Cosmology inside a black hole: adding matter on the brane

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New Insights in Black Hole Physics from Holography, IFT Madrid

Preliminaries

- AdS/CFT correspondence provides a non-perturbative description of quantum gravity (Maldacena 1997, Witten 1998, Gubser *et al.* 1998)

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- Within this framework, big bang/ big crunch cosmologies can emerge from some high-energy black hole microstates (Cooper *et al.* 2019, Antonini *et al.* 2021)

- These closed FRW cosmologies live on an End-of-the-World (ETW) brane inside a one-sided AdS black hole

- More precisely, the $(d+1)$-dimensional model consists of a bulk Einstein–Hilbert action with a cosmological constant $\Lambda = -\frac{d(d-1)}{2L^2}$, together with a d-dimensional brane with tension T.

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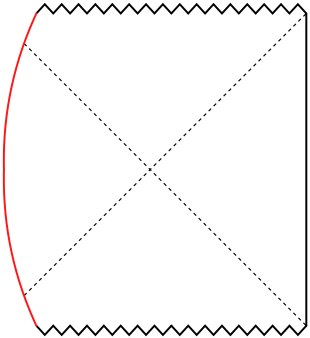
- We consider a vacuum black hole solution in the bulk,

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Sigma_k^2\,, \qquad f(r) = k + \frac{r^2}{L^2} - \frac{\mu}{r^{d-2}}\,,$$

with inverse temperature β,

$$\beta = \frac{4\pi r_h L^2}{dr_h^2 + (d-2)kL^2},$$



Penrose diagram for ETW inside an AdS-Schw. black hole

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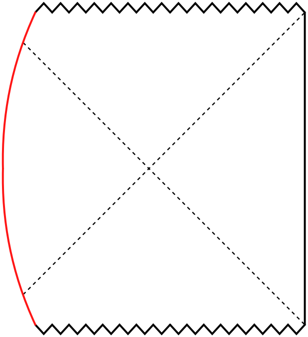
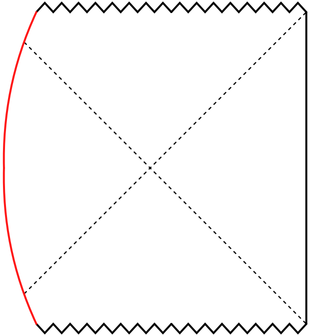


Penrose diagram for ETW inside an AdS-Schw. black hole

- To have an effective cosmological description on the brane, we must take,

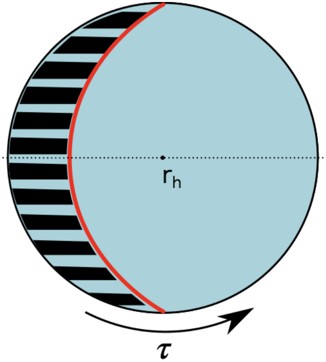
$$\bar{r} \gg r_h \quad (T \approx 1/L \quad \text{and} \quad \frac{1}{\ell^2} = \frac{1}{L^2} - T^2)$$



Penrose diagram for ETW inside an AdS-Schw. black hole

- The world-volume of the ETW brane is FRW cosmology with induced metric,

$$ds^2_{\text{ETW}} = -d\lambda^2 + r(\lambda)^2 d\Sigma_k^2$$



Euclidean ETW brane trajectory

- The braneworld induced metric after the analytic continuation $\lambda \to -i\lambda$,

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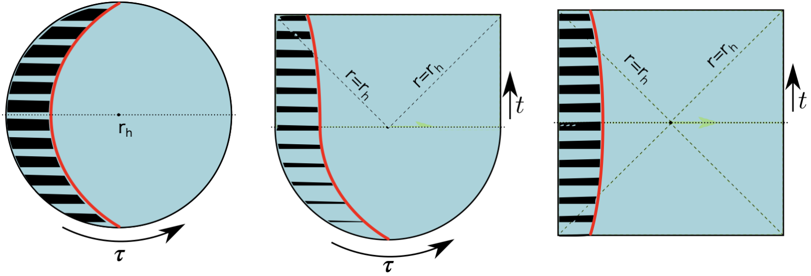
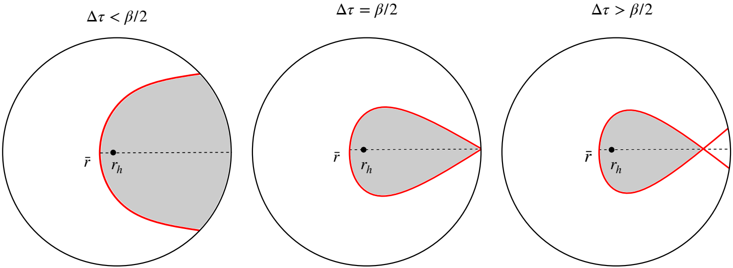


Euclidean construction of the ETW brane inside an AdS black hole (Almheiri *et al.* 2023)

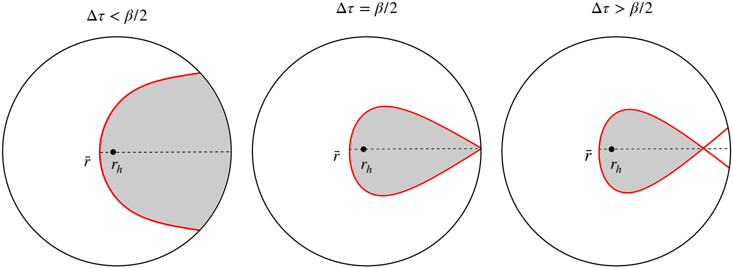
- Existence of the Euclidean solution is essential to prepare the initial state on the $t = 0$ slice through Euclidean time evolution



Euclidean ETW brane configurations for different turning point $\bar{r}$

- The Euclidean ETW brane with tension T reaches the boundary at

$$\tau_{\mp} = \mp\frac{\beta}{2} \pm \int_{\bar{r}}^{\infty} \frac{dr}{f}\frac{H}{\sqrt{f - H^2}} , \qquad H = Tr .$$



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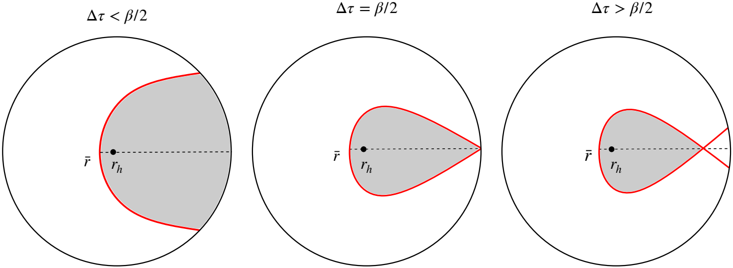

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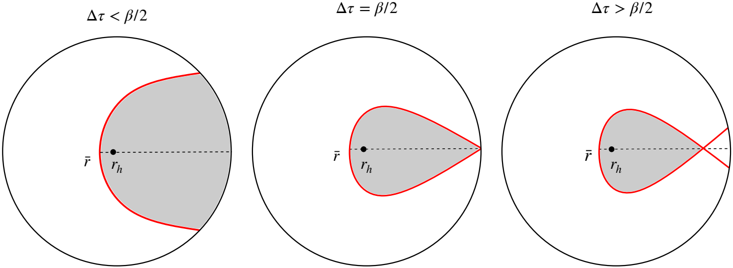

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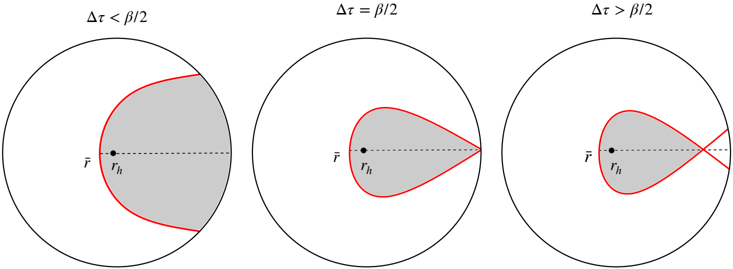
- However, for pure tension braneworlds, this requirement is violated for some region $\bar{r} > r_h$ for $d > 2$!!!



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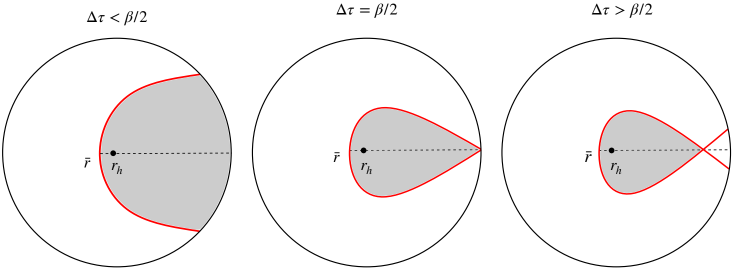
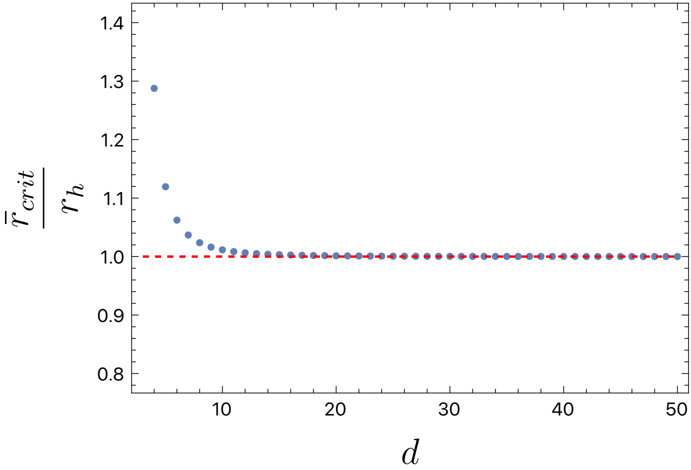


Euclidean ETW brane configurations for different turning point $\bar{r}$

- This corresponds to the regime well before the effective dynamics on the brane can be approximated by Einstein gravity ($\bar{r} \gg r_h$).
- As the brane self-intersects, it becomes disconnected from the boundary and there is no possible way to prepare the $t = 0$ state through Euclidean time evolution.



$\bar{r}_{crit}$ in units of r_h vs spatial dimension d

Outline

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- I will consider explicit matter content such as a perfect fluid, a scalar field and an axion field, which are highly relevant to cosmological modeling.
- I will explain that while the perfect fluid and axion can resolve the self-intersection problem, a homogeneous scalar field can not!

Perfect fluid on the brane

- First, we add a perfect fluid on the brane with stress-energy tensor,

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- Brane induced cosmological constant is,

$$\boxed{\Lambda_{\text{brane}} = -\frac{(d-1)(d-2)}{\ell^2} \quad \text{where} \quad \ell \gg L}$$

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$$\boxed{\left(\frac{\dot{r}}{r}\right)^2 \approx \frac{16\pi G_{brane} TL}{(d-2)(d-1)}\rho - \frac{k}{r^2} - \frac{1}{\ell^2} + \frac{\mu}{r^d}}$$

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- By doing the analytic continuation, we derive the Euclidean version,

$$\left(\frac{\dot{r}}{r}\right)^2 \approx \frac{k}{r^2} + \frac{1}{\ell^2} - \frac{16\pi G_{brane} TL}{(d-2)(d-1)}\rho - \frac{\mu}{r^d}$$

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- By demanding, $r_h \sim L$ and $\bar{r}/\ell > \mathcal{O}(1)$, we can show that for any $d > 2$,

$$\Delta\tau < \beta/2.$$

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- For a concrete example, take dust with $\rho \sim r^{-(d-1)}$,

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- The key point is that the addition of brane matter enables us to solve the turning point equation in the small L regime, and to choose a large value of $\bar{r}$, without changing r_h.

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- The Euclidean scalar equation obtained by the usual sign-flip of the potential with respect to the Lorentzian one:

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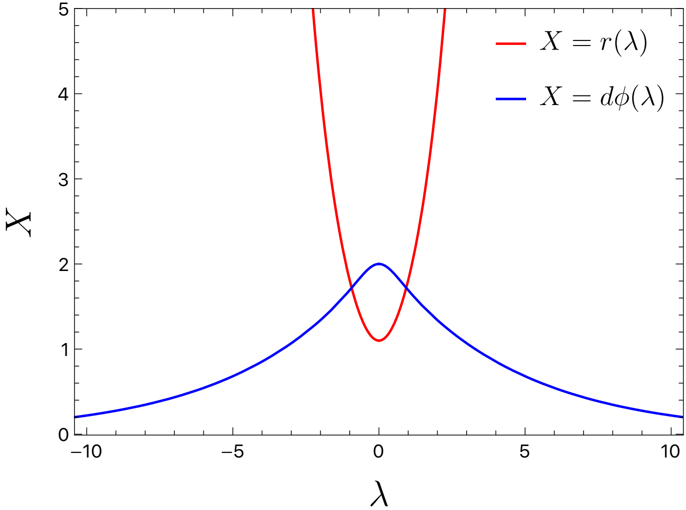

Scalar and the brane profile for the potential $V(\phi) = m^2\phi^2$ with $m^2 = -1/4$

Scalar on the brane

- The cosmological friction implies,

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- Since the pure-tension brane already suffers from self-intersections, the addition of even a small homogeneous scalar field further aggravates the issue.

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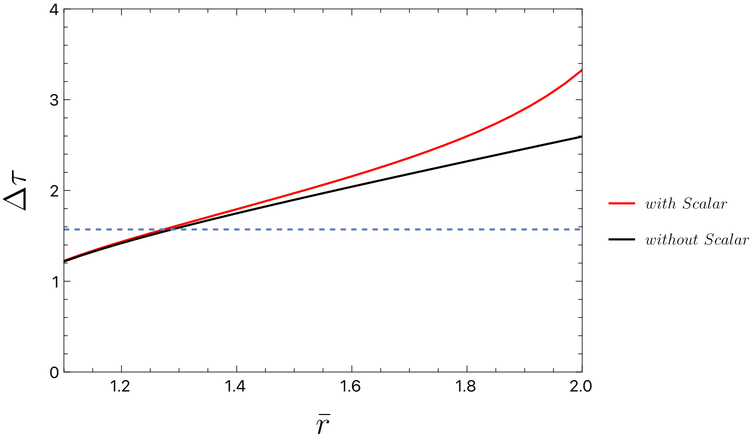

$\Delta\tau$ vs the brane turning point $\bar{r}$

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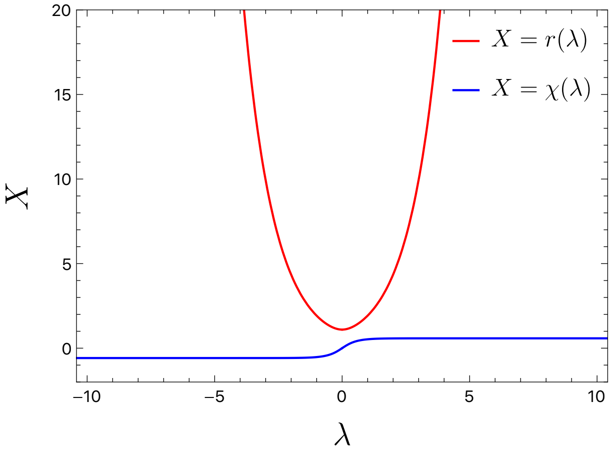

Axion and Euclidean ETW brane profiles

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- While a homogeneous scalar field on the brane does not resolve this problem, an axion field does!

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- While a homogeneous scalar field can not cure the self-intersection problem, an inhomogeneous scalar field may be able to resolve it.

(Betzios & Papadoulaki 2024)

Thank You!