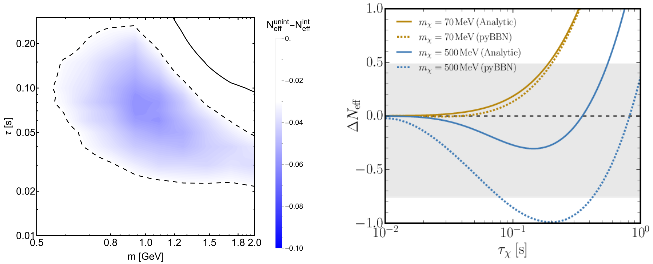


Figure: *Left:* Integrated vs. unintegrated approach for a toy FIP decaying into the EM sector. *Right:* Impact on N_{eff} of a decaying HNL: unintegrated (analytic) vs. discretized (pyBBN) treatment.

Discretized Approach

- Define a momentum grid with fixed step size in comoving momentum space: $\tilde{p} = p_{\text{phys}} \cdot a(T)/a_0$.
- Grid extends from $E_{\text{min}}(T_{\text{min}})$ up to $E_{\text{max}} \cdot a(T_{\text{min}})/a_0$.
- Analytically reduce the collision integral.
- Solve the Boltzmann integro-differential equation on this grid, coupled to cosmic expansion (Friedmann equations).

Pros

- Captures the full evolution of the neutrino plasma.
- Accuracy controlled by grid resolution.

Cons

- Requires strong dimensional reduction of $I_{\text{coll},\alpha}$.
- Computational cost scales as

$$t_{\text{comp}} \propto E_{\nu,\text{max}}^{k+2},$$

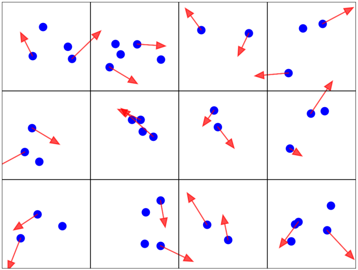
  where k is the reduced dimensionality.
- May produce conflicting results across implementations.

Direct Simulation Monte Carlo (DSMC)

- Original DSMC was used for the simulation of rarefied gas flows
- Particles are treated individually, tracking their state $\{\mathbf{r}_i, \mathbf{v}_i, t\}$
- Volume divided into small cells with N_{cell} particles
- Particles within one cell can collide
- At each iterative timestep Δt

$$N_{\text{sampled}} = \frac{N_{\text{cell}}(N_{\text{cell}} - 1)}{2} \frac{(\sigma v)_{\text{max}} \Delta t}{V_{\text{cell}}}$$

  pairs are sampled for interaction.
- Each interaction is accepted with probability $P_{\text{acc}} = \frac{(\sigma v)}{(\sigma v)_{\text{max}}}$ and the outgoing kinematics is generated



DSMC for Early Universe

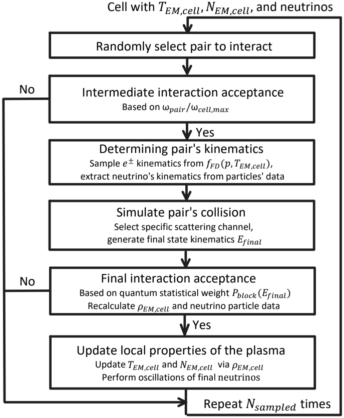
DSMC can be adapted for the Early Universe dynamics:

General idea:

- System is presented as a set of individual particles ($\nu_i/\bar{\nu}_i$ and $\gamma, e^{\pm}$, potentially X, Y... representing BSM species, mesons etc.).
- Isotropy and homogeneity - only momenta degrees of freedom $\{\cancel{\mathbf{r}_i}, \mathbf{v}_i, t\}$.
- System is split into subsets (cells) at each timestep, and only interactions within a cell are considered
- EM particles are in thermal equilibrium represented by $T_{EM}/T_{EM,\text{cell}} \Rightarrow$
  No tracking, we *sample* them at every step.
- Quantum statistics must be taken into account
- Expansion of the Universe is included at each step
  $V_{\text{system}} \to V_{\text{system}}(1+3H\Delta t), \quad E_i \to \frac{E_i}{1+H\Delta t}$

Interaction step

- Initialize the cell with $T_{EM,\text{cell}} = T_{EM}$ and N_{EM}
- N_{cell} neutrinos are picked randomly
- Sample the interaction$\Rightarrow$
- After the N_{sampled} (Δt passed) interactions combine the cells - average the T_{EM}
- In case of presence of extra species - determine their dynamics over Δt + inject neutrinos from decays
- Update the volume of the system and particles energies due to expansion
- Repeat



What it gives?

- Weaker dependence of the complexity on the maximum neutrino energy in the system $\Rightarrow$ significant speed-up compared to the traditional Boltzmann discretization approach.
- Possibility to study very high-energetic injections $\gg$ GeV,
- Cross-checks for Boltzmann solver implementations.
- Easy tracking of the system at each step and more control over microscopics.

Cross-checks and tests

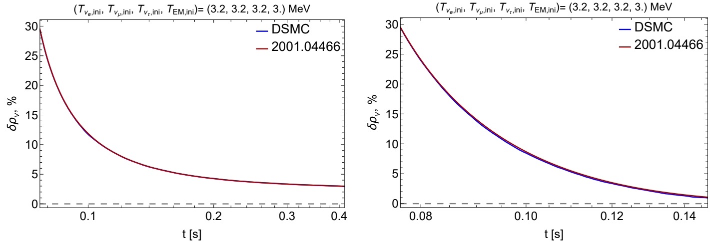


Figure: Energy density evolution if **all** species are assumed with equilibrium distribution (integrated approach) with (*left*) and without (*right*) expansion.

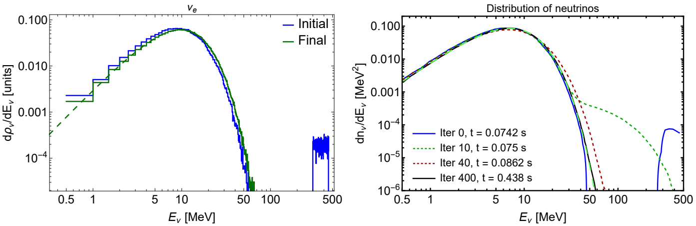


Figure: Approaching the thermal equilibrium in case of high-energy neutrino injection

Cross-checks and tests

- Simulation with $N = 3 \cdot 10^7$ has fluctuations at level $\mathcal{O}(0.1\%)$
- few $\times 100$ is a sufficient number of neutrinos per cell

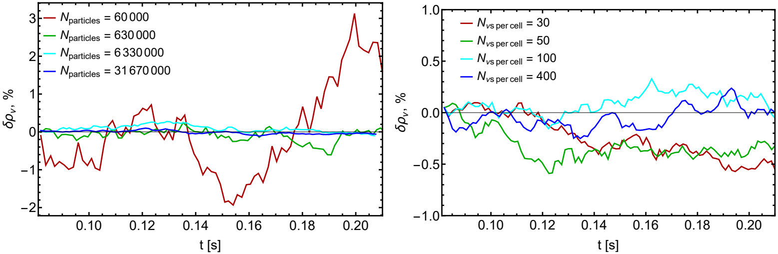
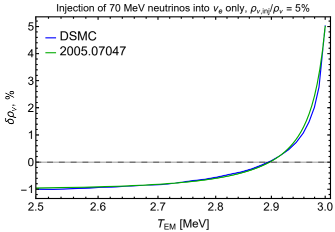


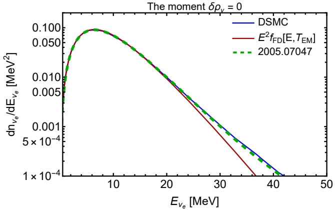
Figure: The temporal evolution of the quantity $\delta\rho_\nu$ when varying numbers of neutrinos per cell $N_{\text{cell},\nu}$ and particles in the system N with equilibrium starting conditions

- Comparison of the DSMC approach with the discretization code for the setup of injection of 70 MeV neutrinos into ν_e.
- presented value $\delta\rho_\nu$:

$$\delta\rho_\nu = \left(\frac{\rho_{\mathrm{EM}}}{\rho_\nu}\right)_{\mathrm{SM}} \frac{\rho_\nu}{\rho_{\mathrm{EM}}} - 1$$

- Injection of neutrinos with $E_\nu \gg T$ eventually leads to **decrease** of N_{eff}



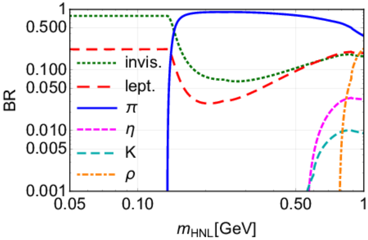


- In recent update of the DSMC [2508.08379], complete SM setup including QED corrections were tested and result $N_{\mathrm{eff}} = 3.0439$ was obtained. It is in a perfect agreement with previous calculations

Metastable particles

- EM and neutrino injections can appear through metastable particles produced in FIPs' decays
- It was common to treat them as instantly decaying
- They can participate in *(i)* annihilations, *(ii)* interactions with nuclei, *(iii)* EM scatterings *(iv)* decays
- Except for EM scatterings $\Gamma_{EM} \gg \Gamma_{\text{ann, nucl, dec}}$ no clear hierarchy

We focus on the dynamics of $\mu^{\pm}, \pi^{\pm}, K^{\pm}, K_L^0$



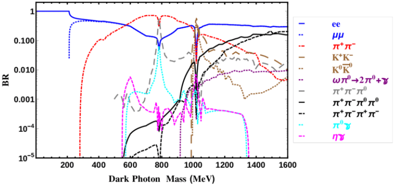


Figure: BR of different FIPs

Evolution of metastable particles

- We solve a system of coupled equations for each $Y = \mu^\pm, \pi^\pm, K^\pm, K_L$.

$$\frac{dn_Y}{dt} + 3Hn_Y = \frac{n_X}{\tau_X} N_Y^X - \frac{n_Y}{\tau_Y} - n_Y n_{\bar{Y}} \langle \sigma_{\text{ann}}^Y v \rangle + \left(\frac{dn_Y}{dt}\right)_{\mathcal{N}} + \sum_{Y' \neq Y} n_{Y'} \Gamma_{Y' \to Y}$$

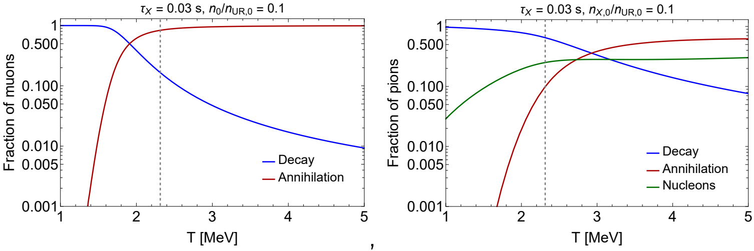


Figure: The yields of muons and pions that would decay, annihilate, or interact with the nucleons if injected by decaying toy-model FIP with BR solely into a $\pi^+\pi^-/\mu^+\mu^-$.

$N_{\rm eff}$ change for toy models and scalar

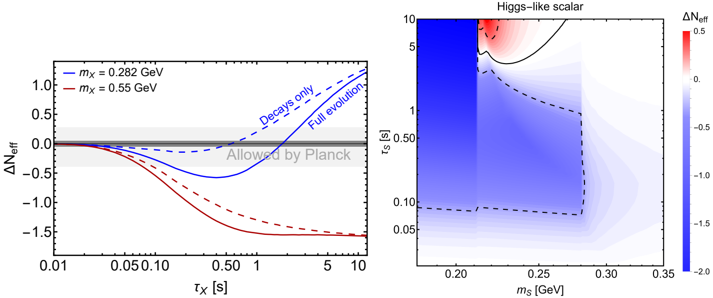


Figure: *Left:* toy model decaying only into pions, *Right:* Higgs-like scalar effect on $N_{\rm eff}$

- Accurate account of Y's evolution change the outcome of $N_{\rm eff}$ value.
- Especially important near the decay mass threshold.

Conclusion

- DSMC presents a new approach of studying the dynamics of neutrinos during their decoupling
- Their non-trivial evolution can lead to unexpected outcomes in terms of $N_{\rm eff}$
- DSMC proposes a cross-check **alternative** for SM BBN/CMB scenario, significantly **more efficient** option for heavy ($m_{\rm FIP} \lesssim$ GeV) FIPs+BBN/CMB and the **only** option to study ultra-high energy neutrino injections $E_\nu \gg$ GeV