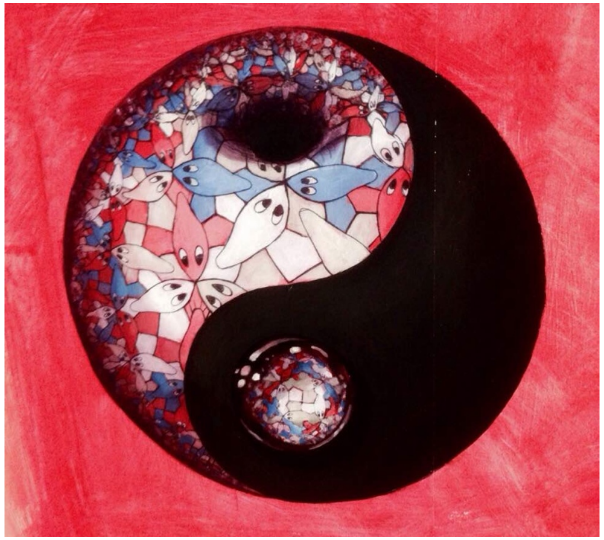
A Josephson wormhole in coupled superconducting Yukawa-SYK metals

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University

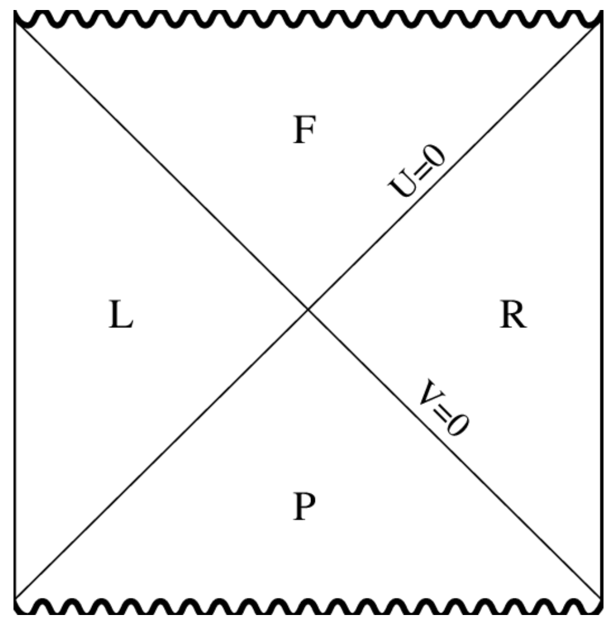


Shankar, Plugge, Steenbergen, Schalm
arXiv:2506.XXXXX

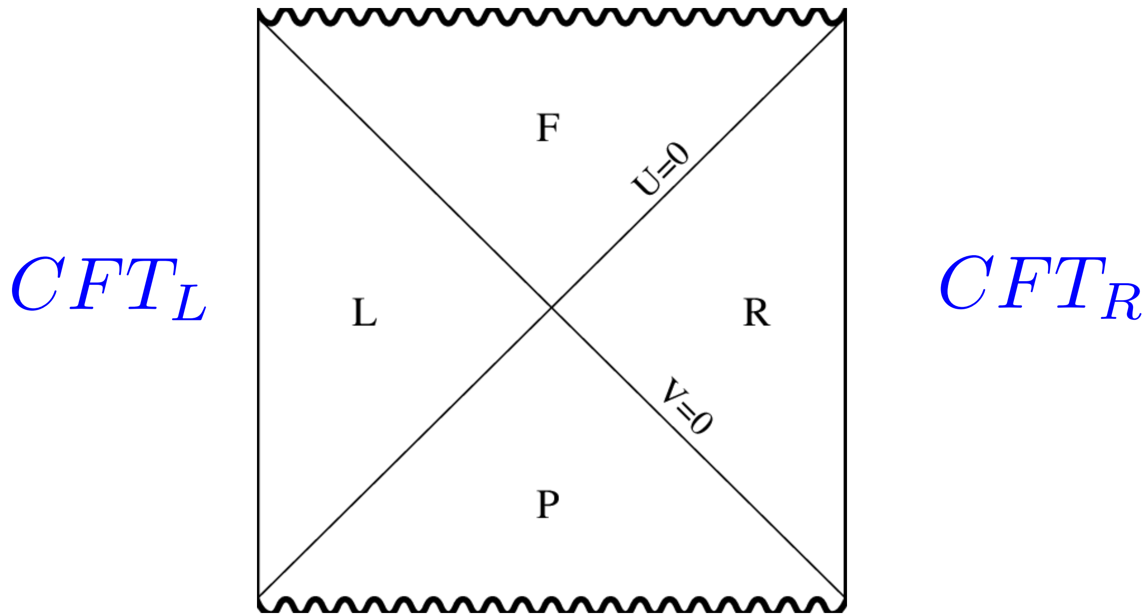
Netherlands Organisation for Scientific Research

Delta Institute for Theoretical Physics

An anti-de-Sitter Black Hole in Kruskal coordinates



An anti-de-Sitter Black Hole in Kruskal coordinates and holography Witten, Son, Herzog



Two observations:

1. The resulting observables are *finite temperature* observables with $T = T_{\rm BH}$

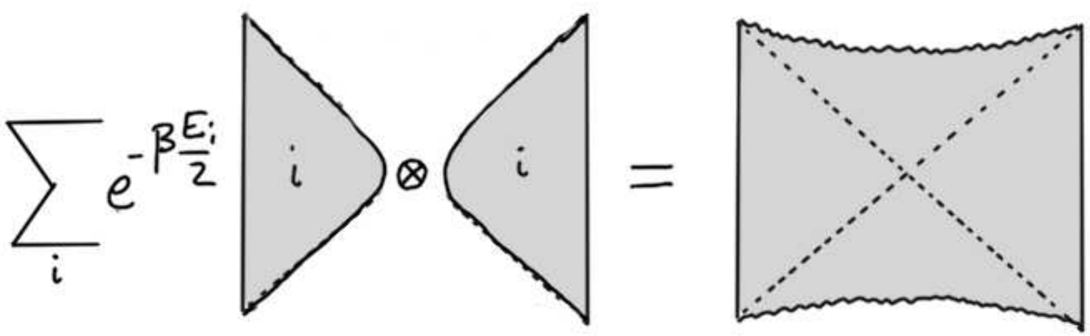
2. These are computed as probabilities on the doubled time contour (Schwinger-Keldysh, in-in formalism)

van Raamsdonk

Spacetime from entanglement

A thermal (mixed) state can also be viewed as a pure state in a "thermofield double" of the theory.

$$|TFD\rangle = \sum \delta_{n_1 n_2} e^{-\frac{\beta}{2} E_{n_1}} |n_1\rangle |n_2\rangle$$



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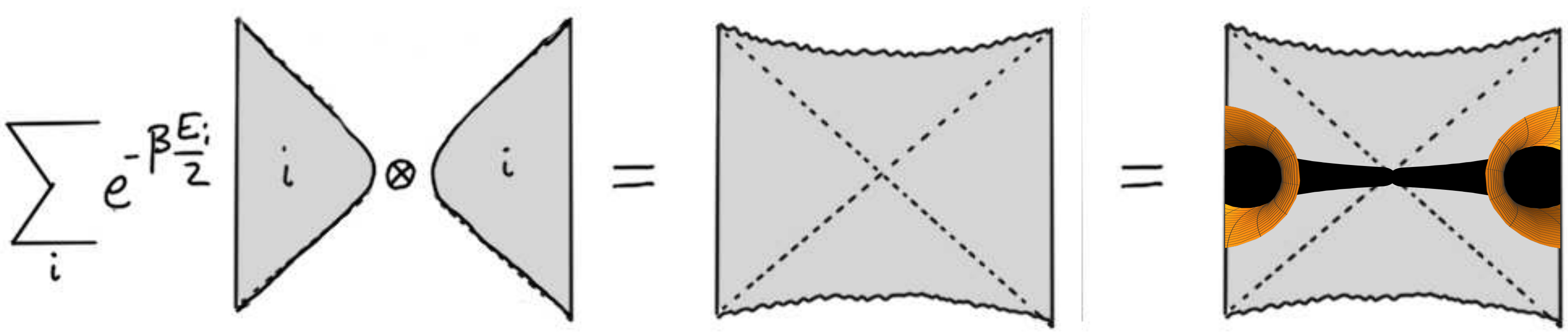
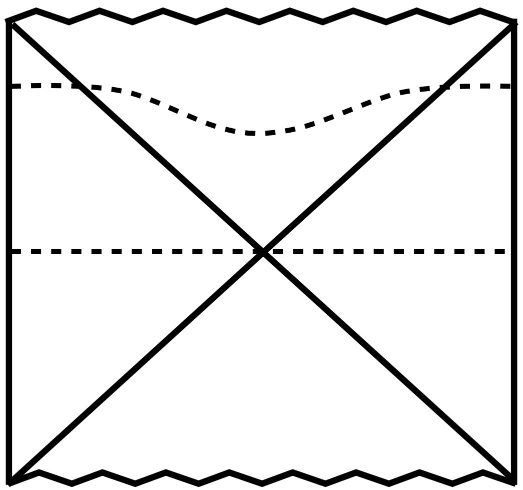


Doubled geometry contains a wormhole:
(Einstein-Rosen bridge)

Maldacena, Susskind

This idea can be tested by computing $CFT_L \times CFT_R$ cross correlation functions

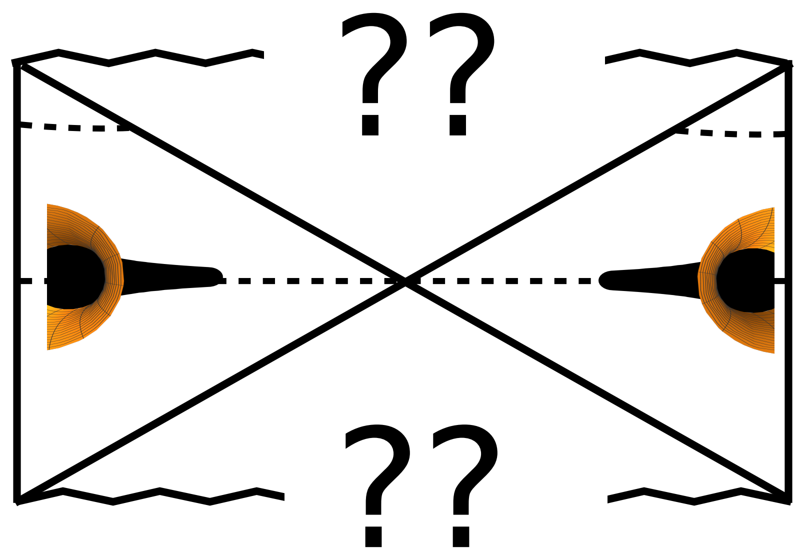
$$|EST> = \sum f(E_n)|n_1\rangle|n_2\rangle$$



Fidkowski, Hubeny, Kleban, Shenker;
Festuccia, Liu;
Romero-Bermudez, Sabella-Garnier, Schalm;
...

Consider in more generality the $CFT_L \times CFT_R$

$$|EST >= \sum f(E_n)|n_1\rangle|n_2\rangle$$



The coupled system with $CFT_L \times CFT_R$ should contain a Thermofield double state in the double-copy system.

Gao, Jefferis, Wall;
Maldacenca Qi;
Cottrell, Freivogel, Hofman, Lokhande

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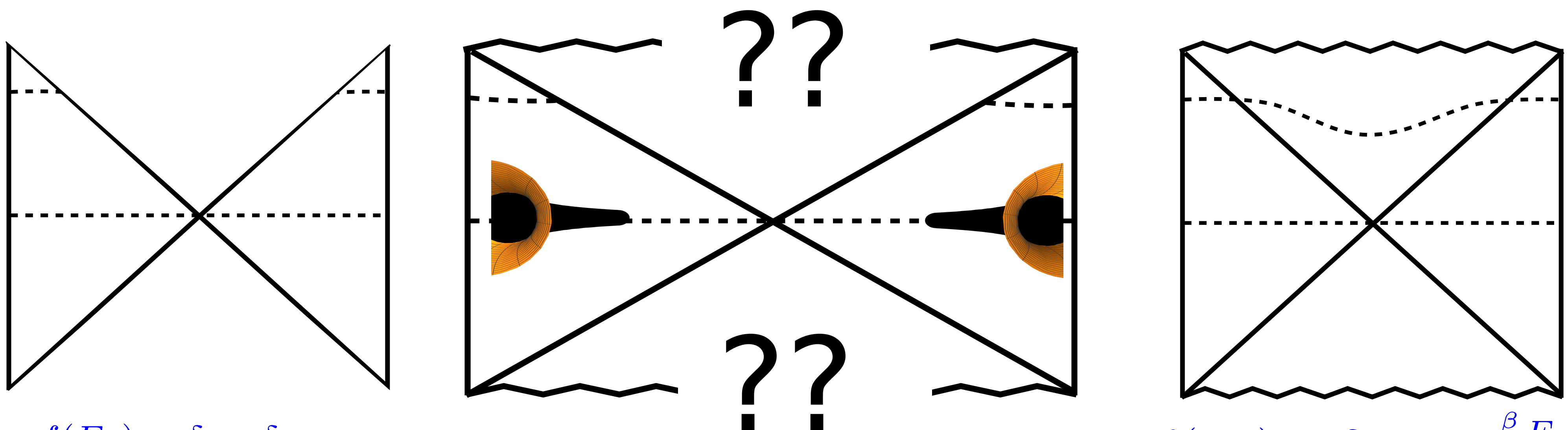

$f(E_n)=\delta_{n1,0}\delta_{n_2,0}$

$f(E_n)=\delta_{n_1 n_2} e^{-\frac{\beta}{2}E_{n_1}}$

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How to build a Thermofield double state.

Cottrell, Freivogel, Hofman, Lokhande

$|TFD\rangle$ is the groundstate of the combined system

$$S = S_L + S_R + \sum_k c_k (\mathcal{O}^k_L - \mathcal{O}^k_R)^2$$

with $\mathcal{O}^k$ a generating set of all operators.

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How to build an (almost) Thermofield double state.

Maldacenca Qi;

The groundstate of the combined system

$$S = S_L + S_R + \lambda \mathcal{O}_L \mathcal{O}_R + \text{c.c.}$$

with $\mathcal{O}$ the lowest dimension operator, is approx. $|TFD\rangle$

How to build a Thermofield double state.

$$S = S_L + S_R + \lambda \mathcal{O}_L \mathcal{O}_R + \text{c.c.}$$

holographist

… *also* describes traversable wormholes in the holographic dual

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$$S = S_L + S_R + \lambda \mathcal{O}_L \mathcal{O}_R + \text{c.c.}$$

condensed matter
physicist

… *also* describes standard tunneling interaction between two contacts in quantum electronics

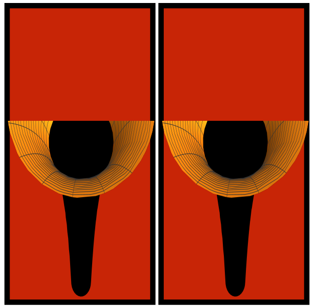
How to build a Thermofield double state.

Maldacena Qi

$$S = S_L + S_R + \lambda \mathcal{O}_L \mathcal{O}_R + \text{c.c.}$$

If this is a relevant deformation, then at high T two decoupled thermal states

$$\rho_{2\text{BH}} = \frac{1}{Z_\beta^2} \sum_{n_1,n_2} e^{-\beta(H_1+H_2)} |n_1, n_2\rangle\langle \bar{n}_1, \bar{n}_2| \ .$$



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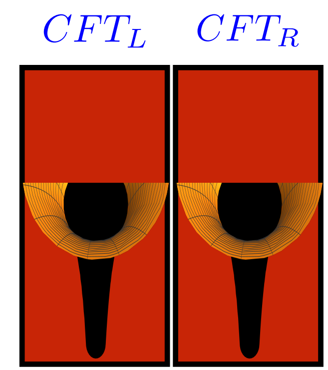

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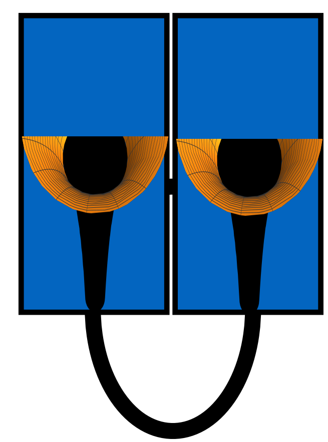
But at low T, the system should be in its $|TFD\rangle$ groundstate



$$\rho_{\text{WH}} = \frac{1}{Z_\beta} \sum_{n_1, n_2} e^{-\frac{\beta}{2}(E_{n_1}+E_{n_2})} |n_1, \bar{n}_1\rangle \langle \bar{n}_2, n_2|$$

- What is the signature of a wormhole, i.e. of being in the TFD state?

Maldacena Qi

BH-WH is 1st order Hawking page transition: no order parameter

- What is the signature of a wormhole, i.e. of being in the TFD state?

Maldacena Qi

BH-WH is 1st order Hawking page transition: no order parameter

- Wormhole emerges from CFT at finite T:

$$H = H_L + H_R + \lambda \mathcal{O}^L_\Delta \mathcal{O}^R_\Delta$$

If CFT = AdS2 (dual to SYK) : unique characteristic:

$$E_n = E_{\rm gap}(1 + \frac{n}{\Delta}) \; + \mathcal{O}(\lambda)$$

$$E_{\rm gap} \sim \lambda^{\frac{1}{2-2\Delta}}$$

Harmonic oscillator-like spectrum implies revivals in linear response.

signal traversing the wormhole

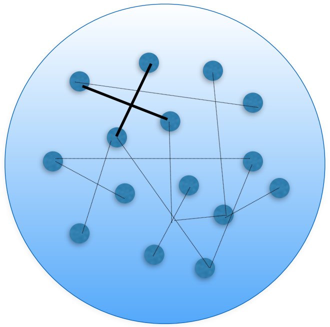
- AdS-CFT: Can model black hole physics with conventional quantum systems.
  - We will use two coupled SYK models:

Sachdev-Ye-Kitaev model: N complex/real fermions with $q = 2p$-point interactions

$$H = J_{i_1 i_2 \ldots i_p j_1 j_2 \ldots j_p} c^\dagger_{i_1} c^\dagger_{i_2} \ldots c^\dagger_{i_p} c_{j_1} c_{j_2} \ldots c_{j_p}$$

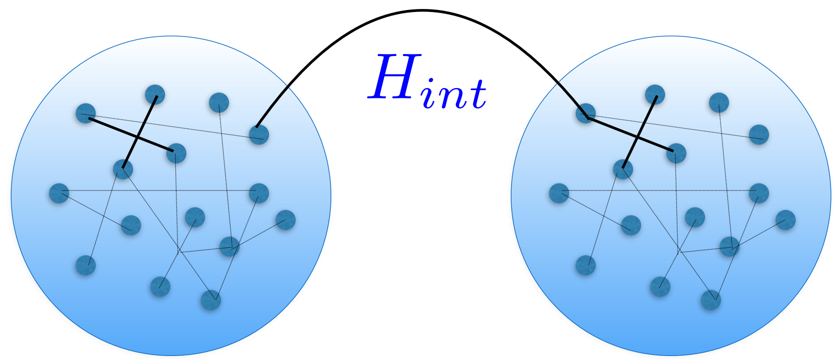
with random disorder averaged interactions

$$\langle J_{i_1 i_2 \ldots i_p j_1 j_2 \ldots j_p} J_{i'_1 i'_2 \ldots i'_p j'_1 j'_2 \ldots j'_p} \rangle = \frac{(p!)^2}{N^{2p-1}} J^2 \delta_{i_1 i'_1} \ldots \delta_{j_1 j'_1}$$

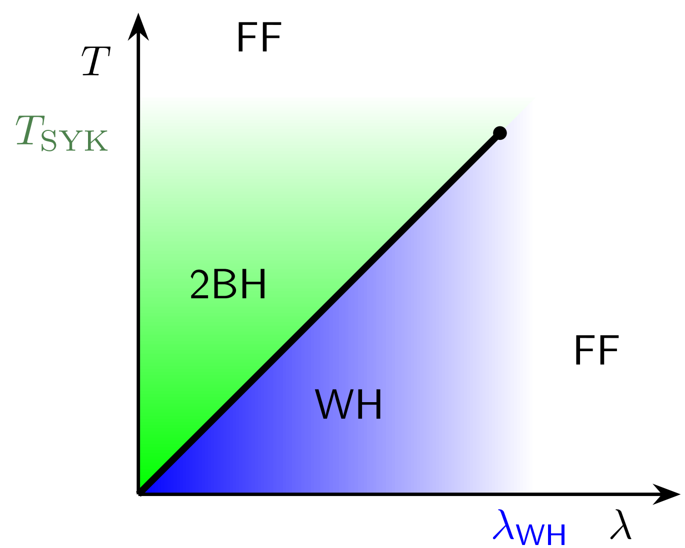


Plugge, Lantagne-Hurturbise, Franz
Sahoo, Plugge, Lantagne-Hurturbise, Franz
Maldacena, Milenkin

- Tunneling coupling two thermal SYK quantum dots



$$H_{\text{int}} = \lambda(c_i^\dagger \psi_i + \psi_i^\dagger c_i)$$ relevant deformation



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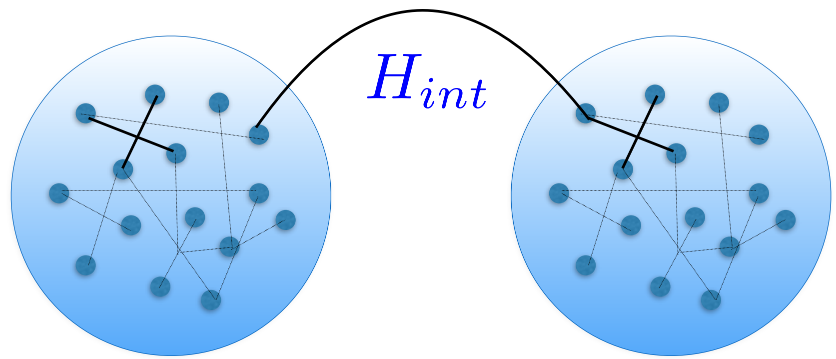

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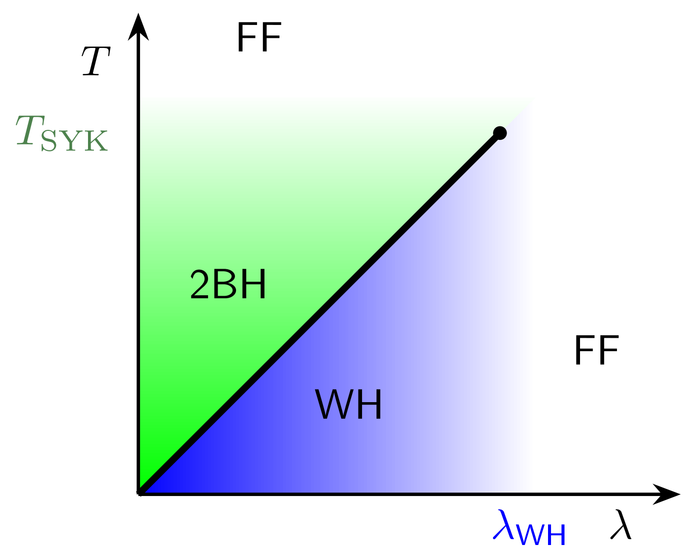


For $N \to \infty$:

Solve Schwinger-Dyson Eqns

after disorder averaging

$$G(i\omega_n) = \frac{1}{-i\omega_n - \mu - \Sigma(i\omega_n)}$$

$$\Sigma(\tau) = -J^2 G(\tau) G(-\tau)$$

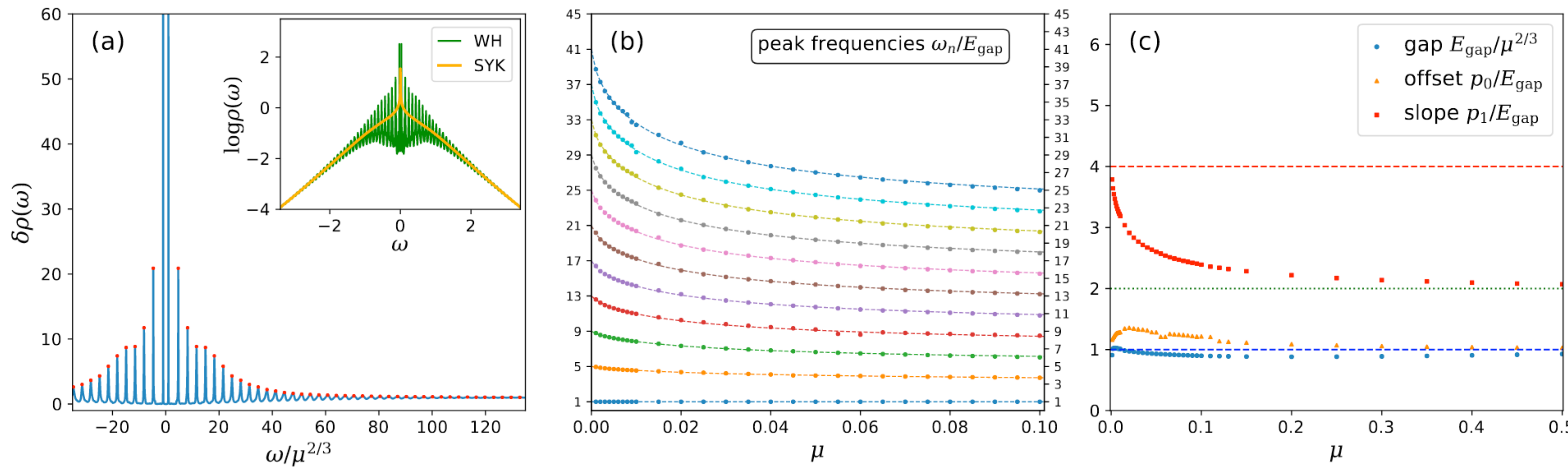


FIG. 1. Spectral features of the MQ model obtained from numerical solution of the SD equations (4) in real time and at temperatures $T \ll \mu$ below the Hawking-Page transition [21] ($T = 2\cdot 10^{-4}$ for $\mu \geq 0.01$, $T = 5\cdot 10^{-5}$ for $\mu < 0.01$). (a) inset: spectral function for SYK (ρ_0, $\mu = 0$) and wormhole (ρ_{LL}, $\mu = 0.004$). Main panel: relative spectral weight $\delta\rho(\omega) = \rho_{LL}/\rho_0$. Red dots indicate positions of the dominant peaks analyzed in (b,c). (b) peak frequencies $\omega_n > 0$, extracted from peak positions of $\delta\rho(\omega)$ in (a). For $\mu \to 0$ and $\omega_n \ll J$ we see a clear approach to the conformal tower $\omega_n = E_{\rm gap}(4n+1)$ [21]. The dashed lines are a simple fit $Y_n(\mu)$, see text. (c) spectral gap $E_{\rm gap} = \omega_0$, and offset p_0 and slope p_1 of linear fits $\omega_n \sim p_0 + p_1 n$.

$$H_{\rm int} = \mu(c_i^\dagger\psi_i + \psi_i^\dagger c_i)$$

$$\mathrm{SYK}_{\mathrm{Majorana}}\ ;\quad \Delta_\psi = \frac{1}{4}$$

Plugge, Lantagne-Hurturbise, Franz
Sahoo, Plugge, Lantagne-Hurturbise, Franz

perturbations “revive” instead of dissipate

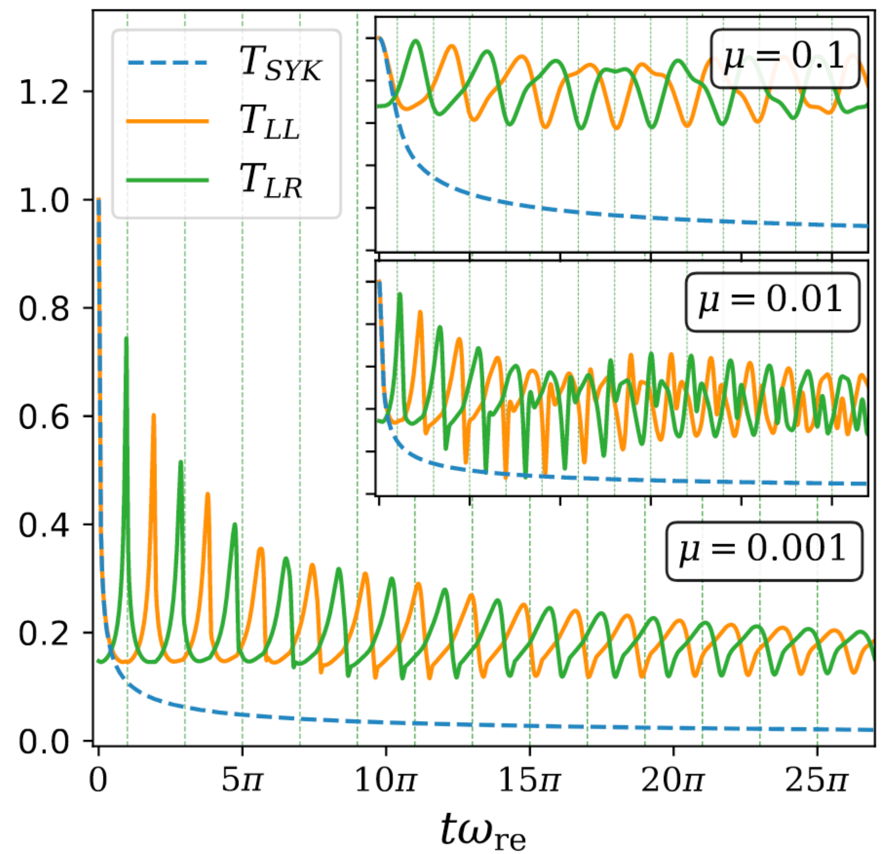


FIG. 2. Revival dynamics in transmission amplitudes $T_{LL,LR}$ [Eq. (6)] at temperature $T < \mu$; $T_{\mathrm{SYK}} = T_{LL}(\mu = 0)$ refers to an uncoupled SYK model. We show data for various μ, with time axes rescaled by $\omega_{\mathrm{re}} = p_1/2\pi$, cf. Fig. 1. Vertical lines indicate times $t_{\mathrm{re},n} = (2n+1)\pi/\omega_{\mathrm{re}}$ for which an excitation $\chi_R^j|\Psi_0\rangle$ is expected to re-assemble on the left side.

Plugge, Lantagne-Hurturbise, Franz
Sahoo, Plugge, Lantagne-Hurturbise, Franz

- The importance of large N

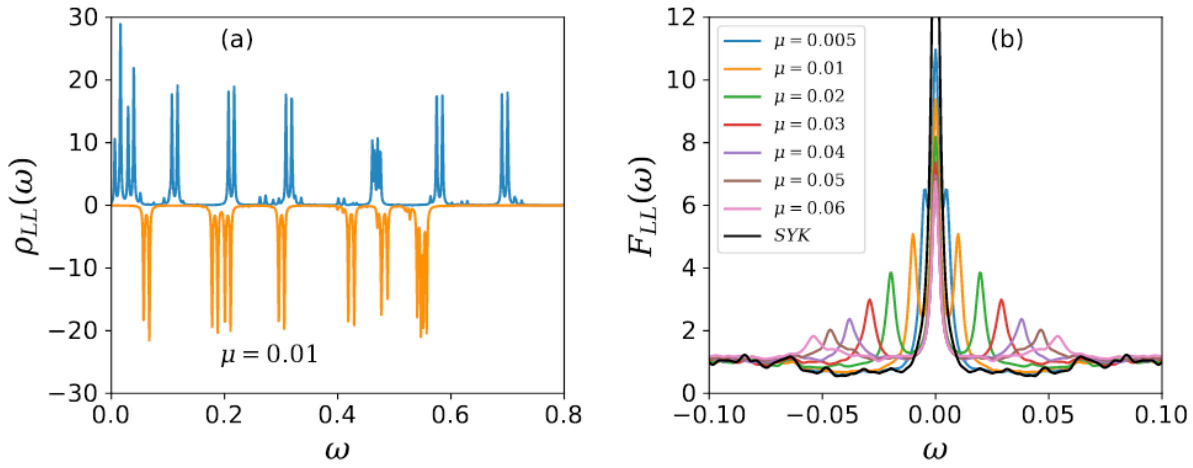
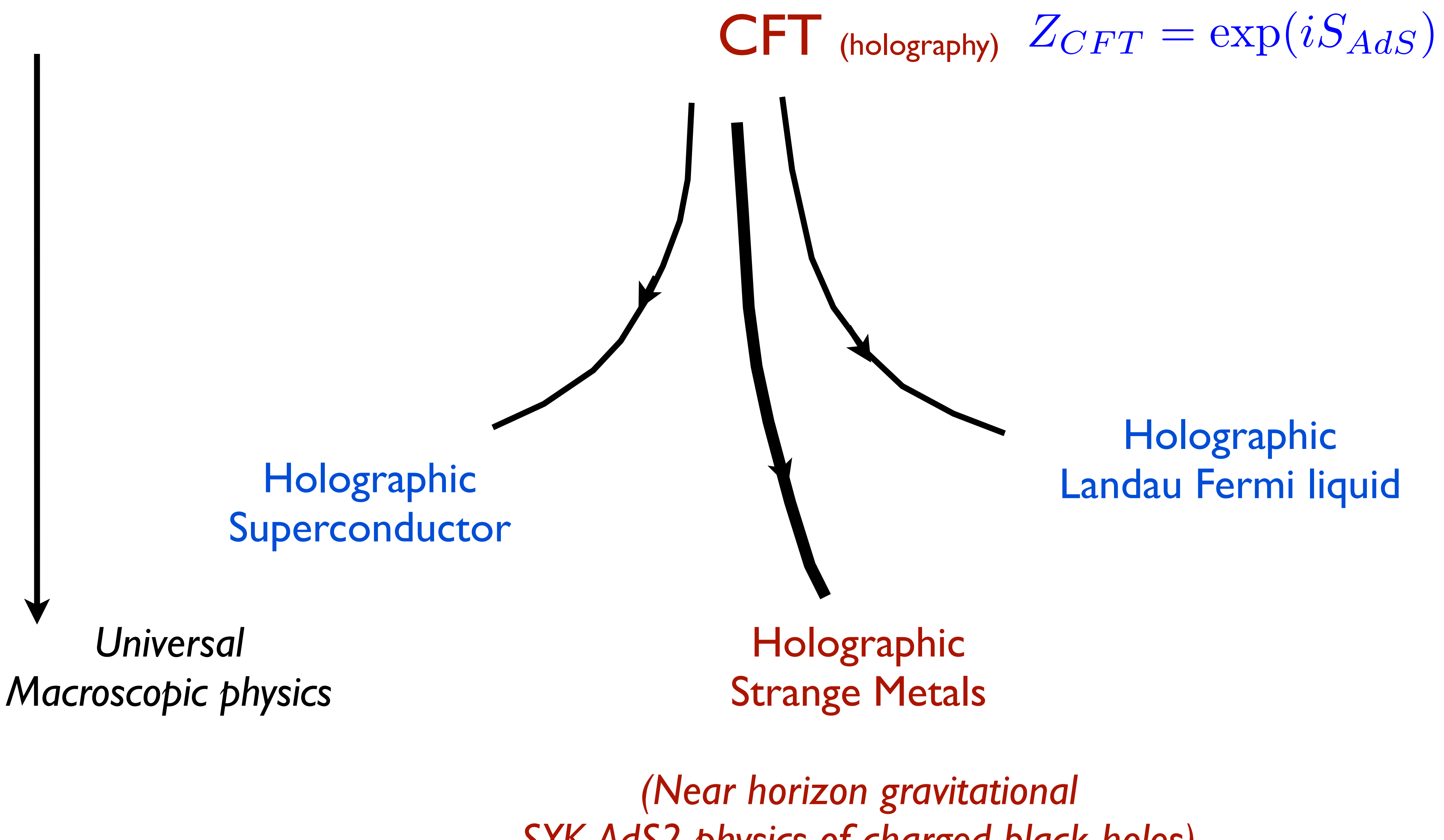
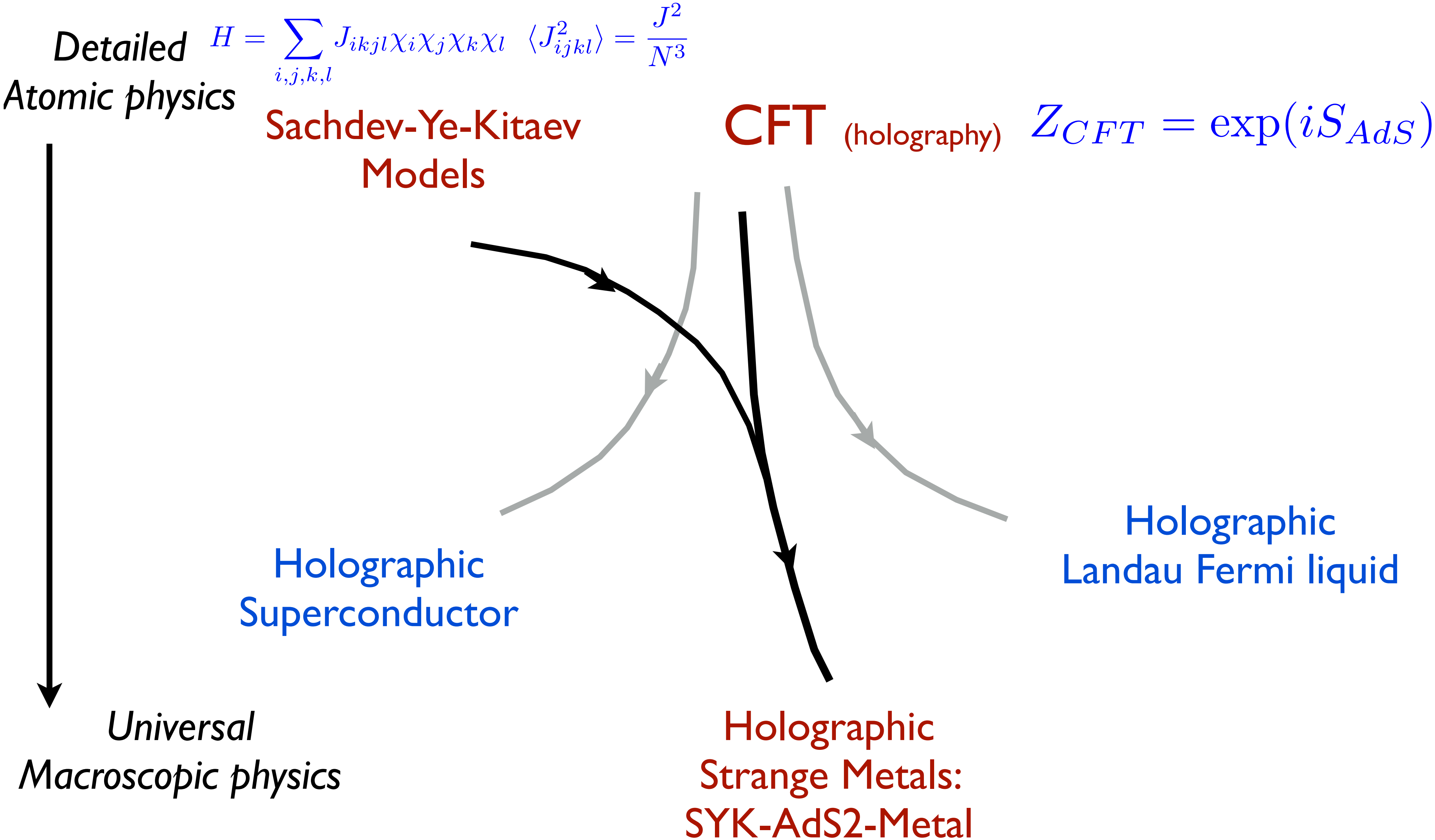


FIG. 3. Spectral properties of the MQ model from exact diagonalization for $2N = 16$ Majorana fermions. (a) spectral function ρ_{LL} for two distinct disorder realizations, showing doublets of states split by ω_D. (b) Auto-correlation $F_{LL}(\omega)$ for various μ, each averaged over 50 disorder realizations.

Can we build a “wormhole” in experiment?


CFT (holography) $Z_{CFT} = \exp(iS_{AdS})$
Holographic Superconductor
Holographic Landau Fermi liquid
Universal Macroscopic physics
Holographic Strange Metals
(Near horizon gravitational SYK-AdS2 physics of charged black holes)


Detailed
Atomic physics
$H = \sum_{i,j,k,l} J_{ikjl} \chi_i \chi_j \chi_k \chi_l \quad \langle J_{ijkl}^2 \rangle = \frac{J^2}{N^3}$
Sachdev-Ye-Kitaev
Models
CFT (holography) $Z_{CFT} = \exp(iS_{AdS})$
Holographic
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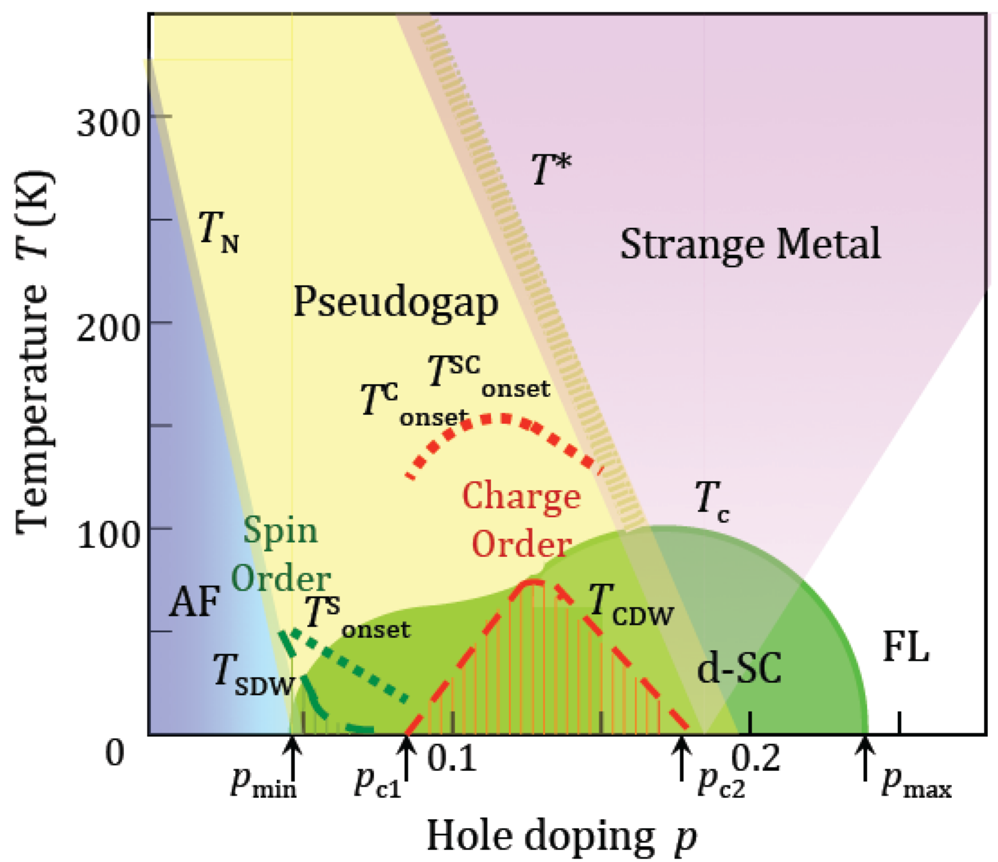
Macroscopic physics
Holographic
Strange Metals:
SYK-AdS2-Metal

The strange metal in high Tc cuprates



Keimer et al,
Nature 518 (2015) 179

2022-2024 (Partial) Breakthrough

arXiv > cond-mat > arXiv:2205.04030

Condensed Matter > Strongly Correlated Electrons

[Submitted on 9 May 2022 (v1), last revised 26 May 2023 (this version, v2)]

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

arXiv > cond-mat > arXiv:2203.04990

Condensed Matter > Strongly Correlated Electrons

[Submitted on 9 Mar 2022 (v1), last revised 6 Jul 2023 (this version, v7)]

Universal theory of strange metals from spatially random interactions

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, Subir Sachdev

arXiv > cond-mat > arXiv:2406.07608

Condensed Matter > Strongly Correlated Electrons

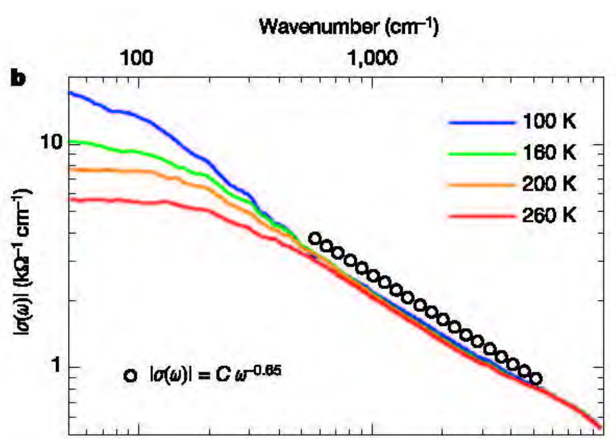
[Submitted on 11 Jun 2024 (v1), last revised 1 Aug 2024 (this version, v3)]

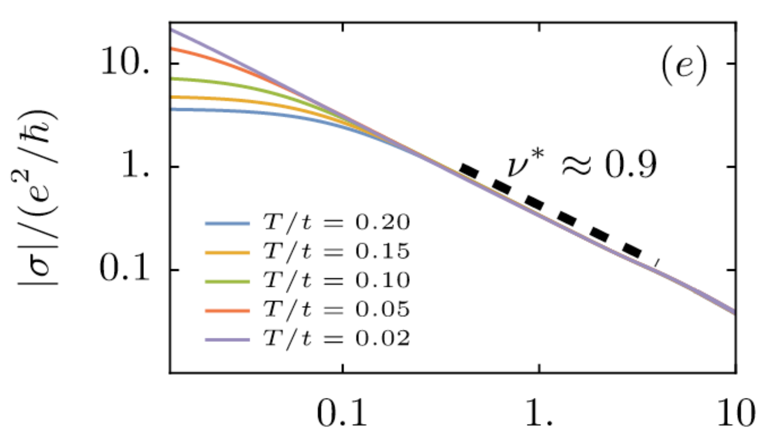
Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Davide Valentinis, Aavishkar A. Patel, Haoyu Guo, Jörg Schmalian, Subir Sachdev, Ilya Esterlis

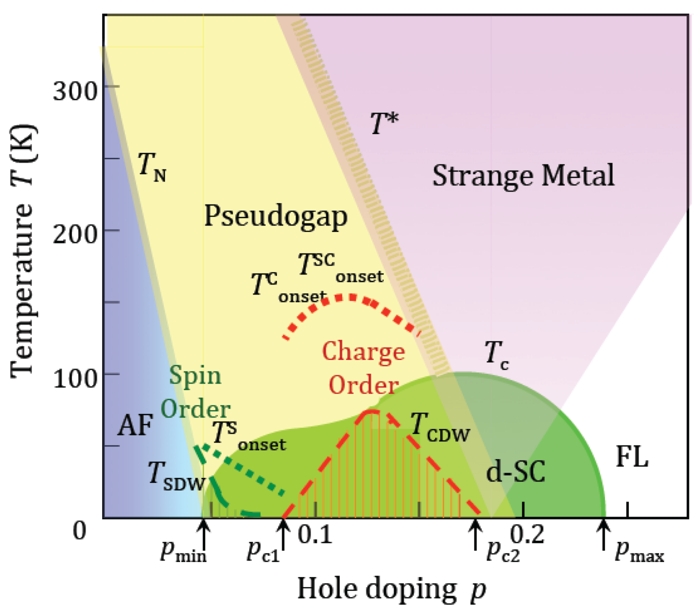
- Linear-in-T resistivity
- Tlog(T) specific heat
- Power-Law optical conductivity

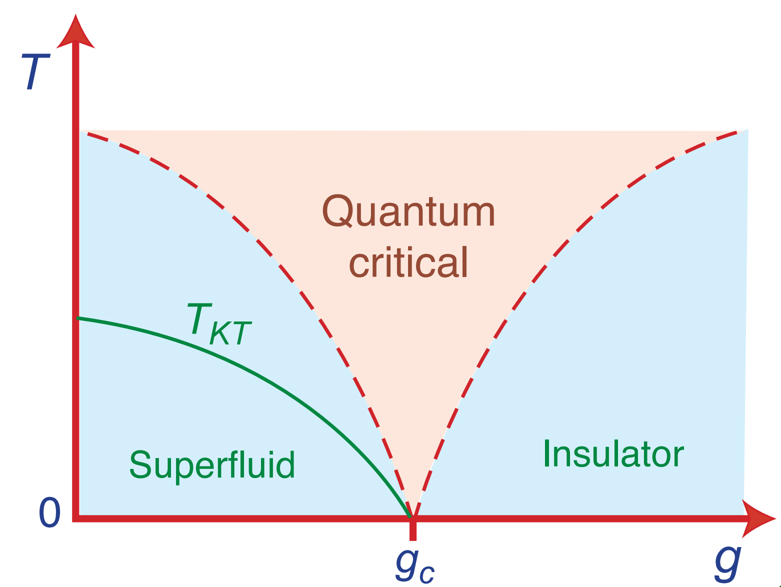
…





- The strangeness of the strange metal arises from the fact that
  - The SYK-AdS is a non-trivial IR Quantum Critical Point
  - Non-trivial IR fixed points are notoriously *unstable*
  - In CMT the predominant instability is superconductivity (spontaneous breaking of $U(1)$)





Sachdev

We have a candidate AdS2 system to build a wormhole:

high T_c cuprate strange metals

But do they in fact support such a state?

We have a candidate AdS2 system to build a wormhole:

high T_c cuprate strange metals

But do they in fact support such a state?

- The TFD of a CFT dual to AdS is dual to (macroscopic) wormhole state should be a more general story.

- But the groundstate is not an AdS2 quantum spin liquid, but a superconductor.

  What happens to the TFD/Wormhole state in that case?
  Diagnosis: difficult.

  - SC is characterized by a gap
  - WH is characterized by a gap
  - Do the characteristic revivals (supported by AdS2 perturbation theory) survive?

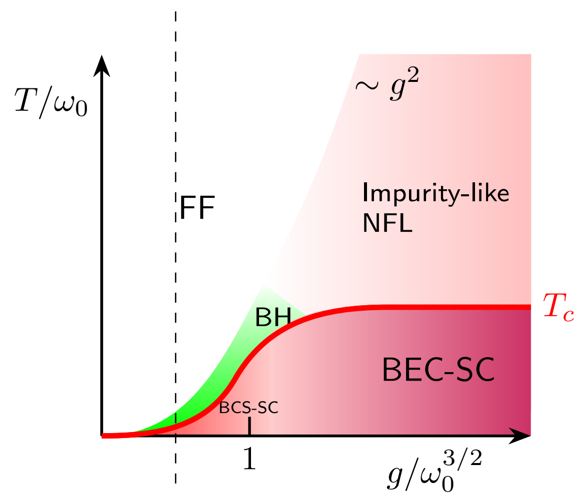
- Yukawa-SYK model (0D disordered electron-phonon model)

$$H_{\text{Y-SYK}} = -\mu \sum_{i=1}^{N} \sum_{\sigma=\uparrow,\downarrow} c_{i,\sigma}^{\dagger} c_{i,\sigma} + \sum_{k=1}^{M} \frac{1}{2}(\pi_k^2 + \omega_0^2 \phi_k^2) + \frac{\sqrt{2}}{N} \sum_{i,j,k} \sum_{\sigma} g_{ijk} c_{i,\sigma}^{\dagger} c_{j,\sigma} \phi_k$$

$$g_{ijk} = g'_{ijk} + i g''_{ijk}$$

$$\langle g'_{ijk} g'_{lmn} \rangle = (1 - \frac{\alpha}{2}) g^2 \delta_{kn} (\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl})$$
$$\langle g''_{ijk} g''_{lmn} \rangle = \frac{\alpha}{2} g^2 \delta_{kn} (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})$$
$$\langle g'_{ijk} g''_{lmn} \rangle = 0$$

For $\alpha < 1$: groundstate is superconducting



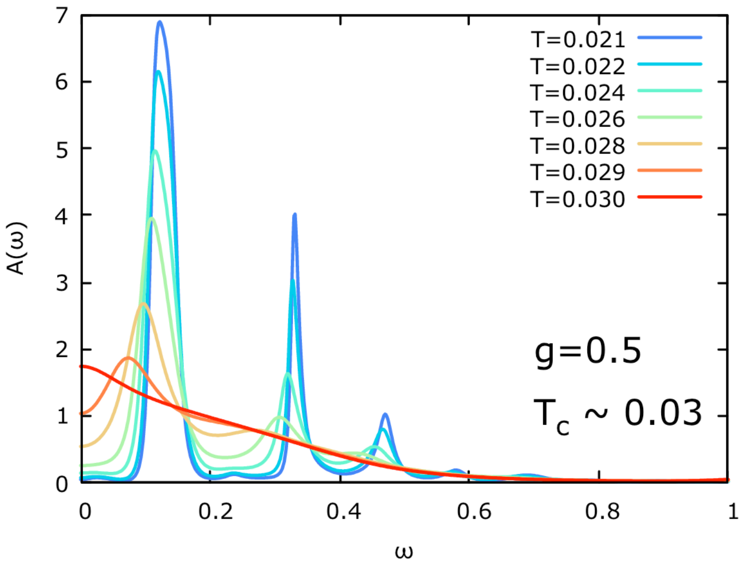
$$G = \langle \psi^{\dagger} \psi \rangle$$
$$F = \langle \psi \psi \rangle$$
$$D = \langle \phi \phi \rangle$$

$$\Delta_{\psi} = 0.42037...$$

Esterlis, Schmalian

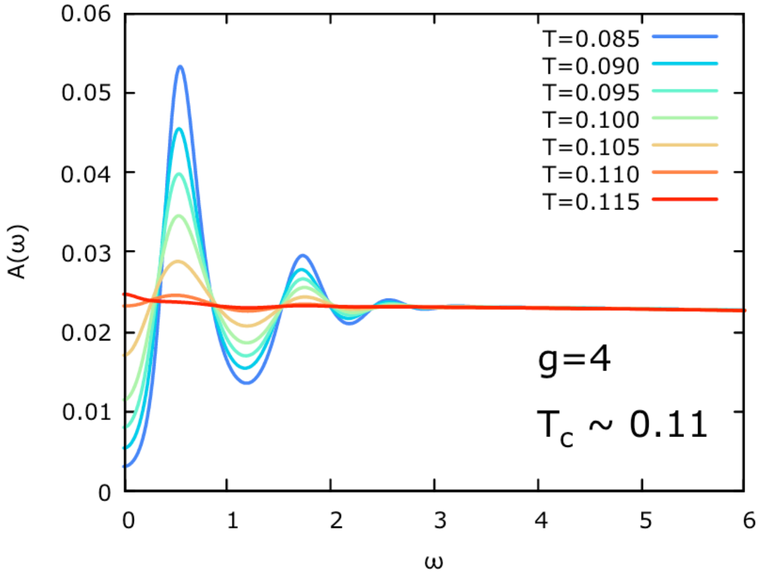
$$A(\omega) = -\frac{1}{\pi}\mathrm{Im}G^R(\omega)$$

BCS-superconductivity



gap opens and closes with T

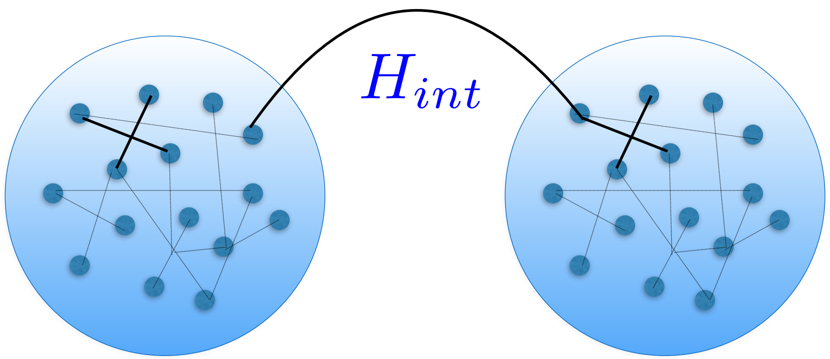
BEC-superconductivity



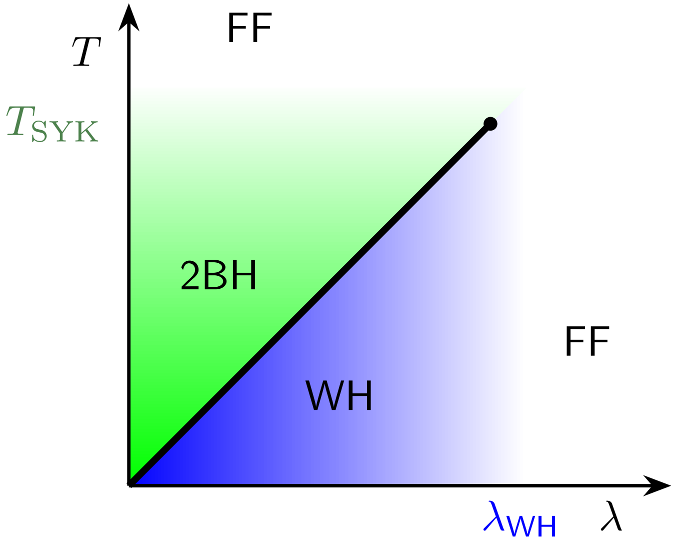
gap fills and empties with T

Shankar, Plugge, Steenbergen, Schalm

- Tunneling coupling two thermal Yukawa-SYK quantum dots



$$H_{\text{int}} = \lambda(c_i^\dagger \psi_i + \psi_i^\dagger c_i)$$



- Numerical solution of Schwinger-Dyson equations

$$\Sigma_{ab}(\tau,\tau') = \kappa g^2 D_{ab}(\tau,\tau') G_{ab}(\tau,\tau')\,,$$
$$\Phi_{ab}(\tau,\tau') = -(1-\alpha)\kappa g^2 F_{ab}(\tau,\tau') D_{ab}(\tau,\tau')\,,$$
$$\Pi_{ab}(\tau,\tau') = -2g^2\left[G_{ab}(\tau,\tau')G_{ba}(\tau',\tau) - (1-\alpha)F_{ab}(\tau,\tau')\bar{F}_{ba}(\tau',\tau)\right]\,.$$

$$\hat{G}(i\omega_n) = \begin{pmatrix} i\omega_n - \Sigma_{11} & -\Phi_{11} & -\lambda - \Sigma_{12} & -\Phi_{12} \\ -\bar{\Phi}_{11} & i\omega_n - \tilde{\Sigma}_{11} & -\bar{\Phi}_{12} & \lambda^* - \tilde{\Sigma}_{12} \\ -\lambda^* - \Sigma_{21} & -\Phi_{21} & i\omega_n - \Sigma_{22} & -\Phi_{22} \\ -\bar{\Phi}_{21} & \lambda - \tilde{\Sigma}_{21} & -\bar{\Phi}_{22} & i\omega_n - \tilde{\Sigma}_{22} \end{pmatrix}^{-\top}$$
$$\hat{D}(i\nu_n) = \begin{pmatrix} \nu_n^2 + \omega_0^2 - \Pi_{11}(i\nu_n) & \Pi_{12}(i\nu_n) \\ \Pi_{21}(i\nu_n) & \nu_n^2 + \omega_0^2 - \Pi_{22}(i\nu_n) \end{pmatrix}^{-\top}$$

$$\text{Nambu basis: } \hat{G} = \begin{pmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \mathcal{G}_{21} & \mathcal{G}_{22} \end{pmatrix} \text{ with each } \mathcal{G}_{ab} = \begin{pmatrix} G_{ab} & F_{ab} \\ \bar{F}_{ab} & \tilde{G}_{ab} \end{pmatrix},$$

Shankar, Plugge, Steenbergen, Schalm

metallic YSYK
(no TRS)

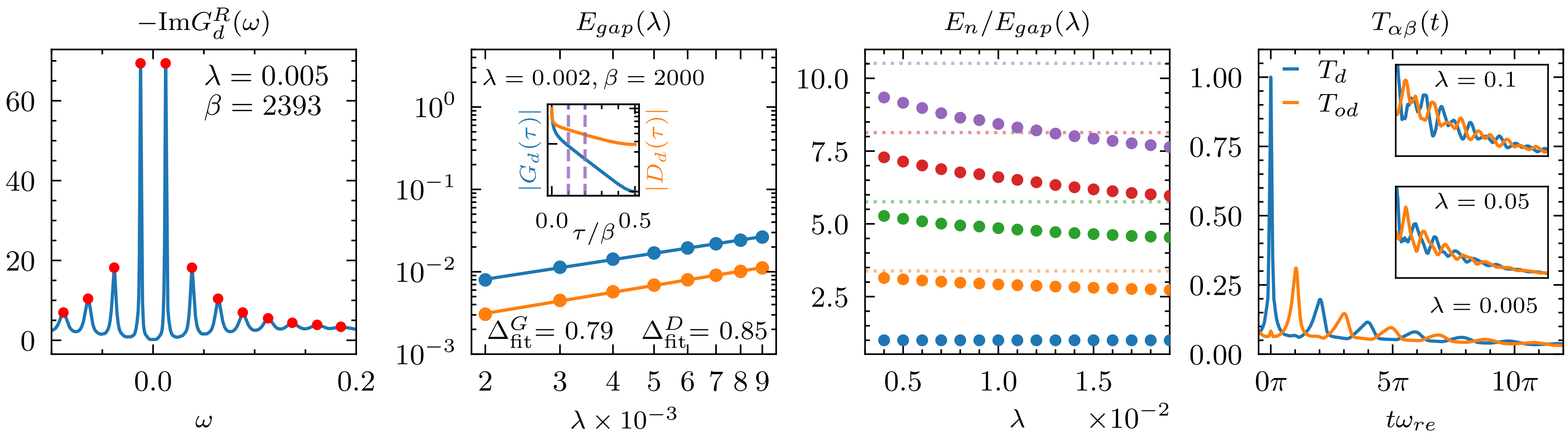
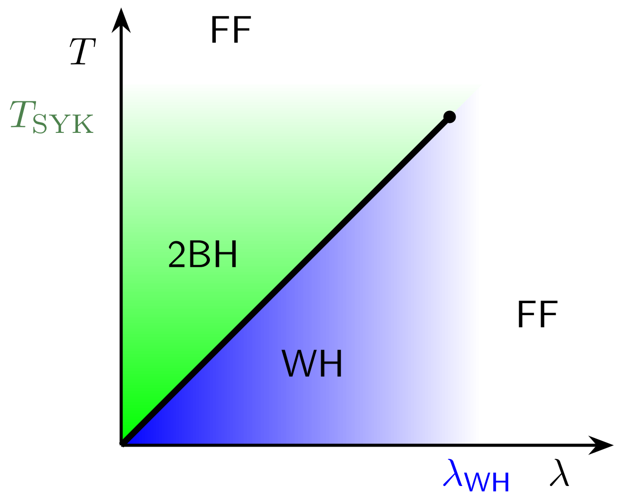
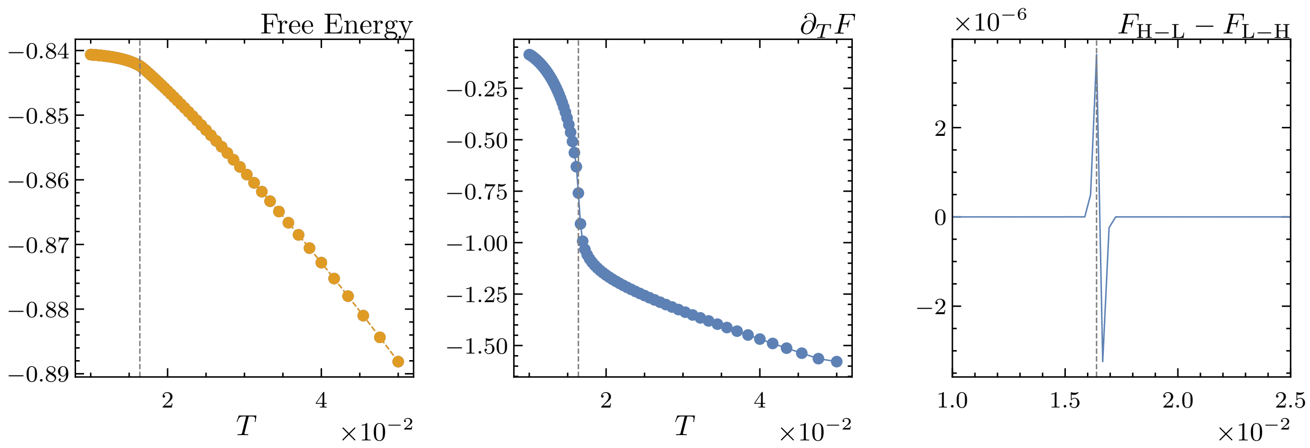


FIG. 2. TFD/wormhole state signatures in two tunnel-coupled metallic YSYK models with $g/\omega_0^{3/2} = 0.5$. **A:** The density of states exhibits the regular linear spacing characteristic of the TFD/wormhole. **B:** The gap in both the fermionic and bosonic Green's function shows the the expected $\lambda^{\frac{1}{2-2\Delta_f}}$ scaling with $1/(2-2\Delta_f) = 0.86$. The inset shows the exponential behavior of the Green's functions and the purple dashed lines indicate the region where the fit was performed. **C:** As λ decreases, the position of the leading and subleading peaks approach the analytical prediction $E_n/E_{\mathrm{gap}} = (1 + \frac{1}{\Delta}n)$ indicated by the dotted lines. **D:** Revival oscillations of the transmission amplitude $T_{\alpha\beta}(t) = \frac{2}{\pi}|G^>_{\alpha\beta}(t)|$, with $iG^>_{\alpha\beta}(\omega) = -[1 - n_F(\omega)]\mathrm{Im}G_{\alpha\beta}(\omega)$. T_d and T_{od} are perfectly out of phase. The characteristic frequency is $\omega_{re} = \frac{p_1}{2\pi}$, where p_1 is the average spacing between the peaks in the spectral function. In the gravitational description, this represents a particle traversing/reemerging from the wormhole.

metallic YSYK
(no TRS)



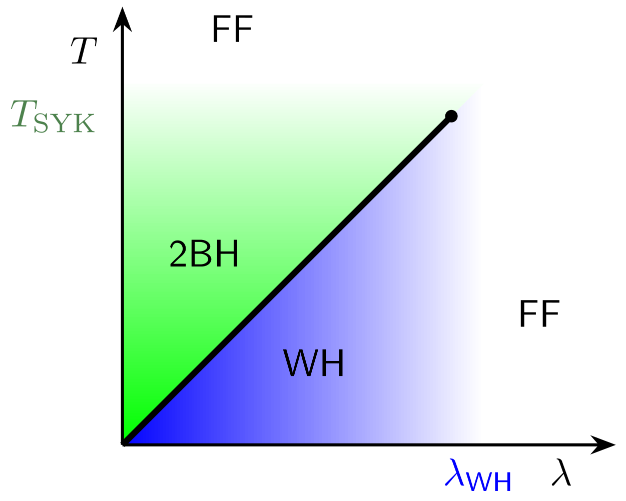


Does the "wormhole" survive (is it detectable) in the superconducting groundstate?

Shankar, Plugge, Steenbergen, Schalm

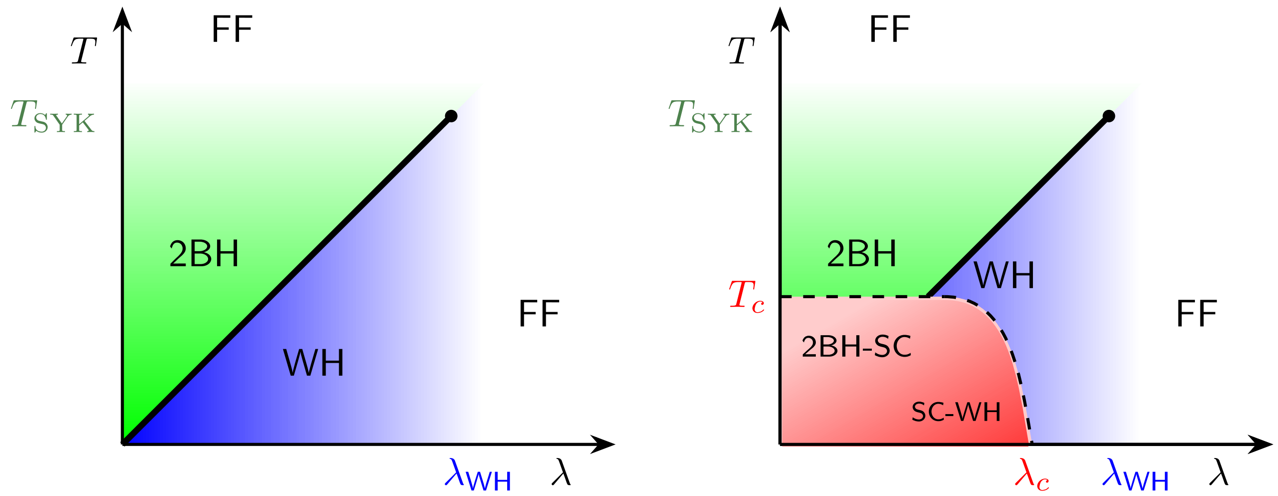
metallic YSYK
(no TRS)

superconducting YSYK
(TRS)



metallic YSYK
(no TRS)

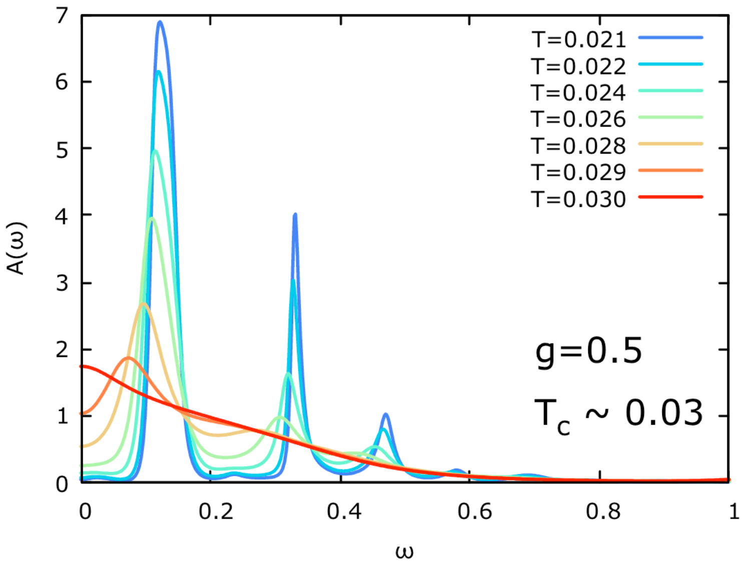
superconducting YSYK
(TRS)



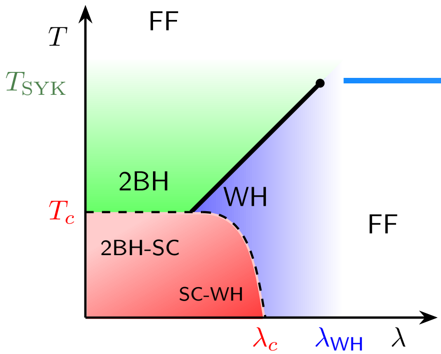
Esterlis, Schmalian

$$A(\omega) = -\frac{1}{\pi}\mathrm{Im}G^R(\omega)$$

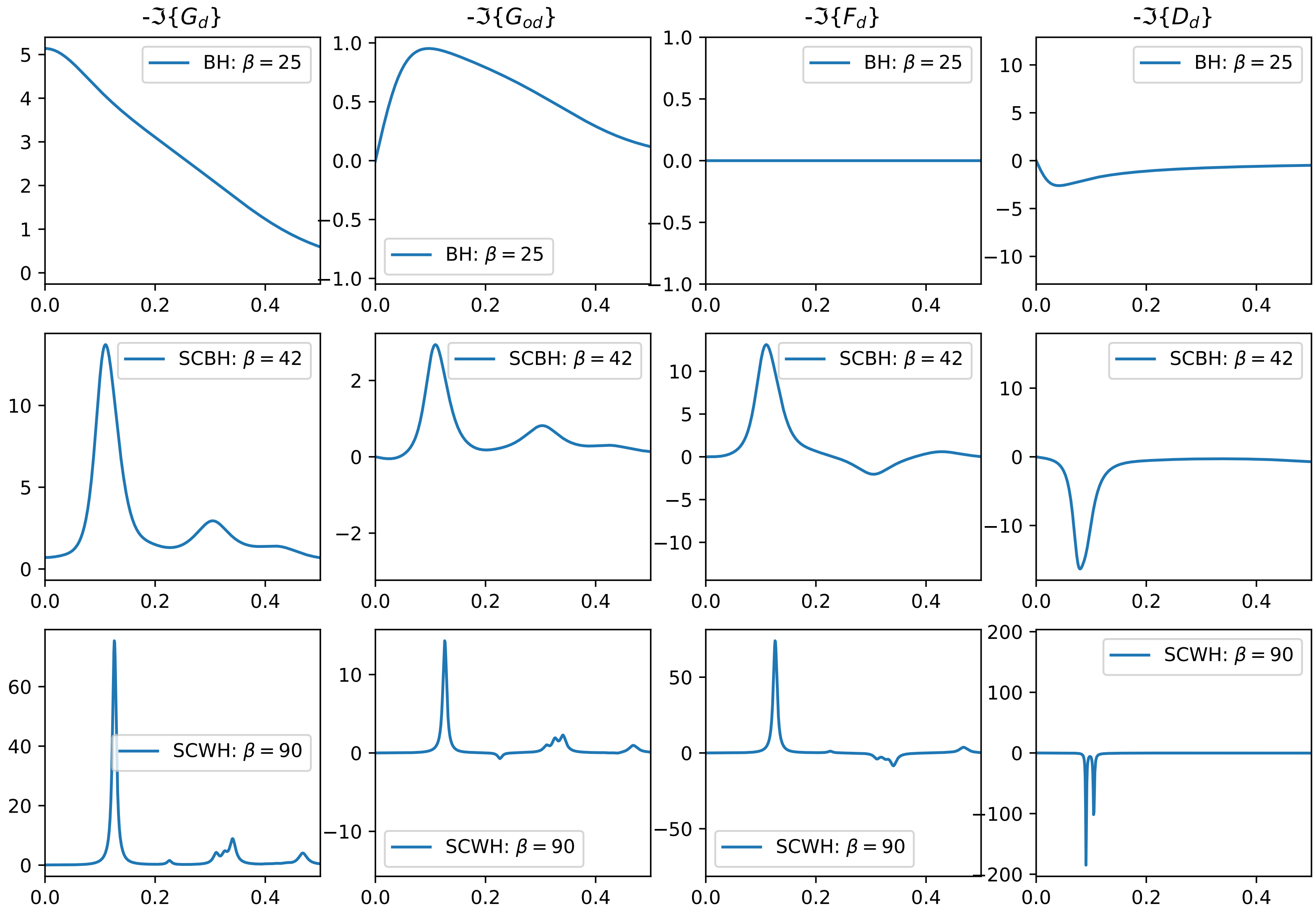
BCS-superconductivity

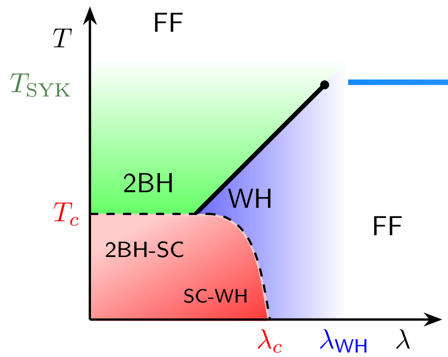


gap opens and closes with T

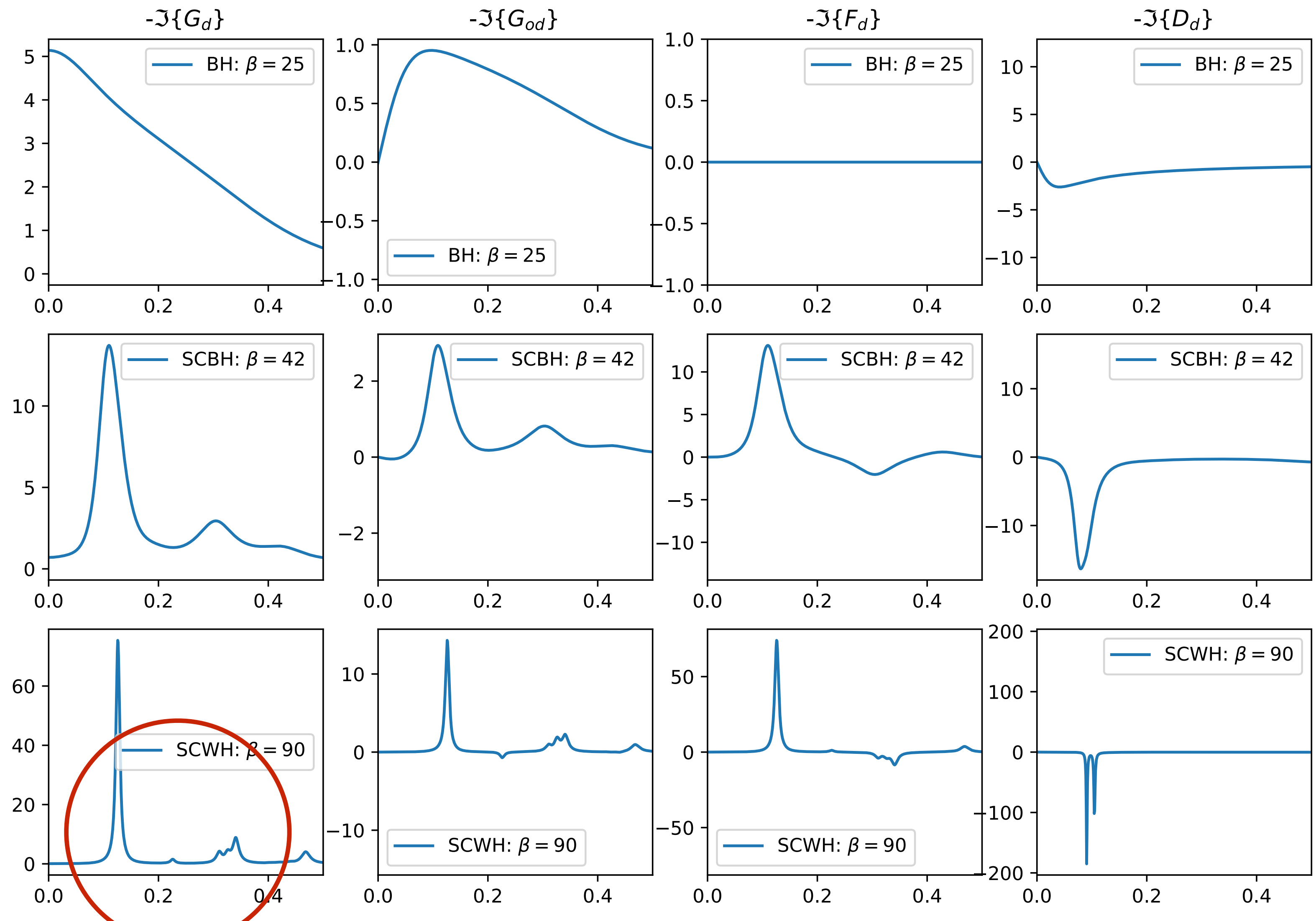


SCWH solution: different phases across temperature (J = 0)



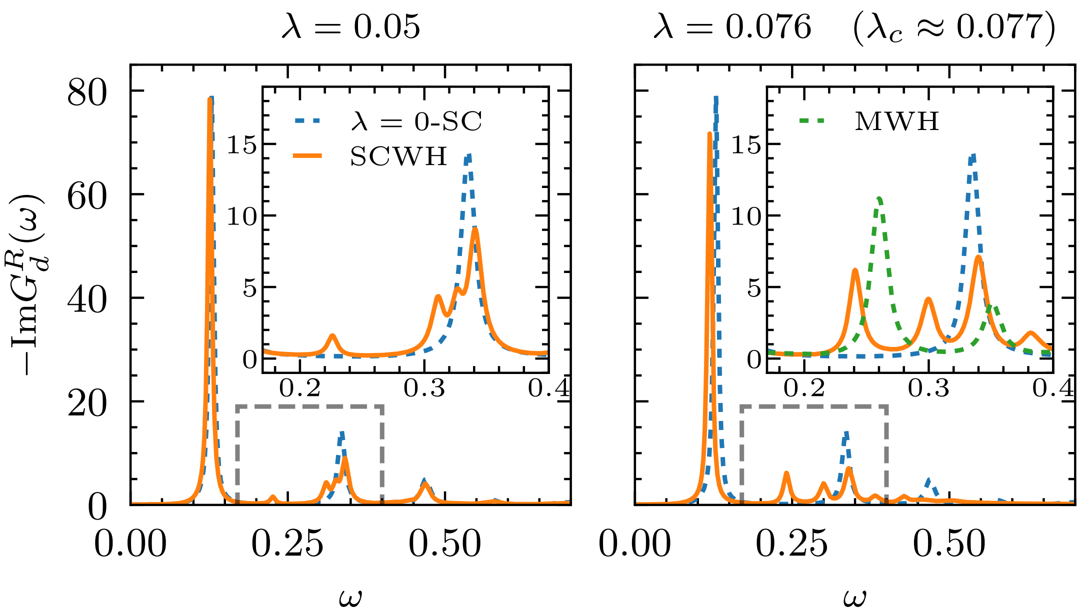


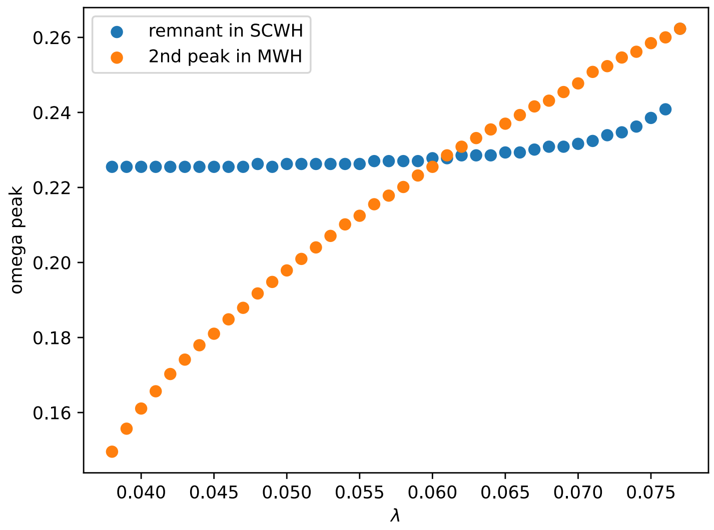
SCWH solution: different phases across temperature (J = 0)



Signatures of a TFD/wormhole in the YSYK superconducting groundstate

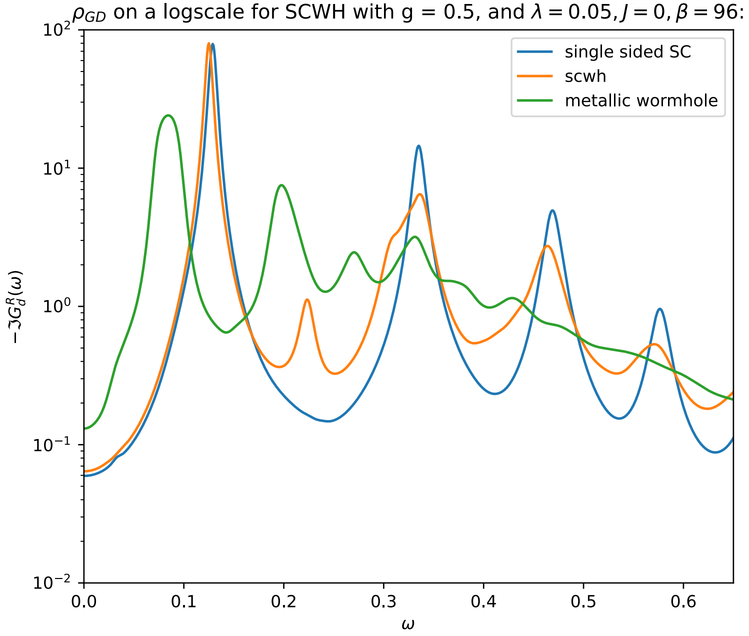
In a small window near λ_c does the new peak behave like the 1st excited WH peak

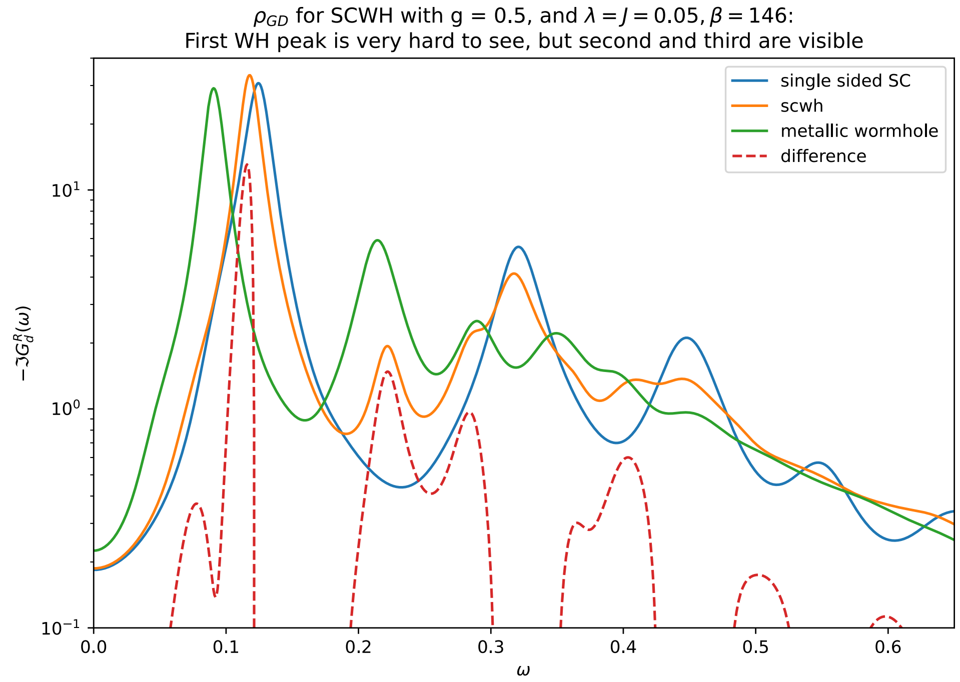




Signatures of a TFD/wormhole in the YSYK superconducting groundstate

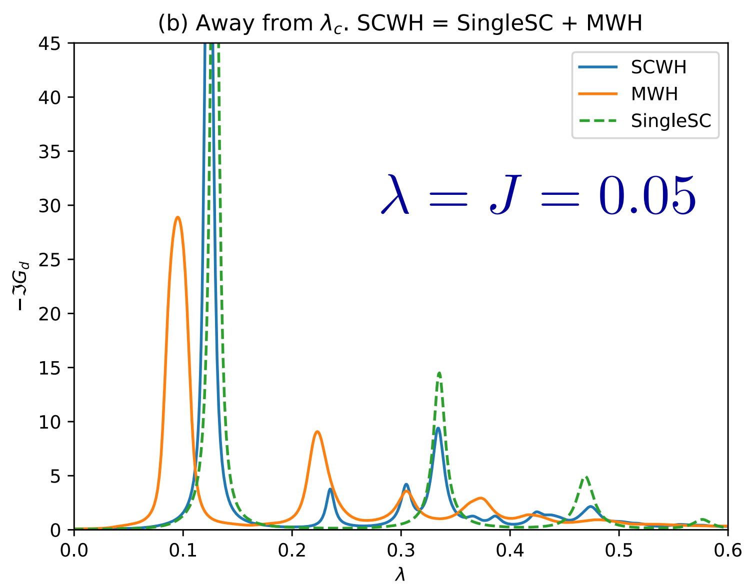
Including a boson tunneling interaction: $\mathcal{L} = \int d\tau \ \ldots + J\phi_L^a \phi_{aR}$

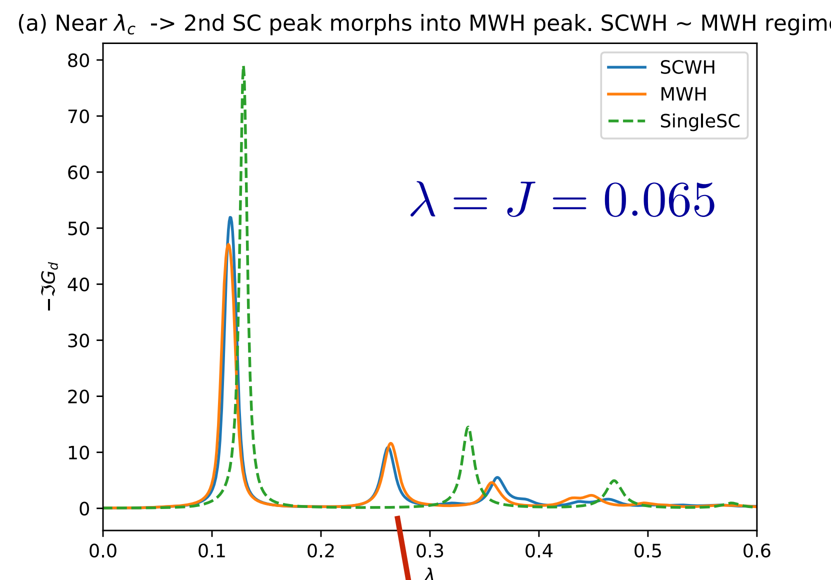


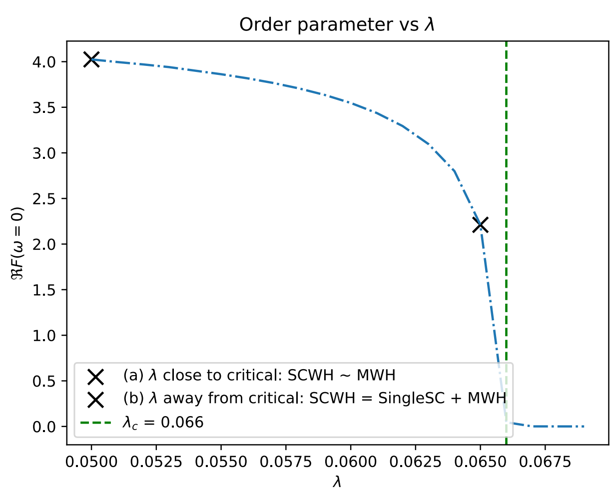


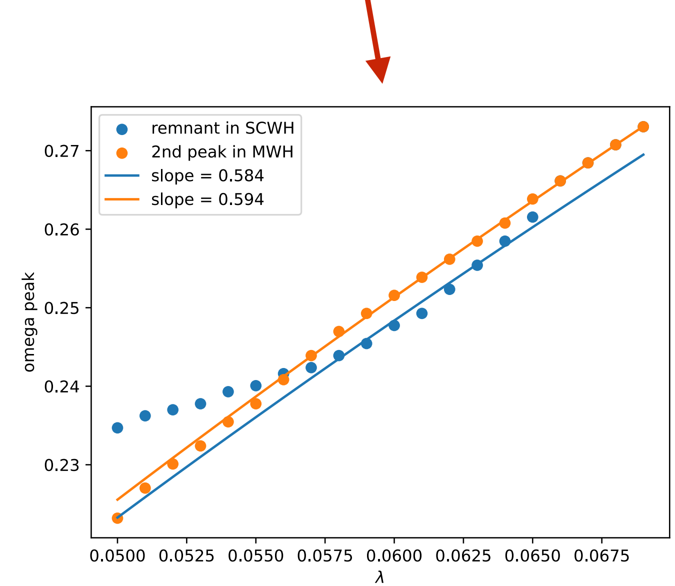
Signatures of a TFD/wormhole in the YSYK superconducting groundstate

Including a boson tunneling interaction: $\mathcal{L} = \int d\tau \ \ldots + J\phi_L^a \phi_{aR}$



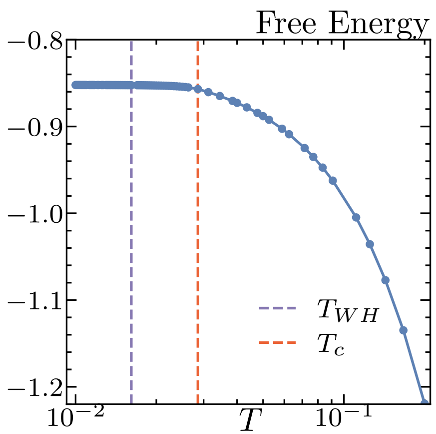


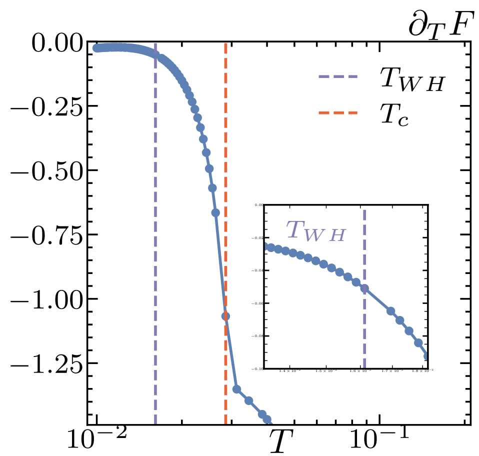


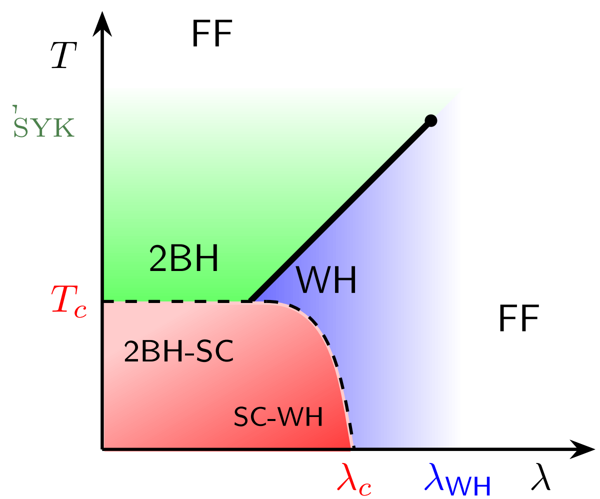


- The SC-wormhole crosses over to the 2BH-SC state .

superconducting YSYK
(TRS)

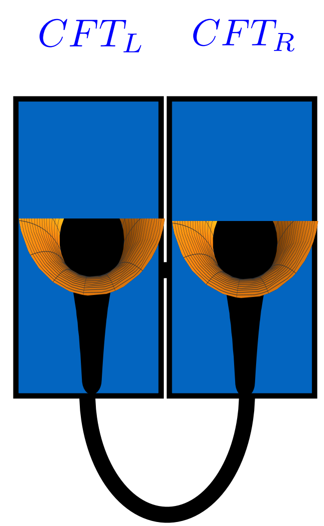




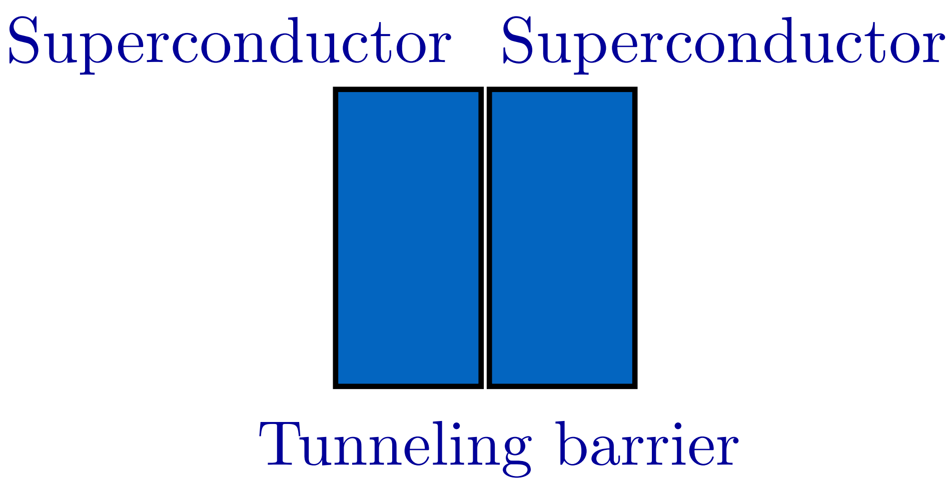


Free energy is continuous between 2BH-SC and SC-WH

- Hybrid combination of superconductor/wormhole state exists

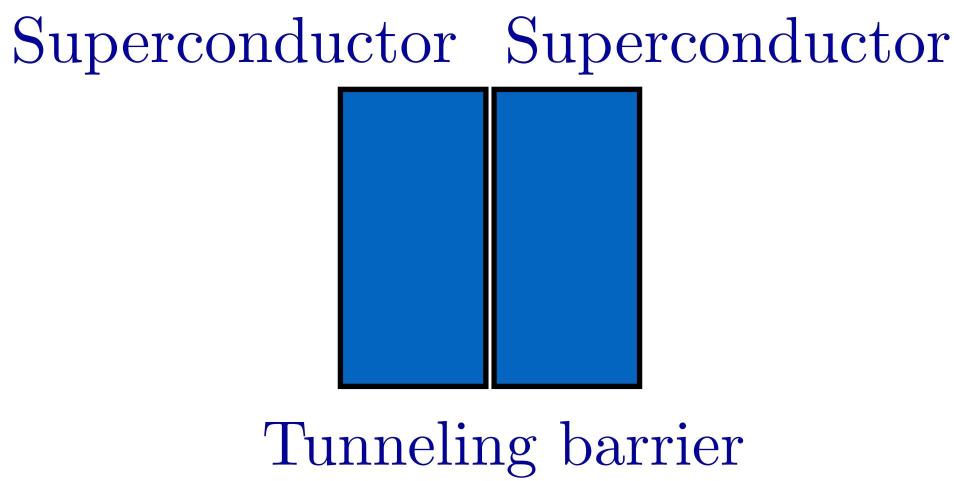


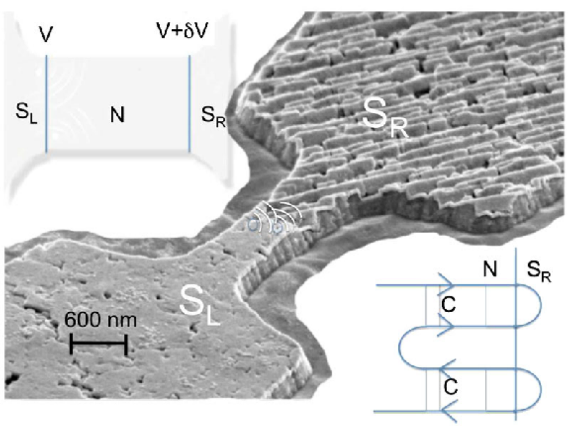
- Hybrid combination of superconductor/wormhole state exists



- Hybrid combination of superconductor/wormhole state exists

Josephson junction





Lucignano et al PRL 105 147001

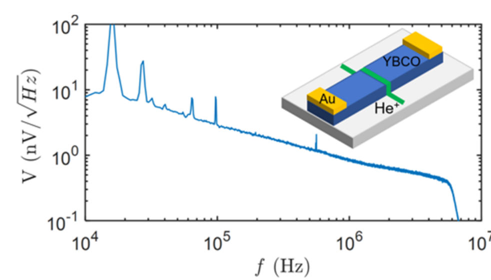
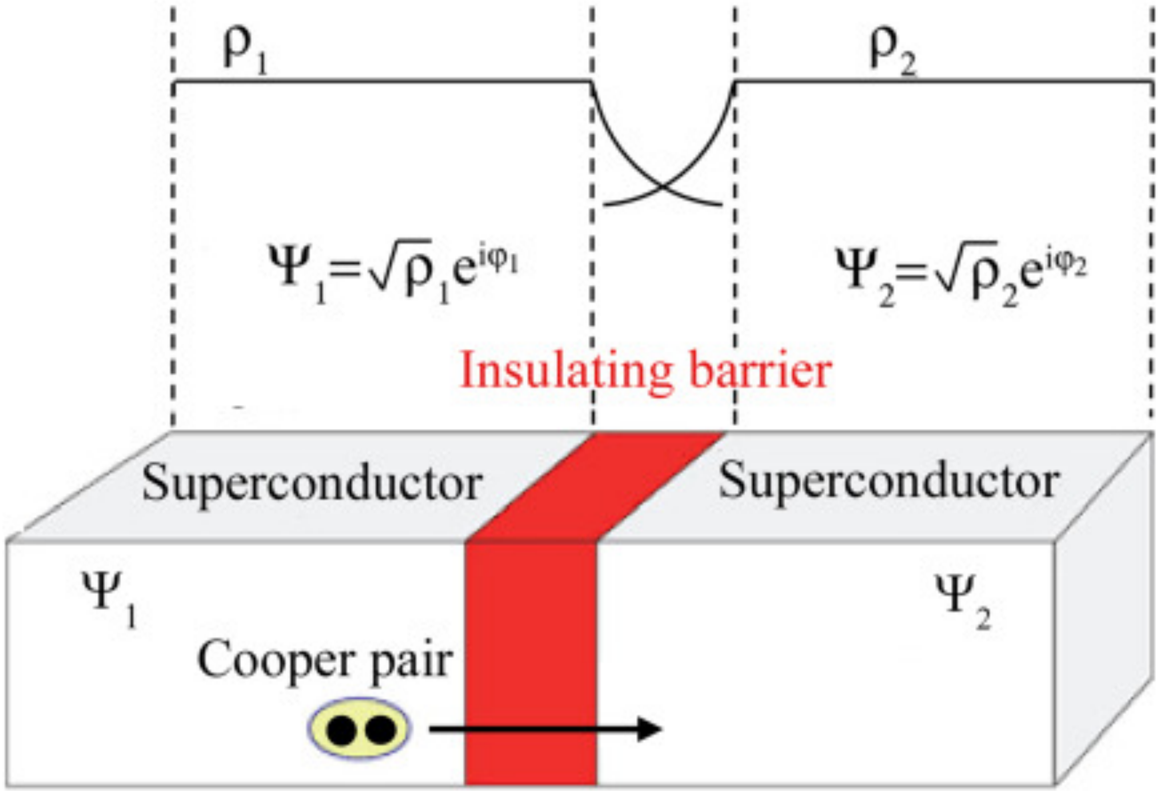


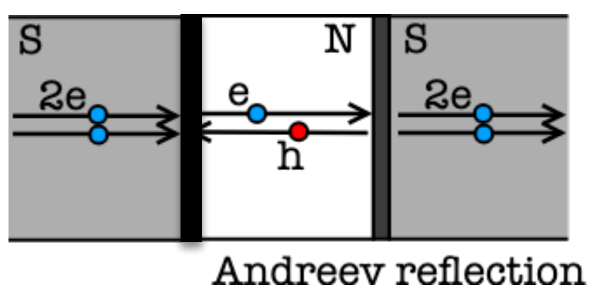
Fig. S8: Shot noise in a Josephson junction made of the high-T_c superconductor $YBa_2Cu_3O_7$ (YBCO) fabricated using a focused ion (He^+) beam. The spectrum is dominated by $1/f$ noise and accurate shot noise cannot be calculated. Inset: schematic of the sample structure.

Niu,….,Allan: arXiv:2306.02397

- Josephson physics (Cooper pair tunneling = Andreev reflection)



Josephson current



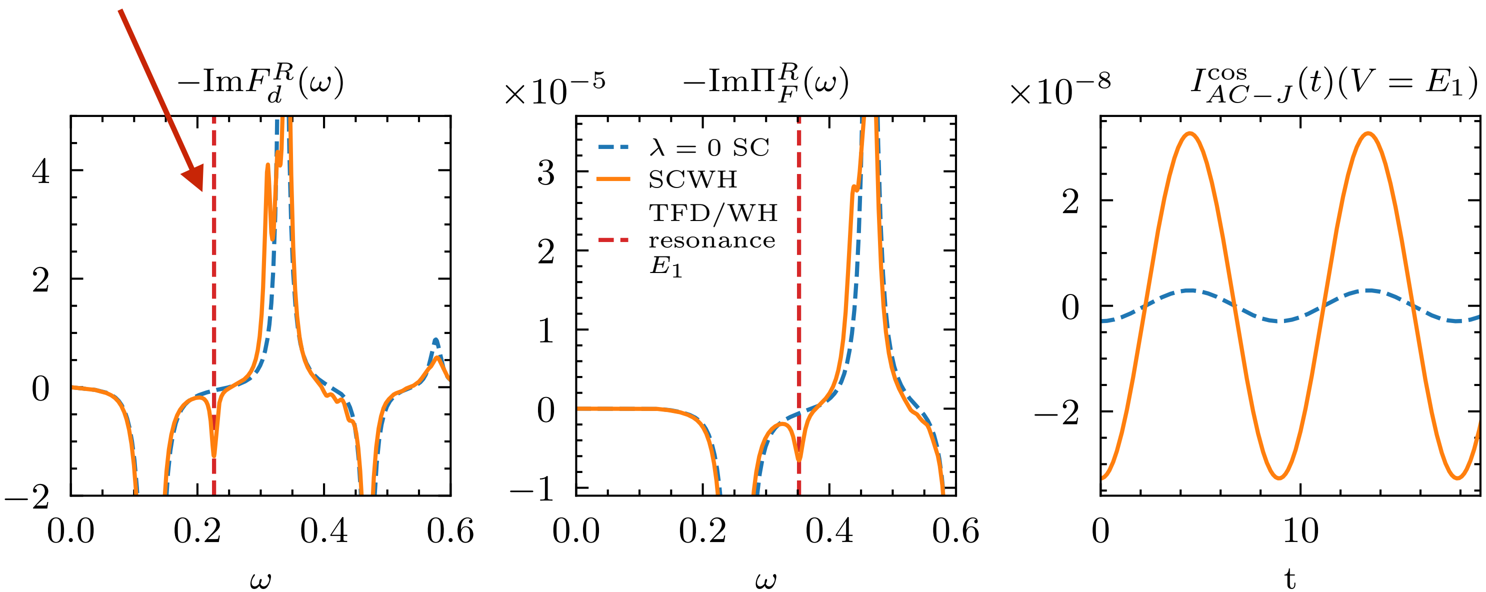
$$G(t,t') = \langle \psi^\dagger(t)\psi(t) \rangle$$
$$F(t,t') = \langle \psi(t)\psi(t') \rangle$$

- The AC Josephson effect can detect these resonances

$$I_{\text{AC-}J}(t) = 2\lambda^2 \left[\mathrm{Re}\Pi_F^R(V)\sin(2Vt) + \mathrm{Im}\Pi_F^R(V)\cos(2Vt)\right]$$

$$\Pi_F(\nu_n) = T\sum_n \bar{F}(\omega_m)F(\omega_m - \nu_n)$$

Andreev revival



Conclusions and Outlook

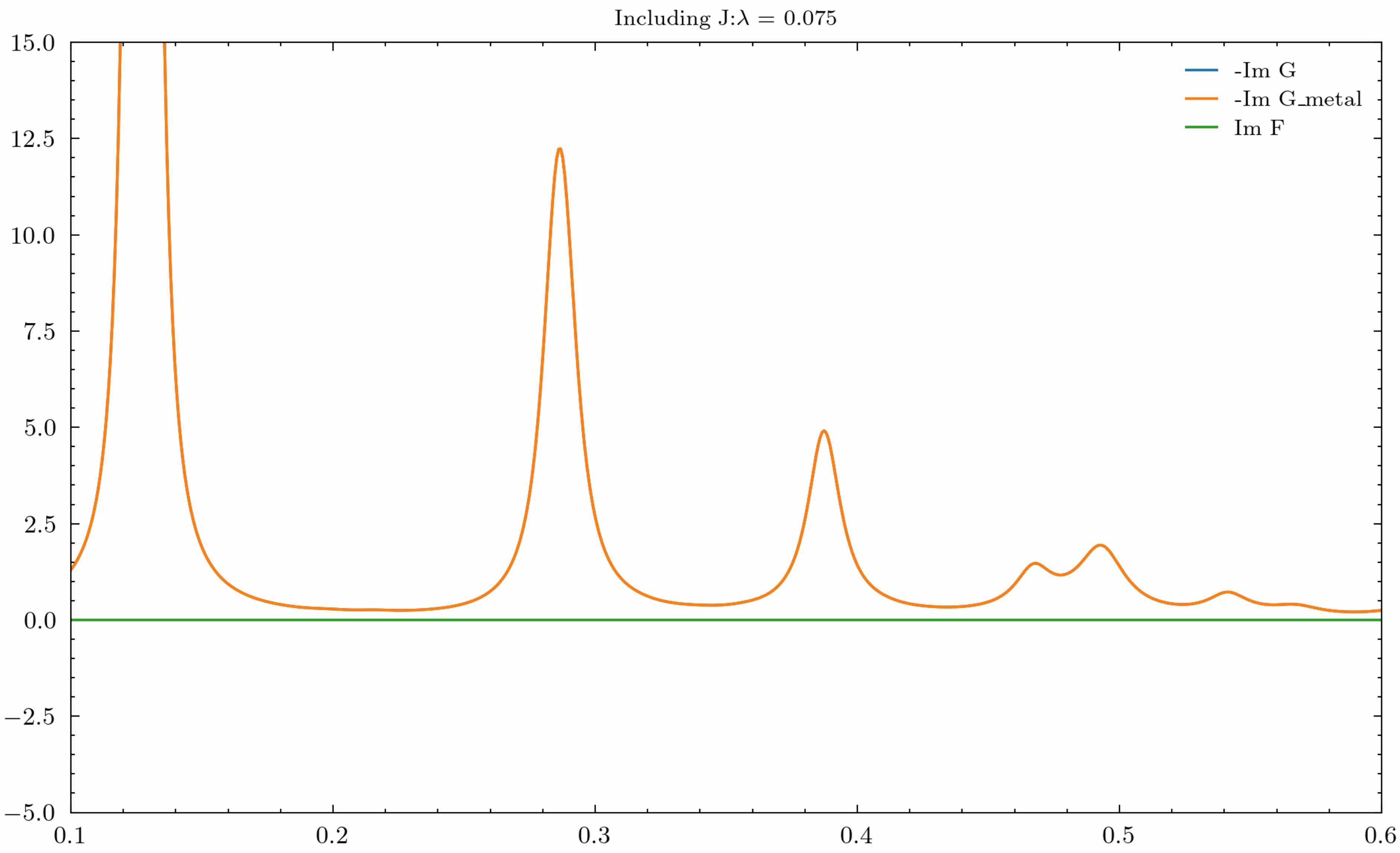
- Despite instability towards superconductivity, detectable signatures of a quantum critical wormhole survive in two coupled Yukawa-SYK quantum dots.

Semi-local quantum liquid/intermediate scale fixed point/ AdS_2 holographic superconductor

- The extra wormhole resonances (Andreev revivals) are detectable in the AC-Josephson junction.

- This *can be expected* to survive in two coupled Yukawa-SYK 2D lattices (universal theory of a strange metal). 2D quantum critical systems are special. To be confirmed…

- Provides an outlook to realize a strange-metal Josephson experiment whose physics is holographically dual to a semi-macroscopic wormhole.


Including J:λ = 0.075
-Im G
-Im G_metal
Im F
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
0.1
0.2
0.3
0.4
0.5
0.6