Holography in 4d Quantum Gravity

David Lowe, Brown University

work with Larus Thorlacius
('22 → present)
and Bing-Nan Liu

Plan:

- Focus on "small" black holes in AdS_4/CFT_3
  - $\rightarrow$ evaporate to nothing, learn about local spacetime physics
  - $\rightarrow$ states are non-geometric (conventional saddle point methods don't work)
  - $\rightarrow$ well-defined states in CFT_3
- Formation/evaporation/state counting via holographic map

- Puzzles / novelties

$\rightarrow$ interactions don't turn off near boundary of AdS_4

$\Rightarrow$ expect CFT_3 to obey Wigner surmise

Quantum chaos

$\longrightarrow$ can apply Eigenstate Thermalization Hypothesis

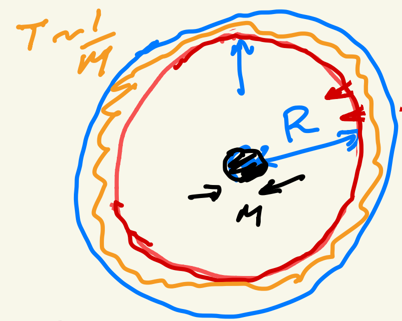
- Harder question $\rightarrow$ how does semiclassical geometry emerge from Hilbert space of non-geometric states
- Suggestion : collective effect captured by gravitational action
- Set up 4d semiclassical gravity problem

Part 1

- Large black holes in AdS well understood → clear saddle for gravitational path integral
  Hawking & Page
- Don't evaporate away
  → radiation falls back into horizon due to Λ_{AdS}
- Behave like remnants
- Focus instead on small black holes

$\longleftarrow \ell_{AdS} \longrightarrow$

$D \downarrow \sim M^3 =$ Page time



thermal gas w/ solid angle constraint

$E = M$

$\sim T^4 R^2 D \, \delta\Omega$

$$\delta\Omega \sim \frac{M^2}{R^2}$$

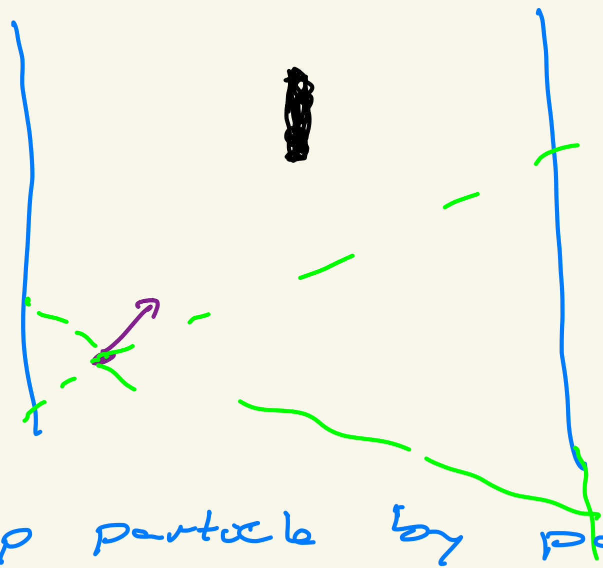
Note R^2 cancels, with $T \sim \frac{1}{M}$

Entropy of black hole $\sim A \sim M^2$

$\sim$ entropy of constrained gas $\sim E/T$
$\sim M^2$

- Key idea: time reverse of typical outgoing state = typical ingoing state
- Can count asymptotic states reliably and map into CFT via HKLL

- Good understanding of CFT_3 states needed via holographic map of HKLL

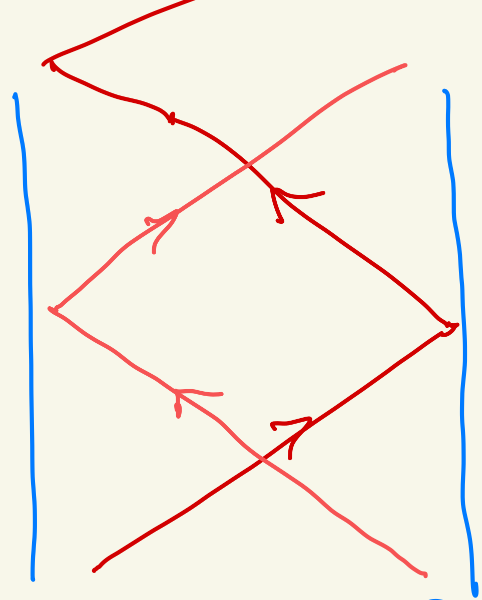


- Build up particle by particle
- Slight change in directions $\Rightarrow$ black hole doesn't form $\to$ not amenable to geometric methods at early/late times $\to$ no semiclassical saddle

Puzzles & Novelties

Part 2

- Perturbative states have well-defined energies $E_n \sim n/R_{AdS}$
- Would naively expect relevant states would have $\delta E \sim \frac{1}{R_{AdS}}$ and enormous degeneracies $\Leftrightarrow$ integrability / amplitudes
- Magic of AdS gives a different answer

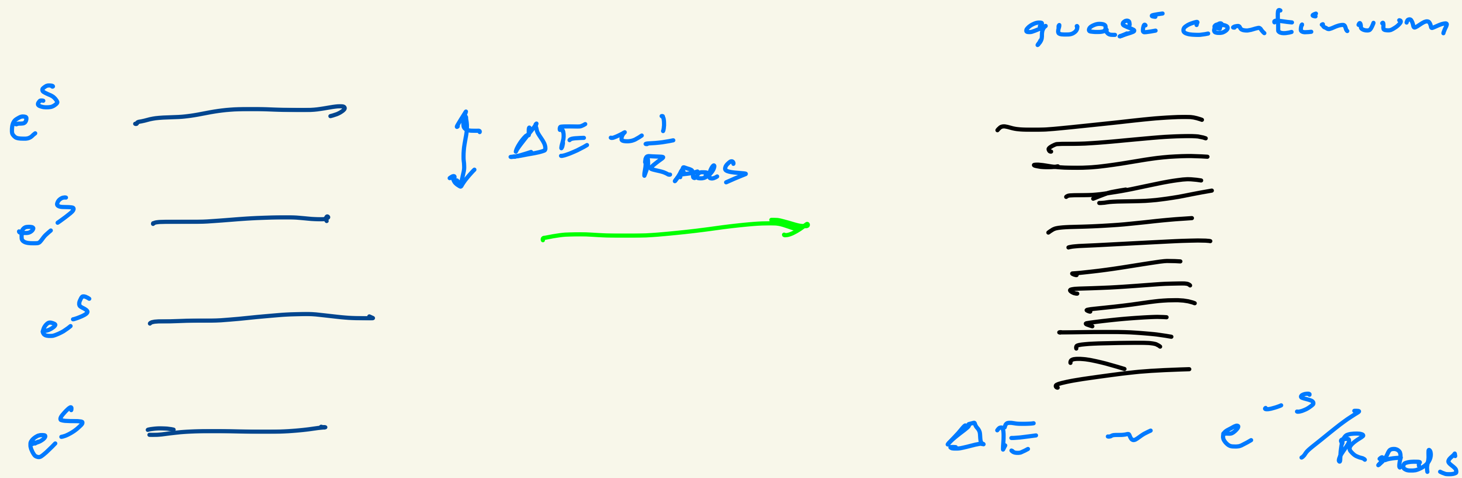


Time average separation of two light rays is finite [we derived detailed 2-body Lagrangian to $\mathcal{O}(G_N)$]

$\longrightarrow$ interaction energy finite

[Unlike in flat spacetime S-matrix]

- Class of 2-body Hamiltonians classically chaotic
- Quantum mechanically → invoke Wigner surmise → energy levels repel in such systems



- Distinctive feature of 4d gravity / CFT_3
- Expect to see effect in Δ's of CFT_3 primary operators [Δ large enough]
- Corrections to energies
$$\delta\Delta \sim \mathcal{O}(1) \quad \text{for} \quad \Delta \gg 1$$
- Suggests random matrix methods should be useful for CFT_3

- Upshot, energy spacings

$$\Delta E \lesssim \frac{1}{e^S}$$

$S \sim$ entropy of black hole

- Implies energy eigenstates delocalized $\Delta t \sim e^S \gg R_{AdS}$

$\rightarrow$ non-geometric

- Nevertheless $\rightarrow$ can make localized superpositions w/

$\delta E \sim \frac{1}{R_{AdS}}$ as above

Eigenstate Thermalization Hypothesis

- This class of superpositions will self-average and rapidly converge to a semiclassical answer
- Can calculate fluctuations using ETH

In energy eigenstate basis

smooth

$$\langle \alpha | \mathcal{O} | \beta \rangle = \bar{\mathcal{O}}_\alpha \, \delta_{\alpha\beta} + e^{-S/2} \, \overline{\mathcal{O}^2_{\alpha\beta}} \, R_{\alpha\beta}$$

random w/ unit variance

- Wavepacket always have delocalized tails $\mathcal{O}(e^{-S/2})$ effects
- Quantum information always accessible at or near boundary of AdS w/ sufficiently non-local probes
- Solution of black hole information problem

Part 3

- New problem: how does 4d semiclassical geometry emerge from CFT
  → small change in initial state disrupts black hole formation
- Don't seem to currently have tools in CFT_3 to analyze this collective effect
  → invoke AdS / CFT to reframe using semiclassical gravity

- Goal : solve $G_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle + \Lambda g_{\mu\nu}$

for collapse / evaporation process

track full time dependence

c.f. David Mateos

- One approach : solve problem of time dependent renormalization of $\langle T_{\mu\nu} \rangle$ and frame as an initial value problem ✓

→ bilocal PDEs (local in time)

- New feature: equations involve 4 derivatives $\times \hbar$

  $\rightarrow$ forced to choose special quantum states where these terms $O(\hbar) \Rightarrow$ small correction

- Rules out most familiar quantum states

  eg Hartle-Hawking $T_{00} \sim T^4$ large radius

  $\rightarrow$ Jean's instability

Boulware : large $\langle T_{\mu\nu} \rangle$ on horizon

Unruh state unique state satisfies these conditions

$$\left[\quad \langle T_{\text{ingoing}} \rangle_{\mathcal{I}^-} = 0 \right.$$

$$\langle T_{\text{outgoing}} \rangle_{\mathcal{I}^+} = \text{Hawking flux}$$

$$\left. u^\mu \langle T_{\mu\nu} \rangle u^\nu \ \text{finite on horizon} \right]$$

- Bilocal equations still too hard → do analog of RST for CGHS

- Build a 4d theory that reproduces trace anomaly

$$\langle T_\mu{}^\mu \rangle = \mathrm{tr}\, N \left[R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} + \ldots \right]$$

Riegert (1982)

$M^2/r^6 \sim T^4$ on horizon

- Idea → try to do analog of Polyakov action (2d)

- Analog of Liouville field satisfies a 4th order equation, conformally invariant

$$\Delta_4 \Phi = \Box(\Box\Phi) + \# R^{\mu\nu} \nabla_\mu \nabla_\nu \Phi + \cdots$$

$$= \text{source}$$

- Upshot is a 4d local Lagrangian that reproduces

$\langle T_\mu{}^\mu \rangle_{\text{renormalized}}$

$\longrightarrow$ predicts

$\langle T_{\mu\nu} \rangle_{\text{renormalized}}$

$$S = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\,(R - \Lambda)$$

$$+ \int d^4x\,\sqrt{-g}\,\Big[\, \frac{b}{2}\,\Phi\,\Delta_4\,\Phi \; + \; \Phi\,\big[\, R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + \cdots \big] \Big]$$

- Bugs / features:

can add any term like

$$\delta S = \int d^4x \sqrt{g} \left[\frac{1}{2} \chi \Delta_4 \chi \right.$$

$$+ \chi \times \text{scale invariant}$$

curvature invariant

and trace anomaly same

- Upshot: if you add term where invariant $= C_{\mu\nu\lambda\rho} C^{\mu\nu\lambda\rho}$ Weyl^2 (to appear)

- Get $\langle T_{\mu\nu} \rangle_{\text{renorm}}$ analytically for Schwarzschild black hole in Unruh state (time indep. $\langle T_{\mu\nu} \rangle$)
  $\longrightarrow$ unique state

- Gives expected result for Hawking radiation for general matter

- Issues :
  4th order theories contain ghosts

→ Theory is linearly stable around flat spacetime

→ Nonlinear instabilities generated w/ wavelength $\sim 2GM$

→ This is the Hawking radiation

- Next steps:
  1) Solve for $\langle T_{\mu\nu} \rangle$ for null collapsing shell
     → issue of pre Hawking radiation
  2) Evolve this state in time numerically
     → 2d 4th order PDE for Schwarzschild

Summary

- Can understand microstate counting of small AdS black holes
- Unitary evolution
- Quantum information delocalized but scrambled
- Hard question → how does semiclassical geometry emerge