Boundary-induced transitions in Möbius quenches of holographic BCFT

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with Alice Bernamonti (Florence U.) Dongsheng Ge (Osaka U.)

Introduction

Boundaries are a generic features in physical systems and arise in a number of situations, with effects that are probed by physical quantities such as partition and correlations functions.

In 2d CFT for instance the presence of a boundary gives additional contributions, which depends on the choice of boundary conditions, to the thermodynamic and entanglement entropy

$$S_{TH} = \frac{c\pi}{3\beta} L + s_{\mathcal{B}} \qquad\qquad S_{EE} = \frac{c}{6} \log \frac{\ell}{\epsilon} + s_{\mathcal{B}}$$

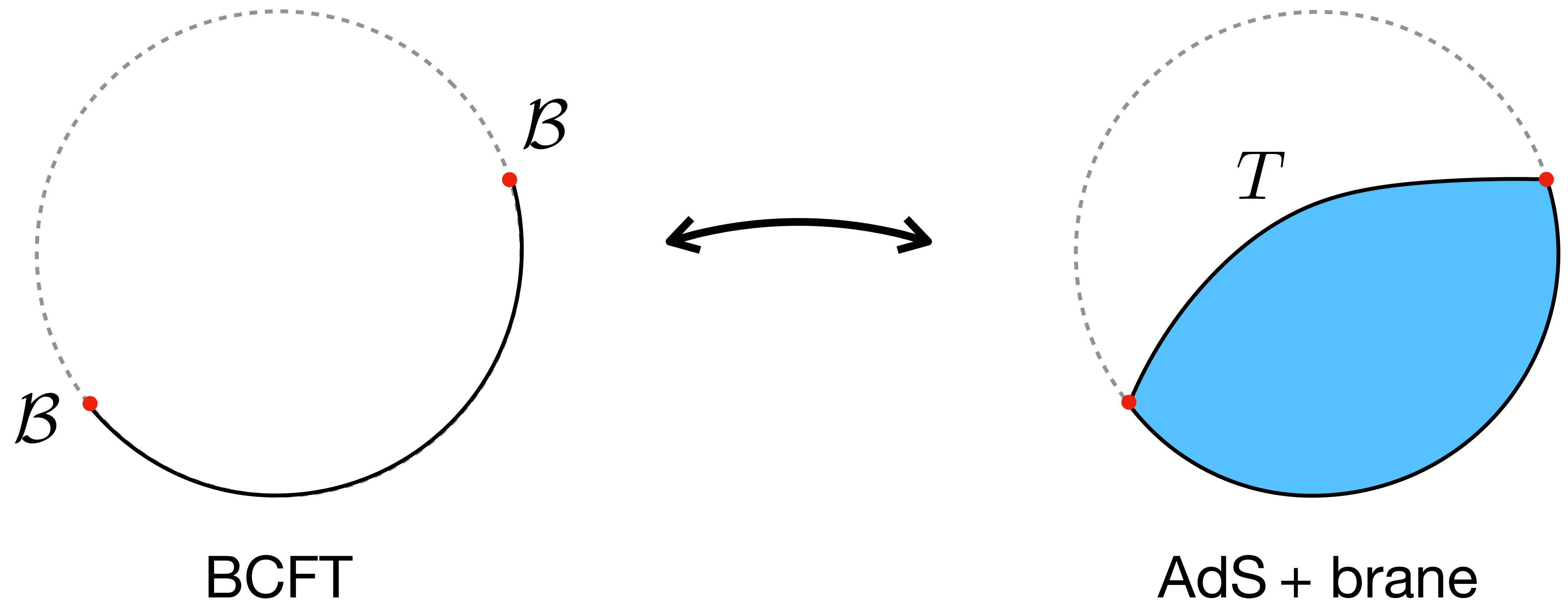
where next to the usual terms there is a contribution from the boundary entropy associated with the corresponding boundary state

When some dynamics is injected into a system, boundary conditions can significantly alter the dynamics and lead to different effects depending on the choice of boundary conditions.

In this talk

Q: How boundary effects can influence the dynamics of entanglement?

- 2d Holographic BCFTs:

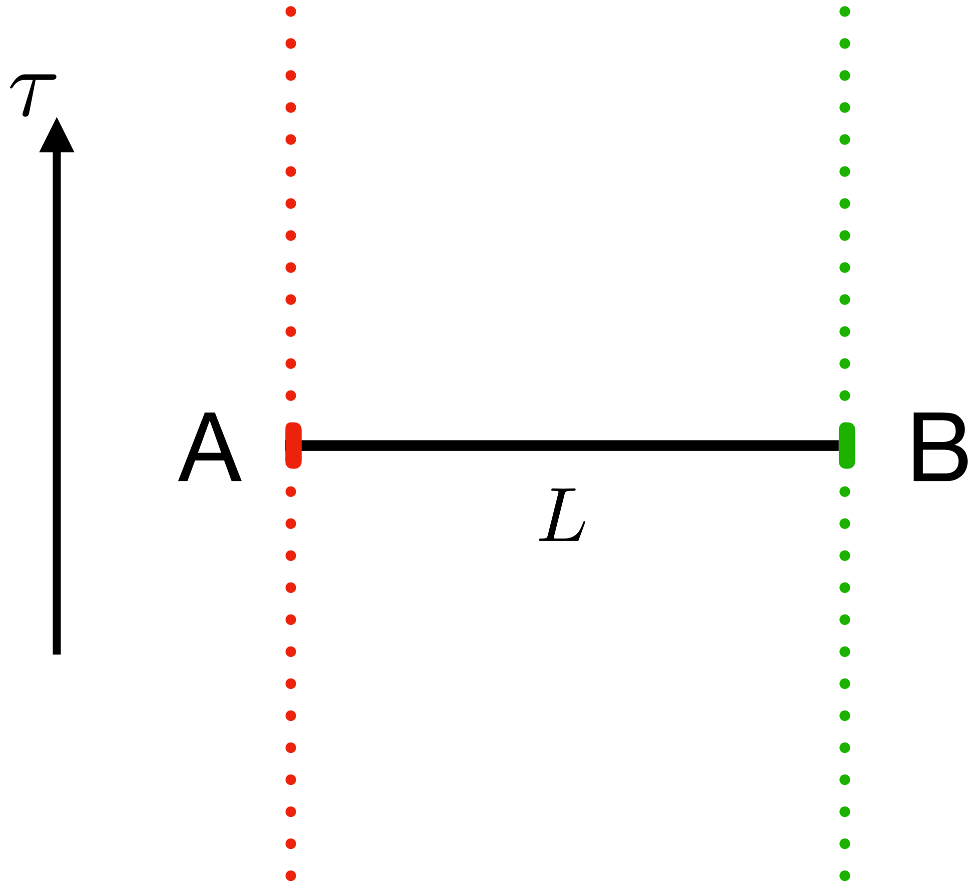


- Dynamics induced by a particular class of spatially inhomogenous quenches: Möbius and SSD Hamiltonian quenches

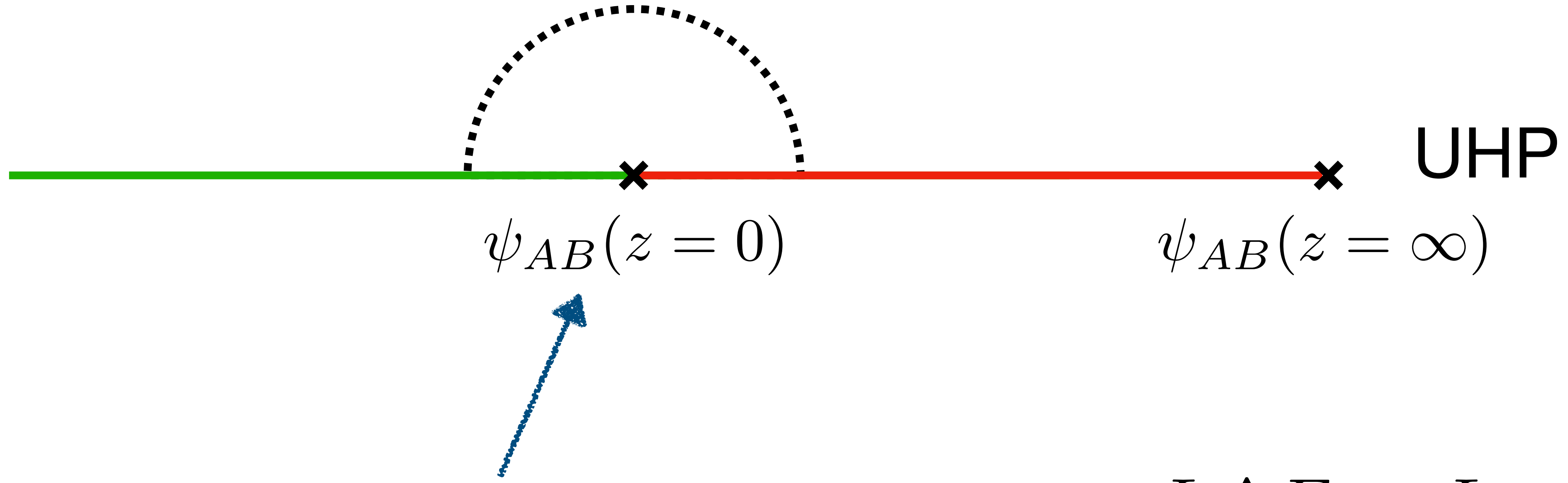
Setup: 2D CFT on a Strip

CFT in 1+1 dimensions on an interval of length L, with different conformal boundary conditions at the two ends of the interval

[Cardy89]
[Lewellen92]
[Affleck92]



$$|\psi_{AB}\rangle$$



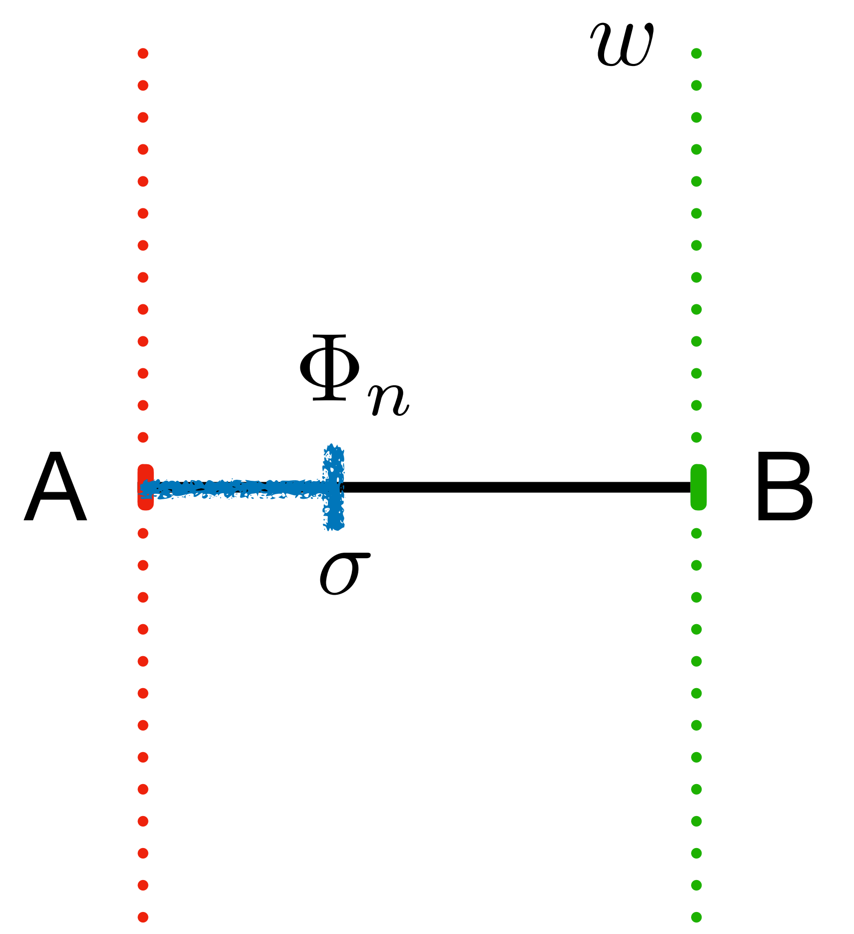
Boundary Condition Changing operator

$$h_\psi = \frac{L\,\Delta E}{\pi} = \frac{L}{\pi}(E_{AB} - E_{AA})$$

$|\psi_{AB}\rangle = \psi_{AB}(0)|A\rangle$ minimal or higher-energy

Entanglement Entropy

Entanglement entropy of a subinterval adjacent to one of the boundaries



$$S = -\mathrm{Tr}\ \rho_\sigma \log \rho_\sigma$$

reduced density matrix

Can be computed as a correlator of twist operators

[CalabreseCardy04]
...

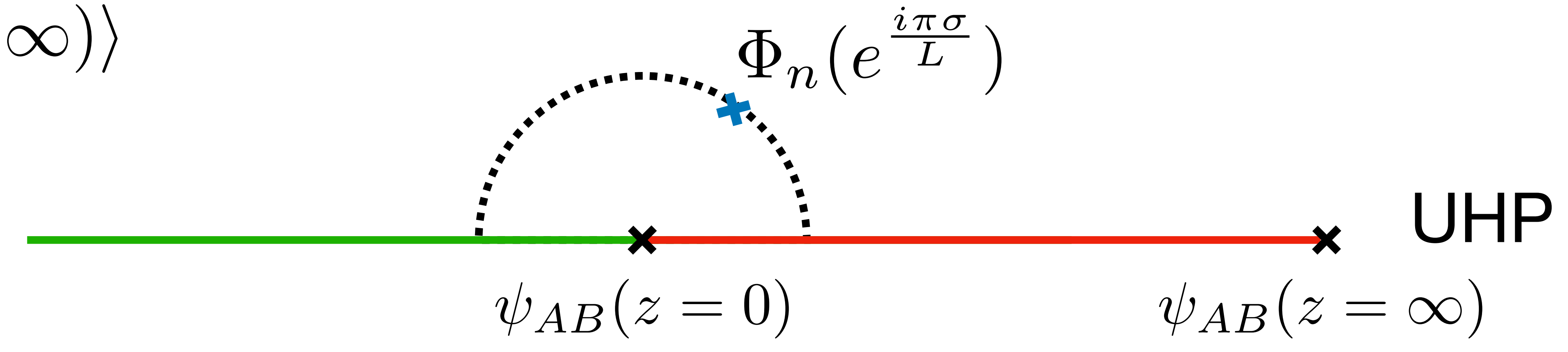
$$S = \lim_{n\to 1} \frac{1}{1-n} \log \langle \psi_{AB} | \Phi_n(w, \bar{w}) | \psi_{AB} \rangle$$

twist operator

$$h_h = \frac{c}{24}\left(n - \frac{1}{n}\right)$$

As opposed to the case with homogeneous BCs, this is not fixed by conformal invariance

$$\langle \psi_{AB} | \Phi_n(w, \bar{w}) | \psi_{AB} \rangle = \left| \frac{\partial z}{\partial w} \right|^{2h_n} \langle \psi_{AB}(0) \Phi_n(z, \bar{z}) | \psi_{AB}(\infty) \rangle$$

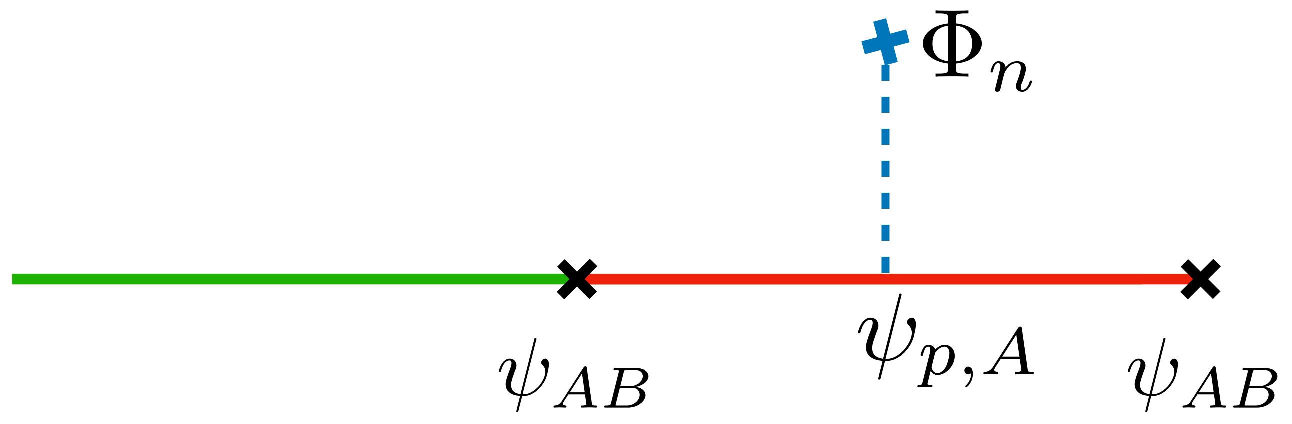


Holographic BCFTs

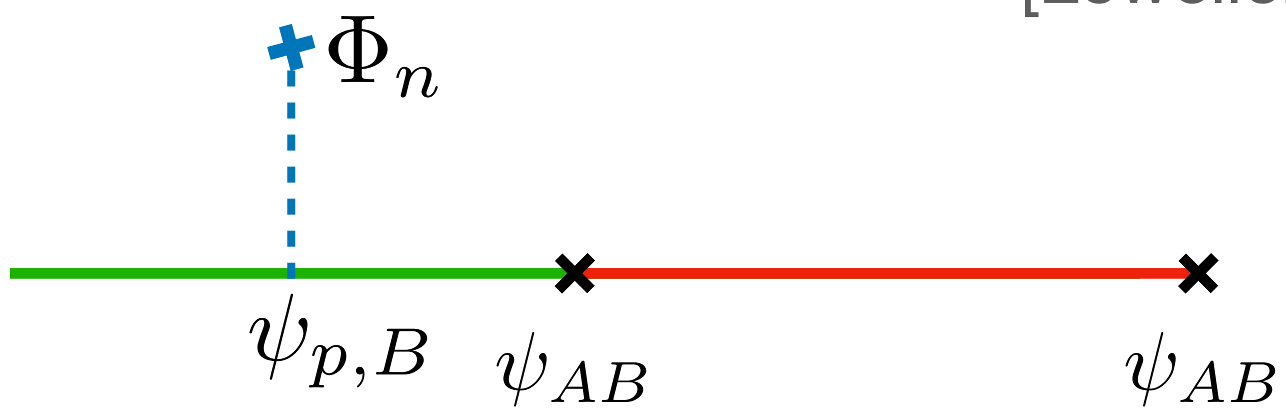
Analogous to a four point function in the full complex plane. Two relevant expansions:

[Lewellen92]

$$\langle \psi_{AB}(0)\Phi_n(z,\bar{z})|\psi_{AB}(\infty)\rangle = \sum_p$$



$$= \sum_p$$



Depends on all the details of the CFT

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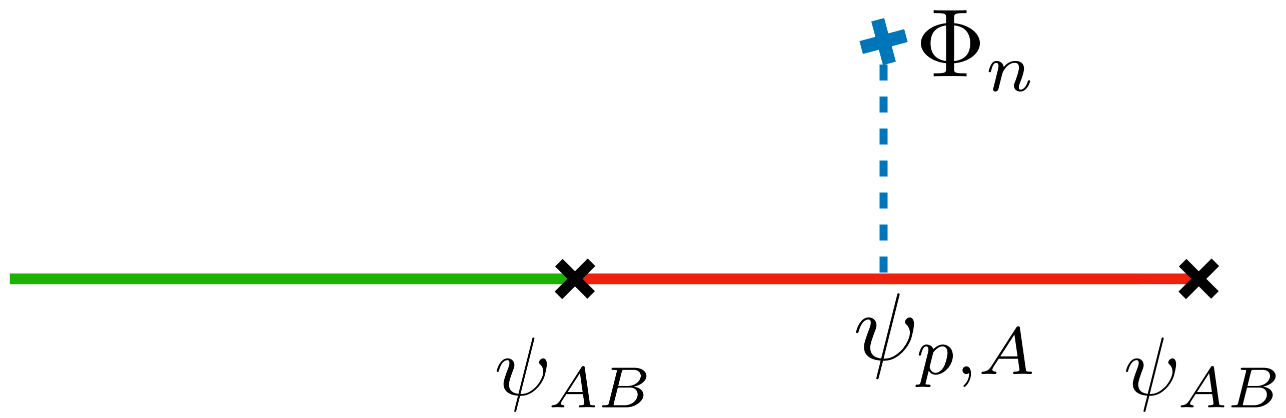

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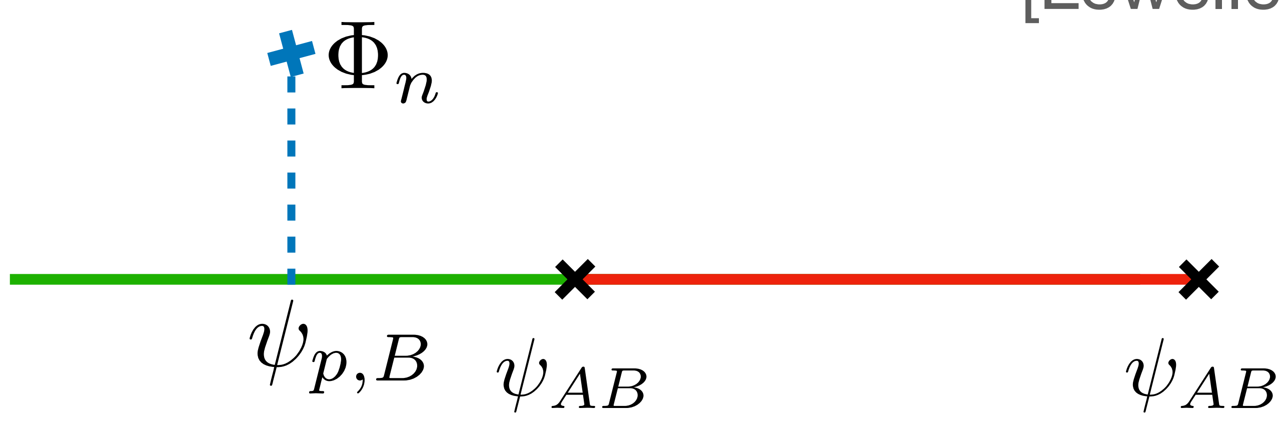


Depends on all the details of the CFT

For holographic CFTs

Large central charge $c \gg 1$

Sparse spectrum of low-dimension operators

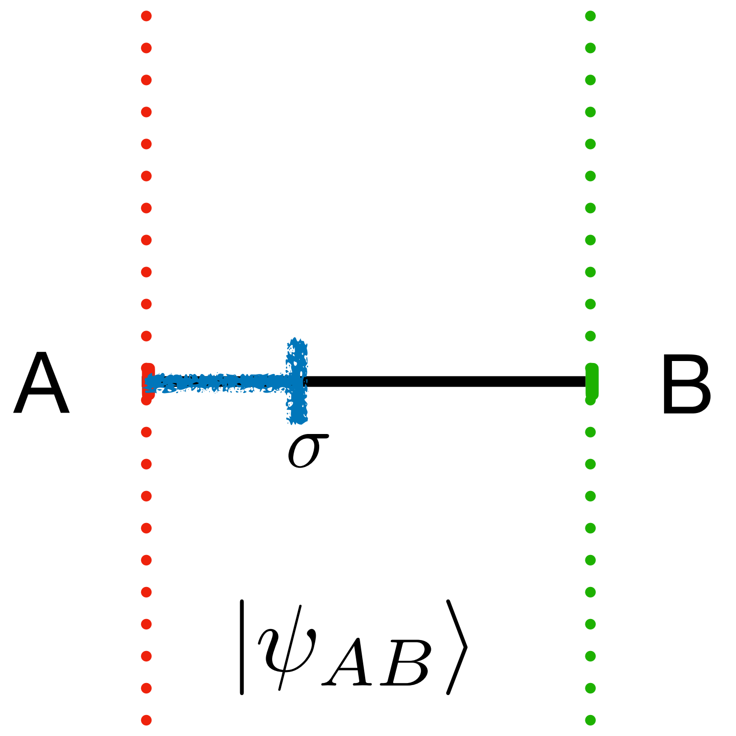
→ Dominant contribution to the computation of the entanglement entropy comes from the exchange of the stress tensor and its conformal family

[FitzpatrickKaplanWalters14]
[Hartman14]
[AsplundBernamontiFGHartman14]
…

Entanglement Entropy in HBCFTs

Retaining the universal contribution from the conformal family of the Identity the entanglement entropy is evaluated as the minimum between two contributions

$$S = \frac{c}{6}\log\frac{2L}{\pi\alpha\epsilon} + \min\left\{\frac{c}{6}\log\sin\frac{\alpha\pi\sigma}{L} + s_A ; \frac{c}{6}\log\sin\frac{\alpha\pi(L-\sigma)}{L} + s_B\right\}$$



$$\alpha = \sqrt{1 - \frac{24h_\psi}{c}}$$

Boundary entropy $s_A = \log\langle 0|A\rangle$

[AffleckLudwig91]

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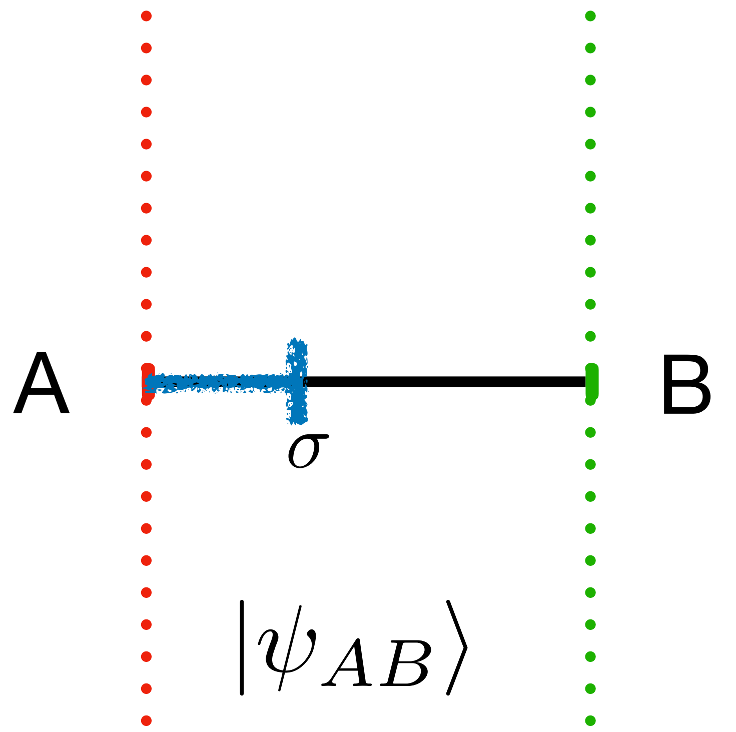


$$\alpha = \sqrt{1 - \frac{24h_\psi}{c}}$$

Boundary entropy $s_A = \log\langle 0|A\rangle \sim c$

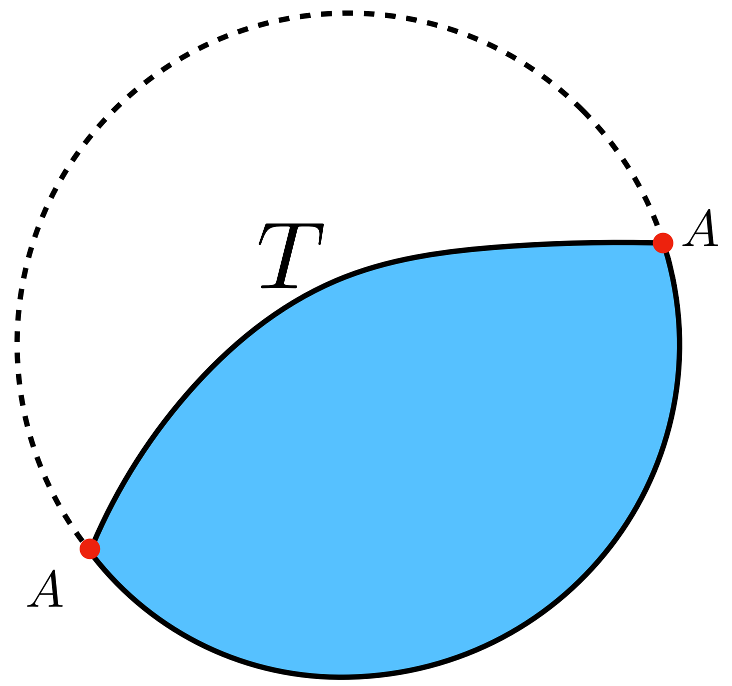
[AffleckLudwig91]

HBCFT

$$s = \frac{c}{3}\mathrm{arctanh}T$$

[Takayanagi11]

...



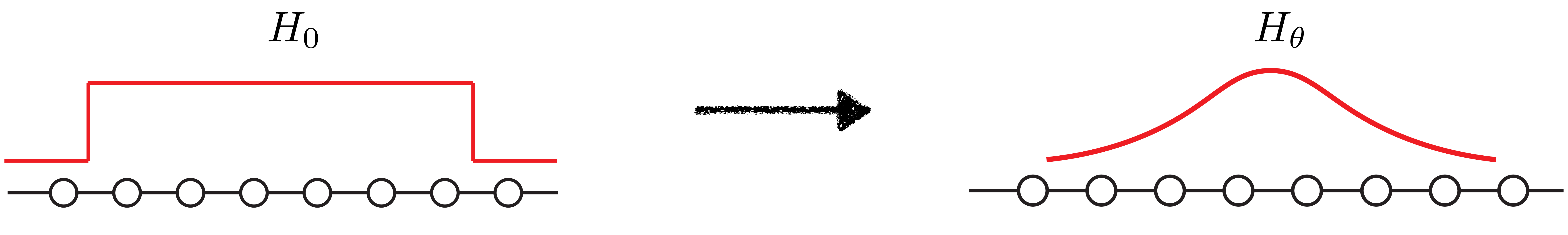
The boundary entropy participates in determining the dominant saddle

Even for a small region adjacent one boundary, the entropy can “probe” the other boundary

Inhomogeneous Quench

Q: What happens when you give the system some dynamics?

Quench the system changing the Hamiltonian



$|\psi_{AB}\rangle$ equilibrium state

$|\psi_{AB}(t)\rangle = e^{-iH_\theta t}|\psi_{AB}\rangle$

non-trivial dynamics

H_θ class of spatially inhomogeneous deformations of H_0

Möbius-like Hamiltonians

Full class of deformed Hamiltonian constructed from the CFT stress tensor

$$H_\theta = H_0 - \frac{\tanh(2\theta)}{2}(H_k^+ + H_k^-)$$

strength of the deformation

$$H_0 = \int_0^L d\sigma\, T_{\tau\tau}(\sigma) = \int_0^L \frac{dw}{2\pi}\left[T(w) + \bar{T}(\bar{w})\right]$$

$$H_k^\pm = \int_0^L \frac{dw}{2\pi}\left[e^{\pm k\pi w/L}T(w) + e^{\mp k\pi \bar{w}/L}\bar{T}(\bar{w})\right]$$

Example: Sine Square Deformation, obtained for *k=2* and $\theta \to \infty$

$$H_{SSD} = \int_0^L d\sigma\, 2\sin^2\left(\frac{\pi\sigma}{L}\right) T_{\tau\tau}(\sigma) = \frac{\pi}{L}\left(L_0 - \frac{L_2 + L_{-2}}{2} - \frac{c}{24}\right)$$

Defined in terms of the $\{L_0, L_{\pm k}\}$ SL(2,R) subalgebra of the full Virasoro

Time Evolution

The action of the Hamiltonian on a primary operator can be obtained explicitly and takes the form of a conformal transformation

$$e^{H_\theta t_E}\mathcal{O}(z,\bar{z})e^{-H_\theta t_E} = \left(\frac{\partial z_{t_E}}{\partial z}\right)^{h_\mathcal{O}} \left(\frac{\partial \bar{z}_{t_E}}{\partial \bar{z}}\right)^{\bar{h}_\mathcal{O}} \mathcal{O}(z_{t_E},\bar{z}_{t_E})$$

$$z_{t_E}^2 = \frac{\frac{\pi t_E}{L}(z^2-1)+z^2}{\frac{\pi t_E}{L}(z^2-1)+1} \qquad \text{SSD}$$

the time t_E after the quench enters as a parameter

For post quench state $\rho(t_E) = e^{-H_\theta t_E}|\psi_{AB}\rangle\langle\psi_{AB}|e^{H_\theta t_E}$

the evaluation of entanglement entropy reduces to the same computation performed for the initial state

$$S(t_E) \sim \langle\psi_{AB}|\, e^{H_\theta t_E}\Phi_n(w,\bar{w})e^{-H_\theta t_E}\,|\psi_{AB}\rangle = \left|\frac{\partial z_{t_E}}{\partial w}\right|^{2h_n} \langle\psi_{AB}(\infty)\Phi_n(z_{t_E},\bar{z}_{t_E})\psi_{BA}(0)\rangle$$

but with time dependent insertion

Entanglement Entropy Evolution

Continuing to real time $t_E \to it$ we obtain the general expression

$$S(t) = \frac{c}{12}\log\left[\left(\frac{2L}{\pi\alpha\epsilon}\right)^2\left(f^2(t)+\sin^2\left(\frac{2\pi\sigma}{L}\right)\right)\right] + \min\left\{\frac{c}{12}\log\sin^2\left(\frac{\alpha}{2}\delta(t)\right)+s_A; \frac{c}{12}\log\sin^2\left(\frac{\alpha}{2}(2\pi-\delta(t))\right)+s_B\right\}$$

SSD Hamiltonian

$$f(t) = -\frac{2\pi^2t^2}{L^2} + \left(1+\frac{2\pi^2t^2}{L^2}\right)\cos\left(\frac{2\pi\sigma}{L}\right)$$

$$e^{i\delta} = \frac{f(t)+i\sin\left(\frac{2\pi\sigma}{L}\right)}{\sqrt{f(t)^2+\sin^2\left(\frac{2\pi\sigma}{L}\right)}}$$

Möbius k=2 Hamiltonian

$$f_\theta(T) = -\sin^2(T)\sinh(4\theta) + \left(\cos^2(T)+\cosh(4\theta)\sin^2(T)\right)\cos\left(\frac{2\pi\sigma}{L}\right)$$

$$e^{i\delta_\theta} = \frac{f_\theta(T)+i\sin\left(\frac{2\pi\sigma}{L}\right)}{\sqrt{f_\theta(T)^2+\sin^2\left(\left(\frac{2\pi\sigma}{L}\right)\right.}}$$

$$T = \frac{\pi t}{L\cosh(2\theta)}$$

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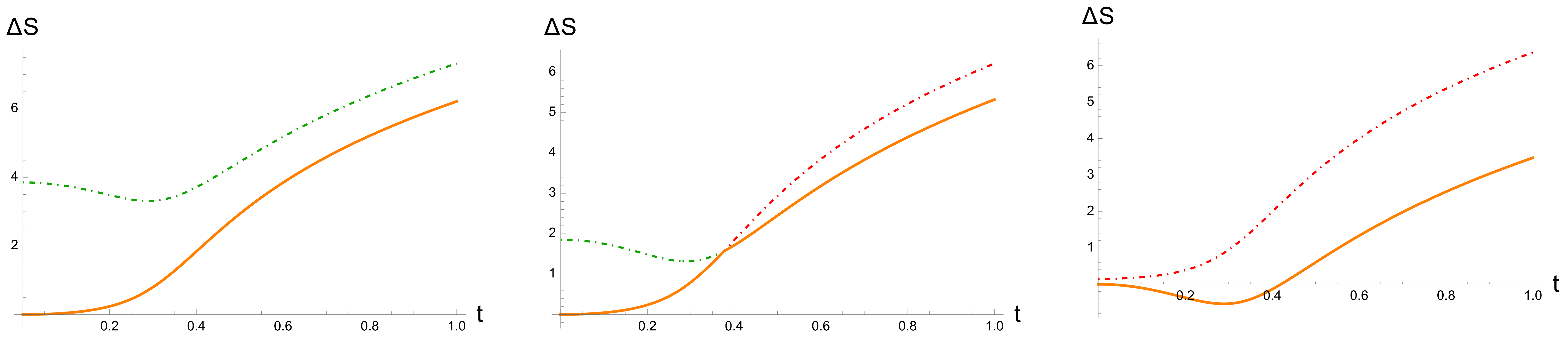
$$T = \frac{\pi t}{L\cosh(2\theta)}$$

Q: Can the quench dynamics change the dominant saddle?

SSD Quench



$s_A \ll s_B$ $\qquad$ $0 < s_A - s_B < \Delta s(\sigma)$ $\qquad$ $s_A \gg s_B$

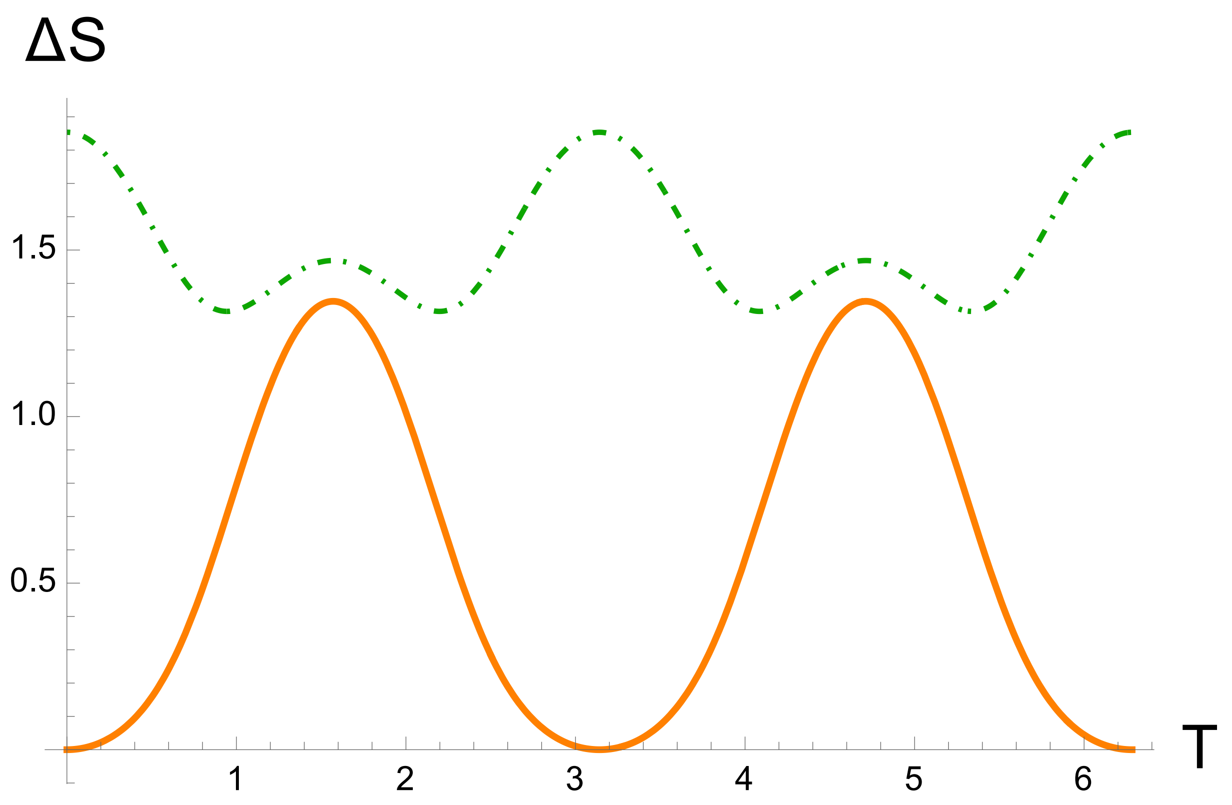
Dynamical phase transition induced by the quench

The transition pattern is determined by relation between the boundary entropies: when the boundary degrees of freedom of A and B are close enough to each other a transition from A to B happens

Evolution of each channel qualitatively similar to the universal case with a single boundary condition

Late time growth with no revivals compatible with an effective infinite length $S(t) \sim \frac{c}{3} \log \frac{t}{\epsilon}$ [WenWu18]

Möbius Quench



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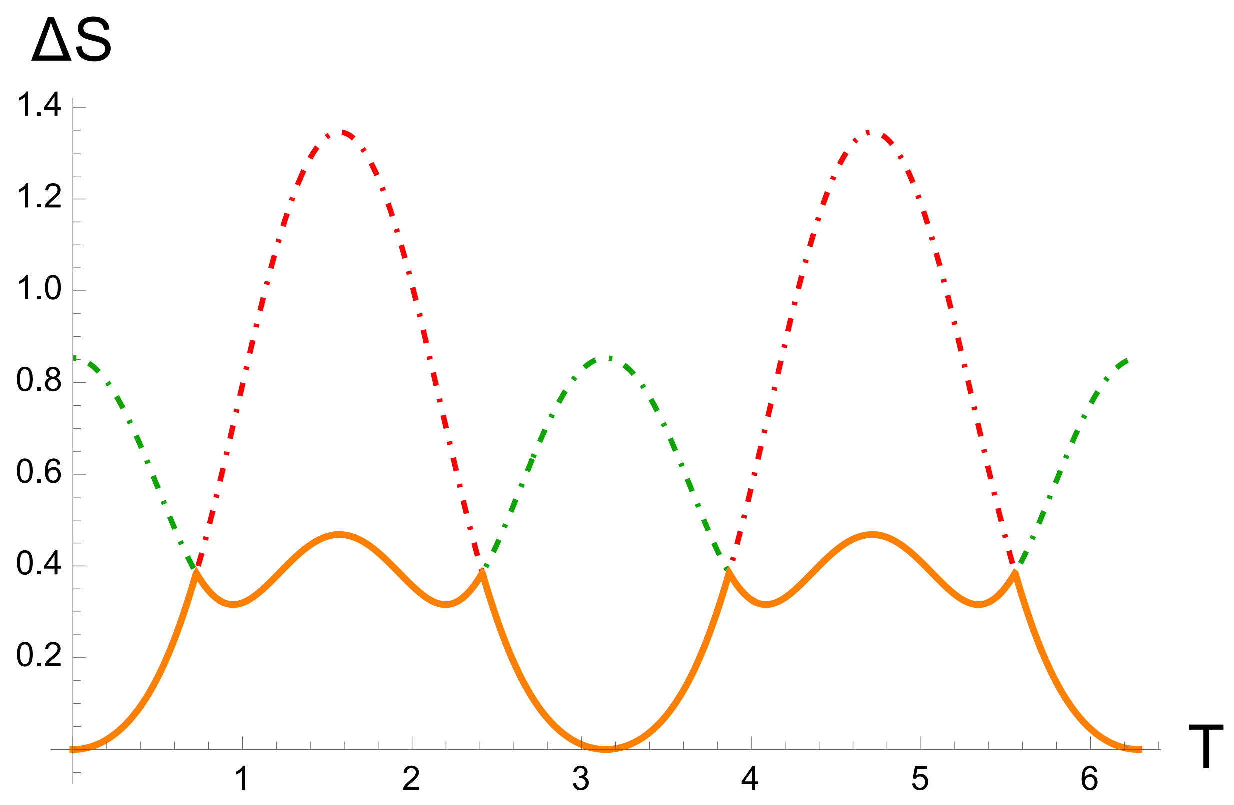
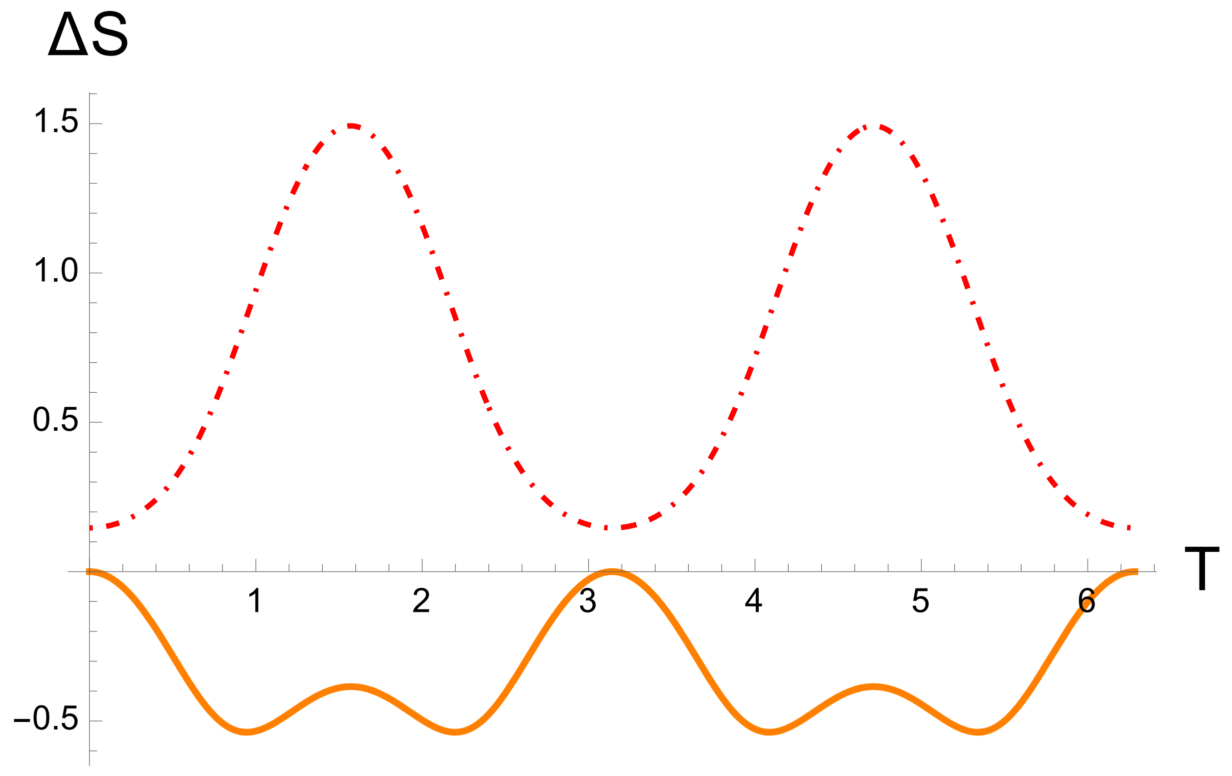


$\tilde{\Delta} s(\sigma, \theta) < s_A - s_B < \Delta s(\sigma)$



$s_A \gg s_B$

Interpolates between the undeformed $(\theta = 0)$ and the SSD $(\theta = \infty)$ case

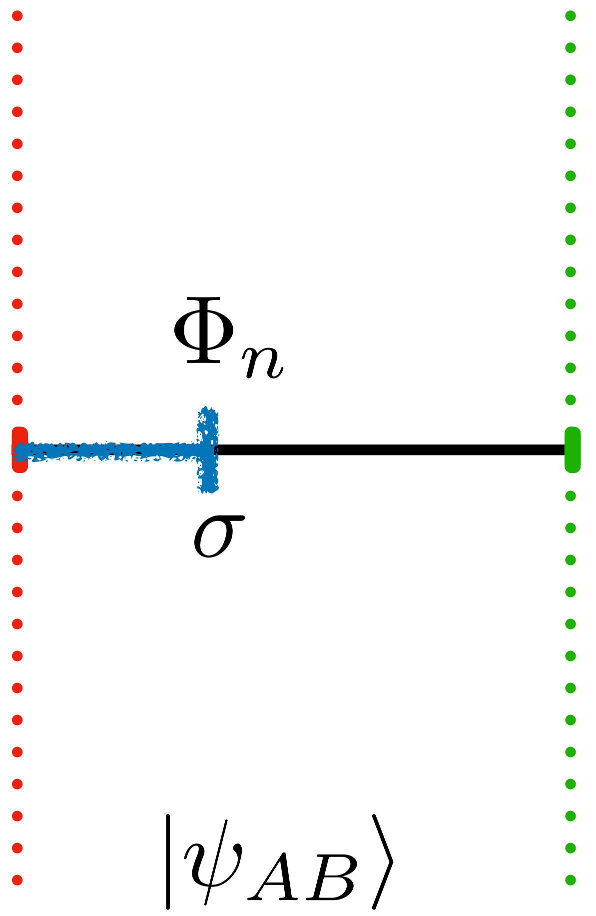
Dynamics compatible with a finite effective size $L_{eff} \sim L \cosh 2\theta$

Alternating pattern of dynamical phase transitions A-B-A over each period similarly determined by the boundary entropies

Holographic Interpretation

For a holographic BCFT this has a dual geometric description

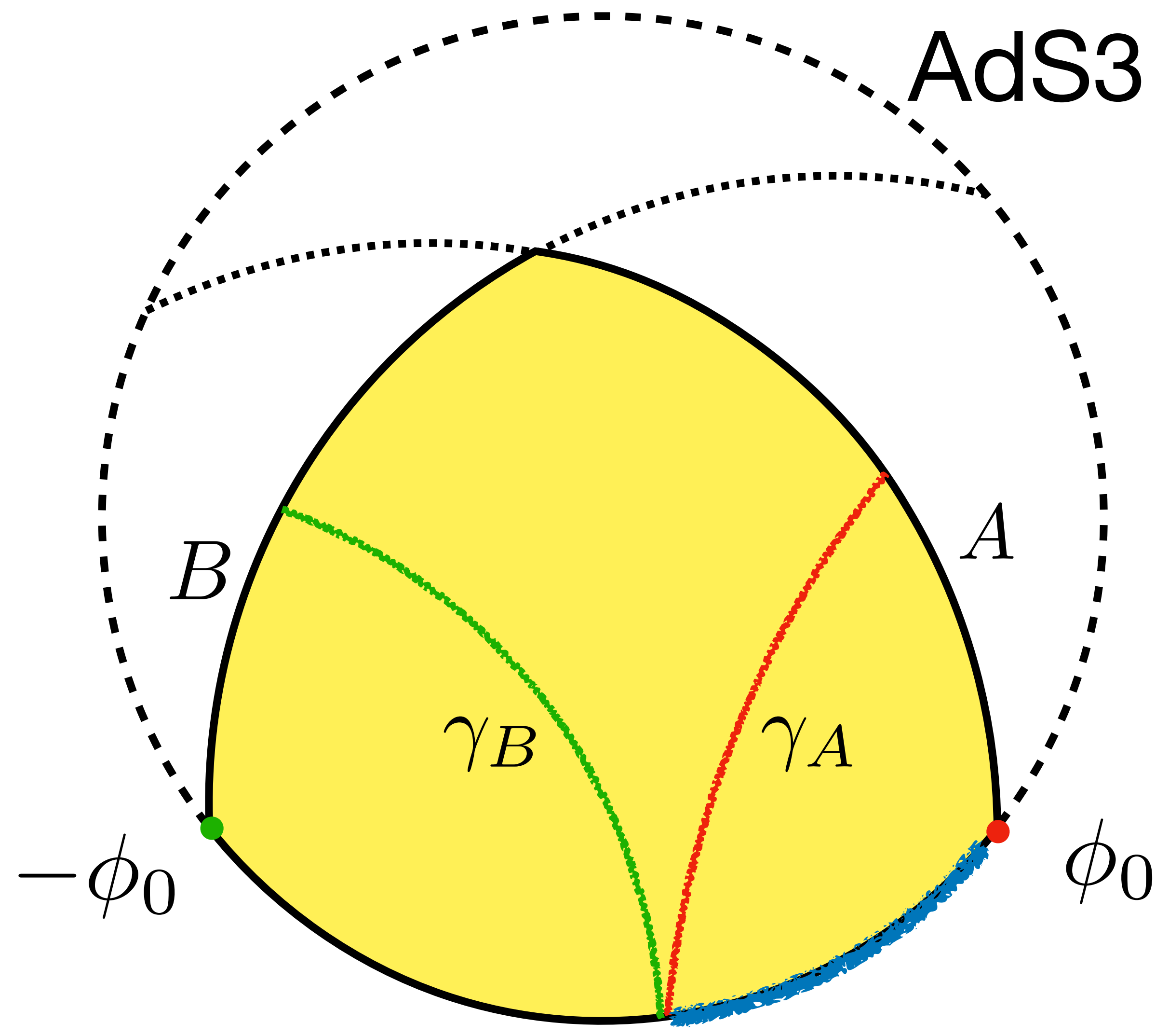
[MiyajiMurdia22]



$$\alpha = \sqrt{1 - \frac{24 h_\psi}{c}} = \frac{2\phi_0}{\pi}$$

$$s_{A,B} = \frac{c}{3} \operatorname{arctanh} T_{A,B}$$

$$S = \frac{Area \gamma}{4G_N} = \frac{\min Area\{\gamma_A, \gamma_B\}}{4G_N}$$



The post quench computation can be obtained extending the CFT map to a bulk diffeomorphism, with some subtlety

[Banados99]
[Roberts12]

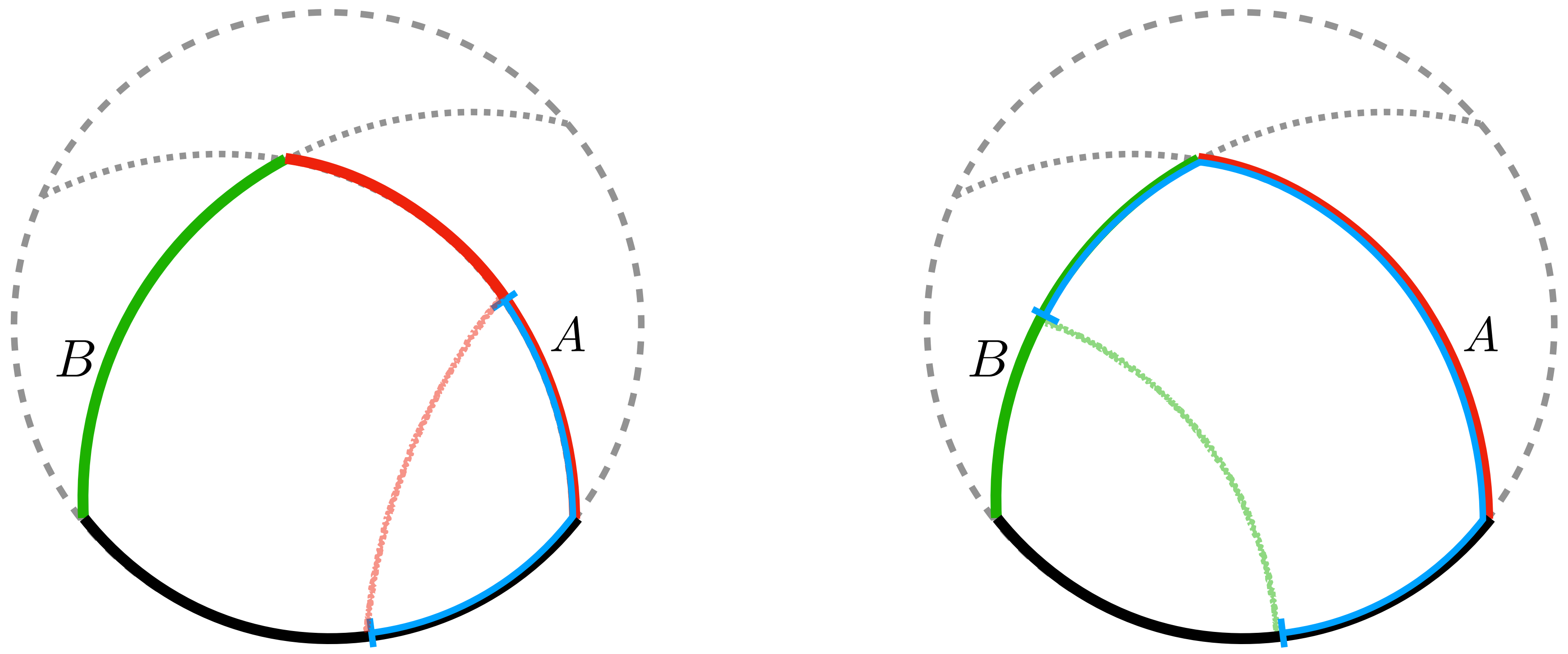
Concluding Remarks

In holographic BCFTs, there is a phase transition in the entanglement entropy induced by the inhomogeneous quench and linked to mixed boundary conditions.

Does this phase transition distinguish holographic BCFTs from non-holographic ones?

AdS geometry dual to the post-quench time dependent state?

Double holography: two different gravitational AdS2 spacetimes separated by a defect. Quantum extremal surface dynamically jumping from one spacetime to the other



Thank you!