Black hole mergers in AdS

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Schwarzschild AdS_4

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2$$ with lapse function ($G_N = 1$): $$f(r) = 1 - \frac{2m}{r} + \left(\frac{r}{\ell}\right)^2$$

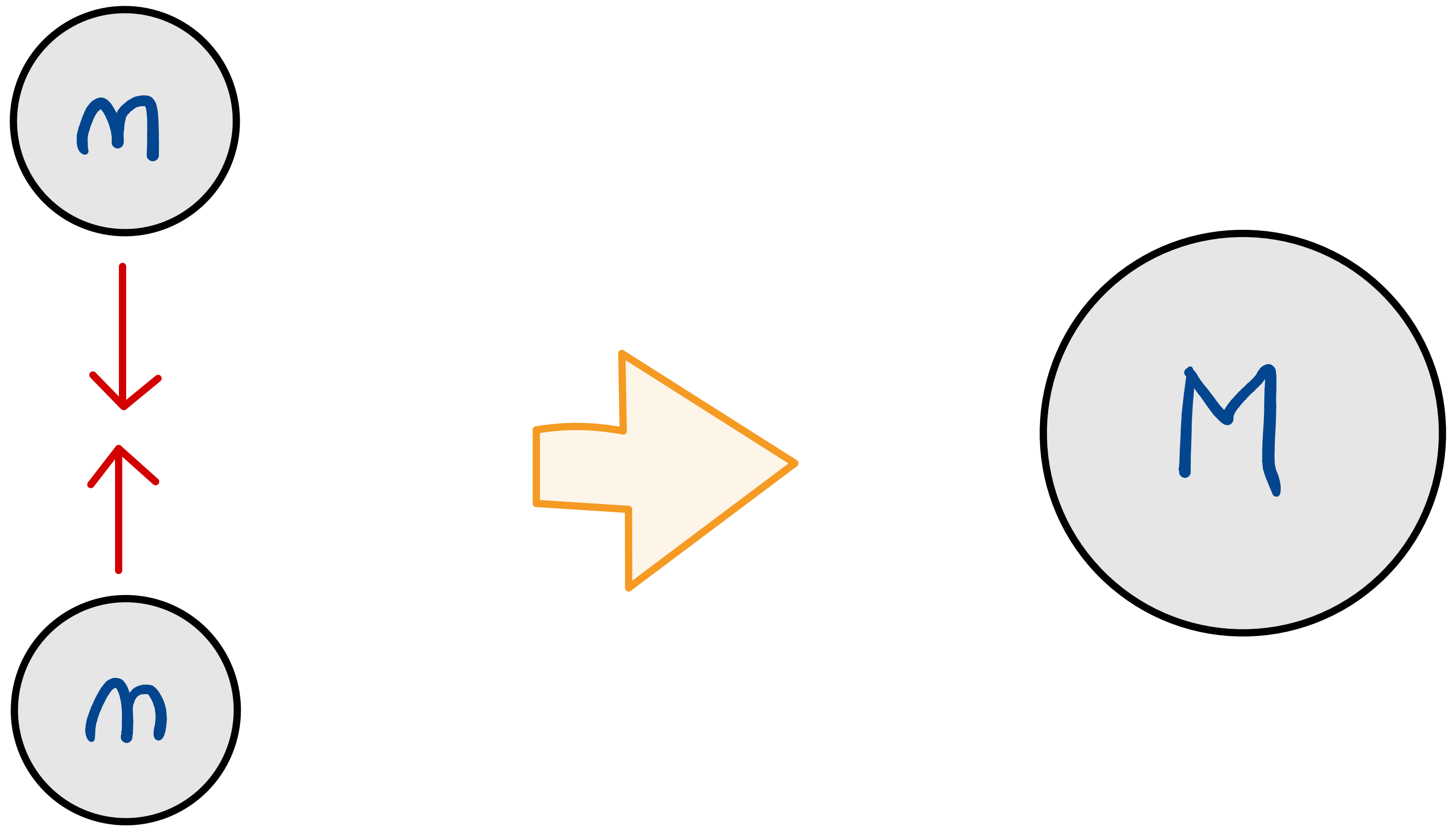
Entropy $$S = \pi r_{\mathrm{H}}^2$$

Temperature $$T = \frac{1}{4\pi}\left(\frac{1}{r_{\mathrm{H}}} + \frac{3r_{\mathrm{H}}}{\ell^2}\right)$$

Large black holes are thermodynamically stable for

$r_{\mathrm{H}} \geq \ell$ and $T \geq \dfrac{1}{\ell\pi}$

Head-on collision in AdS



Rényi entropy in the canonical ensemble

For a thermal density matrix $\rho = \dfrac{e^{-\beta H}}{Z^{\mathrm{c}}(\beta)}$

$$F(\beta) \equiv -\frac{1}{\beta} \log Z^{\mathrm{c}}(\beta)$$

in the canonical ensemble $S_n(\beta) \equiv \dfrac{1}{1-n} \log \mathrm{Tr}\rho^n = \dfrac{n\beta}{1-n}\Big(F(\beta) - F(n\beta)\Big) = \dfrac{n\beta}{1-n} \displaystyle\int_{n\beta}^{\beta} \frac{d\tilde{\beta}}{\tilde{\beta}^2} S(\tilde{\beta})$

$$S(\beta) = \beta^2 \frac{\partial F}{\partial \beta}$$

$$\boxed{S_n(\beta) = -\frac{\pi \ell^2}{3}\left\{1 + \frac{2}{9b^2 n^2(1-n)}\left[n^3 - 1 + (1-3b^2)^{3/2} n^3 - (1-3b^2n^2)^{3/2}\right]\right\}}$$ with $b \equiv \dfrac{\beta}{2\pi\ell}$

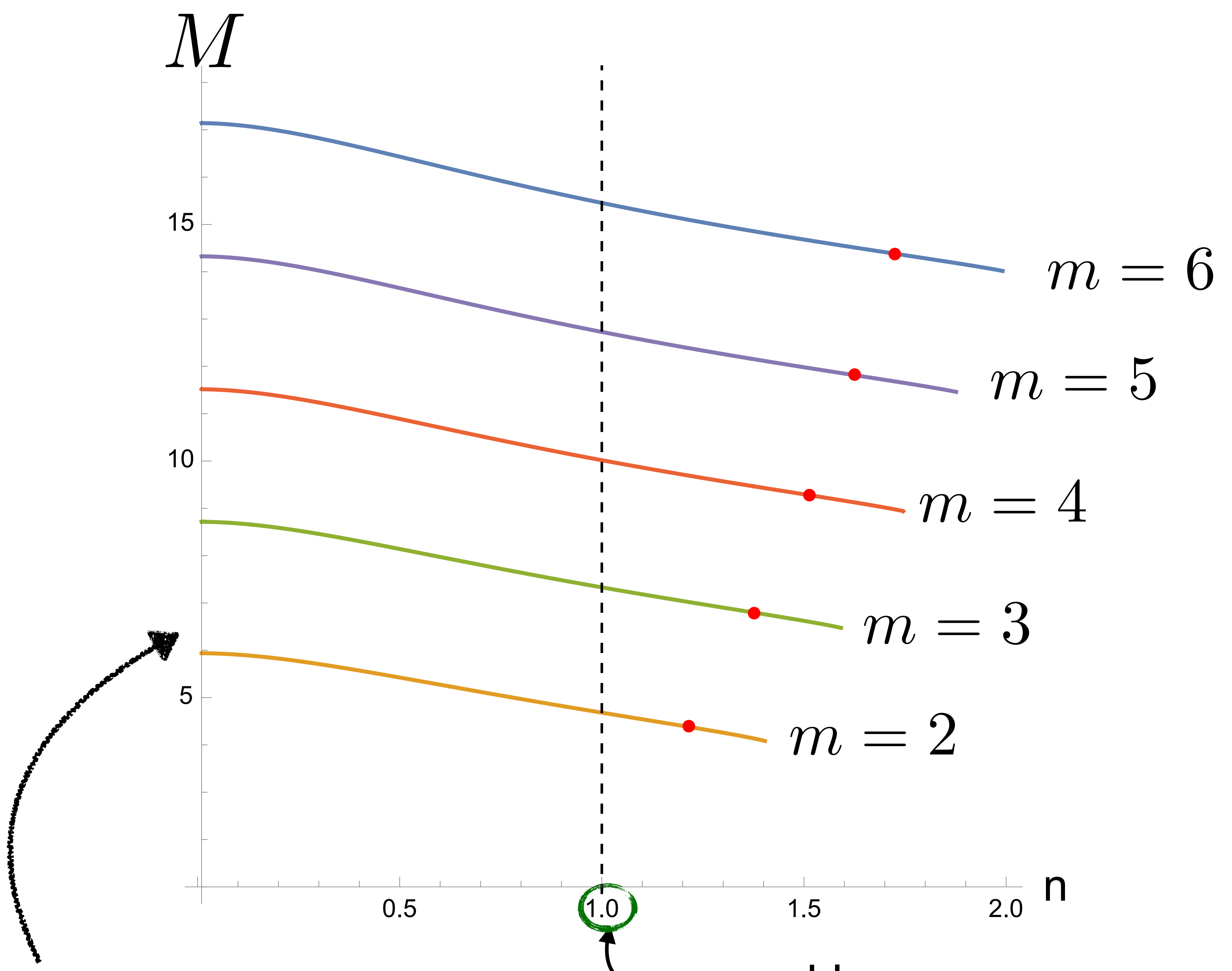
$$\lim_{n\to 1} S_n(\beta) = S(\beta) = \frac{\pi\ell^2}{9b^2}\left(1 + \sqrt{1-3b^2}\right)^2$$

ATT! For a given b, this holds up to $n = \dfrac{1}{2b}$ to remain above the Hawking-Page transition of the replica manifold.

1. Bounds on the final mass

For initial conditions $\rho \approx \rho_1 \otimes \rho_2$ ➡ Rényi laws $S_n(M) \geq 2 S_n(m)$

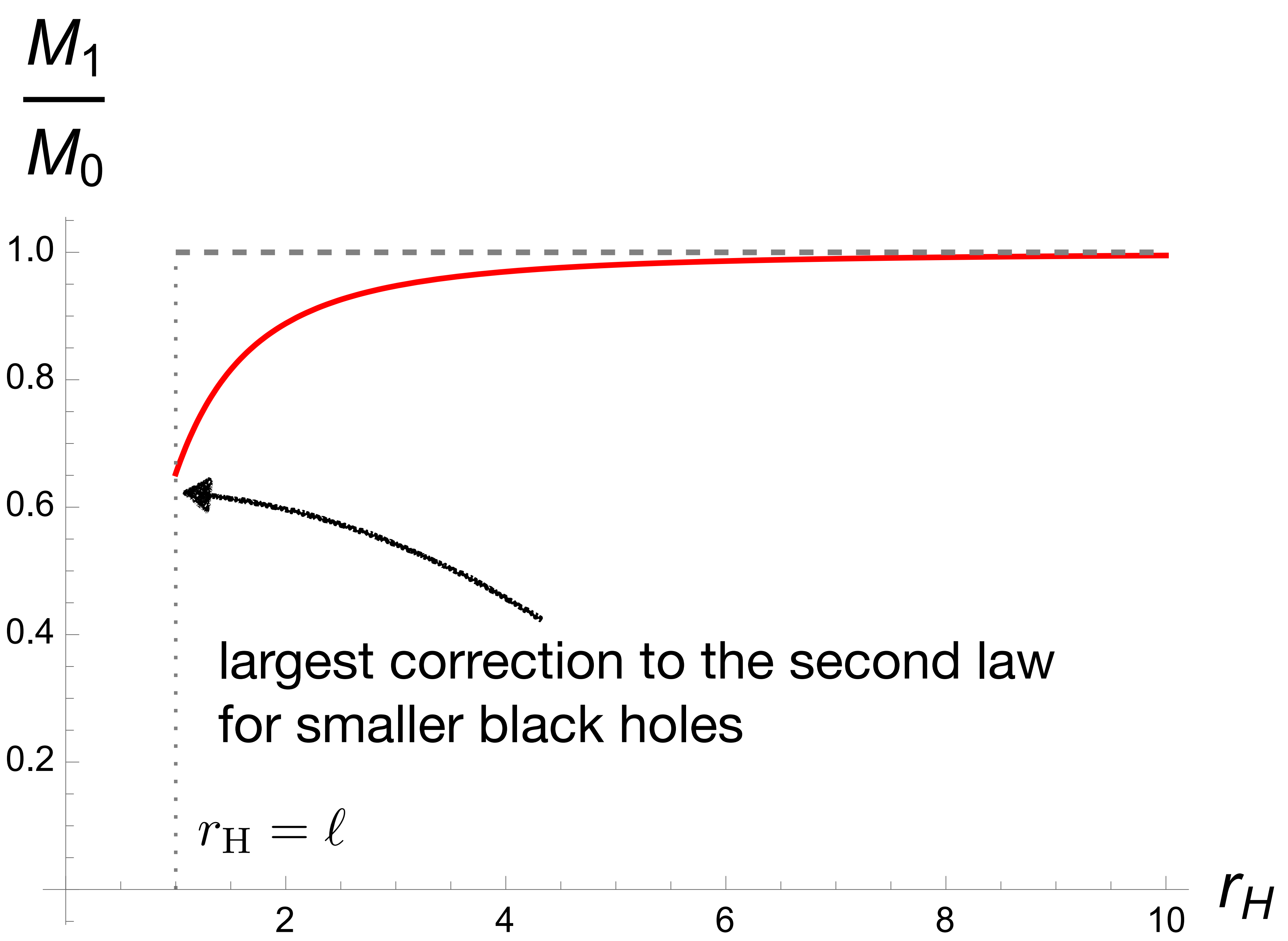
Lower bound on final mass $(\ell = 1)$



stronger bounds
n → 0

second law

Ratio of lower bounds $(\ell = 1)$

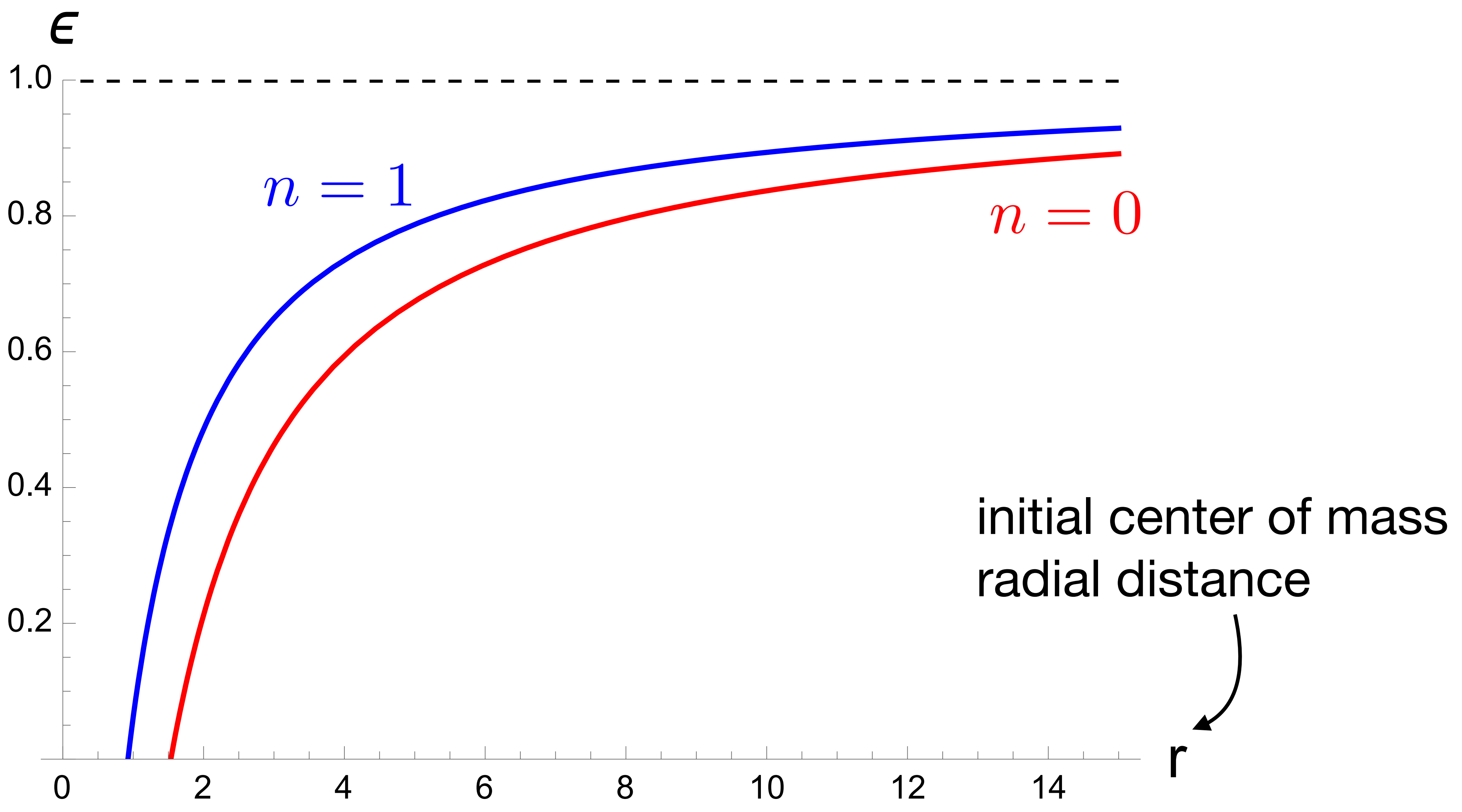


2. Bounds on gravitational radiation

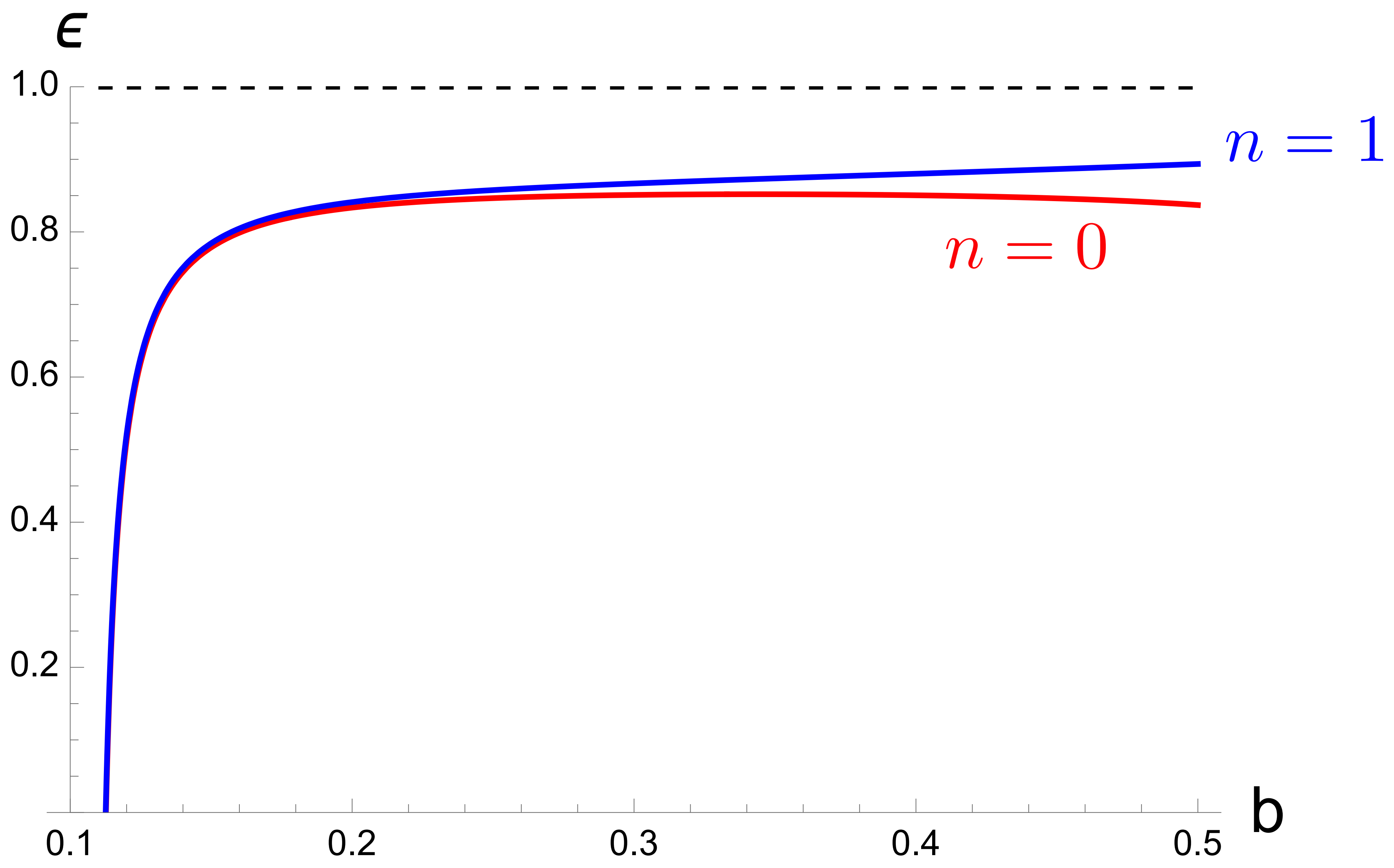
Efficiency with which the total energy E can be converted into gravitational radiation

$$\epsilon = 1 - \frac{M}{E} \qquad \text{with} \qquad E \approx 2U_{\mathrm{AdS}} + U_{\mathrm{Newton}}$$

Efficiency upper bounds



$b = \frac{1}{2}\,, \quad \ell = 1$

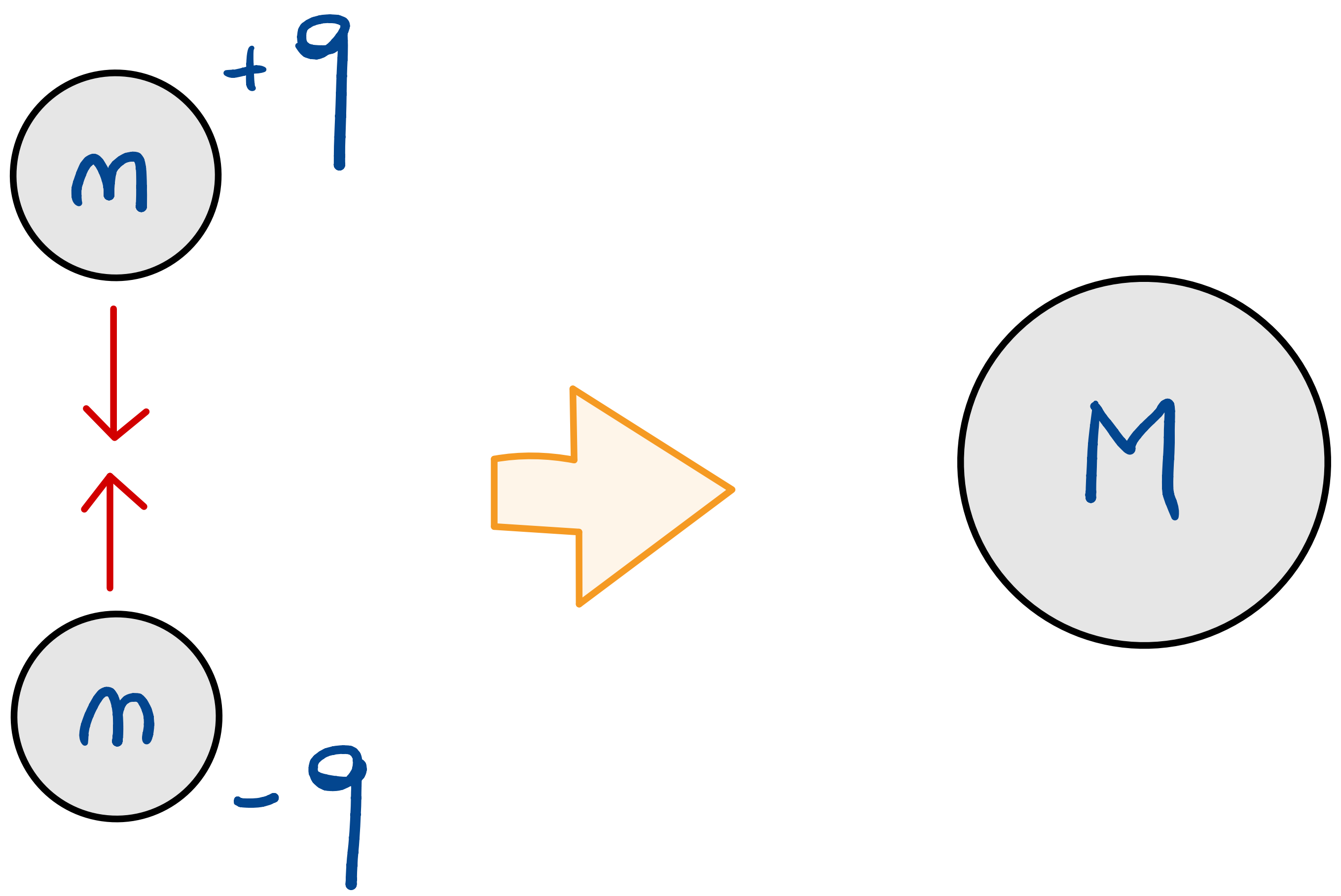


$r = 10\,, \quad \ell = 1$

Extremal supersymmetric mergers in AdS4

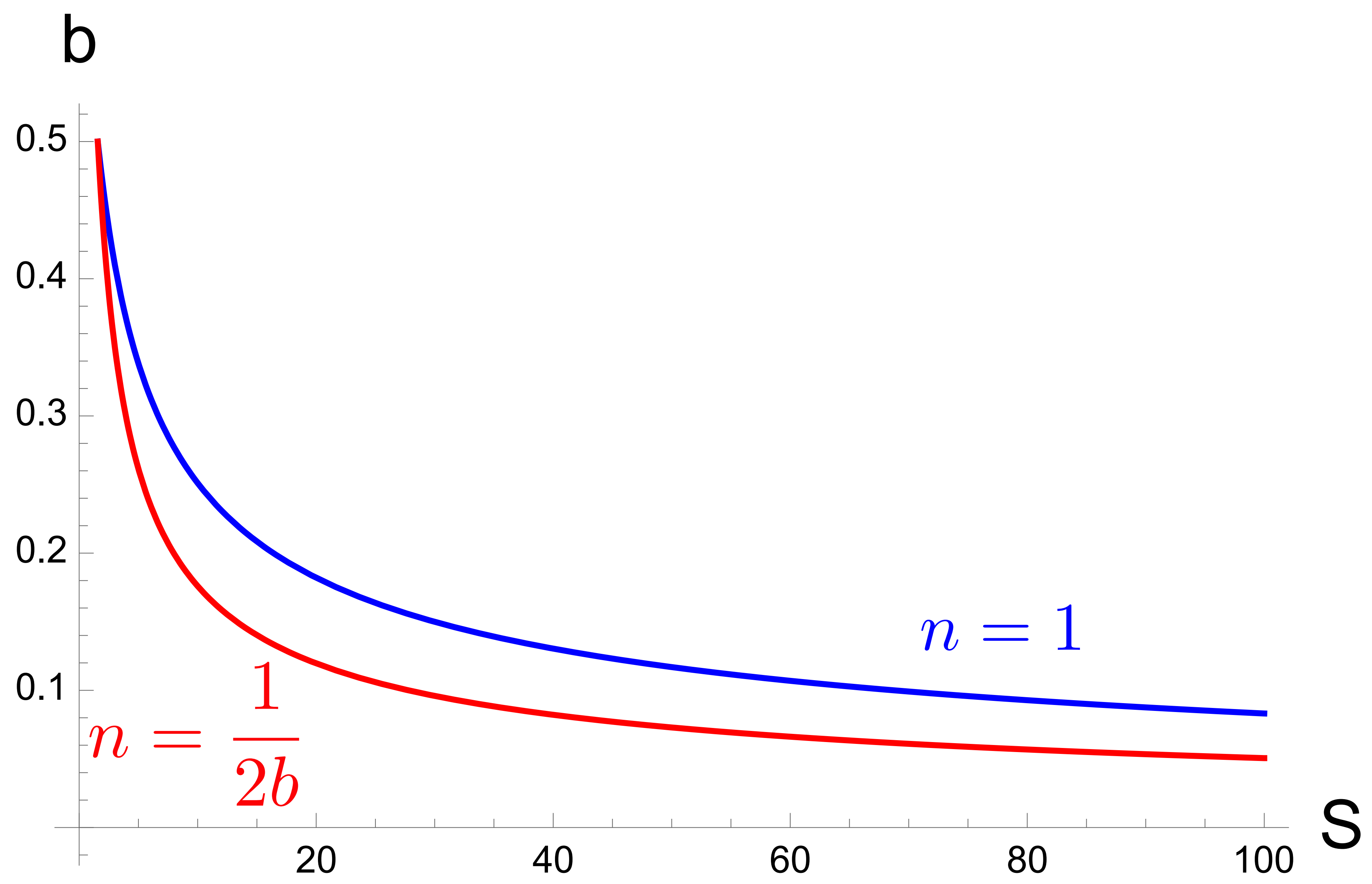
Zero temperature Rényi entropies $S_n(\beta \to \infty) = S(\beta \to \infty)$

Coalescence of oppositely charged/rotating extremal AdS4 black holes



Rényi laws $S_n(\beta) \geq 2S(\beta \to \infty)$ → optimal bound $n = \frac{1}{2b}$

Upper bounds on final
reduced inverse temperature



Near-extremal mergers in AdS$_4$

emergent IR scale

One-loop corrections give: $S^{\text{ne}}(\beta \gg 1, J) = S_{\text{cl}} + a_0 \log S_{\text{cl}} + 4\pi^2 \frac{\beta_q}{\beta} + \frac{3}{2} \log \frac{\beta_q}{\beta} + \mathcal{O}(\beta^{-2})$

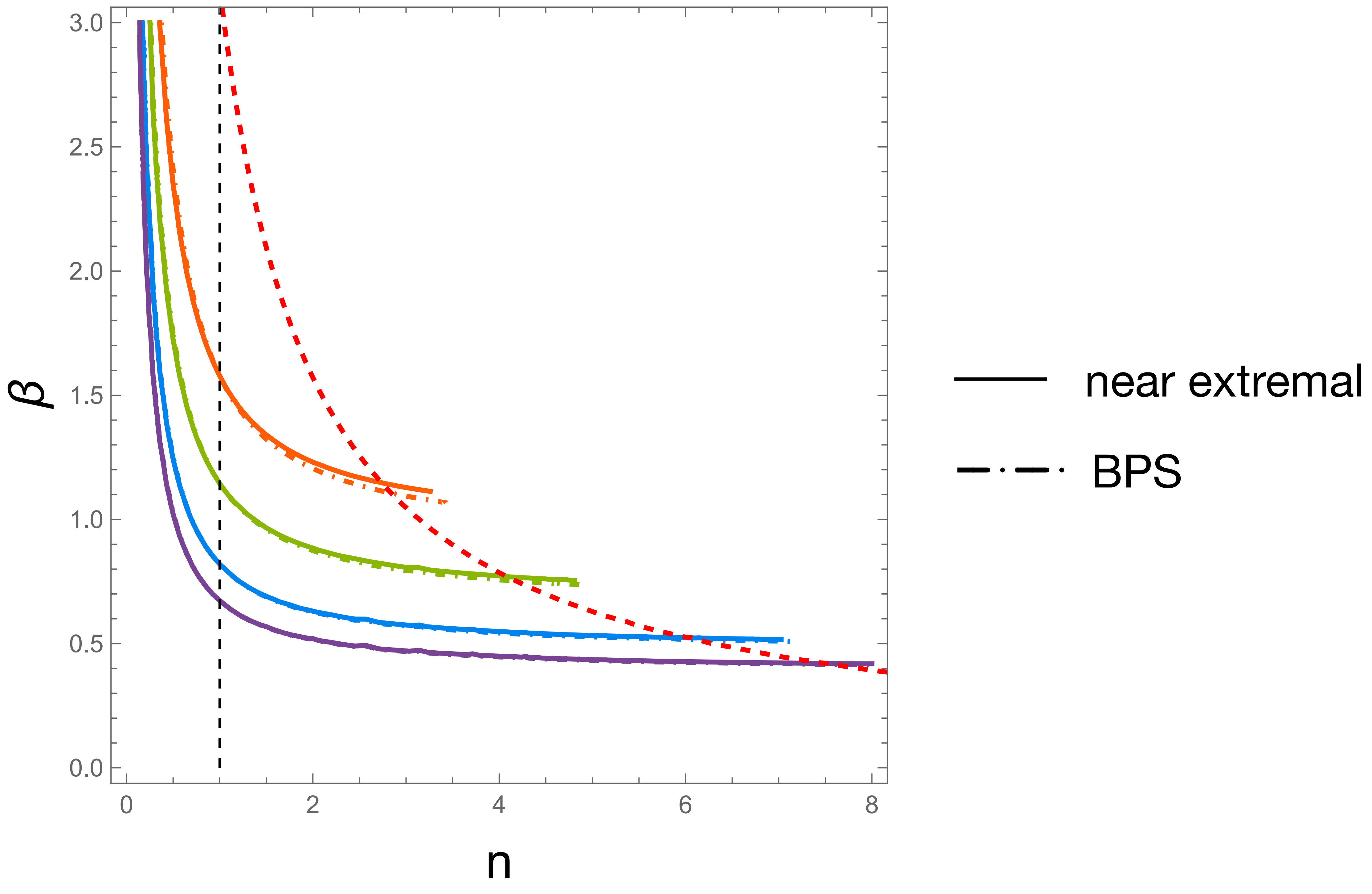
[Rakic, Rangamani, Turiaci 2024]
[Kapec, Sheta, Strominger, Toldo 2024]

$$S_n(\beta \gg 1, J) \simeq S^{\text{ne}}(\beta \gg 1, J) - \frac{3}{2}\left(1 + \frac{\log n}{1-n}\right)$$

Rényi laws $S_n(\beta) \geq 2S_n(\beta \gg 1, J)$ ➡ optimal bound $n = \frac{1}{2b}$

Upper bounds on final inverse temperature

The n-dependence slightly relaxes the constraint as compared to the supersymmetric case

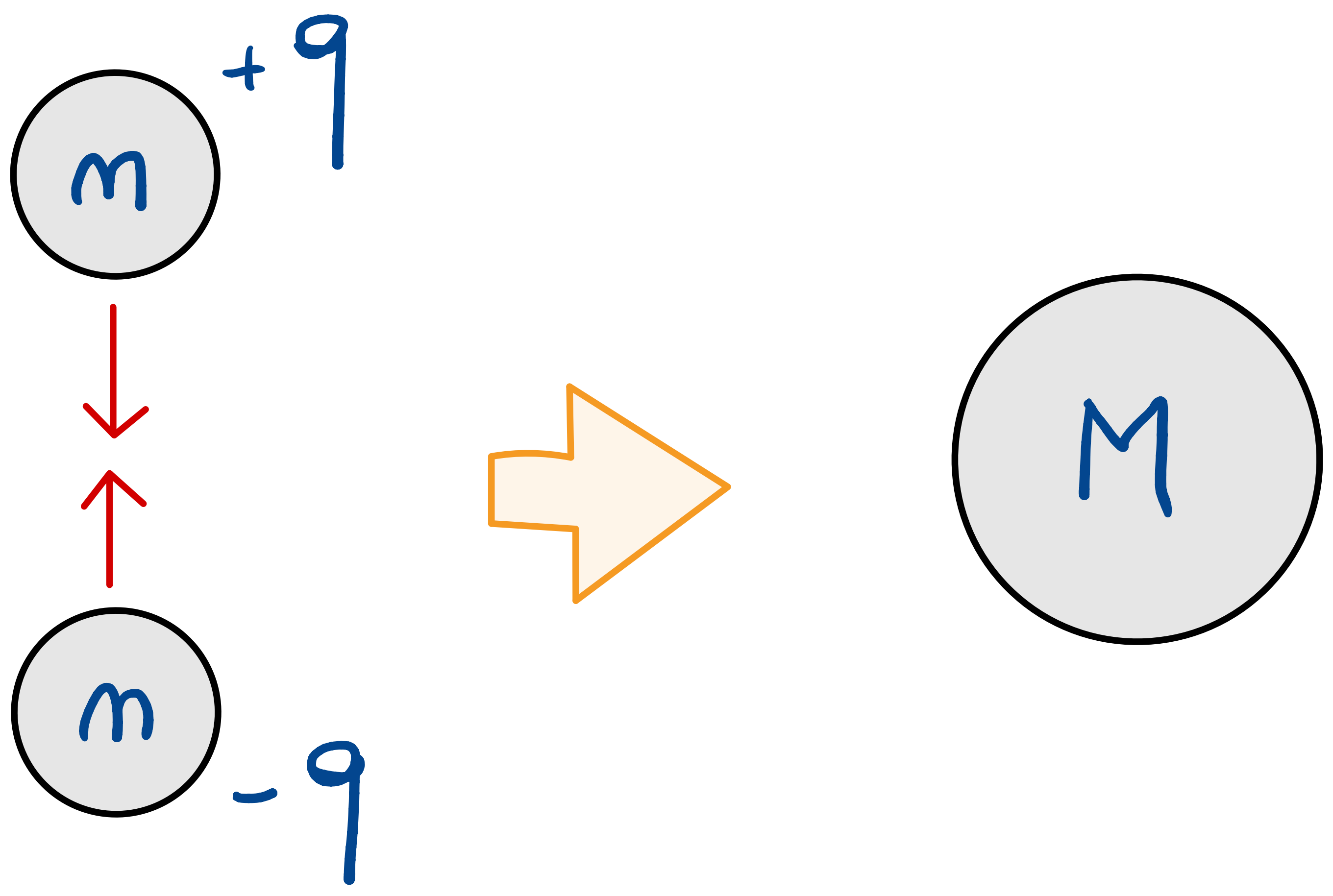


Cold to hot mergers in AdS$_D$

High temperature Rényi entropies $S_n(\beta) = \frac{1}{D-1}\frac{1-n^{1-D}}{1-\frac{1}{n}}S(\beta)$ with $\beta \to 0$

Coalescence of oppositely charged/rotating extremal susy AdS$_D$ black holes

Rényi laws $S_n(\beta) \geq 2S(\beta \to \infty)$ ➡ stronger bounds $n \gg 1$



Ratio of lower bounds on M

$$\frac{M_{n\to\infty}}{M_{n=1}} = (D-1)^{\frac{D-1}{D-2}}$$

D	$\frac{M_{n\to\infty}}{M_{n=1}}$
3	4
4	5.2
5	6.3

Hawking-Rényi entropy in canonical ensemble

For a thermal density matrix $$\rho = \frac{e^{-\beta H}}{Z^{\mathrm{c}}(\beta)}$$

in the canonical ensemble $$\tilde{S}_n(\beta) = S(n\beta)$$

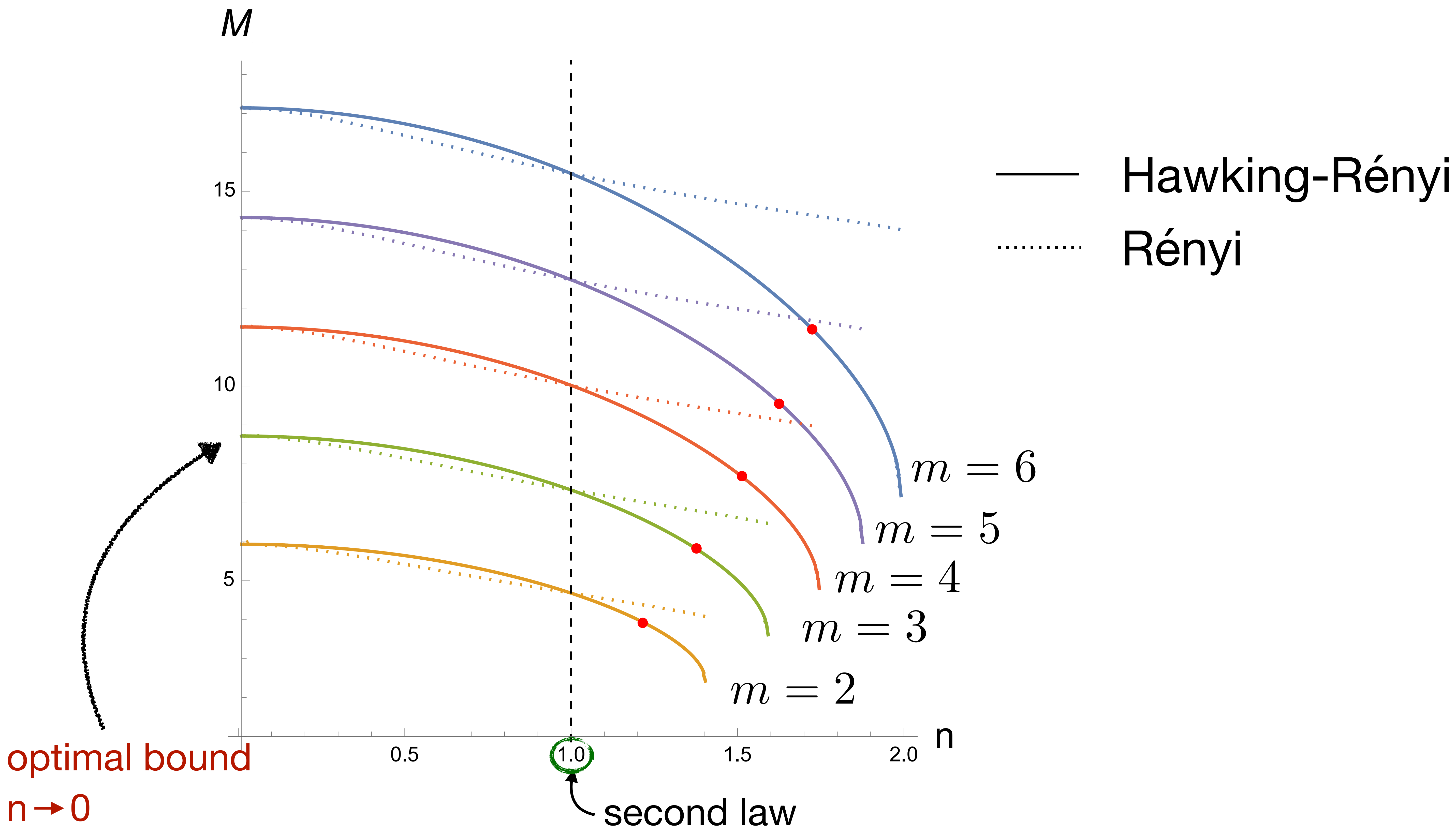
$$\Rightarrow \boxed{\tilde{S}_n(\beta) = \frac{\ell^2\pi}{9b^2n^2}\left(1+\sqrt{1-3b^2n^2}\right)^2}$$ with $$b \equiv \frac{\beta}{2\pi\ell}$$

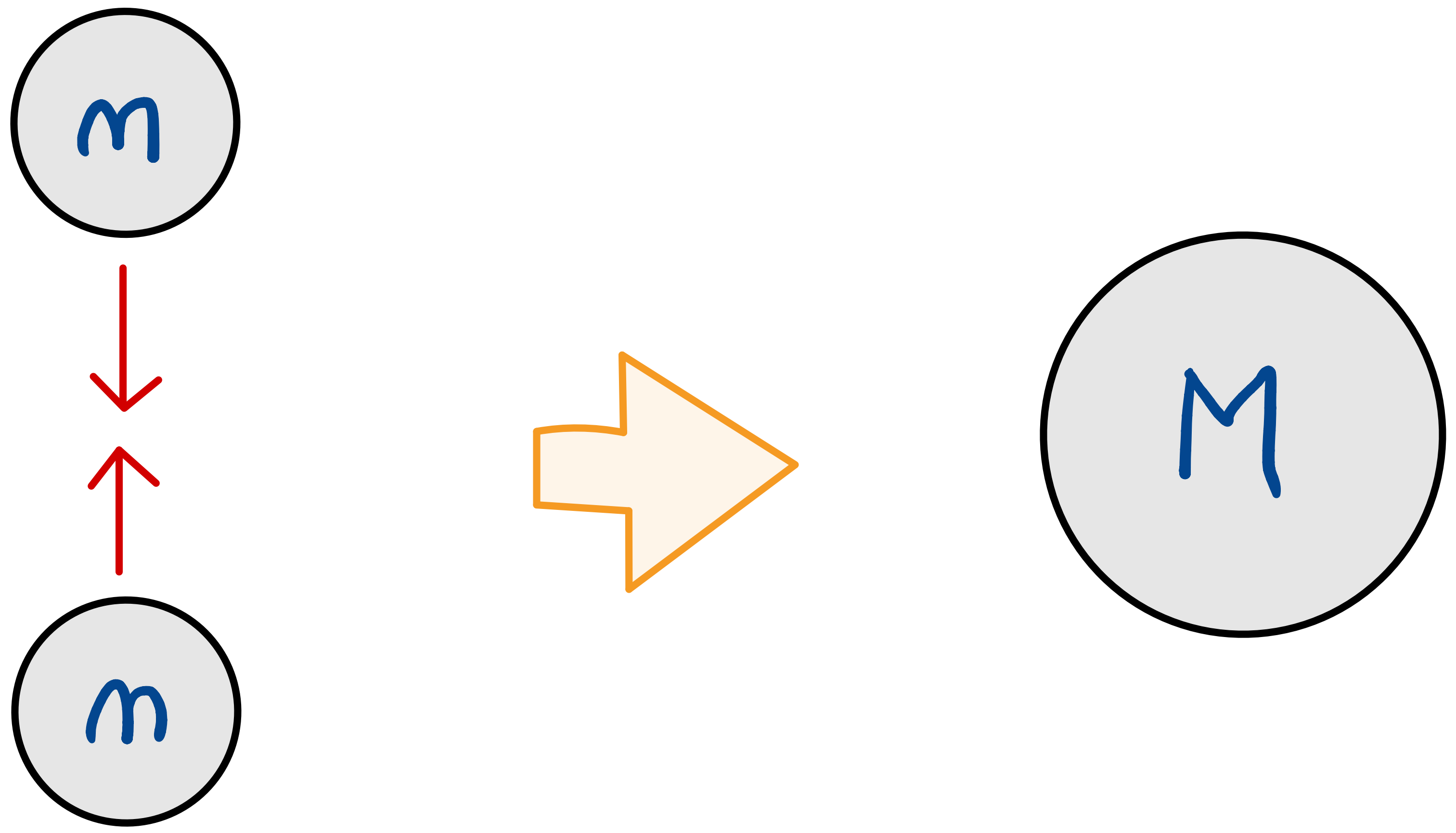
ATT! For a given b, this holds up to $n = \dfrac{1}{2b}$ to remain above the Hawking-Page transition of the replica manifold.

Head-on mergers in AdS$_4$

Hawking - Rényi laws $\tilde{S}_n(M) \geq 2\tilde{S}_n(m)$

Lower bound on final mass $(\ell = 1)$





For n < 1, more constraining than Rényi entropy laws

Optimal bound coincides with Hartley entropy bound

Extensions

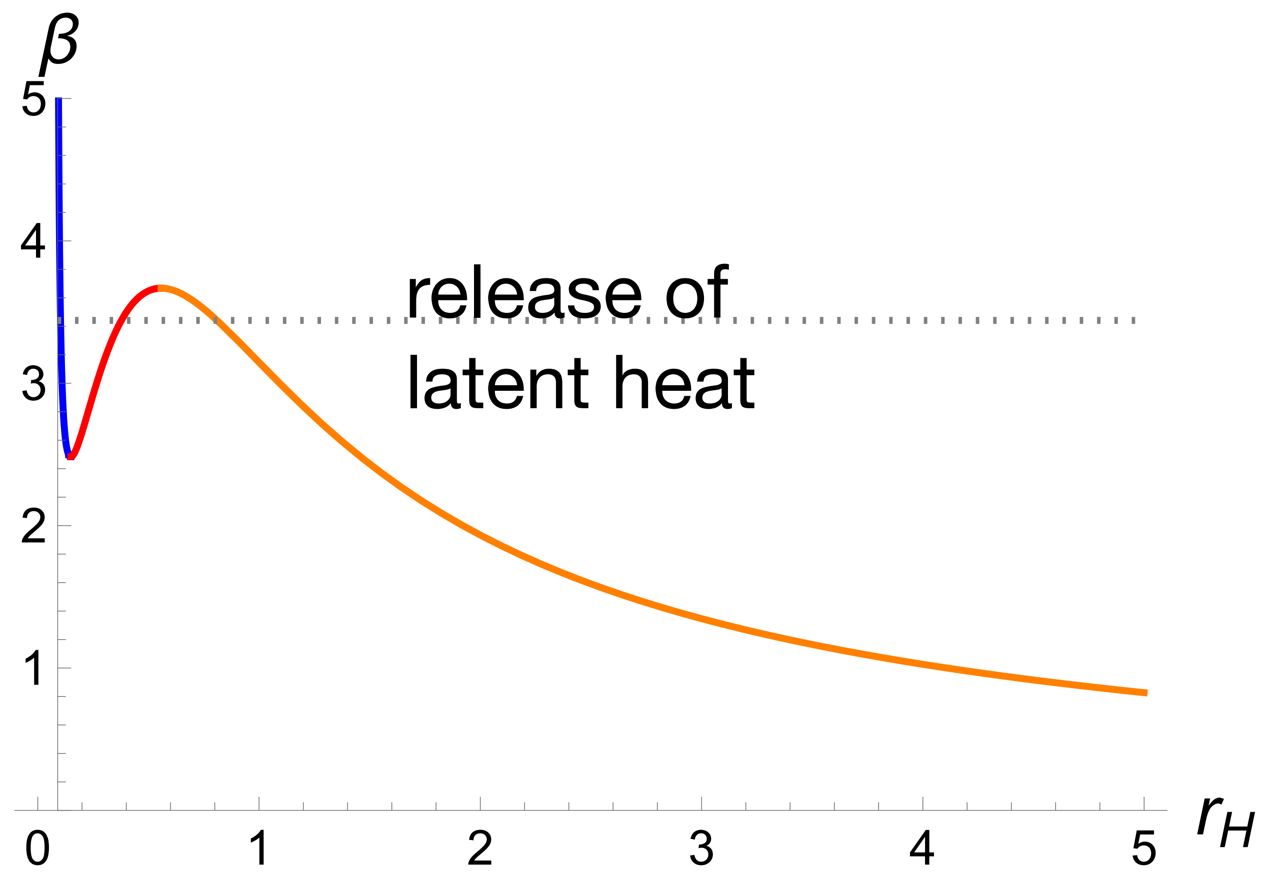
In collaboration with F. Galli, R. C. Myers, I. Reyes, E. Rizza

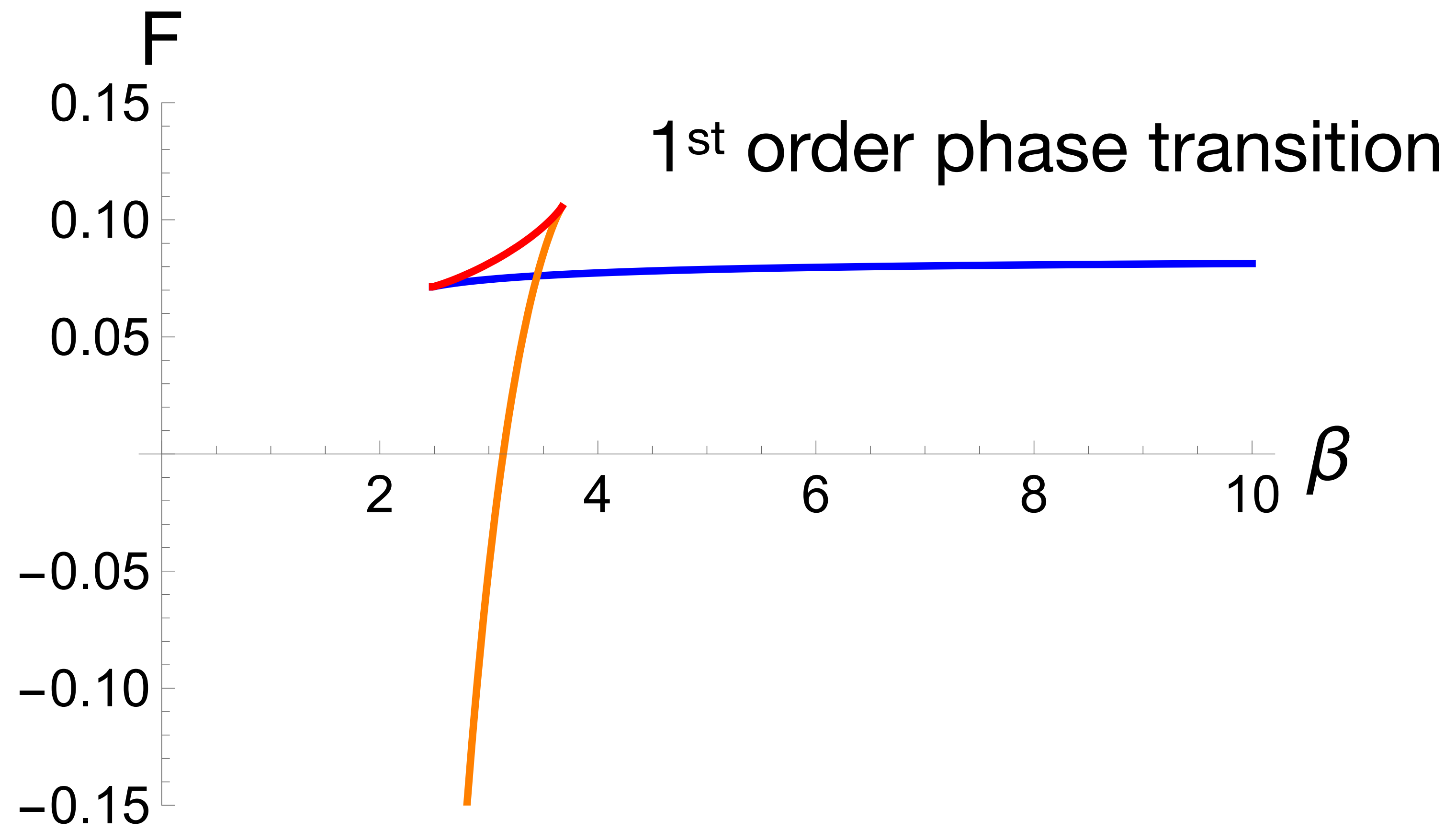
Charged AdS$_4$ black holes

[Bernamonti, Galli, Rizza]

Lapse function ($G_N = 1$): $f(r) = 1 - \frac{2m}{r} + \left(\frac{q}{r}\right)^2 + \left(\frac{r}{\ell}\right)^2$

Canonical ensemble: fixed $q = \frac{\ell}{12} < q_{\text{crit}} = \frac{\ell}{6}\,, \quad (\ell = 1)$





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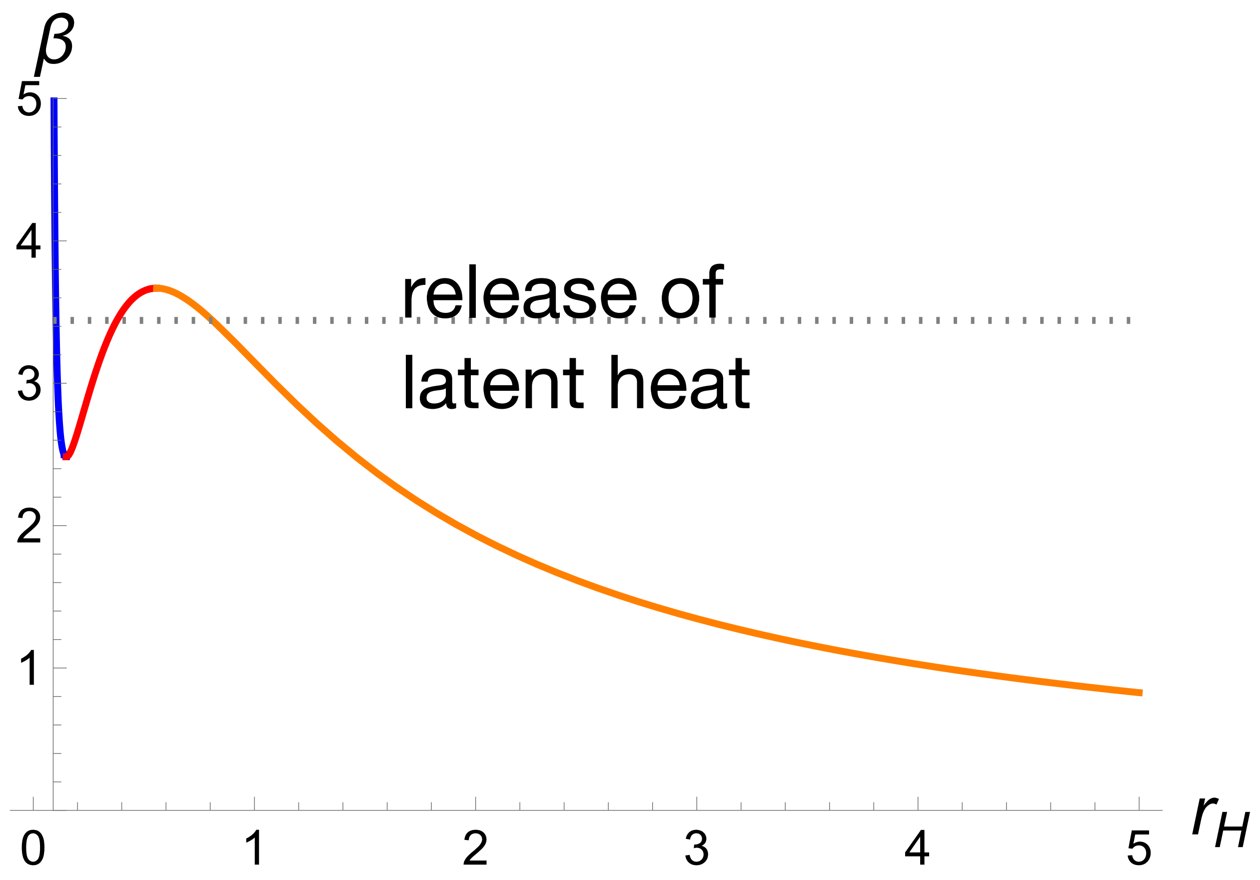
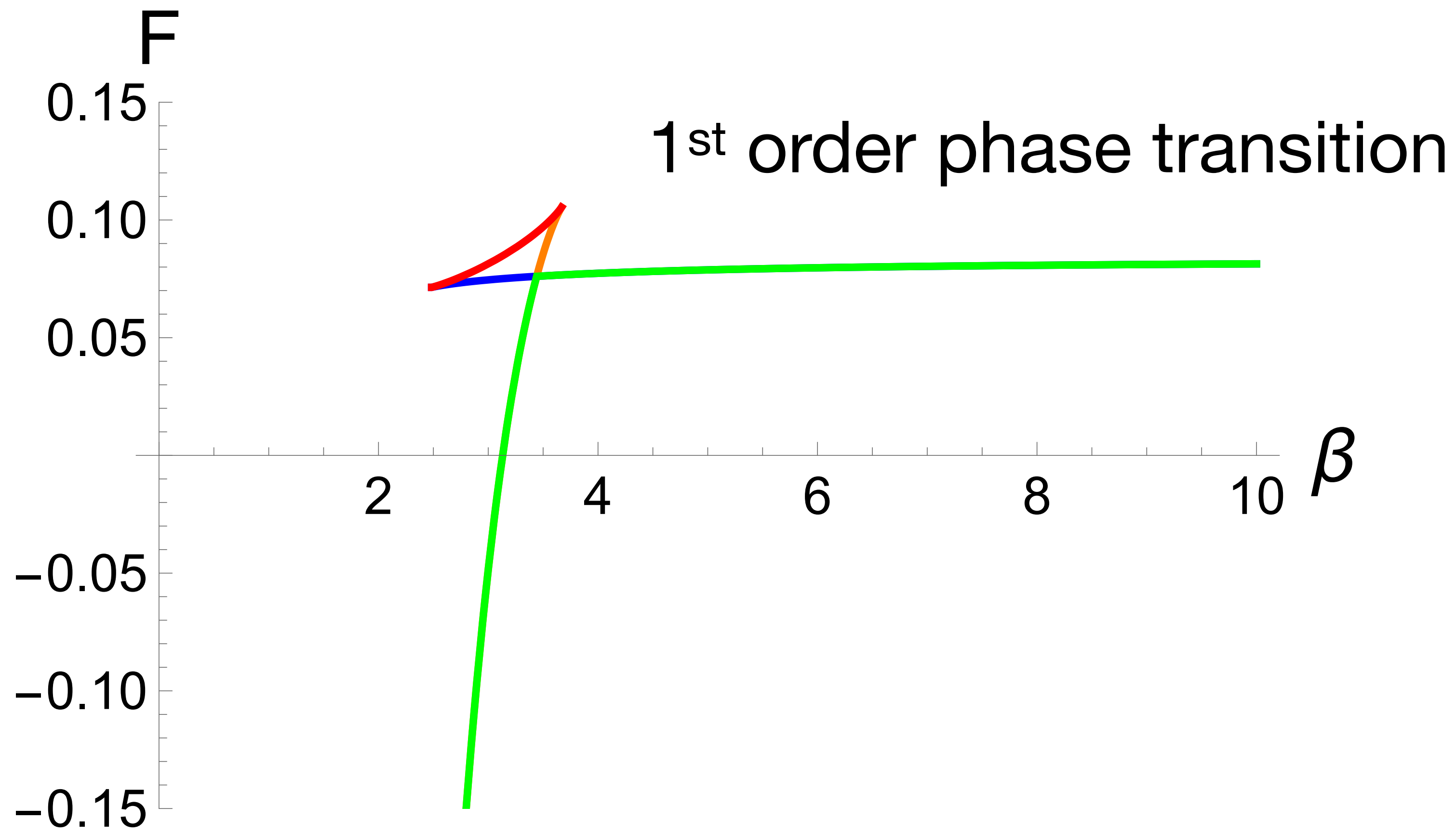
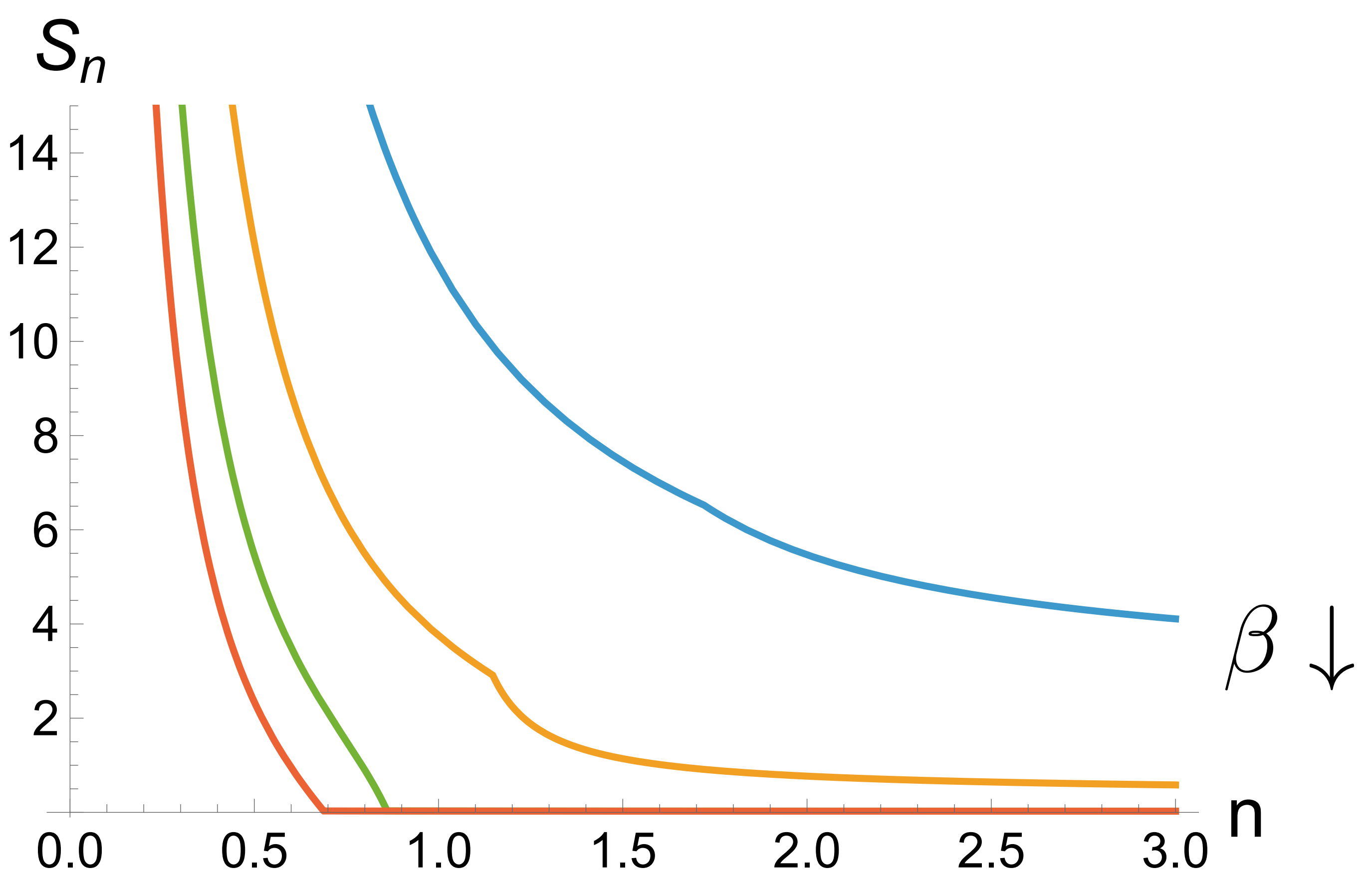






$$\beta = 2, 3, 4, 5$$

Richer phase structure of Rényi entropies and second laws

Black hole mergers in 4D flat space

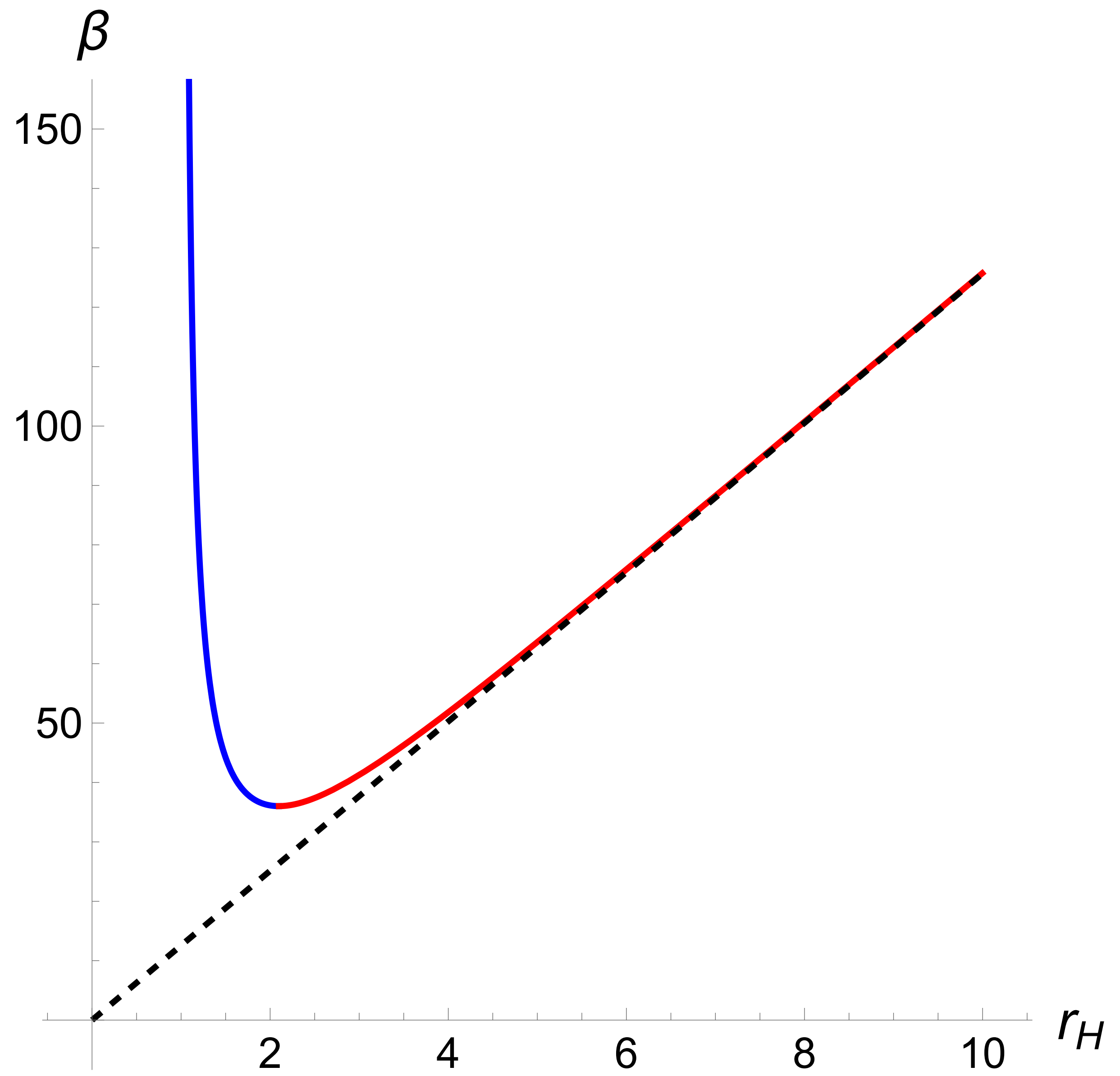
[Bernamonti, Galli, Myers, Reyes]

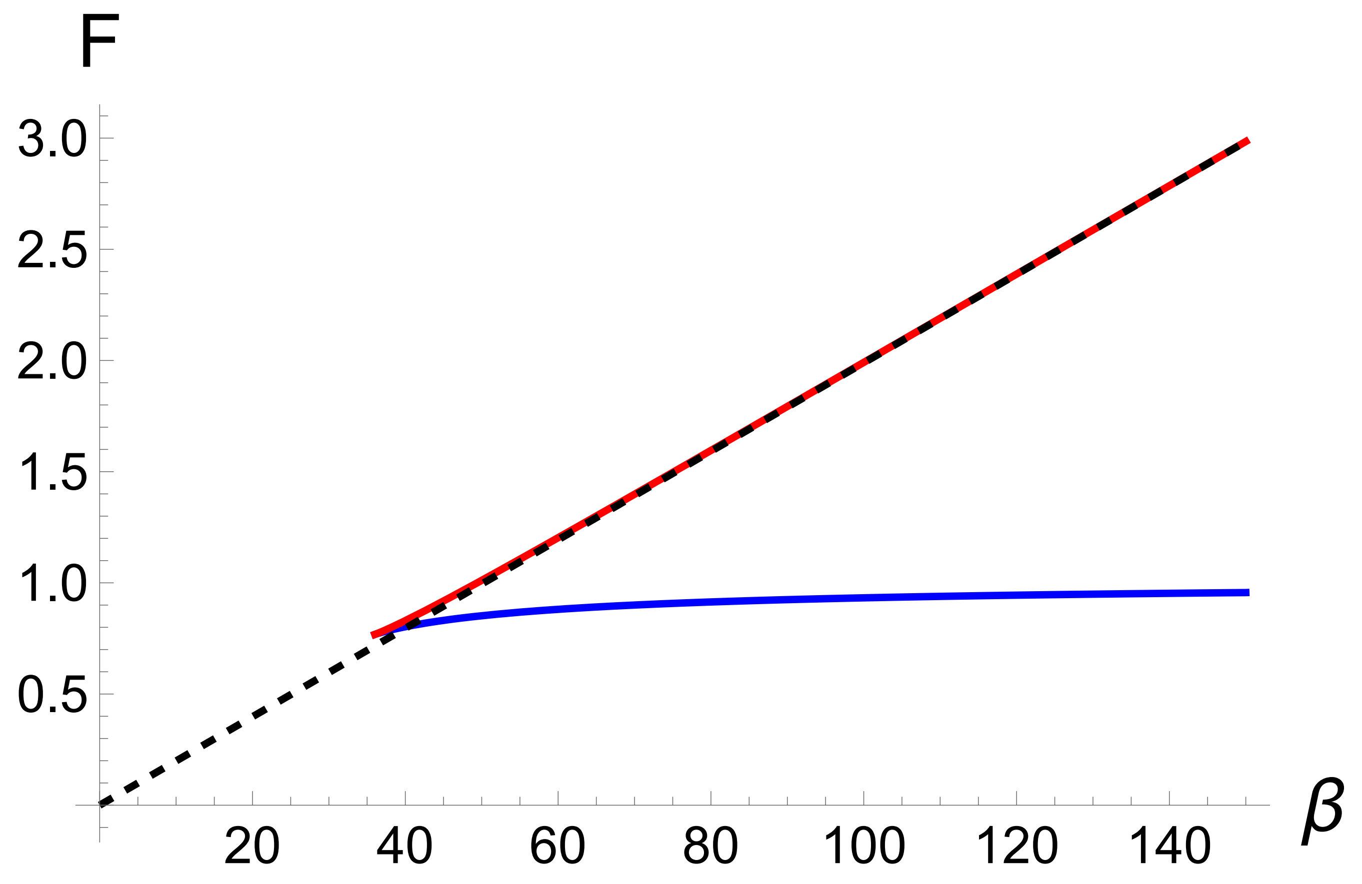
In flat space the infinite temperature black hole is thermodynamically unstable

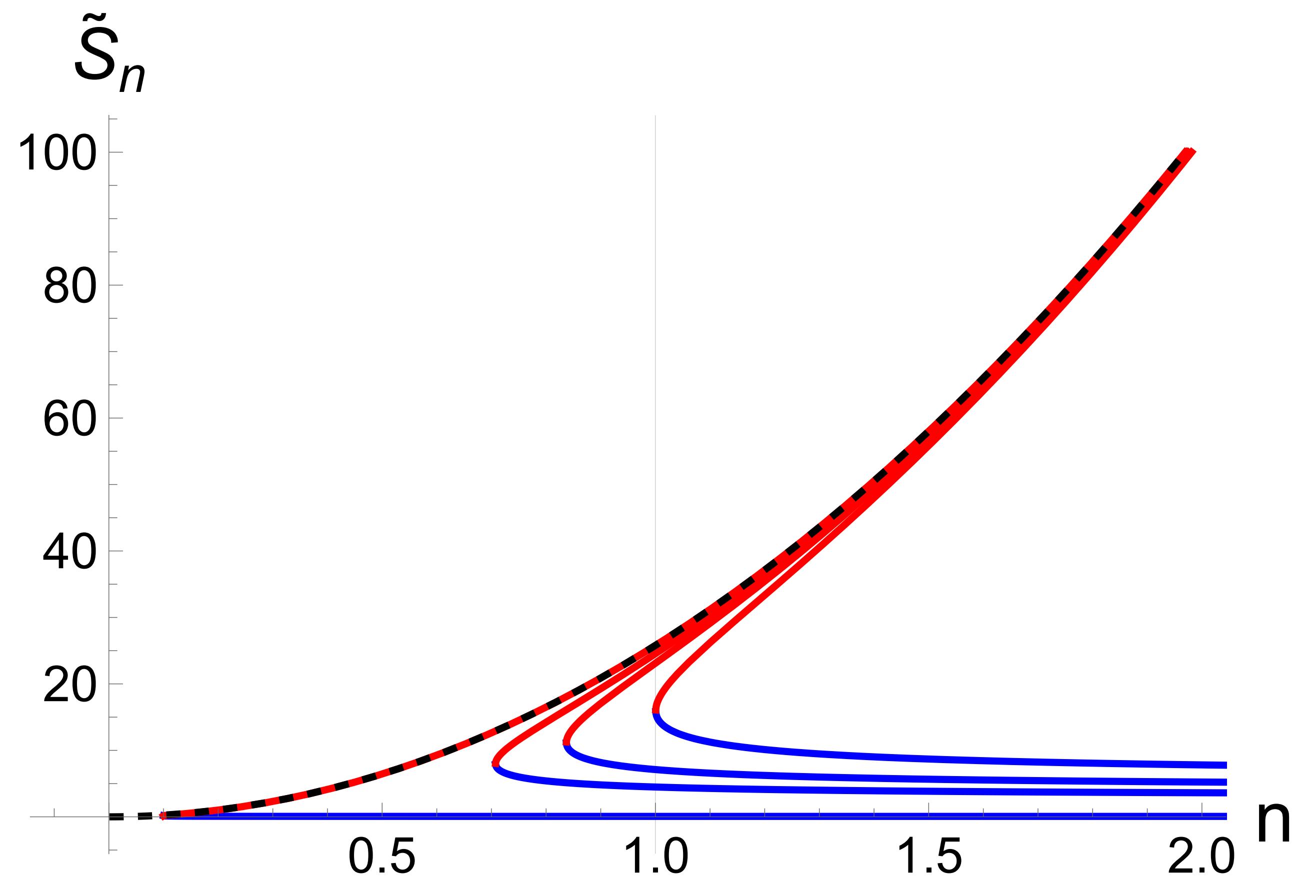
$S_n(\rho) \leq S_n(\rho')$ Rényi second laws ✗

$\tilde{S}_n(\rho) \leq \tilde{S}_n(\rho')$ Hawking-Rényi second laws ✓

Kerr black holes in canonical ensemble (fixed J):







New constraints of flat space black hole mergers

Perspectives

- Rényi laws put tighter constraints on gravitational dynamics than second law.
- Ensemble inequivalence: ensemble dependent physical predictions
- Pin down bounds on processes that are informative for numerical relativity simulations, *e.g.* Kerr-AdS$_4$ superradiant instability [Green,Holland,Ishibashi,Wald 2015] [Chesler 2021] and astrophysical mergers
- Rényi mutual information
- Via the area theorem the Hawking-Rényi laws have a geometric interpretation in the replica manifold. Is there a geometric interpretation also for the Rényi second laws?
- Entanglement spectrum
- Explore the broader family of monotones (*h divergences*). Can we trade EOM for constraints? [Lostaglio, Korzekwa 2022]
- Connection through AdS/CFT to QGP formation in heavy-ion collisions. [Bantilan, Romatschke 2014]

Thank you!