THE UNIVERSITY of EDINBURGH

CENTRE FOR THEORETICAL PHYSICS · Higgs

Spectral duality relation in thermal CFTs

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New Insights in Black Hole Physics from Holography

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Based on

Sašo Grozdanov, M.V.:

2406.19790

2505.14229

Recap

Dualities of perturbations around 4d black holes

⇓ AdS/CFT in large-N limit

Duality relation for retarded correlators: $\underbrace{G_+(\omega)G_-(\omega)}_{\text{same theory}} = \dfrac{\omega^2}{\omega_*^2} - 1$

⇓ Thermal product formula
[Dodelson, Iossa, Karlsson, Zhiboedov, 2023]

Spectral duality relation: $S(\omega) - S(-\omega) = 2i\lambda \sinh \dfrac{\beta\omega}{2}$

$$S(\omega) = \left(1 + \frac{\omega}{\omega_*}\right) \prod_n \left(1 - \frac{\omega}{\omega_n^+}\right)\left(1 + \frac{\omega}{\omega_n^-}\right)$$

ω_*: algebraically special frequency

ω_n^+, ω_n^-: QNMs

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algebraically special frequency

QNMs

Algebraically special solutions

Space is algebraically special if $C_{\mu\nu\rho[\sigma}l_{\alpha]}l^\nu l^\rho = 0$ for some null l^μ.

- all static black holes are algebraically special
- algebraically special perturbations only possible with $\omega = \pm\,\omega_*$

Einstein-Maxwell + massless scalar:

$\Rightarrow$ Schwarzschild: $\omega_* = i\dfrac{k^4}{12M}$

$\Rightarrow$ Reissner-Nordström: $\omega_* = i\dfrac{k^4/6M}{1 \pm \sqrt{1 + 4Q^2k^2/9M^2}}$

$\Rightarrow$ Linear-axion model: $\omega_* = i\dfrac{k^4 + k^2m^2}{12M}$

} Simple computation, non-trivial physics

- On the boundary: $G_+(\omega_*) = G_-(-\omega_*) = 0$
- Classifies pole-skipping points

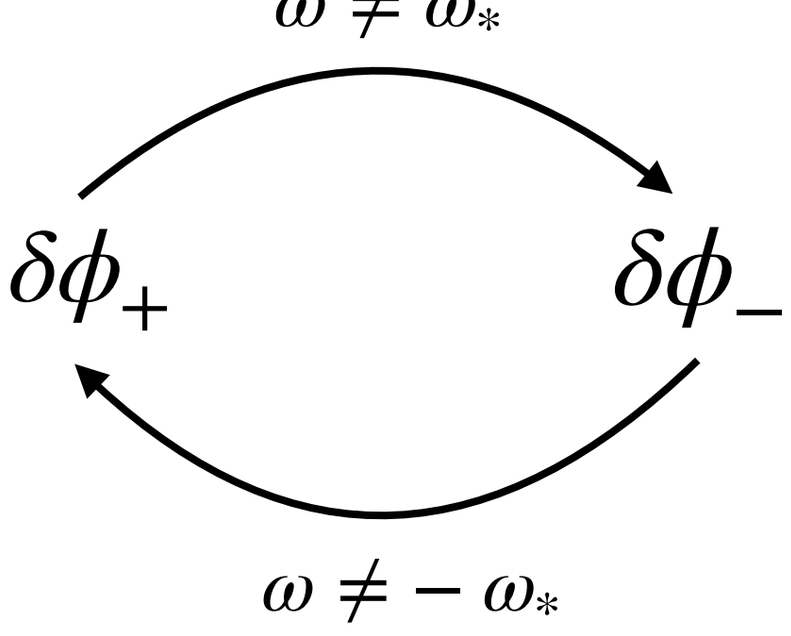
Bulk Dualities

Background: $ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\gamma_{AB}dx^A dx^B$

maximally symmetric $2d$ metric

Linear perturbations (Einstein-Maxwell): $g \to g + \delta g, A \to A + \delta A$

We can map between solutions of the channels:



Darboux transformations/SUSY QM

⇕

‘Mixed’ electric-magnetic-type duality

These transformations leave background alone

Self-dual limit

$$\omega_* \to \infty$$

- $Q \to 0$: Maxwell/electric-magnetic duality
  [Herzog, Kovtun, Sachdev, Son, 2007]

- M $\to$ 0 (pure dS/flat/AdS): graviton electric-magnetic duality
  [Leigh, Petkou, 2003]

- self-dual limit of the linear axion model: emergent SL(2,$\mathbb{R}$) $\times$ SL(2,$\mathbb{R}$)
  [Davison, Goutéraux, 2014]

This limit is holographically understood:
particle-vortex self-duality and its $T^{\mu\nu}$ analogue

Summary

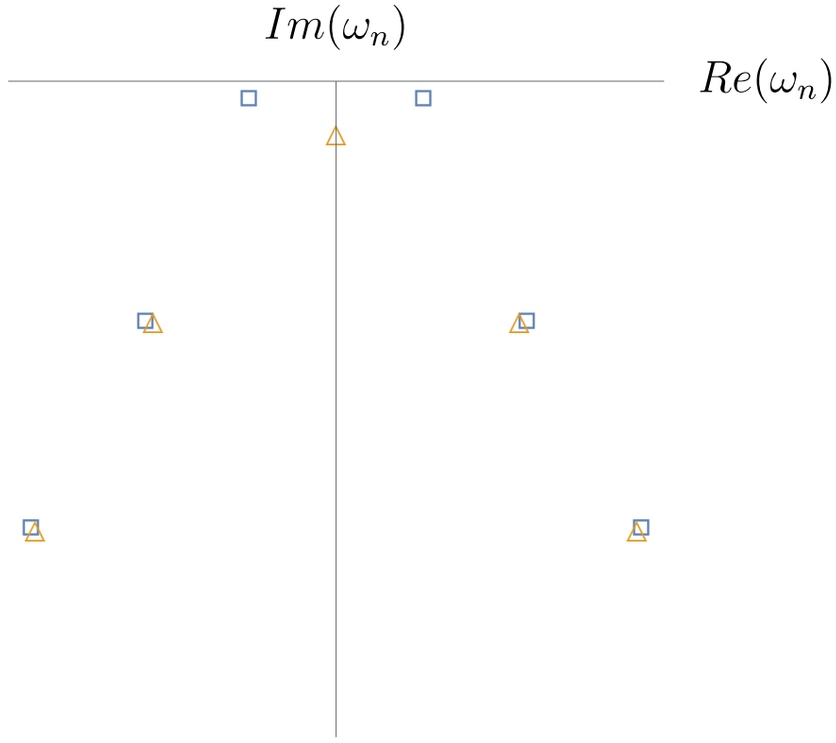
- Such dualities are a very special property of 4d gravity
- Controlled by the algebraically special frequency ω_*
- Their geometry is now better understood
- QNMs:

  $\Rightarrow$ asymptotically flat/dS:

  $$\omega_n^+ = \omega_n^-$$

  $\Rightarrow$ asymptotically AdS $\longrightarrow$

  $$S(\omega) - S(-\omega) = 2i\lambda \sinh \frac{\beta\omega}{2}$$



- QFT meaning unclear
- The extent of generality unclear