Harris Relevant Disordered Charged Horizons

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Based on work to appear
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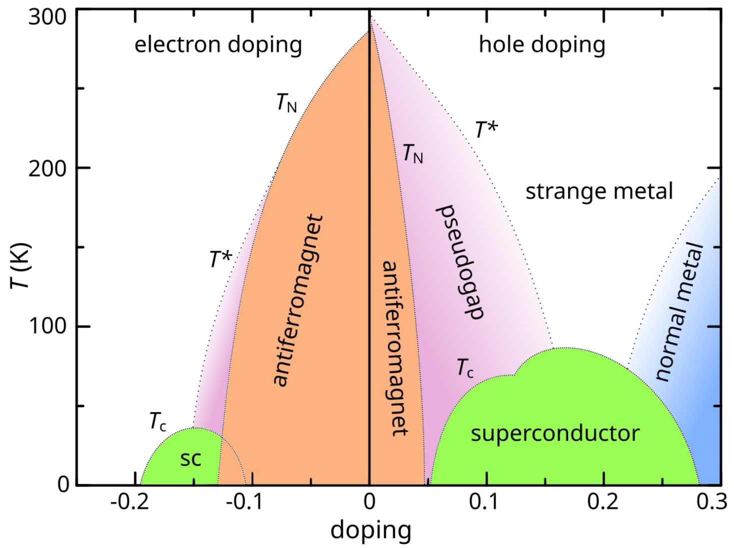
Disordered systems

Cool Condensed Matter Phenomenology with no clean counterpart, specially when combined with **strong interactions**:

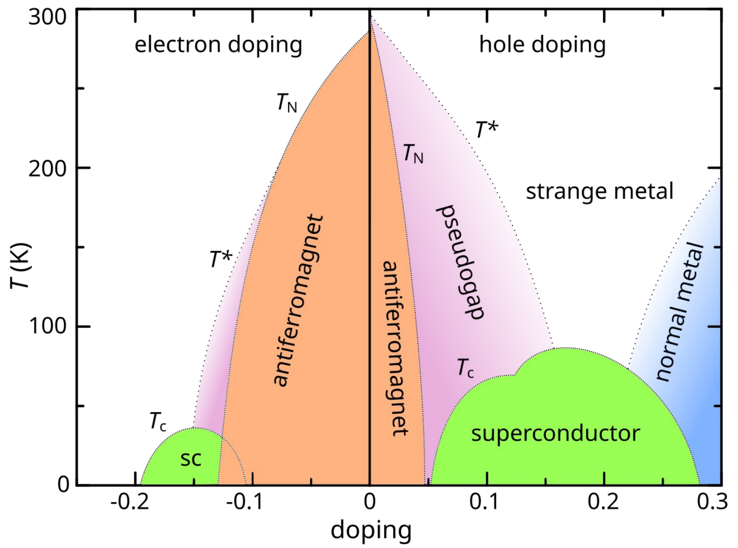
- Finite transport
- Griffiths phase transitions [Vojta '10]
- Anderson Localisation [Anderson '58]
- Disordered fixed points
- High Tc superconductivity



Disordered systems

We need to understand how disorder affects our predictions:

- Does disorder change qualitatively our observables (conductivities, entropy, 2-point functions...) ?
- Are IR fixed points stable under disorder?



What is disorder?

In (perturbative) QFT, we describe disorder by deforming a theory with an operator with a **space-dependant coupling**

$$S = S_0 + \int d^d x \; \gamma(x) \mathcal{O}_\Delta(x)$$

Observables are **disorder-averaged quantities** calculated by summing over different choices of $\gamma(x)$, using a measure that fixes disorder correlations and strength.

$$\langle \ldots \rangle_D = \int D\gamma \; P[\gamma] \, (\ldots)$$

[Vojta '13 ;
Aharony, Komargodski, Yankielowicz '15]

We will focus on white Gaussian disorder along n spatial directions.

$$\langle \gamma(x) \rangle_D = 0 \qquad\qquad \langle \gamma(x)\gamma(y) \rangle_D = V^2 \delta^{(n)}(x-y)$$

[Harris ’74]

Harris Criterion

For **Gaussian disorder** we can do the disorder-averaging path integral using the replica trick. We end up with m copies of the clean theory deformed by an operator with an homogeneous coupling, the disorder strength.

$$\tilde{S} = \sum^{m} S_0 + \int d^d x \; V^2 \; \tilde{\mathcal{O}}$$

$$[\ \tilde{\mathcal{O}}\ ] = 2\Delta - (d - n)$$

$$\begin{array}{c} \mathrm{UV} \\ \Big\downarrow \\ \mathrm{IR}^{(\mathrm{Clean})} \end{array}$$

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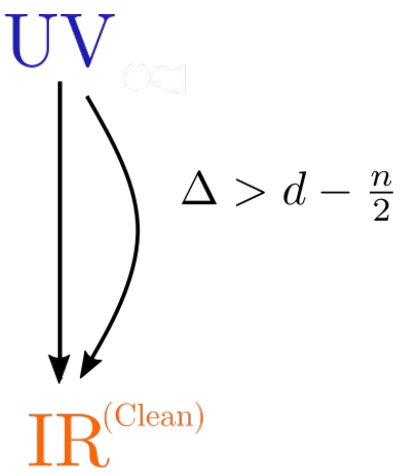

[Harris '74]

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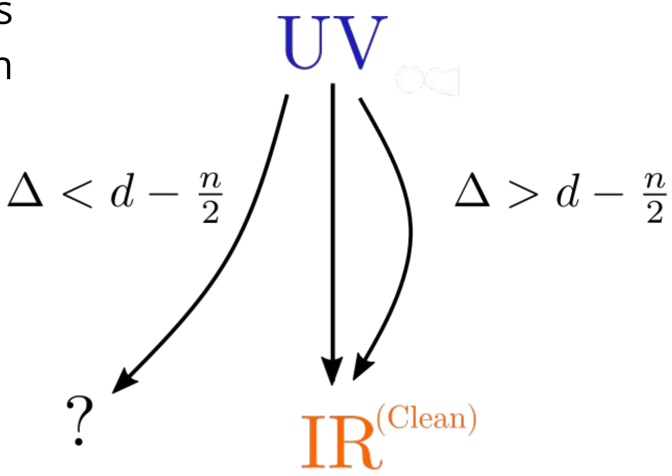


Holographic setup

$$AdS_{d+1}$$

Holographic setup

$$AdS_{d+1}$$

$$\Big\downarrow \; \mu = \mu_0$$

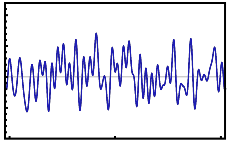
$$AdS_2 \times \mathbb{R}^{d-1}$$

$$\mathcal{S} - \mathcal{S}_0 \propto T$$

$$\sigma = \infty$$

Holographic setup

$$AdS_{d+1}$$

$\mu =$ 

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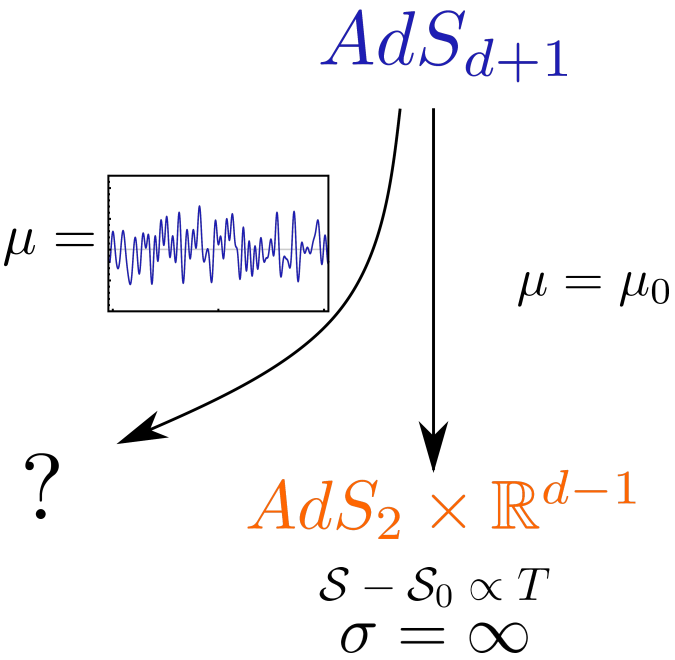

Minimal setup with a $U(1)$ current and AdS_{d+1} asymptotics

$$S = \int d^{d+1}x \, \sqrt{-g} \left(\mathcal{R} - 2\Lambda - \frac{1}{4} F^2 \right)$$

Disorder is introduced via an space-dependant B.C. along one direction $n = 1$ for the gauge field. This produces **Harris relevant** disorder

Holographic setup

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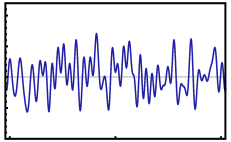
$\mu =$ 

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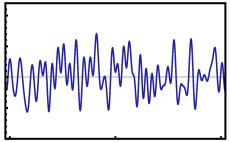
$$A|_{\partial AdS_3} = \mu(x) \log(z) dt$$

$$A|_{\partial AdS_4} = \mu(x) dt$$

$$\text{with} \quad \mu(x) = \mu_0 + V \sqrt{\frac{k_{UV}}{N}} \sum_j^N \cos\left(\frac{j}{N} \, k_{UV} \, x + \delta_j \right)$$

Holographic setup

AdS_{d+1}

$\mu =$ 

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Disordered Regime

$$\mu(x) = \mu_0 + V\sqrt{\frac{k_{UV}}{N}}\sum_j^N \cos\left(\frac{j}{N}\ k_{UV}\ x + \delta_j\right)$$

Only reproduces (white) Gaussian disorder in the regime $[k_{IR}, k_{UV}]$.

For our horizon to feel the disorder we must consider Black Holes with

$$\frac{k_{IR}}{\mu_0} < \frac{T}{\mu_0} < \frac{k_{UV}}{\mu_0}$$

$$k_{UV} \to \infty$$

$$k_{IR} \to 0$$

Disordered Regime

$$\mu(x) = \mu_0 + {\color{green}V}\sqrt{\frac{k_{UV}}{N}}\sum_j^N \cos\left(\frac{j}{N}\,k_{UV}\;x + \delta_j\right)$$

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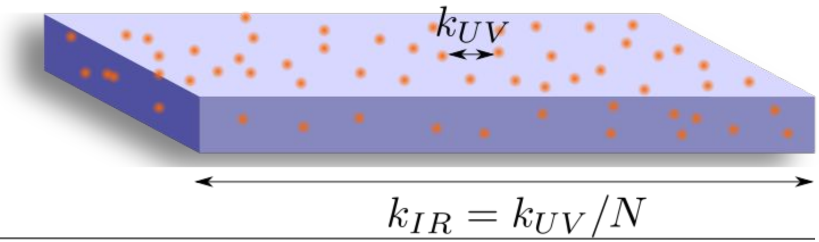
$$\cancel{\begin{aligned} k_{UV} &\to \infty \\ k_{IR} &\to 0 \end{aligned}}$$



Disordered Regime

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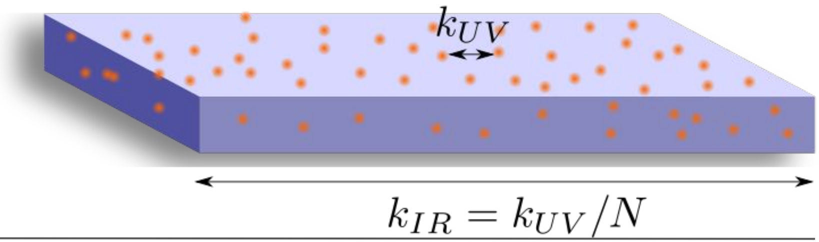
$$\frac{k_{IR}}{\mu_0} < \frac{T}{\mu_0} < \frac{k_{UV}}{\mu_0}$$

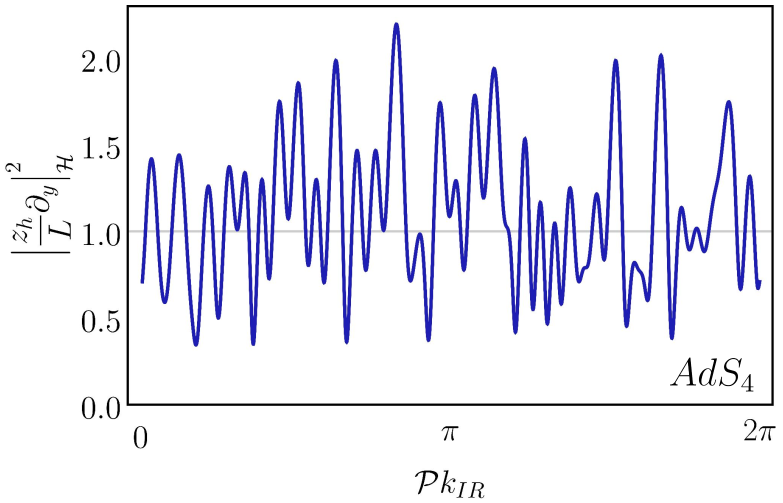
$$\cancel{k_{UV} \to \infty}$$
$$\cancel{k_{IR} \to 0}$$

$$k_{UV} = \text{const.}$$
$$k_{IR} \to 0 \ (N \to \infty)$$

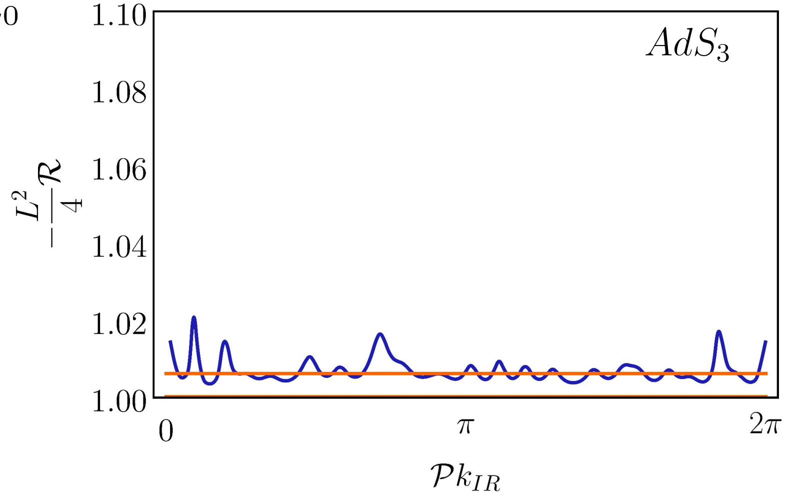


AdS_4 and AdS_3 Disordered Charged Horizons

$$\frac{T}{\mu_0} \sim \frac{k_{UV}}{\mu_0}$$



How distances contract/expand in the direction perpendicular to the disorder as a function of proper distance along the horizon.



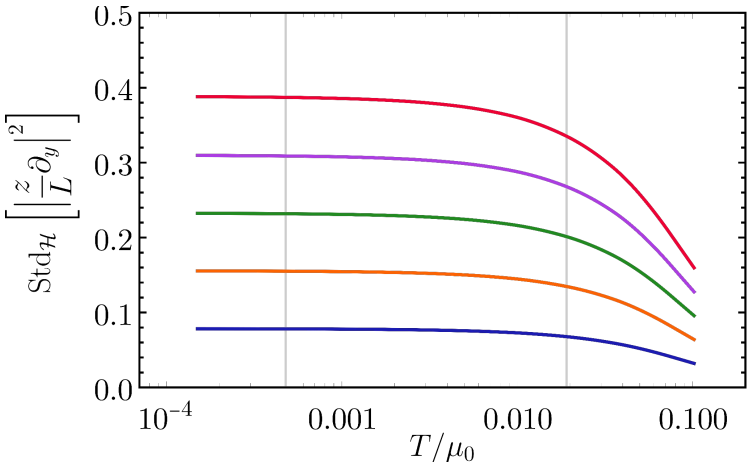
Blue: value of the Ricci scalar along the horizon. Orange: value of the clean Ricci scalar at the same temperature

AdS_4 Fate of the IR fixed point

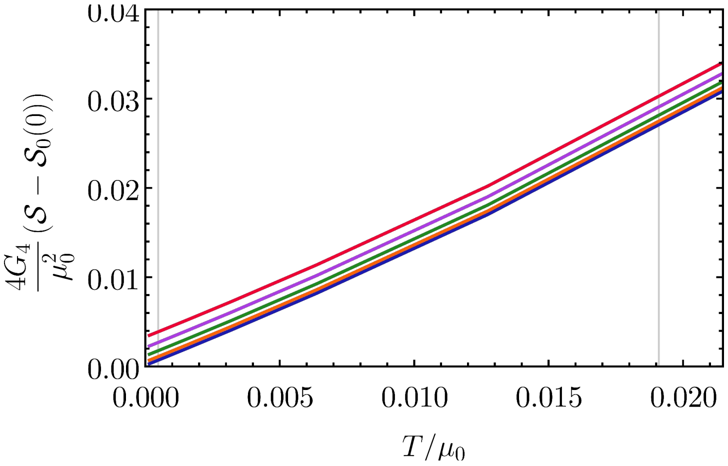
$$\int_{\mathcal{H}} X = \frac{1}{\int dx \sqrt{\gamma_{\mathcal{H}}}} \int dx \sqrt{\gamma_{\mathcal{H}}} \;\; X\big|_{z=1}$$

$$\mathrm{Std}_{\mathcal{H}}\left[X\right] = \int_{\mathcal{H}} X^2 - \left(\int_{\mathcal{H}} X\right)^2$$

$$\mathcal{S} - \mathcal{S}_V \propto T$$



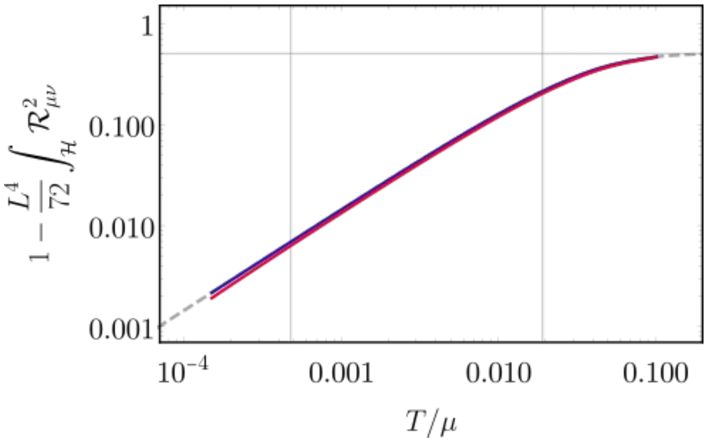
Disorder gets stabilized within the disordered regime



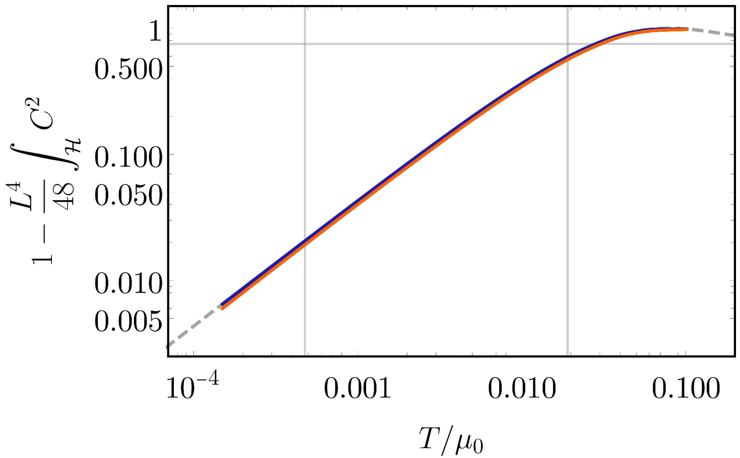
Entropy gets shifted by a disorder dependant constant value.

AdS_4 Fate of the IR fixed point (?)

The average geometry goes like the clean case.



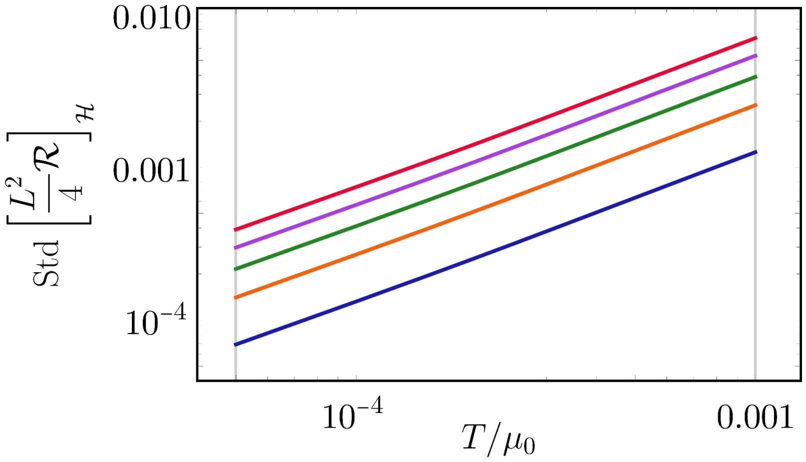
Average value of the square of the Ricci tensor at the horizon.



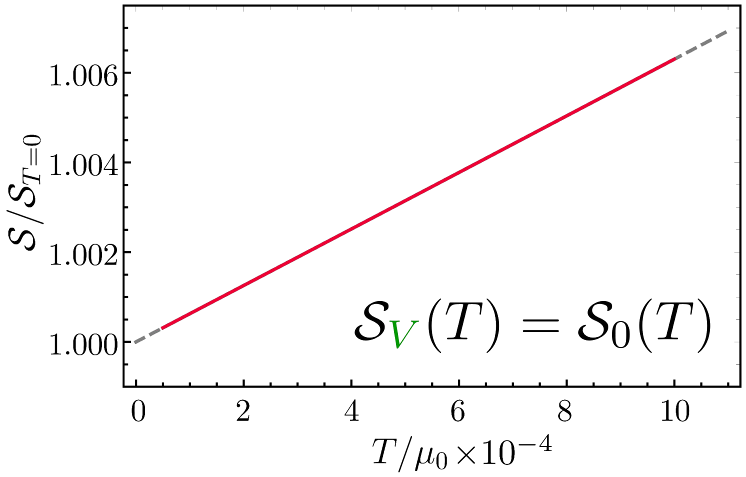
Average value of the square of the Weyl tensor at the horizon.

[Schrauth, Portela, Goth ’18; Fayfar, Bretaña, Montfrooji ’21; Topchyan, Nuding, Klümper, Sedrakyan ’24]

AdS_3 Fate of the IR fixed point (??)



Inhomogeneity dies off as we lower the temperature.

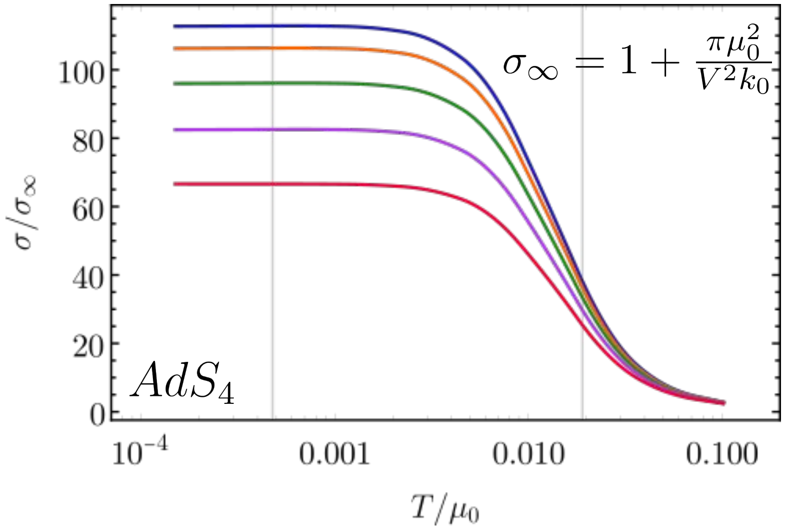


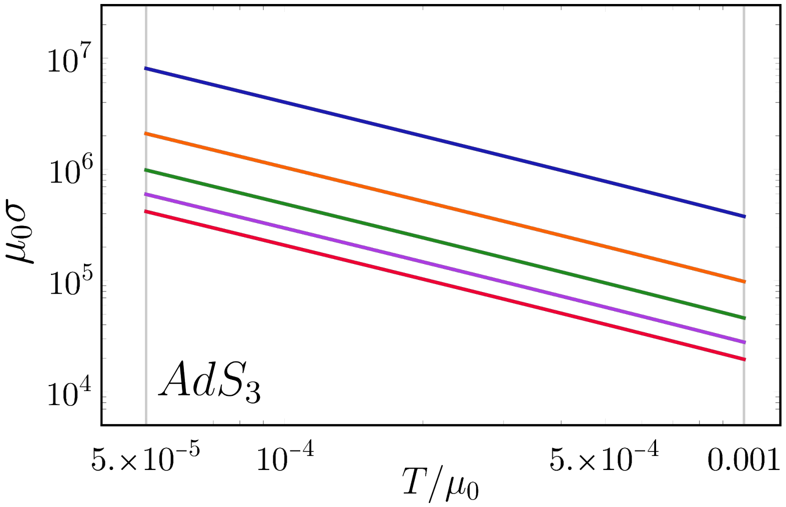
Entropy is independent of disorder for all temperatures.

AdS_4 and AdS_3 DC conductivities

[Donos, Gauntlett 14']

$$\sigma = \frac{1}{Z}\left(1 + \frac{\langle\rho\rangle^2}{\langle\rho^2\rangle - \langle\rho\rangle^2 + \langle\Upsilon^2\rangle}\right)$$





Summary

- We obtain **disordered horizons with Harris-relevant disorder**
- Asymptotically AdS_4
  - Inhomogeneity stabilizes *within* the disordered regime + *finite transport coefficients*
  - In the IR we find a *continuous family* of disordered fixed points labeled by the disorder strength
  - Still *retains characteristics of the clean fixed poin*t: average geometry, entropy scaling with temperature.
- Asymptotically AdS_3
  - IR goes back to the clean case, **explicit violation of the Harris criterion**.

Thank you!

Numerical Implementation

$$\begin{cases} 0 \\ H_5 dy^2 \end{cases}$$

$$ds^2 = \frac{L^2}{z^2}\left(-H_1 f(z)dt^2 + \frac{H_2}{f(z)}dz^2 + H_3(dx + H_4 dz)^2 + ds_\perp^2\right)$$

The ansatz doesn't fully fix the gauge. We use DeTurck trick to have a well defined boundary problem

Clean metric

$$\mathcal{R}_{\mu\nu} \;\to\; \mathcal{R}_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} \text{ with } \xi^\nu = g^{\alpha\beta}(\Gamma(g)^\nu_{\alpha\beta} - \Gamma(\bar{g})^\nu_{\alpha\beta})$$

[Headrick, Kitchen, Wiseman '09]

Harris Relevance

To change the IR we must turn on a Harris relevant disorder $\Delta < d - \frac{n}{2}$

We source disorder via the chemical potential $\mu(x)$ associated with a conserved current

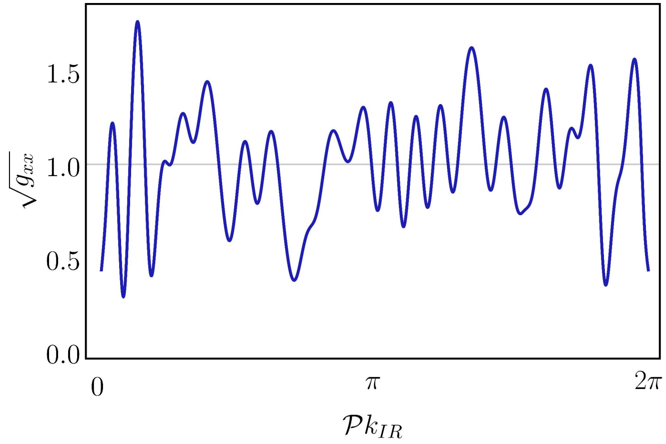
$$\Delta_\mu = d - 1$$

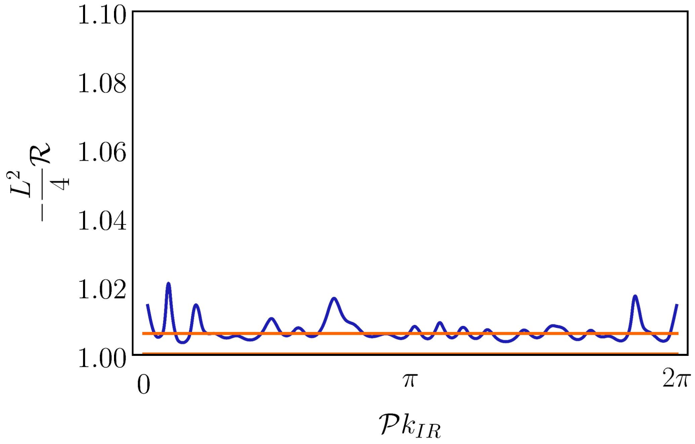
$$\Delta_\mu < d - \tfrac{n}{2} \rightarrow n < 2$$

We only add disorder along one direction ($n = 1$), this ensures that it is Harris-relevant $\forall d$.

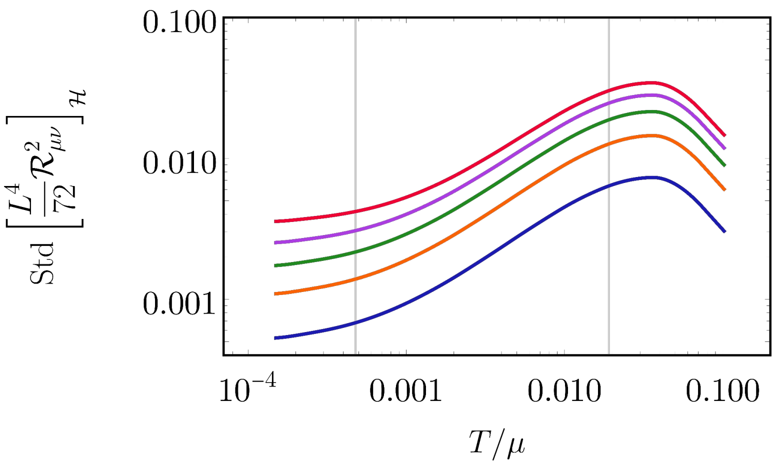
AdS_3 Disordered Charged Horizons

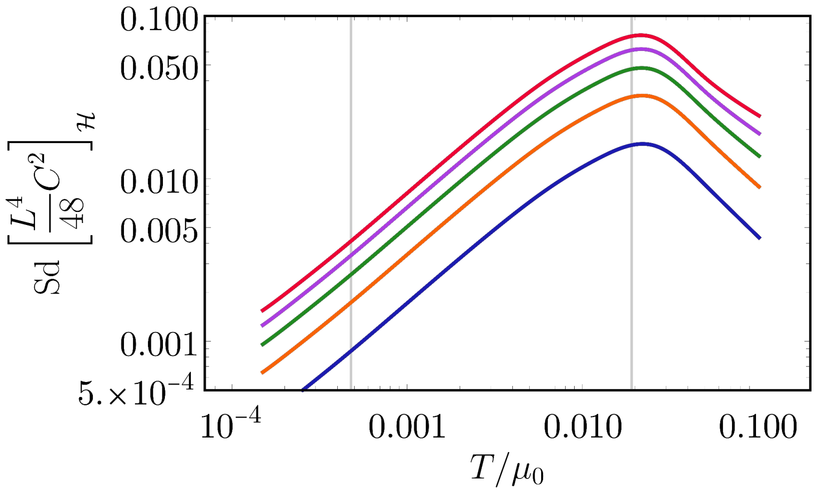
$$\frac{T}{\mu_0} \sim \frac{k_{UV}}{\mu_0}$$





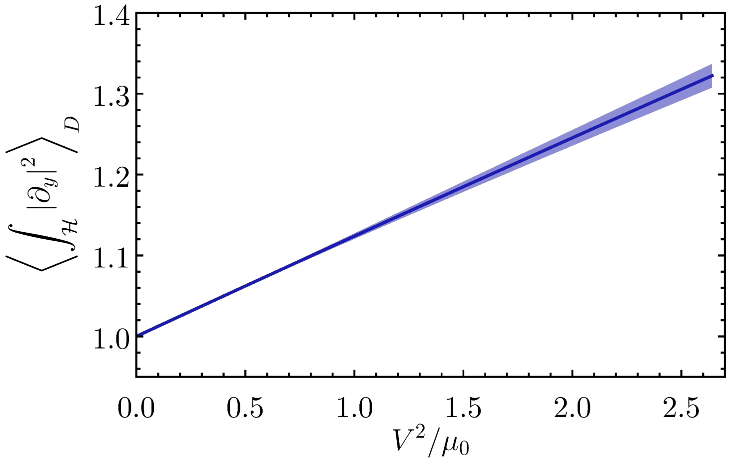
AdS_4 Fate of the IR fixed point (Std)



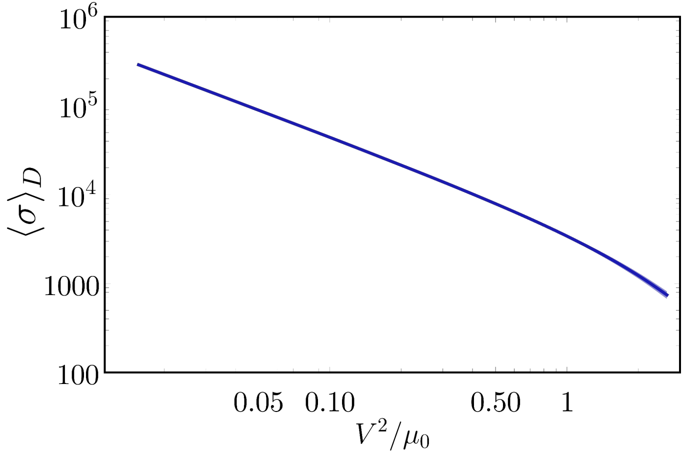


Realisation dependance

Integration along the spatial coordinate kills off the dependance on the random phases. Disorder averaging doesn't change qualitatively our results.



$$\frac{T}{\mu_0} \sim \frac{k_{IR}}{\mu_0}$$

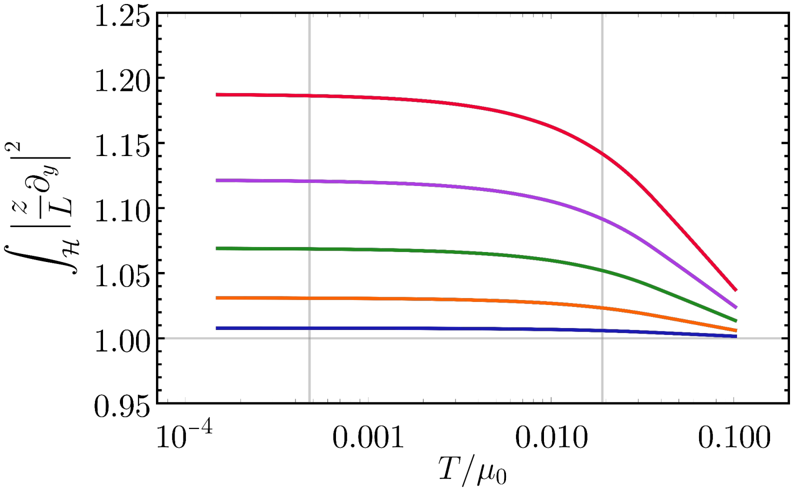


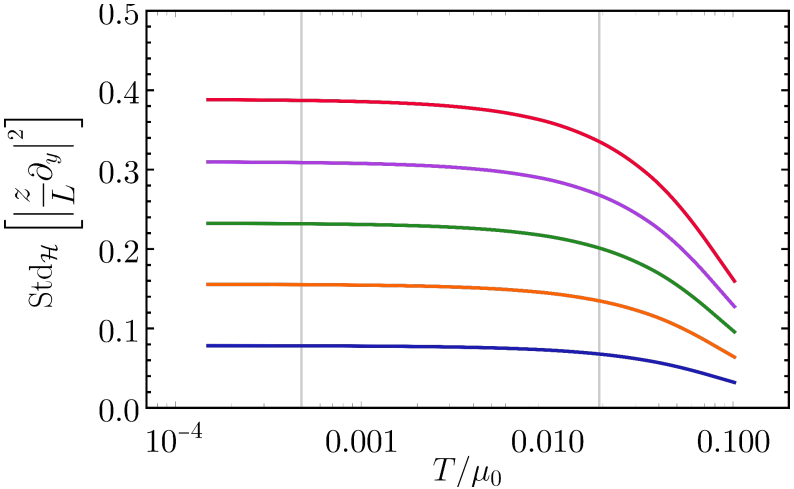
AdS_4 **Fate of the IR fixed point**

$V/\sqrt{\mu} = 1.25$
$V/\sqrt{\mu} = 1$
$V/\sqrt{\mu} = 0.75$
$V/\sqrt{\mu} = 0.5$
$V/\sqrt{\mu} = 0.25$

$$\int_{\mathcal{H}} X = \frac{1}{\int dx \sqrt{\gamma_{\mathcal{H}}}} \int dx \sqrt{\gamma_{\mathcal{H}}} \;\; X\big|_{z=z_h}$$

$$\mathrm{Std}_{\mathcal{H}}\left[X\right] = \int_{\mathcal{H}} X^2 - \left(\int_{\mathcal{H}} X\right)^2$$



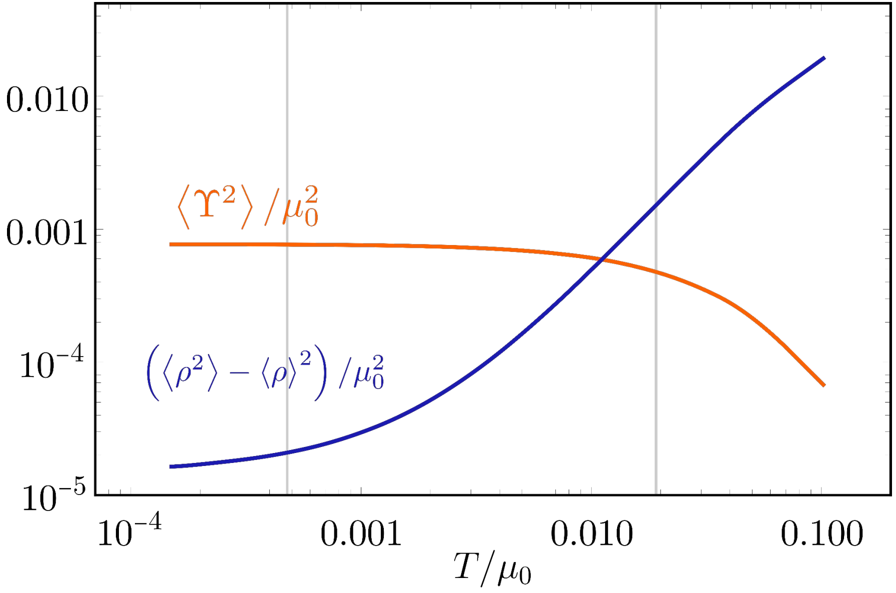


AdS_4 DC conductivities

$$\sigma = \frac{1}{Z}\left(1 + \frac{\langle\rho\rangle^2}{\langle\rho^2\rangle - \langle\rho\rangle^2 + \langle\Upsilon^2\rangle}\right)$$

$$\rho = \frac{A_t'}{h_{tt}} \quad \text{and} \quad \Upsilon^2 = \frac{1}{g_{xx}}\left(\frac{\partial_x g_{yy}}{g_{yy}}\right)^2$$

At low temperatures the dominant contribution to the resistivity comes from the inhomogeneous geometry.



AdS_3 DC conductivities

[Donos, Gauntlett 14']

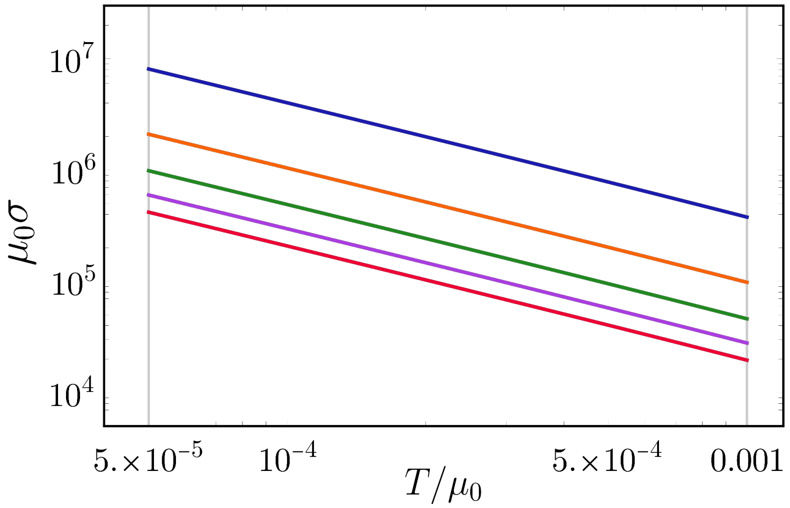
In the DC conductivity we don't have the contribution from the transverse geometry.

As we lower the temperature electrical conductivity diverges.

$$\sigma = \frac{1}{Z}\left(1 + \frac{\langle\rho\rangle^2}{\langle\rho^2\rangle - \langle\rho\rangle^2}\right)$$

$$\langle ... \rangle = \frac{1}{Z}\int dx\sqrt{g_{xx}}(...) \text{ with } Z = \int dx\sqrt{g_{xx}}$$

$$\rho = \frac{A_t'}{H_1}$$



AdS_4 DC conductivities

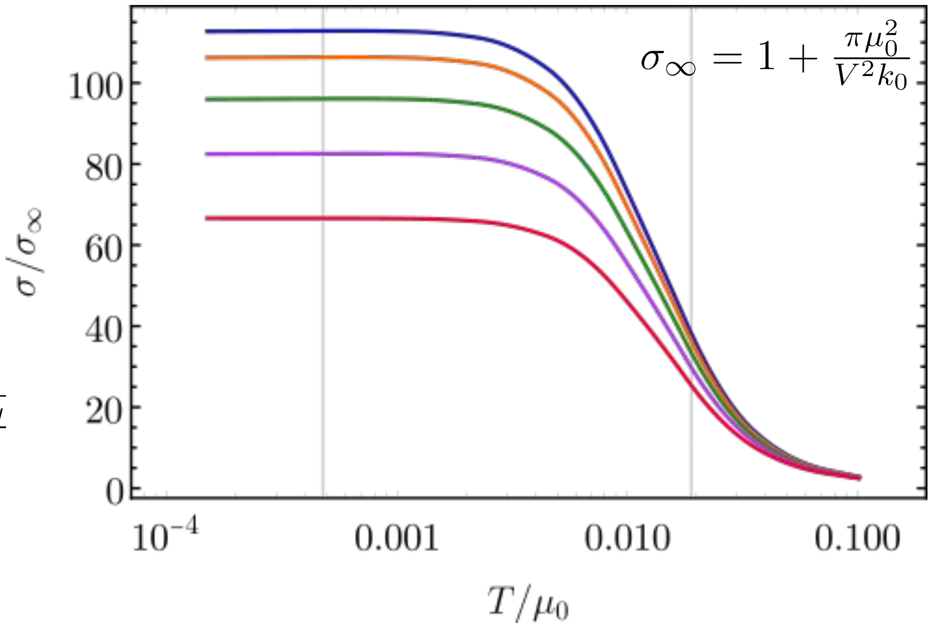
[Donos, Gauntlett 14']

We can obtain DC transport coefficients from integrals over the horizon.

$$\sigma = \frac{1}{Z}\left(1 + \frac{\langle\rho\rangle^2}{\langle\rho^2\rangle - \langle\rho\rangle^2 + \langle\Upsilon^2\rangle}\right)$$

$$\langle\ldots\rangle = \frac{1}{Z}\int dx \frac{\sqrt{g_{xx}g_{yy}}}{g_{yy}}(\ldots) \text{ with } Z = \int dx \frac{\sqrt{g_{xx}g_{yy}}}{g_{yy}}$$

$$\rho = \frac{A_t'}{h_{tt}} \quad \text{and} \quad \Upsilon^2 = \frac{1}{g_{xx}}\left(\frac{\partial_x g_{yy}}{g_{yy}}\right)^2$$



Future directions

- What happens in AdS_5?
- What if we change the disorder (correlations, source, directions...) ?
- Is there any (simple) way to predict how disorder will affect our IR fixed point à la Harris ?
- Can we obtain our disorder transport coefficients directly from the (homogeneous) average geometry (massive gravity) ?