Quantum Corrections in Near-Extremal Black Holes: Thermodynamics, Dynamics and Applications

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2501.08252, S. Maulik, X. Meng and LPZ
2410.11487, Xiao-Long Liu, Jun Nian and LPZ
2401.16507, S Maulik, LPZ, A. Ray, J. Zhang
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Motivation: Microstate Counting

- Microscopic entropy of rotating, electrically charged, AdS_{d+1} black holes using the superconformal index in CFT_d.
- Entropy of supersymmetric AdS_5 black holes.

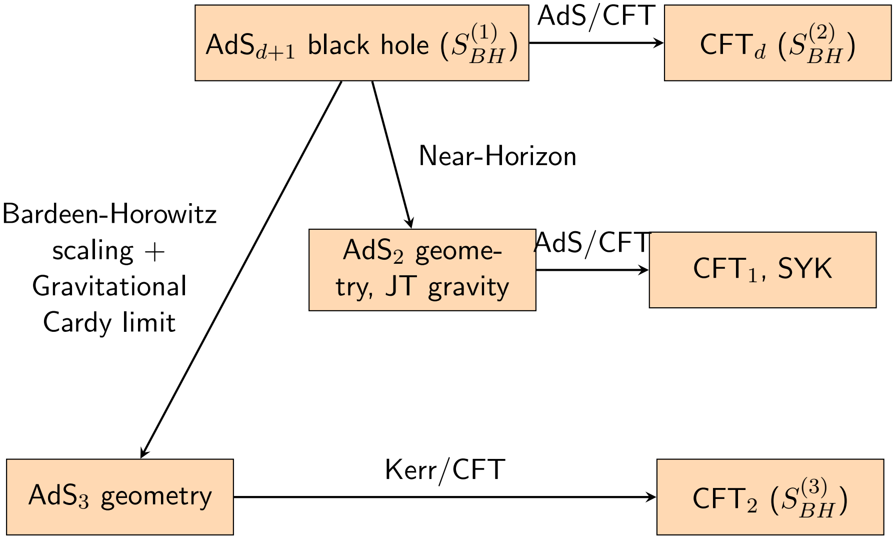
$$S_{BH} = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1 + J_2)}.$$

- $\mathcal{N} = 4$ SYM superconformal index on $S^1 \times S^3$

$$\mathcal{I}(\beta, \Delta_I, \omega_i) = \mathrm{Tr}\left((-1)^F e^{-\beta E} e^{-\sum_{i=1}^{3}\Delta_I Q_I} e^{-\sum_{i=1}^{2}\omega_i J_i}\right).$$

- $E = \{\mathcal{Q}, \mathcal{Q}^\dagger\}$, Q_I-charges in $U(1)^3 \in SO(6)_R$, J_i angular momenta.
- Some sub-leading corrections are understood $S = \frac{A}{4G} + \alpha \log\left(\frac{A}{G}\right)$, where α is the result of a one-loop computation determined by massless fields running in the loop and zero modes.
- How reliable are low-dimensional (CFT_2 and CFT_1), low-energy approaches to the entropy and the dynamics of near-extremal black holes?

Microstate Counting of Supersymmetric Black Holes



Outline

- A puzzle in the thermodynamics of near-extremal black holes and its resolution
- Universality in $\frac{3}{2}\log T_{\text{Hawking}}$ corrections for near-extremal rotating black hole thermodynamics from the one-loop gravitational partition function
- Vanishing $\log T_{\text{Hawking}}$ contributions in supersymmetric black holes
- Quantum corrections to Hawking radiation of rotating black holes
- Quantum corrections to the Holographic Strange Metal

Low temperature black holes are thermodynamically unstable

- Large degeneracy is *per se* not an issue in supersymmetric black holes.
- The low temperature breakdown of black hole thermodynamics for extremal black holes was pointed out more than thirty years ago: $S \sim S_0 + \alpha T, M \sim M_0 + \beta T^2$

  [Preskill, Schwarz,Shapere,Trivedi, Wilczek, 1991] [Maldacena, Michelson, Strominger 1998] [Page 2000] .
- At very low temperatures, the emission of one Hawking quantum can drastically change the temperature of the near-extremal black hole.

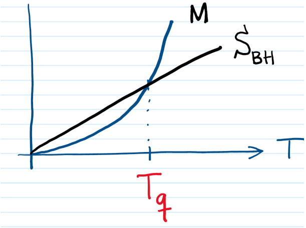


Figure: Limitations of the Statistical Description of Black Holes [PSSTW '91]

Jackiw-Teitelboim physics as a quantum resolution

- The resolution to this puzzle did not require knowledge of the full gravitational path integral (quantum gravity) and was first achieved by a careful treatment of certain zero modes in the extremal solution [Iliesiu, Turiaci, 2003.02860], [Heydeman, Iliesiu, Turiaci, Zhao, 2011.01953], [Boruch,Heydeman,Iliesiu,Turiaci,2203.01331].
- The realization that temperature effectively acts as a coupling constant whereby the high-temperature regime is classical while the very low-temperature regime is quantum and strongly coupled was understood first in the context of two-dimensional Jackiw-Teitelboim (JT) gravity [almheiri,Polchinski, 1402.6334],[Jensen, 1605.06098], [Maldacena, Stanford, Yang, 1606.01857].

JT in higher dimensional black holes

- The near-horizon region of higher-dimensional, near-extremal black holes contains a JT subsector that dominates the full path integral [Iliesiu, Turiaci, 2003.02860], [Heydeman, Iliesiu, Turiaci, Zhao, 2011.01953], [Boruch,Heydeman,Iliesiu,Turiaci,2203.01331].
- Embedding quantum aspects of JT gravity in spherically symmetric near-extremal black holes is almost immediate. The near-horizon geometry of extremal Reissner-Nordstrom is $\mathrm{AdS}_2 \times S^p$.
- Embedding JT in rotating black holes with focus on classical aspects: [Castro, Larsen, Papadimitriou,1807.06988], [Moitra,Sake,Trivedi,Vishal, 1905.10378], [Castro, Pedraza, Toldo, Verheijden, 2106.00649].

Rotating black holes at one-loop: JT physics without KK

- Recently $\log T$ correction to the thermodynamics of Kerr spacetime was addressed in [Kapec-Sheta-Strominger-Toldo 2310.00848, Rakic-Rangamani-Turiaci 2310.04532]
- The near-horizon extremal Kerr solution admits a family of normalizable zero modes corresponding to reparametrizations of the boundary time, just as in JT gravity.
- The path integral over these zero modes leads to an infrared divergence in the one-loop approximation to the Euclidean partition function.
- This divergence can be regulated by turning on a small but finite temperature correction in the geometry

  [Iliesiu, Murthy, Turiaci, 2209.13608], [Banerjee, Saha, 2303.12415], [Banerjee, Saha, Srinivasan, 2311.09595].
- The resulting finite-temperature geometry lifts the eigenvalues of the zero modes, rendering the path integral infrared finite and leading to the thermodynamic-altering correction to the near-extremal black hole: $\frac{3}{2}\log T_{\mathrm{Hawking}}$.

Universality in Logarithmic Temperature Corrections to Near-Extremal Rotating Black Hole Thermodynamics in Various Dimensions, 2401.16507



Figure: Sabyasachi Maulik, Augniva Ray and Jingchao Zhang

Universality of $\frac{3}{2}\log T_{\text{Hawking}}$ from tensor modes

- 4D: Kerr-AdS_4, Kerr-Newman-AdS_4 and the rotating black hole in $\mathcal{N} = 4$ gauged supergravity with two scalars and two electric charges turned on.
- 5D: Asymptotically flat Myers-Perry black hole and the Kerr-AdS_5 black hole, $U(1)^3$ gauged supergravity.
- Universally find that tensor modes contribute $\frac{3}{2}\log T_{\text{Hawking}}$ to the low-temperature thermodynamics. Root cause:
  - The universal presence of a $SL(2,\mathbb{R})$ subgroup of isometries in the near-horizon geometry.
  - A set of cancellations in the Lichnerowicz operator.
- These two conditions hold for near-extremal black holes in asymptotically flat and asymptotically AdS spacetimes of various dimensions.

Kerr-AdS$_4$

$$I_{\rm grav} = -\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\,(R-2\Lambda) - \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^3y\sqrt{h}\,K.$$

$$\begin{aligned}
ds^2 &= -\frac{\Delta_r}{\Sigma^2}\left(dt - \frac{a}{\Xi}\sin^2\theta d\phi\right)^2 + \frac{\Sigma^2}{\Delta_r}dr^2 + \frac{\Sigma^2}{\Delta_\theta}d\theta^2 \\
&+ \frac{\Delta_\theta}{\Sigma^2}\sin^2\theta\left(adt - \frac{(r^2+a^2)}{\Xi}d\phi\right)^2, \\
\Delta_r &= (r^2+a^2)\left(a+\frac{r^2}{L^2}\right) - 2Mr\,, \quad \Delta_\theta = 1-\frac{a^2}{L^2}\cos^2\theta\,, \\
\Xi &= 1-\frac{a^2}{L^2}\,, \qquad \Sigma = \sqrt{r^2+a^2\cos^2\theta}\,.
\end{aligned}$$

Mass, angular momentum, entropy, first law

- Mass, angular momentum and entropy:

$$E = \frac{M}{\Xi^2}, \qquad J = \frac{M\,a}{\Xi^2}, \quad S = \frac{\pi\left(r_+^2 + a^2\right)}{\Xi},$$

- Surface gravity κ, and temperature

$$T = \frac{\kappa}{2\pi} = \frac{r_+\left(1 + \frac{a^2}{L^2} + \frac{3r_+^2}{L^2} - \frac{a^2}{r_+^2}\right)}{4\pi\left(r_+^2 + a^2\right)}.$$

- Angular velocity and first law:

$$\Omega = \frac{a\left(1 + \frac{r_+^2}{L^2}\right)}{r_+^2 + a^2}, \qquad dE = TdS + \Omega dJ.$$

Low temperature Kerr-AdS$_4$

- The *extremal limit* is defined as the limit when the two horizons coalesce into one, and the temperature becomes zero.
- The low temperature instability from the classical parameters defines T_q.
- J receives no T corrections – canonical ensemble.

$$
\begin{aligned}
M &= \frac{r_0\left(L^2+r_0^2\right)^2}{L^2\left(L^2-r_0^2\right)} + \frac{4\left(\pi^2 L^4 r_0^3 + 2\pi^2 L^2 r_0^5 + 9\pi^2 r_0^7\right)\left(L^2+r_0^2\right)}{\left(L^2-r_0^2\right)\left(L^4+6L^2r_0^2-3r_0^4\right)} T^2 + \ldots \\
S &= \frac{2\pi r_0^2}{1-\frac{3r_0^2}{L^2}} + \frac{8\pi^2 r_0^3\left(1-\frac{r_0^2}{L^2}\right)}{\left(1-\frac{3r_0^2}{L^2}\right)\left(1+\frac{6r_0^2}{L^2}-\frac{3r_0^4}{L^4}\right)} T + \ldots \\
J &= \frac{L r_0^2\sqrt{\left(L^2-r_0^2\right)\left(L^4+3L^2r_0^2\right)}}{\left(L^2-3r_0^2\right)^2}.
\end{aligned}
$$

Near-horizon geometry and ensemble choice

- 'Bardeen-Horowitz' like scaling transformation $\{(r,t,\theta,\phi)\to(y,\tau,\theta,\varphi)\}$ where $\lambda\equiv T$

$$r = r_+(T) + \frac{4\pi L^2 r_0^2\left(L^2+r_0^2\right)}{L^4+6L^2r_0^2-3r_0^4}\,T\,(y-1)\ ,$$
$$t = -\frac{i\tau}{2\pi T}\,,\qquad \theta=\theta\,,$$
$$\phi = \varphi - i\tau\left(\frac{\left(L^2-3r_0^2\right)\sqrt{\frac{L^4+3L^2r_0^2}{L^2-r_0^2}}}{4\pi L^3 r_0}\frac{1}{T}-1\right).$$

- T as a physical regulator, other regulators in QNM [Kapec et al.], [Arnaudo, Bonelli, Tanzini].

Near-horizon geometry

- The leading-order zero-temperature metric:

$$
\begin{aligned}
ds^2_{(0)} &= \bar{g}_{\mu\nu} dx^\mu dx^\nu \\
&= g_1(\theta)\left(\frac{dy^2}{y^2-1} + \left(y^2-1\right) d\tau^2\right) + g_2(\theta) d\theta^2 \\
&+ g_3(\theta)\left(d\varphi + i g_4(y)\left(y-1\right) d\tau\right)^2 ,
\end{aligned}
$$

- AdS_2 is squashed, g_1, and fibered over $S^1 \sim \varphi$, g_4. The θ dependence prevents reduction to 2d.
- The linear in temperature deformation, by expanding M and a:

$$
ds^2_{(1)} \;\; = \;\; T g^{(1)}_{\mu\nu} dx^\mu dx^\nu \, .
$$

Quantum corrections via path integral

- Fluctuations around the background: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$Z = \exp(-I(\bar{g})) \int D[h] \exp\left[- \frac{1}{64\pi} \int d^4x \sqrt{\bar{g}}\ \tilde{h}^{\mu\nu} \Delta^{\Lambda} h_{\mu\nu} \right].$$

- Gauge symmetry and gauge fixing $\left(g_{\mu\nu} \mapsto g_{\mu\nu} + \partial_{(\mu} f_{\nu)}\right)$:

$$\mathcal{L}_{GF} = \frac{1}{32\pi} \bar{g}_{\mu\nu} \left(\bar{\nabla}_{\alpha} h^{\alpha\mu} - \frac{1}{2} \bar{\nabla}^{\mu} h^{\alpha}{}_{\alpha} \right) \left(\bar{\nabla}_{\beta} h^{\beta\nu} - \frac{1}{2} \bar{\nabla}^{\nu} h^{\beta}{}_{\beta} \right).$$

- The linearized kinetic operator (Lichnerowicz) for $h_{\mu\nu}$:

$$\begin{aligned} \Delta^{\Lambda} h_{\mu\nu} &= \frac{1}{32\pi} \bigg(- \bar{\nabla}^2 h_{\mu\nu} + 2 \bar{R}_{\mu\rho} h^{\rho}{}_{\nu} - 2 \bar{R}_{\mu\rho\nu\sigma} h^{\rho\sigma} \\ &- 4 \left(\bar{R}_{\nu\sigma} - \frac{1}{4} \bar{g}_{\nu\sigma} \bar{R} \right) h^{\sigma}{}_{\mu} - 2\Lambda h_{\mu\nu} \bigg). \end{aligned}$$

Diffeomorphisms and zero modes

- The $\log T$ corrections arise from the zero modes of the Lichnerowicz operator: $\Delta^{\Lambda} h_{\mu\nu} = 0$.
- An Ansatz for the vector field generating the zero modes as diffeomorpshisms.

$$\xi^{(n)} = e^{in\tau}\left(f_1(y)\frac{\partial}{\partial y} + f_2(y)\frac{\partial}{\partial \tau} + f_3(y)\frac{\partial}{\partial \varphi}\right)$$

- The vector generates diffeomorphisms given by $h_{\mu\nu} = \mathcal{L}_{\xi} g^{(0)}_{\mu\nu}$.

Diffeomorphisms and zero modes

- The zero mode, $h^{(n)}_{\mu\nu} dx^\mu dx^\nu =$

$$\sqrt{\frac{3|n|\,(n^2-1)\left(L^6+3L^4r_0^2-21L^2r_0^4+9r_0^6\right)}{64\pi^2 L}}$$
$$\frac{L^2-r_0^2+\left(L^2+3r_0^2\right)\cos^2\theta}{L^4+6L^2r_0^2-3r_0^4}$$
$$e^{in\tau}\left(\frac{y-1}{y+1}\right)^{\frac{|n|}{2}}\left(-d\tau^2+2i\frac{|n|}{n}\frac{d\tau dy}{y^2-1}+\frac{dy^2}{\left(y^2-1\right)^2}\right)\,, \quad |n|\geq 2\,.$$

- One can check $(\Delta^\Lambda h)_{\mu\nu}=0\,, \qquad \mu,\nu\in\{y,\tau,\theta,\varphi\}$.
- The zero modes are generated by diffeomorphisms $h^{(n)}_{\mu\nu}\propto \mathcal{L}_\xi \bar{g}_{\mu\nu}$
- The zero mode extends in (τ,y) but depends also on θ.
- The physics of (AdS_2) JT in higher dimensions without KK reduction.

Diffeomorphisms and zero modes

- The diffeomorphisms generated by the vector field ξ are large gauge transformations and they can not be gauged away.
- Note that the vector field ξ does not die off at the asymptotic boundary of AdS_2, they are $\mathcal{O}(1)$ ($y = \cosh\eta$):

$$\xi^{(n)}\Big|_{\eta\to\infty} = \frac{e^{in\tau}}{2|n|\left(n^2-1\right)}\left(|n|\partial_\eta + i\partial_\tau - \partial_\varphi\right)\,,$$

- The zero modes $\{h^{(n)}_{\mu\nu}\}$ themselves are normalizable and are part of the physical spectrum and need to be integrated over in the path integral.

Diffeomorphisms and zero modes

- The diffeomorphism generating vector fields at the boundary

$$\xi \sim \varepsilon'(\tau)\partial_\eta - \varepsilon(\tau)\partial_\tau - i\varepsilon(\tau)\partial_\varphi \,, \quad \varepsilon(\tau) = \sum_{n=2}^{\infty} f_n \exp(in\tau)\,.$$

- The vector field generates the reparametrization

$$\eta \to \eta + \varepsilon'(\tau)\,, \qquad \tau \to \tau - \varepsilon, \qquad \varphi \to \varphi - i\varepsilon$$

.

- The vector fields generating the large diffeomorphisms are the Schwarzian modes which act on the boundary [Iliesiu, Murthy, Turiaci, 2209.13608] [Maldacena, Stanford, Yang, 1606.01857].

Log T corrections from zero modes

- The pertubatively deformed eigenvalue problem ($\{h_n, \lambda_n^0\}$ is the eigenspectrum and $\{\delta h_n, \delta\lambda_n\}$ is the respective correction)

$$\left(\Delta^\Lambda + \delta\Delta^\Lambda\right)\left(h_n + \delta h_n\right) = \left(\lambda_n^0 + \delta\lambda_n\right)\left(h_n + \delta h_n\right),$$

$$\delta\lambda_n = \frac{3n\left(1 - \frac{r_0^2}{L^2}\right)}{64\, r_0} T, \quad n \geq 2\,.$$

- The zero modes $\lambda_n^0 = 0$ leads to the $\log T$ corrections:

$$\delta \log Z\Big|_{\log T} = -\log\left[\prod_{n=2}^{\infty}\left(\frac{3n\left(1 - \frac{r_0^2}{L^2}\right) T}{64\, r_0}\right)\right].$$

Final result

$$\delta \log Z\Big|_{\log T} = \frac{3}{2}\log\left(\frac{T}{T_q}\right) + O(1), \quad T_q \equiv \frac{64\, r_0}{3\left(1-\frac{r_0^2}{L^2}\right)}\,.$$

- Note that on taking the large L limit above, one recovers the result for the Kerr black hole in flat space presented in [Kapec-Sheta-Strominger-Toldo 2310.00848, Rakic-Rangamani-Turiaci 2310.04532].
- We confirm the independence of the $\log T$ correction with respect to the ensemble (canonical versus grand canonical).
- JT physics, $Z_{\text{JT}} = \left(\frac{T}{T_q}\right)^{3/2} e^{S_0+\alpha\, T}$, without KK reduction.

Kerr-Newman-AdS$_4$

- Einstein-Maxwell with negative cosmological constant:

$$I = -\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(R - 2\Lambda - F^2\right) + I_{Boundary}.$$

- The Lichnerowicz operator:

$$\begin{aligned}
h_{\alpha\beta}\Delta_L^{\alpha\beta,\mu\nu}h_{\mu\nu} &= h_{\alpha\beta}(\Delta_{EH}^{\alpha\beta,\mu\nu} - 2\Lambda\Delta_1^{\alpha\beta,\mu\nu} - 2\Delta_F^{\alpha\beta,\mu\nu})h_{\mu\nu}, \\
h_{\alpha\beta}\Delta_F^{\alpha\beta,\mu\nu}h_{\mu\nu} &= h_{\alpha\beta}(-\frac{1}{8}F^2\left(2g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\beta}g^{\mu\nu}\right) + F^{\alpha\mu}F^{\beta\nu} \\
&+ 2F^{\alpha\gamma}F^{\mu}{}_{\gamma}g^{\beta\nu} - F^{\alpha\gamma}F^{\beta}{}_{\gamma}g^{\mu\nu})h_{\mu\nu}
\end{aligned}$$

- At zero temperature ($g = \bar{g}, A = \bar{A}$), the above operator has the following zero modes $h_{\mu\nu}^{(n)}dx^\mu dx^\nu$ ($n \geq 2$):

$$c_n e^{in\tau}\left(\frac{-1+y}{1+y}\right)^{\frac{n}{2}}\left(a^2 + 2r_0^2 + a^2\cos(2\theta)\right)$$
$$\left(d\tau^2 - 2i\frac{|n|}{n}\frac{d\tau dy}{y^2-1} - \frac{dy^2}{(y^2-1)^2}\right)$$

Zero modes, one-loop action

- The expression of the operator is intractable, but the integration is straightforward and the result is simple

$$\delta\lambda_n = \int dx^4\sqrt{\bar{g}}h^{(n)*}_{\alpha\beta}\delta\Delta_L^{\alpha\beta,\mu\nu}h^{(n)}_{\mu\nu} = \frac{3nr_0T}{8(a^2+3r_0^2)}, \quad n \geq 2.$$

- The contribution of the extremal zero modes to the low-temperature partition function is therefore

$$\delta\log Z = \log(\prod_{n\geq 2}\frac{\pi}{\delta\lambda_n}) = \frac{3}{2}\log(\frac{T}{T_q}) + \mathcal{O}(1).$$

Other explicit theories

- Kerr-Newmann-AdS black hole in $\mathcal{N} = 4$ gauged supergravity.

$$
\begin{aligned}
\mathcal{L}_4 =& R * 1 - \frac{1}{2} * \mathrm{d}\zeta \wedge \mathrm{d}\zeta - \frac{1}{2} e^{2\zeta} * \mathrm{d}\chi \wedge \mathrm{d}\chi \\
& - \frac{1}{2} e^{-\zeta} * F_{(2)2} \wedge F_{(2)2} - \frac{1}{2} \chi F_{(2)2} \wedge F_{(2)2} \\
& - \frac{1}{2\left(1+\chi^2 e^{2\zeta}\right)} \left(e^{\zeta} * F_{(2)1} \wedge F_{(2)1} - e^{2\zeta} \chi F_{(2)1} \wedge F_{(2)1} \right) \\
& + g^2 \left(4 + 2\cosh\zeta + e^{\zeta}\chi^2 \right) * 1,
\end{aligned}
$$

where ζ and χ are the dilaton and axion.

- Myers-Perry black holes in five dimensions
- Kerr-AdS$_5$ Black hole

Why is this universal?

- Extremal black holes in their near horizon geometry always have an AdS_2 factor, possibly fibered over some compact directions.
- The explicit form of the near-horizon isometry group was first explicitly presented in [Bardeen, Horowitz, 9905099].
- H. K. Kunduri, J. Lucietti and H. S. Reall, Near-horizon symmetries of extremal black holes, Class. Quant. Grav. 24 (2007) 4169–4190, [0705.4214] **Lemma** : The metric

$$ds^2 = \Gamma(\rho)\left[A_0 r^2 dv^2 + 2dvdr\right] + d\rho^2 + \gamma_{ij}(\rho)(dx^i + k^i r dv)(dx^j + k^j r dv)$$

has isometry group $SL(2,\mathbb{R}) \times U(1)^{D-3}$ if $A_0 \neq 0$.

Universality of $\log T$ corrections

- Technically, the $\log T$ corrections arise as corrections to the eigenspectrum of the Lichnerowicz operator.
- Perturbation theory: We know the eigenspectrum $\{h_n, \lambda_n^0\}$ of the Lichnerowicz operator $\bar{\Delta}$ evaluated at the *NHEK-AdS*: $\bar{\Delta}\, h_n = \lambda_n^0 h_n$.
- Move slightly away from extremality, i.e., we turn on a small temperature T. This induces a change in the metric $\bar{g} \to g = \bar{g} + T\,\delta g + O(T^2)$. This in turn induces a change in the Lichnerowicz operator $\bar{\Delta} \to \Delta = \bar{\Delta} + \delta\Delta(T)$ and the eigenspectrum $\{h_n + \delta h_n(T), \lambda_n^0 + \delta\lambda_n(T)\}$.
- The corrected eigenvalue is the expectation value of the corrected operator:

$$\delta\lambda_n(T) = \int d^4x\sqrt{\bar{g}}\,\left(h^{(n)}\right)_{\alpha\beta}(\delta\Delta)^{\alpha\beta,\mu\nu}\left(h^{(n)}\right)_{\mu\nu}.$$

Universality of $\log T$ corrections

- One-loop fluctuations equal determinat $(\delta\lambda_n(T) = n f_1 T + O\left(T^2\right))$:

$$\log Z \sim \log \prod_n \frac{1}{\delta\lambda_n(T)}$$

- The range for $n \geq 2$. The near horizon geometry necessarily has an AdS_2 factor. The eigenfunctions h_n^0 of the Lichnerowicz operator are generated by diffeomorphisms: $h_n \sim \mathcal{L}_{\xi^{(n)}}\bar{g}$. Now, due to the presence of the AdS_2 throat, $\mathcal{L}_{\xi^{(n)}}\bar{g}$ vanish for $n = \pm 1, 0$ as they correspond to the isometry of $\mathrm{AdS}_2 \subset \mathrm{Diff}(S^1)$.
- Evaluating the infinite product via zeta-function regularization

$$\log Z \sim \frac{3}{2}\log T + \dots$$

Universality of $\log T$ corrections

- Key ingredients:

  1. Presence of an AdS_2 throat in the near horizon geometry [Lemma]
  2. Regularized value of an infinite product
  3. The $\log T$ term receives no corrections from the matter sector.

- The contributions come from only two terms in $h_{\alpha\beta}\delta\Delta^{\alpha\beta,\mu\nu}h_{\mu\nu}$:
  - $h_{\alpha\beta}\ \delta\left(\frac{1}{2}g^{\alpha\mu}g^{\beta\nu}\Box h_{\mu\nu}\right)$
  - $h_{\alpha\beta}\ \delta\left(R^{\alpha\mu\beta\nu}\right)h_{\mu\nu}$
- The above terms are completely determined by the geometry which is universal.
- Claim in progress [Jingchao Zhang]: The above cancellation is true also for dS asymptotics. (See talk by Watse Sybesma)

Supersymmetric black holes

- $\mathcal{N} = 2$ supergravity

$$
\begin{aligned}
I &= \int d^4x \sqrt{g}\, (\mathcal{L}_b + \mathcal{L}_f)\,, \\
\mathcal{L}_b &= R - F_{\mu\nu}F^{\mu\nu}, \\
\mathcal{L}_f &= -\frac{1}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - \frac{1}{2}\bar{\varphi}_\mu \gamma^{\mu\nu\rho} D_\nu \varphi_\rho + \frac{1}{2} F^{\mu\nu}\bar{\psi}_\mu \varphi_\nu \\
&+ \frac{1}{4} F_{\rho\sigma}\bar{\psi}_\mu \gamma^{\mu\nu\rho\sigma}\varphi_\nu - \frac{1}{2}F^{\mu\nu}\bar{\varphi}_\mu \psi_\nu - \frac{1}{4}F_{\rho\sigma}\bar{\varphi}_\mu \gamma^{\mu\nu\rho\sigma}\psi_\nu\,.
\end{aligned}
$$

- The gauge-fixing terms for the fermionic part $(\psi_\mu \to \psi_\mu + \gamma_\mu \epsilon)$

$$
\mathcal{L}_{gf,\,f} = \frac{1}{4}\bar{\psi}_\mu \gamma^\mu \gamma^\nu D_\nu \gamma^\rho \psi_\rho + \frac{1}{4}\bar{\varphi}_\mu \gamma^\mu \gamma^\nu D_\nu \gamma^\rho \varphi_\rho\,.
$$

The Reissner-Nordstrom background

- Asymptotically flat, electrically charged solution.

$$
\begin{aligned}
ds^2 &= f d\tau^2 + \frac{dr^2}{f} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \\
A &= -i\frac{Q}{r_+}(1 - \frac{r_+}{r})d\tau,
\end{aligned}
$$

- All computations can be performed for the dyonic solution.
- The near-horizon geometry is $\mathrm{AdS}_2 \times S^2$.

Vector contribution

- Action

$$I = -\int d^4x\sqrt{|g|}F^{\mu\nu}F_{\mu\nu}.$$

- The gauge fixing term (ζ is a constant) and the kinetic operator:

$$\begin{aligned}\mathcal{L}_{gf,v} &= -\frac{2}{\zeta}(\nabla_\mu A^\mu)^2,\\ \delta A_\mu \Delta^{\mu\nu}\delta A_\nu &= 2\delta A_\mu\left(g^{\mu\nu}\Box + \left(\frac{1}{\zeta}-1\right)g^{\mu\rho}g^{\nu\sigma}\nabla_\rho\nabla_\sigma\right)\delta A_\nu.\end{aligned}$$

- The vector zero modes [Camporesi-Higuchi, '94]

$$\delta A = d\Phi^{(\ell)}, \quad \Phi^{(\ell)} = \frac{1}{\sqrt{2\pi|\ell|}}\left[\frac{\sinh\eta}{1+\cosh\eta}\right]^{|\ell|}e^{i\ell\theta}, \quad \ell = \pm1,\pm2,\pm3,\cdots$$

Vector contribution

- Turning on a small temperature, we can regularize these zero modes and obtain the lifted eigenvalues

$$\Lambda_\ell = \delta\Lambda_\ell = \frac{3\pi}{a}\ell T.$$

- Performing the zeta-function regularization:

$$\delta \log Z \sim -2 \times \frac{1}{2}\log(\prod_{\ell\geq 1}\Lambda_\ell) \sim \frac{1}{2}\log T.$$

Fermionic contribution

- Regularization of the zero modes by turning on a small temperature. Substitute $g = \bar{g} + \delta g$ instead of $\bar{g}$ and keep to $O(T)$and find the lifted eigenvalues
$$\Lambda_k = \delta\Lambda_k \propto (2k+1)\, T.$$
- Fermionic fluctuations lead to Fermionic determinants as product of eigenvalues
$$\delta \log Z \sim \log(\prod_{k\geq 1} \Lambda_k).$$
- Zeta function regularization can be applied to the infinite product. We have
$$\delta \log Z \sim -\frac{1}{2} \log T.$$
- Four zero modes would give us a total contribution
$$\delta \log Z \sim -2 \log T.$$

$\log T$ cancellation in susy black holes

- Graviton contribution: $\frac{3}{2}\log T_{\text{Hawking}}$
- Gravitino contribution: $4 \times \left(-\frac{1}{2}\right)\log T_{\text{Hawking}}$
- Vector contribution: $\frac{1}{2}\log T_{\text{Hawking}}$

- Vanishing total contribution: $\left(+\frac{3}{2} - 2 + \frac{1}{2}\right)\log T_{\text{Hawking}}$
- WIP: Asymptotically AdS and rotating black holes in various dimensions.
- The main technical difficulty is the construction of the fermionic zero modes in the rotating case.
-

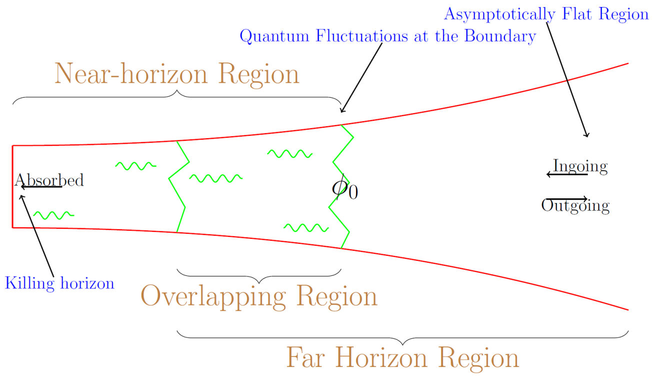
Open Issues

- Near-horizon versus the full geometry. What is the nature of these zero modes in the full geometry? [Kolanowski, Marolf, Rakic,Rangamani,Turiaci, 2409.16248]
- Regularization, finite T, QNM [Arnaudo-Bonelli-Tanzini,2412.16057, 2405.13830, 2506.08959]
- Applications to de-Sitter black holes [Blacker,Castro,Sybesma,Toldo, 2503.14623] [Maulik, Mitra,Mukherjee,Augniva Ray, 2503.08617]
- Logarithmic temperature corrections in supersymmetric black holes. The precise cancellation from the higher-dimensional and lower-dimensional points of view [LPZ, Jingchao Zhang].

Quantum-Corrected Hawking Radiation from Near-Extremal Kerr-Newman Black Holes, 2501.08252

Figure: Sabyasachi Maulik and Xin Meng

Effective Quantum Corrections



Quantum corrected Hawking Radiation

- [Brown, Iliesiu, Pennington,Usatyuk, 2411.03447]
- The higher dimensional black hole usually carries conserved charges: electric charge or conserved angular momentum.
- The low energy action with global $U(1)$ symmetry [Thomas Mertens]:

$$\begin{aligned} I_{Sch\times U(1)} &= -C\int_0^{\frac{1}{T_H}} d\tau \{\tan\left(\pi T_H f\left(\tau\right)\right), \tau\} \\ &+ \frac{K}{2}\int_0^{\frac{1}{T_H}} d\tau \left(\partial\tau\phi - i\left(2\pi\mathcal{E}T_H\right)\partial_\tau f\right)^2 . \end{aligned}$$

- The parameters: C, K, and $\mathcal{E}$ are specified by their connection to the thermodynamics of the original four dimensional black hole:

$$C = \frac{1}{4\pi^2}\left(\frac{\partial S}{\partial T}\right)_{T_H\to 0}, \quad \mathcal{E} = \frac{1}{2\pi}\left(\frac{\partial S}{\partial Q}\right)_{T_H\to 0}, \quad K = \left(\frac{dQ}{d\mu}\right)_{T_H\to 0}.$$

K is the charge susceptibility or compressibility of the four dimensional black hole.

Density of States

- The partition function obtained from the 2D action

$$Z\left(\beta_H, \mu\right) = \underbrace{e^{S_0}\left(\frac{2\pi C}{\beta_H}\right)^{\frac{3}{2}} e^{\frac{2\pi^2 C}{\beta_H}}}_{\text{Schwarzian sector}} \underbrace{\sum_n e^{-\beta_H\left(\frac{n^2}{2K}-\mu n\right)}}_{U(1)\text{ sector}},$$

- Density of States

$$\rho\left(E, q\right) = 2C\ e^{S_0}\sinh\left(2\pi\sqrt{2C\left(E-\frac{q^2}{2K}\right)}\right).$$

Correlation functions

- To compute the emission rate, we also require the matrix element of an operator $\mathcal{O}$ on the AdS_2 boundary between the initial and final energy eigenstates - which are the states of the black hole and radiation before and after emission.

$$
\begin{aligned}
|\langle E_f,\omega|\,\mathcal{O}(0)|\,E_i\rangle|^2 &= \frac{C}{\pi^2}\,\delta_{q_1,q_2+q} \\
&\times \frac{\Gamma\left(\Delta \pm i\sqrt{2C\left(E_i-\frac{q_1^2}{2K}\right)} \pm i\sqrt{2C\left(E_f-\frac{q_2^2}{2K}\right)}\right)}{(2C)^{2\Delta}\;\Gamma\left(2\Delta\right)}.
\end{aligned}
$$

Coupling and Fermi's Golden Rule

- Scalar field for example and coupling

$$\phi(t,z) = \phi_{\mathsf{bdy}}\, z^{1-\Delta} + O\left(z^{\Delta}\right),$$

$$I = I_{\mathsf{Sch}\times U(1)} + \int dt\, \phi_{\mathsf{bdy}}(t)\mathcal{O}(t).$$

- Spontaneous transition rate

$$\Gamma_{i\to f} = |\phi_0|^2 \left|\langle E_f,\omega|\, \mathcal{O}(0)|\, E_i\rangle\right|^2 \delta\left(E_i - E_f - \omega\right).$$

- Full spontaneous emission rates

$$\Gamma_{\mathsf{spon.}} = \int d\omega\, \omega \int dE_f\, \rho\left(E_f,\omega\right) \Gamma_{i\to f}.$$

- The rate of change of energy is calculated by

$$\frac{dE}{dt\, d\omega} = \omega \int dE_f\, \rho\left(E_f\right) \Gamma_{i\to f}.$$

Brown, Iliesiu, Pennington,Usatyuk, 2411.03447

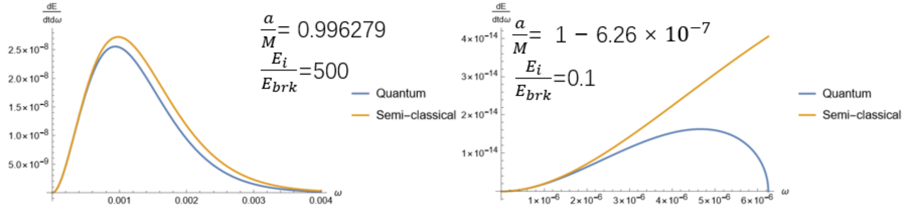


Figure: Comparing the emission, $\frac{dE}{dtd\omega}$, in the quantum-corrected and semi-classical framework for a scalar particle with quantum numbers $s = 0, l = 0, m = 0$.

Scalar field with angular momentum

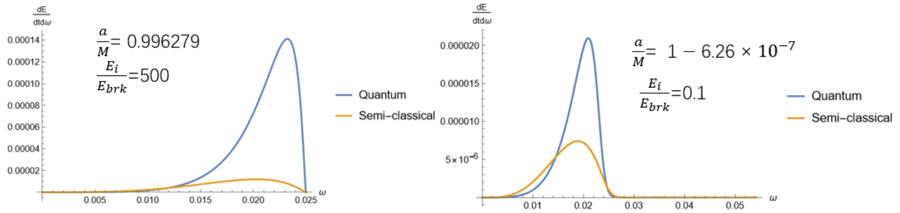


Figure: Comparing the emission rate, $\frac{dE}{dtd\omega}$, in quantum-corrected theory and semi-classical theory for a scalar particle emission with positive angular momentum $s = 0$, $l = 1$, $m = 1$ for black hole at relatively higher temperature and lower temperature

Rotation is different

- Unlike the generic suppression of particle emission in the Reissner-Nordström case, we uncover that for particles with non-vanishing angular momentum, the quantum-corrected emission can be substantially enhanced with respect to the standard semiclassical result.
- Photon and gravitons emission are problematic due to potential complex effective conformal dimensions.
- For spherically symmetric black holes $\Delta(\ell) = \ell(\ell+1) - s(s+1)$, for rotating, through the spin-weighted spheroidal function $\Delta(a\omega)$.
- Difficulties in accommodating higher-spin fields in the JT framework?

Recent Developments/Ongoing work

- The evaporation of charged black holes [Brown, Iliesiu, Penington and Usatyuk, 2411.03447]
- Quantum-Corrected Hawking Radiation from Near-Extremal Kerr-Newman Black Holes [Maulik Sabyasachi, Xin Meng, LPZ].
- The evaporation of black holes in supergravity [Lin, Iliesiu, Usatyuk, 2504.21077]
- Black hole absorption [Emparan,2501.17470], [Biggs, 2503.02051]
- Quantum Corrections in the Low-Temperature Fluid/Gravity Correspondence [Nian-PZ-Yue], towards $\frac{\eta}{s}$ corrections.
- Applications for the evolution of astrophysical black holes: PBH, evaporation and Dark Matter.

Quantum Corrections to Holographic Strange Metal at Low Temperature, 2410.11487

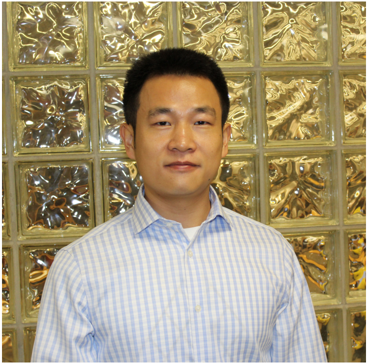


Figure: Xiao-Long Liu and Jun Nian

The Phase Diagram of Cuprates

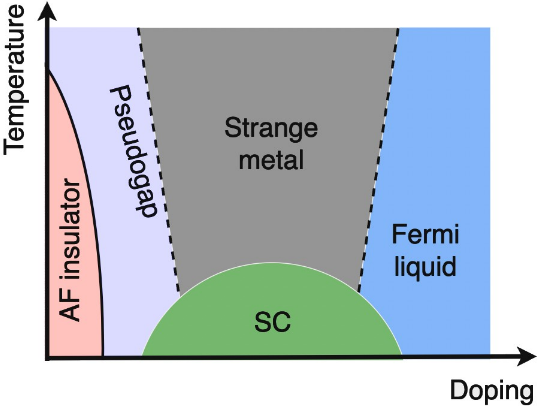


Figure: The phase diagram of cuprates.

The Phase Diagram of Cuprates

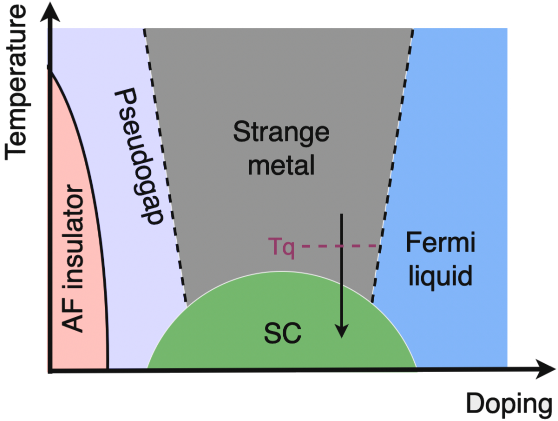


Figure: The phase diagram of cuprates.

Gravity Setup

- Simplest bottom-up holographic setup to a theory with stress-energy tensor and conserved current: Einstein-Hilbert with negative cosmological constant and Maxwell field.
- The charged black brane

$$ds^2 = \frac{r^2}{R^2}\left[-f(r)dt^2 + dx_i^2\right] + \frac{R^2}{r^2}\frac{dr^2}{f(r)}\,,$$

$$f(r) = 1 + \frac{Q^2}{r^4} - \frac{M}{r^3}, \qquad A_t = \mu\left(1 - \frac{r_0}{r}\right).$$

Near-horizon limit

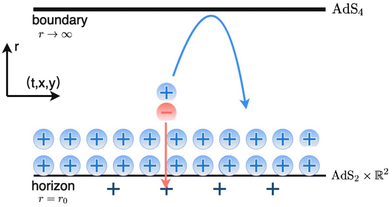
$$r - r_* = \lambda \frac{R_2^2}{\zeta}, \qquad r_0 - r_* = \lambda \frac{R_2^2}{\zeta_0}, \qquad t = \lambda^{-1}\tau,$$

$$ds^2 = \frac{R_2^2}{\zeta^2}\left[-\left(1-\frac{\zeta^2}{\zeta_0^2}\right)d\tau^2 + \frac{d\zeta^2}{1-\frac{\zeta^2}{\zeta_0^2}}\right] + \frac{r_*^2}{R^2}d\vec{x}^{\,2},$$

$$A_\tau = \frac{g_F}{2\sqrt{3}}\left(\frac{1}{\zeta} - \frac{1}{\zeta_0}\right).$$

- The strict $\lambda \to 0$ limit. Beware of order of limits!
- A ζ-reparametrization of AdS_2

Holographic Strange Metal



- The mechanism of holographic strange metal: [Faulkner, Iqbal, Liu McGreevy and Vegh, Science 329 (2010) 1043]
- The negatively charged particle falls into the horizon, the positive scatters leading to a finite density positively charged gas hovering outside.
- Treat the bulk spinor as a probe.

Fermionic field

- In the near-horizon AdS_2, one can also consider the action for a spinor field Ψ:

$$S = \int d^2x\sqrt{-g}\, i\,\left(\bar{\Psi}\Gamma^{\alpha}D_{\alpha}\Psi - m\bar{\Psi}\Psi + i\tilde{m}\bar{\Psi}\,\Gamma\,\Psi\,\right)\,.$$

- We obtain the fermionic operator retarded Green's function in the near-horizon boundary CFT_1:

$$\mathcal{G}_R(\omega,T) = (2\pi T)^{2\ell-1}\, g\left(\frac{\omega}{T},\frac{k}{\mu}\right)\,,$$

$$\ell \equiv \frac{1}{\sqrt{6}}\sqrt{m^2R^2 + \frac{3k^2}{\mu^2} - \frac{q^2}{2} + \frac{1}{2}}.$$

- There might not be a sugra field with such ℓ

3d/1d Gluing

- The Green's functions in the UV CFT$_3$, G_R^{3d}, and in the IR CFT$_1$, $\mathcal{G}_R$, are related [Faulkner et al 2009wj].

$$G_R^{3d}(\omega, k) = \frac{h_1}{k - k_F(\omega, T) - \Sigma(\omega, k)}\,,$$

- $\Sigma(\omega, T) = h_2\,\mathcal{G}(\omega, T)$, and $k_F(\omega, T)$ is approximately the Fermi momentum k_F for low ω and T.
- $h_{1,2}$ are positive constants that can be fixed numerically.
- We exploit this 3d/1d gluing as a shortcut to including quantum corrections.

Linear Resistivity

- The conductivity from Kubo formula

$$\sigma \equiv \sigma(\Omega) = \frac{1}{i\Omega}\langle J_y(\Omega) J_y(-\Omega)\rangle = \frac{1}{i\Omega} G_R^{yy} \, .$$

- The DC conductivity at the leading order in small T is given by

$$\sigma_{DC} = \alpha T^{1-2\ell} \, .$$

- For $\ell = 1$, this corresponds to the marginal Fermi liquid.

Quantum Corrections to Holographic Strange Metal

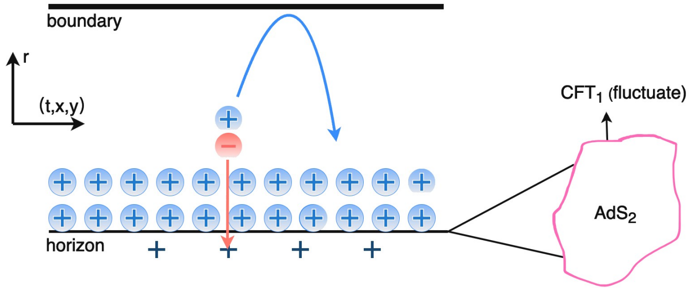


Figure: The mechanism of introducing quantum fluctuations in the AdS_2 throat region, or equivalently in the boundary CFT_1, in contrast to the classical case of [Faulkner, Science].

Throat region

1. The main AdS/CFT formula
$$Z_{\rm CFT}[J] = Z_{\rm bulk}[\phi_0],$$
ϕ_0 is the boundary value of the bulk field dual to the operator that couples to the source J.
2. The holographic computation of the (2+1)-dimensional Green's function, G_R^{3d}, is connected to the (0+1)-dimensional retarded Green's function of the throat, $\mathcal{G}_R^{1d}$
3. Approximate the retarded Green's function of the throat by using the coupling in the throat region to be φ_0.

$$\begin{aligned}\frac{\delta^2 Z_{\rm JT}^{2d}[\varphi_0]}{\delta\varphi_0^2} &= \frac{\delta^2}{\delta\varphi_0^2}\int [Dg_{\mu\nu}][D\phi]\,\exp\Big[-S(g_{\mu\nu},\phi;\varphi_0)\Big] \\ &\approx \frac{\delta^2}{\delta\varphi_0^2}\int [Df]\,\exp\Big[-S_{\rm 2d}(f(t))-S_{\rm matter}[\varphi_0]\Big] \\ &= \int [Df]\,e^{-S_{2d}[f(t)]}\,G_R(t) = \langle G_R(t)\rangle .\end{aligned}$$

Effective action

- The effective action for both gravity and gauge fluctuations is given by [Sachdev et al, Mertens et al.]:

$$S_{eff}[f,\Lambda] = -C\int_0^\beta d\tau \left\{\tan\frac{\pi}{\beta}f(\tau),\tau\right\} - \frac{K}{2}\int_0^\beta d\tau \,\left[\Lambda'(\tau) - i\mu f'(\tau)\right]^2$$

- $\Lambda(\tau) \equiv \int_{r_0}^{\infty} A_r(r,\tau)\, dr$ denotes the gauge fluctuation, and the coupling constant K is the compressibility of the boundary quantum system
- $\mathrm{AdS}_2 \times \mathbb{R}^2 \to AdS_2 \times T^2$ leads to $U(1)\times U(1)$ gauge field with

$$M_{U(1)\times U(1)} \ll C^{-1} \simeq K^{-1}.$$

Quantum corrections

- The quantum-corrected Green's function as a path integral with the effective action:

$$\langle \mathcal{G}(\tau_1,\tau_2)\rangle = \int [\mathcal{D}f][\mathcal{D}\Lambda]\, e^{-S_{eff}[f,\Lambda]}\, \mathcal{G}'(\tau_1,\tau_2)\,.$$

- Introducing $\tilde{\Lambda}(\tau) = \Lambda(\tau) - i\mu f(\tau)$ leads to a decoupling of the f-dependent and the Λ-dependent factors:

$$\begin{aligned}\langle \mathcal{G}(\tau_1,\tau_2)\rangle &= \langle e^{i(\tilde{\Lambda}(\tau_1)-\tilde{\Lambda}(\tau_2))}\rangle \cdot \left\langle \left(\frac{\sqrt{f'(\tau_1)f'(\tau_2)}}{\frac{\beta}{\pi}\sin\frac{\pi}{\beta}\,|f(\tau_1)-f(\tau_2)|} \right)^{2\ell} \right\rangle \\ &= \langle \mathcal{G}_f\rangle \cdot \langle \mathcal{G}_{\tilde{\Lambda}}\rangle\,,\end{aligned}$$

- The $\mathrm{SL}(2,\mathbb{R})$ part, $\langle \mathcal{G}_f\rangle$, and the $\mathrm{U}(1)$ part, $\langle \mathcal{G}_{\tilde{\Lambda}}\rangle$.

DC Conductivity

- In the high-temperature limit ($\beta \to 0$), the quantum fluctuations can be neglected. We should recover linear resistivity with no quantum fluctuations from gravity.
- We first take the high-temperature limit ($\beta \to 0$) and then gradually lower the temperature by adding terms of higher orders in β to see how the resistivity deviates from the linear scaling law.
- This is an intrinsically perturbative approach driven by the phenomenology of the problem.

Results

- Rewrite the quantum-corrected Green's function:

$$\langle \mathcal{G}(\omega, T)\rangle = F_2(\beta, \ell; \omega, q, K) \cdot \left[\frac{\beta}{2C} + \frac{1-2\ell}{2\pi^2}\left(\frac{\beta}{2C}\right)^2\right]^{1-2\ell}$$
$$= F_1(\beta, \ell; \omega, q, K) \cdot \left(\frac{\beta}{2C}\right)^{1-2\ell'}$$

- A renormalized parameter ℓ' represent the quantum-corrected ℓ.

$$\ell' = \frac{1}{2} - \frac{\ln\left(\frac{F_2(\beta,\ell;\omega,q,K)}{F_1(\beta,\ell;\omega,q,K)}\right) + (1-2\ell)\ln\left[\frac{\beta}{2C} + \frac{1-2\ell}{2\pi^2}\left(\frac{\beta}{2C}\right)^2\right]}{2\ln\frac{\beta}{2C}},$$

Frame Title

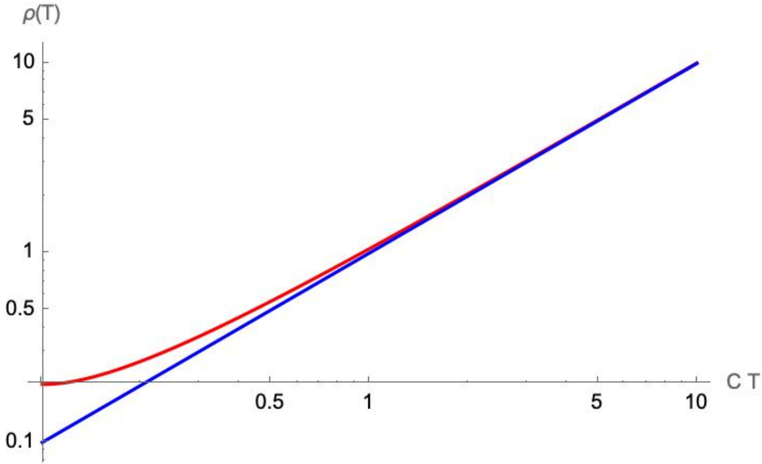


Figure: The uncorrected (blue) and quantum-corrected (red) DC resistivities as functions of CT (with a special choice of parameters $\ell = 1$, $q = 1$, $K = 10^4$, and $\omega = 10^{-3}$).

A little Pheno?

- The entropy of a near-extremal AdS_4 black hole:

$$S = S_0 + 4\pi^2 CT + \frac{3}{2}\log(CT)$$
$$= S_0 + \left(\frac{C_p}{T}\right)_{T=0} \cdot T + \frac{3}{2}\log(CT)\,,$$

- S_0 the AdS_4 black hole entropy in the extremal limit, C_p denotes the heat capacity.
- Heat capacity is linear in T at low temperatures with proportionality constant $\gamma \equiv C_p/T$ - the Sommerfeld coefficient.
- For a class of strange metal ($Ba_4Nb_{1-x}Ru_{3+x}O_{12}$, $|x| < 0.20$), which takes values in the range $[164\,\mathrm{mJ}/(\mathrm{mol}\cdot\mathrm{K}^2),\, 275\,\mathrm{mJ}/(\mathrm{mol}\cdot\mathrm{K}^2)]$ for $T \in [50\,\mathrm{mK},\, 30\,\mathrm{K}]$.
- Based on these data, an estimate of the quantum temperature in this case is $T_q \equiv 1/\gamma \simeq [30\,\mathrm{K},\, 50\,\mathrm{K}]$, which, in principle, allows an experimental detection of quantum corrections.