Quantum corrections to the path integral of near extremal de Sitter black holes

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Based on [2503.14623] with **Matthew Blacker, Alejandra Castro, and Chiara Toldo**

Swedish Research Council

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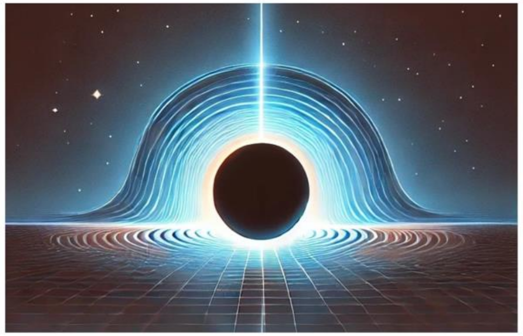
The Nordic Institute for Theoretical Physics

New Insights in Black Hole Physics from Holography




2 1204 Vasudevan / Enuganti
2 1203 Guinet
2 1202 Andronico
2 1201 Santolaria / Gómez Lucas
1 1104 Black / Brane
1 1103 Feng
1 1102 Murtazina
1 1101 Christian / Rüchardt

Motivation of this talk

- Black hole conference, why de Sitter spacetime?

- Euclidean path integral provides many quantum corrected insights, in absence of theory of quantum gravity.

- Utilise recent insights of computing path integral corrections using low energy effective theory

$$Z_{\text{low T}} \sim \exp\left(-I(\bar{g}, \bar{A}) + \# \log T\right)$$

- For black holes it has shown corrected thermo, density of states, black hole evaporation, …

See also Leopoldo's talk

- One hand: investigate applicability this method
  Other hand: learn something about de Sitter spacetime

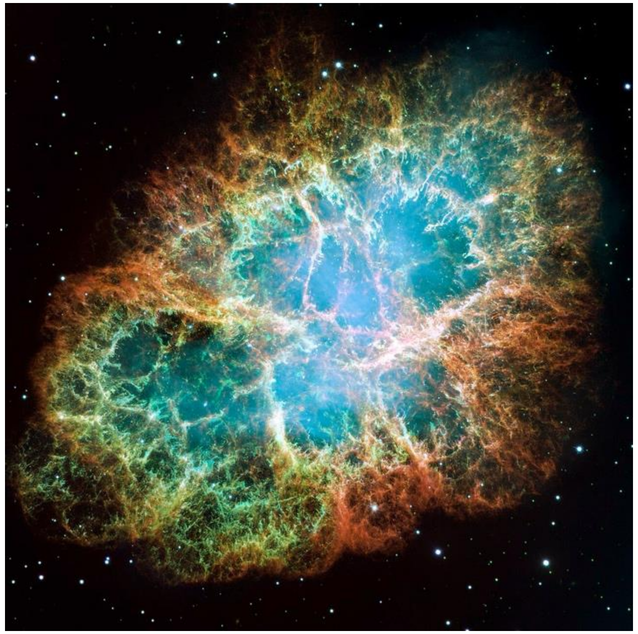
- Corrections to de Sitter partition function can help for matching predictions from UV, de Sitter thermo, holography, …

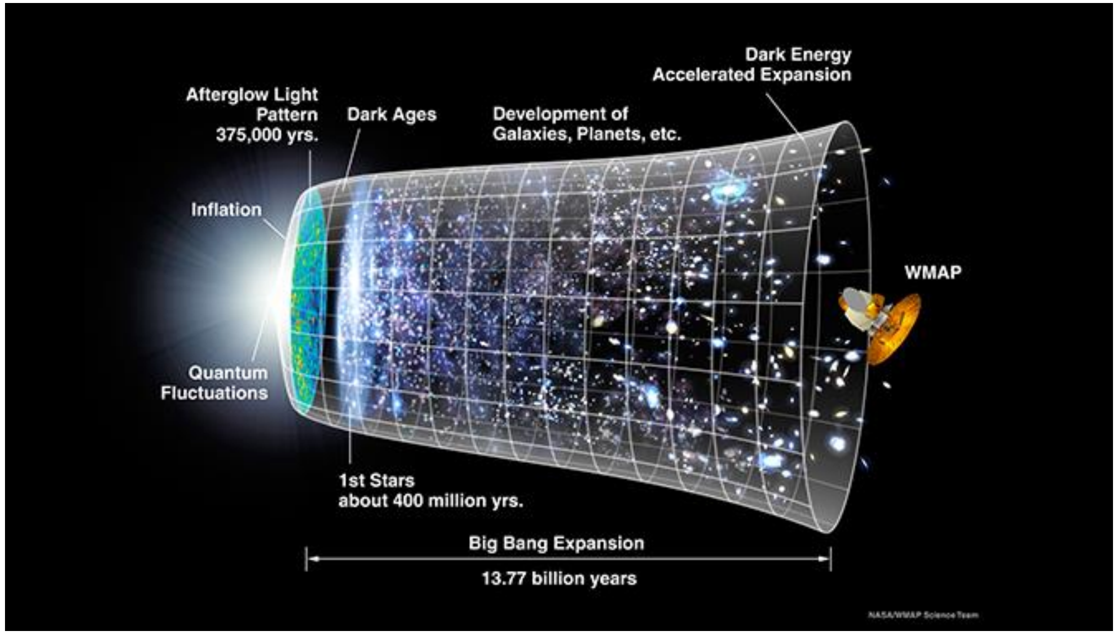
Plan for today
1 de Sitter spacetime basics,
2 Black Holes in de Sitter spacetime,
3 Essence of log-T corrections,
4 De Sitter black holes and log-T corrections

de Sitter spacetime basics

A rapidly expanding universe

Current expanding phase — e.g. Supernova redshift observations, baryonic accoustic osscilations



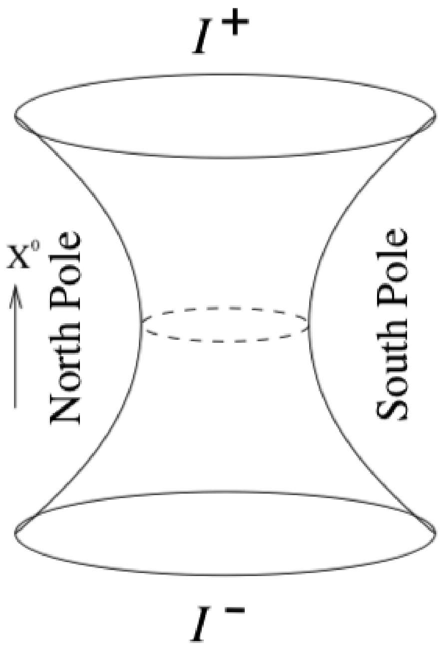


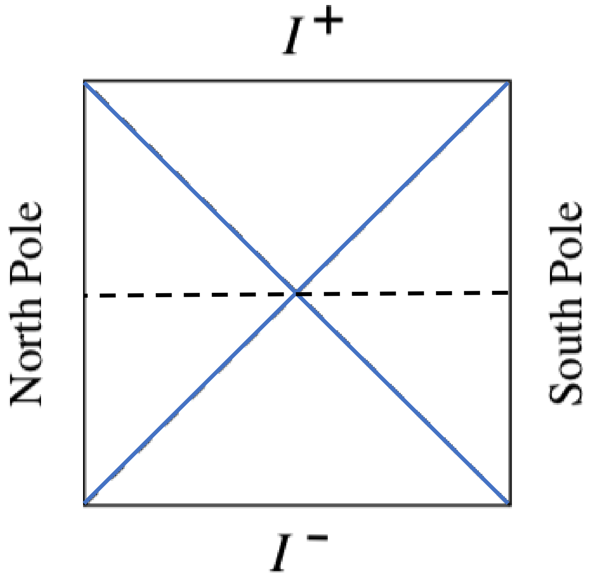
Past expanding phase — e.g. CMB observations; COBE, WMAP

See e.g. review by ['23 Galante]

De Sitter geometry

D-dimensional de Sitter space as a hypersurface in D+1 Minkowski $-X_0^2 + X_1^2 + \ldots + X_D^2 = \ell^2$





Dashed line represents S^{D-1}

Every interior point represents S^{D-2}

Poles represent a point

No single observer at pole can access all of space

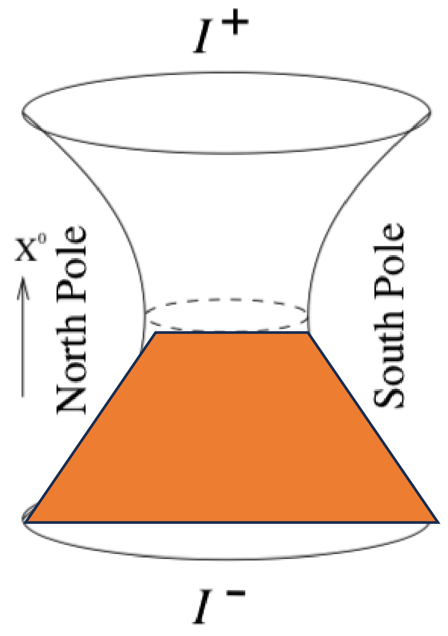
Solution to EE with positive cosmological constant

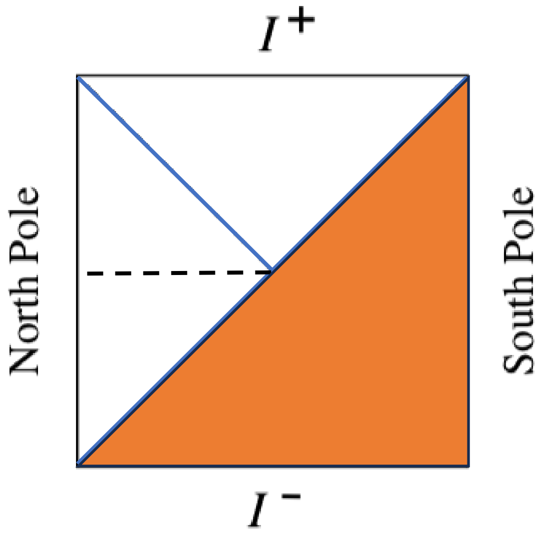
Figures from [Spradlin, Strominger, Volovich'01]

Connection to rapid expansion: planar coordinates

hypersurface in D+1 Minkowski

$$-X_0^2 + X_1^2 + \ldots + X_D^2 = \ell^2$$

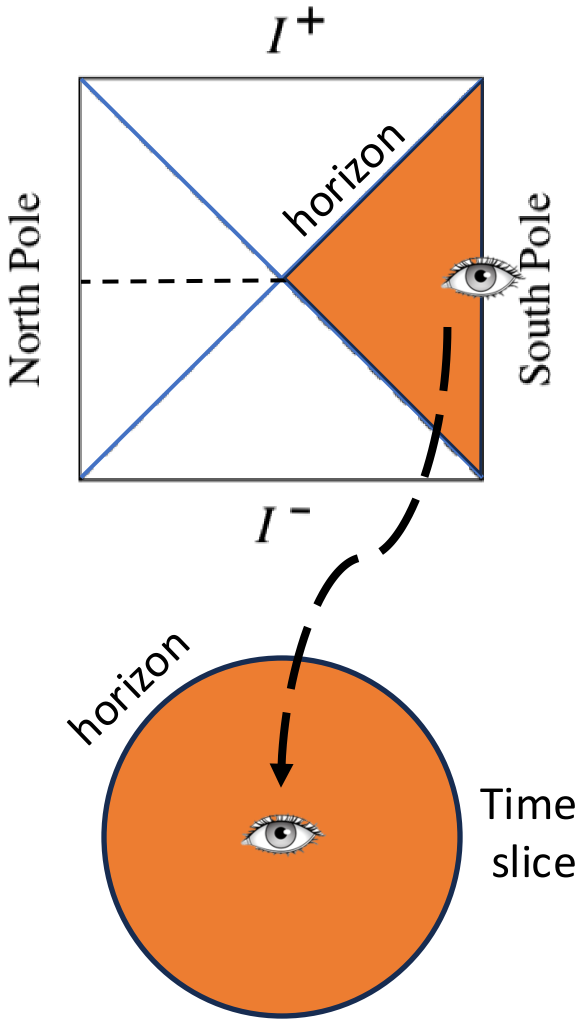




$$\begin{aligned}
X_0 &= \ell \sinh \frac{t}{\ell} - \frac{x_i x^i}{2\ell} e^{-\frac{t}{\ell}}\,, \\
X_i &= x^i e^{-\frac{t}{\ell}}\,, \\
X_D &= \ell \cosh \frac{t}{\ell} - \frac{x_i x^i}{2\ell} e^{-\frac{t}{\ell}}\,, \\
ds^2 &= \; -\mathrm{d}t^2 + e^{-2\frac{t}{\ell}} \mathrm{d}x_i \mathrm{d}x^i\,.
\end{aligned}$$

Recognise from FLRW

Connection to thermodynamics: static coordinates patch



$$X_0 = \sqrt{\ell^2 - r^2}\,\sinh\frac{t}{\ell}\,,$$
$$X_i = r^i w^i\,,$$
$$X_D = \sqrt{\ell^2 - r^2}\,\cosh\frac{t}{\ell}\,,$$
$$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{1 - \frac{r^2}{\ell^2}} + r^2 d\Omega_{d-2}^2\,.$$

Timelike Killing symmetry

Some similarities with BH horizon

- ★ Covers part spacetime
- ★ $T_{\mathrm{dS}} = \dfrac{1}{2\pi\ell}$
- ★ $S_{\mathrm{dS}} = \dfrac{\mathrm{Area}}{4G_N}$

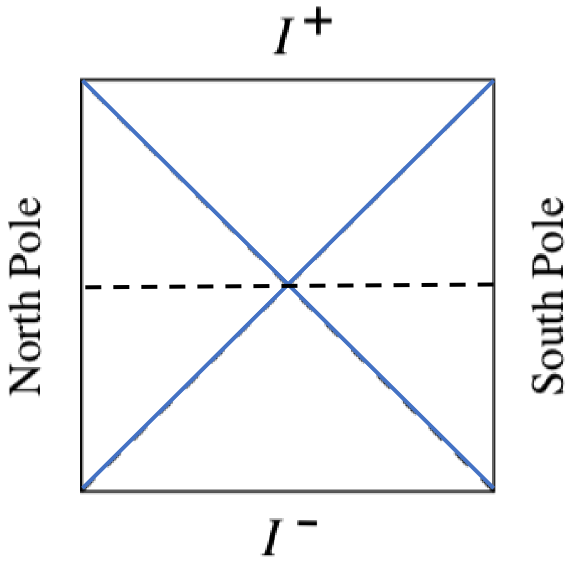
Some differences with BH horizon

- ★ Observer dependent
- ★ Set by hyperparameter
- ★ First law minus sign

[Gibbons, Hawking ‘77]

Aside: Some holographic attempts

Absence of string theoretical guide or sensible decoupling

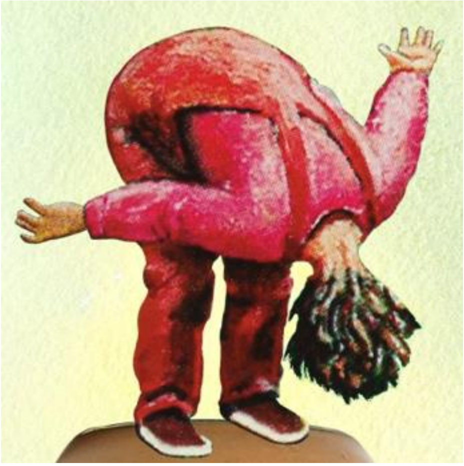


dS/CFT – e.g. higher spin realisations

e.g. [Strominger ‘01][Anninos, Hartman, Strominger '11]

Static patch – e.g. 2d gravity/SYK and role observer

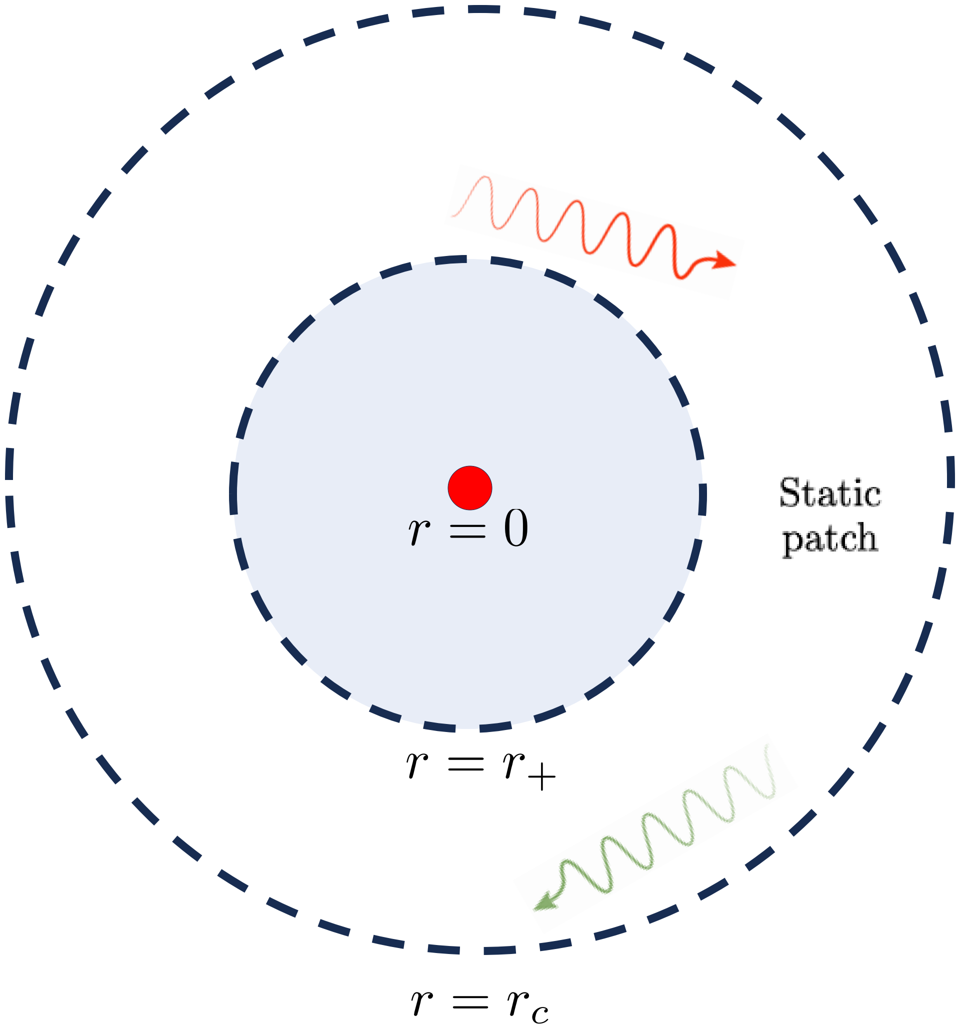
e.g. works by Susskind, and Verlinde and others

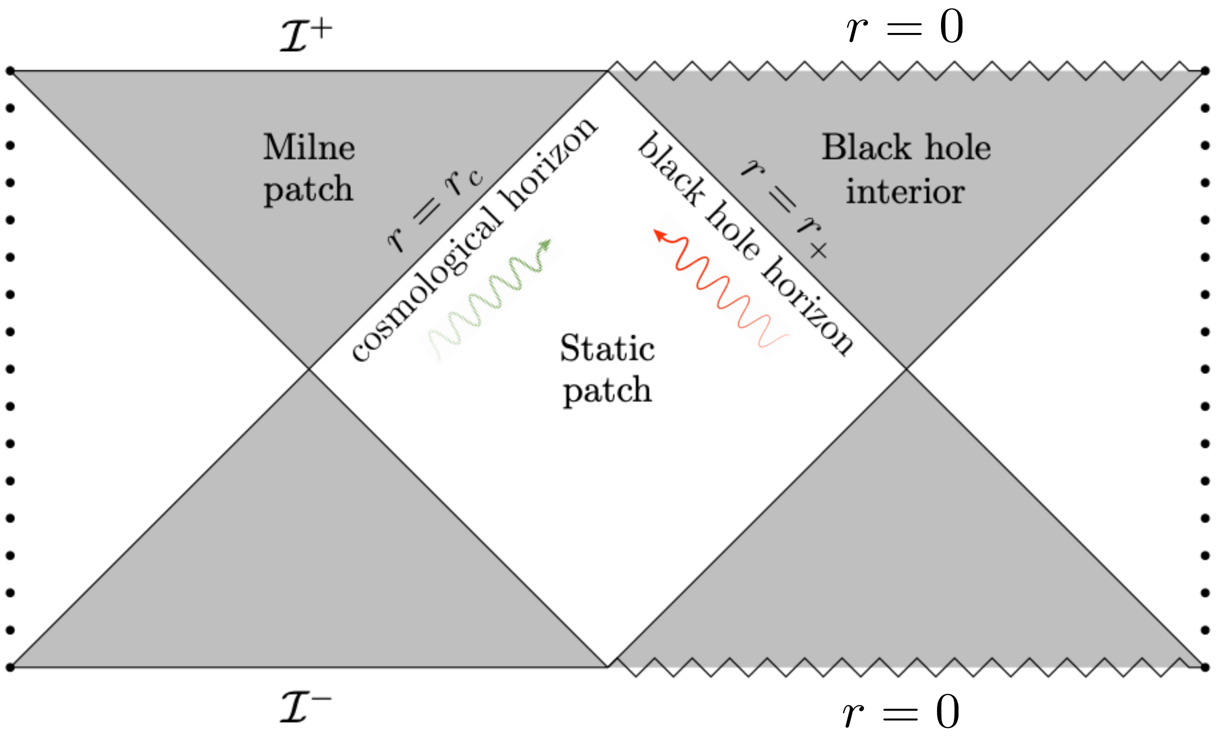


Perhaps describe different aspects of de Sitter?

Black Holes in de Sitter spacetime

Schwarzschild-de Sitter

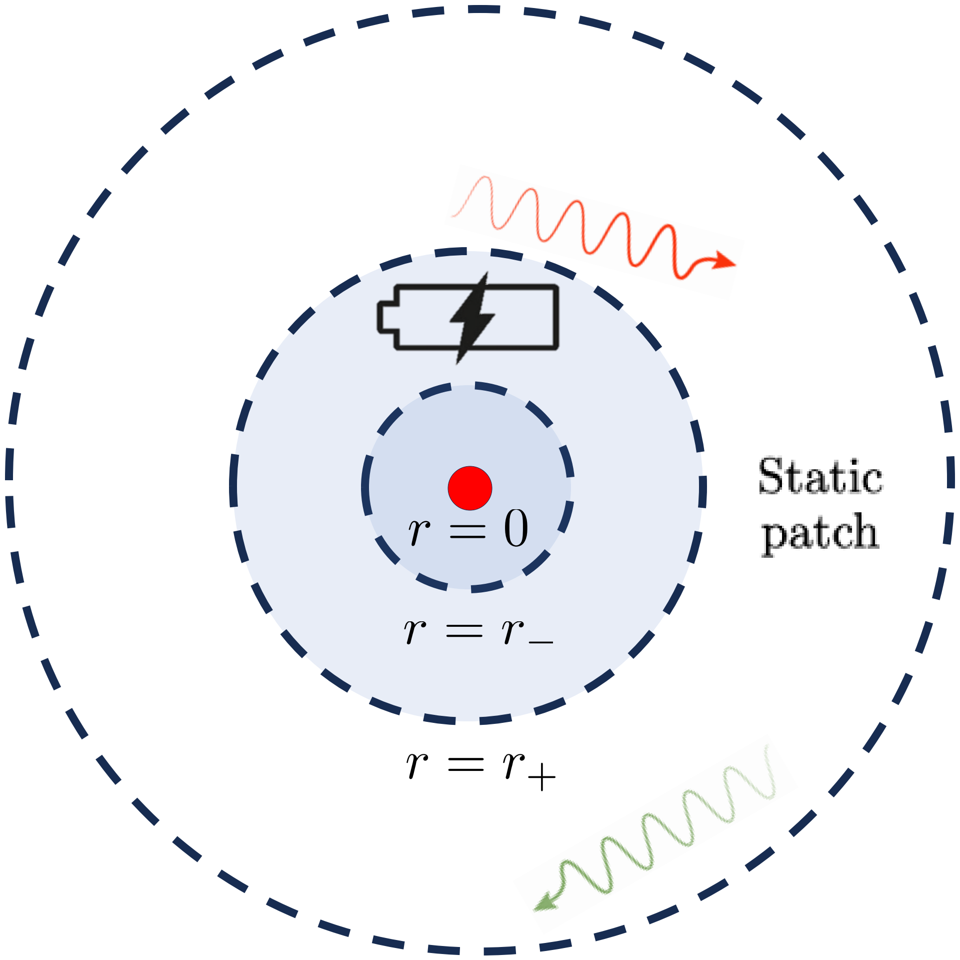


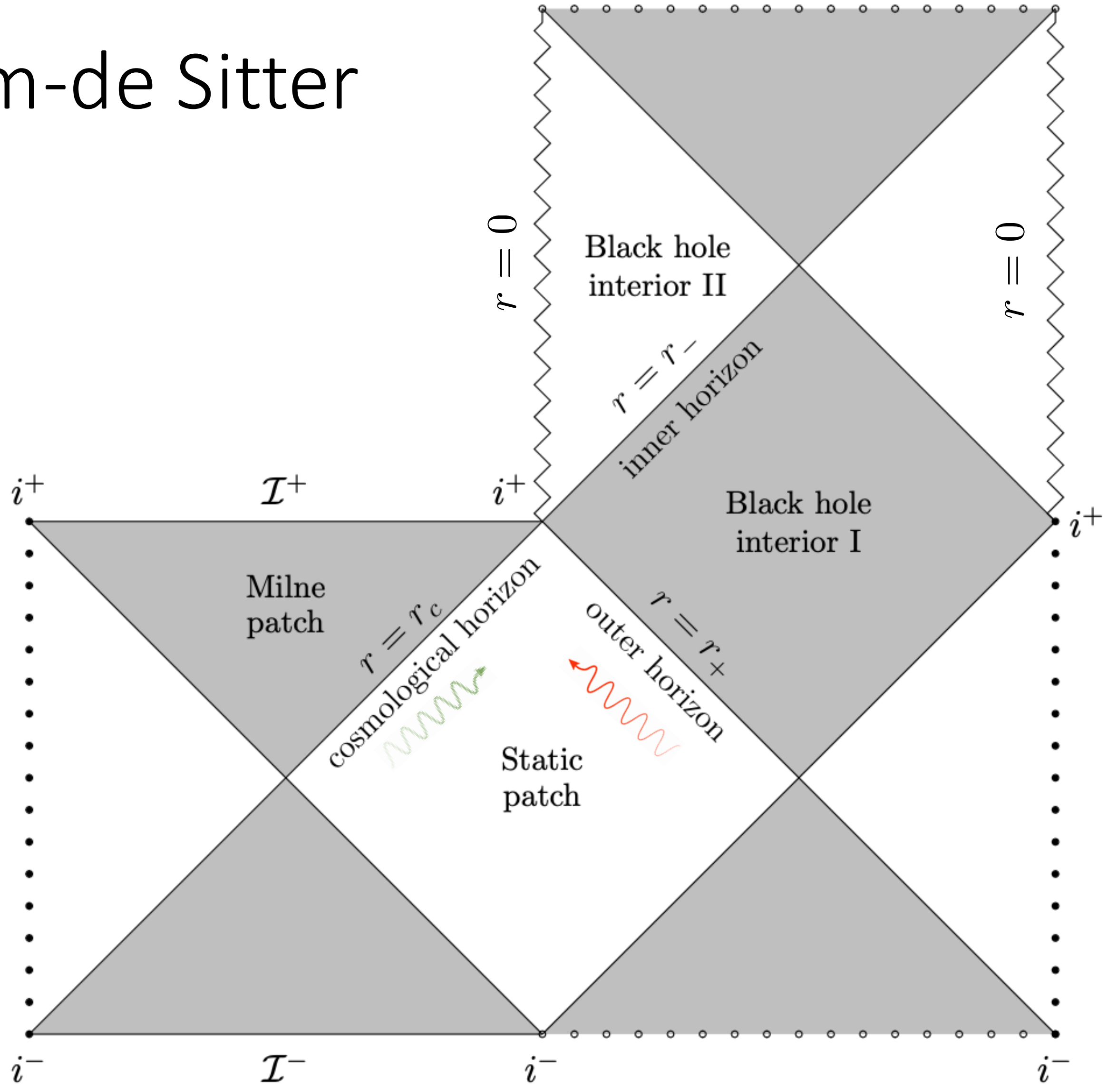


Nariai limit: $r_+ = r_c$

[Nariai '50]

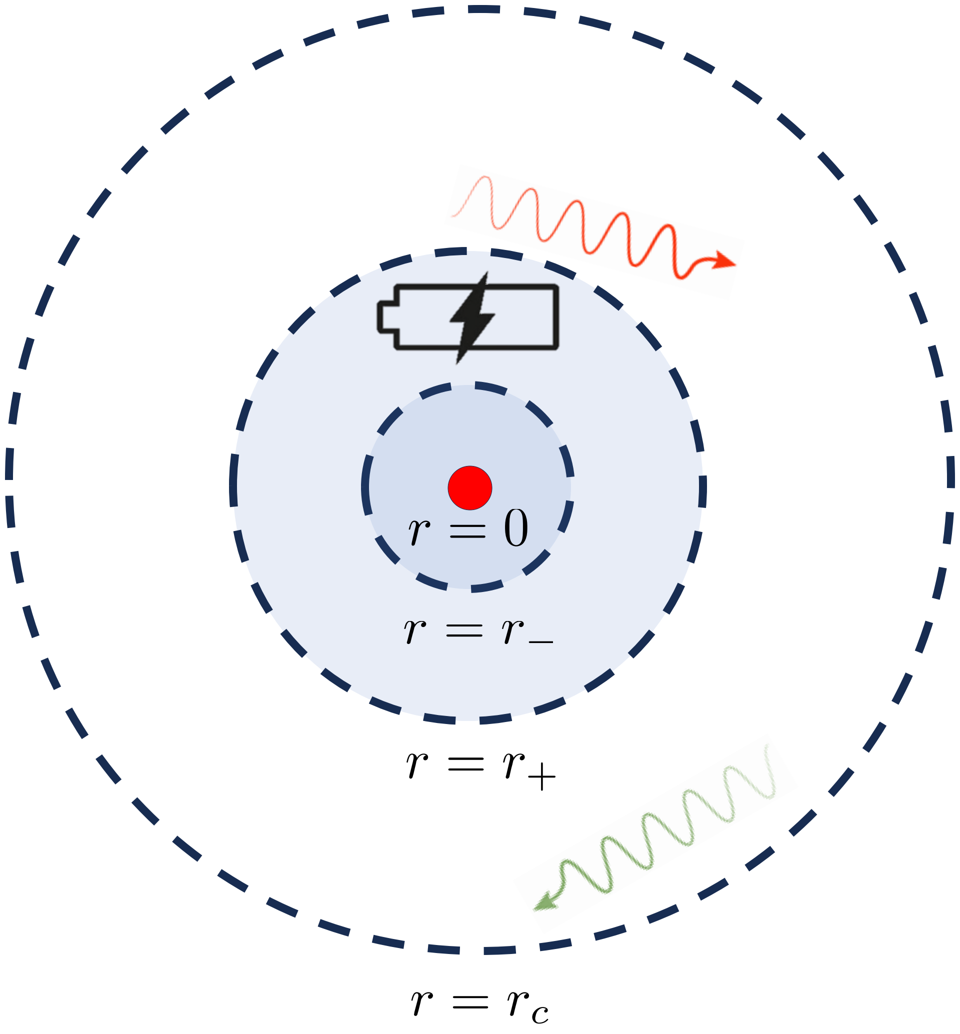
Reissner-Nordström-de Sitter

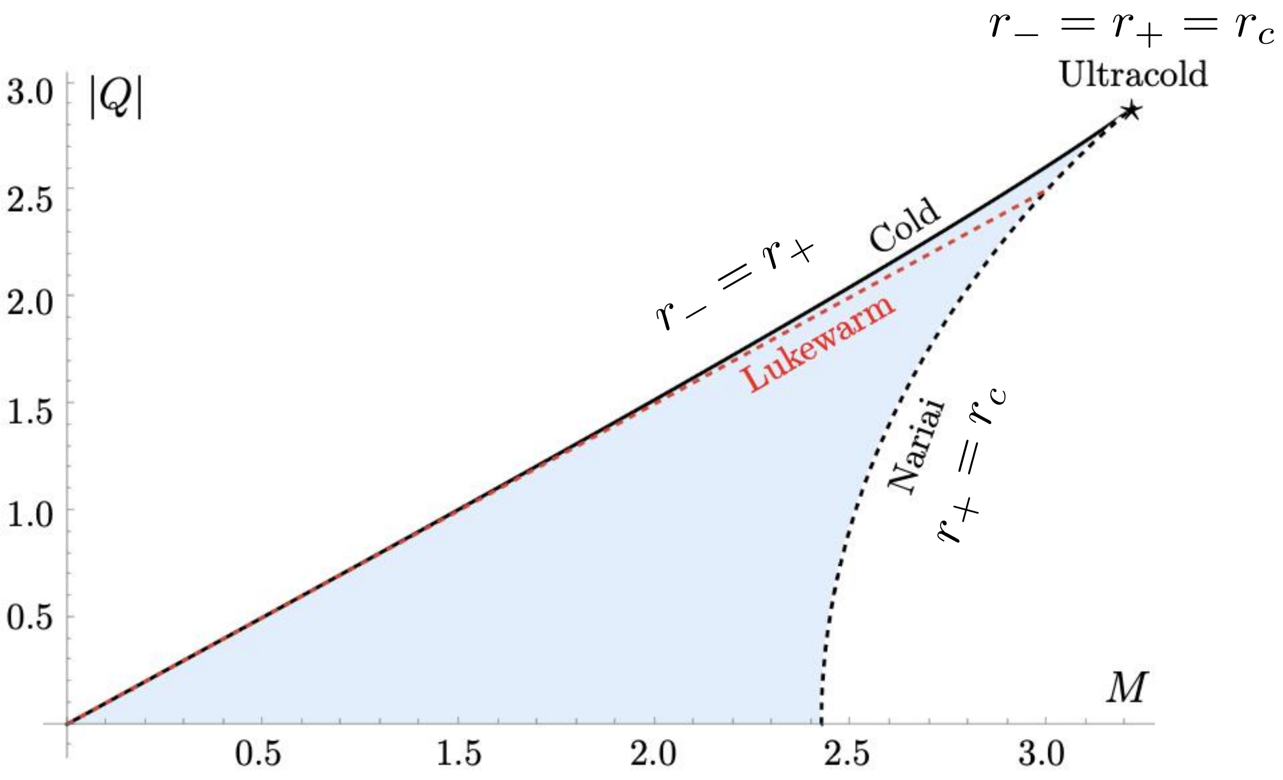




More than only Nariai limit

Decoupling limits and the "Shark Fin"





Cold	Nariai	Ultracold
$AdS_2 x S^2$	$dS_2 x S^2$	$Mink_2 x S^2$

See e.g. [Castro, Mariani, Toldo '22]
or [Climent, Hennigar, Panella, Svesko]

Essence of log-T corrections

Setting up the Euclidean path integral

Consider Einstein-Hilbert-Maxwell action: $$Z = \int [Dg]\,[DA]\,e^{-I[g,A]}\,, \qquad I[g,A] = I_{\rm EM} + I_{\rm boundary} + I_{\rm gauge}\,.$$

Dirichlet Metric, Neumann Gauge field; transeverse-traceless + Lorenz

Background and fluctuations respectively: $$g = \bar{g} + h \qquad A = \bar{A} + \frac{1}{2}a \qquad\qquad Z \approx Z_0 = \exp\left(-I[\bar{g},\bar{A}]\right)$$

Which up to second order we can collect as: $$Z \approx \exp\left(-I[\bar{g},\bar{A}]\right) \int [Dh][Da] \exp\Big[- \int d^4x \sqrt{\bar{g}} \Big(h^* D[\bar{g},\bar{A}] h + a^* P[\bar{g},\bar{A}] a + \left(h^* O_{\rm int}[\bar{g},\bar{A}] a + \text{h.c} \right) \Big) \Big]$$

Evaluate determinants

[Gibbons, Hawking '77]

Approach for evaluating the path integral

$$Z \approx \exp\left(-I[\bar{g}, \bar{A}]\right) Z_{\text{z.m.}} \, Z_{\text{n.z.m.}} \qquad\qquad Z_{\text{z.m.}} = \sum_{\text{zero-modes}} \int [Dh]_{\text{zero-modes}} \int [Da]_{\text{zero-modes}}$$

simplifies near extremality!

Extremal RN: the background geometry $AdS_2 \times S^2$ will have an infinite set of zero modes due to diffeos that act on the AdS_2 geometry, and diffeos on S^2, which are not removed by gauge-fixing. At zero temperature these will cause an IR divergence.

we will regularise the zero modes by slightly moving away from extremality (giving them a mass), by heating up black hole:

Dominant at low T → $Z_{\text{z.m.}} \sim e^{\# \log T}$ $\qquad Z_{\text{n.z.m.}} \sim e^{T^{a>0}}$

See e.g. [Charles, Larsen'20][Iliesiu, Turiaci'21][Heydeman, Ilisesiu, Turiaci, Zhao'22][Iliesu, Murthy, Turiaci '22][Maulik, Pando Zayas, Ray,Zhang'24]

How to get to Log-T? Characterisation of zero modes

Geometry at extremality $\mathcal{M} \times S^2,$

Conventions for background geometry
$$\bar{g}_{\mu\nu} dx^\mu dx^\nu = \bar{g}_{ab} dx^b dx^a + \bar{g}_{ij} dx^i dx^j \ ,$$
$$\bar{A} = \bar{A}_a dx^a \ ,$$

Gauge transformations that are not fixed by gauge fixing but annihilated by D and P:

Fluctuations generated by diffeo: $h_{ab} = \mathcal{L}_\zeta \bar{g}_{ab} \ ,\qquad \zeta = \zeta^a \partial_a \ . \qquad \zeta^a \zeta_a \to \infty.$

Inner product to be finite and nonzero: $\langle h^{(k)} | h^{(k')} \rangle \equiv \int d^4x \sqrt{\bar{g}}\, (h_{\mu\nu}{}^{(k)})^* \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} h^{(k')}_{\alpha\beta}$

Use 2d gravity techniques to compute these modes

options

(i) tensor modes, which are diffeomorphisms on M.
(ii) vector modes, which correspond to diffeomorphism that deform S^2 along M.
(iii) U(1) gauge transformations acting on A.

Turning the crank explicitly for tensor mode in Reissner-Nordström

We are now able to compute the tensor mode contribution

Extremal charge

Explicit metric at extremality: $ds^2 = Q^2(-\sinh^2\eta\, dt^2 + d\eta^2) + Q^2(d\theta^2 + \sin^2\theta d\phi^2)$

Yielding (Wick rotated): $h^{(n)}_{\mu\nu} dx^\mu dx^\nu = \frac{\sqrt{|n|(n^2-1)}}{2\pi} \frac{(\sinh\eta)^{|n|-2}}{(1+\cosh\eta)^{|n|}} e^{in\tau}(d\eta^2 + 2i\frac{n}{|n|}\sinh\eta\, d\eta d\tau - \sinh^2\eta d\tau^2)$

Turning on the temperature: $\frac{\delta g_{\mu\nu} dx^\mu dx^\nu}{4\pi Q^3 T} = (2+\cosh\eta)\tanh^2\frac{\eta}{2}(d\eta^2 - \sinh^2\eta d\tau^2) + \cosh\eta(d\theta^2 + \sin^2\theta d\phi^2)$

Recall: $Z \approx \exp\left(-I[\bar g, \bar A]\right) \int [Dh][Da] \exp\Big[-\int d^4x\sqrt{\bar g}\Big(h^* D[\bar g, \bar A]h + a^* P[\bar g, \bar A]a\Big)\Big]$

Low temperature zero mode part: $\delta \log Z \sim -\sum_{n\geq 2} \log\langle h^{(n)}|\delta D|h^{(n)}\rangle = \log\left(\prod_{n\geq 2}\frac{16Q}{nT}\right) = \log\left(\frac{1}{64\sqrt{2\pi}}\frac{T^{3/2}}{Q^{3/2}}\right)$

see e.g. [Banerjee, Saha ‘23]

Log-T corrections for near extremal RN

Final result (input explicit modes, zeta regularisation)

$$Z_{\text{low-}T} \sim \int \mathcal{D}h_{\text{z.m.}}\mathcal{D}a_{\text{z.m.}}e^{-\delta\Lambda_h(T)\langle h_{\text{z.m.}}|h_{\text{z.m.}}\rangle-\delta\Lambda_a(T)\langle a_{\text{z.m.}}|a_{\text{z.m.}}\rangle}$$

Eigenvalue:

$$\delta\Lambda_h(T)=\frac{\langle h_{\text{z.m.}}|\delta D|h_{\text{z.m}}\rangle}{\langle h_{\text{z.m.}}|h_{\text{z.m}}\rangle}$$

For example for near extremal RN

Near-horizon geometry	Partition function $\mathcal{Z}_{\text{low }T}\propto$
$EAdS_2\times S^2$	T^3

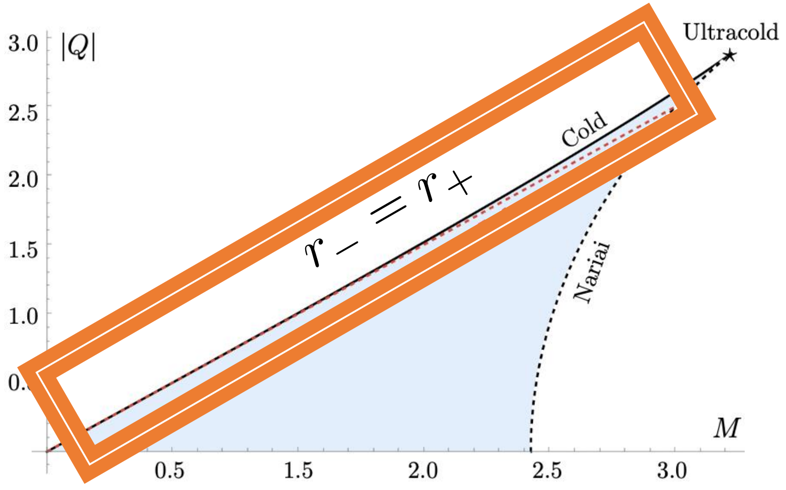
Tensor modes		Vector modes		$U(1)$ modes	
$\langle h^0\|h^0\rangle$	$\lambda_h(T)$	$\langle h^0\|h^0\rangle$	$\lambda_h(T)$	$\langle a^0\|a^0\rangle$	$\lambda_a(T)$
$+$	$+$	$+$	$+$	$+$	0

see e.g. [Banerjee, Saha '23]

De Sitter black holes and log-T corrections

Cold results

Cold
AdS_2xS^2

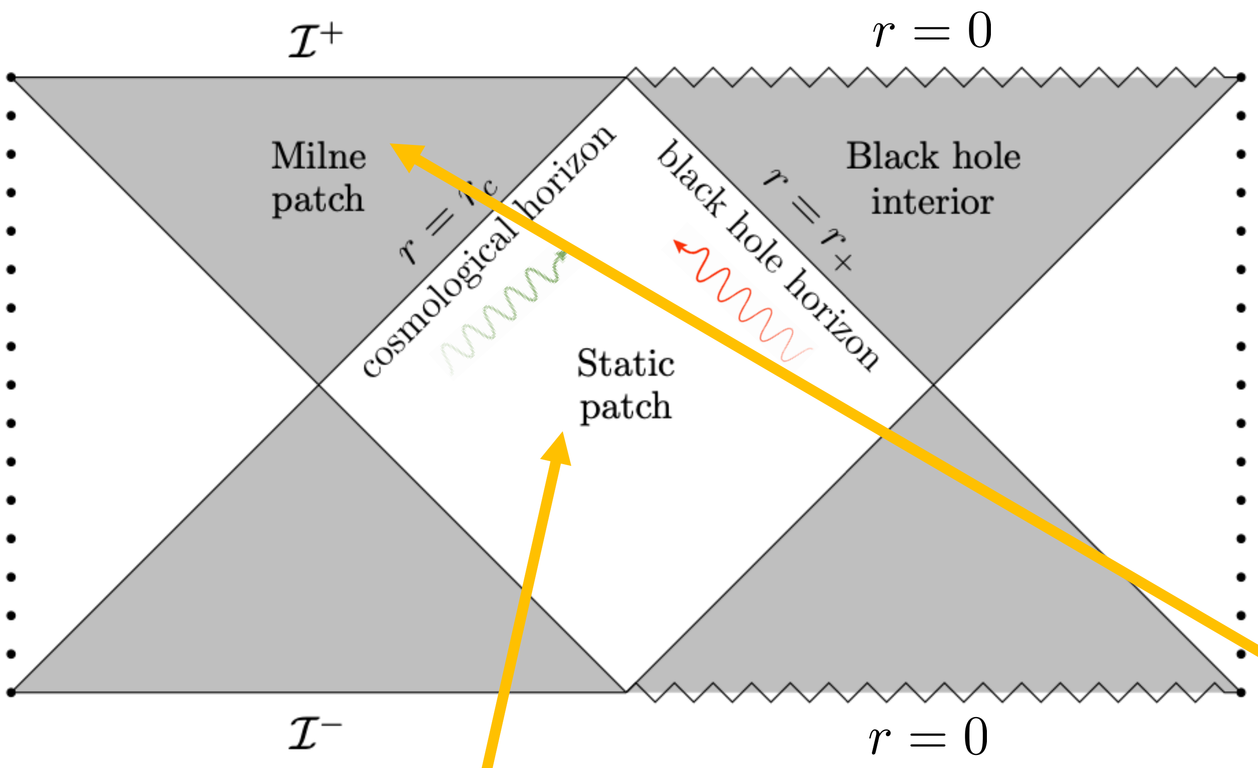


$$Z_{\text{low-}T} \sim \int \mathcal{D}h_{\text{z.m.}}\mathcal{D}a_{\text{z.m.}}e^{-\delta\Lambda_h(T)\langle h_{\text{z.m.}}|h_{\text{z.m.}}\rangle-\delta\Lambda_a(T)\langle a_{\text{z.m.}}|a_{\text{z.m.}}\rangle}$$

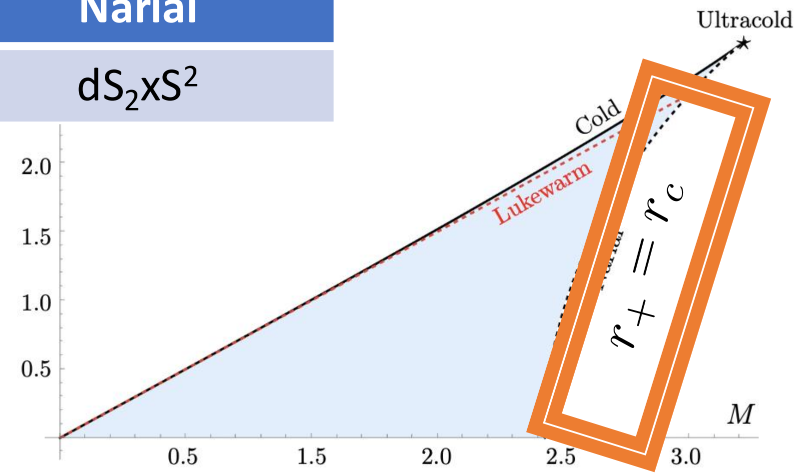
Decoupling limits of RN-dS$_4$	Near-horizon geometry	Partition function $\mathcal{Z}_{\text{low }T} \propto$
Cold	$EAdS_2 \times S^2$	$T^{7/2}$
vanilla RN	$EAdS_2 \times S^2$	T^3

Tensor modes		Vector modes		$U(1)$ modes	
$\langle h^0\|h^0\rangle$	$\lambda_h(T)$	$\langle h^0\|h^0\rangle$	$\lambda_h(T)$	$\langle a^0\|a^0\rangle$	$\lambda_a(T)$
+	+	+	+	+	+
+	+	+	+	+	0

Nariai results



Nariai
dS_2xS^2



S^2 x S^2 Euclidean geometry
No zero mode candidate

$(-EAdS)^2$ x S^2 Euclidean geometry
zero mode candidates
(-,-,+,+) signature

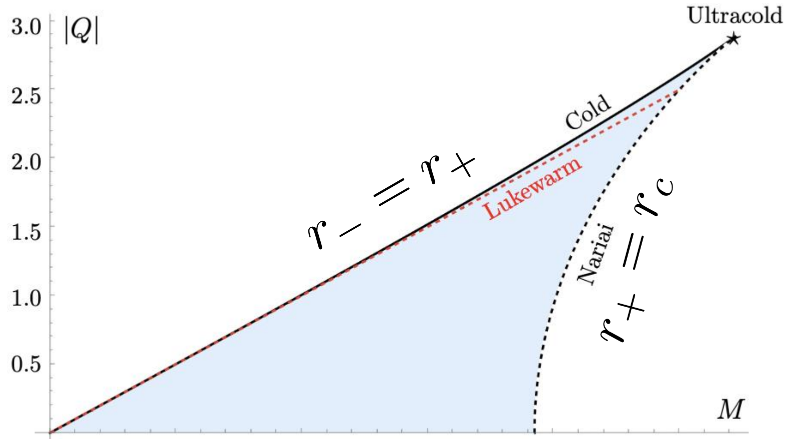
Nariai results

$$Z_{\text{low-}T} \sim \int \mathcal{D}h_{\text{z.m.}}\mathcal{D}a_{\text{z.m.}}\, e^{-\delta\Lambda_h(T)\langle h_{\text{z.m.}}|h_{\text{z.m.}}\rangle - \delta\Lambda_a(T)\langle a_{\text{z.m.}}|a_{\text{z.m.}}\rangle}$$

Eucl. geometry	Decoupling limits	Tensor modes		Vector modes		Gauge modes		**Partition function** $Z_{\text{low-T}} \sim$
		$\langle h_{\text{z.m.}}\|h_{\text{z.m.}}\rangle$	$\delta\Lambda_h(T)$	$\langle h_{\text{z.m.}}\|h_{\text{z.m.}}\rangle$	$\delta\Lambda_h(T)$	$\langle a_{\text{z.m.}}\|a_{\text{z.m.}}\rangle$	$\delta\Lambda_a(T)$	
$(-\text{EAdS})^2 \times S^2$ (-,-,+,+)	SdS_4 Nariai	$+$	$-$	$-$	$-$			unresolved
$(-\text{EAdS})^2 \times S^2$ (-,-,+,+)	RNdS_4 Nariai	$+$	$-$	$-$	$\pm$	$-$	$+$	unresolved
$\text{EAdS}^2 \times (-S^2)$ (-,-,+,+)	SdS_4 Nariai (Complexified)	$+$	$+$	$-$	$+$			T^3
$-(\text{EAdS}^2 \times S^2)$ (-,-,-,-)	RNdS_4 Nariai (Complexified)	$+$	$+$	$+$	$+$	$-$	$-$	unresolved

['25 Blacker, Castro, Sybesma, Toldo]

$$Z_{\text{low T}} \sim \exp\left(-I(\overline{g}, \overline{A}) + \#\log T\right)$$



Decoupling limits	Tensor modes		Vector modes		Gauge modes	
	$\langle h_{\text{z.m.}} \vert h_{\text{z.m.}} \rangle$	$\delta\Lambda_h(T)$	$\langle h_{\text{z.m.}} \vert h_{\text{z.m.}} \rangle$	$\delta\Lambda_h(T)$	$\langle a_{\text{z.m.}} \vert a_{\text{z.m.}} \rangle$	$\delta\Lambda_a(T)$
RNdS_4 Cold	$+$	$+$	$+$	$+$	$+$	$+$
SdS_4 Nariai	$+$	$-$	$-$	$-$		
SdS_4 Nariai (Complexified)	$+$	$+$	$-$	$+$		
RNdS_4 Nariai	$+$	$-$	$-$	$\pm$	$-$	$+$
RNdS_4 Nariai (Complexified)	$+$	$+$	$+$	$+$	$-$	$-$

Decoupling limits	Near-horizon geometry	Partition function $Z_{\text{low-}T} \propto$
RNdS_4 Cold	$\text{EAdS}_2 \times S^2$	$T^{7/2}$
SdS_4 Nariai	$-\text{EAdS}_2 \times S^2$	unresolved
SdS_4 Nariai (Complexified)	$\text{EAdS}_2 \times (-S^2)$	T^3
RNdS_4 Nariai	$(-\text{EAdS}_2) \times S^2$	unresolved
RNdS_4 Nariai (Complexified)	$-(\text{EAdS}_2 \times S^2)$	unresolved

- Negative norms, eigenvalues
- Complements JT predictions; tensor modes not enough [Maldacena, Turiaci, Yang '19][Cotler, Jensen Maloney'19]
- Log T in Static patch?
- DHS; away from near horizon [Denef, Hartnoll, Sachdev '10] e.g. [Kapec, Law, Toldo '24][Arnaudo, Bonelli, Tanzini '24]
- ultracold limit?

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Back up slides

Identifying the quadratic contributions

Let us take a step back and consider in detail:

$$Z \approx \exp\left(-I[\bar{g},\bar{A}]\right) \int [Dh][Da] \exp\Big[-\int d^4x\sqrt{\bar{g}}\Big(h^* D[\bar{g},\bar{A}]h + a^* P[\bar{g},\bar{A}]a + \left(h^* O_{\text{int}}[\bar{g},\bar{A}]a + \text{h.c}\right)\Big)\Big]$$

graviton

$$\begin{aligned} h^*_{\alpha\beta}\, D^{\alpha\beta,\mu\nu}[\bar{g}]\, h_{\mu\nu} = & -\frac{1}{16\pi} h^*_{\alpha\beta}\Big(\frac{1}{4}\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}\bar{\Box} - \frac{1}{8}\bar{g}^{\alpha\beta}\bar{g}^{\mu\nu}\bar{\Box} + \frac{1}{2}\bar{R}^{\alpha\mu\beta\nu} + \frac{1}{2}\bar{R}^{\alpha\mu}\bar{g}^{\beta\nu} \\ & -\frac{1}{2}\bar{R}^{\alpha\beta}\bar{g}^{\mu\nu} - \frac{1}{4}\bar{R}\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu} + \frac{1}{8}\bar{R}\bar{g}^{\alpha\beta}\bar{g}^{\mu\nu} + \frac{1}{8}\bar{F}^2\left(2\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu} - \bar{g}^{\alpha\beta}\bar{g}^{\mu\nu}\right) \\ & -\bar{F}^{\alpha\mu}\bar{F}^{\beta\nu} - 2\bar{F}^{\alpha\gamma}\bar{F}^{\mu}_{\gamma}\bar{g}^{\beta\nu} + \bar{F}^{\alpha\gamma}\bar{F}^{\beta}_{\gamma}\bar{g}^{\mu\nu} - \frac{\Lambda}{2}\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu} + \frac{\Lambda}{4}\bar{g}^{\alpha\beta}\bar{g}^{\mu\nu}\Big)h_{\mu\nu} \end{aligned}$$

photon

$$a^*_\mu\, P^{\mu\nu}\, a_\nu = -\frac{1}{32\pi} a^*_\mu\left(\bar{g}^{\mu\nu}\bar{\Box} - \bar{R}^{\mu\nu}\right) a_\nu$$

mixed

$$h^*_{\alpha\beta}\, O^{\alpha\beta\mu}_{\text{int}}\, a_\mu = \frac{1}{16\pi} h^*_{\alpha\beta}\left(4\bar{g}^{\alpha[\mu}\bar{F}^{\nu]\beta} + \bar{F}^{\mu\nu}\bar{g}^{\mu\nu}\right)\nabla_\mu a_\nu$$

Infinitely many eigenvalues in this determinant are zero or negative

Turning the crank explicitly for tensor mode in Reissner-Nordström

We are now able to compute the tensor mode contribution

Extremal charge

Explicit metric at extremality: $ds^2 = Q^2(-\sinh^2\eta\, \mathrm{d}t^2 + \mathrm{d}\eta^2) + Q^2(d\theta^2 + \sin^2\theta d\phi^2)$

Yielding (Wick rotated): $h^{(n)}_{\mu\nu} dx^\mu dx^\nu = \frac{\sqrt{|n|(n^2-1)}}{2\pi} \frac{(\sinh\eta)^{|n|-2}}{(1+\cosh\eta)^{|n|}} e^{in\tau}(\mathrm{d}\eta^2 + 2i\frac{n}{|n|}\sinh\eta\, \mathrm{d}\eta \mathrm{d}\tau - \sinh^2\eta \mathrm{d}\tau^2)$

Turning on the temperature: $\frac{\delta g_{\mu\nu} dx^\mu dx^\nu}{4\pi Q^3 T} = (2+\cosh\eta)\tanh^2\frac{\eta}{2}(d\eta^2 - \sinh^2\eta d\tau^2) + \cosh\eta(d\theta^2 + \sin^2\theta d\phi^2)$

$$\delta Z_{\text{tensor}} = -\sum_{n\geq 2}\log\delta\Lambda_n = \log\left(\prod_{n\geq 2}\frac{16Q}{nT}\right) = \log\left(\frac{1}{64\sqrt{2\pi}}\frac{T^{3/2}}{Q^{3/2}}\right)$$

$$\delta Z_{\text{vector}} \sim \frac{3}{2}\log T$$

see e.g. [Banerjee, Saha '23]

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