The Cost of Complexity Equals Anything

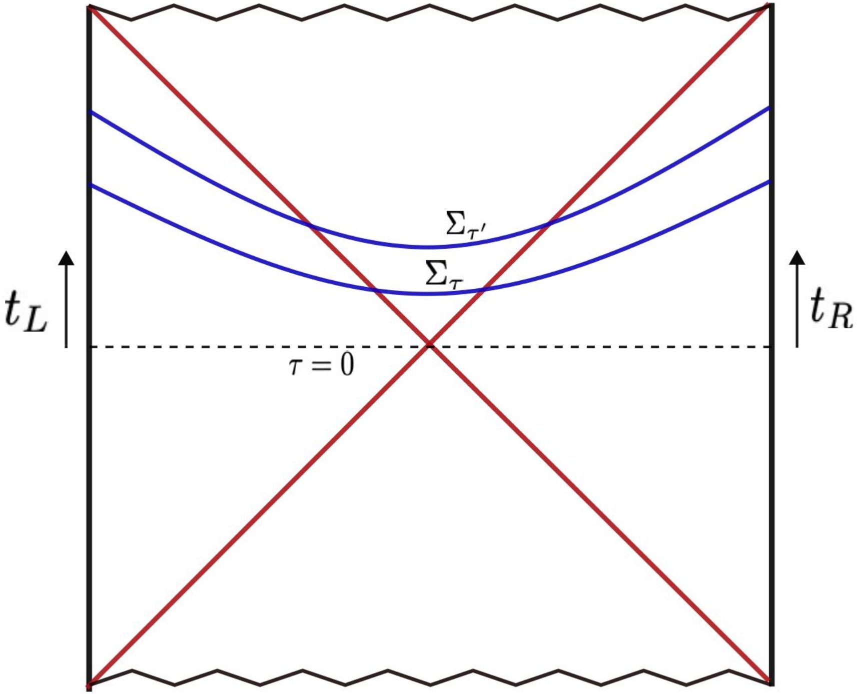
Ongoing work with Carlos Pérez-Pardavila

Rafael Carrasco
IFT UAM-CSIC
June 17th, 2025

Universidad Autónoma de Madrid

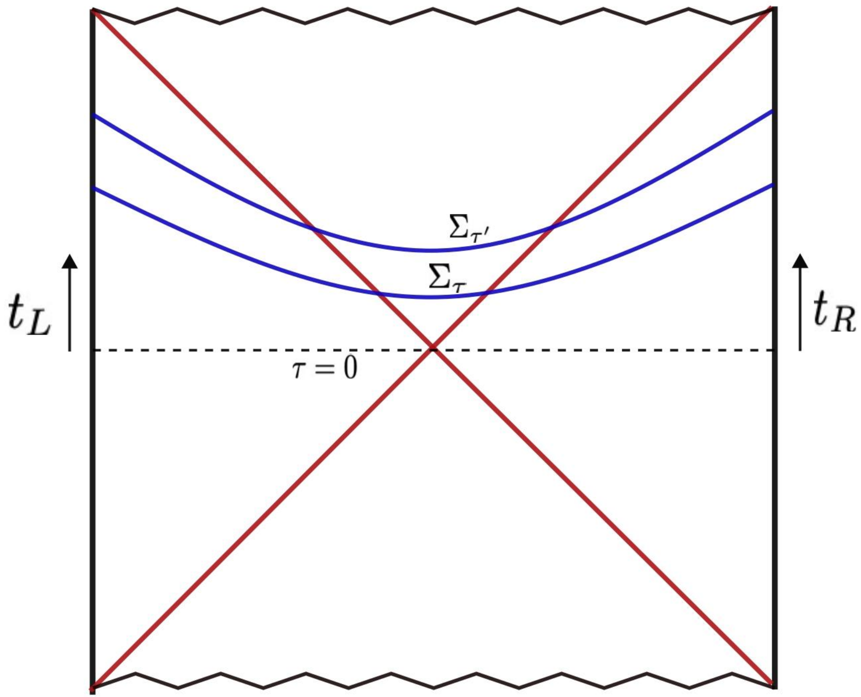
Why Complexity?

- BH thermo. suggest nature is holographic (Hawking, '75).



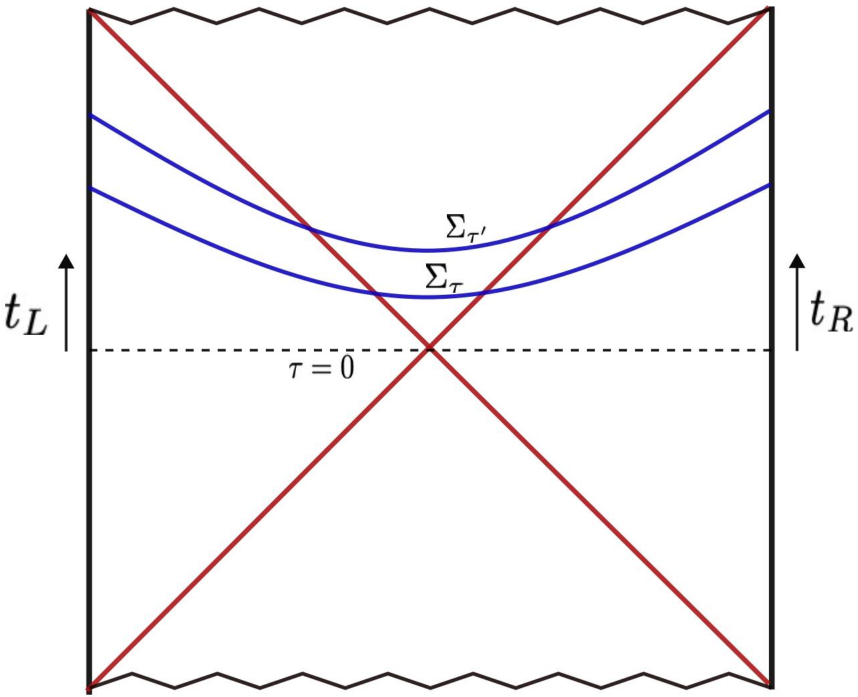
Why Complexity?

- BH thermo. suggest nature is holographic (Hawking, '75).
- EE provides with insight about the holographic nature of gravity.



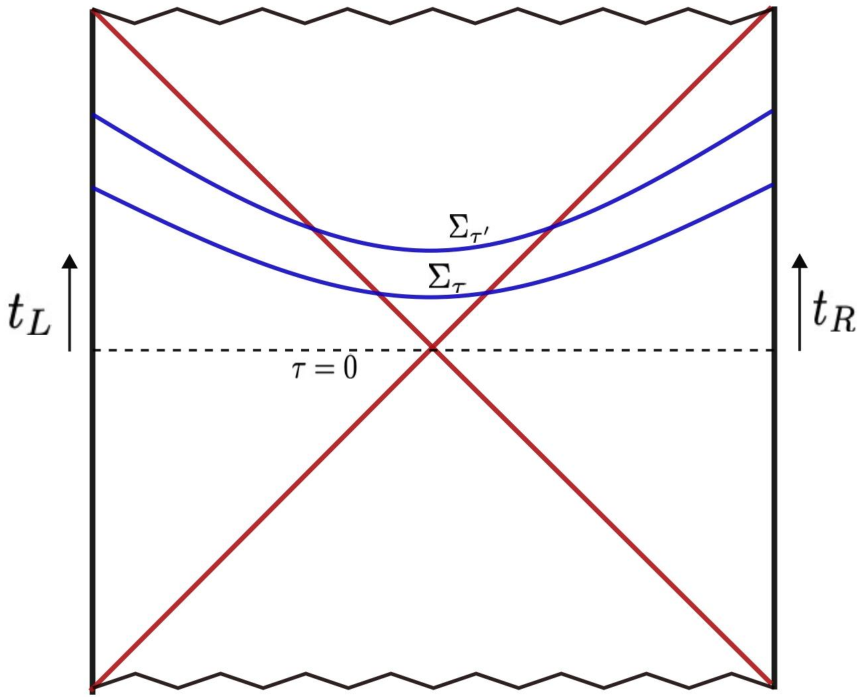
Why Complexity?

- BH thermo. suggest nature is holographic (Hawking, '75).
- EE provides with insight about the holographic nature of gravity.
- EE is not enough: cannot capture late time growth of ER bridges (Susskind, '14).



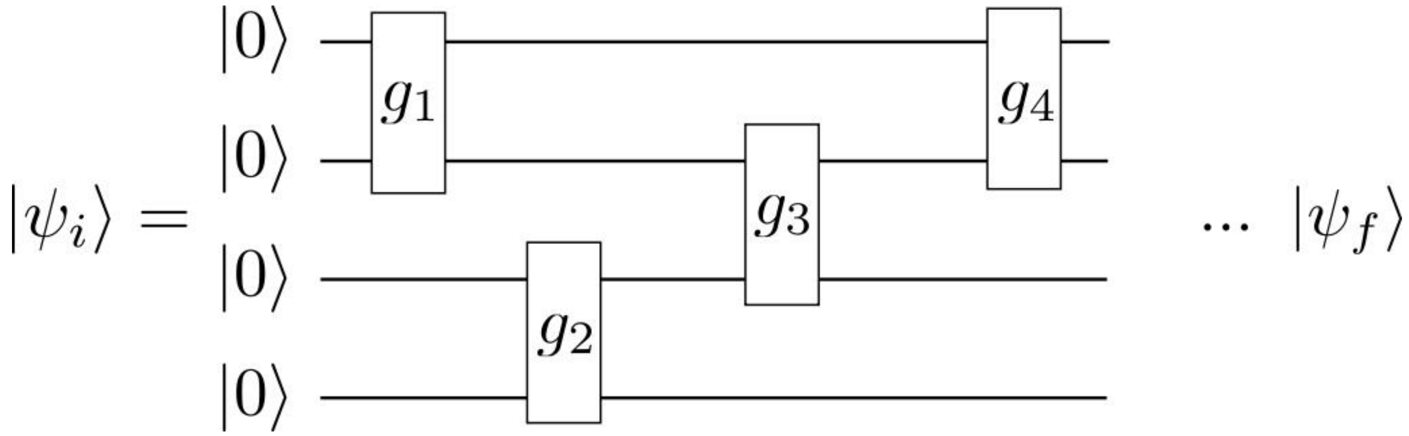
Why Complexity?

- BH thermo. suggest nature is holographic (Hawking, ‘75).
- EE provides with insight about the holographic nature of gravity.
- EE is not enough: cannot capture late time growth of ER bridges (Susskind, ‘14).
- We need **COMPLEXITY.**



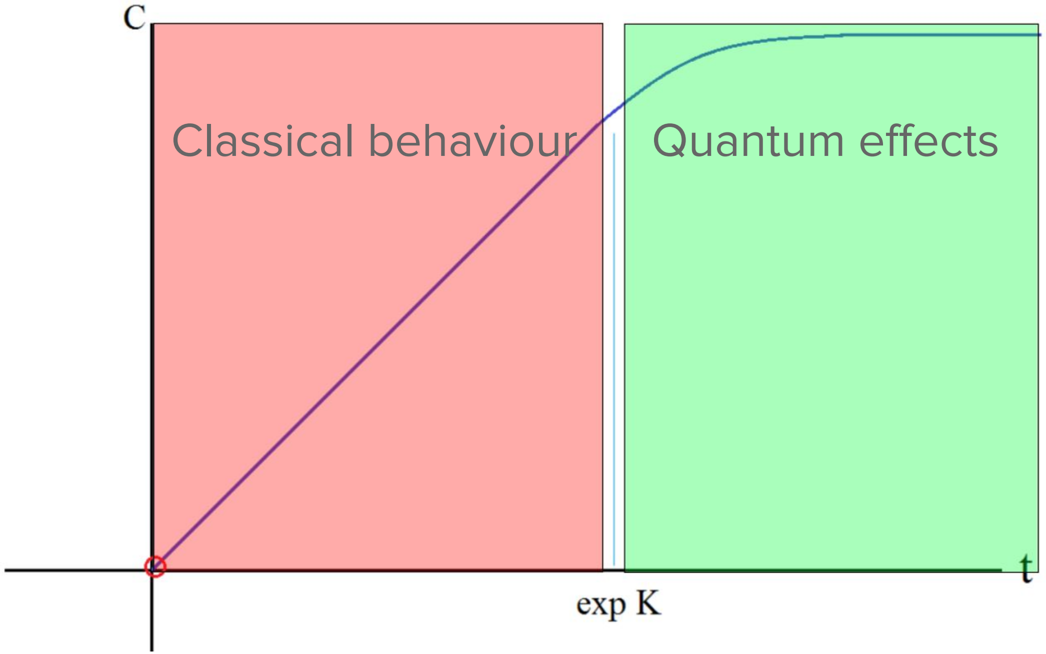
What is complexity?

- In circuit theory: Min number of ops. needed to create a target state from a ref. state.



Properties of Complexity

- Complexity displays linear growth.

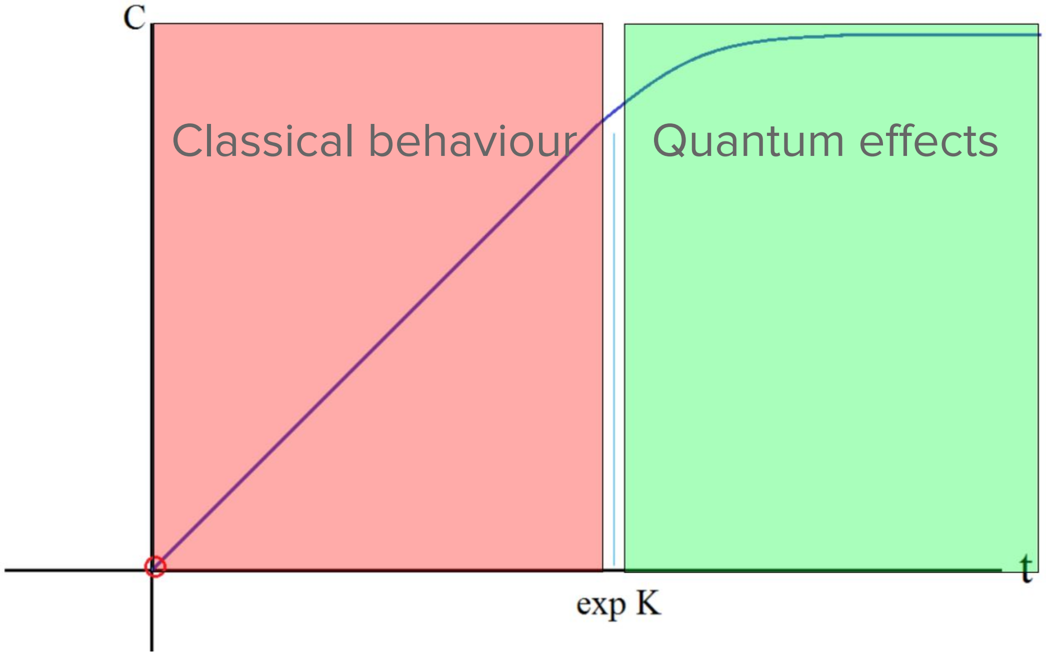


Properties of Complexity

- Complexity displays linear growth.

Another property:

- Switchback effect.



Complexity in QFTs

- How can complexity be determined in a QFT?

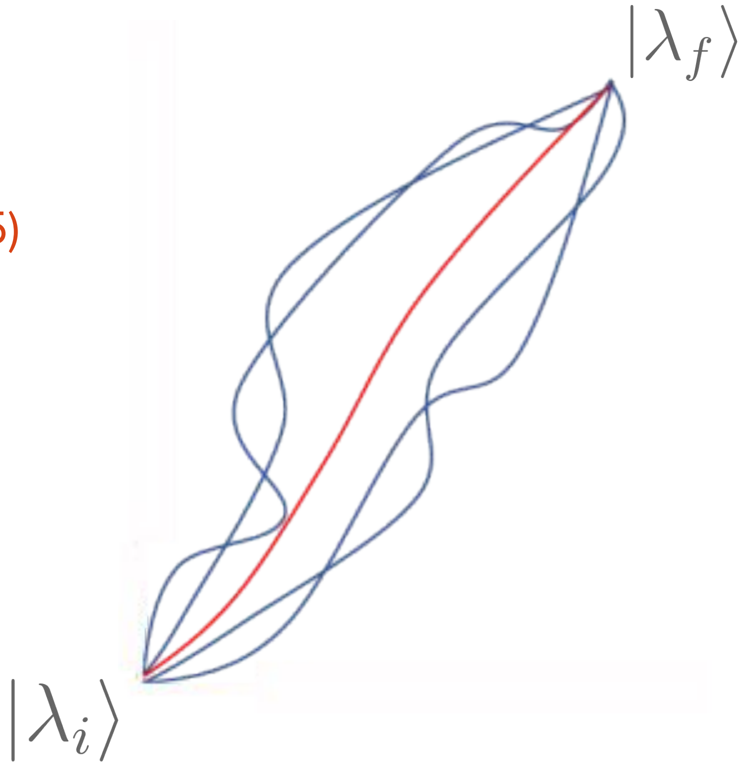
Complexity in QFTs

- How can complexity be determined in a QFT?

- Geometrization of space of states: (Nielsen, '05)

$$\mathcal{C} = \int_{s_i}^{s_f} ds \mathcal{F}[\gamma_{\mu\nu}, x^\mu, \dot{x}^\mu]$$

$\mathcal{F}$ = **Cost function**



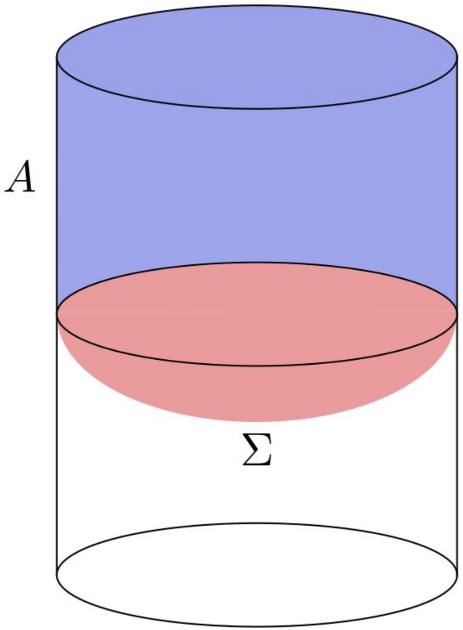
Complexity in the bulk dual: CV

Complexity in the bulk dual: CV

Several proposals for complexity duals in AdS:

- C=Volume

$$\mathcal{C} = \max_{\Sigma \sim \sigma} \frac{\mathrm{Vol}(\Sigma)}{G_N \ell}$$

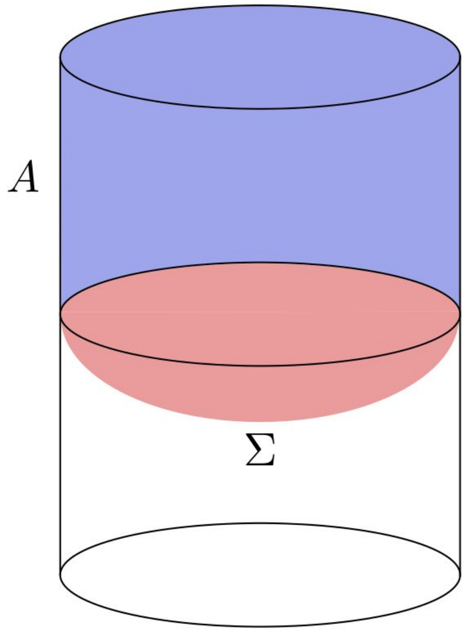


Complexity in the bulk dual: CV

Several proposals for complexity duals in AdS:

- C=Volume

$$\mathcal{C} = \max_{\Sigma \sim \sigma} \frac{\mathrm{Vol}(\Sigma)}{G_N \ell}$$



Other proposals:

- C=Action
- C=Volume 2.0

Complexity in the bulk dual: C-Any

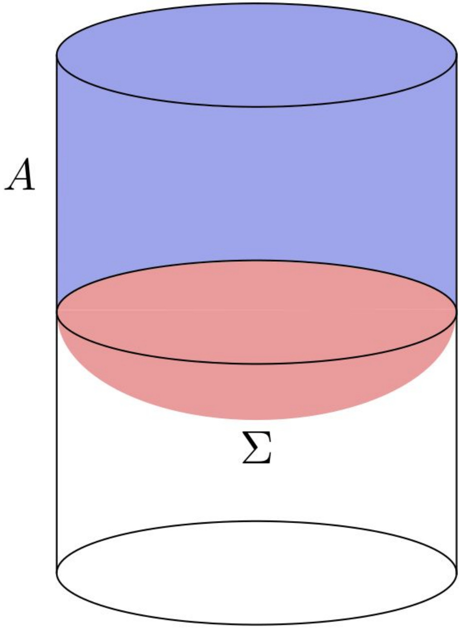
Most general proposal satisfying the requirements is **“Complexity Equals Anything”** (Belin,... ‘21)

Complexity in the bulk dual: C-Any

Most general proposal satisfying the requirements is **"Complexity Equals Anything"** (Belin,... '21)

- Find Σ extremizing:

$$\mathcal{I} = \int_{\Sigma} d^d\sigma \sqrt{h} F(R, K, ...)$$



Complexity in the bulk dual: C-Any

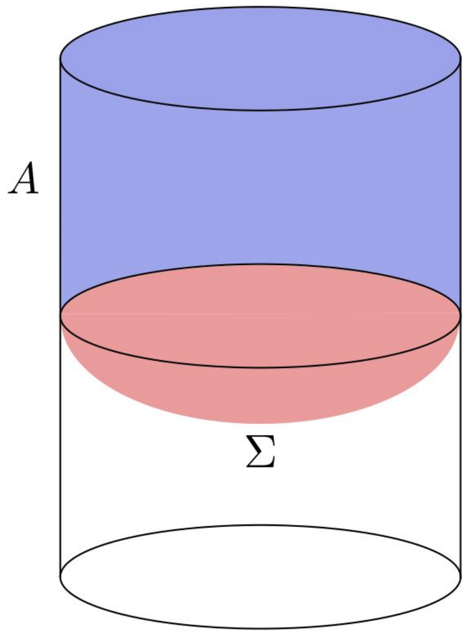
Most general proposal satisfying the requirements is **"Complexity Equals Anything"** (Belin,... '21)

- Find Σ extremizing:

$$\mathcal{I} = \int_\Sigma d^d\sigma \sqrt{h} F(R, K, ...)$$

- Evaluate:

$$\mathcal{C}_F = \int_\Sigma d^d\sigma \sqrt{h} F(R, K, ...)$$



Cost Function (Opt. curves in space of states)

↓

Complexity in CFT

C-Any. observables

↓

Dual complexity candidate

Cost Function (Opt. curves in space of states)

?

C-Any. observables

Complexity in CFT

Dual complexity candidate

Can optimal curves in space of states be read from a C-Any observable?

Can optimal curves in space of states be read from a C-Any observable?

But before doing so, let us do a quick recap on classical mechanics…

Review of classical mechanics

- System with Hamiltonian H.

Review of classical mechanics

- System with Hamiltonian H.
- Evolution of a function f is given by the Poisson bracket:

$$\dot{f} = \{f, H\}$$

Review of classical mechanics

- System with Hamiltonian H.
- Evolution of a function f is given by the Poisson bracket:

$$\dot{f} = \{f, H\}$$

- Define: $\begin{cases} \Omega(X, Y) = \text{Symplectic 2-form} \\ X_H = \text{Hamiltonian vector field} \end{cases}$ $\quad \delta H = \Omega(X_H, \cdot)$

Review of classical mechanics

- System with Hamiltonian H.
- Evolution of a function f is given by the Poisson bracket:

$$\dot{f} = \{f, H\}$$

- Define: $\begin{cases} \Omega(X,Y) = \text{Symplectic 2-form} \\ X_H = \text{Hamiltonian vector field} \\ X_f = \text{Vector field associated to } f \end{cases}$

The first two define: $\delta H = \Omega(X_H, \cdot)$

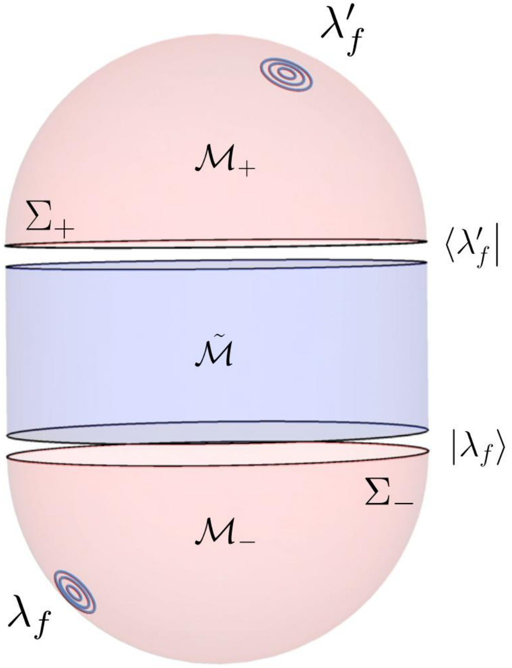
$$\{f, H\} = \Omega(X_f, X_H)$$

State preparation in the CFT

- We can prepare a state using a Euclidean path integral (Skenderis,...'08), (Botta-Cantcheff,...'16)

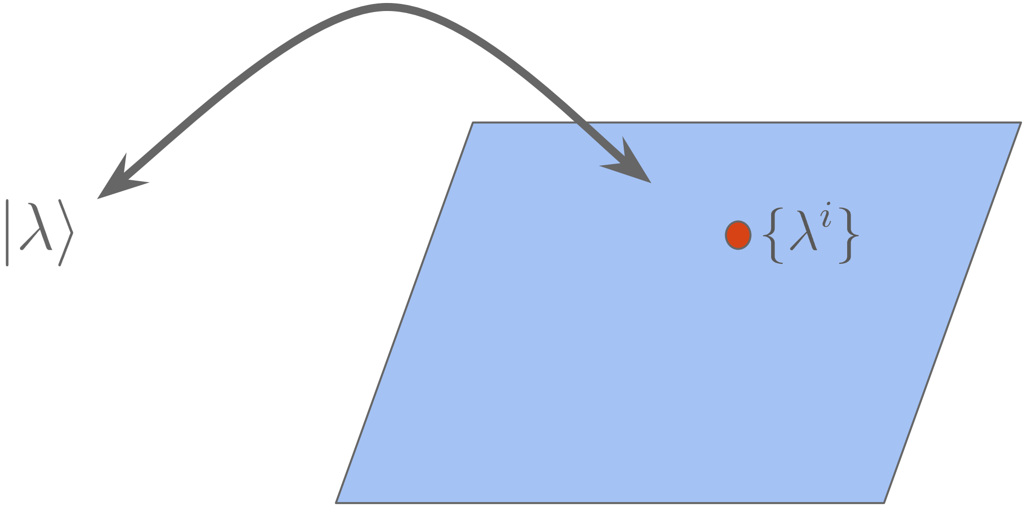
$$|\lambda_f\rangle = \mathcal{T} e^{-\int_{\tau<0} \lambda_\alpha \mathcal{O}_\alpha} |0\rangle$$

- Here:
  - λ_α : source,
  - $\mathcal{O}_\alpha$: associated CFT operator,
  - $|0\rangle$: HH vacuum.



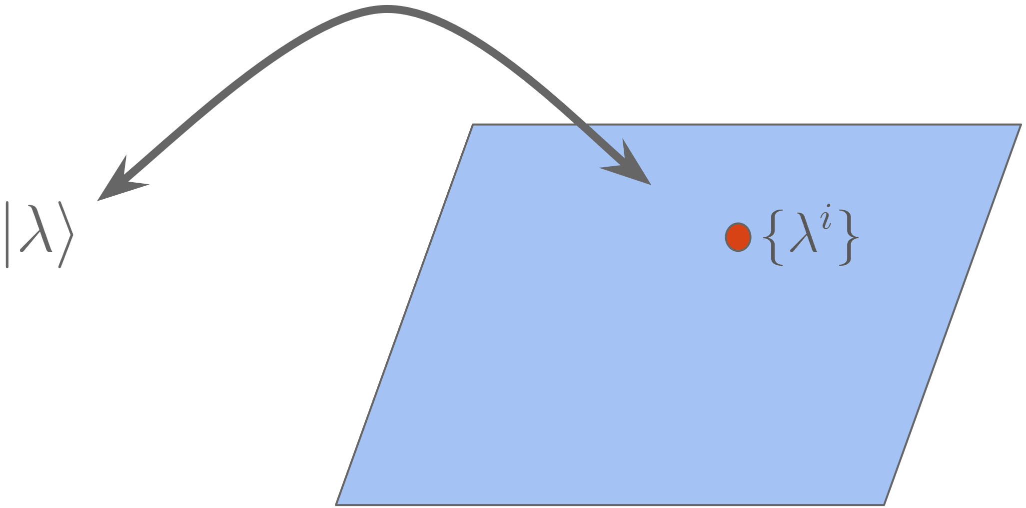
Geometrization of the space of sources

- States can be mapped onto a space of coordinates $\{\lambda^i\}$ (Belin,... '18).



Geometrization of the space of sources

- States can be mapped onto a space of coordinates $\{\lambda^i\}$ (Belin,... '18).



  - Metric: g_{ab}
  - Symplectic form: $\Omega_{\rm bdry}$

We still miss a vector field generating optimal curves in space of sources!

Relation to the bulk

- Bulk is also endowed with a symplectic form: Ω_{bulk}

Relation to the bulk

- Bulk is also endowed with a symplectic form: Ω_{bulk}

- The holographic dictionary says (on-shell):

$$Z_{\mathrm{CFT}}[\lambda^a] = Z_{\mathrm{bulk}}[\phi^{(0)} = \lambda]$$

Relation to the bulk

- Bulk is also endowed with a symplectic form: Ω_{bulk}

- The holographic dictionary says (on-shell):

$$Z_{\text{CFT}}[\lambda^a] = Z_{\text{bulk}}[\phi^{(0)} = \lambda]$$

- It turns that: $\Omega_{\text{bdry}} = \Omega_{\text{bulk}}.$ (Belin,... '18)

Relating bulk and bdry "Hamiltonians"

- Given a C-Any observable W, Peierls' formalism gives X_W, s.t. (Peierls, '52)

$$\Omega_{\text{bulk}}(X_W, \cdot) = \delta W$$

Relating bulk and bdry "Hamiltonians"

- Given a C-Any observable W, Peierls' formalism gives X_W, s.t. (Peierls, '52)

$$\Omega_{\text{bulk}}(X_W, \cdot) = \delta W$$

- This vector field can be pushed to the boundary: $\tilde{X}_W$

Relating bulk and bdry “Hamiltonians”

- Given a C-Any observable W, Peierls’ formalism gives X_W, s.t. (Peierls, ‘52)

$$\Omega_{\text{bulk}}(X_W, \cdot) = \delta W$$

- This vector field can be pushed to the boundary: $\tilde{X}_W$

- Equivalence between bulk and boundary:

$$\delta W = \Omega_{\text{bulk}}(X_W, \cdot) = \Omega_{\text{bdry}}(\tilde{X}_W, \cdot) = \delta \mathcal{C}$$

Relating bulk and bdry "Hamiltonians"

- $\tilde{X}_W$ generates the evolution on the space of sources.

- In vacuum AdS, considering GR:

(Bulk) $$\delta V = \delta \left(\int \sqrt{h} T_{\mu}{}^{\mu} \right)$$ **(Bdry)**

Summary

- Disconnection between boundary and bulk definition of complexity.
- Exploit the equivalence of symplectic structures.

Future (actually current) directions

- Is this method feasible?
  - Apply this formalism to more complex proposals.
- Which is the cost function?
- Extract universal information for any observable.

Thank you so much for your attention!

The Cost of Complexity Equals Anything

Ongoing work with Carlos Pérez-Pardavila

Rafael Carrasco
IFT UAM-CSIC
June 17th, 2025

Universidad Autónoma de Madrid