Black Hole Entropy and Wormhole Instabilities.

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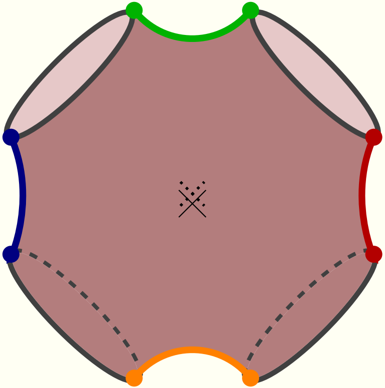

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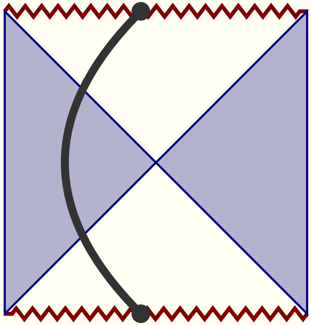
One particular success of holography in the context of Black Hole (BH) physics was explaining the Euclidean Gravity approach pioneered by Gibbons and Hawking.

The key result in Euclidean gravity was the derivation of the formula,

$$S_{BH} = \frac{A_{\mathcal{H}}}{4\,G_N}\,.$$

A microscopic interpretation of the degrees of freedom remains elusive from a semiclassical perspective.

In 2212.02447 V. Balasubramanian, A. Lawrence, J. Magán & M. Sasieta provided an alternative derivation of the Bekenstein-Hawking result.



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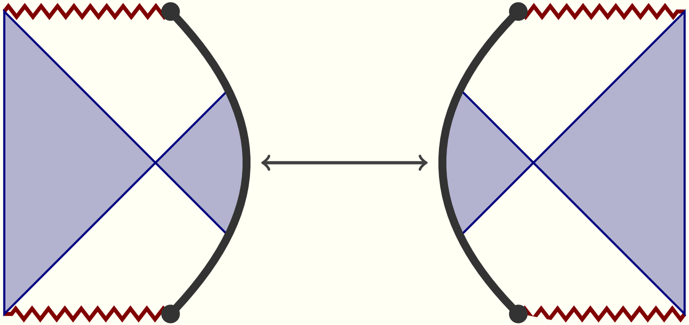

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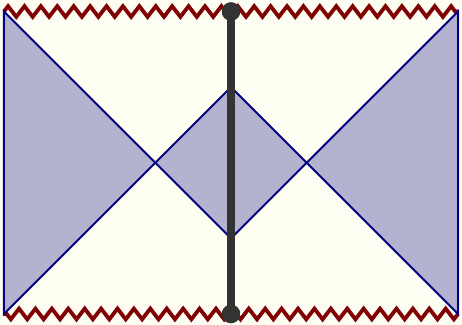
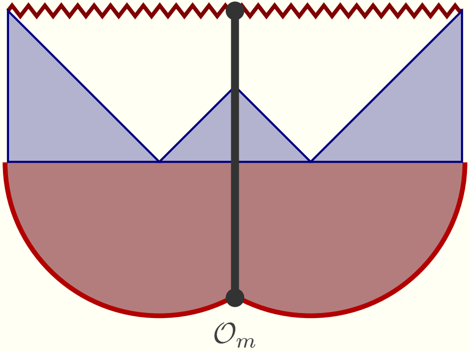
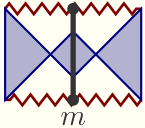
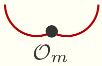
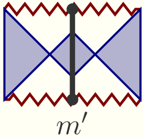
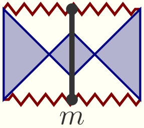
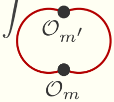


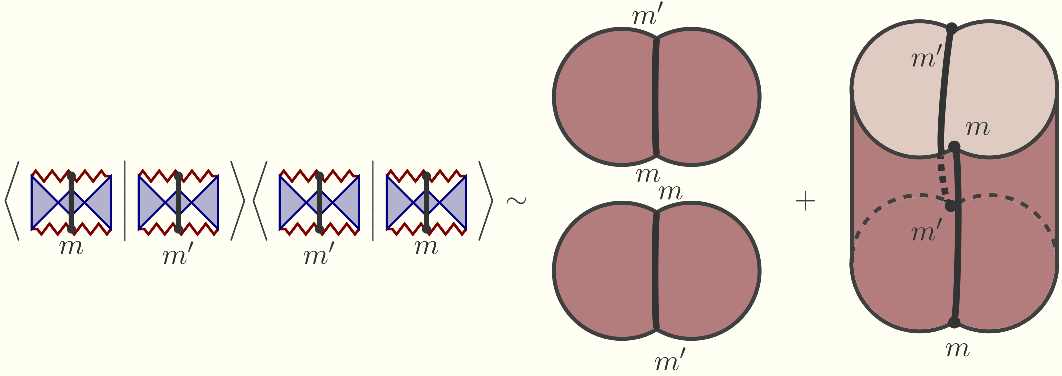


$\mathcal{N}$:= number of states.

$\Big|$  $\Big\rangle = \int$  $\mathcal{D}g \; \exp\left[-I_E[g]\right]$

$\mathcal{G}_{m,m'} := \Big\langle$  $\Big|$  $\Big\rangle = \int$  $\mathcal{D}g \; \exp\left[-I_E[g]\right]$

The Euclidean Gravitational Path Intergal (EGPI) reveals a surprise, as the higher moments of $\mathcal{G}_{m\,m'}$ receive contributions from manifolds that connect the different asymptotic boundaries, i.e., *Euclidean Wormholes*.



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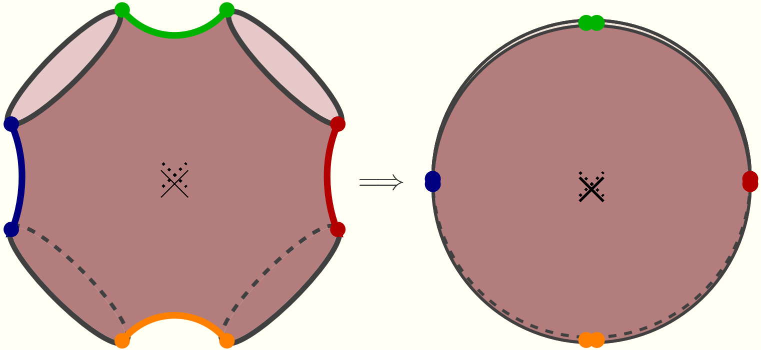
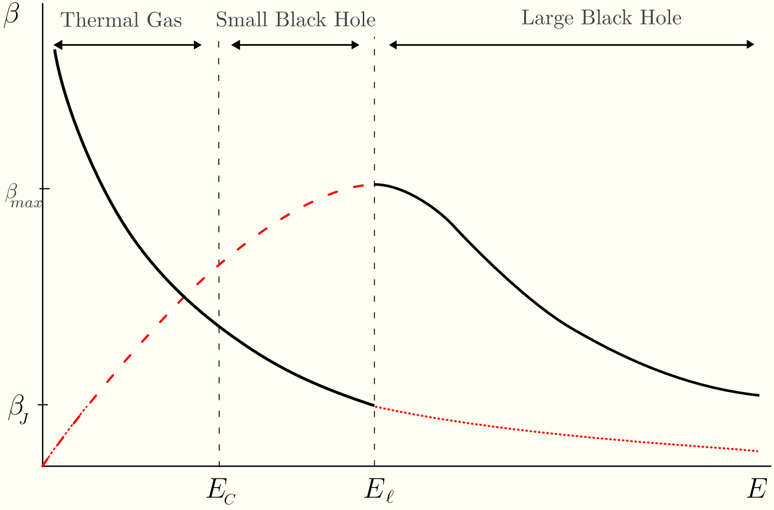
In the m, $m' \to \infty$ *Universality limit*, overlaps and n-moments simplify:



$$\simeq (Z(n\beta))^2 \ .$$

It is necessary to consider these contributions for all non-negative n!

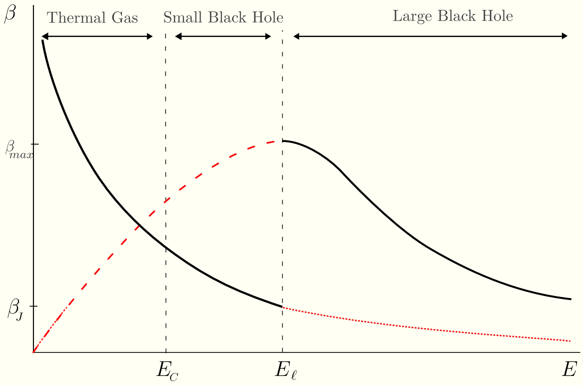
$$\forall \beta > 0 \qquad \exists\, n \in \mathbb{N}, \quad \text{s.t.} \quad n\,\beta > \beta_{max}\,.$$

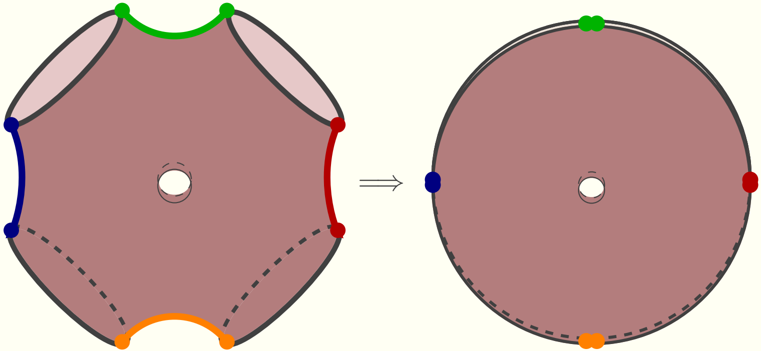


In order to be able to obtain the rank, it is necessary to consider *microcanonical states*. After reducing the support of the states to an energy window one obtains that,

$$\mathrm{rank}\,(\mathcal{G}) = \min\left\{e^{S_{BH}},\, \mathcal{N}\right\}\,.$$

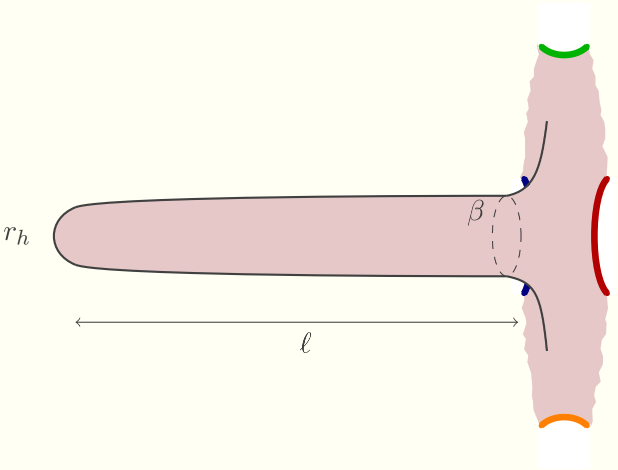
This microcanonical projection is agnostic to whether the BH is *large* or *small*.



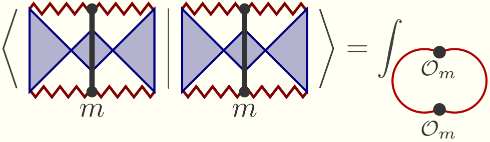


Manifolds degenerate into products of thermal manifolds, i.e., *small/large Black Holes or thermal AdS* inheriting their properties.

Small Black Holes present a single negative eigenvalue for the quadratic operator for graviton fluctuations associated with a thermodynamical instability (Phys. Rev. D 25, 330 D. J. Gross, M. J. Perry & L. G. Yaffe).

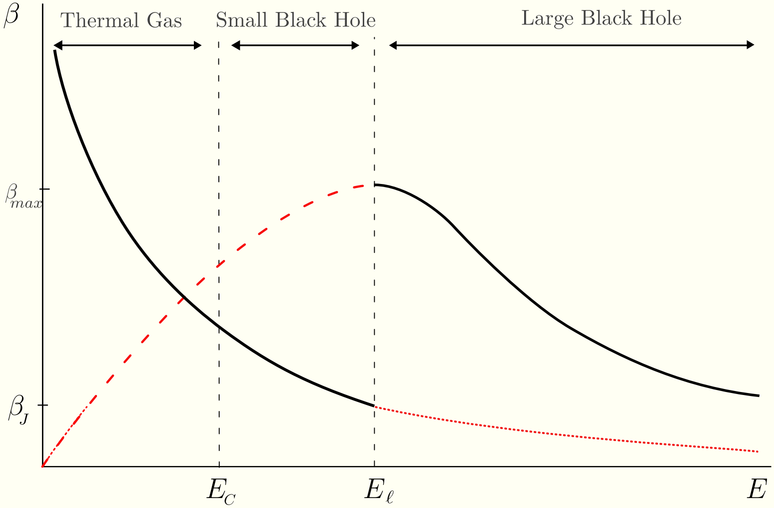


The *Jeans instability* corresponds to a tachyonic mass at the 1-loop in the graviton propagator. It is present for both *small Black Holes* and *thermal AdS*.



$$\mathcal{D}g\ \exp\left[-I_E[g]\right] \simeq \frac{e^{-I[g]}}{\sqrt{\det\left[\Delta^{1L}\right]}}$$

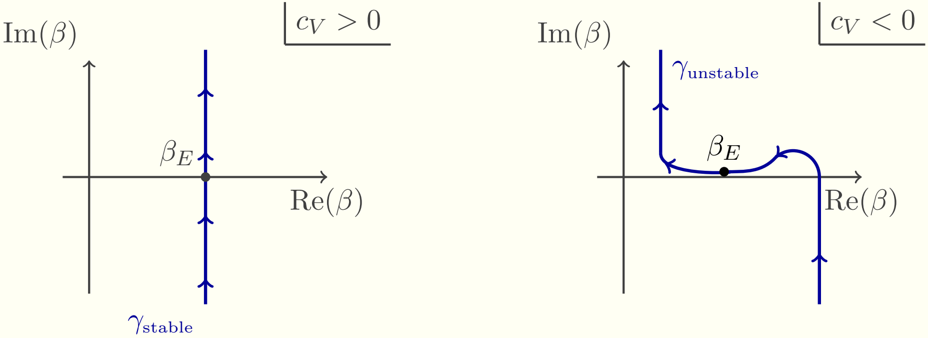
Potential imaginary factors jeopardize the interpretation of the EGPI as computing a positive-definite norm.



The Jeans instability is never present on the dominant manifold in either microcanonical band.

$$\mathcal{N}_{\mathcal{W}} := \int_{\mathcal{W}} dE\, \Omega(E) = \int_{\mathcal{W}} dE\, \frac{1}{2\pi i} \int d\beta\, e^{\beta E}\, Z(\beta)$$

The steepest descent contour depends on the specific heat of $Z(\beta)$.



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Outlook

1. Euclidean methods to understand Quantum Gravity $\rightarrow$ inclusion of Euclidean Wormholes. Their inclusion, however, raises new questions.
2. We have identified a number of aspects that signal the need for a careful treatment of the microcanonical projections. This more careful treatment shows consistency for the different possible microcanonical bands.
3. This raises the interesting question of whether the method can compute entropies in bands where the entropy scales as $\mathcal{O}\left(G_N^0\right)$.