Black Hole Singularities from Holographic Complexity

Based on arXiv:2504.10194 and related ongoing work

Vyshnav Mohan

University of Iceland

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Introduction

Using the null energy condition, Penrose proved that a globally hyperbolic, inextendible spacetime manifold M is null geodesically incomplete if it contains a *trapped surface*.

[Penrose 1965]

Wall showed that the energy condition can be given up in favour of the generalized second law, which states that the generalized entropy always increases.

[Wall 2010]

The generalized second law reflects the thermal properties of the black Hole.

Significant advancements in the last decade have further refined the singularity theorem, resulting in wider applicability

[Bousso 2025]

The study of holographic complexity has also led us to another version of the second law of black hole thermodynamics.

Can we use holographic complexity to arrive at a singularity theorem?

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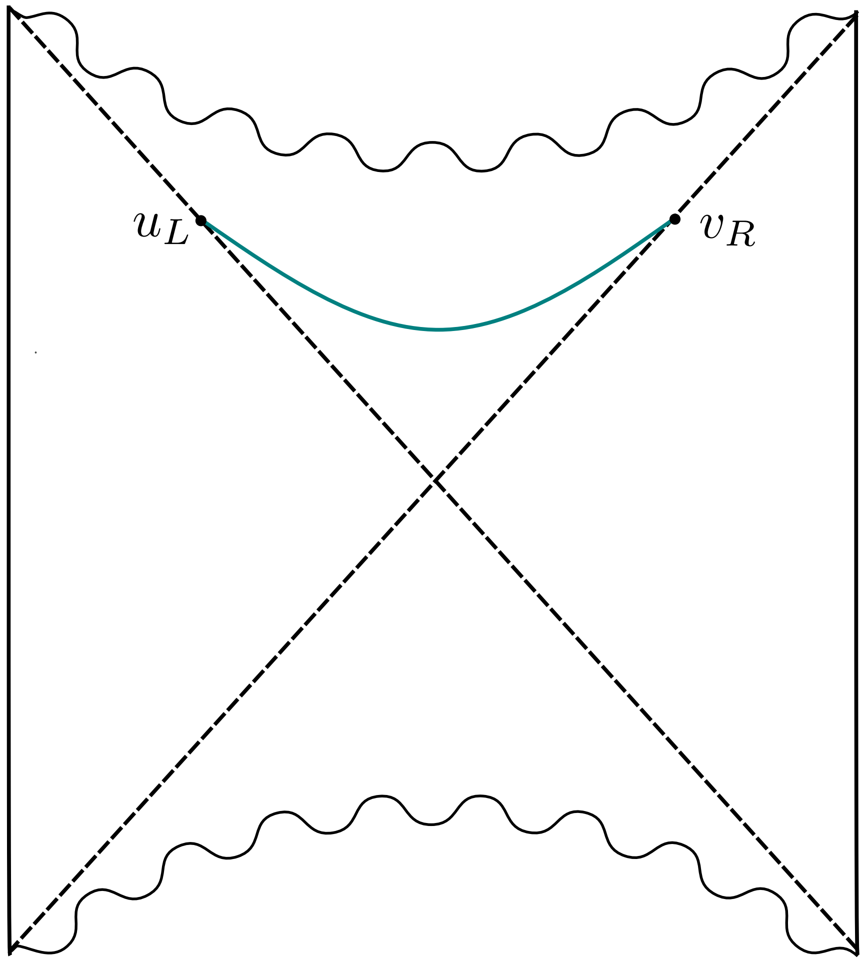
Can we use holographic complexity to arrive at a singularity theorem?

YES!!!

Complexity=Volume

[Susskind 2014]
[Stanford, Susskind 2014]



Complexity=Volume

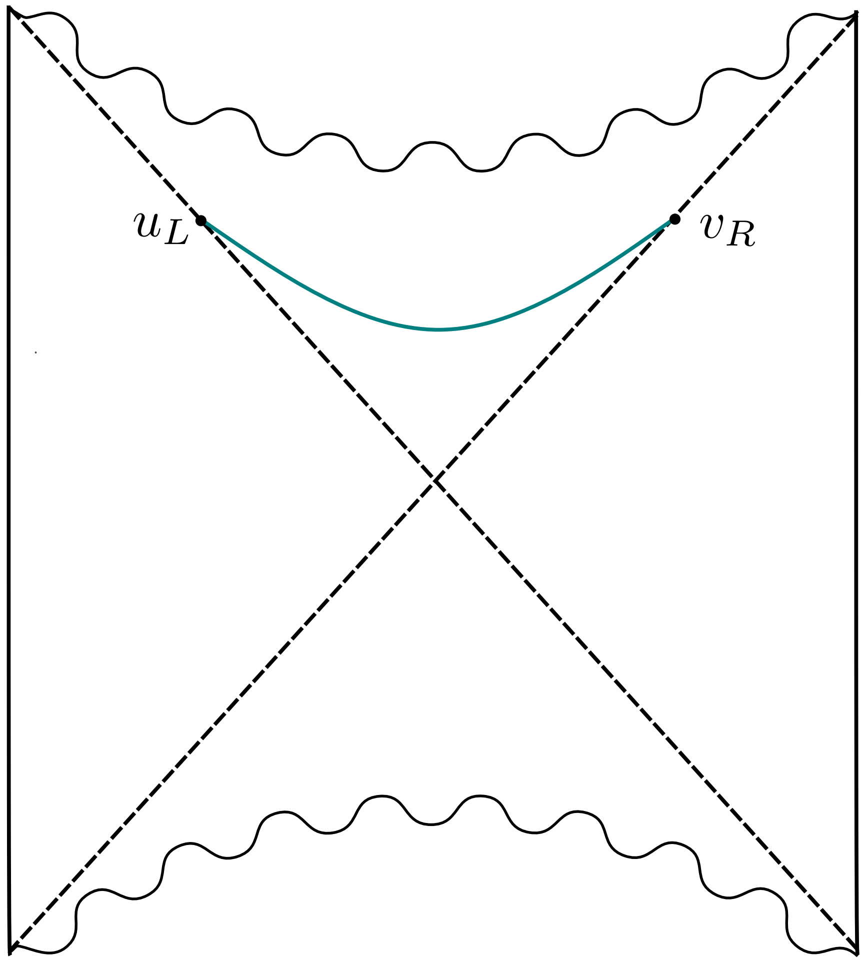
[Susskind 2014]

[Stanford, Susskind 2014]

Stanford and Susskind proposed that the complexity of a black hole is given by the volume of boundary anchored codimension-1 extremal surfaces in the bulk:

$$C = \frac{\mathcal{V}}{G_N r_0}$$

Here r_0 is the characteristic length of the system.



Second Law of Complexity

Second Law of Complexity

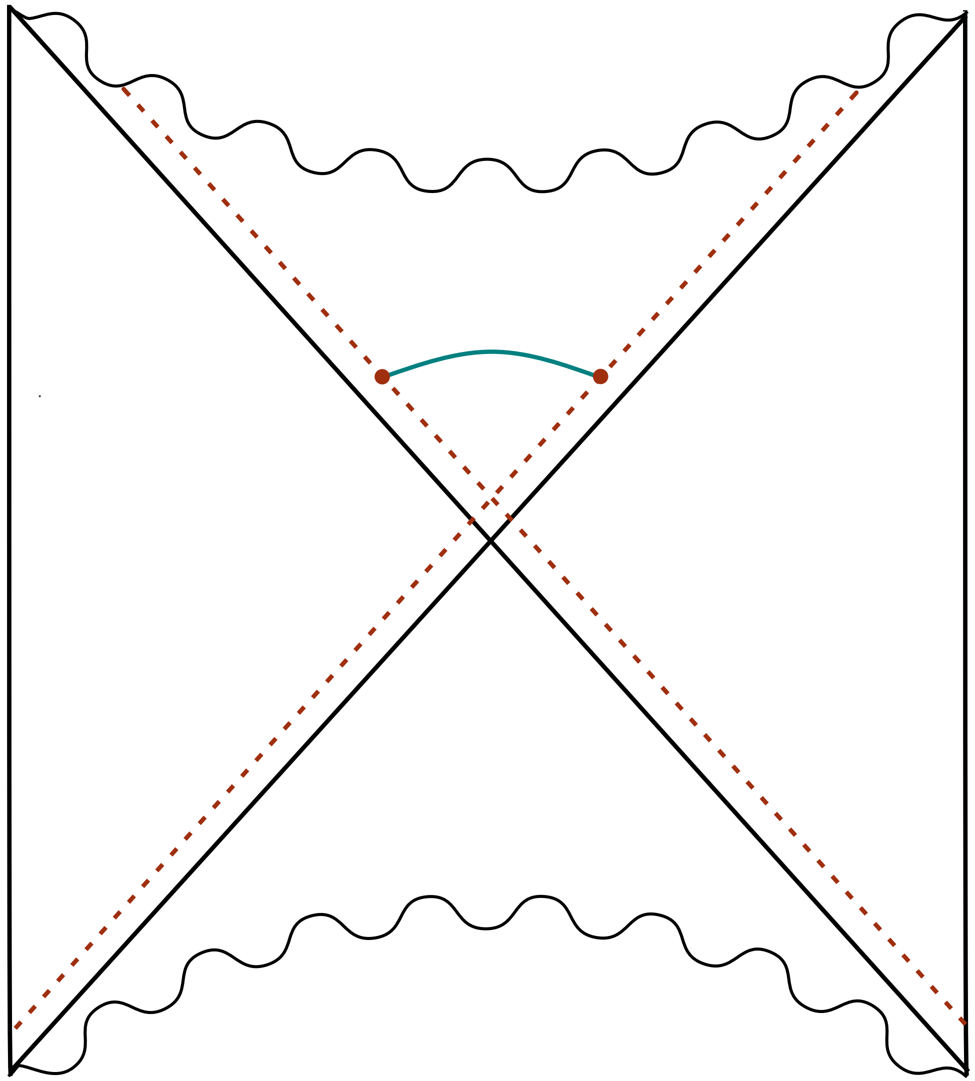
Second Law of Complexity (SLC): The volume of extremal surfaces anchored to a causal horizon always increases as we move the anchor point along the surface into the future.

[Brown Susskind Zhao 2016]

[Brown Susskind 2017]

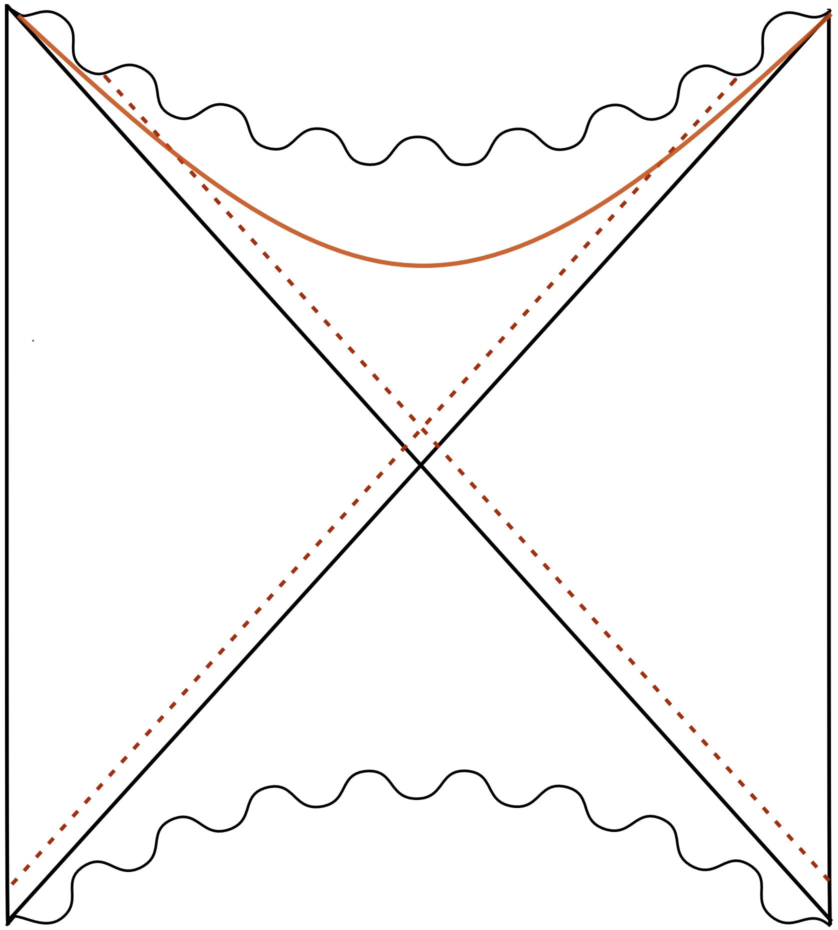
Growth of Null Anchored Extremal Surfaces

[VM 2025]



The extremal surfaces are anchored to an infalling null sheet N.

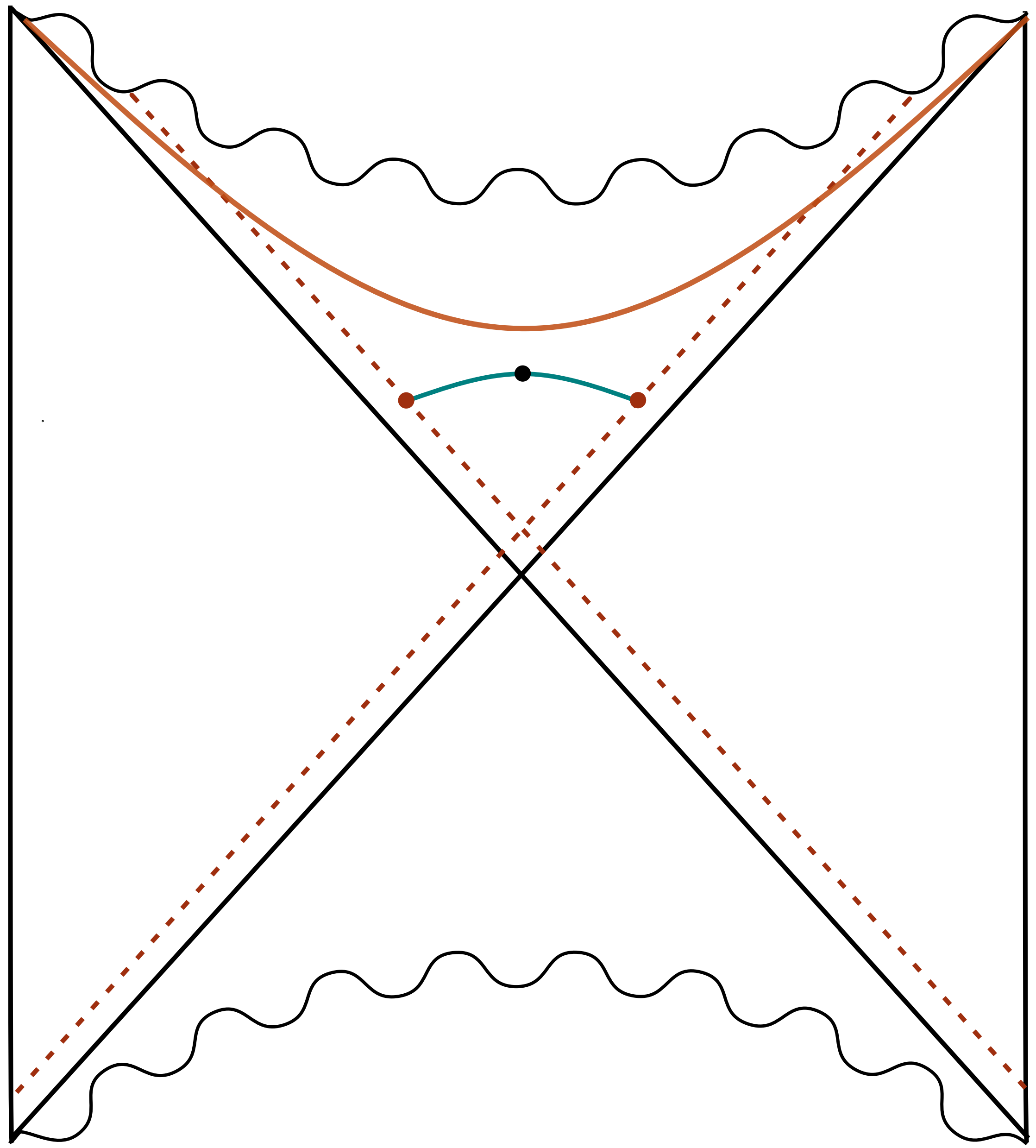
Observation



- There are two distinct cases, depending on the position of the anchor points relative to a special surface inside the black hole.
- We refer to this surface as the accumulation surface.
- Interestingly, such a surface is not unique to this setup—it appears in other black hole solutions as well.

Case 1

Anchor Points are in the exterior of the accumulation surface



Case 1

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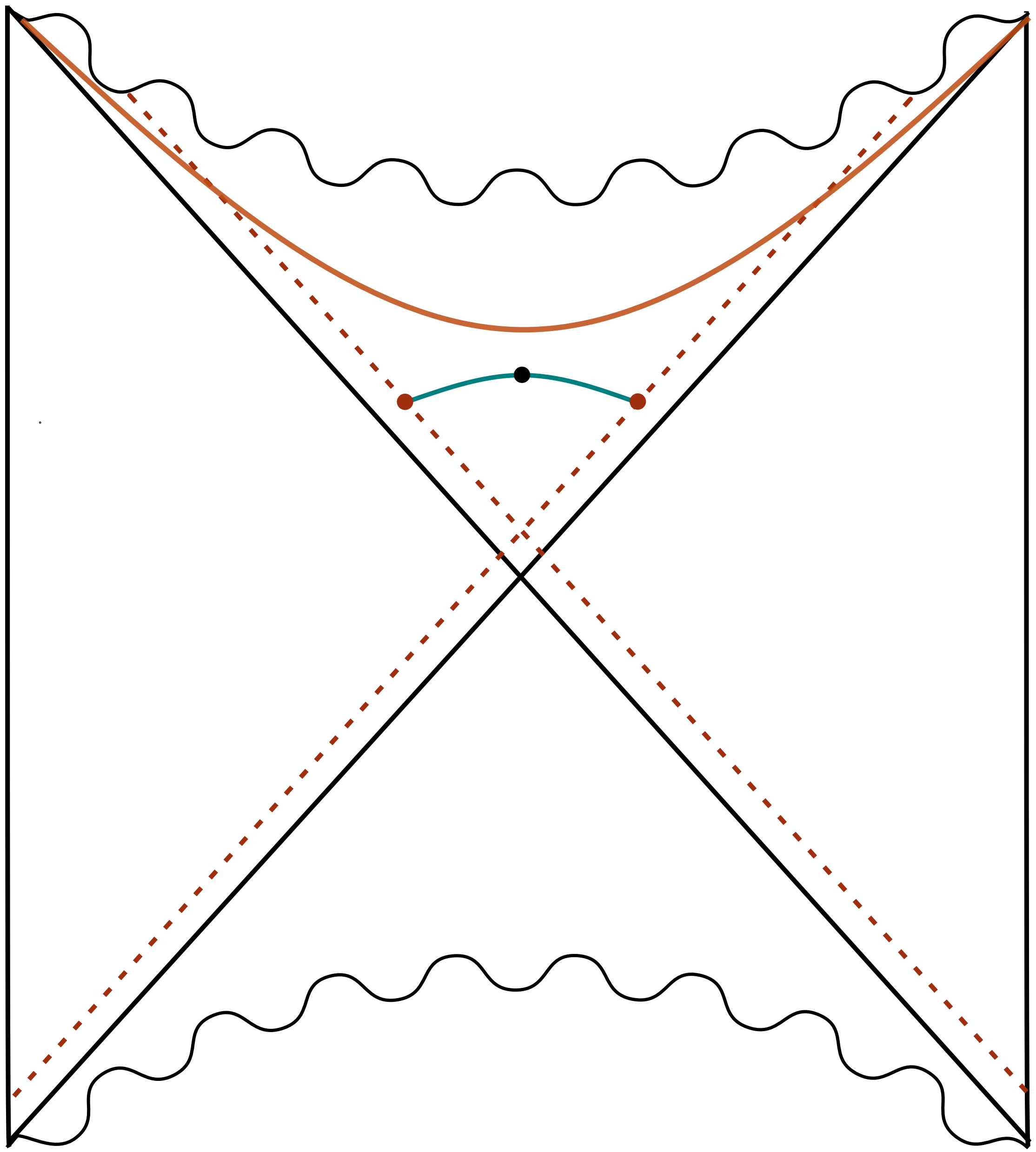
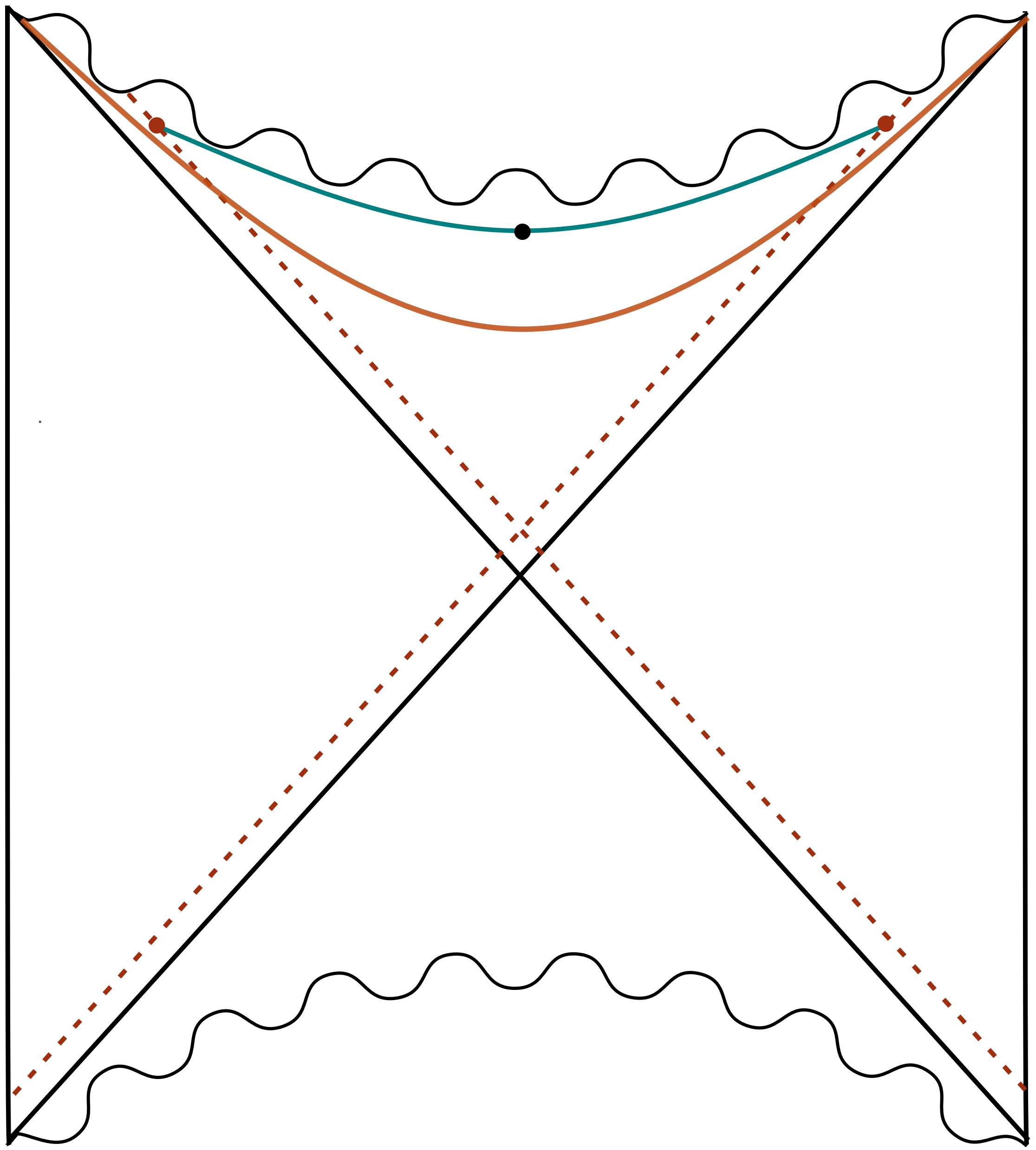


Volume Increases when we move the anchor points to the future

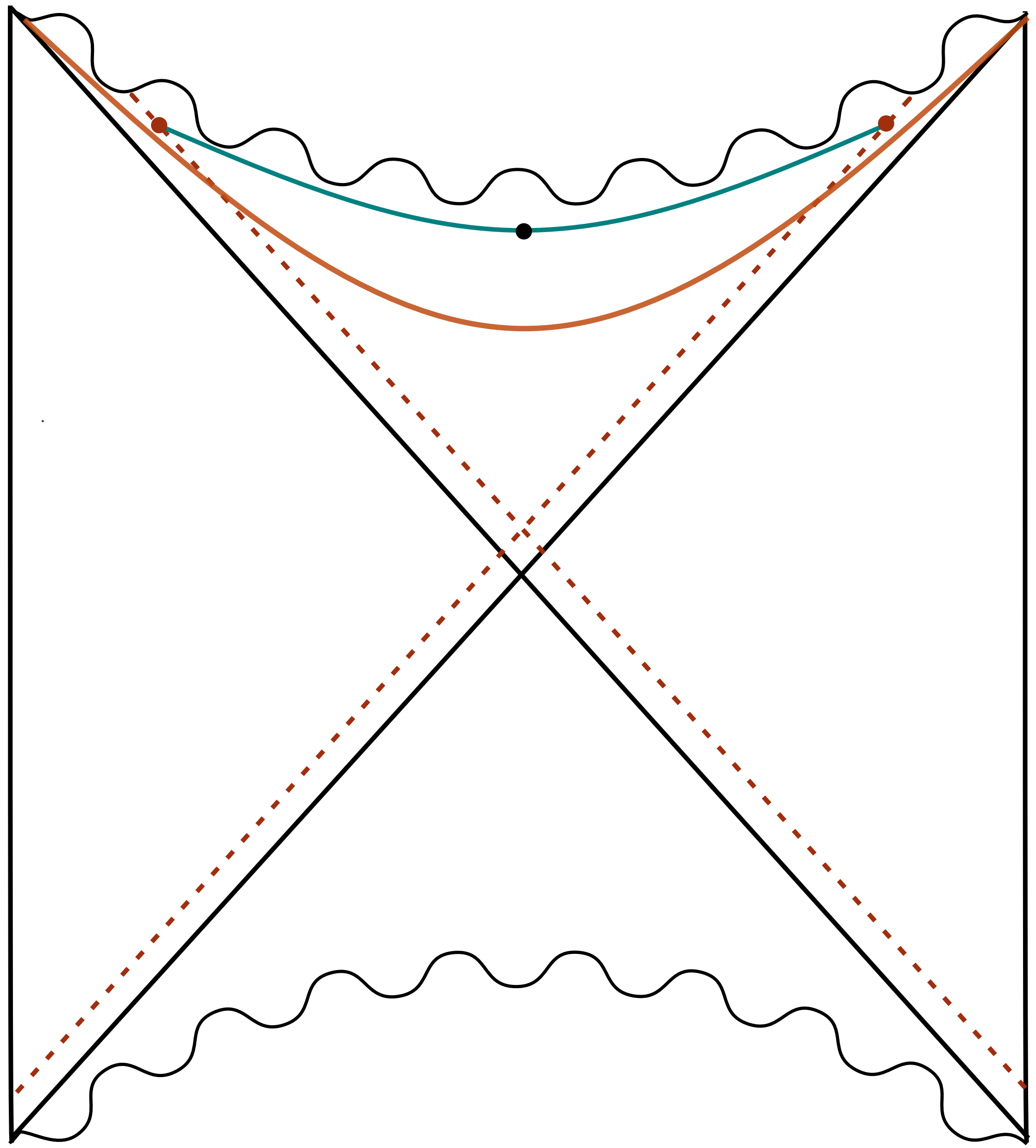
Case 2

Anchor Points are in the interior of the accumulation surface



Case 2

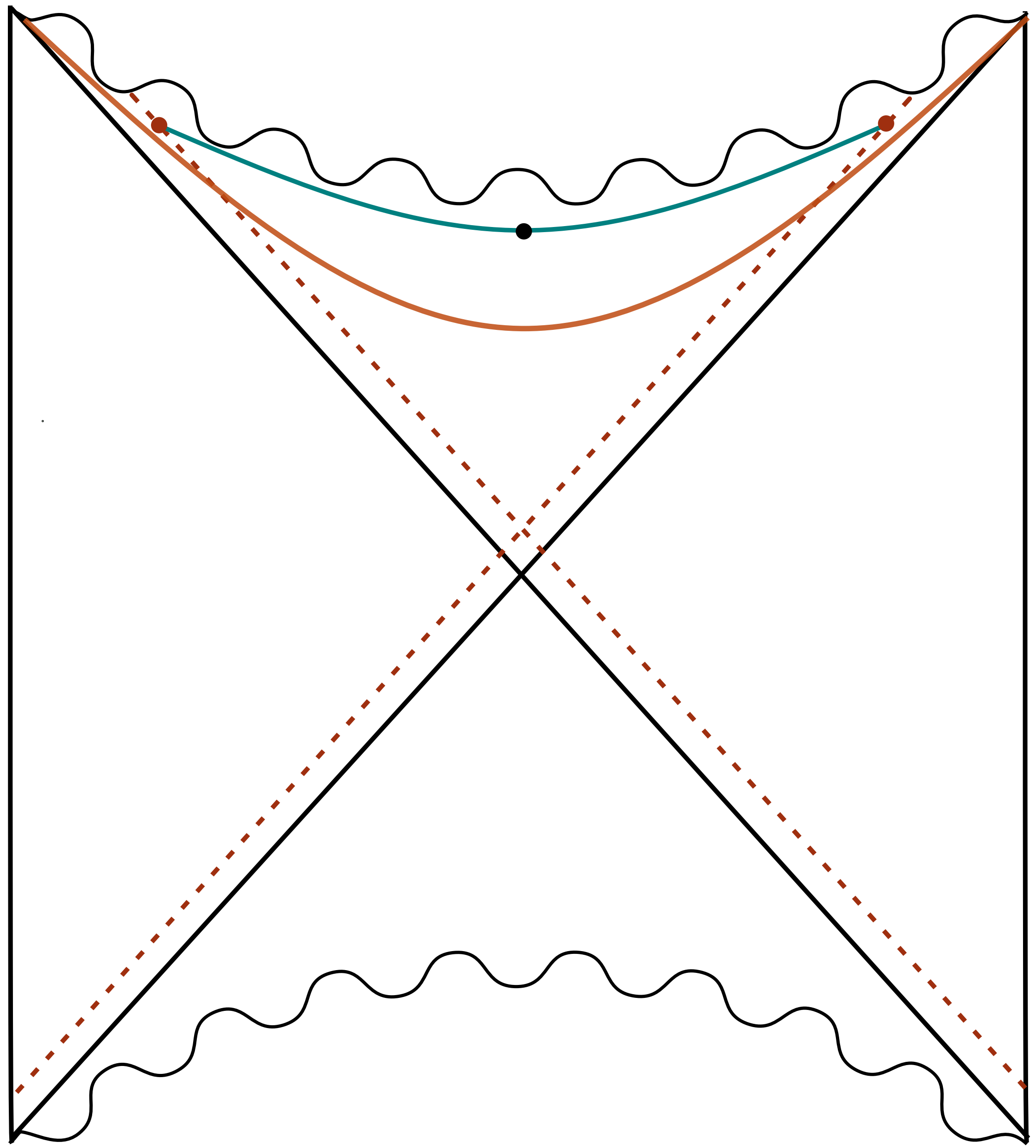
Anchor Points are in the interior of the accumulation surface



Volume **Decreases** when we move the anchor points to the future

Case 2

Anchor Points are in the interior of the accumulation surface



We will refer to these surfaces as Trapped extremal surfaces

Singularity theorem

<u>Theorem</u>

Consider a globally hyperbolic spacetime M with a trapped extremal surface T.

Then, the spacetime is null geodesically incomplete.

Proof (Sketch)

The statement of the SLC is very powerful because it provides a bulk condition on the growth rate of volumes anchored to causal horizons.

An immediate implication of the SLC is that if there are trapped extremal surfaces somewhere in the spacetime, then they cannot be anchored to a causal horizon.

This means that the outward directed null sheet N onto which we anchored the trapped extremal surface is bounded.

If M is null geodesically complete, then N becomes compact. This is problematic because it would mean that non-compact Cauchy slice would evolve into a compact space N. Therefore, M is geodesically incomplete.

So what?

Quantum Corrections and Resolution of Singularities (W.I.P)

The trapped extremal surface starts appearing immediately behind the accumulation surface.

The volume calculation in this region can be mapped to a geodesic length calculation in JT gravity. [Gautason VM Thorlacius 2025]

The length of these geodesics can be non-perturbatively defined in JT gravity using its matrix model description.

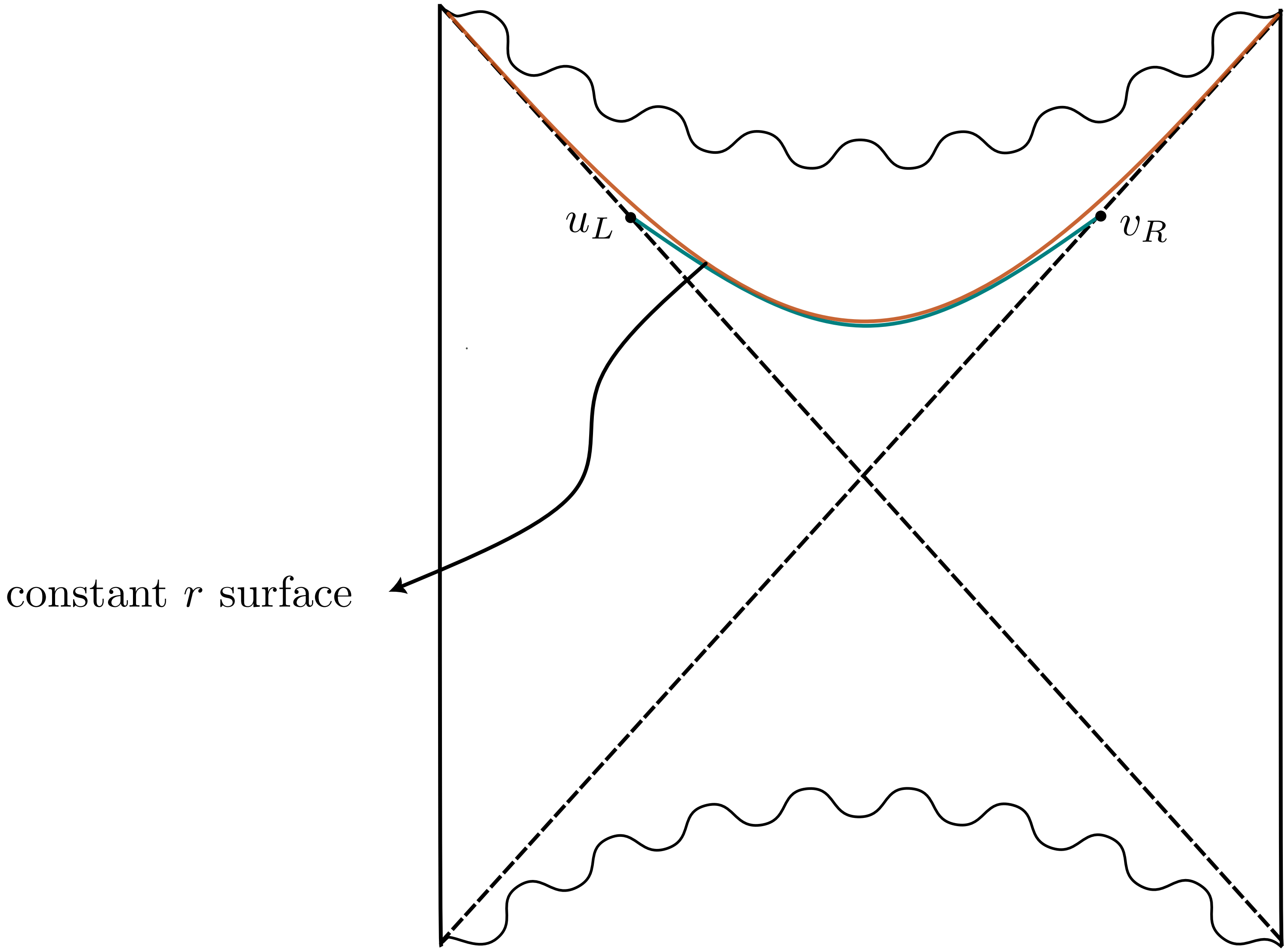
When we add explicit non-perturbative quantum gravity corrections, the trapped extremal surfaces disappear!

Vanishing of a signature of the black hole singularity

Thank You!

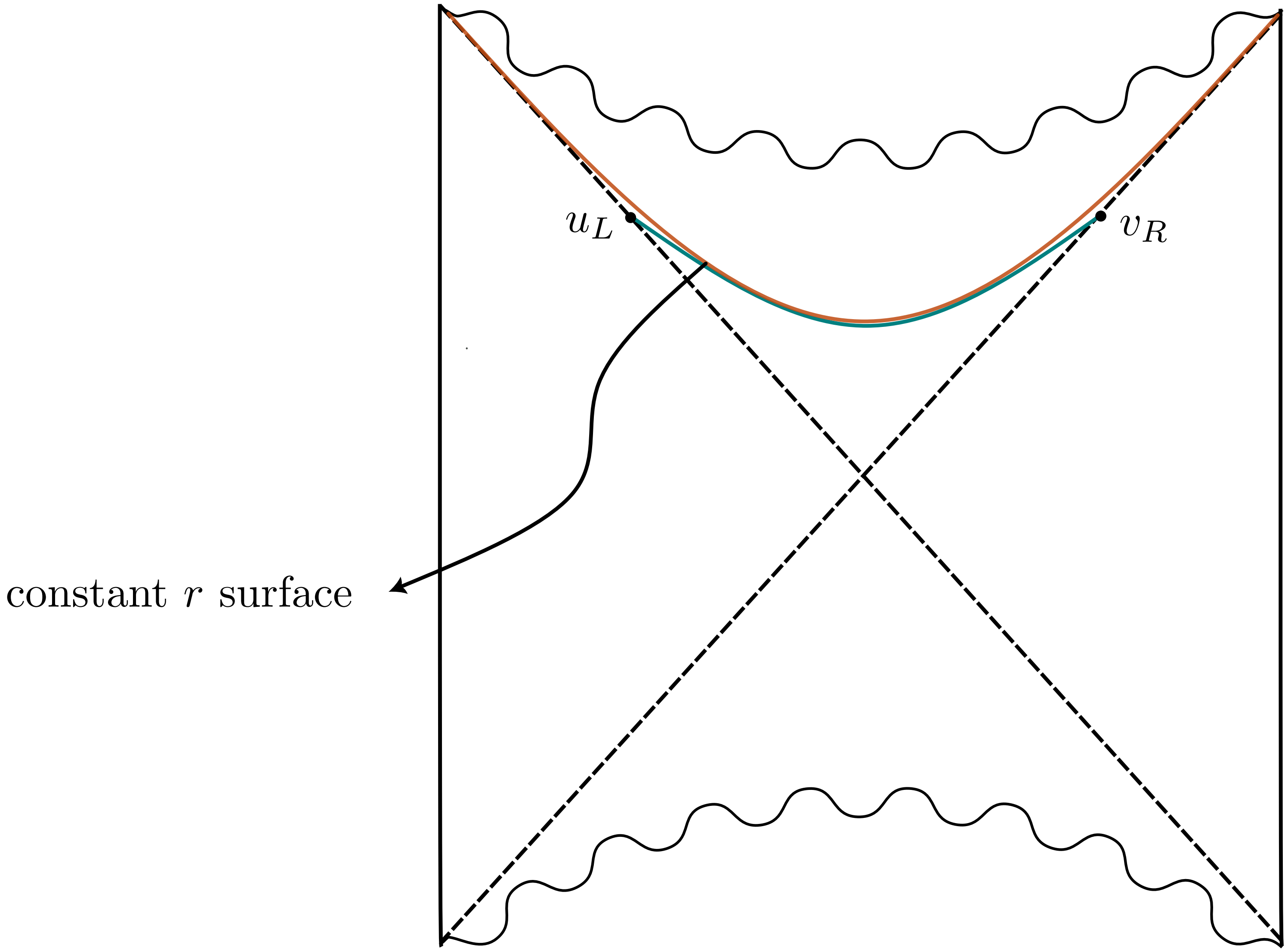
Backup Slides

Late-time behaviour



Accumulation surface

Late-time behaviour



Accumulation surface

Leads to a late-time linear growth

$$\mathcal{V} \propto v_R$$

Causal Horizon

Consider a future-infinite timelike worldline W. We define a future-causal horizon as the boundary of the past of the worldline, that is,

$$H_{\mathrm{fut}} = \partial I^{-}(W)$$

[Jacobson Parentani 2003]