Krylov spread complexity as holographic complexity beyond JT gravity

based on 2412.17785 with M.P. Heller and J. Papalini

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GHENT UNIVERSITY

New Insights in Black Hole Physics from Holography @IFT - Tim Schuhmann - 17.06.25

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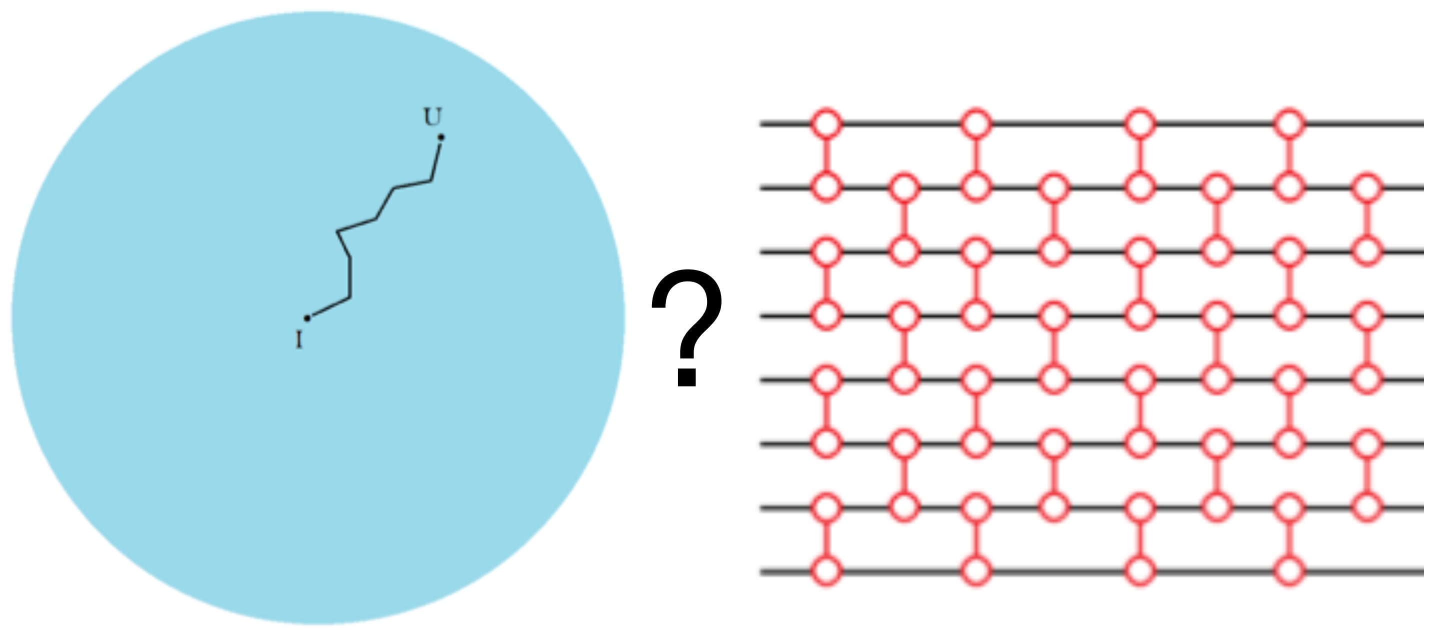
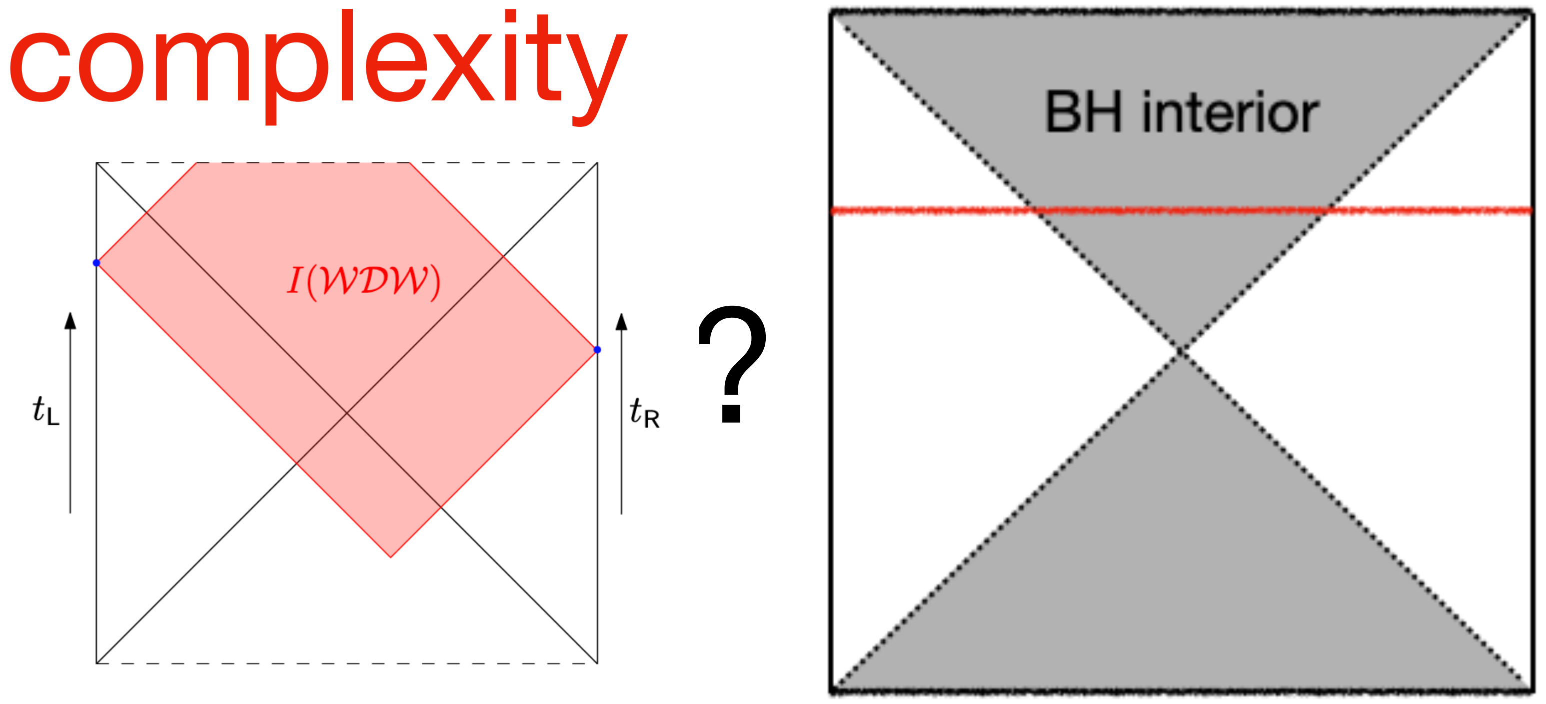
Conjecture [Susskind ’14]	Boundary complexity 	Holographic (bulk) complexity 

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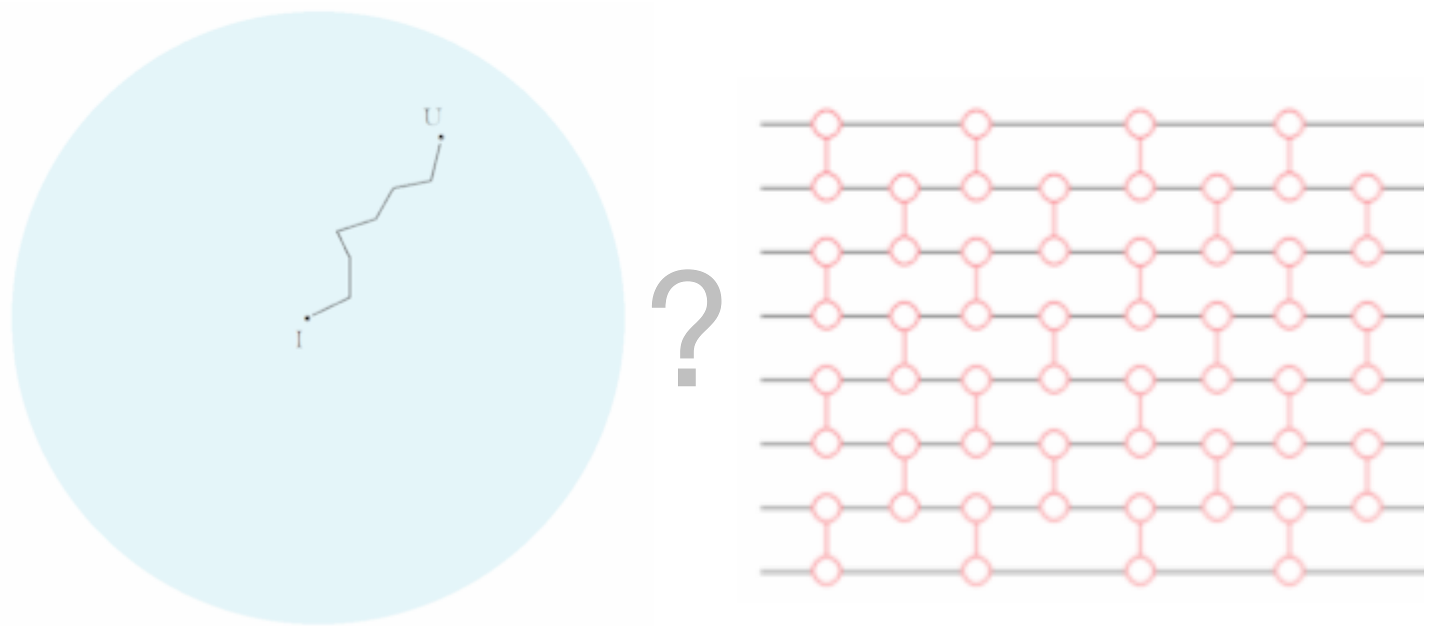
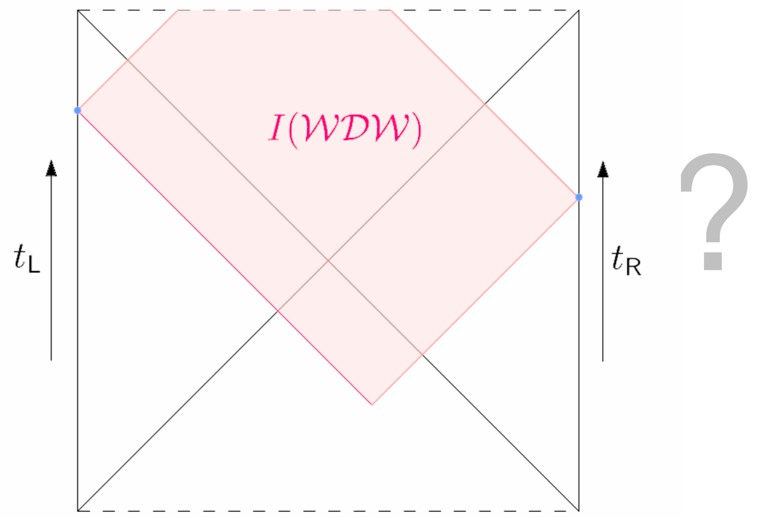
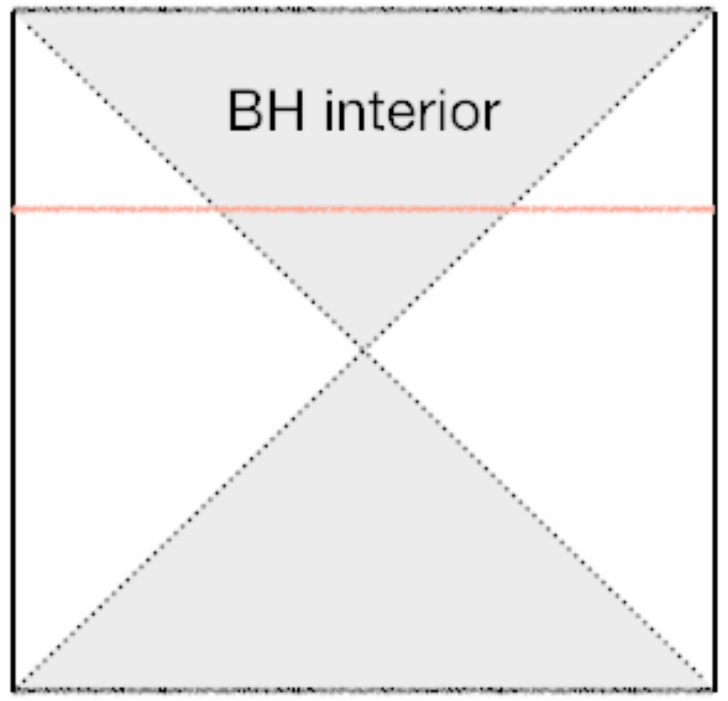
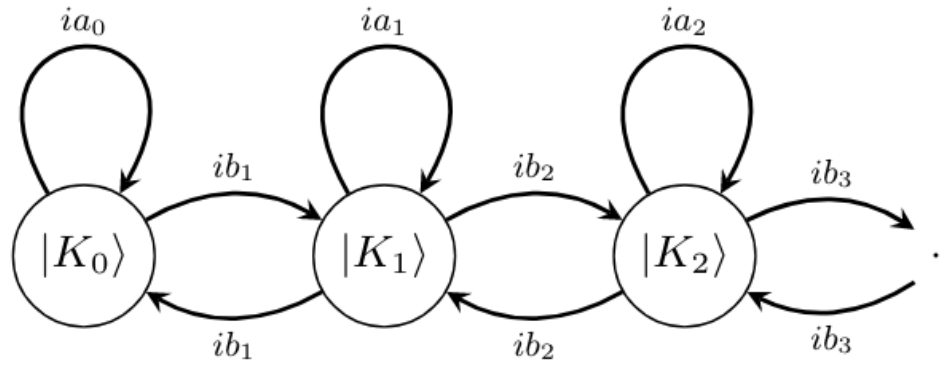
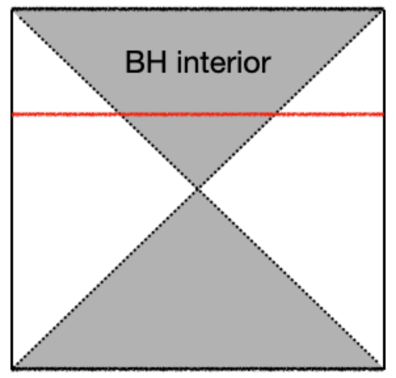
Conjecture [Susskind ’14]	Boundary complexity 	Holographic (bulk) complexity  
Precise verification in soluble model [Rabinovici et al. ’23]	Krylov spread complexity  $\|K_n\rangle = \|n\rangle$ triple-scaled SYK in semiclassical limit $C_K(t)$	Classical wormhole volume $\langle L\rangle \sim \log\cosh(\# t)$  classical JT gravity (d=2)

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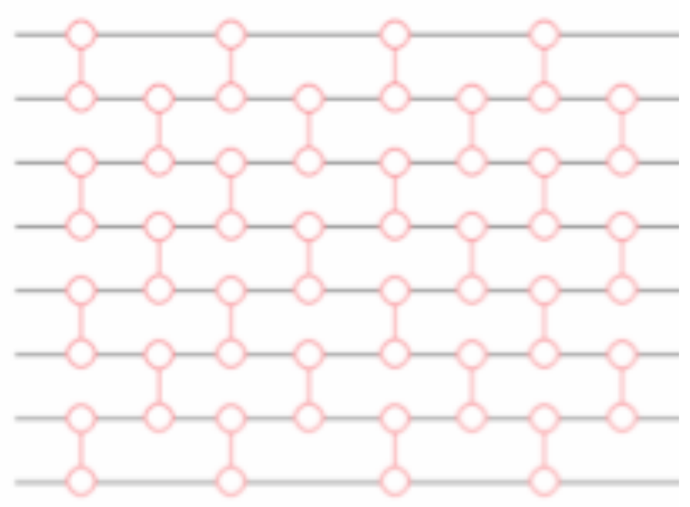
Conjecture

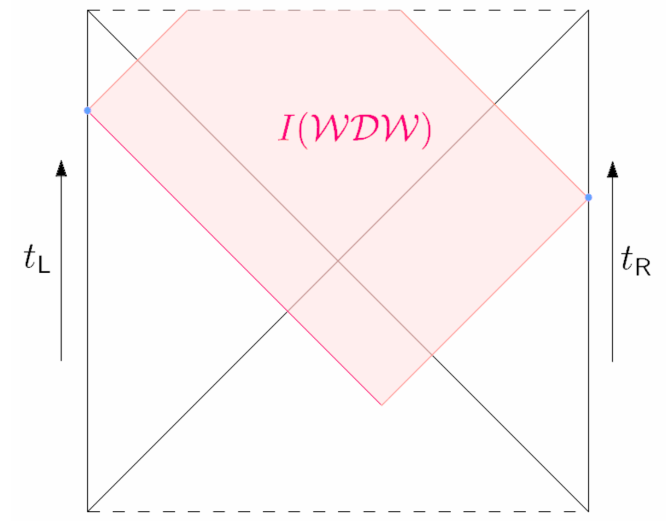
[Susskind ’14]

Boundary complexity

?



Holographic (bulk) complexity



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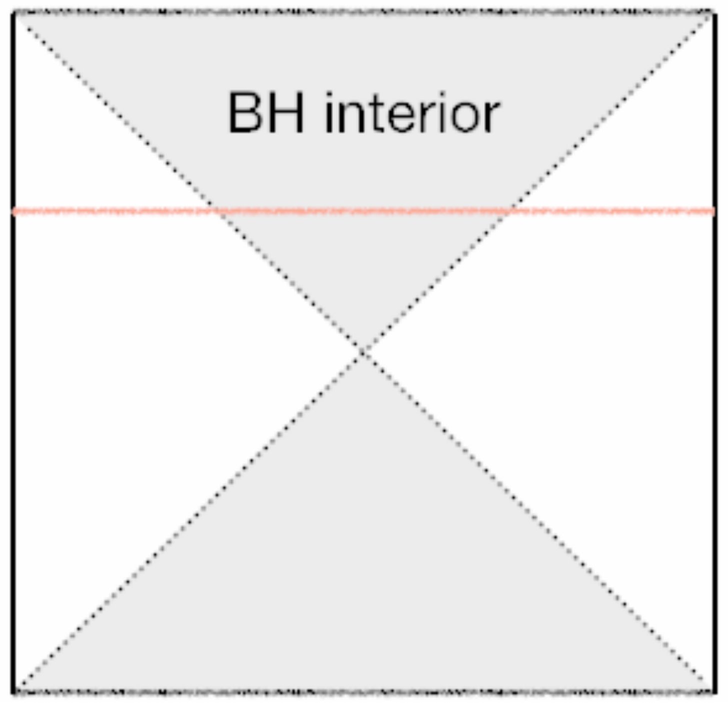
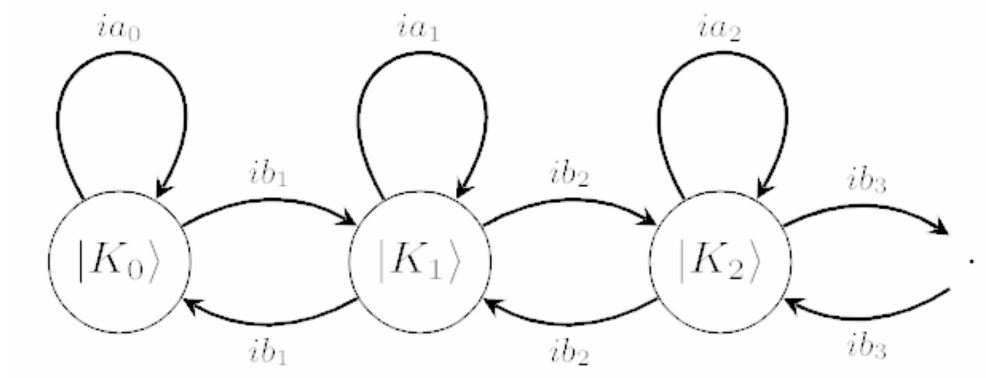


Precise verification in soluble model

[Rabinovici et al. ’23]

Krylov spread complexity



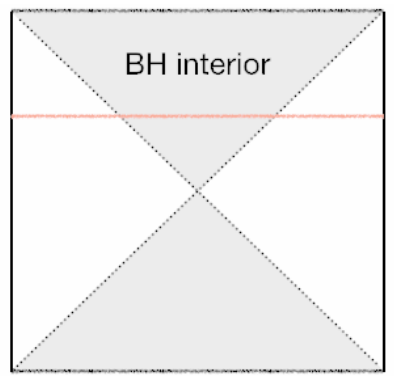
$|K_n\rangle = |n\rangle$

triple-scaled SYK in semiclassical limit

$C_K(t)$

Classical wormhole volume

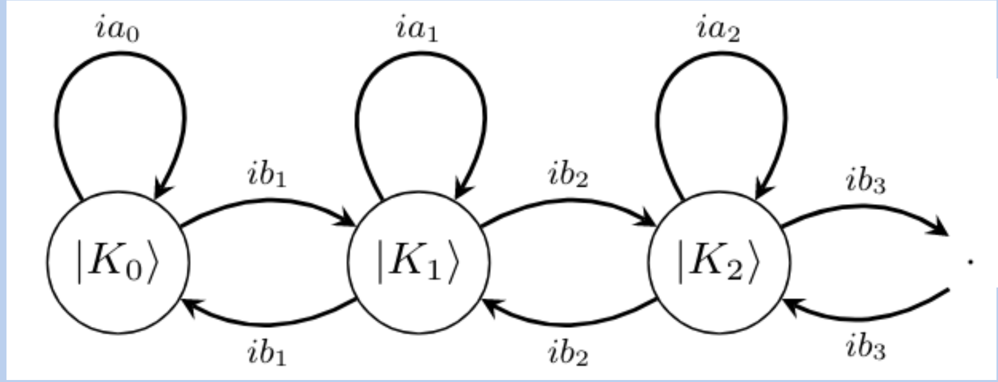
$\langle L \rangle \sim \log\cosh(\# t)$



classical JT gravity (d=2)

<u>This talk</u>

Krylov spread complexity



$|K_n\rangle = |n\rangle$

double-scaled SYK (finite T, quantum)

$C_K(t)$

Quantum generalization of wormhole volume

$$\langle L \rangle = \left[-\partial_\Delta (2pt\ .fn)\right]_{\Delta=0}$$

quantum (disk level) sine-dilaton gravity (d=2)

Bdy setup: Double-scaled SYK model (N fermions with p-body interaction)

$$H_{\rm SYK} = i^{p/2} \sum_{1\le i_1<\cdots<i_p\le N} J_{i_1\cdots i_p}\psi_{i_1}\cdots\psi_{i_p}, \quad N,p\to\infty \text{ while } \left|\log q\right| = p^2/N \text{ fixed}$$

$\Rightarrow$ Coupling averaged multi-point functions of H_{SYK} computable via auxiliary quantum system of chord states $|n\rangle$ governed by transfer matrix $\hat{T}$ [Berkooz et al. '18]

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Bulk dual: Sine-dilaton gravity [Blommaert, Mertens, Yao, Papalini, Levine, Parmentier '23 '24 '25]

Ad: Thomas Mertens' talk on Thursday

$$\int \mathcal{D}g\mathcal{D}\Phi\ \exp\left(\frac{1}{2\left|\log q\right|}\int \mathrm{d}^2x\sqrt{g}\left(\Phi R + \sin(2\Phi)\right) + \text{bdy.}\right)$$

⇒ $\hat{H}_{grav}$ from canonical quantization of E_{ADM} coincides with DSSYK $\hat{T}$

⇒ Holographic dictionary relates bulk length with chord nr. $\hat{L} = 2\left|\log q\right|\hat{n}$

Krylov spread complexity: Expectation value of the position of a state $|\psi(t)\rangle$ spreading over 'minimal basis of evolution / Krylov basis' $|K_n\rangle$ [Balasubramanian et al. '22]

$$C_K(t) = \sum_{n=0}^{\infty} n\,|\langle K_n|\psi(t)\rangle|^2, \;\; |\psi(t)\rangle = e^{-iHt}|R\rangle, \; |K_n\rangle = orthonorm(\{H^n|R\rangle\})$$

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KEY RESULT: Sine-dilaton length $\langle \hat{L} \rangle$ equals DSSYK Krylov complexity $C_K(t)_\beta$

$$\left[-\partial_\Delta (2pt\,.fn)\right]_{\Delta=0} = \langle \hat{L} \rangle = C_K(t)_\beta$$

valid @finite temperature (β) & (disk) quantum level (q) | [Iliesiu, Mezei, Sarosi '22]

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KEY RESULT: Sine-dilaton length $\langle\hat{L}\rangle$ equals DSSYK Krylov complexity $C_K(t)_\beta$

$$\langle\hat{L}\rangle = \left[-\partial_\Delta(2pt\,.fn)\right]_{\Delta=0} = -\,\partial_\Delta\left[Z_\beta^{-1}\langle L=0|\,e^{-\tau\hat{H}_{grav}}e^{-\Delta\hat{L}}e^{-(\beta-\tau)\hat{H}_{grav}}|L=0\rangle\right]_{\Delta=0}$$

$$= \left|\log q^2\right|\sum n\left|\langle n|Z_\beta^{-1/2}e^{-i\hat{T}(t-i\beta/2)}|0\rangle\right|^2 = \left|\log q^2\right| C_K(t)_\beta$$

@$\beta \geq 0$ & $0 \leq q \leq 1$ | (2-par. gen. of [Rabinovici et al. '23] = $(\beta \to 0, q \to 1)$ case)

Insight 1

Choice of reference state $|R\rangle$

Two inequivalent options for combined Lorentzian + Euclidean evolution

$$C_K \text{ for } e^{-iT(t-i\beta/2)}|0\rangle \quad \neq \quad C_K \text{ for } e^{-iTt}e^{-\beta T/2}|0\rangle$$

$$\equiv |R\rangle \qquad\qquad \equiv |R\rangle$$

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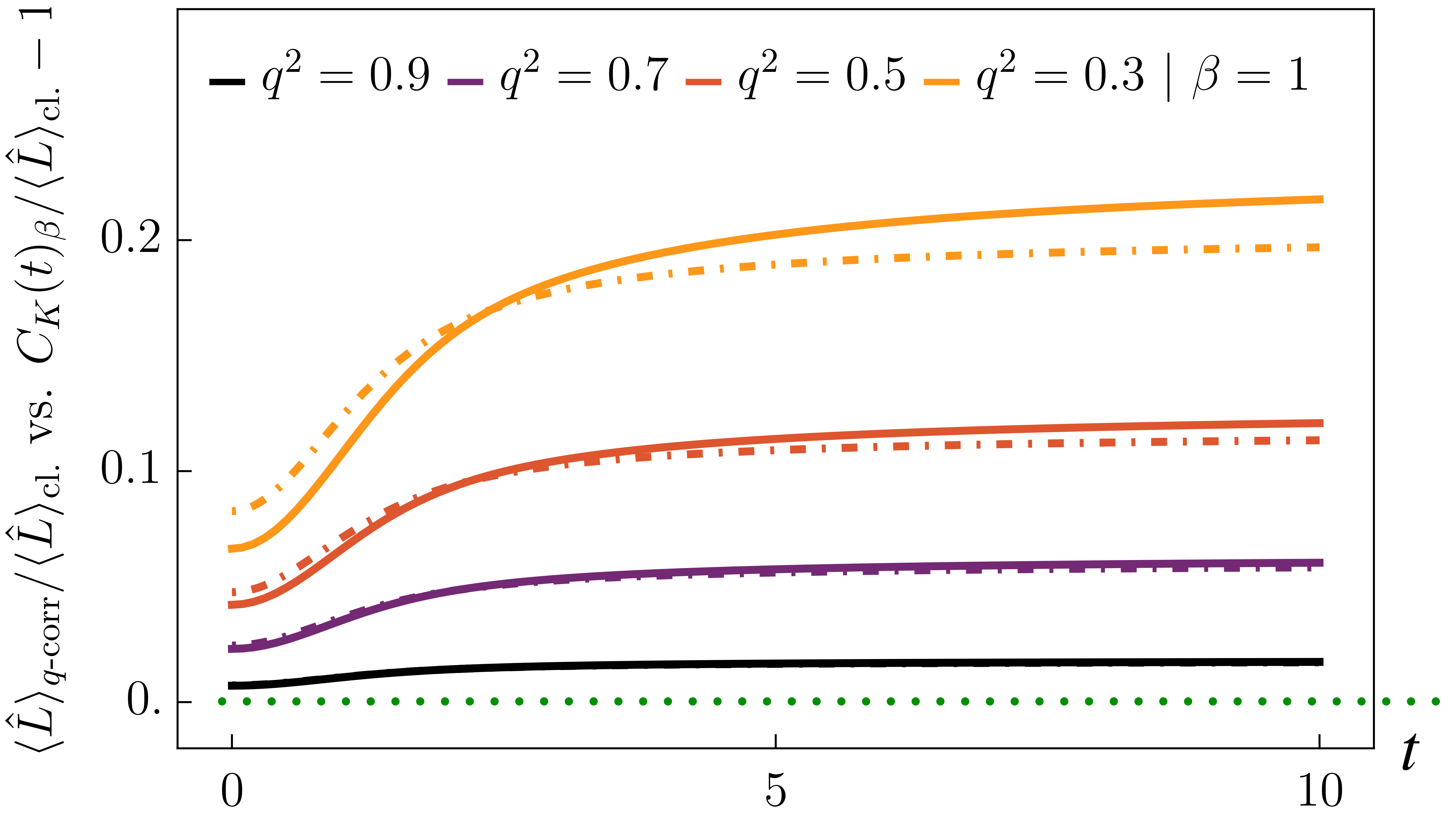
Starting from $\langle \hat{L} \rangle$ on the gravity side, we match this

$\Rightarrow$ Gravity demands to assign complexity also to euclidean state preparation

Insight 2

Our result is valid for any $q \Rightarrow$ Consistently, agrees with first quantum correction on top of the classical sine-dilaton gravity length [Bossi et al. '25] around $q \approx 1$



Solid:
numerical C_K

Dot-dashed: 1st q-corr. to sine-dilaton length

Baseline:
classical length in sine-dilaton

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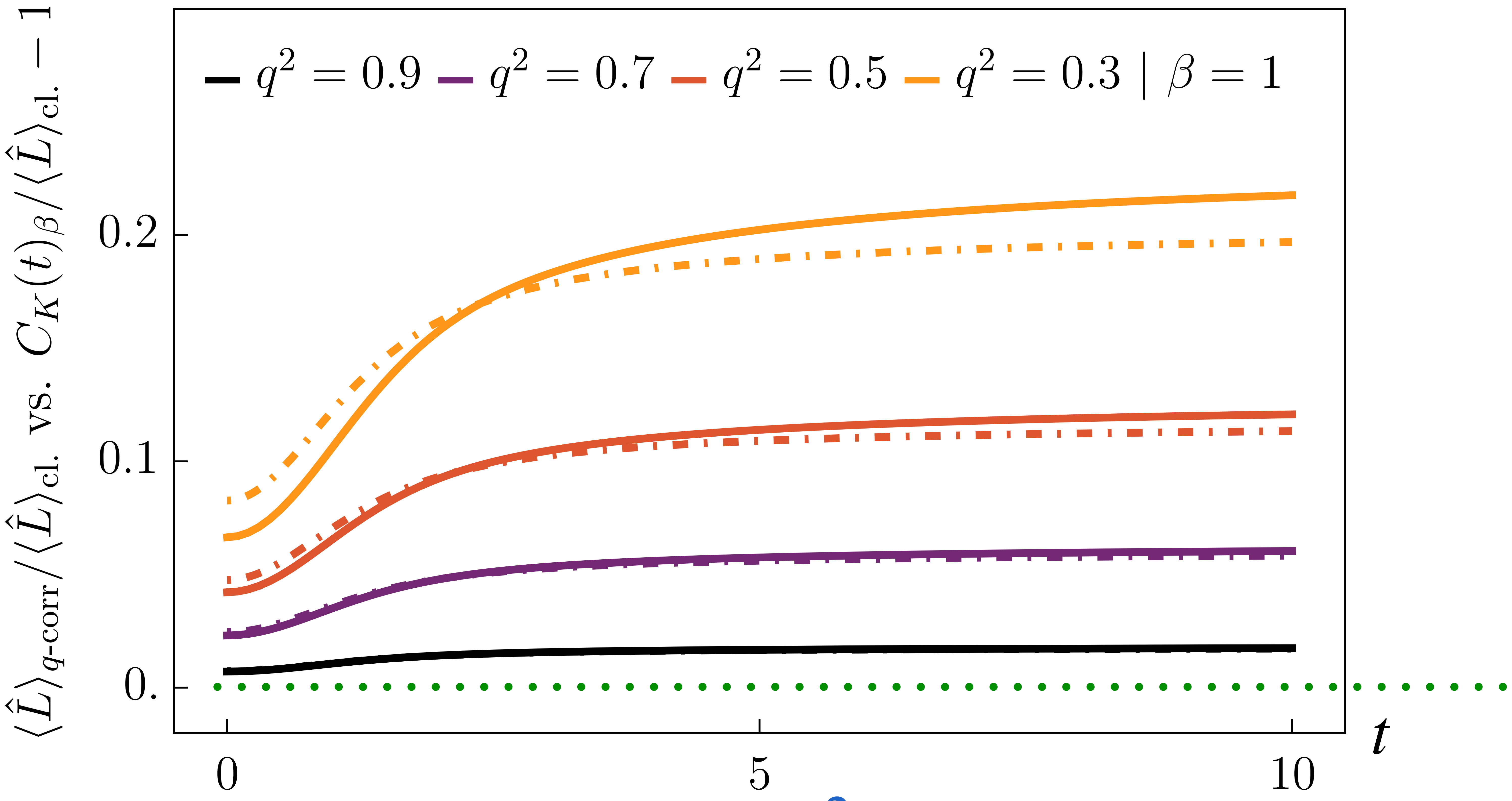

Solid: numerical C_K

Dot-dashed: 1st q-corr. to sine-dilaton length

Baseline: classical length in sine-dilaton

First quantum correction transitions from t^2 early to t late (like C_K does usually)

$\Rightarrow$ **Speculation:** First quantum correction as Krylov spread complexity of bulk fields

Outlook

This talk: $e^{-iT(t-i\beta/2)}|0\rangle$ C_K specified by $H = T_{DSSYK}$, $|R\rangle = |0\rangle$

Q: Precise influence of reference state choice on bulk manifestation?

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Q: Precise influence of reference state choice on bulk manifestation?

- Operator insertions in reference state: $|R\rangle = \mathcal{O}_\Delta |0\rangle$
  $\Rightarrow$ Dual manifestation as length in sine-dilation shockwave geometry?
  [Ambrosini, Rabinovici, Sanchez-Garrido, Shir, Sonner ’24], [Xu ’24], [Aguilar-Gutierrez ’25], [Aguilar-Gutierrez, Xu WIP]

- 1-parameter families:
  $|R(n)\rangle = |n\rangle$ and
  $|R(\beta)\rangle = Z_\beta^{-1/2} e^{-\beta\hat{T}/2}|0\rangle$
  $\Rightarrow$ Manifestation as C_{any} bulk duals in sine-dilaton?
  [Cáceres, Carrasco, Patil, Pedraza, Svesko ‘25]