Higher-Form (Quasi)Hydrodynamics from Holography: Deformations and Dualities

(to appear soon)

André O. Pinheiro (Heriot-Watt University)

Madrid, June 19, 2025

why?

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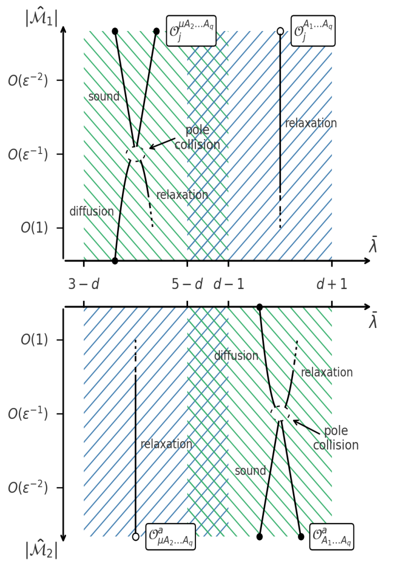
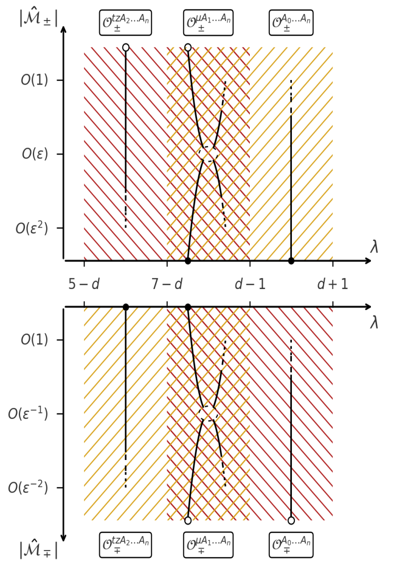


$|\hat{\mathcal{M}}_1|$
$\mathcal{O}_j^{\mu A_2 \dots A_q}$
$\mathcal{O}_j^{A_1 \dots A_q}$
$O(\varepsilon^{-2})$
$O(\varepsilon^{-1})$
$O(1)$
sound
pole collision
relaxation
relaxation
diffusion
$\bar{\lambda}$
$3-d$
$5-d$
$d-1$
$d+1$
$\bar{\lambda}$
$O(1)$
diffusion
relaxation
$O(\varepsilon^{-1})$
pole collision
relaxation
sound
$O(\varepsilon^{-2})$
$|\hat{\mathcal{M}}_2|$
$\mathcal{O}^a_{\mu A_2 \dots A_q}$
$\mathcal{O}^a_{A_1 \dots A_q}$



$|\hat{\mathcal{M}}_\pm|$
$\mathcal{O}_\pm^{tzA_2 \dots A_n}$
$\mathcal{O}_\pm^{\mu A_1 \dots A_n}$
$\mathcal{O}_\pm^{A_0 \dots A_n}$
$O(1)$
$O(\varepsilon)$
$O(\varepsilon^2)$
λ
$5-d$
$7-d$
$d-1$
$d+1$
λ
$O(1)$
$O(\varepsilon^{-1})$
$O(\varepsilon^{-2})$
$|\hat{\mathcal{M}}_\mp|$
$\mathcal{O}_\mp^{tzA_2 \dots A_n}$
$\mathcal{O}_\mp^{\mu A_1 \dots A_n}$
$\mathcal{O}_\mp^{A_0 \dots A_n}$

Hydrodynamics

Hydrodynamics = EFT for many-body systems near equilibrium at finite temperature.

Landau & Lifshitz, ´59

Global symmetries $\Rightarrow$ IR/slow variables: locally conserved charge densities, ρ.

large wavelength
$\lambda \equiv k^{-1}$
perturbation

global equilibrium

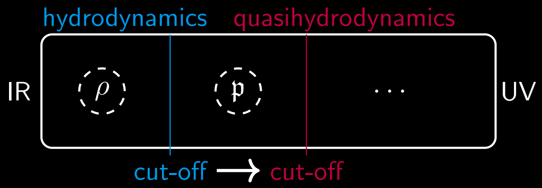
local equilibrium

relaxation over
long time $\tau_\rho(k)$:
$\lim_{k\to 0} \tau_\rho = \infty$

$$\omega(k) \sim \frac{-i}{\tau_\rho} \xrightarrow[k\to 0]{} 0$$

Quasihydrodynamics

Grozdanov, Lucas & Poovuttikul, ´19



$$\lim_{k \to 0} \omega(k) = \frac{-i}{\tau_{\mathfrak{p}}} \neq 0$$

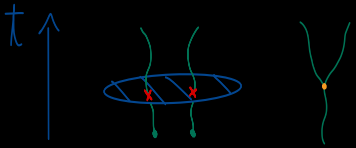
Densities of approximately conserved charges $\in \mathfrak{p}$

weakly-broken symmetry

(Continuous) higher-form symmetries

Gaiotto, Kapustin, Seiberg & Willett, ´15

0-form symmetry (d=3): $\partial_\mu j^\mu = \ell \tilde{j}$



$\ell = 0$ $\qquad$ $\ell \neq 0$

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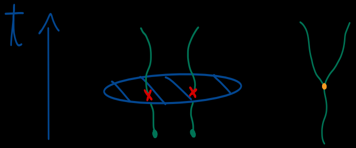
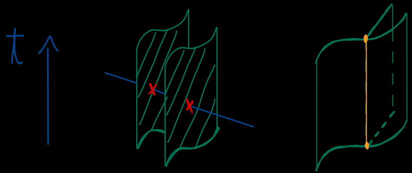


$\ell = 0$ $\ell \neq 0$

1-form symmetry (d=3): $\partial_\mu j^{[\mu\nu]} = \ell \tilde{j}^\nu$



$\ell = 0$ $\ell \neq 0$

Higher-form symmetries and holography

$(p-1)$-form symmetry: $\boxed{\mathrm{d} * j = 0}$

- $j \in \Omega^p(\partial \mathrm{AdS})$

$\bar{\mathcal{S}}[A]: \quad A \sim A + \mathrm{d}\xi$

- $A \in \Omega^p(\mathrm{AdS})$

Broken higher-form symmetries and holography

Approximate $(p-1)$-form symmetry:

$$\boxed{*\mathrm{d} * j = \ell \tilde{j}}$$

- $j \in \Omega^p(\partial \mathrm{AdS})$
- $\tilde{j} \in \Omega^{p-1}(\partial \mathrm{AdS})$
- $\ell \ll 1$

$$\mathcal{S}[A, \tilde{A}] : \quad (A, \tilde{A}) \sim (A, \tilde{A}) + (\mathrm{d}\xi, \Theta\xi)$$

- $A \in \Omega^p(\mathrm{AdS})$
- $\tilde{A} \in \Omega^{p-1}(\mathrm{AdS})$
- $\Theta = \Theta[A, \tilde{A}]$

Broken higher-form symmetries and holography

Approximate $(p-1)$-form symmetry:

$$\boxed{*\mathrm{d}*j = \ell\tilde{j}}$$

- $j \in \Omega^p(\partial\mathrm{AdS})$
- $\tilde{j} \in \Omega^{p-1}(\partial\mathrm{AdS})$
- $\ell \ll 1$

$$\mathcal{S}[A,\tilde{A}]: \quad (A,\tilde{A}) \sim (A+\mathrm{d}\xi, \tilde{A}+\Theta\xi)$$

- $A \in \Omega^p(\mathrm{AdS})$
- $\tilde{A} \in \Omega^{p-1}(\mathrm{AdS})$
- $\Theta = \Theta[A,\tilde{A}]$

linearisation

theory of a single massive p-form B
with mass squared $m^2 \ll 1$

$\Rightarrow$ approximate symmetry

Massless p-form ($d \geq 2$)

Poincaré patch of AdS_{d+1}: $$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{r^2} + r^2 \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu$$

$$\bar{S}_{reg} = \int \mathrm{d}^{d+1}x \sqrt{|g|} \frac{F_{a_0 \ldots a_p} F^{a_0 \ldots a_p}}{p+1} - \bar{S}_{ct}$$

$$\bar{S}_{ct} = \int \mathrm{d}^d x \, r^{\bar{\lambda}+1} \left[\frac{F_{r\mu_1 \ldots \mu_p} F_r{}^{\mu_1 \ldots \mu_p}}{1 - \bar{\lambda}} + \frac{r^{-4}}{p+1} \frac{F_{\mu_0 \ldots \mu_p} F^{\mu_0 \ldots \mu_p}}{\bar{\lambda} - 3} \right] + \ldots$$

where $F_{a_0 \ldots a_p} = \partial_{[a_0} A_{a_1 \ldots a_p]}$.

$$\boxed{\bar{\lambda} \equiv d + 1 - 2p}$$

$0 \leq p \leq d - 1$

Holographic dictionary ($\bar{\lambda} \equiv d + 1 - 2p \geq 2$)

$\mathcal{O}_j \in \Omega^p(\partial \text{AdS}) \longrightarrow$ conserved current: $\mathrm{d} * \langle \mathcal{O}_j \rangle = 0$

Double-trace deformation: $\mathcal{M}_1 \int \mathcal{O}_j \wedge * \mathcal{O}_j$

Standard Quantisation

Witten, ´01

Holographic dictionary ($\bar{\lambda} \equiv d + 1 - 2p \geq 2$)

Standard Quantisation

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Witten, ´01

Alternative Quantisation

$\mathcal{O}^a \in \Omega^p(\partial \mathrm{AdS}) \longrightarrow \langle \mathcal{O}^a \rangle \sim \langle \mathcal{O}^a \rangle + \mathrm{d}\chi$

Goldstone of spontaneously broken higher-form symmetry: $\mathrm{d} * \langle * \mathrm{d} \mathcal{O}^a \rangle = 0$

Double-trace deformation: $\mathcal{M}_2 \int \mathrm{d}\mathcal{O}^a \wedge * \mathrm{d}\mathcal{O}^a$

Holographic dictionary ($\bar{\lambda} \equiv d + 1 - 2p < 2$)

Alternative Quantisation

$\mathcal{O}_j \in \Omega^p(\partial \mathrm{AdS}) \longrightarrow$ conserved current: $\mathrm{d} * \langle \mathcal{O}_j \rangle = 0$

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Finite temperature holography

Son & Starinets, ´02

Herzog & Son, ´03

Isotropic AIAdS black brane:

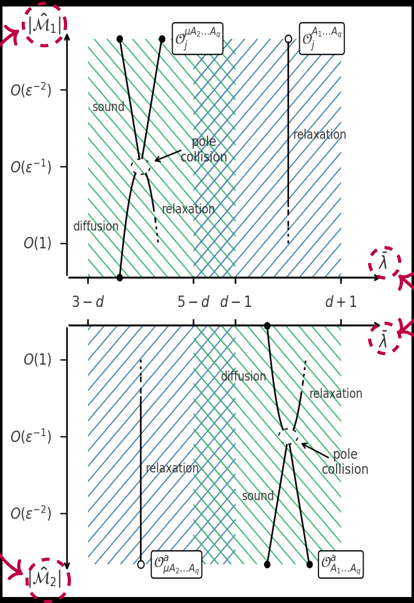
$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{f(r)} - f(r)\mathrm{d}t^2 + r^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

- $T = f'(r_h)/4\pi$
- $x^\mu \equiv (t, z, x^A)$
- $k^\mu = (\omega, k, 0, ..., 0)$

$\mathcal{O}_j$

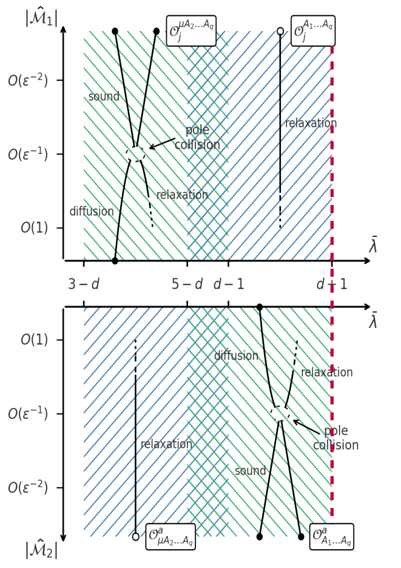
deformation temperature

$\mathcal{O}^a$

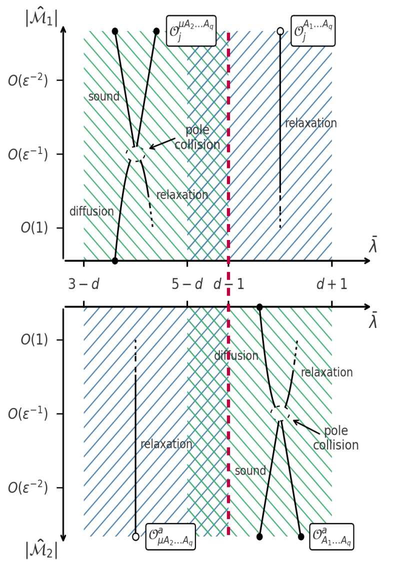


$\bar{\lambda} = d + 1 - 2p$

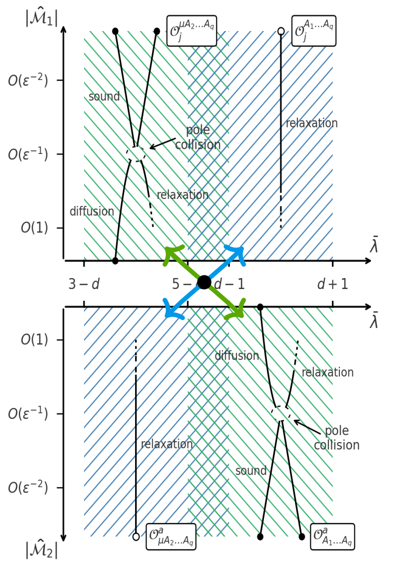
$k \sim \varepsilon \ll 1$



scalar: $p = 0 \Rightarrow \bar{\lambda} = d + 1$



1-form: $p = 1 \Rightarrow \bar{\lambda} = d - 1$



electric-magnetic duality =

$$\bar{\lambda} \leftrightarrow 4 - \bar{\lambda}$$
$$\oplus$$
$$\mathcal{O}_j \rightleftarrows \mathcal{O}^a$$
$$\oplus$$
$$\hat{\mathcal{M}}_1 \leftrightarrow \hat{\mathcal{M}}_2$$

Witten, ´03
Leigh & Petkou, ´03
Yee, ´04
Haro & Gao, ´07
Herzog, Kovtun, Sachdev & Son, ´07
Hartnoll & Herzog, ´07
Haro & Petkou, ´08

Massive p-form ($d \geq 2$)

$$S_{reg} = \int \mathrm{d}^{d+1}x \sqrt{|g|} \left[\frac{(\mathrm{d}B)_{a_0 \dots a_p} (\mathrm{d}B)^{a_0 \dots a_p}}{(p+1)!} + m^2 \frac{B_{a_1 \dots a_p} B^{a_1 \dots a_p}}{p!} \right] + S_{ct}$$

$$S_{ct} = \Delta_- \int \mathrm{d}^d x \, r^{\lambda - 3} \frac{B_{\mu_1 \dots \mu_p} B^{\mu_1 \dots \mu_p}}{p!} + \dots$$

where

$$\Delta_\pm = \frac{\lambda - 3 \pm \sqrt{(\lambda - 3)^2 + 4m^2}}{2}$$

$$\boxed{\lambda \equiv d + 3 - 2p}$$

$0 \leq p \leq d$

Holographic dictionary ($\lambda \equiv d + 3 - 2p > 3$)

$\mathcal{O}_- \in \Omega^p(\partial \mathrm{AdS})$ ⟶ approximately conserved current: $\mathrm{d} * \langle \mathcal{O}_- \rangle \sim O(m^2)$

Double-trace deformation: $\mathcal{M}_- \int \mathcal{O}_- \wedge *\mathcal{O}_-$

Standard Quantisation

Holographic dictionary ($\lambda \equiv d+3-2p > 3$)

$\mathcal{O}_- \in \Omega^p(\partial \mathrm{AdS})$ ⟶ approximately conserved current: $\mathrm{d} * \langle \mathcal{O}_- \rangle \sim O(m^2)$

Double-trace deformation: $\mathcal{M}_- \int \mathcal{O}_- \wedge * \mathcal{O}_-$

Standard Quantisation

$\mathcal{O}_+ \in \Omega^p(\partial \mathrm{AdS})$ ⟶ pseudo-goldstone: $\mathrm{d} * \langle * \mathrm{d} \mathcal{O}_+ \rangle \sim O(m^2)$

Double-trace deformation: $\mathcal{M}_+ \int \mathcal{O}_+ \wedge * \mathcal{O}_+$

Alternative Quantisation

Holographic dictionary ($\lambda \equiv d + 3 - 2p < 3$)

$\mathcal{O}_- \in \Omega^p(\partial\text{AdS}) \longrightarrow$ approximately conserved current: $\mathrm{d} * \langle \mathcal{O}_- \rangle \sim O(m^2)$

Double-trace deformation: $\mathcal{M}_- \int \mathcal{O}_- \wedge *\mathcal{O}_-$

Alternative Quantisation

$\mathcal{O}_+ \in \Omega^p(\partial\text{AdS}) \longrightarrow$ pseudo-goldstone: $\mathrm{d} * \langle *\mathrm{d}\mathcal{O}_+ \rangle \sim O(m^2)$

Double-trace deformation: $\mathcal{M}_+ \int \mathcal{O}_+ \wedge *\mathcal{O}_+$

Standard Quantisation

$1 \leq p \leq d-1$

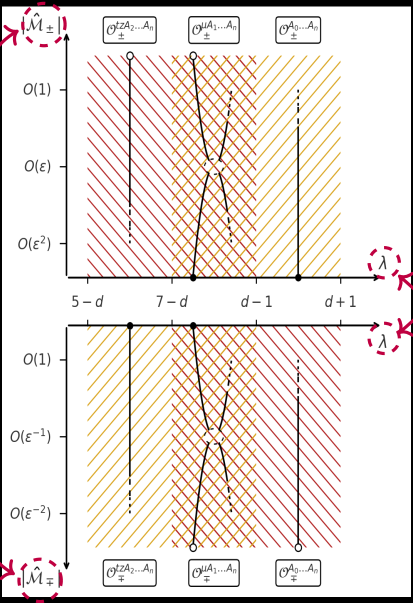
$\lambda < 3$

$\mathcal{O}_\pm$

$\lambda > 3$

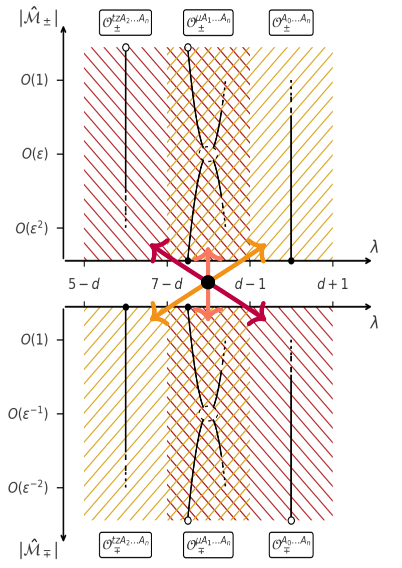
deformation
temperature

$\mathcal{O}_\mp$



$\lambda = d + 3 - 2p$

$k \sim m \sim \varepsilon \ll 1$



massive duality $=$

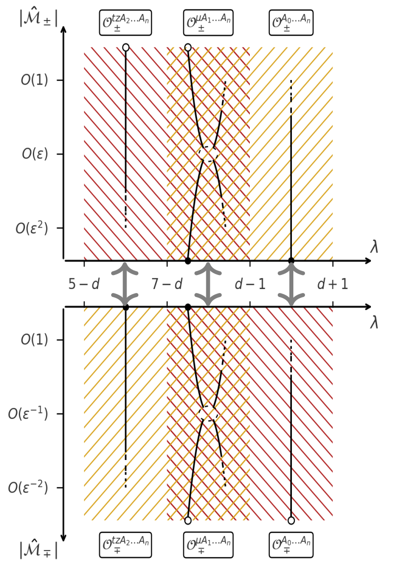
$$\lambda \leftrightarrow 6-\lambda$$

$$\oplus$$

$$\mathcal{O}_\pm \rightleftarrows \mathcal{O}_\mp$$

$$\oplus$$

$$\begin{cases} \hat{\mathcal{M}}_+ \leftrightarrow m^2 \hat{\mathcal{M}}_-, \lambda < 3 \\ \hat{\mathcal{M}}_+ \leftrightarrow m^{-2} \hat{\mathcal{M}}_-, \lambda > 3 \end{cases}$$



"strong/weak coupling" duality =

$$\mathcal{O}_\pm \leftrightarrow \mathcal{O}_\mp$$
$$\oplus$$
$$\hat{\mathcal{M}}_\pm \leftrightarrow 1/\hat{\mathcal{M}}_\mp$$

Thank you!