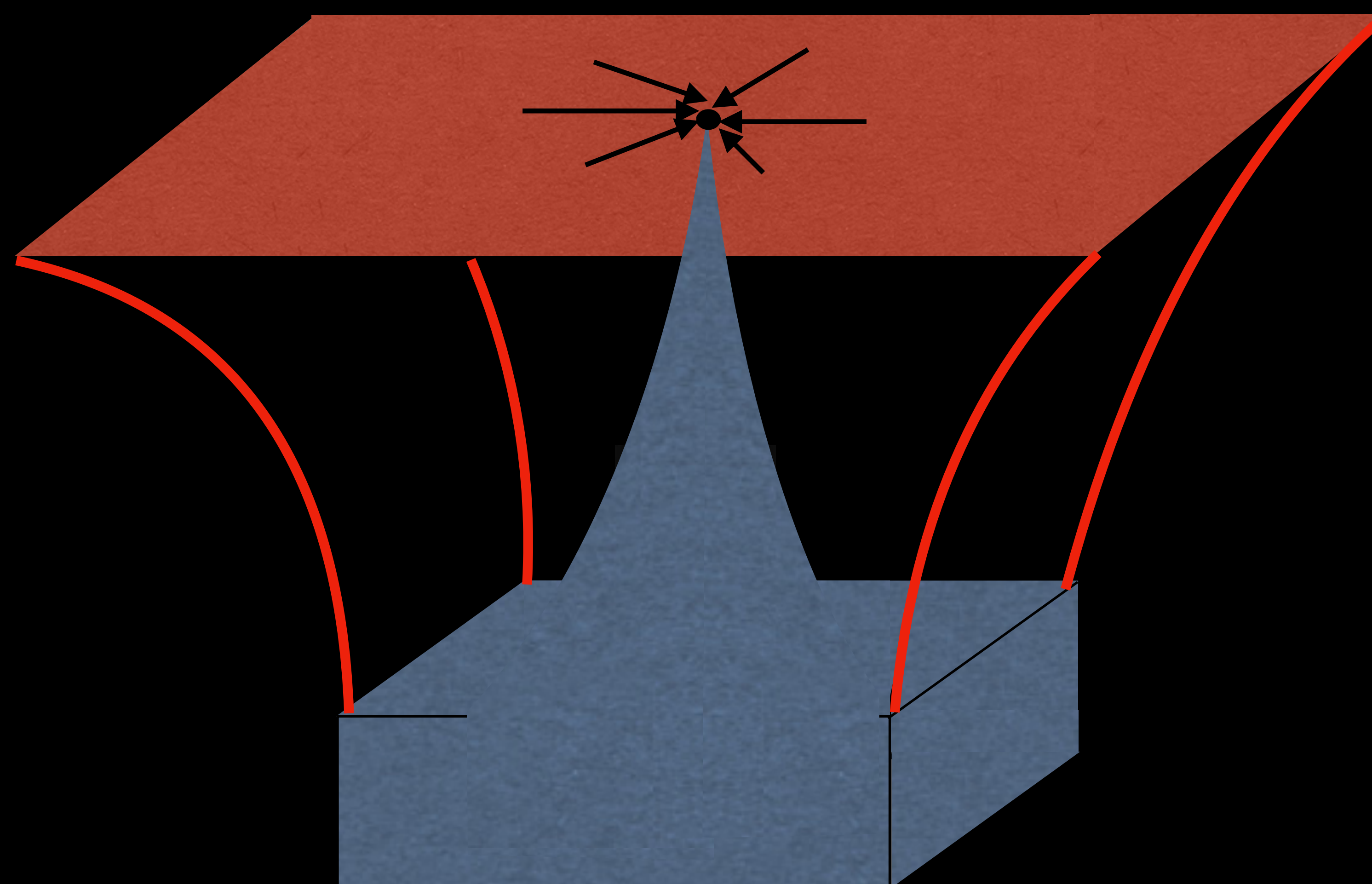
Gravitational Collapse at the Boundary



David Mateos

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erc European Research Council

MINISTERIO DE CIENCIA E INNOVACIÓN

EXCELENCIA MARÍA DE MAEZTU

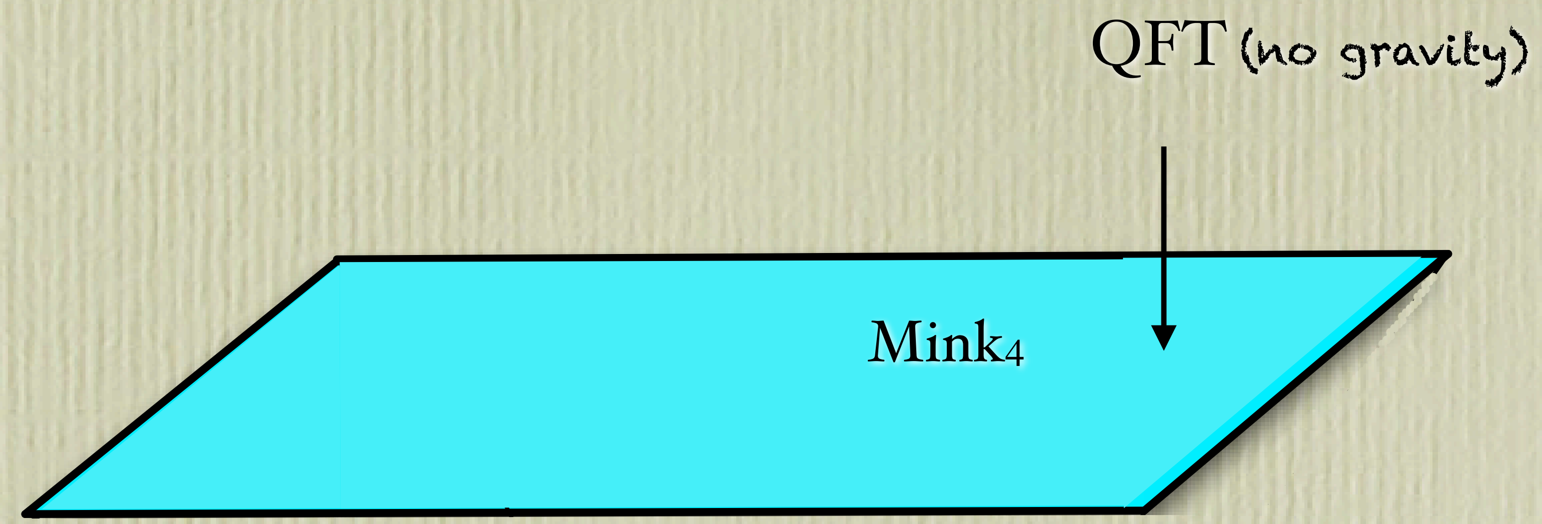
AGAUR Agència de Gestió d'Ajuts Universitaris i de Recerca

Plan

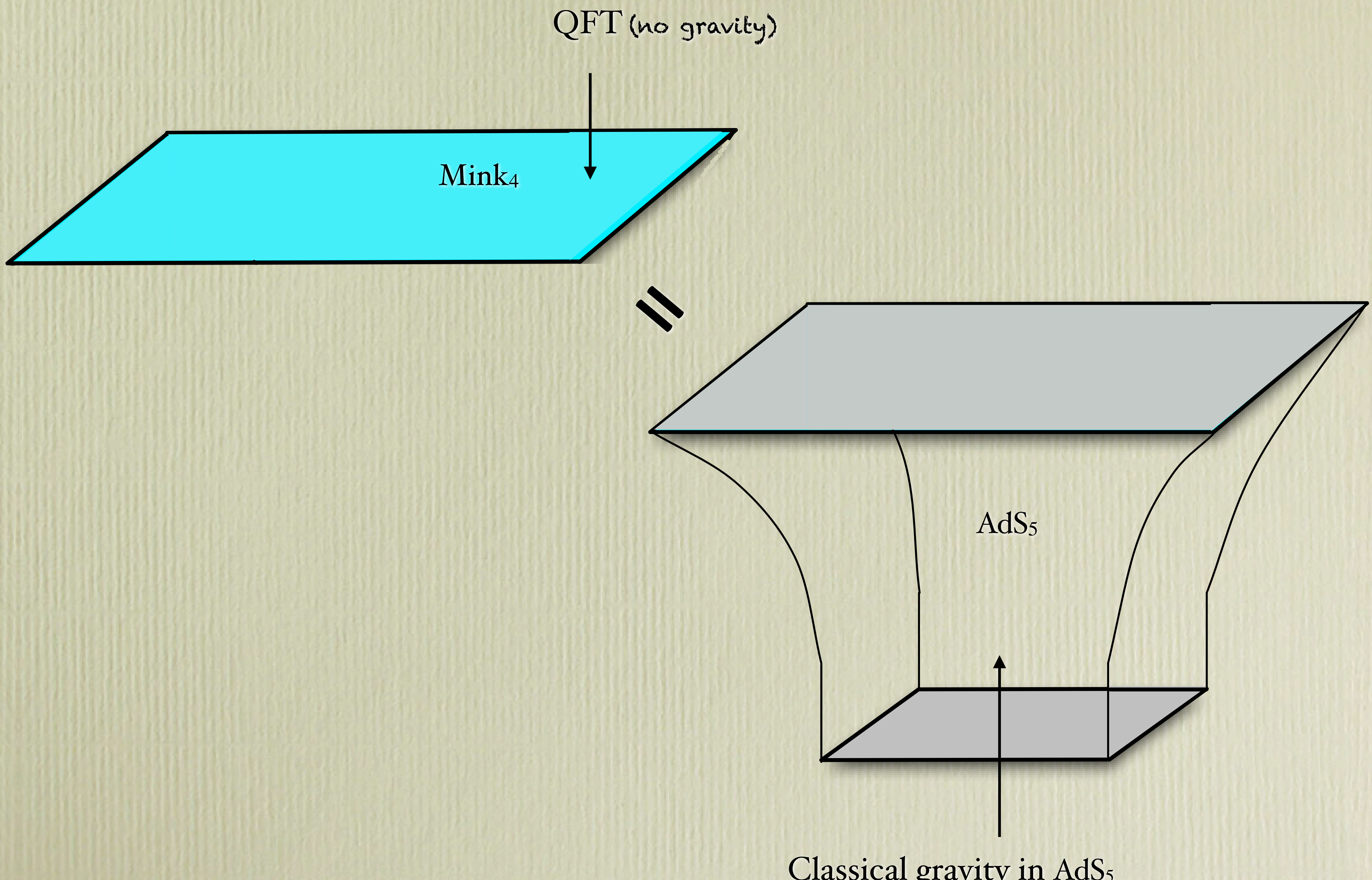
- Holography (standard framework)
- Holography with dynamical boundary gravity
- Gravitational collapse at the boundary
- Can we see evaporation?
- Behind the horizon: Approaching the singularity

Holography
(standard framework)

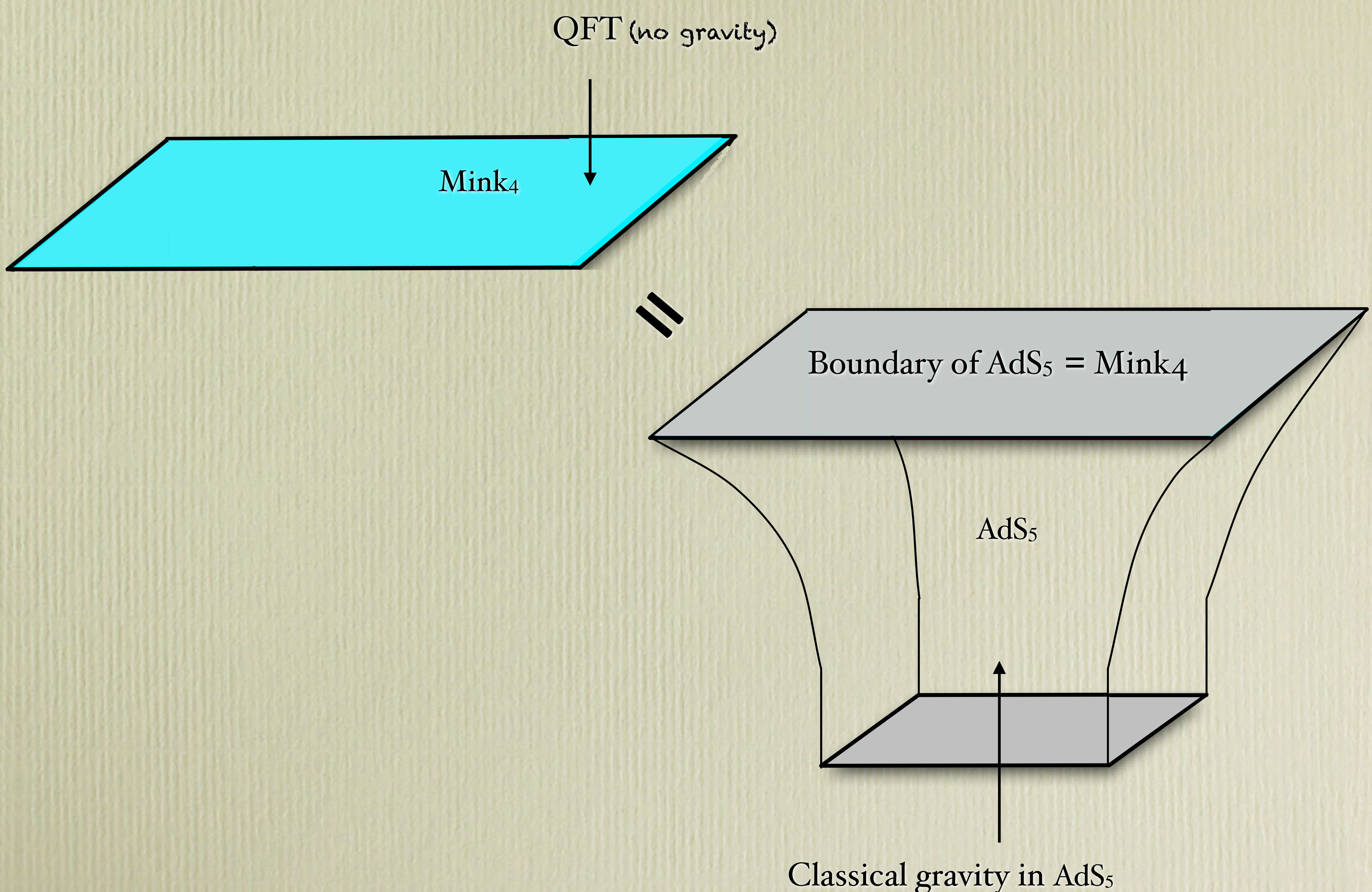
Holography



Holography

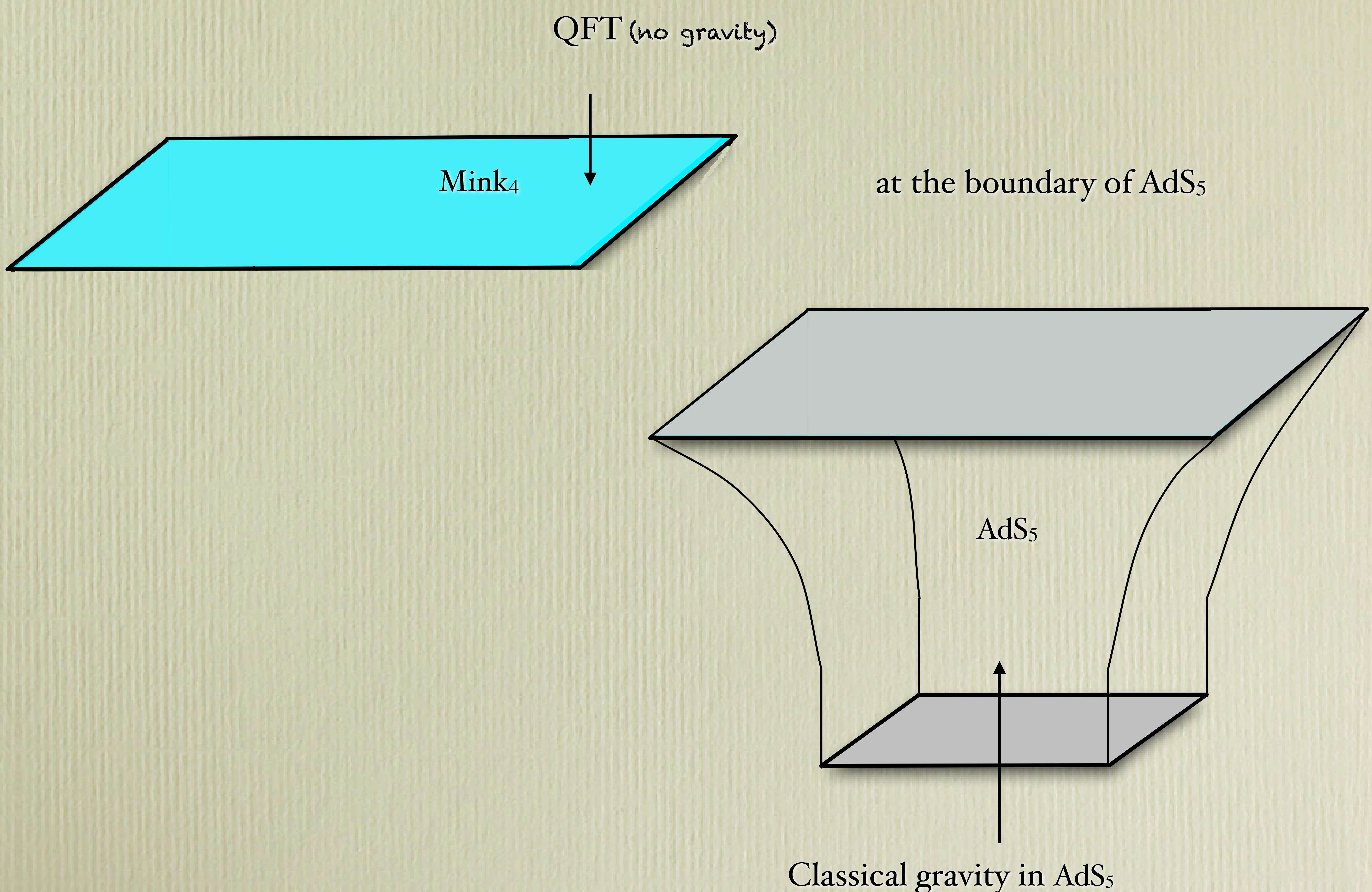

QFT (no gravity)
Mink_4
=
AdS_5
Classical gravity in AdS_5

Holography

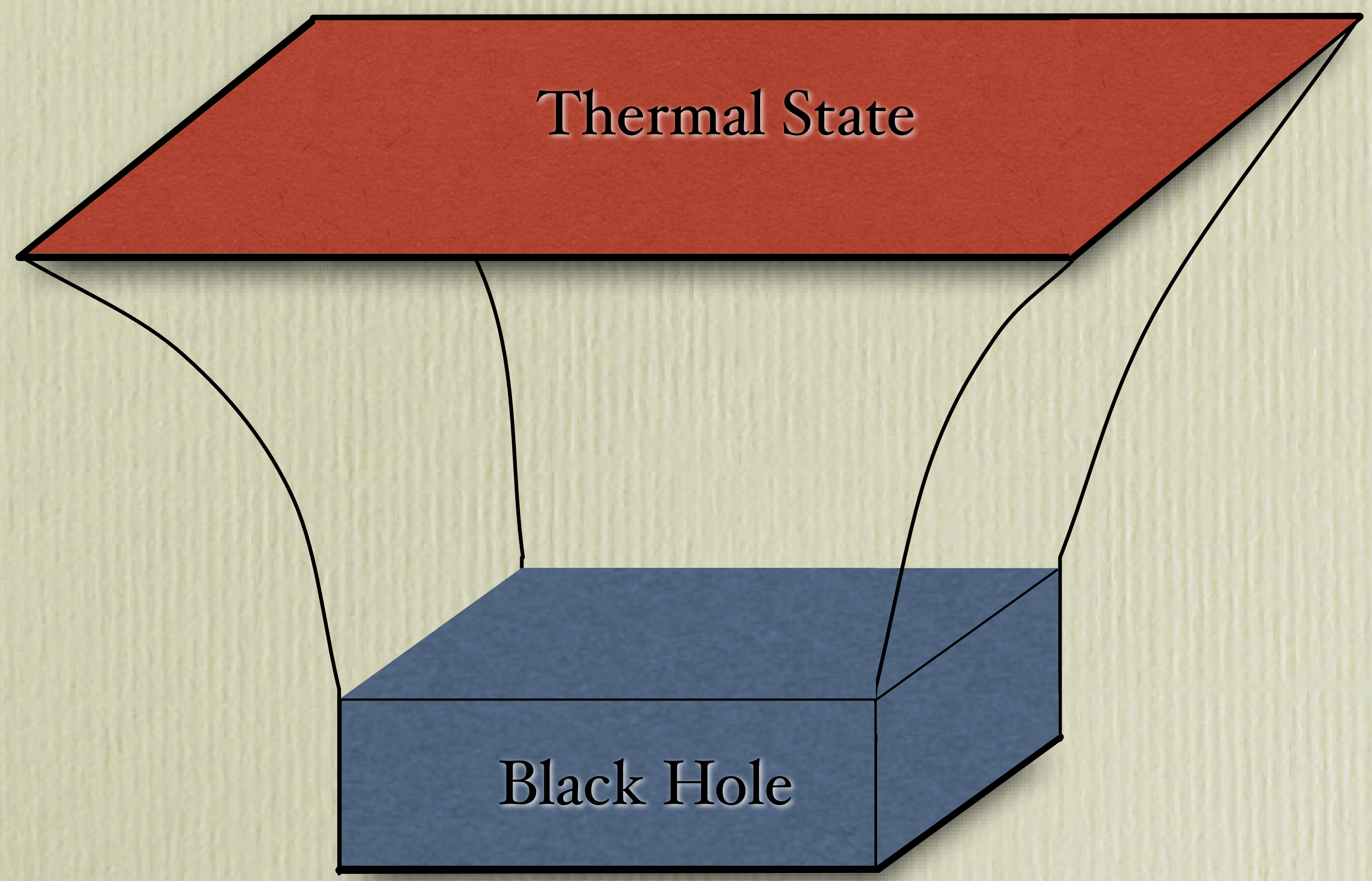


QFT (no gravity)
Mink4
=
Boundary of AdS5 = Mink4
AdS5
Classical gravity in AdS5

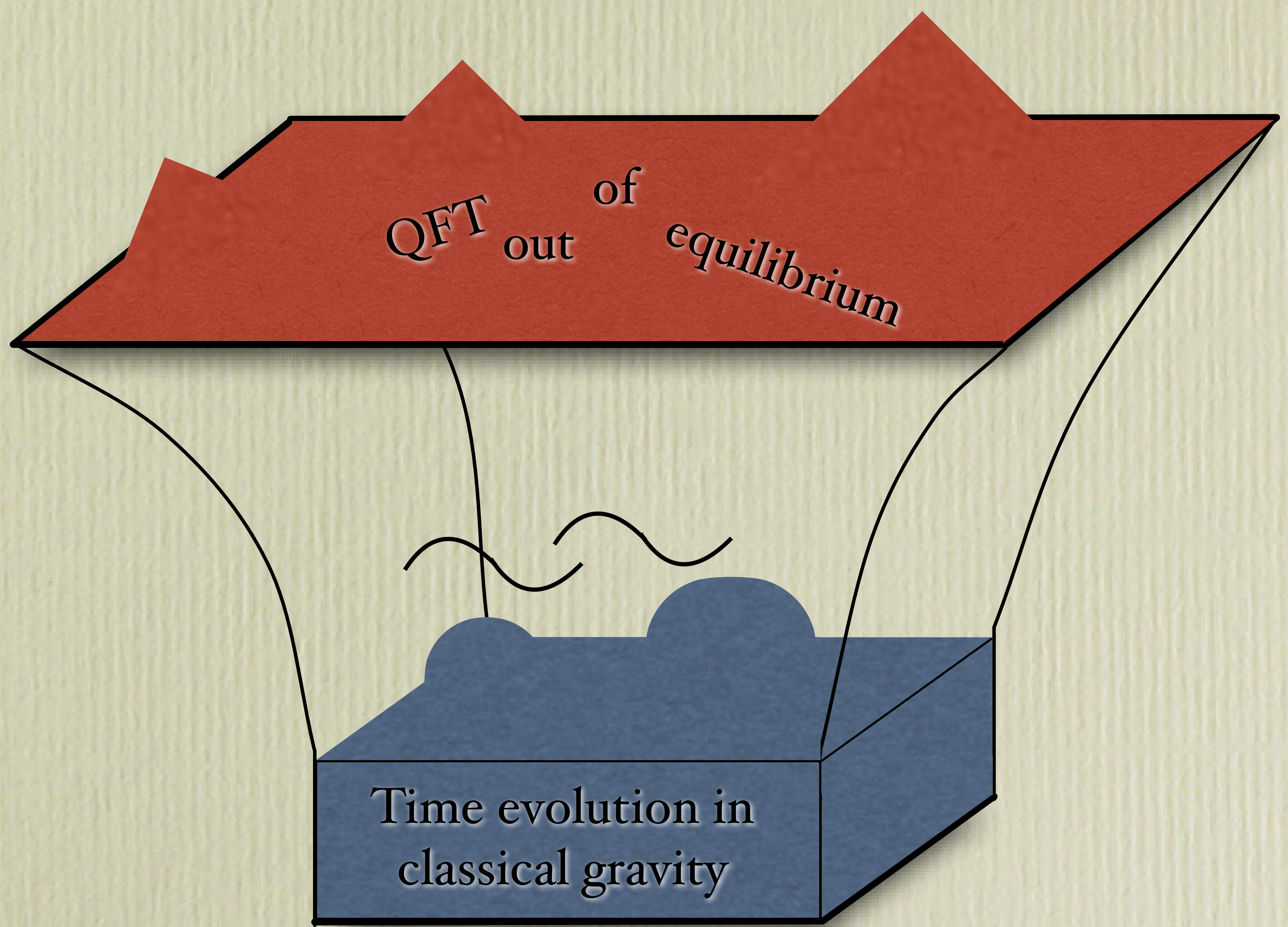
Holography


QFT (no gravity)
Mink_4
at the boundary of AdS_5
AdS_5
Classical gravity in AdS_5

Thermal physics = Black hole physics



The power of holography

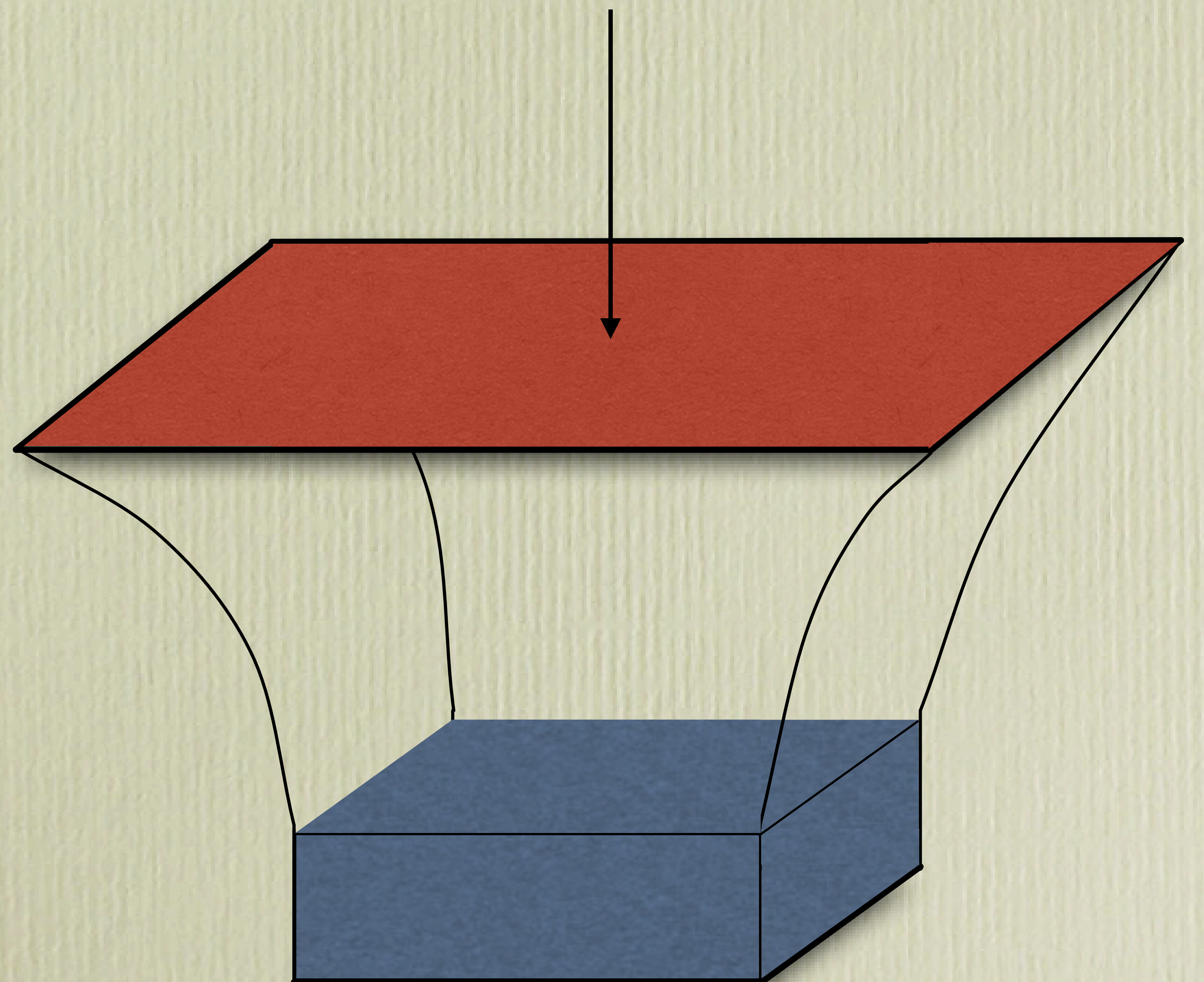


Holography with Dynamical Boundary Gravity

Dynamical gravity at the boundary

- The framework I have just described corresponds to:

Out-of-equilibrium quantum matter in Minkowski space



Dynamical gravity at the boundary

- But many problems require:

Out-of-equilibrium quantum matter + Classical dynamical gravity

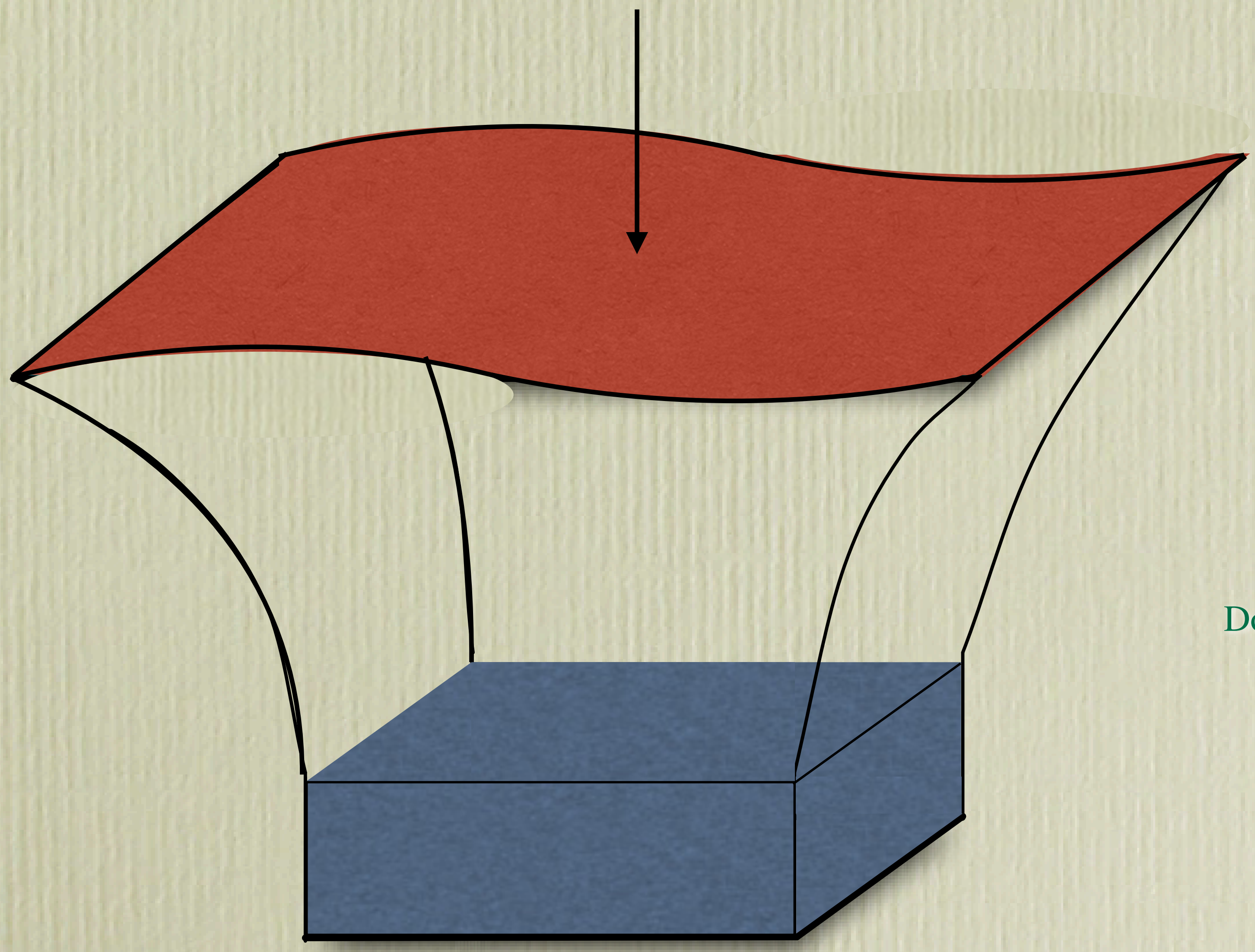
$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \quad = \quad 8\pi G \, \langle T_{\mu\nu} \rangle$$

- Cosmology
- Astrophysics
- Inflation
- (P)reheating
- Black hole formation & evaporation
- Etc

Dynamical gravity at the boundary

- So we would like a new holographic framework:

Out-of-equilibrium quantum matter + Classical dynamical gravity

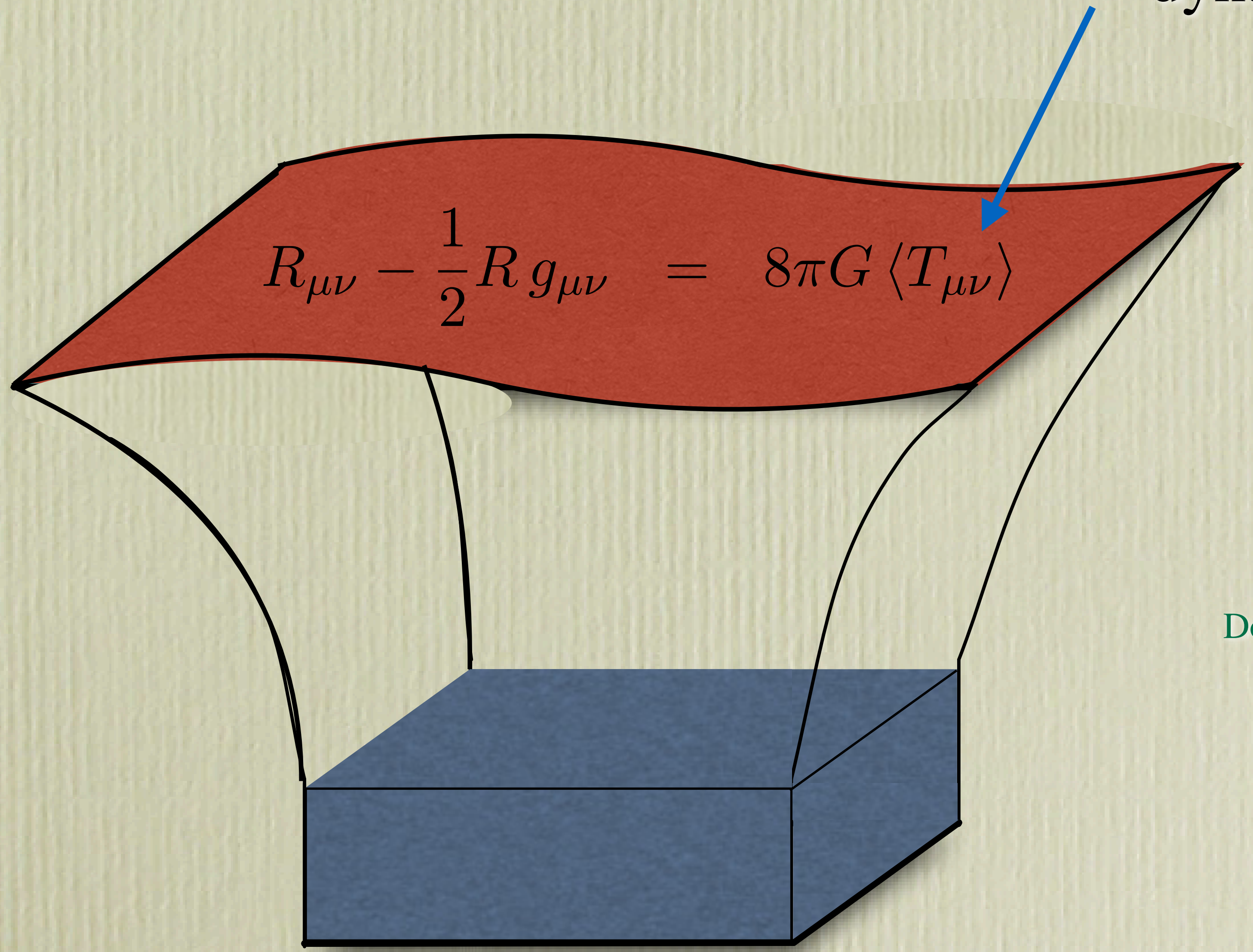


Some related work
Gubser '99
Csaki, Graesser, Kolda & Terning '99
Kehagias & Kiritsis '99
Cline, Grojean & Servant '99
Csaki, Graesser, Randall & Terning '99
Dvali, Gabadadze, & Porrati '00
Karch & Randall '00
Kiritsis '05
Compere & Marolf '08
Apostolopoulos, Siopsis & Tetradis '08
Erdmenger, Ghoroku & Meyer '11
Dong, Horn, Matsuura, Silverstein & Torroba '12
Banerjee, Bhowmick, Sahay & Siopsis '12
Fischetti, Kastor & Traschen '14
Buchel '16
Buchel '17
Emparan, Frassino & Way '20
Ghosh, Kiritsis, Nitti & Witkowski '20
Penin, Skenderis & Withers '22

Dynamical gravity at the boundary

- In this framework the boundary geometry obeys:

Renormalised QFT stress tensor in curved dynamical background



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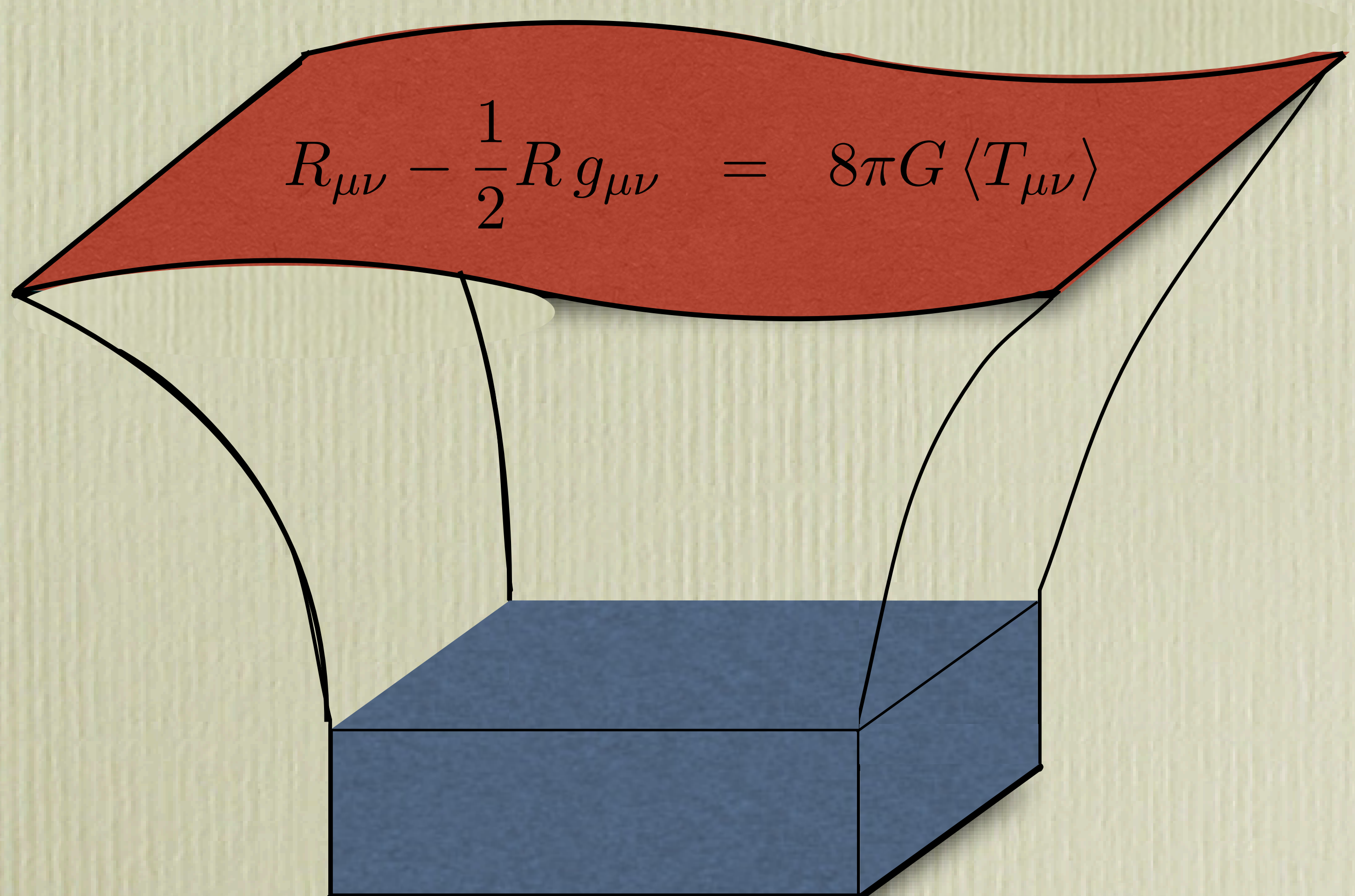
Dynamical gravity at the boundary

- These equations follow from the action:

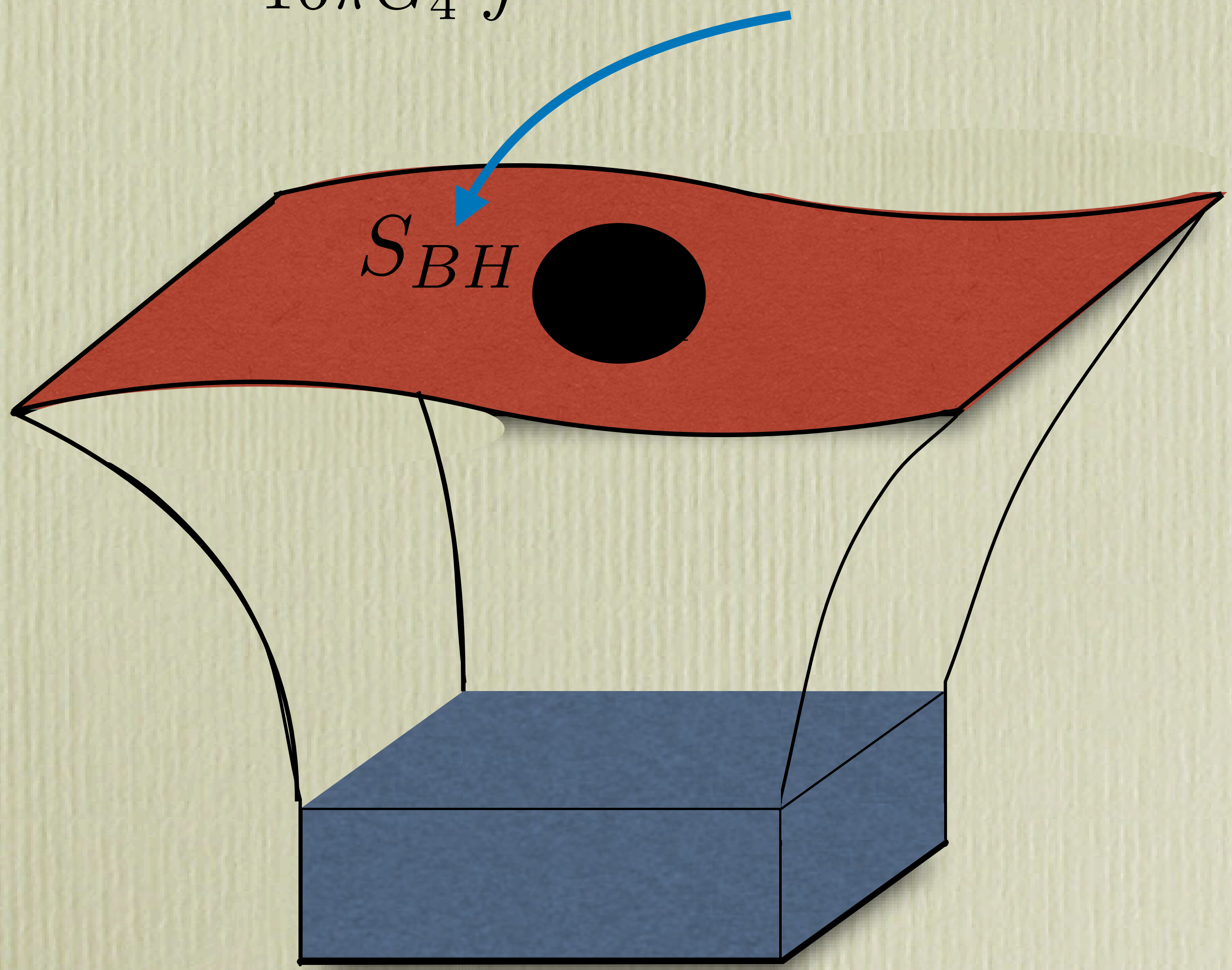
$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\rm GH} - \log Z_{\rm QFT}[g]$$



Dynamical gravity at the boundary

- So if we form a BH at the boundary:

$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\rm GH} - \log Z_{\rm QFT}[g]$$



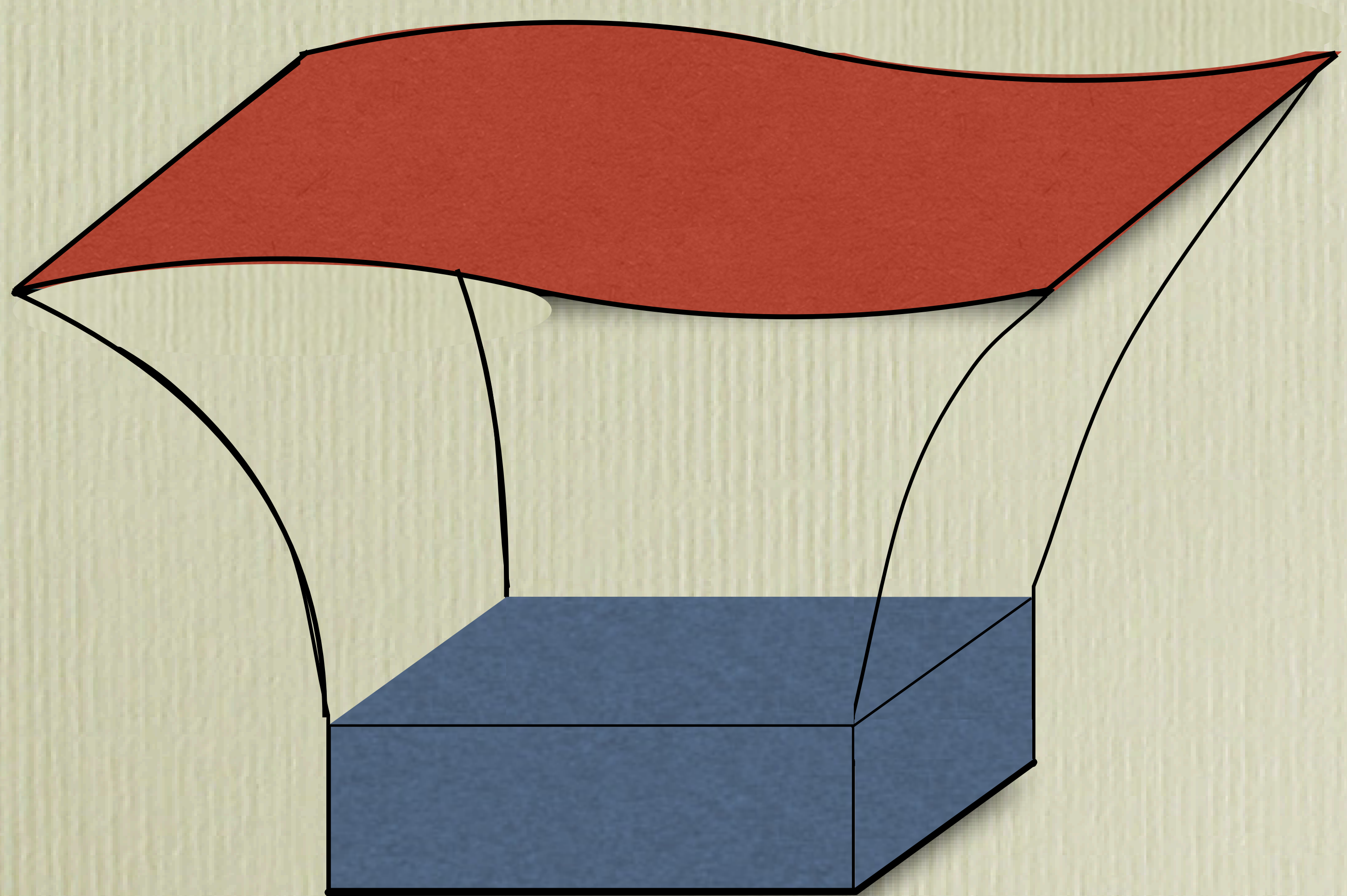
Dynamical gravity at the boundary

- We are interested in the case in which we can use:

$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\mathrm{GH}} - \log Z_{\mathrm{QFT}}[g]$$

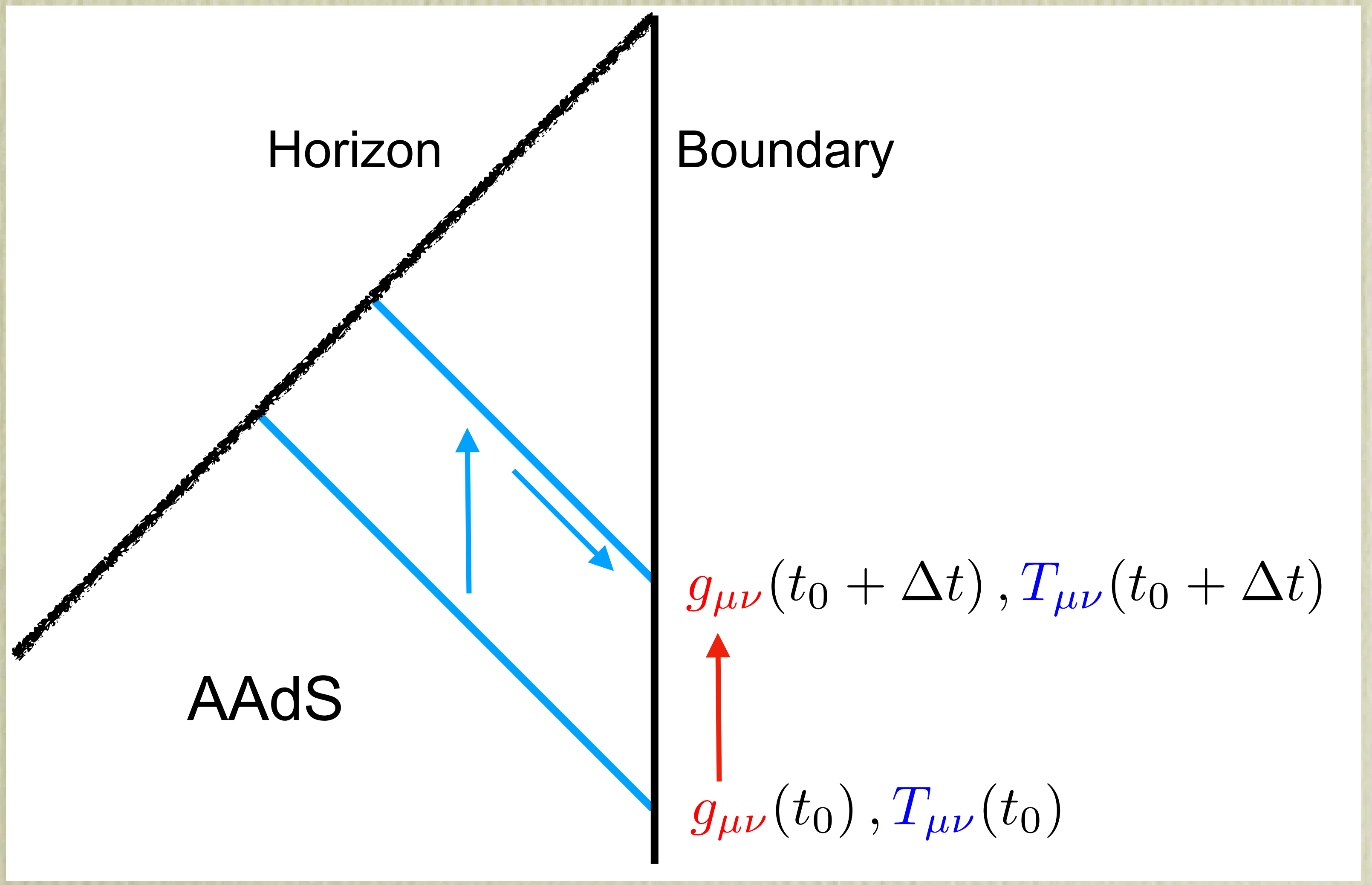
Holography

$$S^{5D}_{EH} + S^{5D}_{GH} + S_{ct}$$



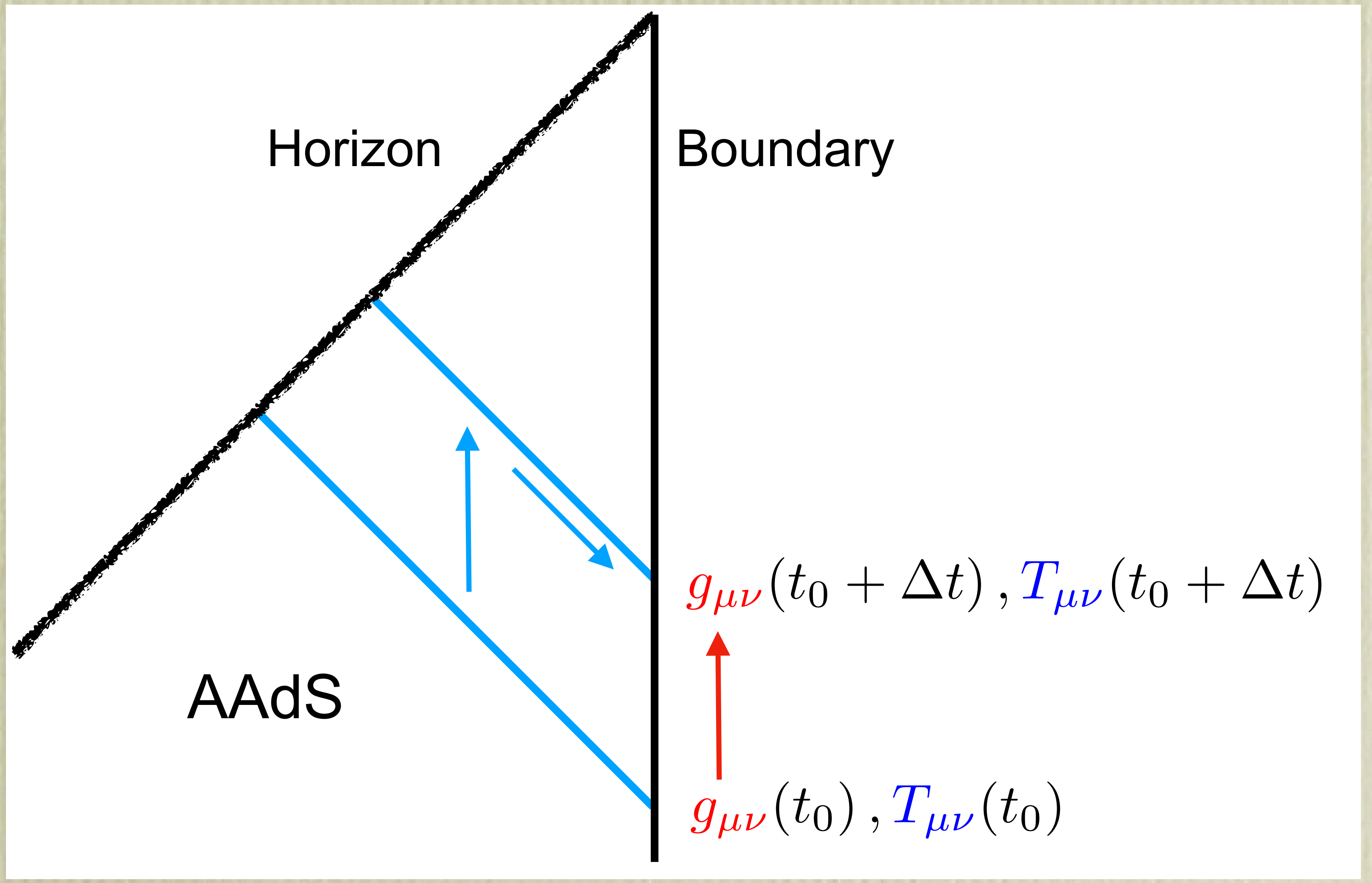
Dynamical gravity at the boundary

- From this perspective, the 5D bulk is just a tool to compute the 4D stress tensor:



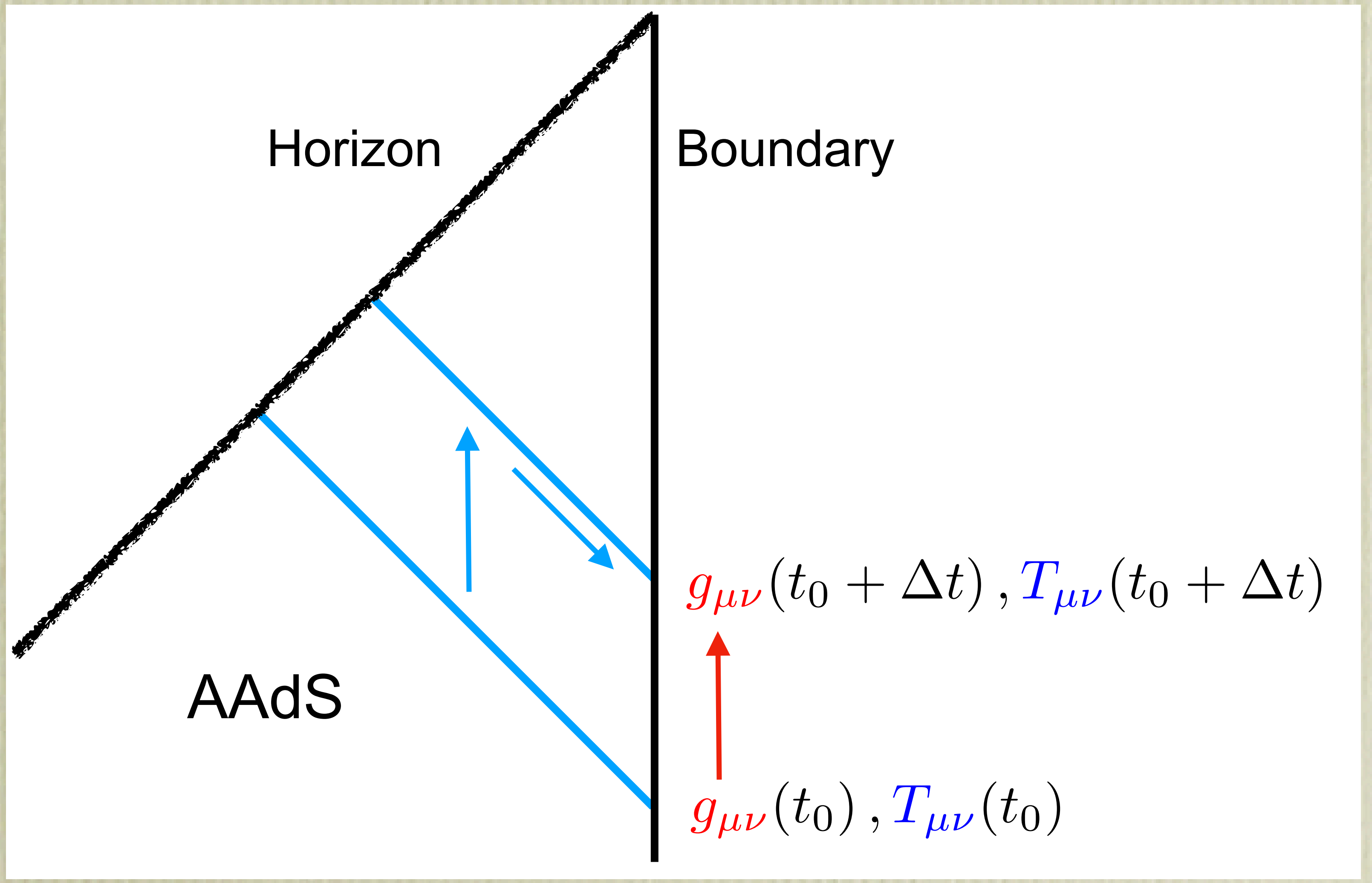
Dynamical gravity at the boundary

- This is the standard framework (and explains why the counterterms are the same) ...



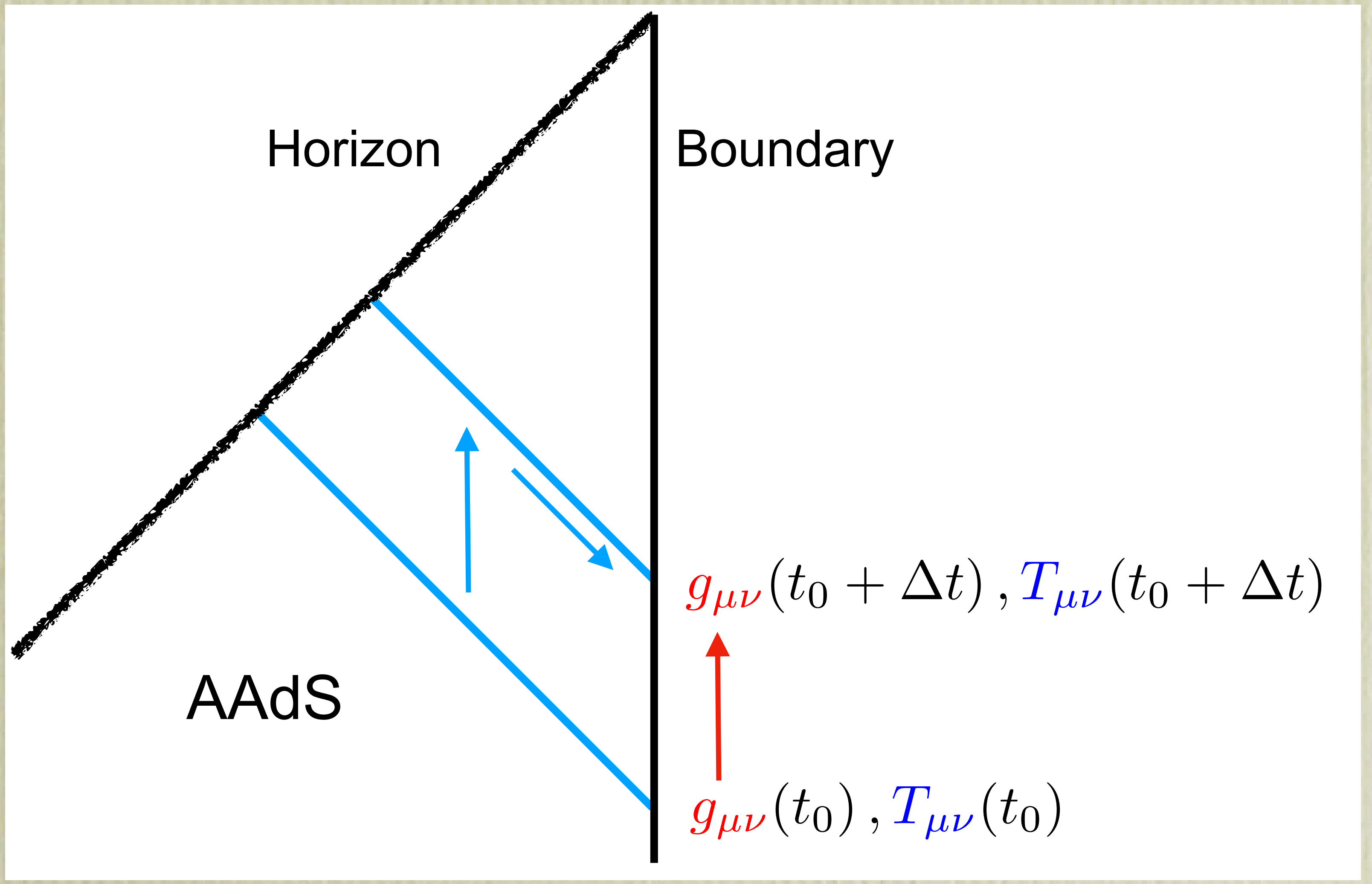
Dynamical gravity at the boundary

... except that the 4D metric is not prescribed a priori but evolved coupled to the stress tensor.



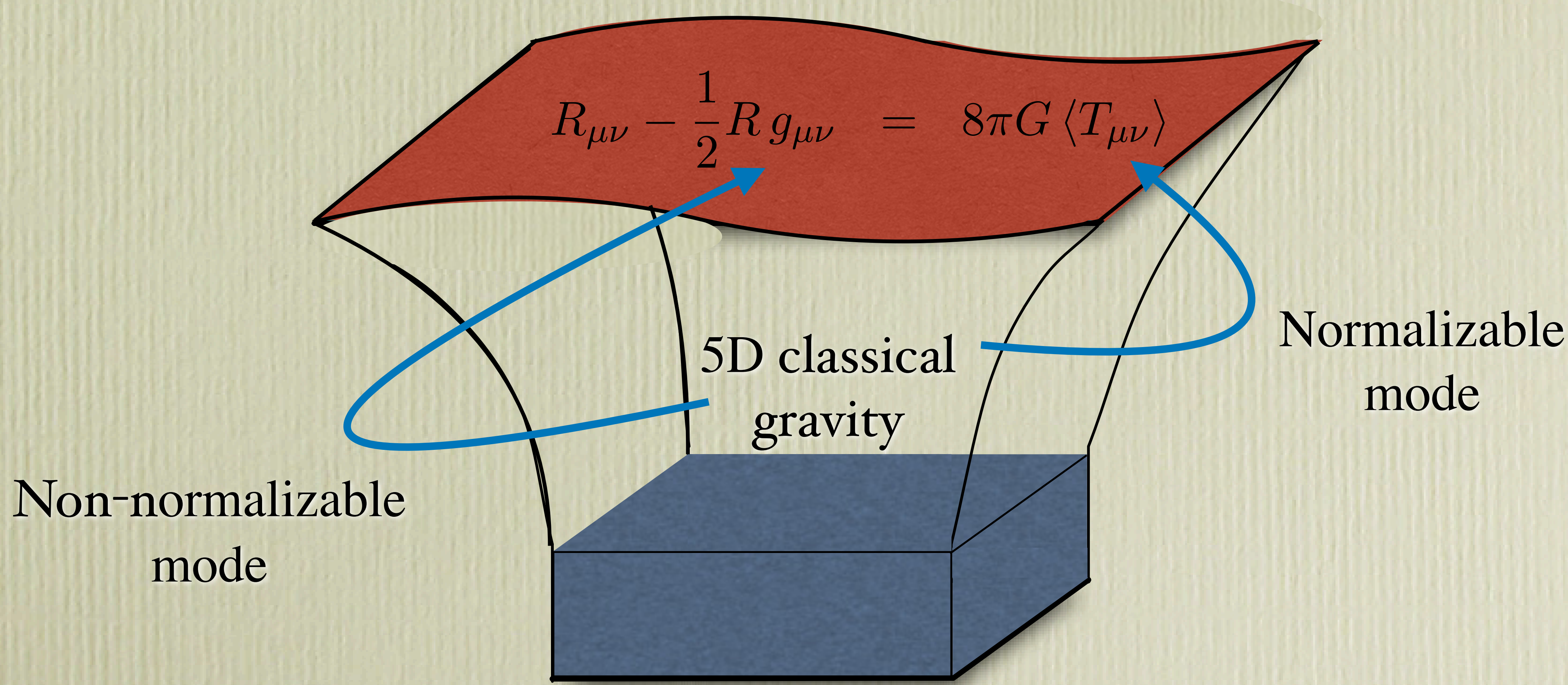
Dynamical gravity at the boundary

- This means we have to solve 4D gravity coupled to 5D gravity ...



Dynamical gravity at the boundary

... with mixed boundary conditions for 5D gravity:



Dynamical gravity at the boundary

- This can introduce pathologies in the theory, e.g. tachyons and ghosts.

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Gravity + QFT is an effective theory (even if QFT is UV-complete)

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N^2 d.o.f.

Gravity + QFT is an effective theory (even if QFT is UV-complete)

with cut-off $$M_{\text{species}} \sim \frac{M_{\text{Planck}}}{N}$$

Dvali '10

Dynamical gravity at the boundary

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N^2 d.o.f.

Gravity + QFT is an effective theory (even if QFT is UV-complete)

with cut-off $$M_{\text{species}} \sim \frac{M_{\text{Planck}}}{N}$$

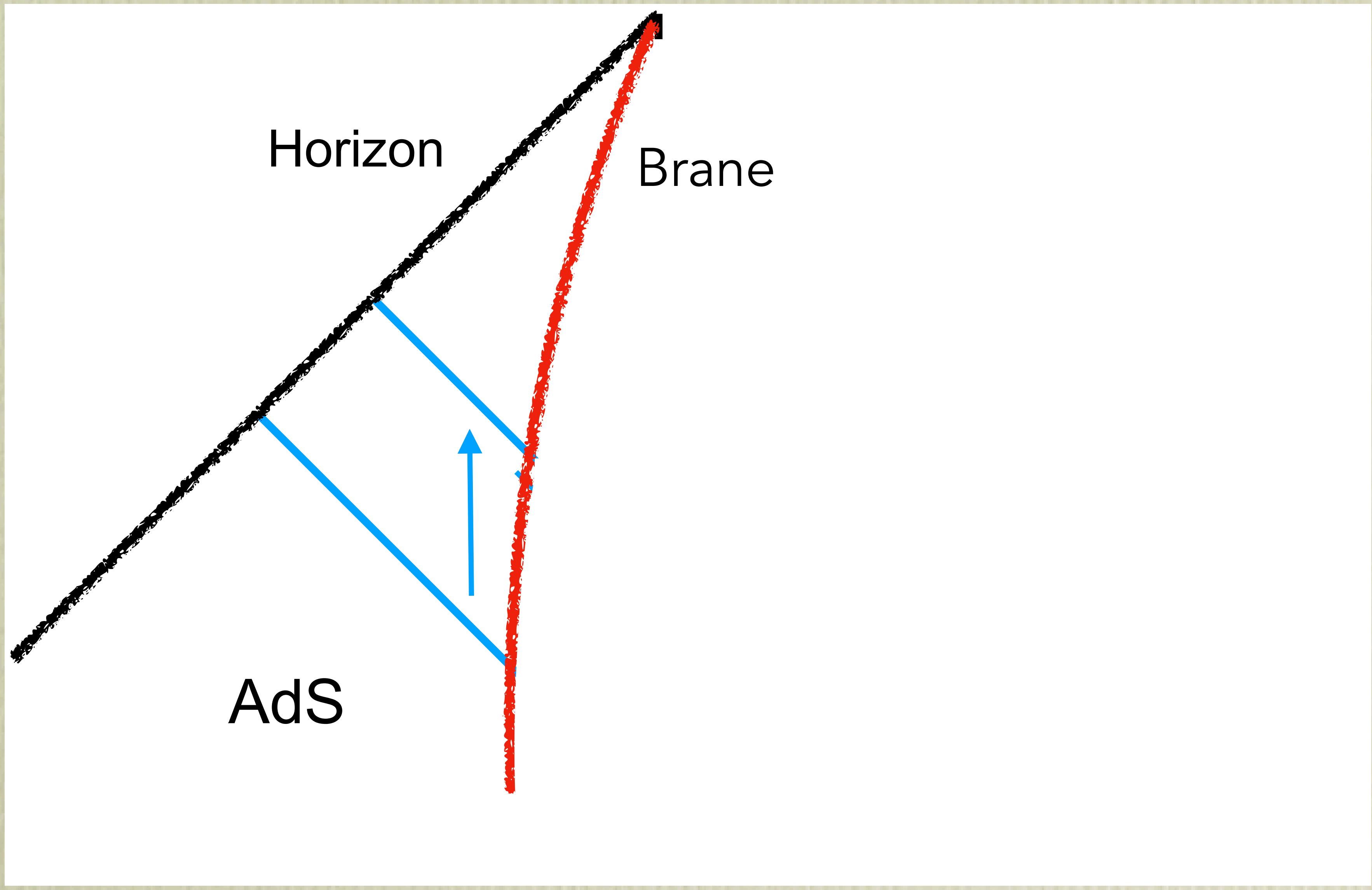
Dvali '10

and one finds $$M_{\text{pathologies}} \geq M_{\text{species}}$$

Chester & Loeb '20
Ghosh, Kiritsis, Nitti & Nourry '23

Dynamical gravity at the boundary

- Clarification for the experts: This is *not* brane-world-holography.



Dynamical gravity at the boundary

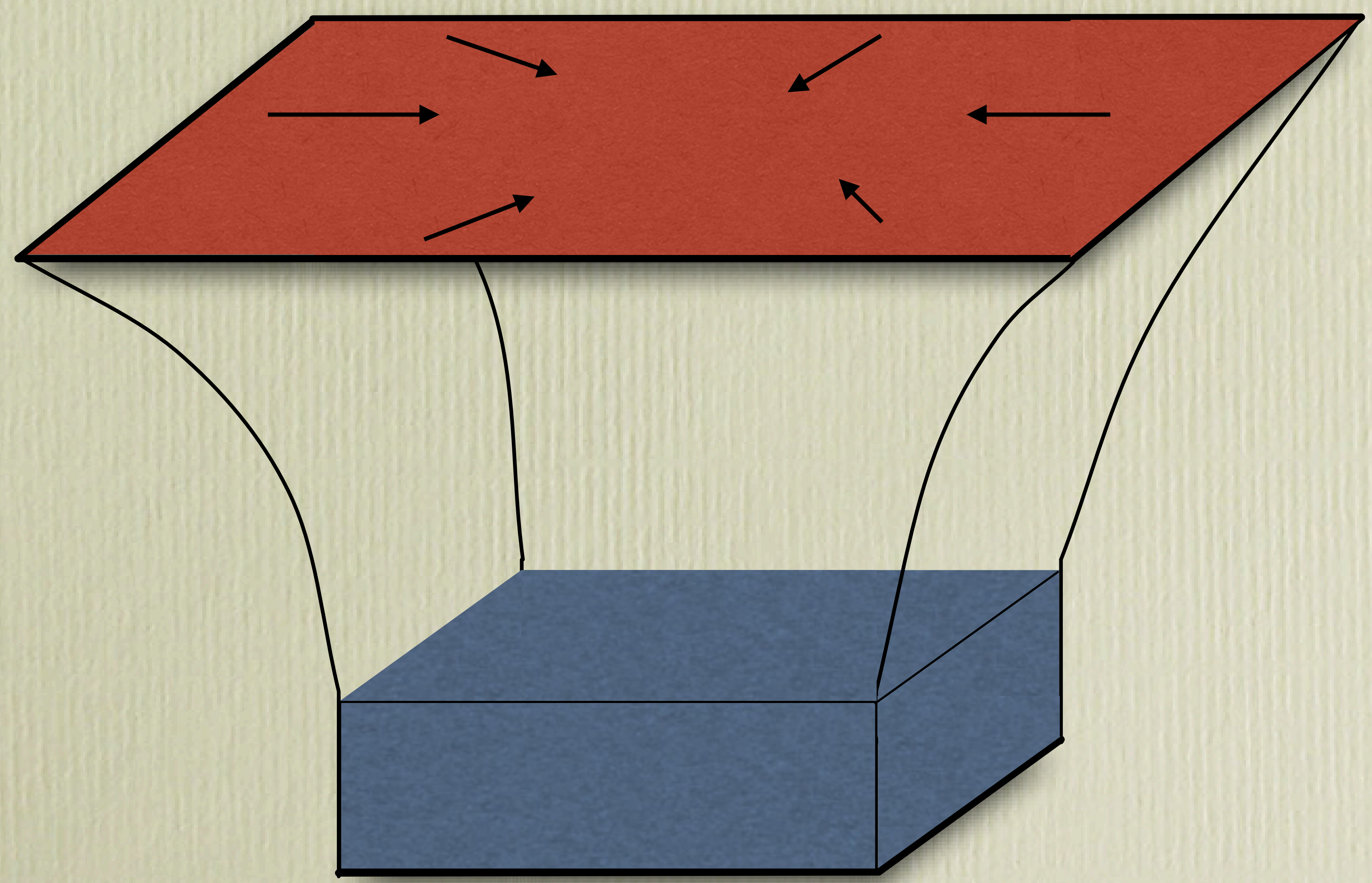
- Has been successfully applied in very symmetric cases:
  - De Sitter — Casalderrey, Ecker, DM & van der Schee '21
  - Inflation+Reheating: — Ecker, Kiritsis & van der Schee '22
- Today we will apply it to the hallmark of dynamical gravity: Gravitational collapse

Gravitational Collapse at the Boundary

Gravitational collapse at the boundary

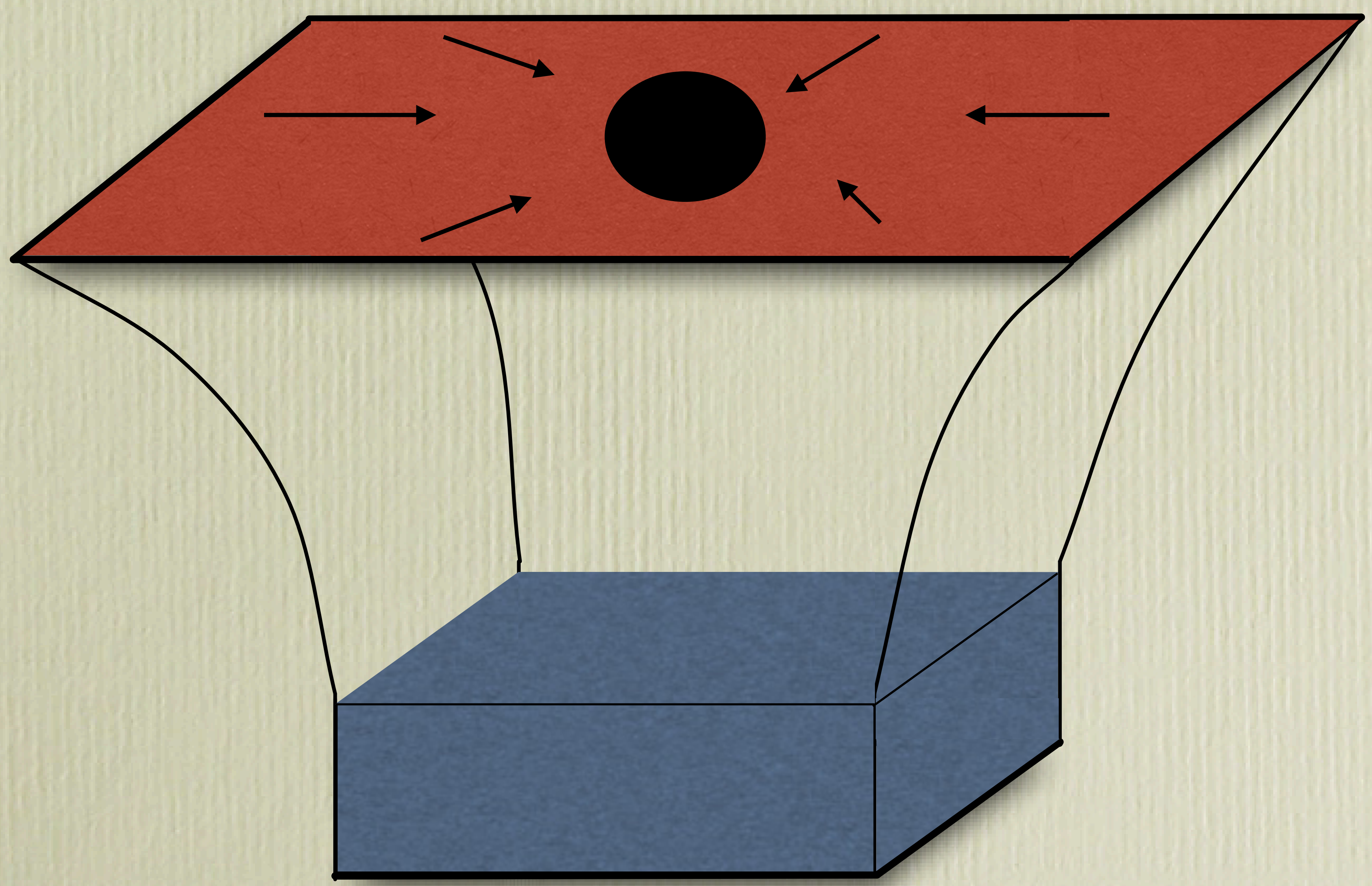
DM, Serantes & Sole (in preparation)

- Consider matter collapsing...



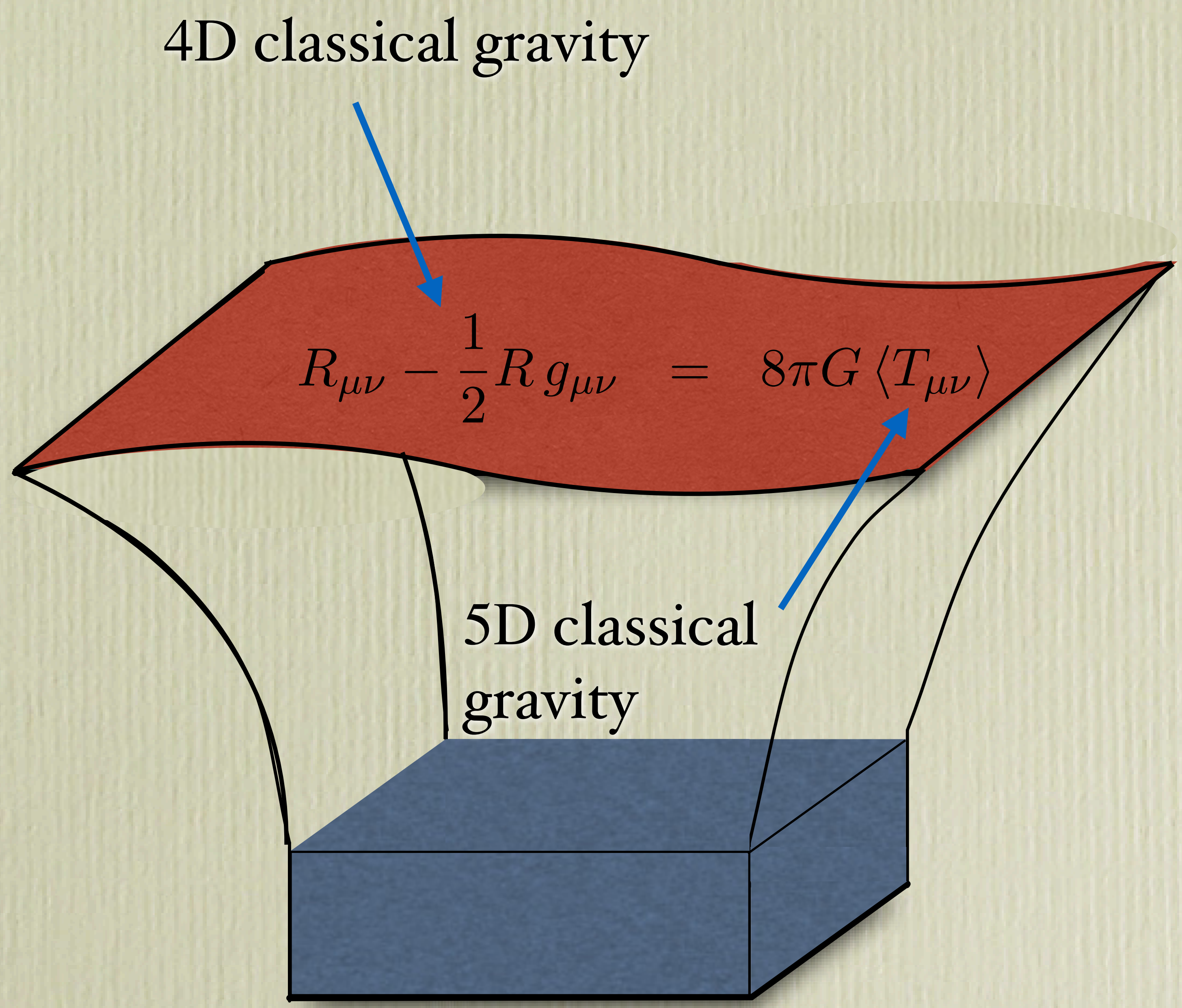
Gravitational collapse at the boundary

- Consider matter collapsing... to form a BH:



Gravitational collapse at the boundary

- This is a hard problem:

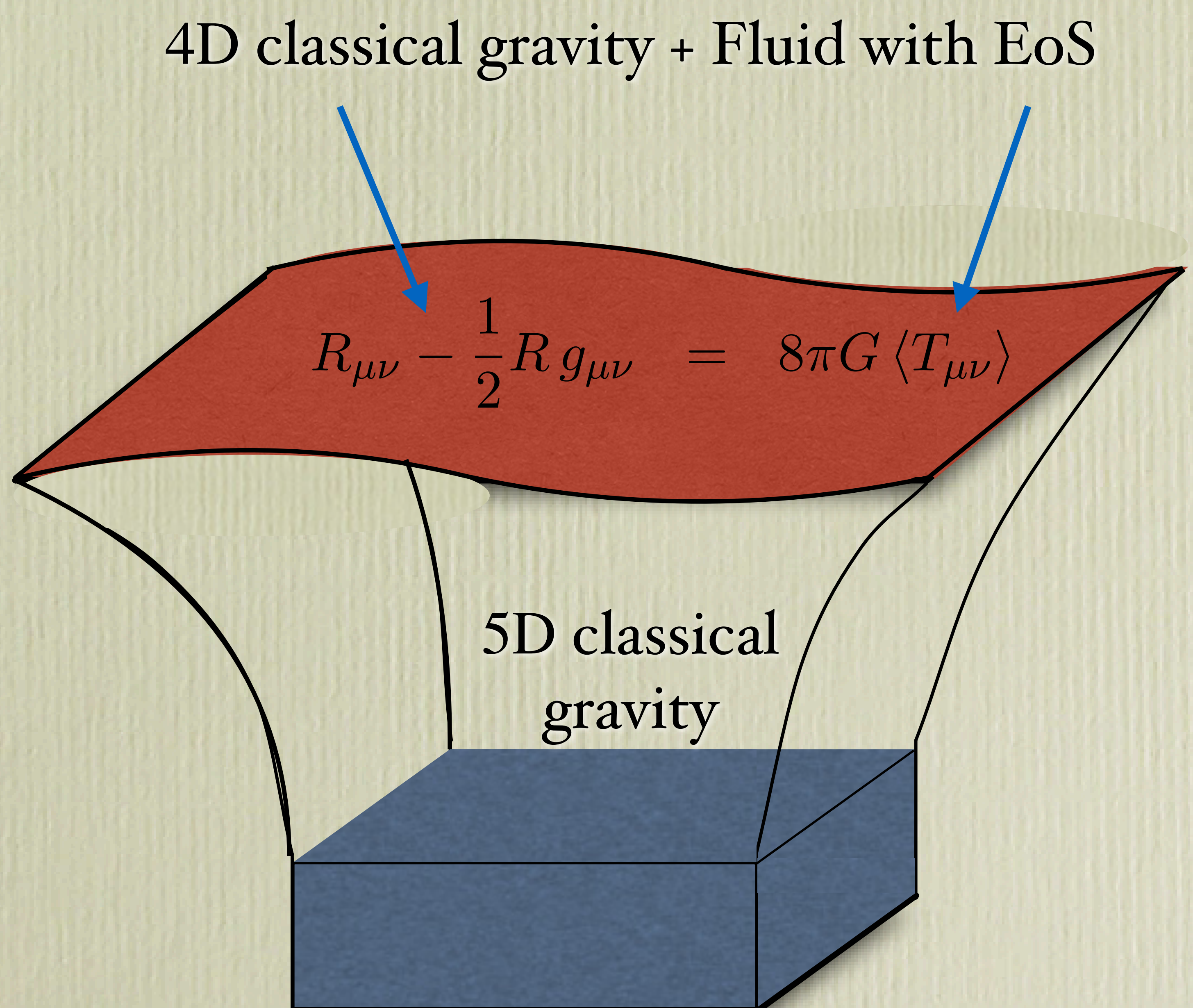


Gravitational collapse at the boundary

- We will work in the limit in which the matter can be described as a fluid (for some time).
- This is a very physical limit.
- It simplifies the problem dramatically for 2 reasons.

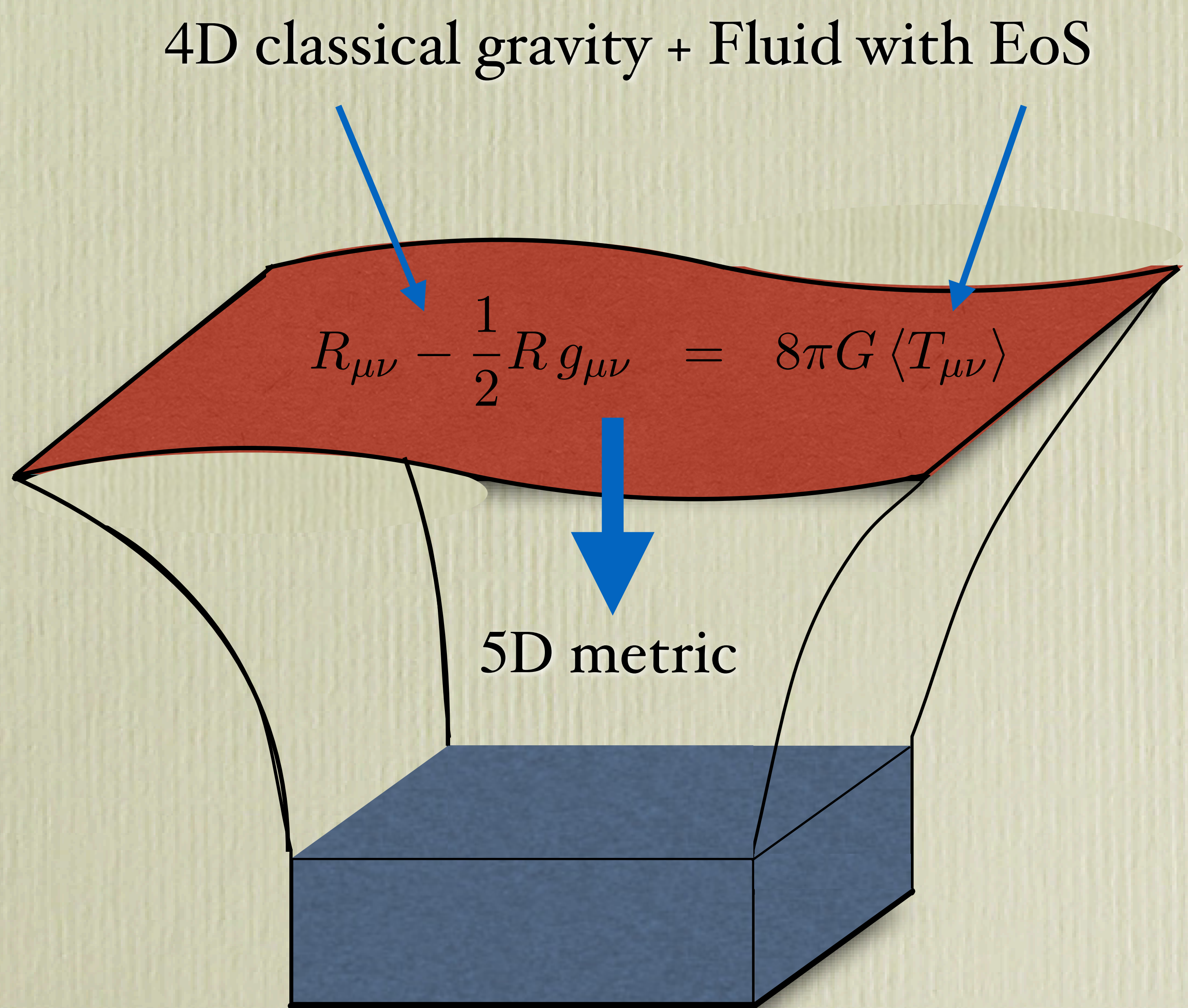
Gravitational collapse at the boundary

- First, we can evolve the boundary independently.



Gravitational collapse at the boundary

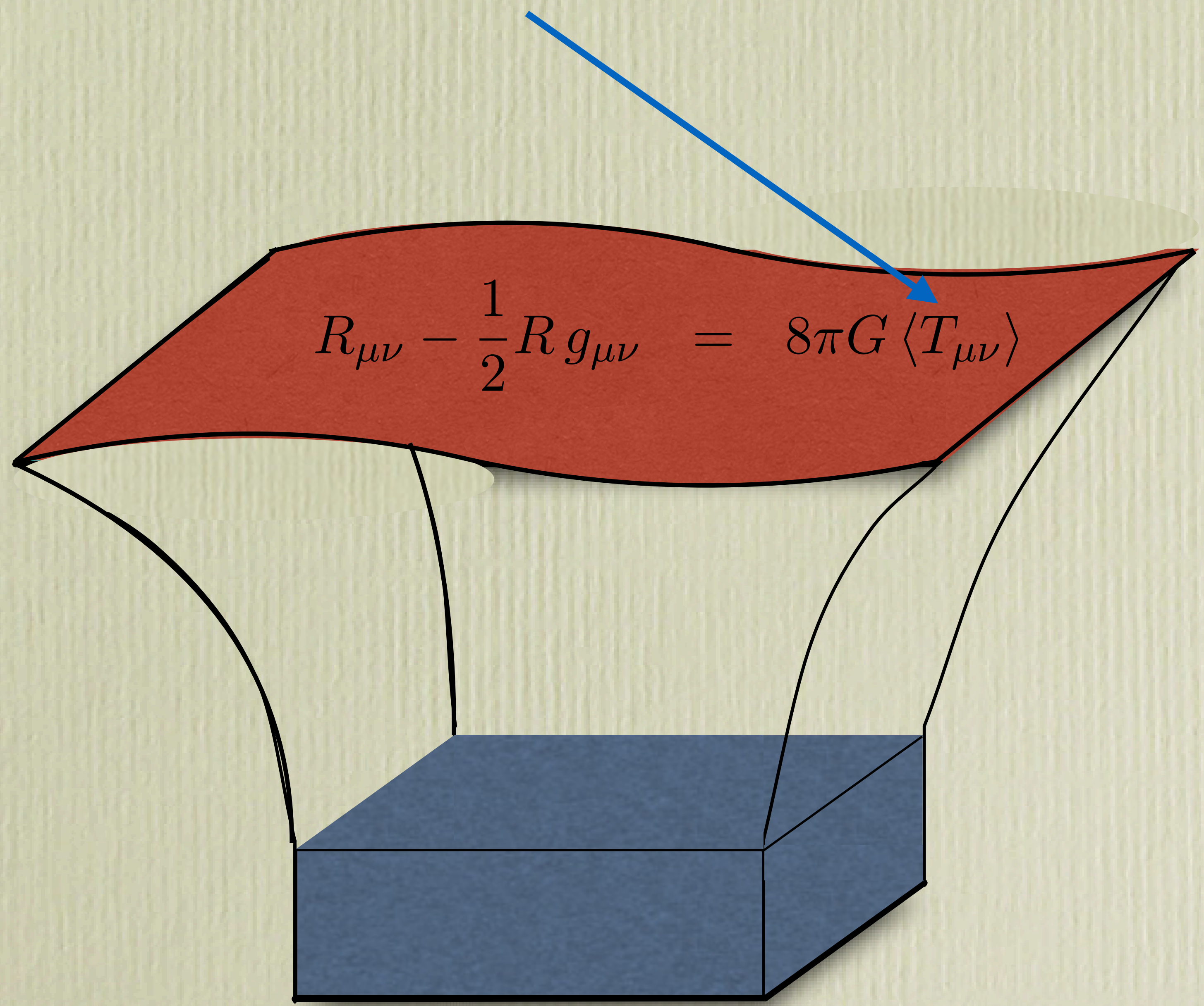
- Second, we can use fluid/gravity to construct bulk solution.



Gravitational collapse at the boundary

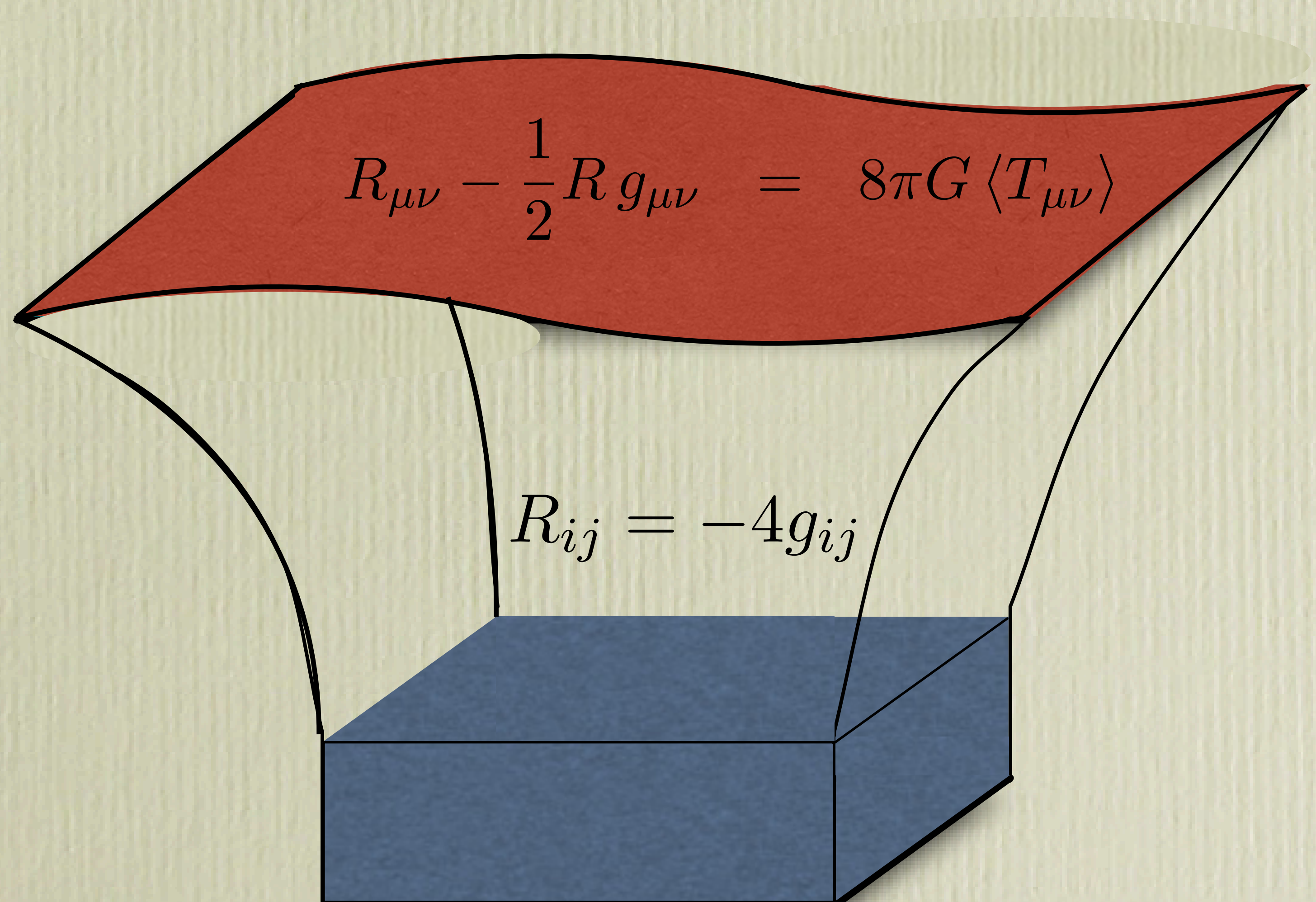
- Will also assume a *conformal* and *ideal* fluid:

$$T^{\mu\nu} = (\mathcal{E} + P)\, u_\mu u_\nu + P g^{\mu\nu}\,, \qquad P = \mathcal{E}/3$$



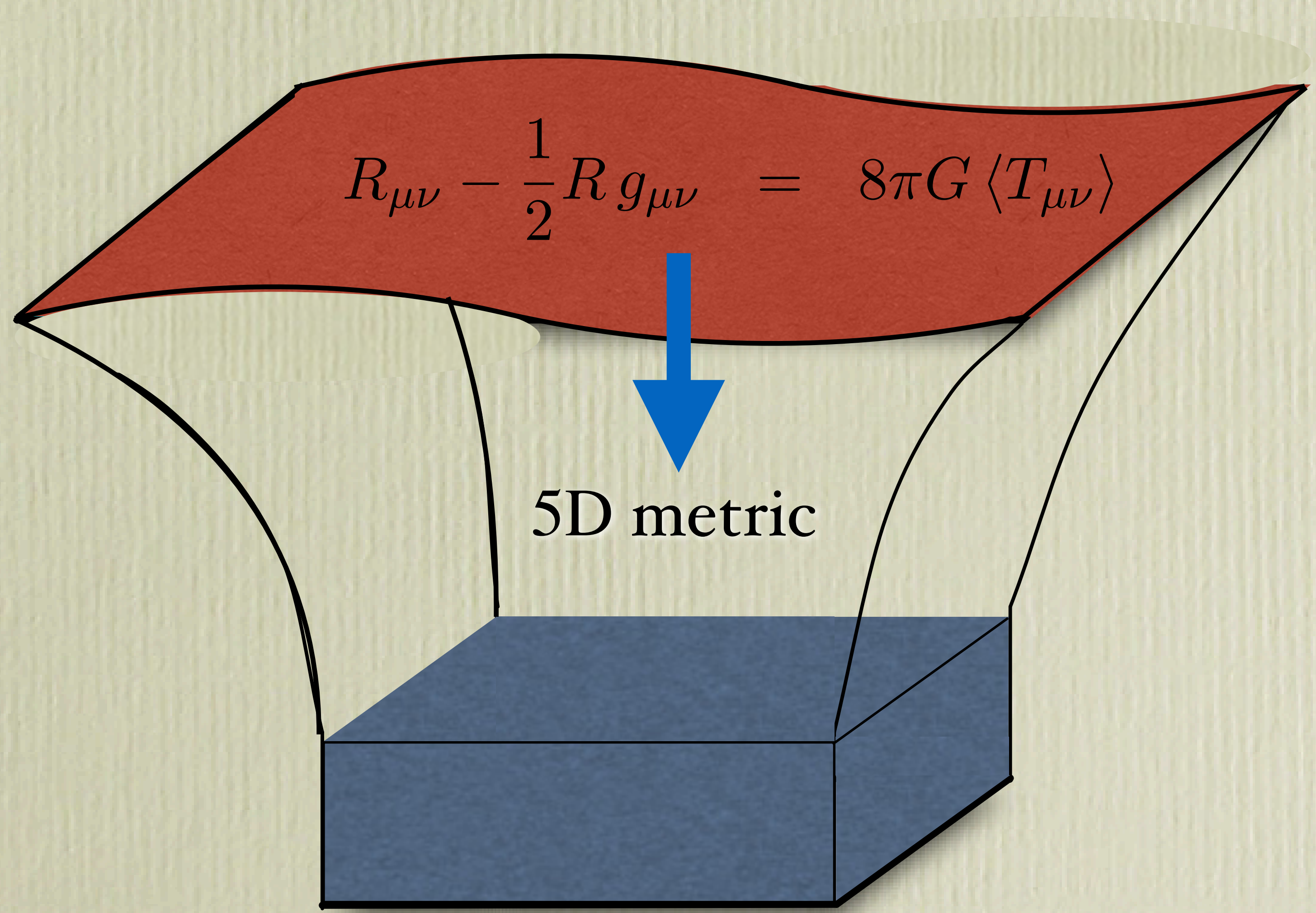
Gravitational collapse at the boundary

- Not essential but:
  - *Conformal* simplifies EoS and implies bulk is *pure gravity*.



Gravitational collapse at the boundary

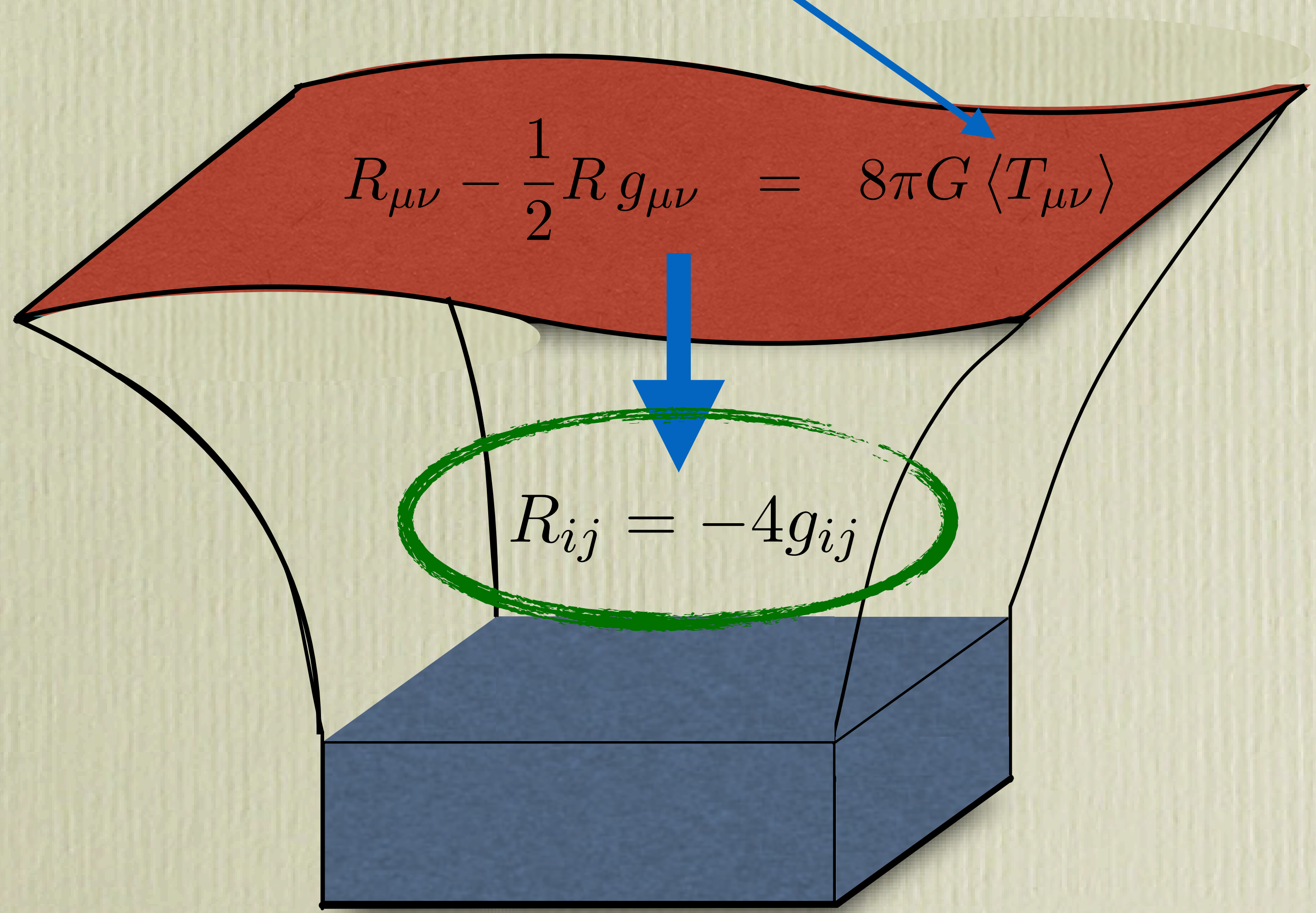
- Not essential but:
  - *Conformal* simplifies EoS and implies bulk is *pure gravity*.
  - *Ideal* simplifies hydro evolution and fluid/gravity map.



Gravitational collapse at the boundary

- After we find the solution we will verify the assumptions:

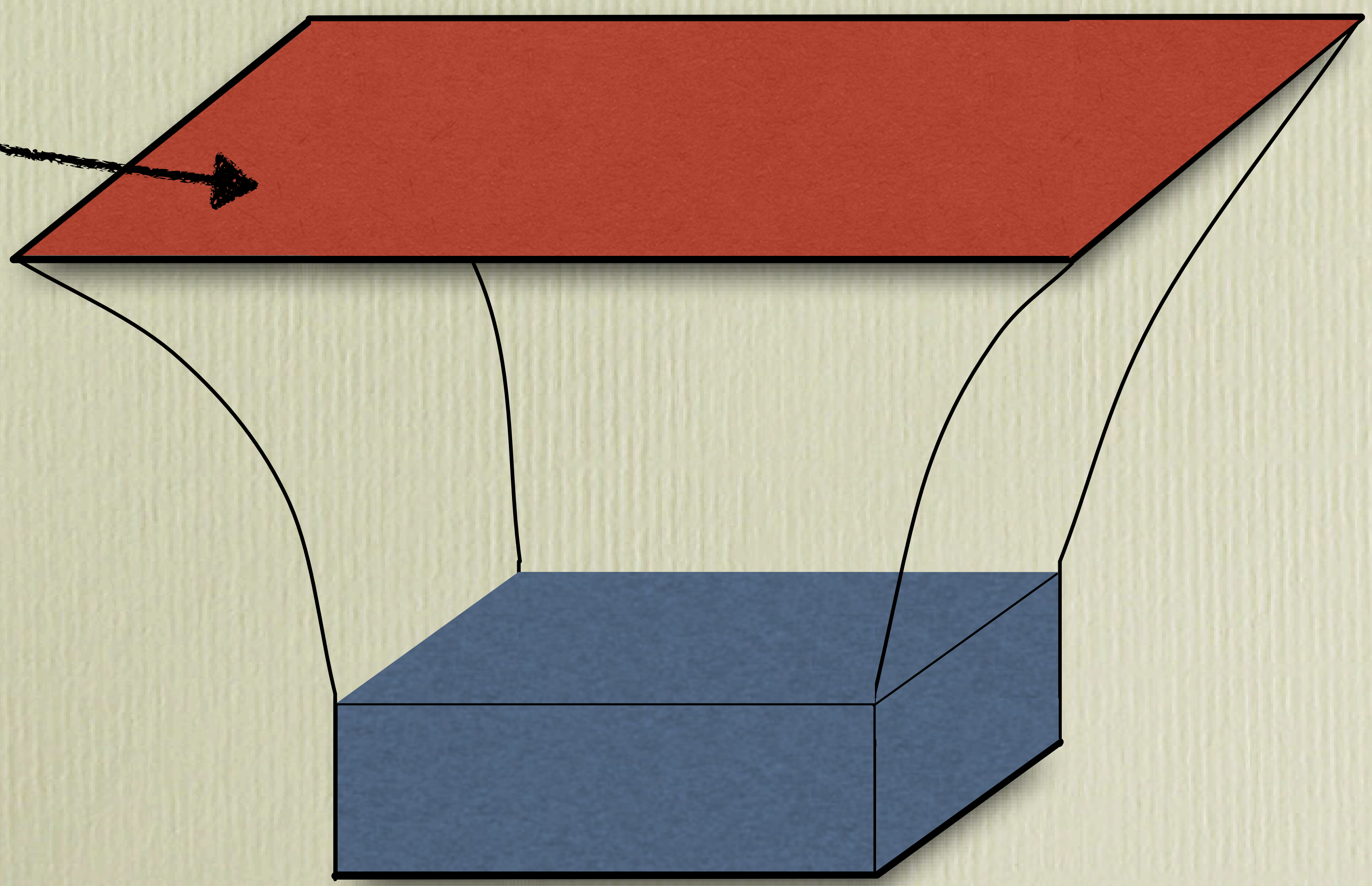
$$T^{\mu\nu} = (P + \mathcal{E})\, u^\mu u^\nu + P g^{\mu\nu}$$



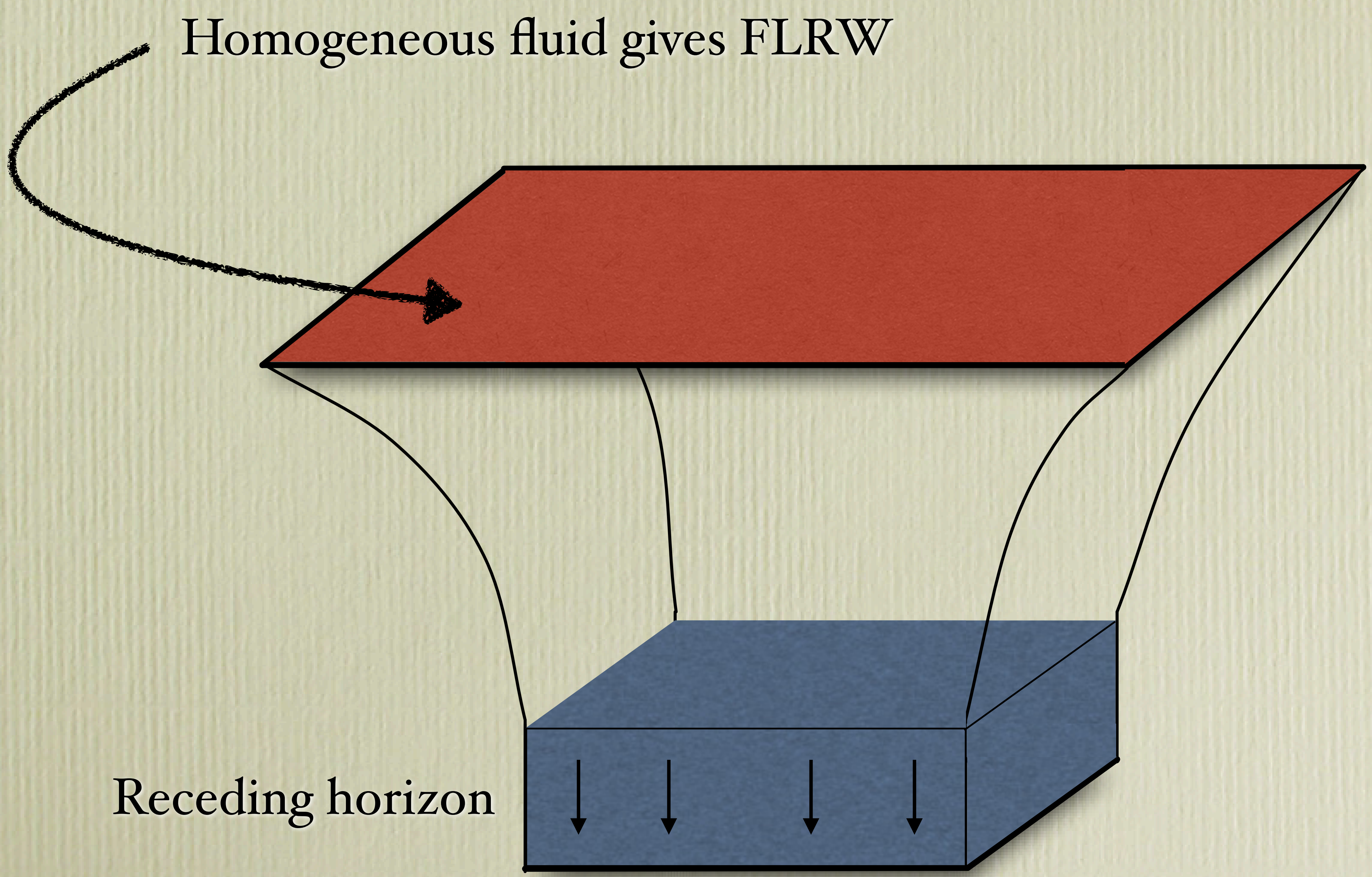
Gravitational collapse at the boundary

- Before details, here is an overview:

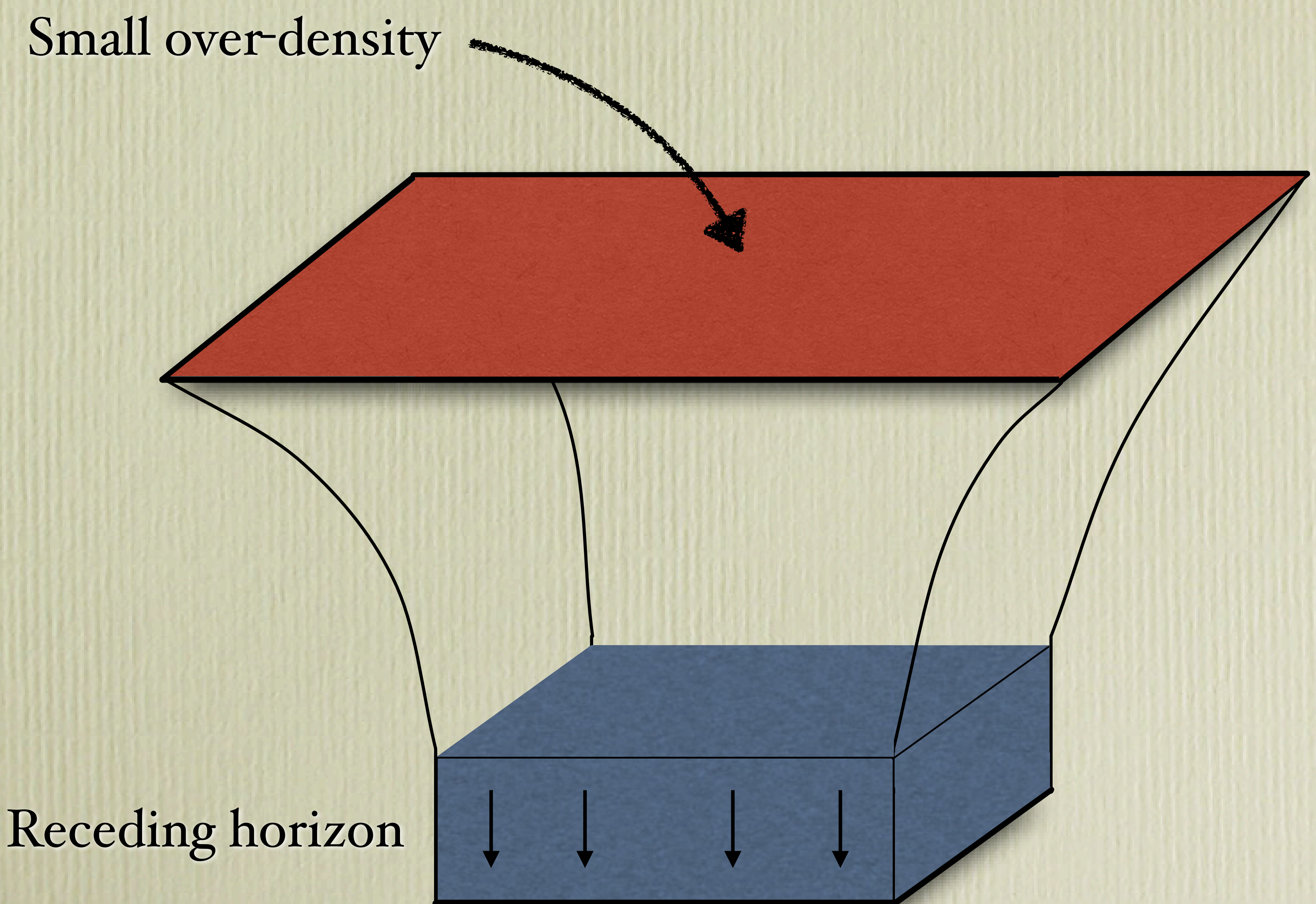
Homogeneous fluid gives FLRW



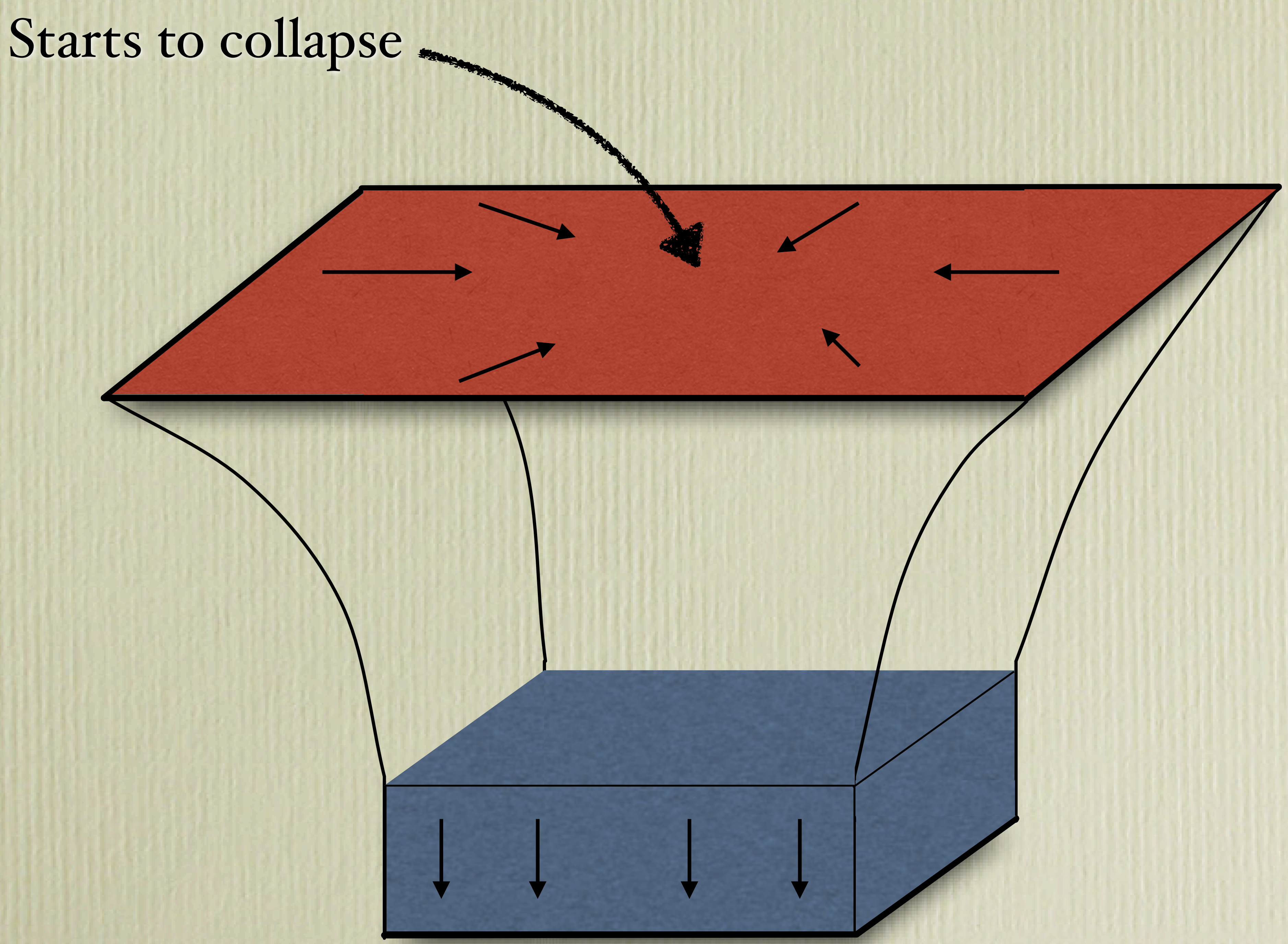
Gravitational collapse at the boundary



Gravitational collapse at the boundary

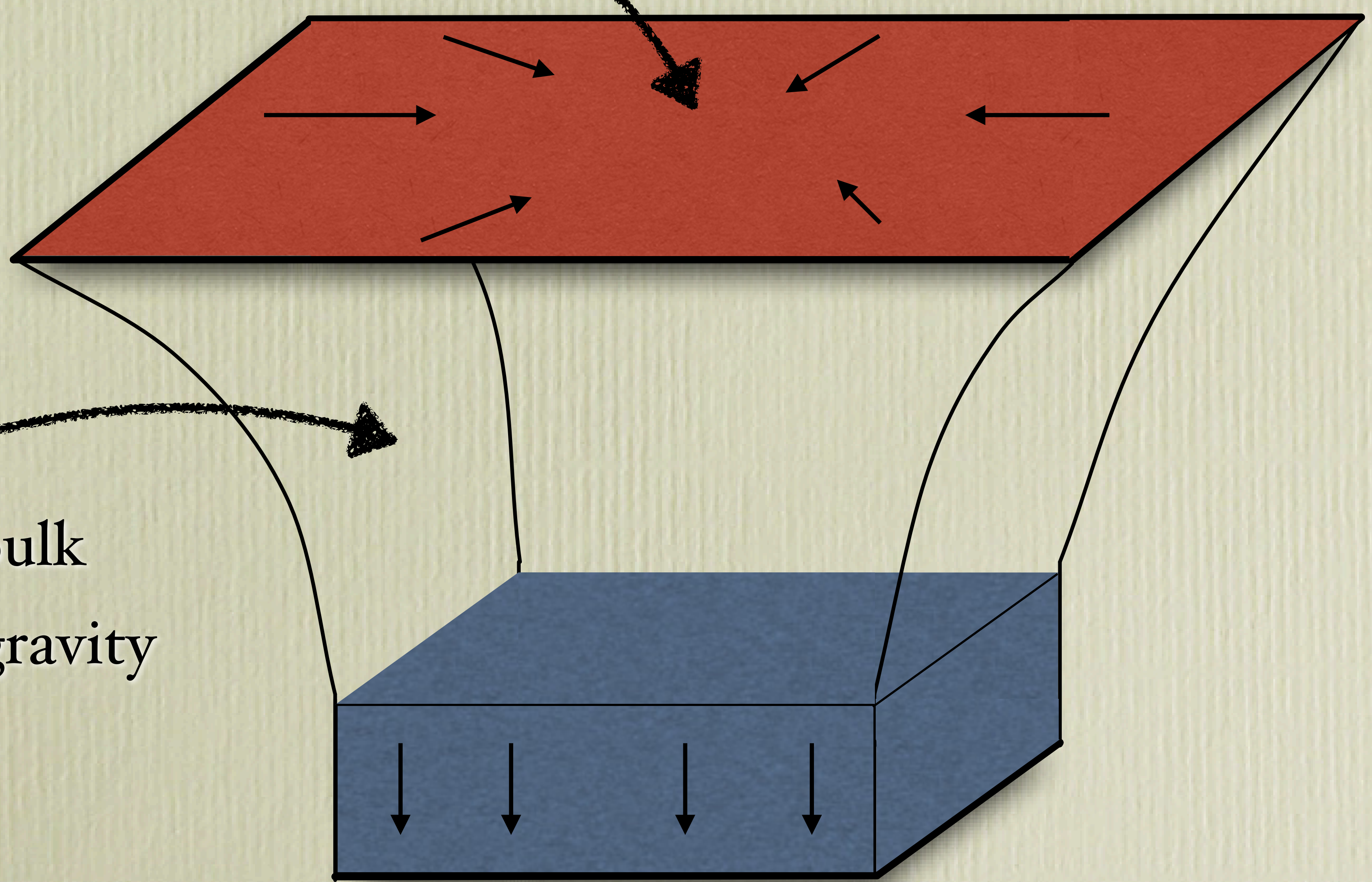


Gravitational collapse at the boundary



Gravitational collapse at the boundary

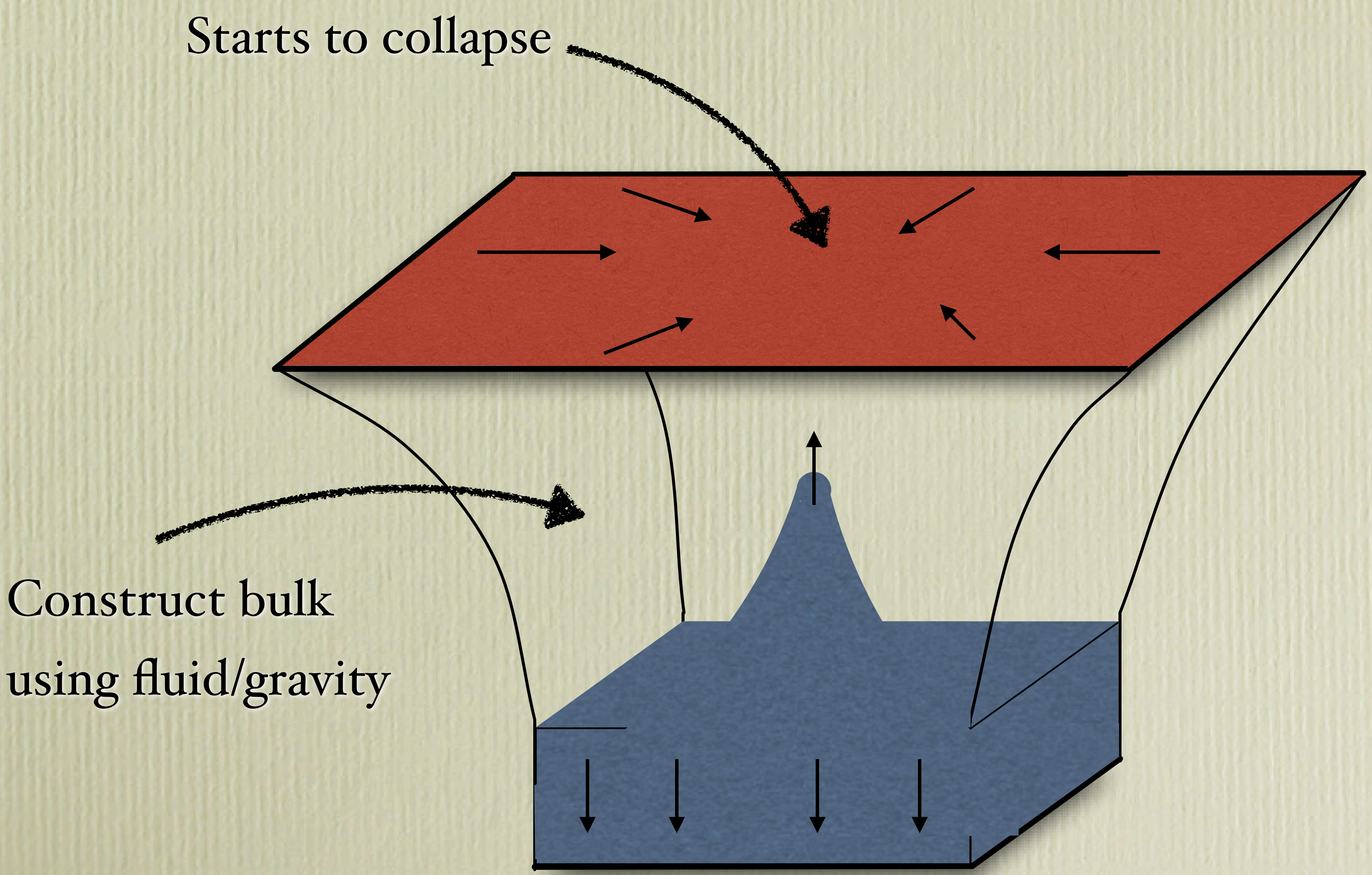
Starts to collapse



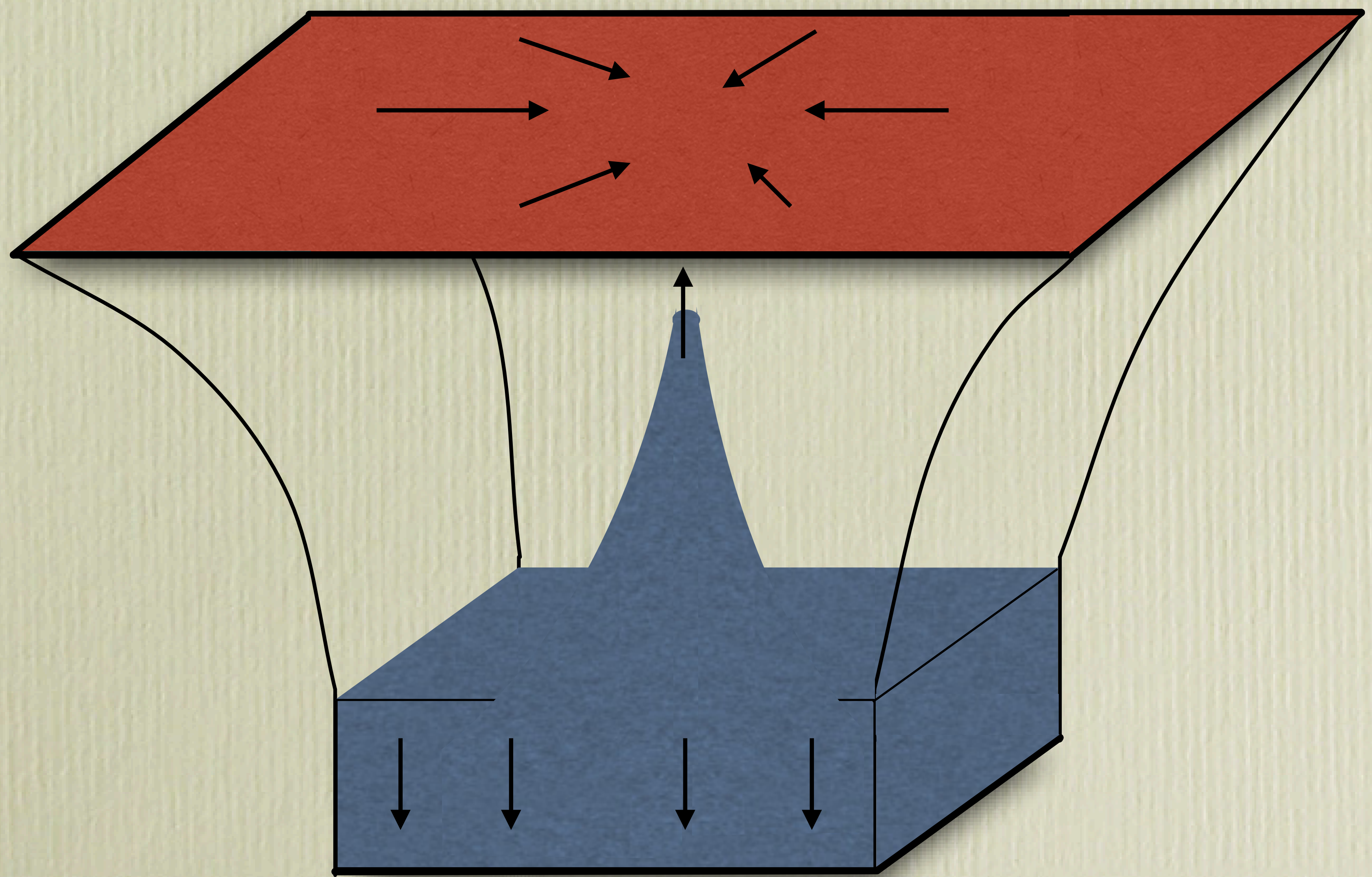
Construct bulk

using fluid/gravity

Gravitational collapse at the boundary



Gravitational collapse at the boundary



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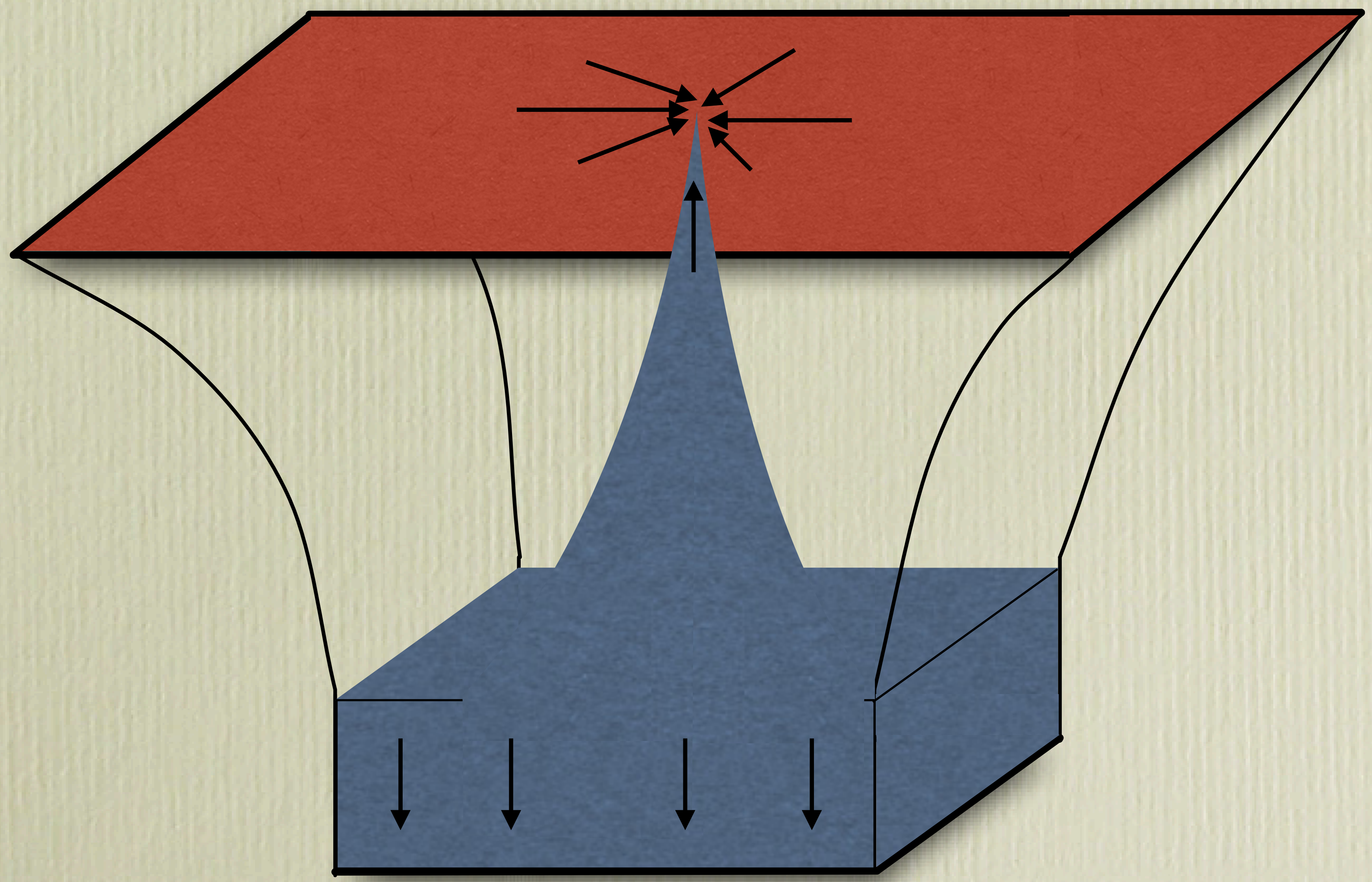
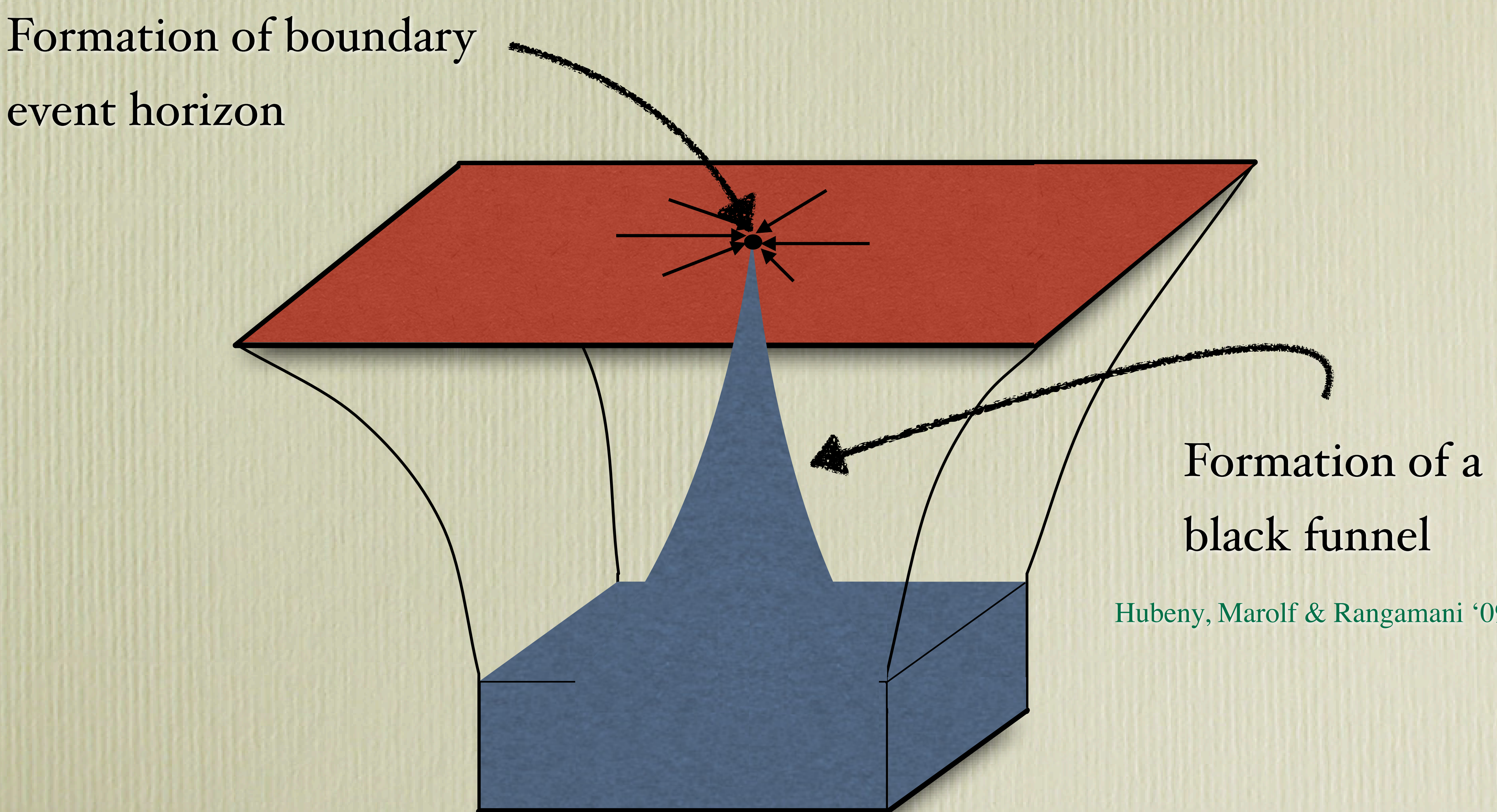


Gravitational collapse at the boundary

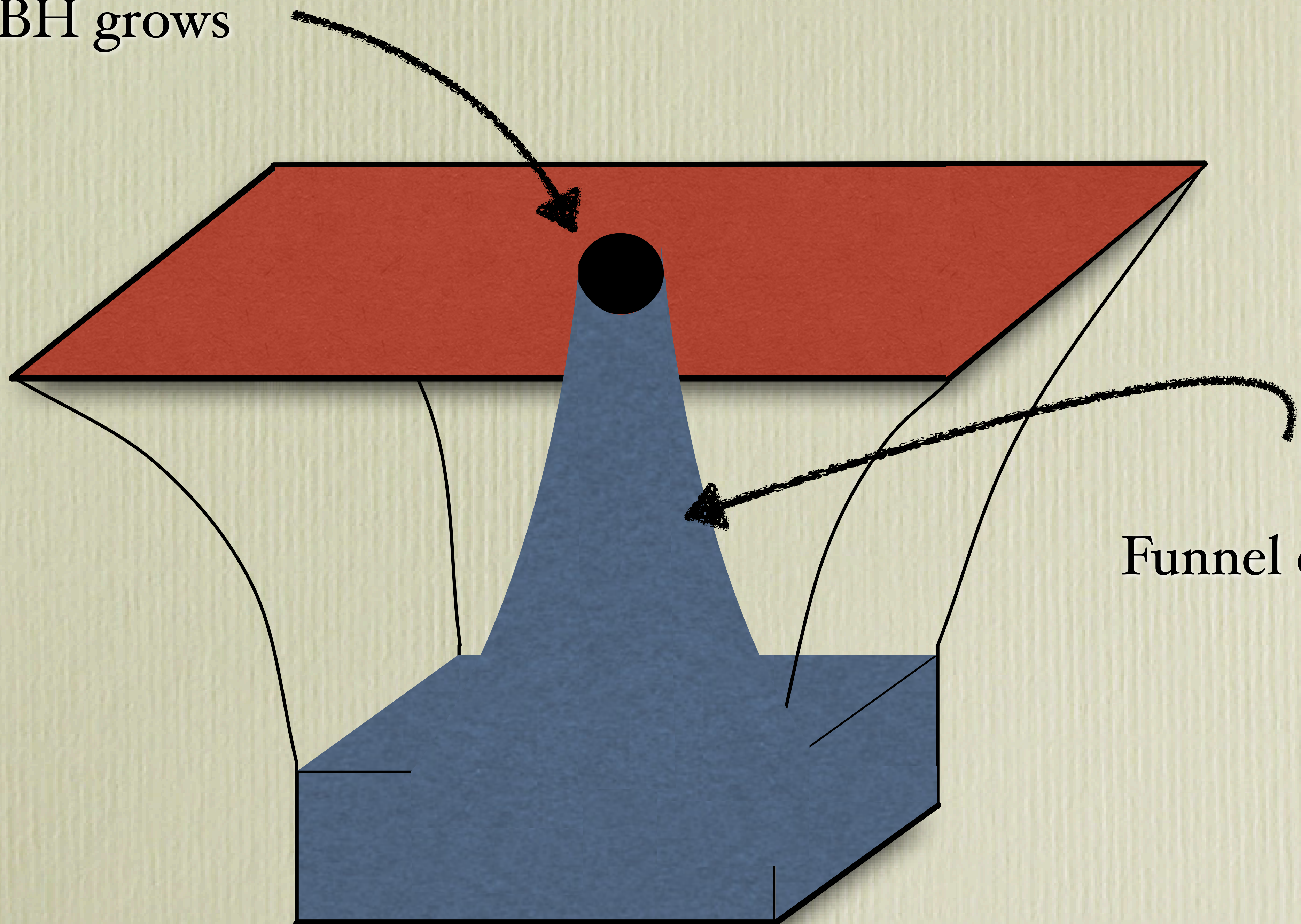
(cf Emparan, Luna, Suzuki, Tomasevic & Way ’23)



Gravitational collapse at the boundary

(cf Emparan, Luna, Suzuki, Tomasevic & Way '23)

Boundary BH grows

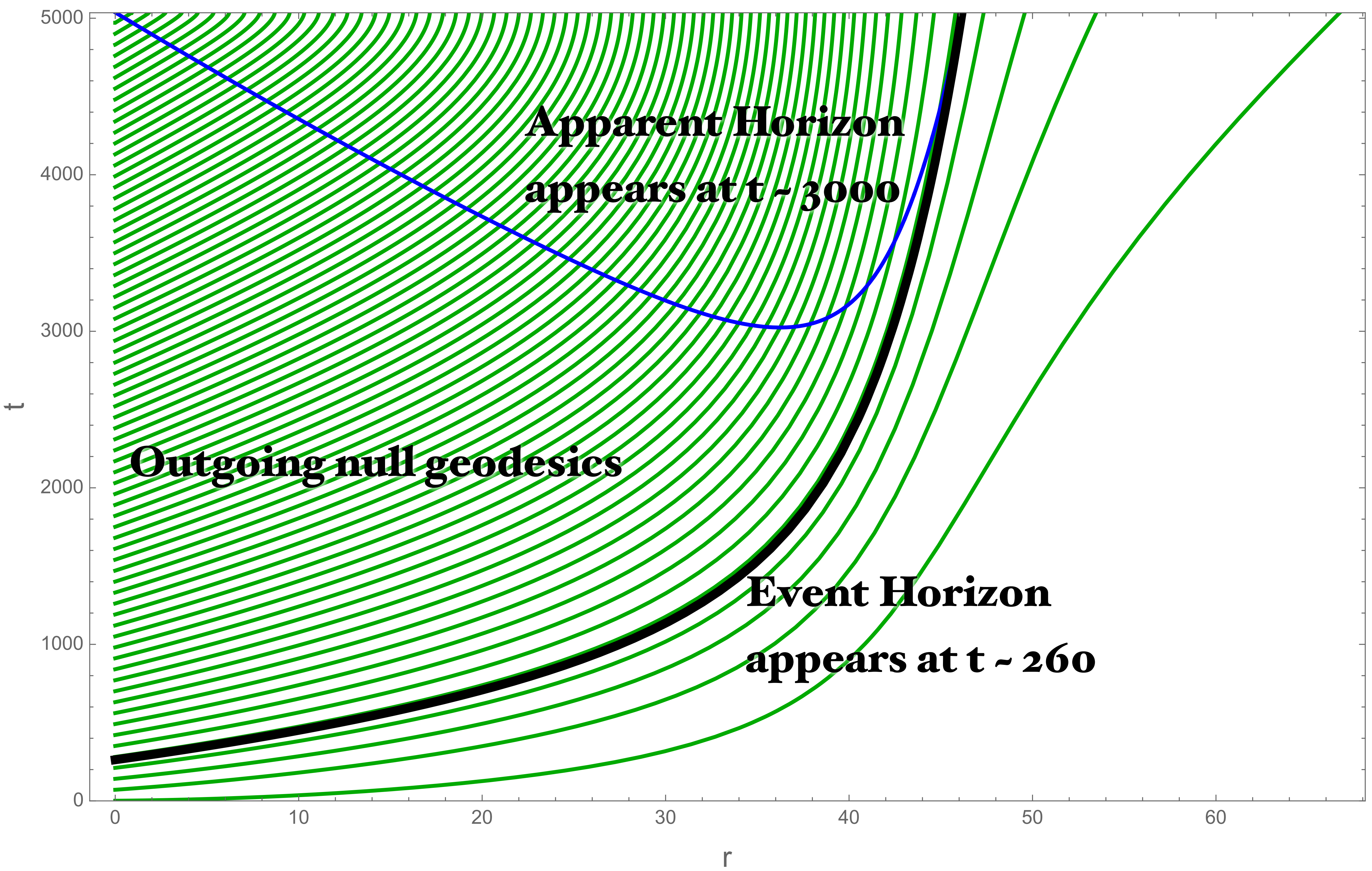


Funnel expands

Gravitational collapse at the boundary

- Calculation is identical to PBH formation.
- With the solution in hand we can analyse its causal structure.

Boundary dynamics



Corrections to ideal fluid

- At leading order there are two corrections:

$$T_{ij} \quad = \quad T_{ij}^{\mathrm{ideal}} - \eta \, \nabla_{\langle i} v_{j \rangle} - \xi \, \left(\nabla_k v^k \right) \delta_{ij} + O(\nabla^2)$$

Corrections to ideal fluid

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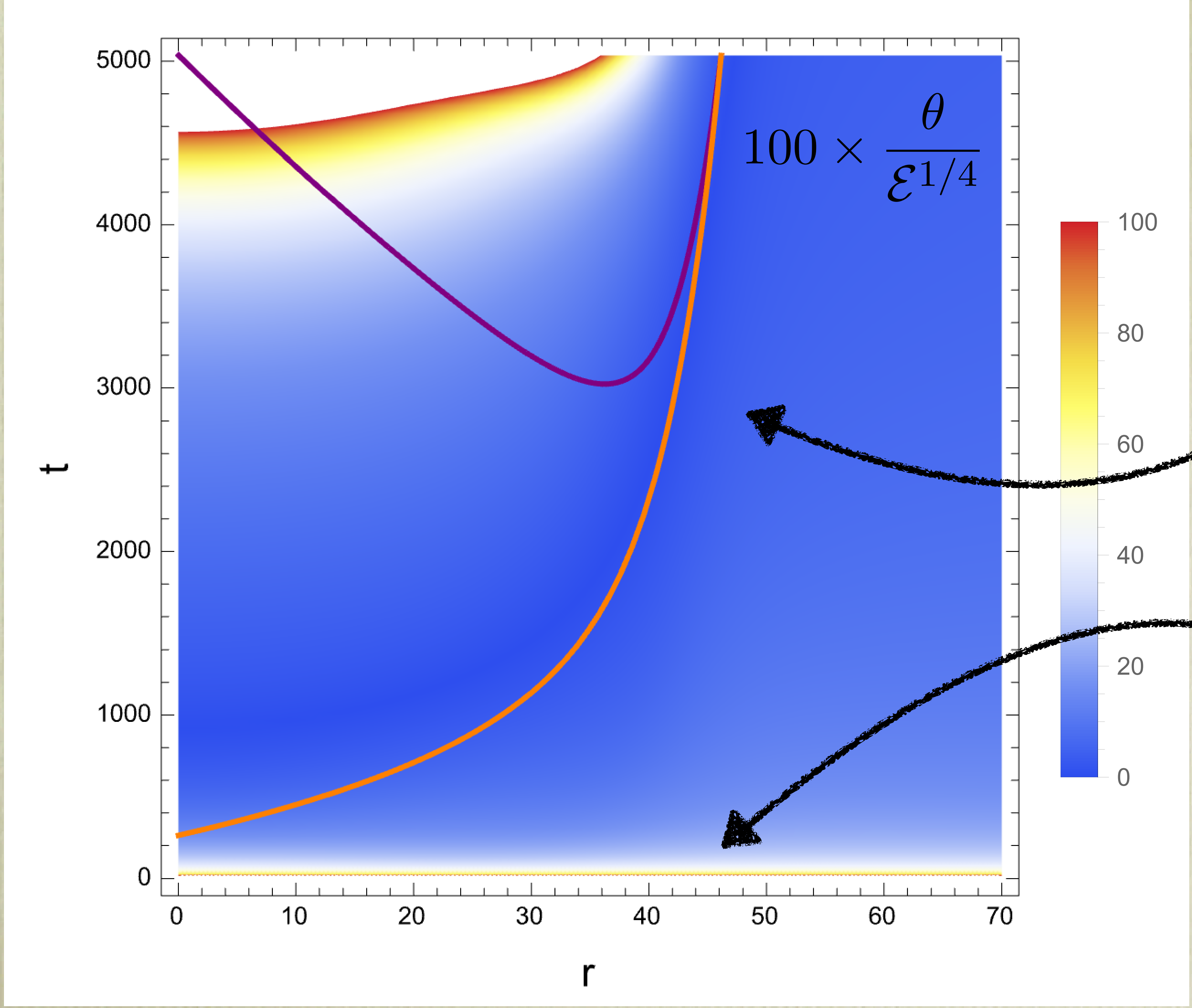
$$T_{ij} \;=\; T_{ij}^{\rm ideal} - \eta\,\nabla_{\langle i}v_{j\rangle} - \xi\,\left(\nabla_k v^k\right)\delta_{ij} + O(\nabla^2)$$

$$\sigma^2 \;\equiv\; \left(\nabla_{\langle i}v_{j\rangle}\right)\left(\nabla^{\langle i}v^{j\rangle}\right)$$

$$\theta^2 \;\equiv\; \left(\nabla^k v_k\right)^2$$

- Will plot: $100 \times \dfrac{\sigma}{\mathcal{E}^{1/4}}\,, \qquad 100 \times \dfrac{\theta}{\mathcal{E}^{1/4}}$

Corrections to ideal fluid

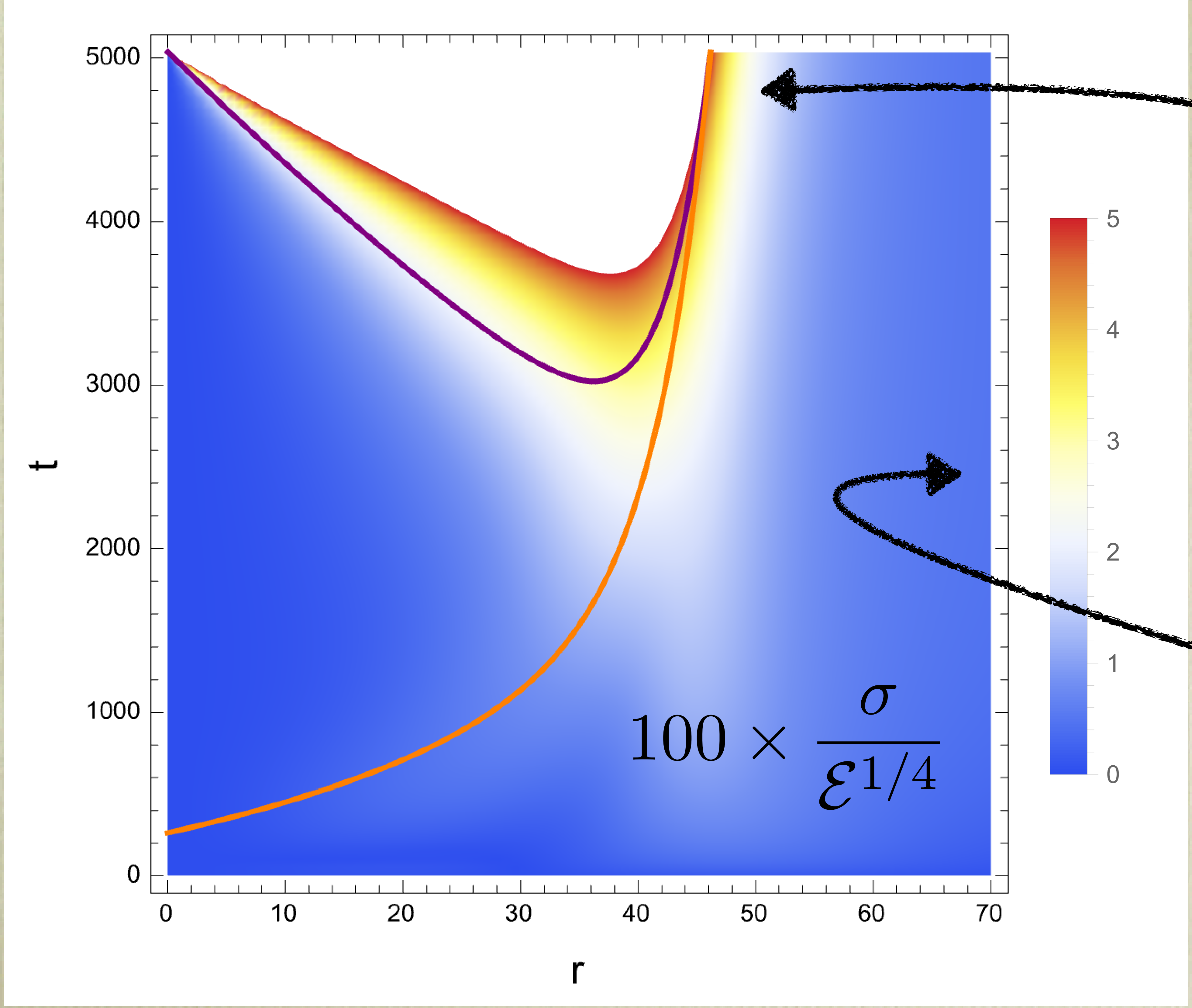


Small elsewhere outside the EH

In FLRW:

$$\frac{\theta}{\mathcal{E}^{1/4}} \sim \frac{1}{\sqrt{t}}$$

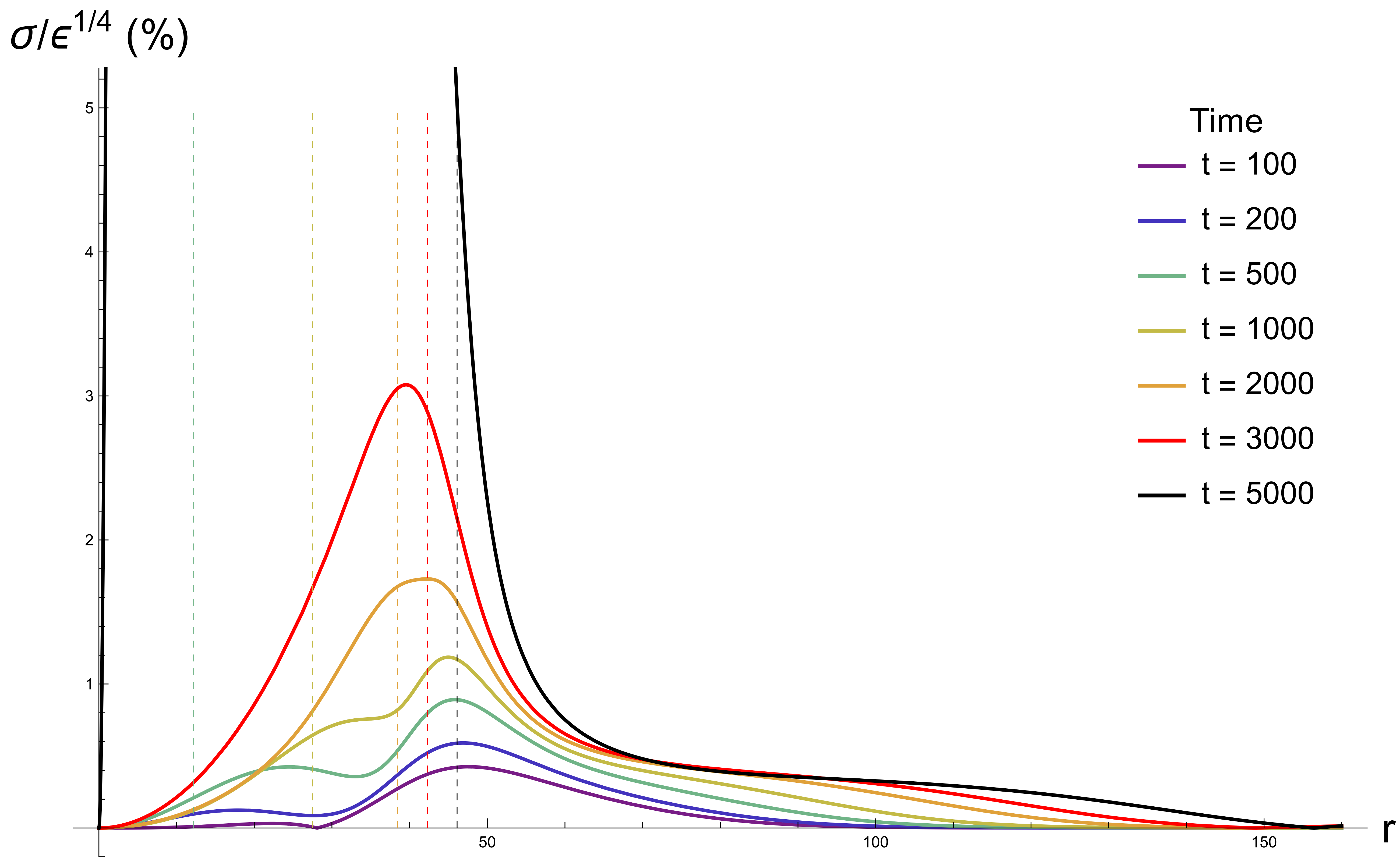
Corrections to ideal fluid



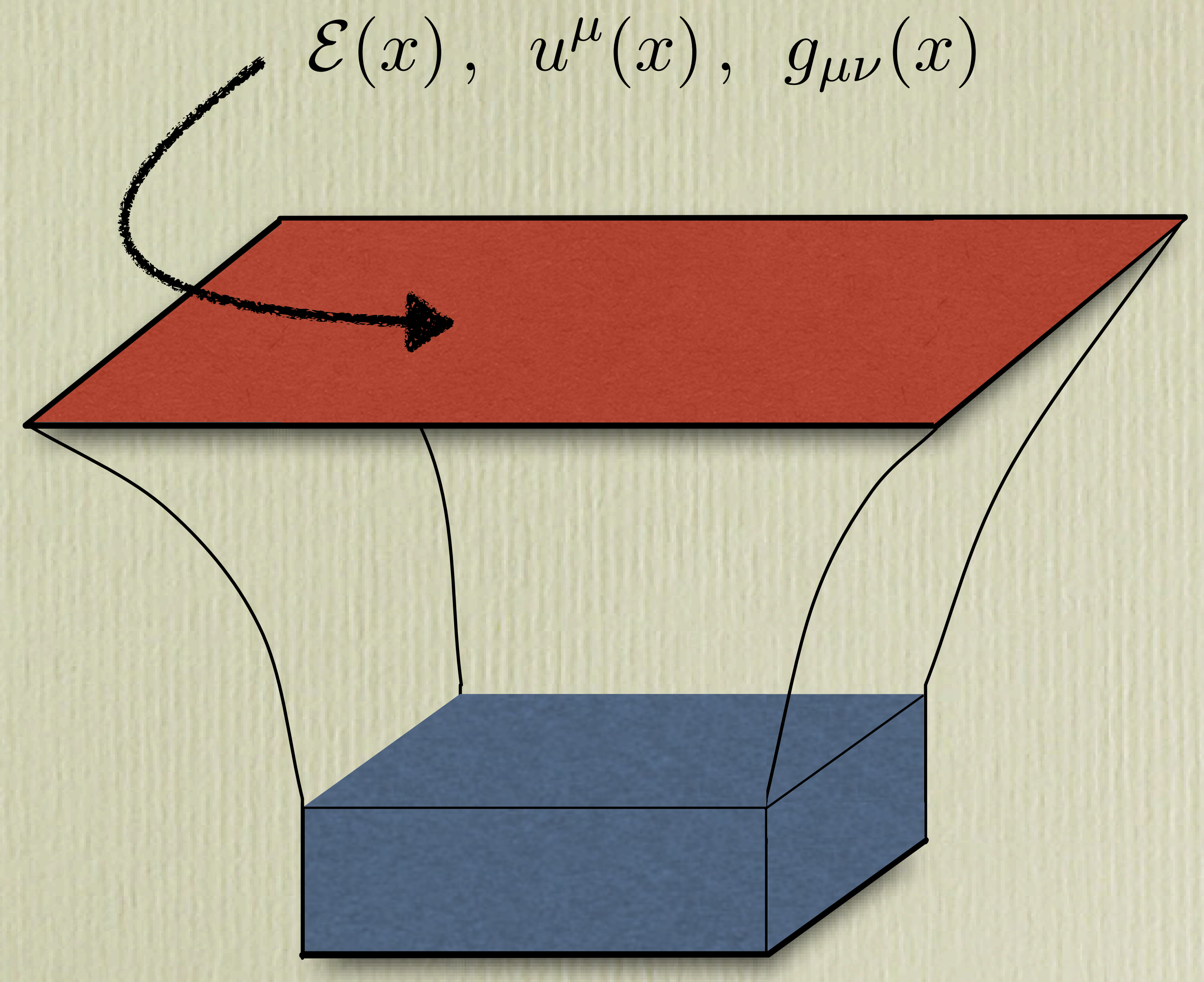
Small but seems to grow near the horizon at late times

Vanishes in FLRW

Corrections to ideal fluid

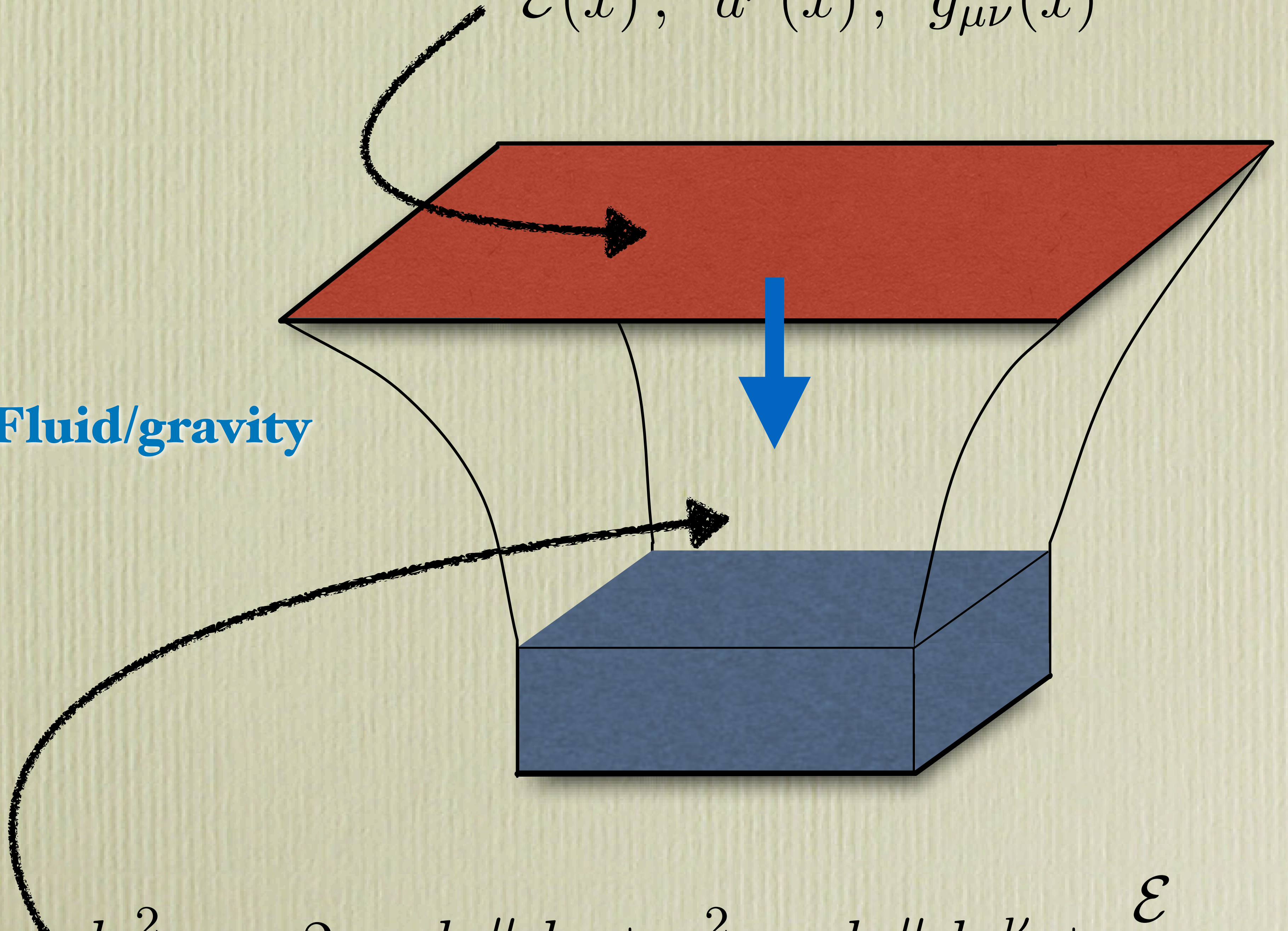


Construction of the bulk solution



Construction of the bulk solution

$$\mathcal{E}(x)\,,\quad u^\mu(x)\,,\quad g_{\mu\nu}(x)$$

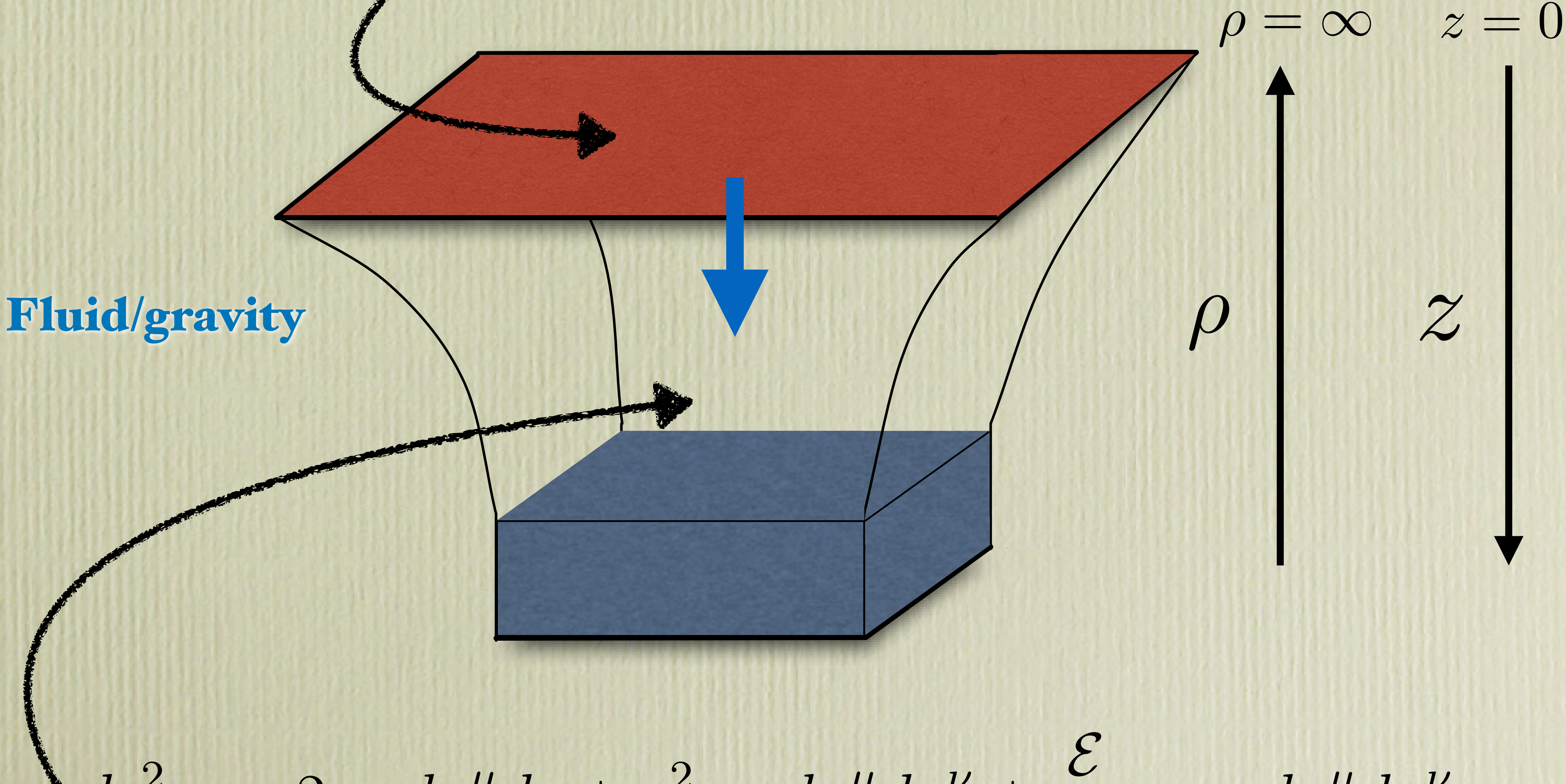


Fluid/gravity

$$ds^2 = -2u_\mu\, dx^\mu d\rho + \rho^2 g_{\mu\nu} dx^\mu dx^\nu + \frac{\mathcal{E}}{\rho^2} u_\mu u_\nu\, dx^\mu dx^\nu$$

Construction of the bulk solution

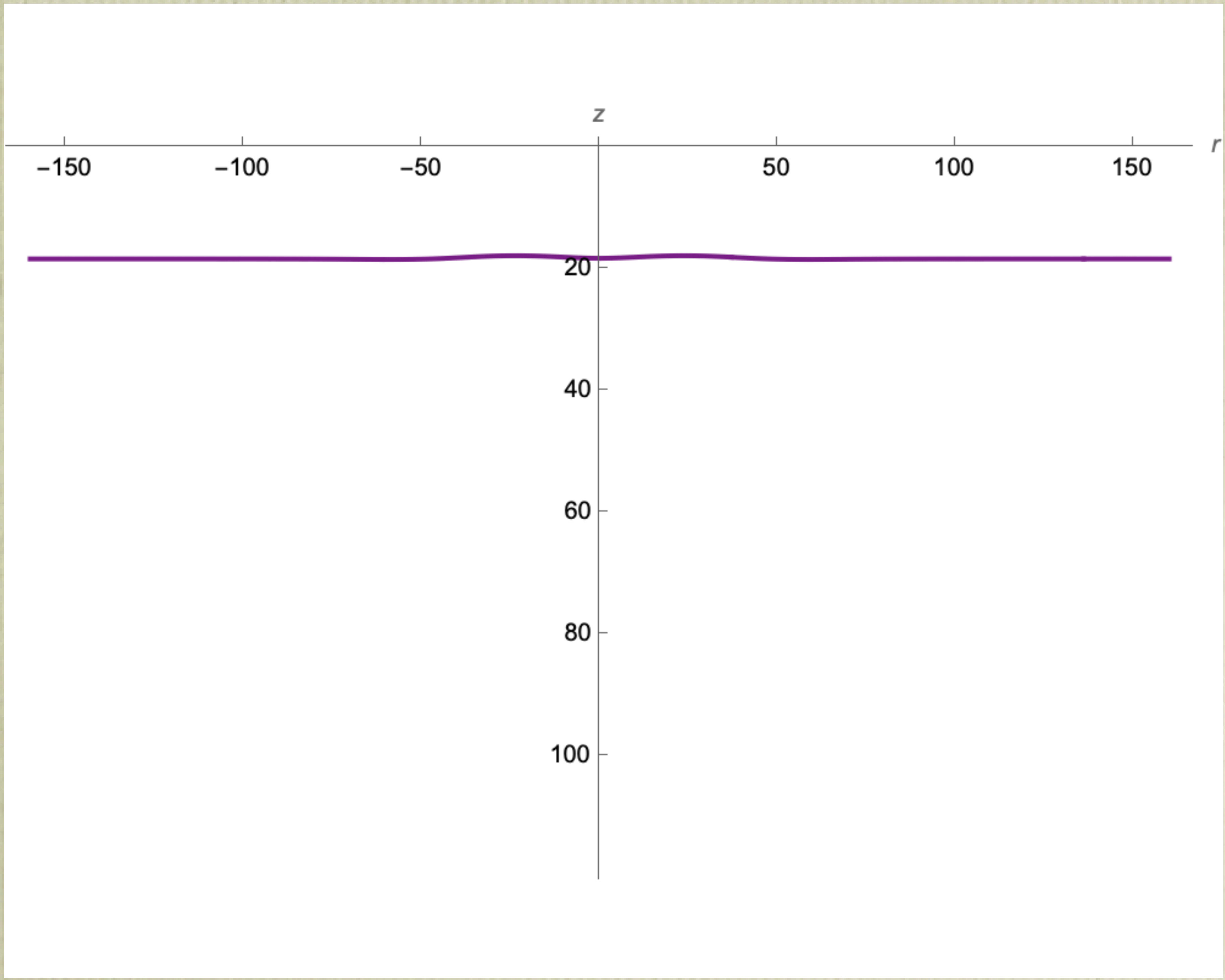
$$\mathcal{E}(x)\,,\quad u^\mu(x)\,,\quad g_{\mu\nu}(x)$$



$$ds^2 = -2u_\mu\, dx^\mu d\rho + \rho^2 g_{\mu\nu} dx^\mu dx^\nu + \frac{\mathcal{E}}{\rho^2} u_\mu u_\nu\, dx^\mu dx^\nu$$

Causal structure

- Shooting geodesics from each point we determine the EH:



Causal structure

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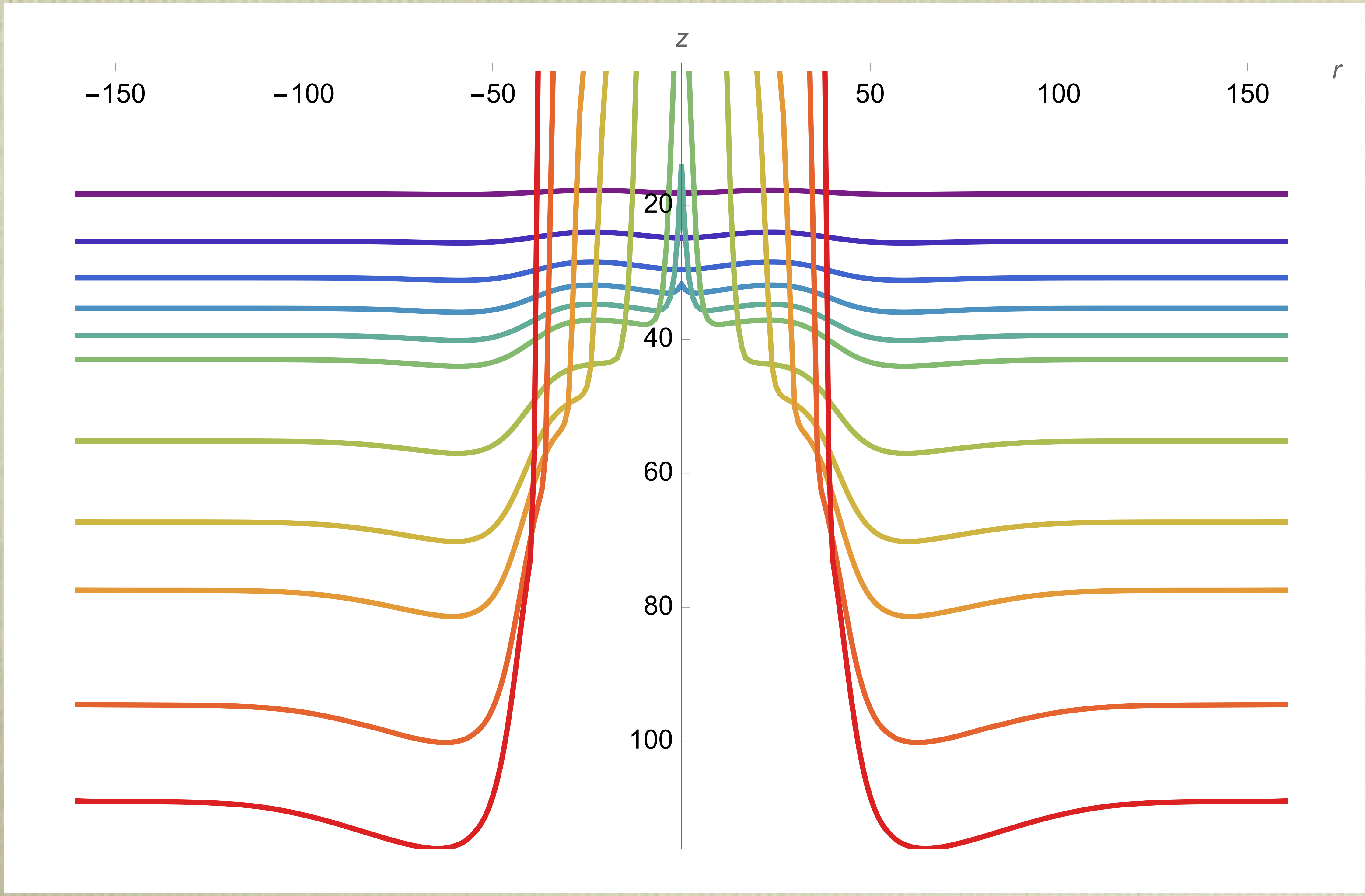
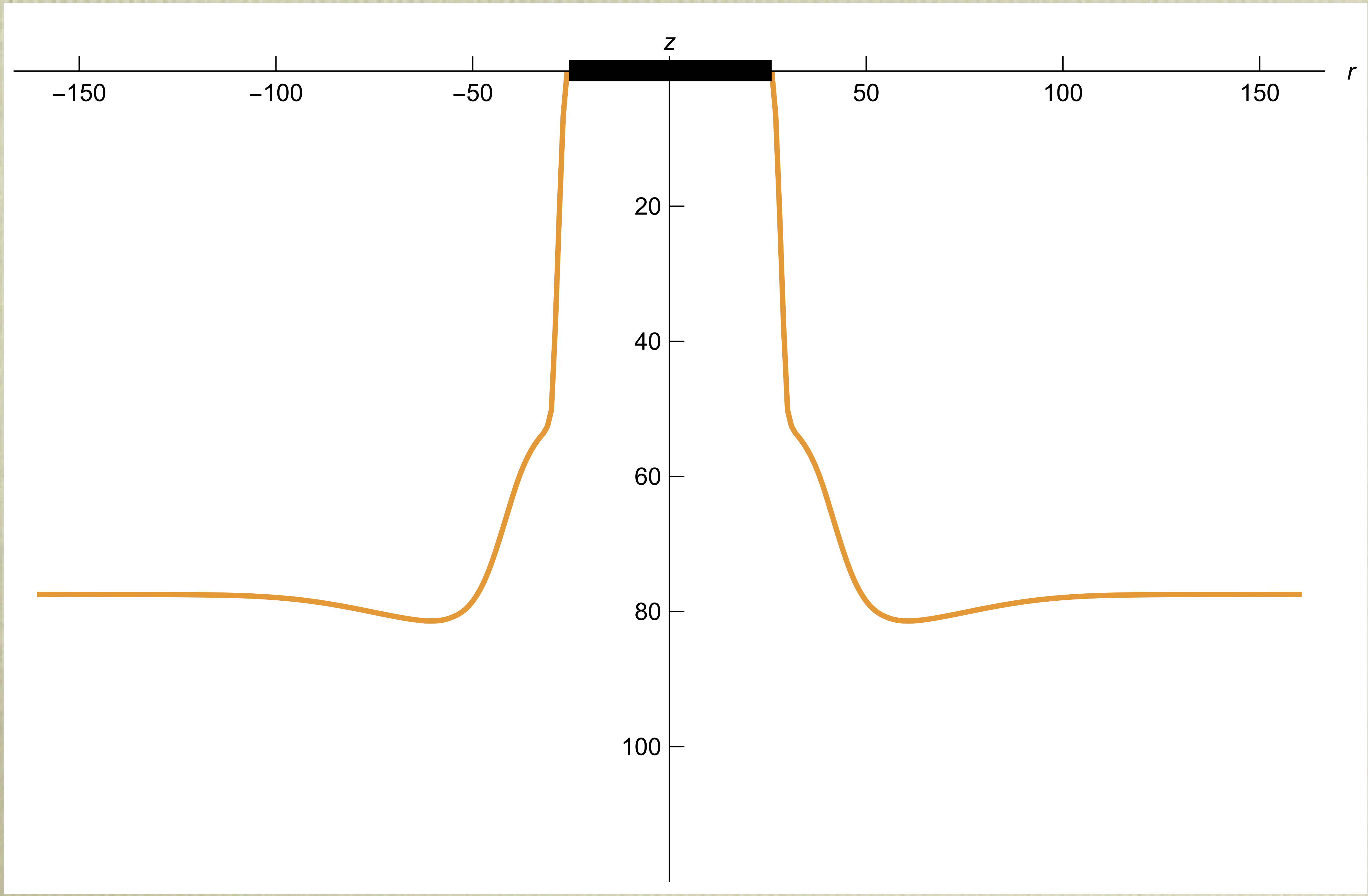


Causal structure

- Bulk EH agrees with boundary EH at all times:

  (within our precision)



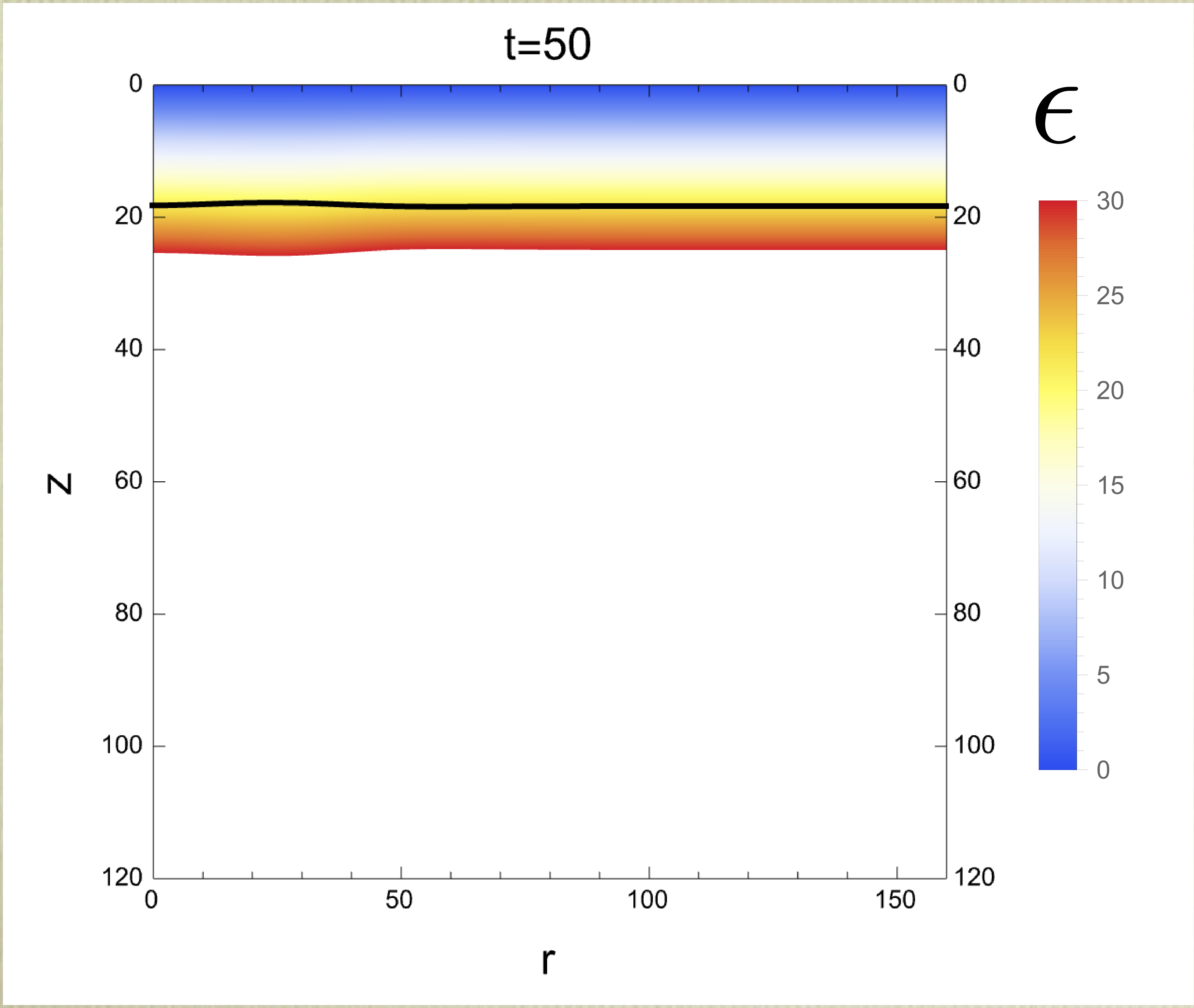
How good is the bulk solution?

- The exact metric would obey $R = -20$.

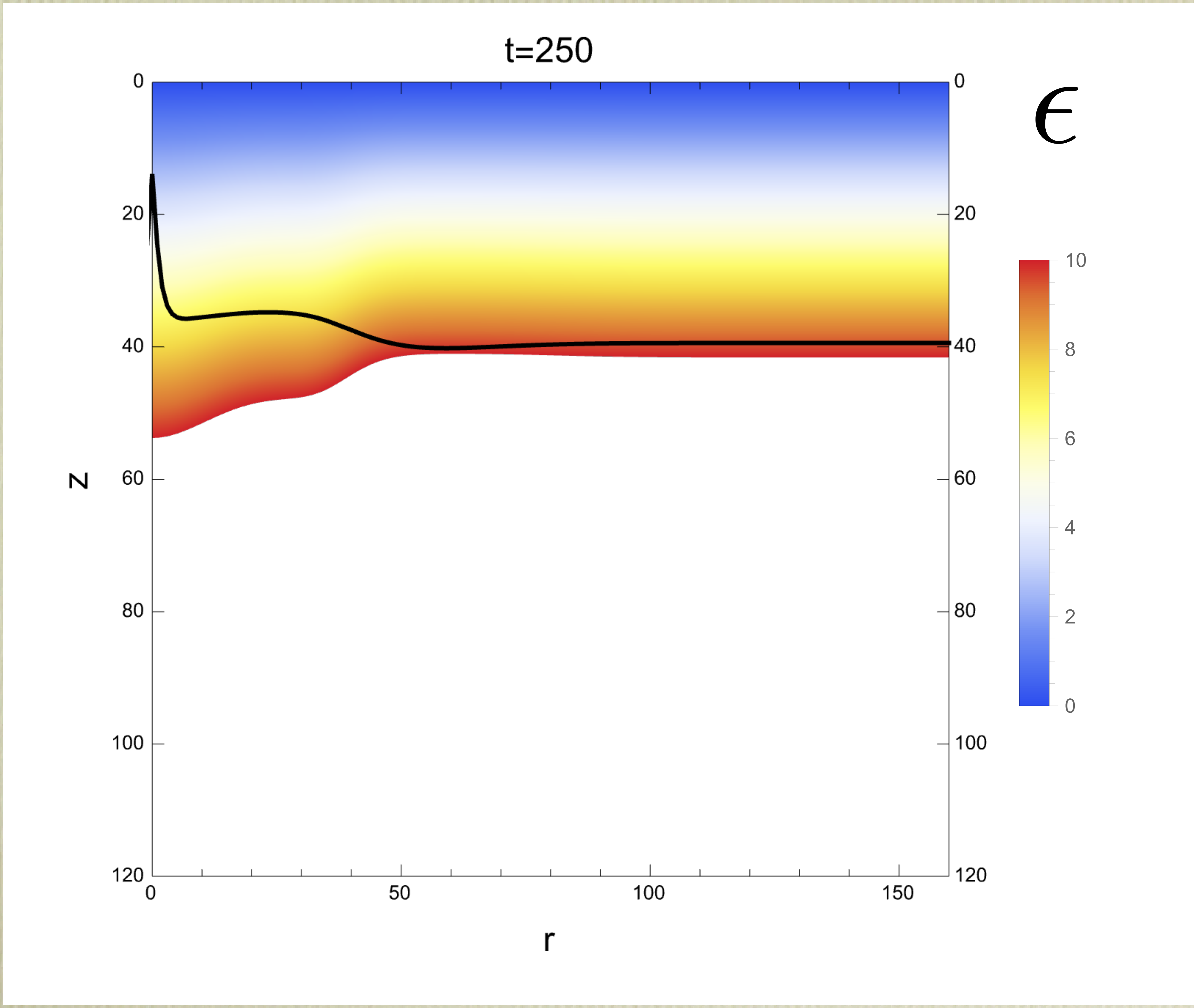
- So let us define the error as:

$$\epsilon = 100 \times \frac{R + 20}{20}$$

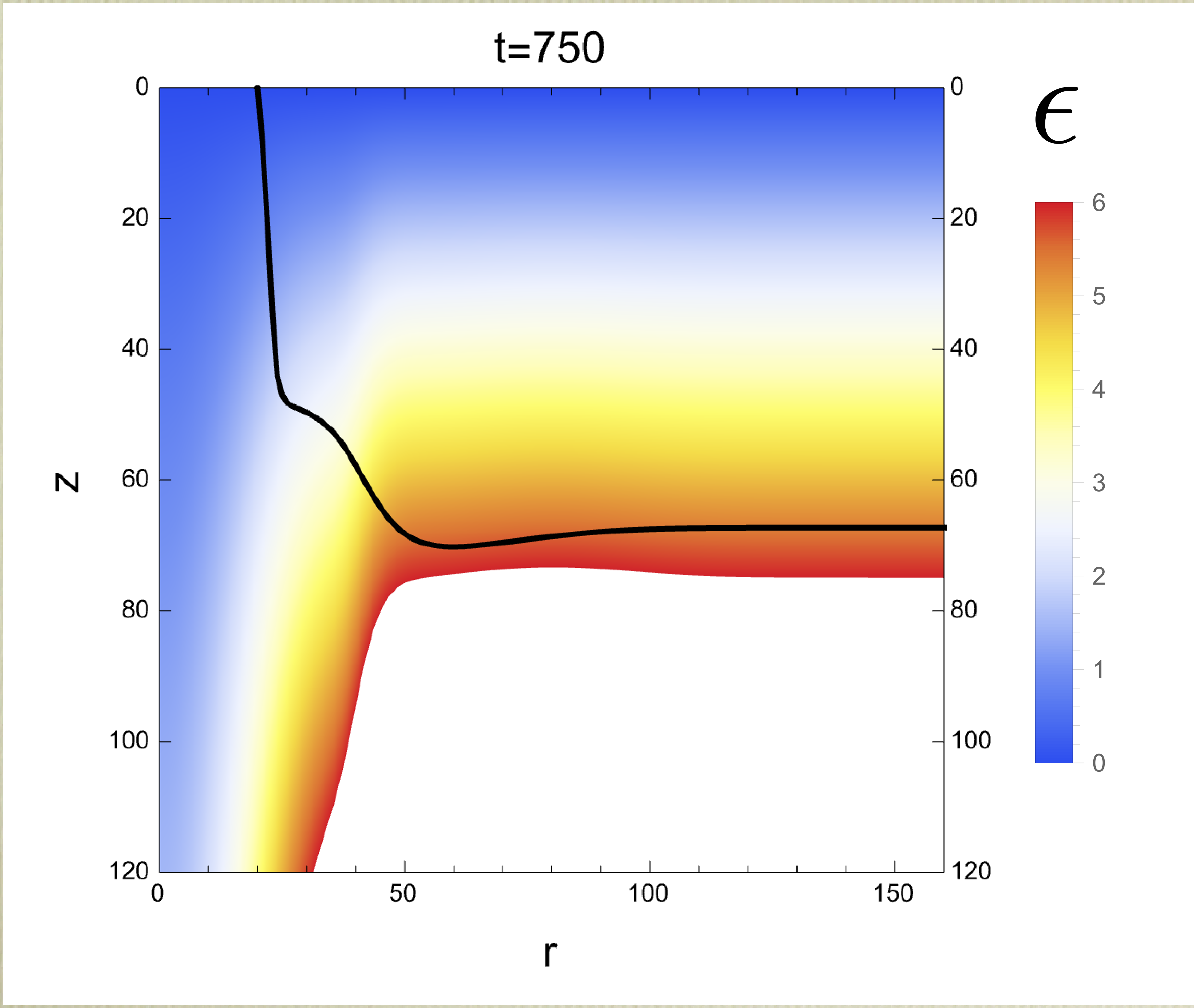
How good is the bulk solution?



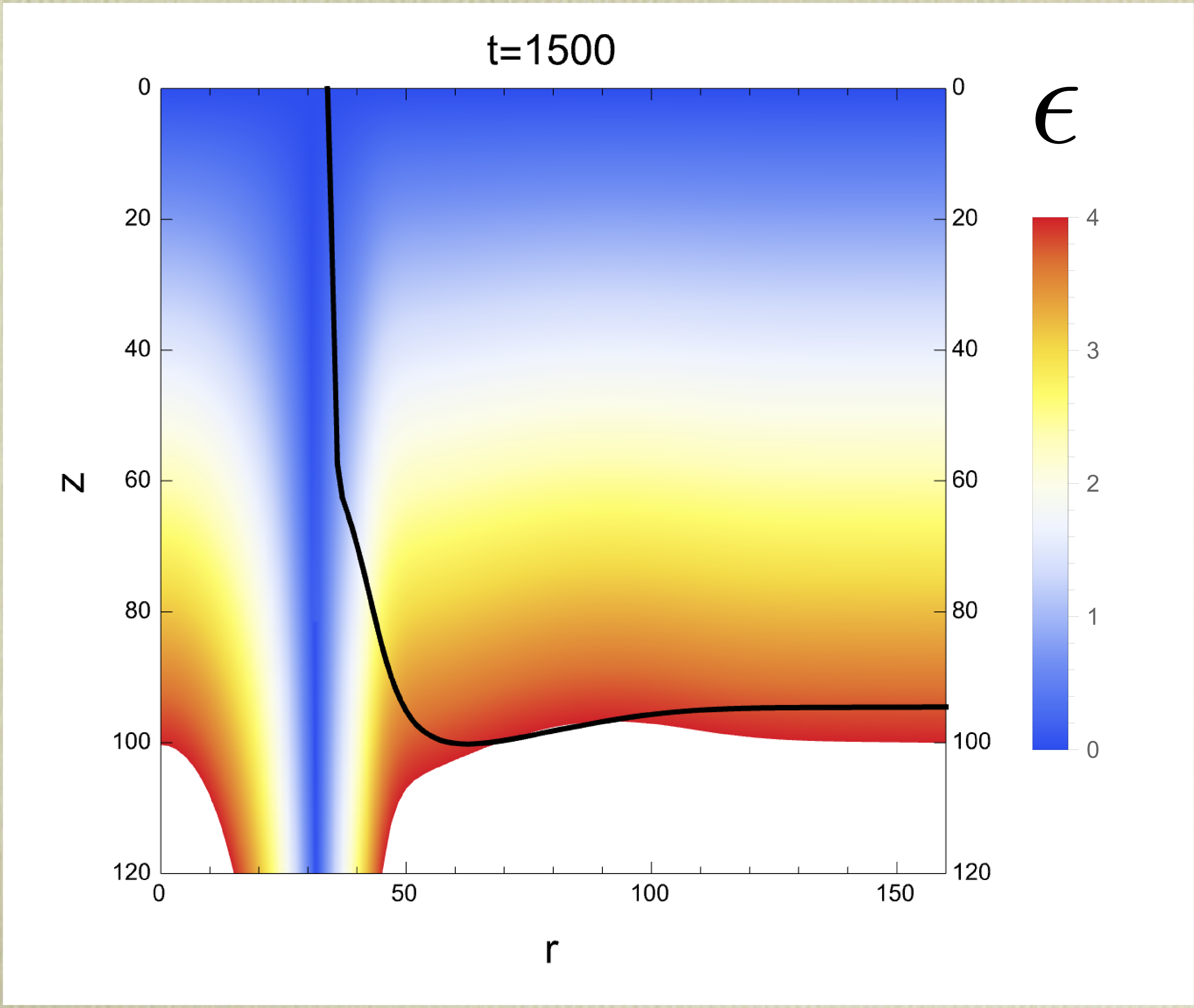
How good is the bulk solution?



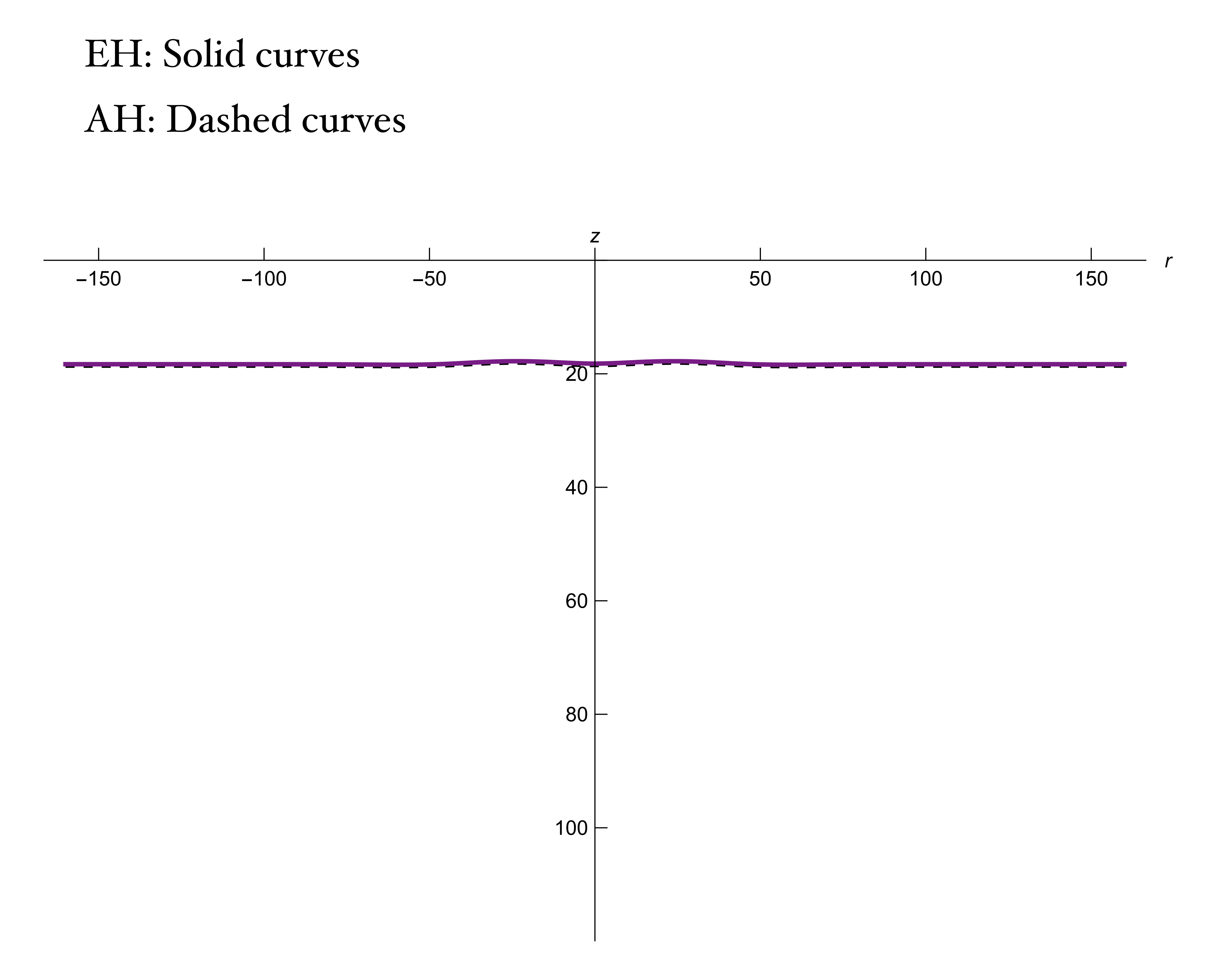
How good is the bulk solution?



How good is the bulk solution?



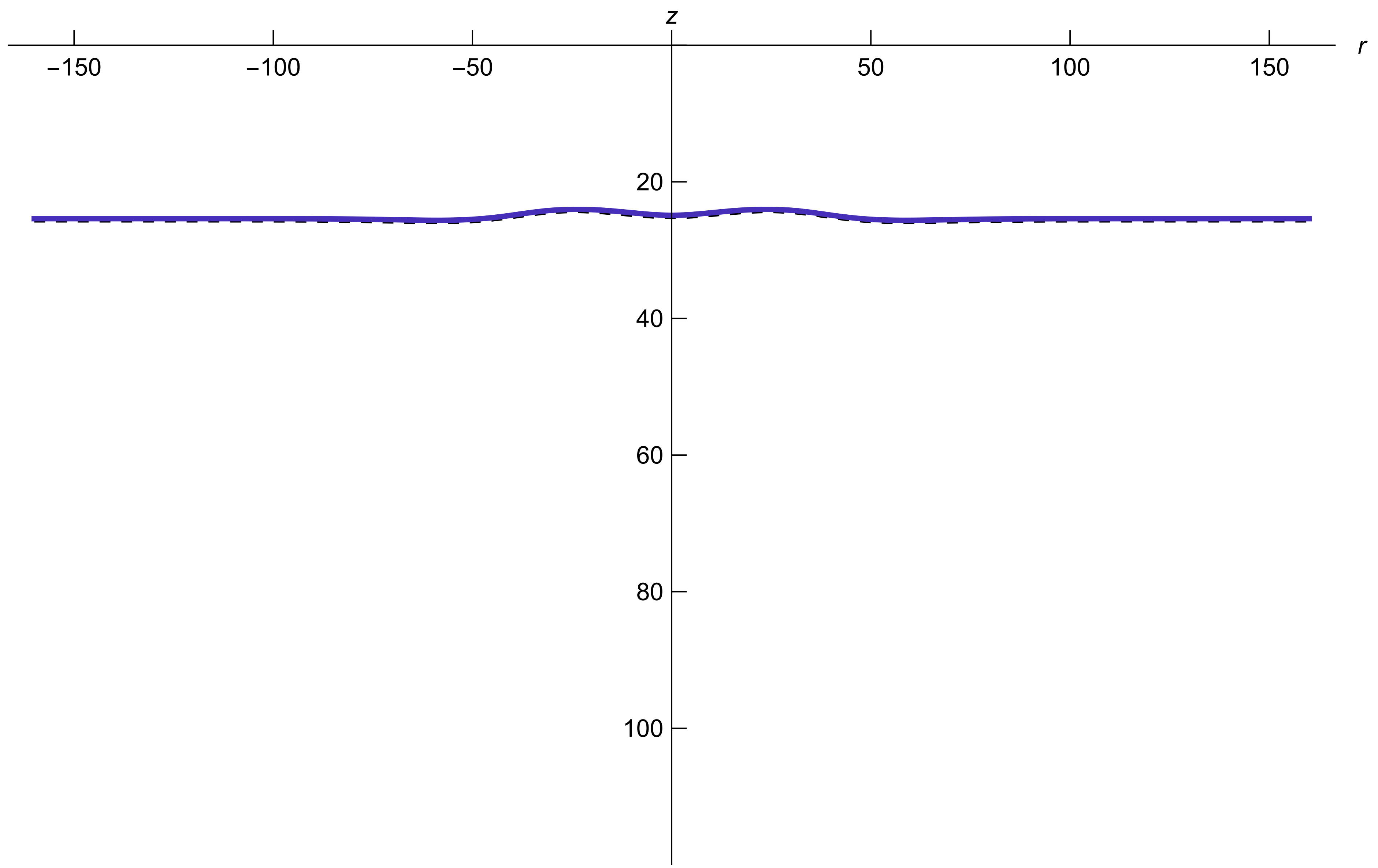
Event Horizon (EH) vs Apparent Horizon (AH)



Event Horizon (EH) vs Apparent Horizon (AH)

EH: Solid curves

AH: Dashed curves



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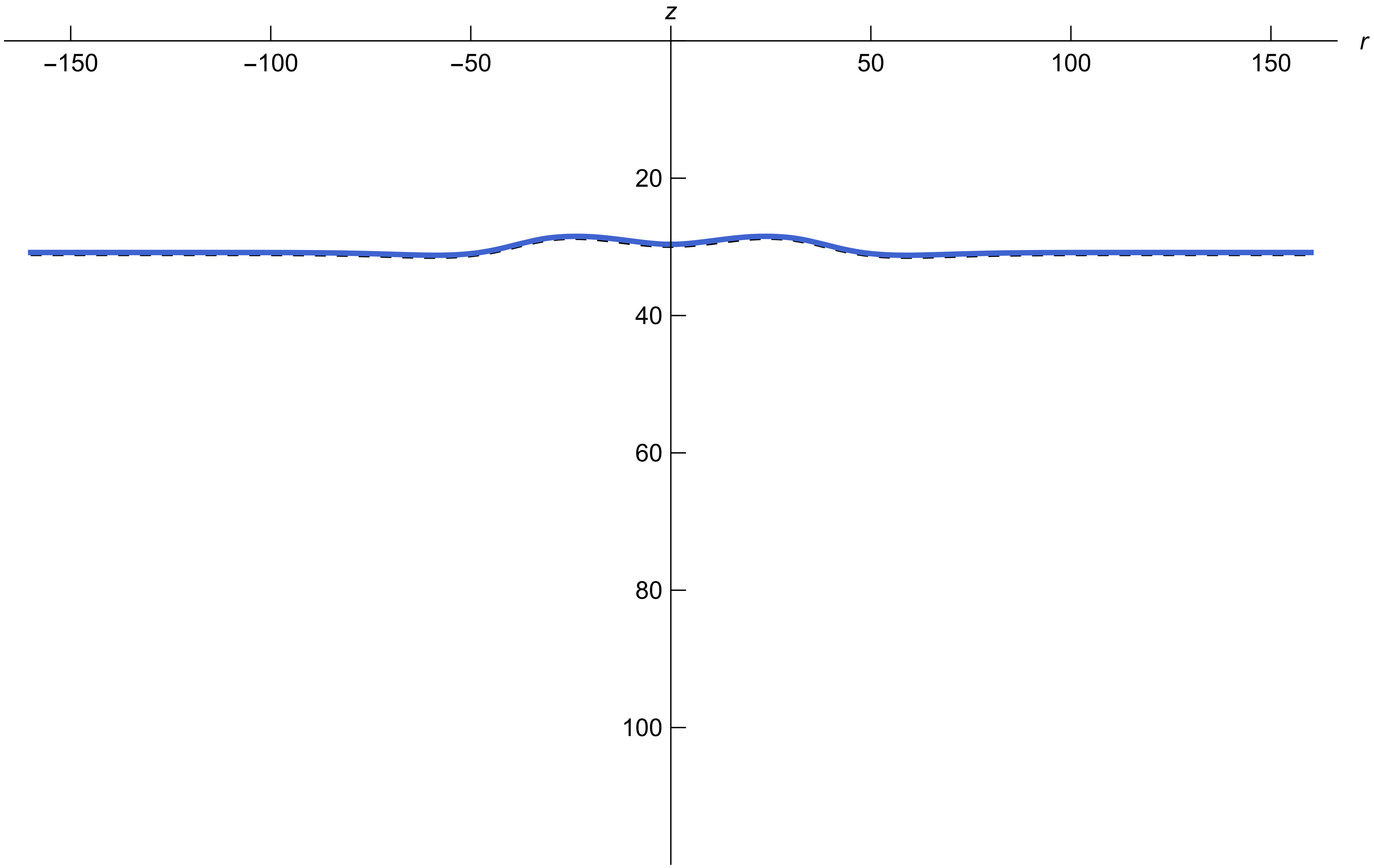

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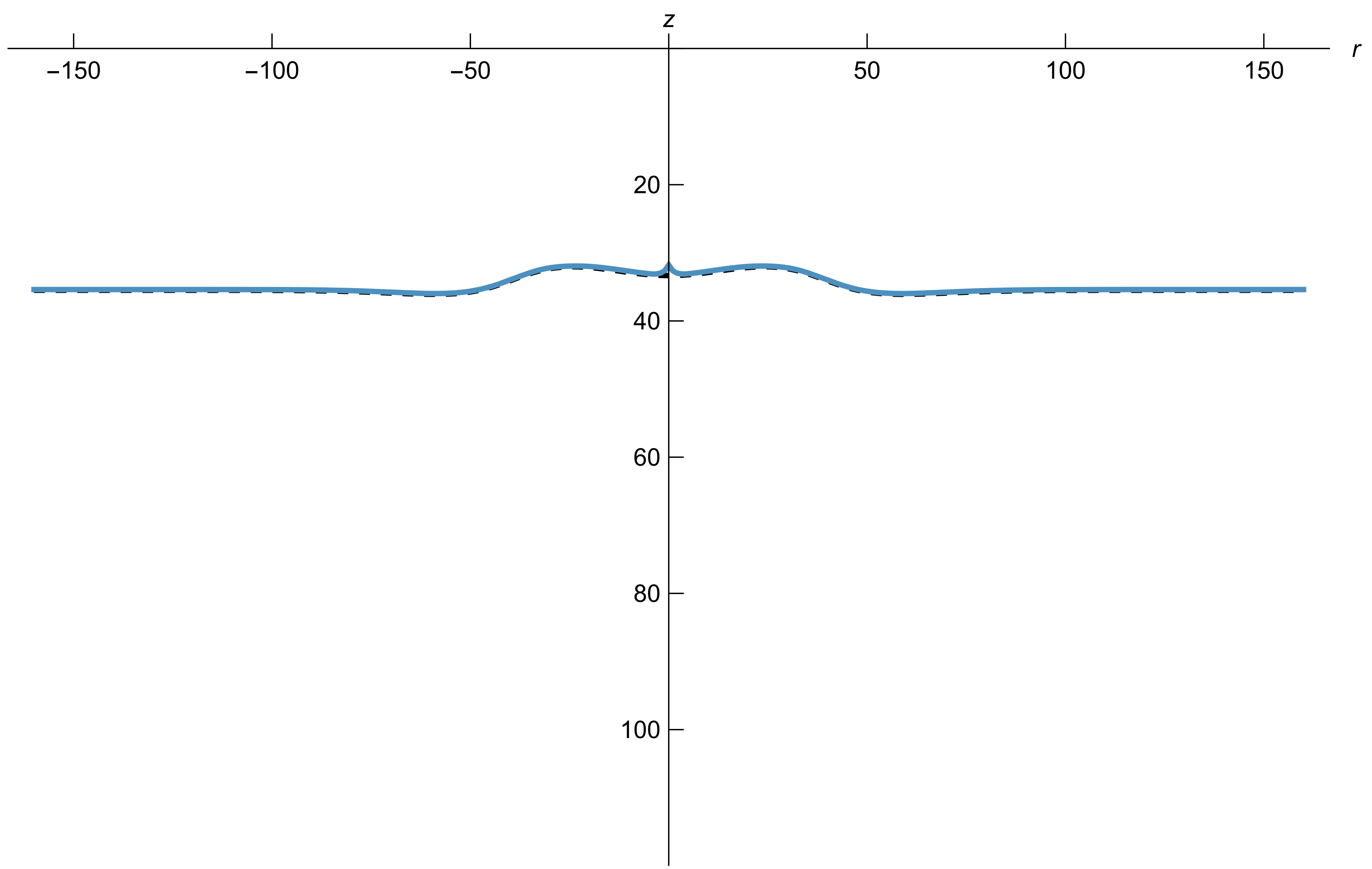

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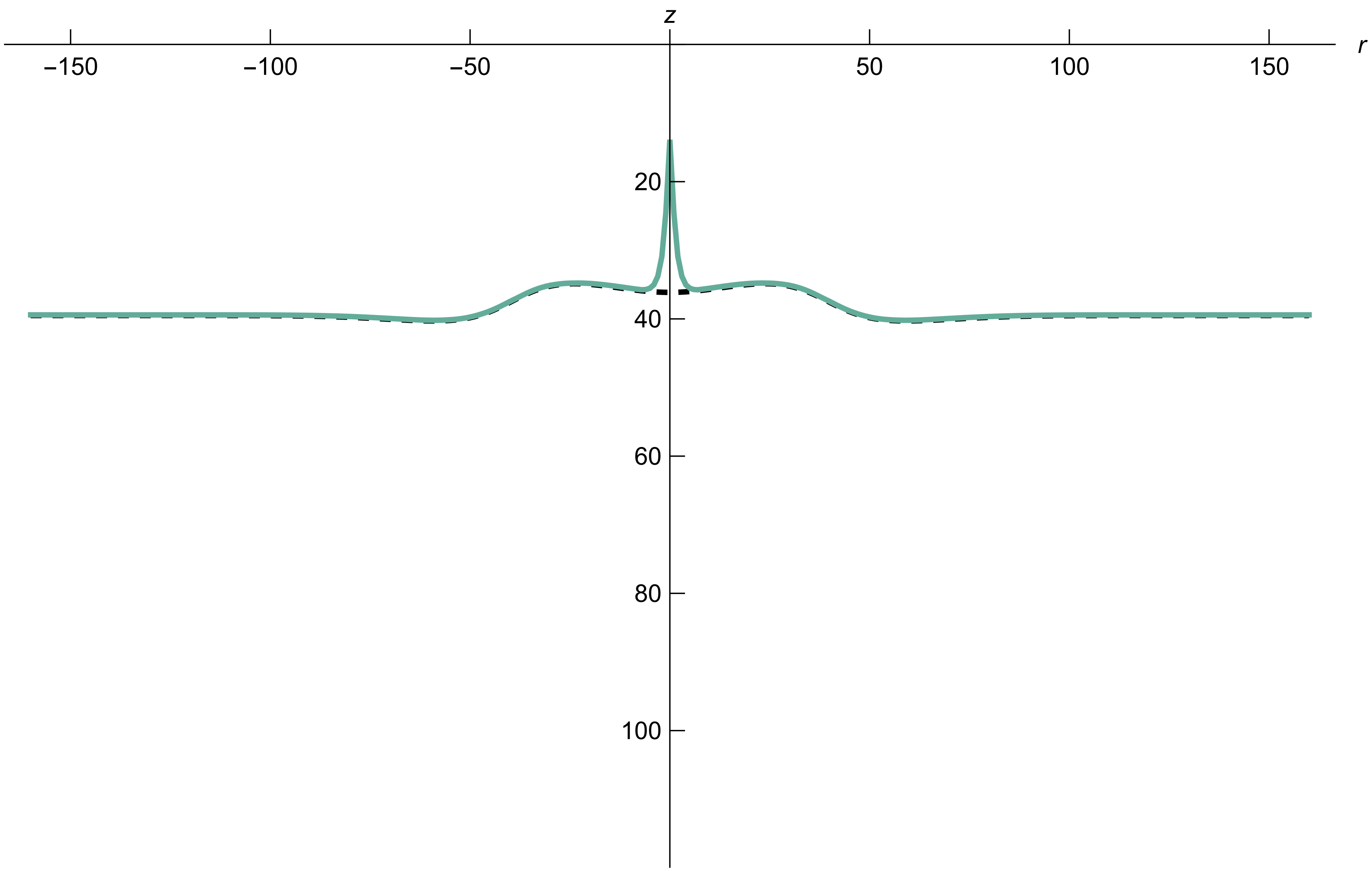
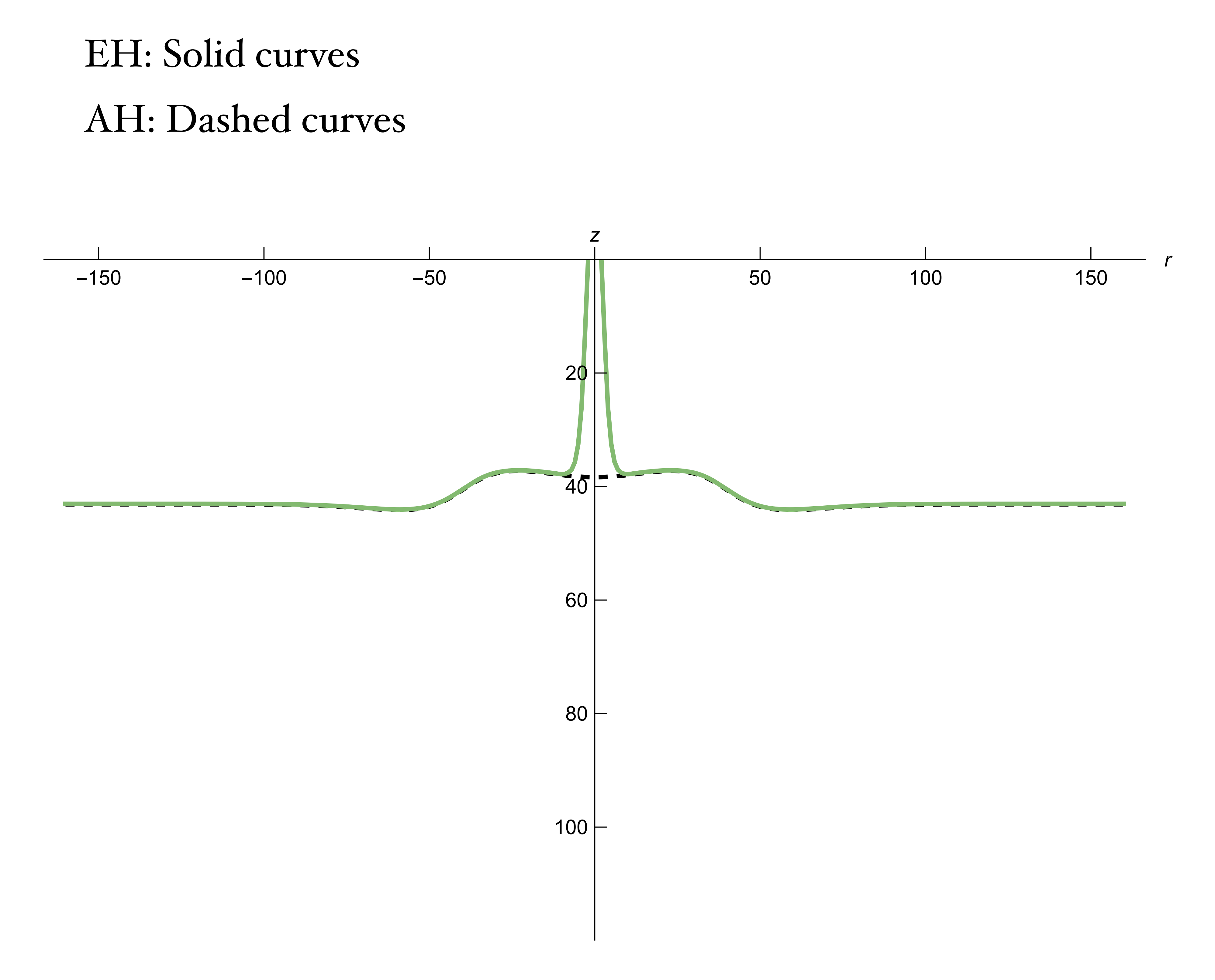
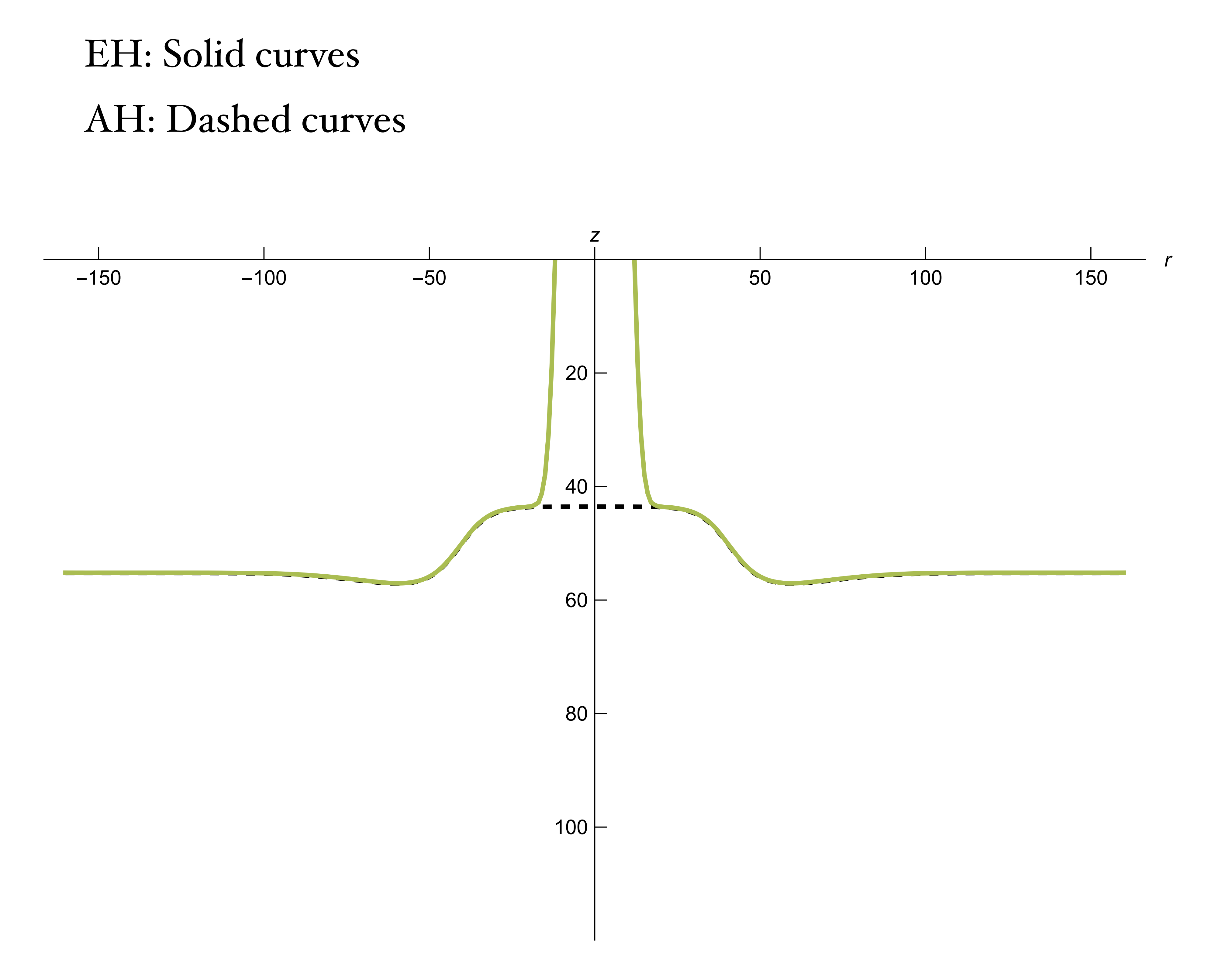


Event Horizon (EH) vs Apparent Horizon (AH)



Event Horizon (EH) vs Apparent Horizon (AH)



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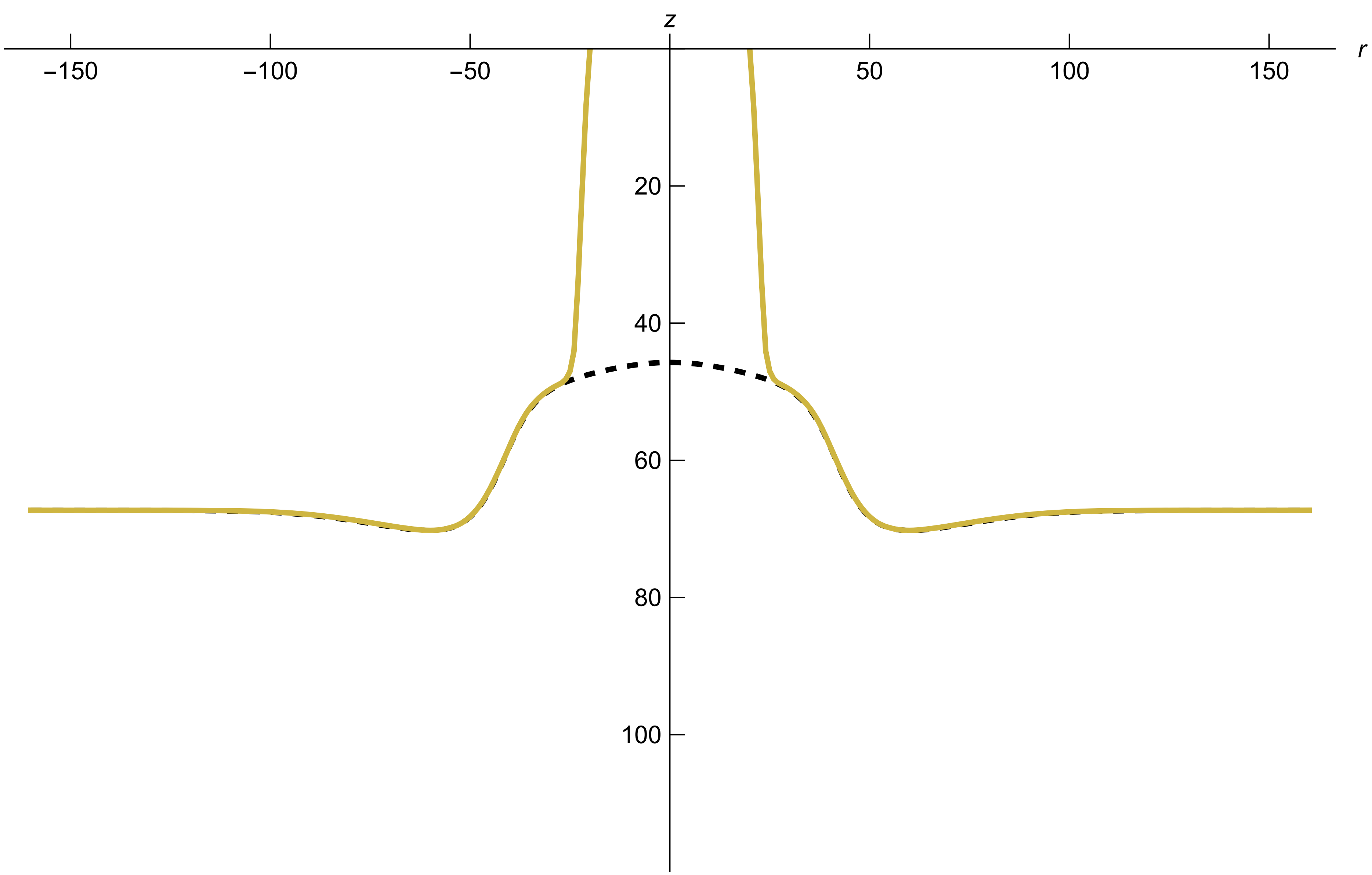

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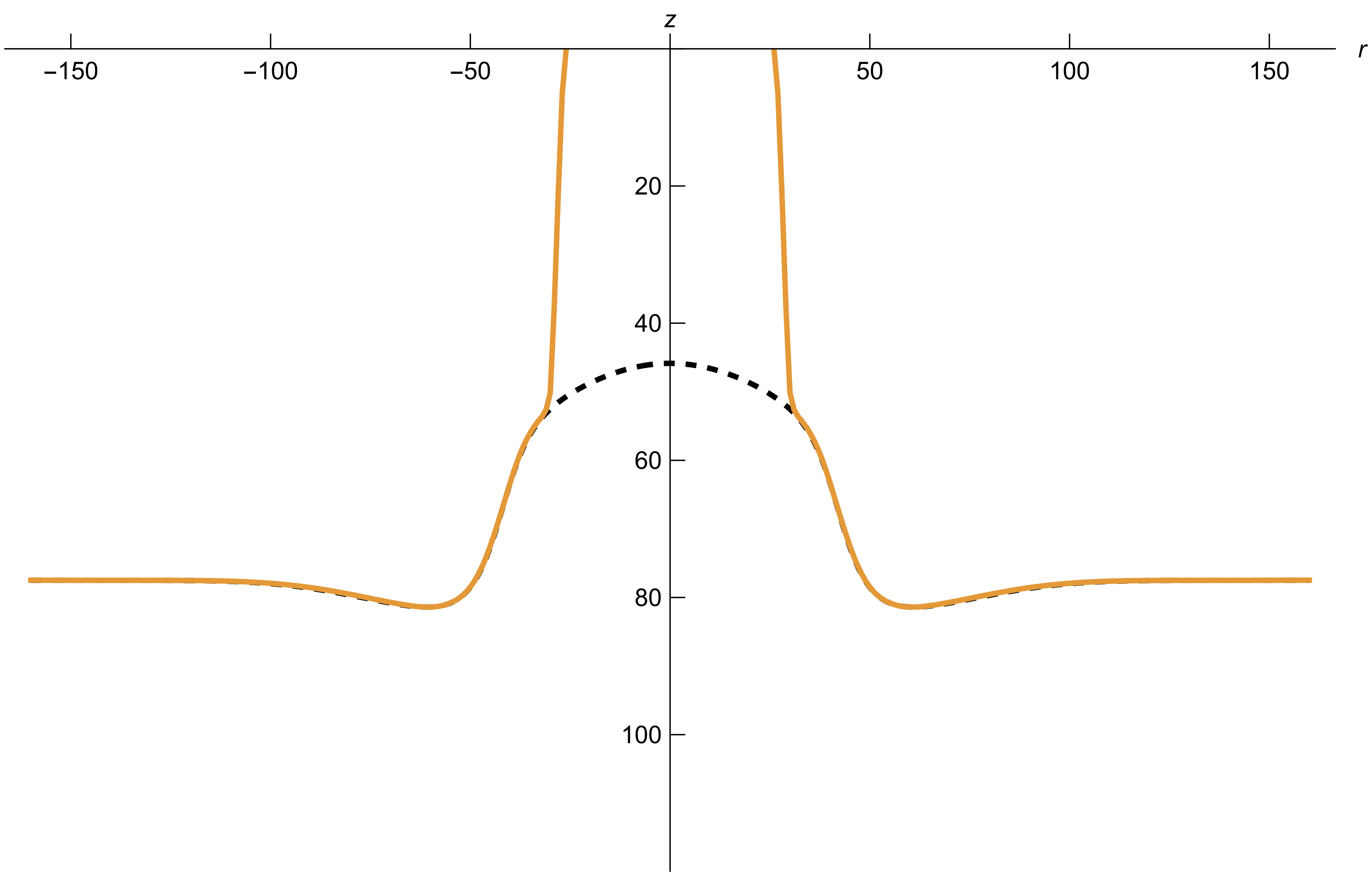
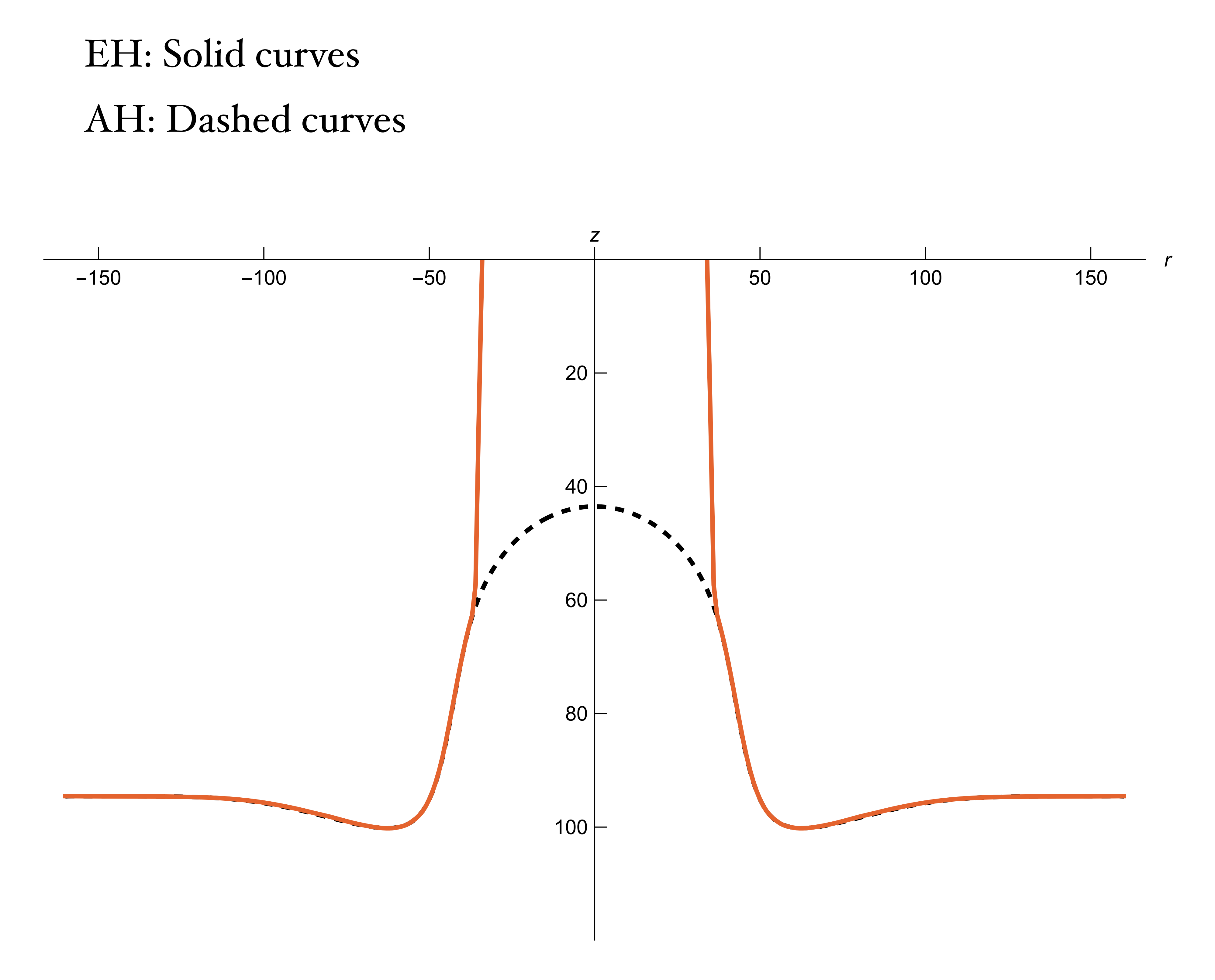


Event Horizon (EH) vs Apparent Horizon (AH)



Event Horizon (EH) vs Apparent Horizon (AH)

EH: Solid curves

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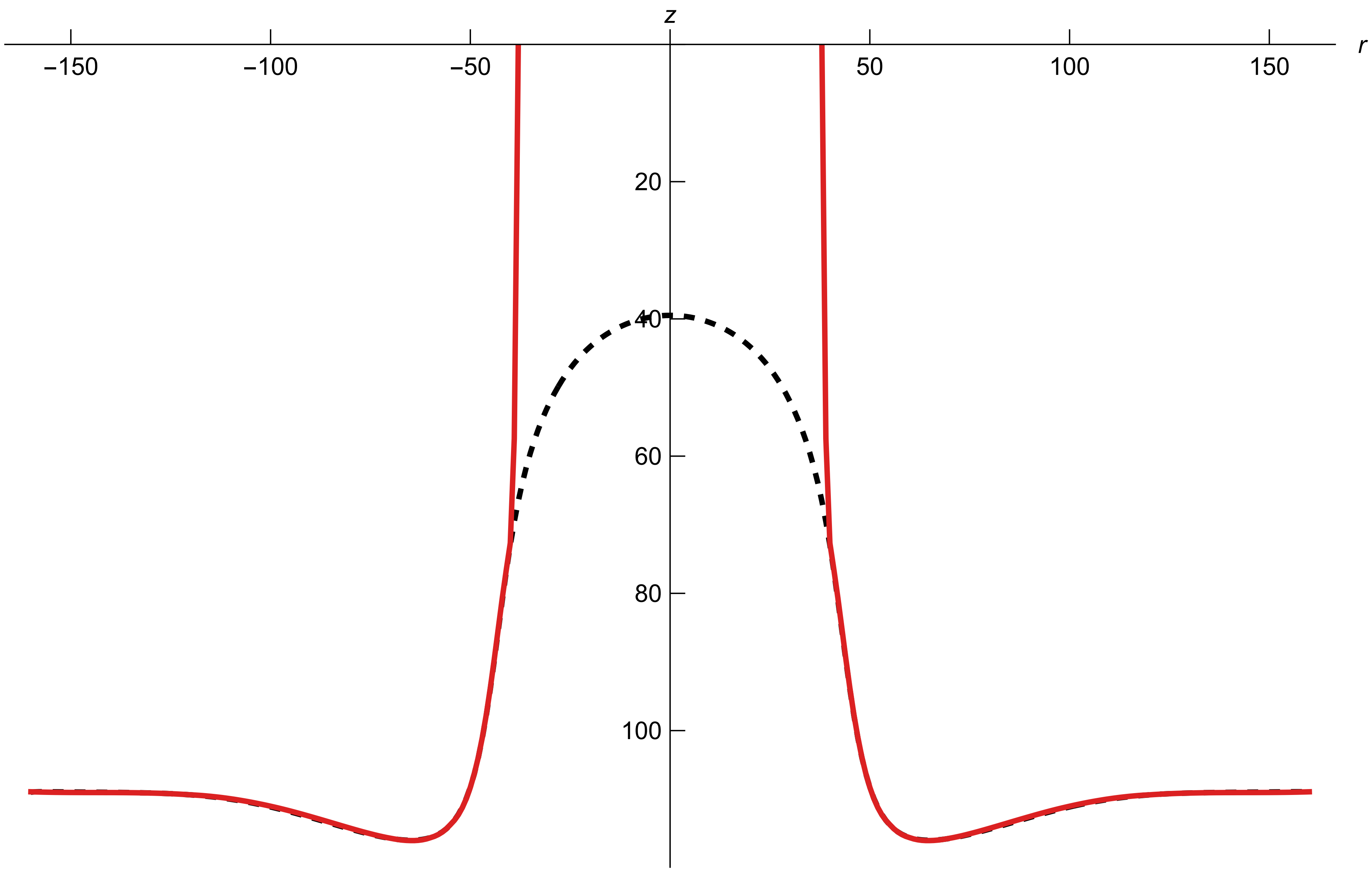


Entropy Density

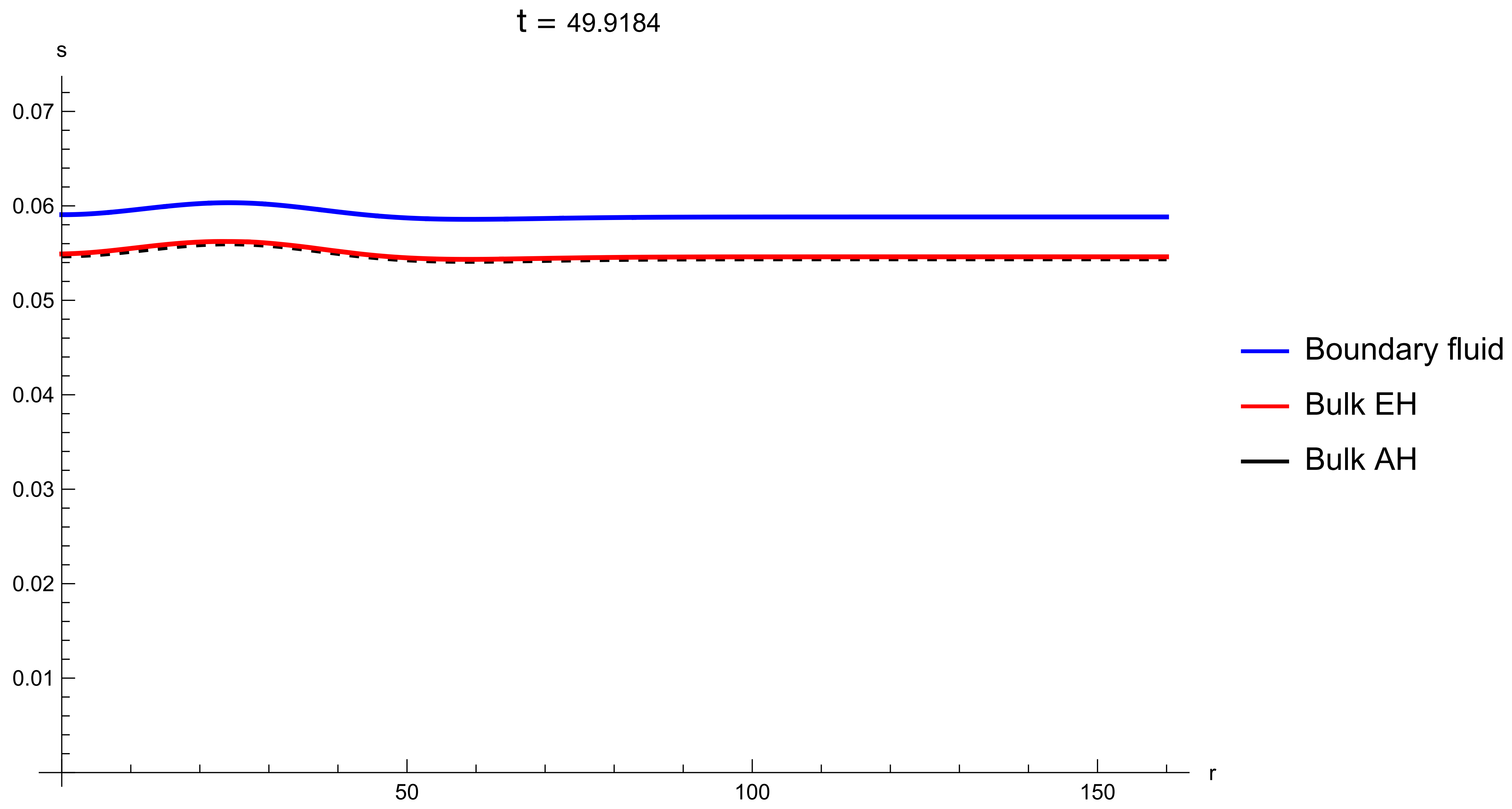
- Which horizon is related to the entropy density at the boundary?

- In general this is ill-posed because entropy is only defined in equilibrium.

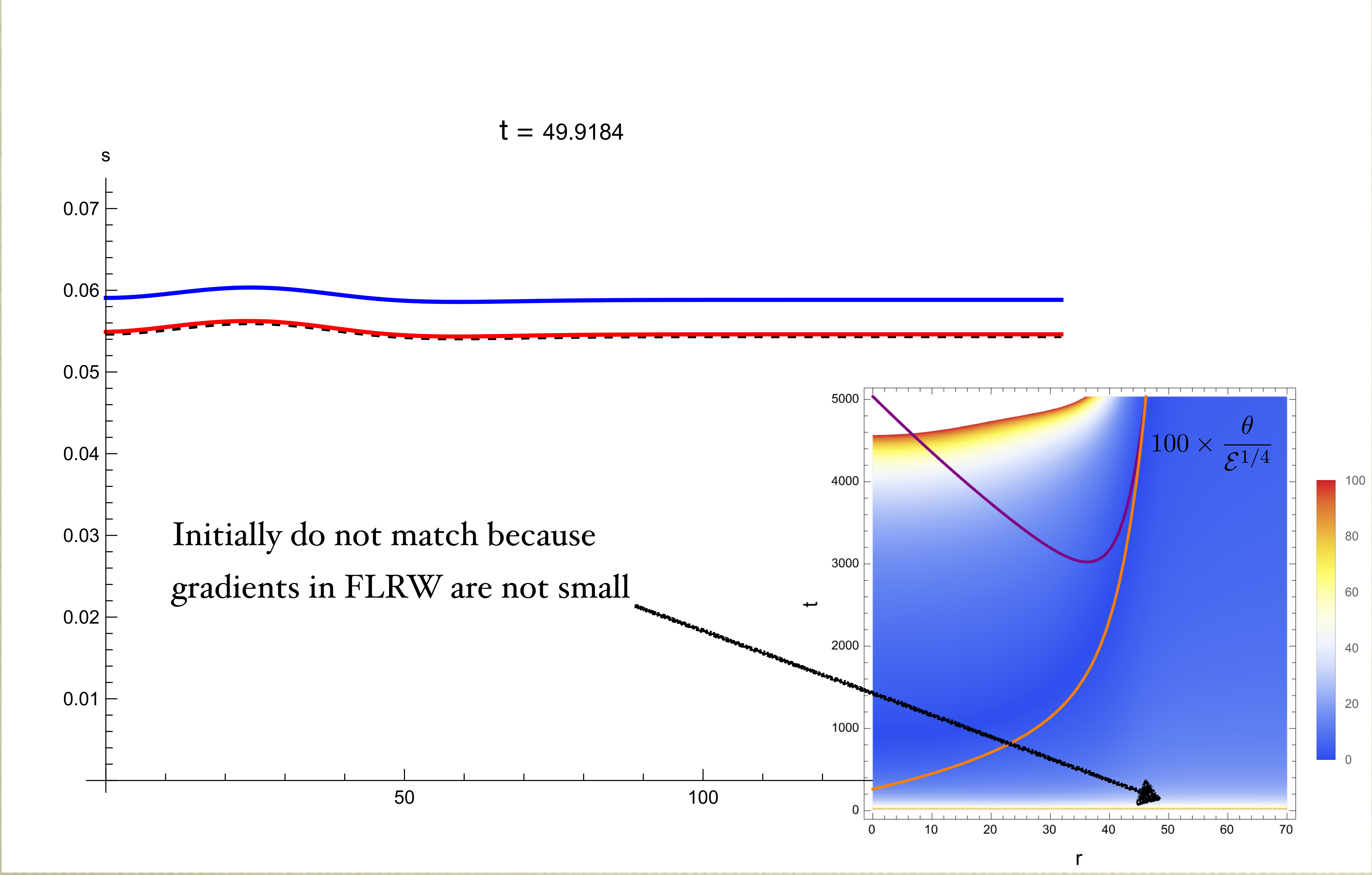
- However, an ideal fluid is in local thermal equilibrium.

- So compare the fluid entropy with the AH / EH area densities.

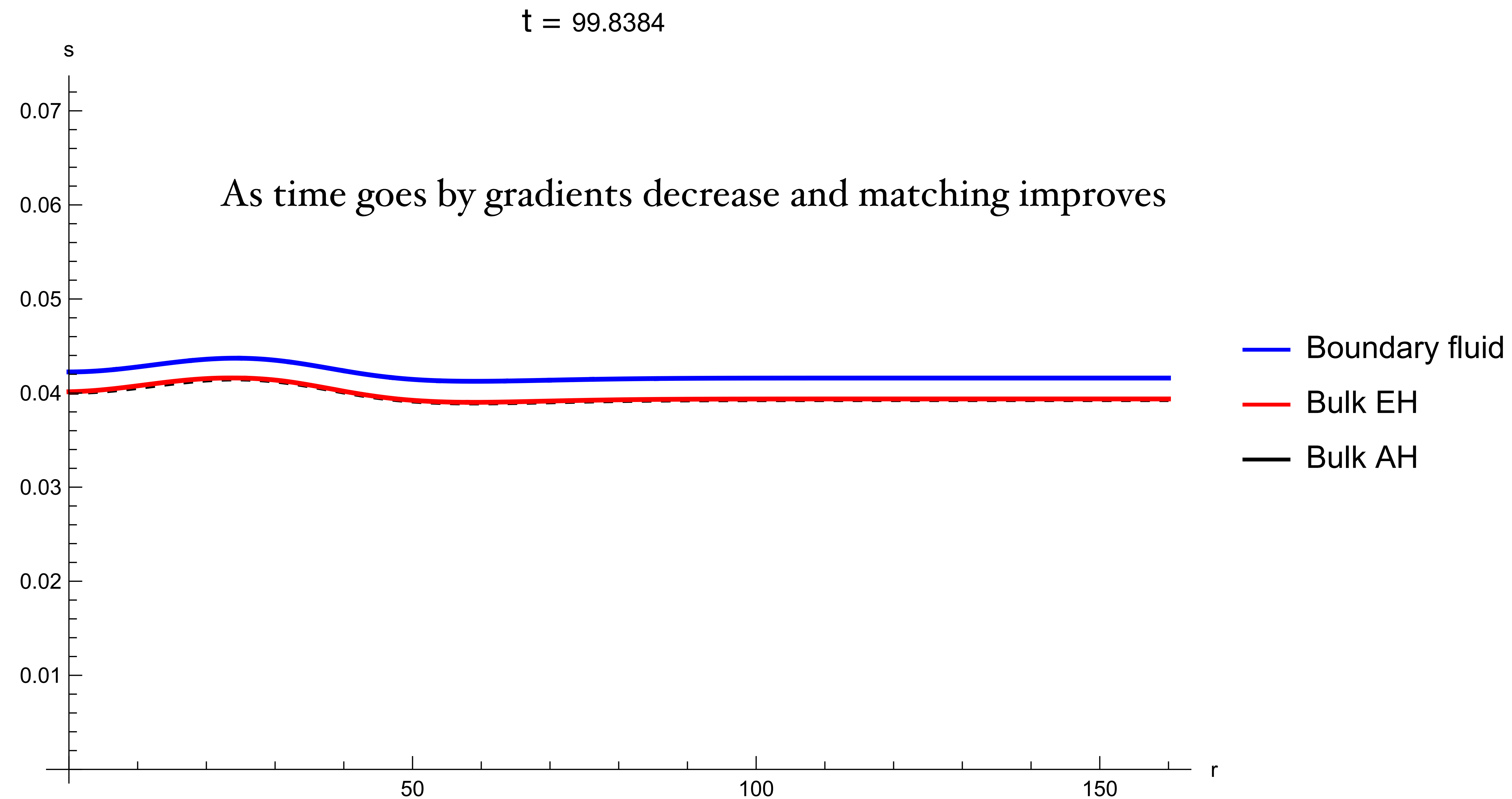
Entropy Density



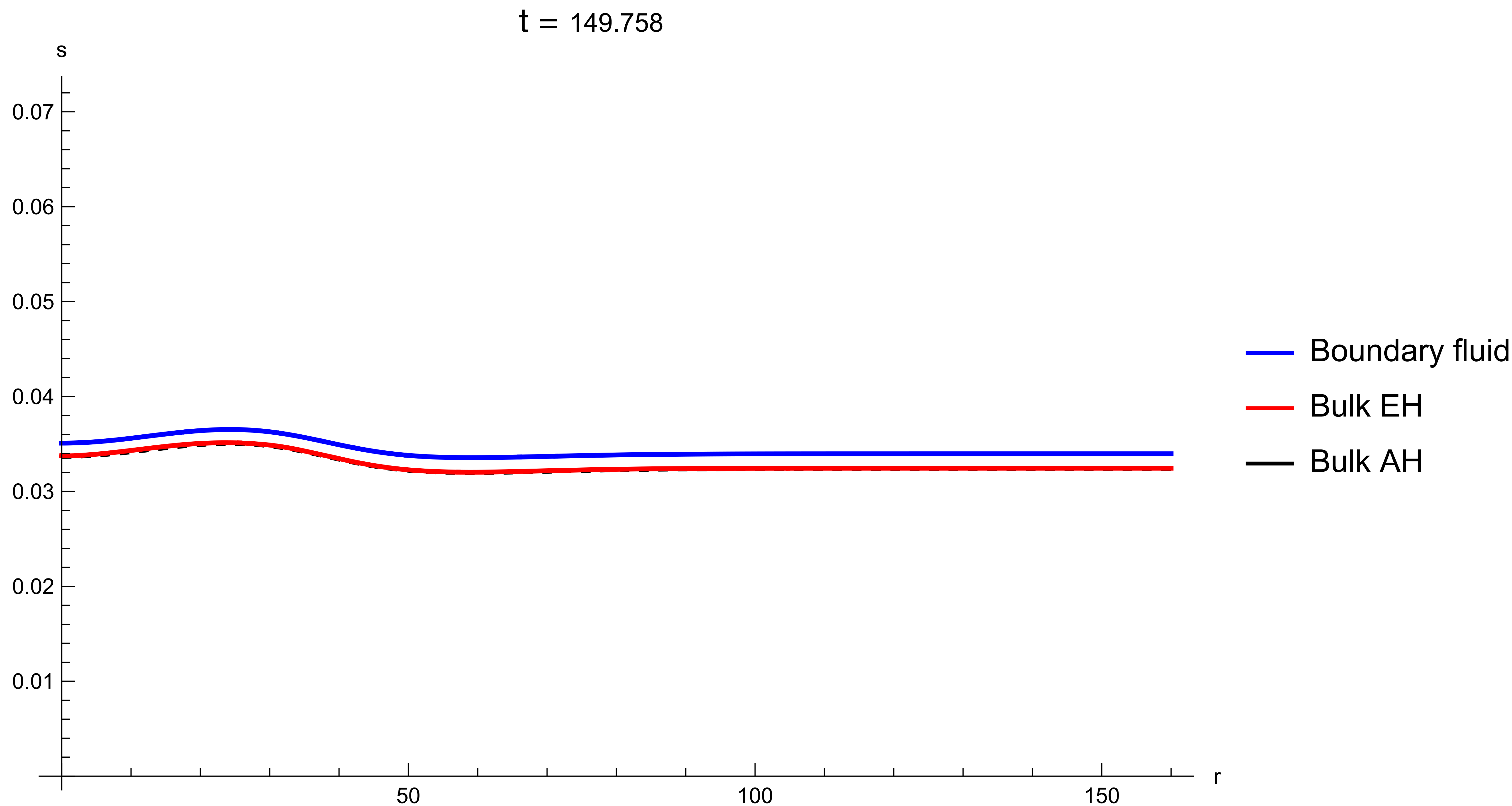
Entropy Density



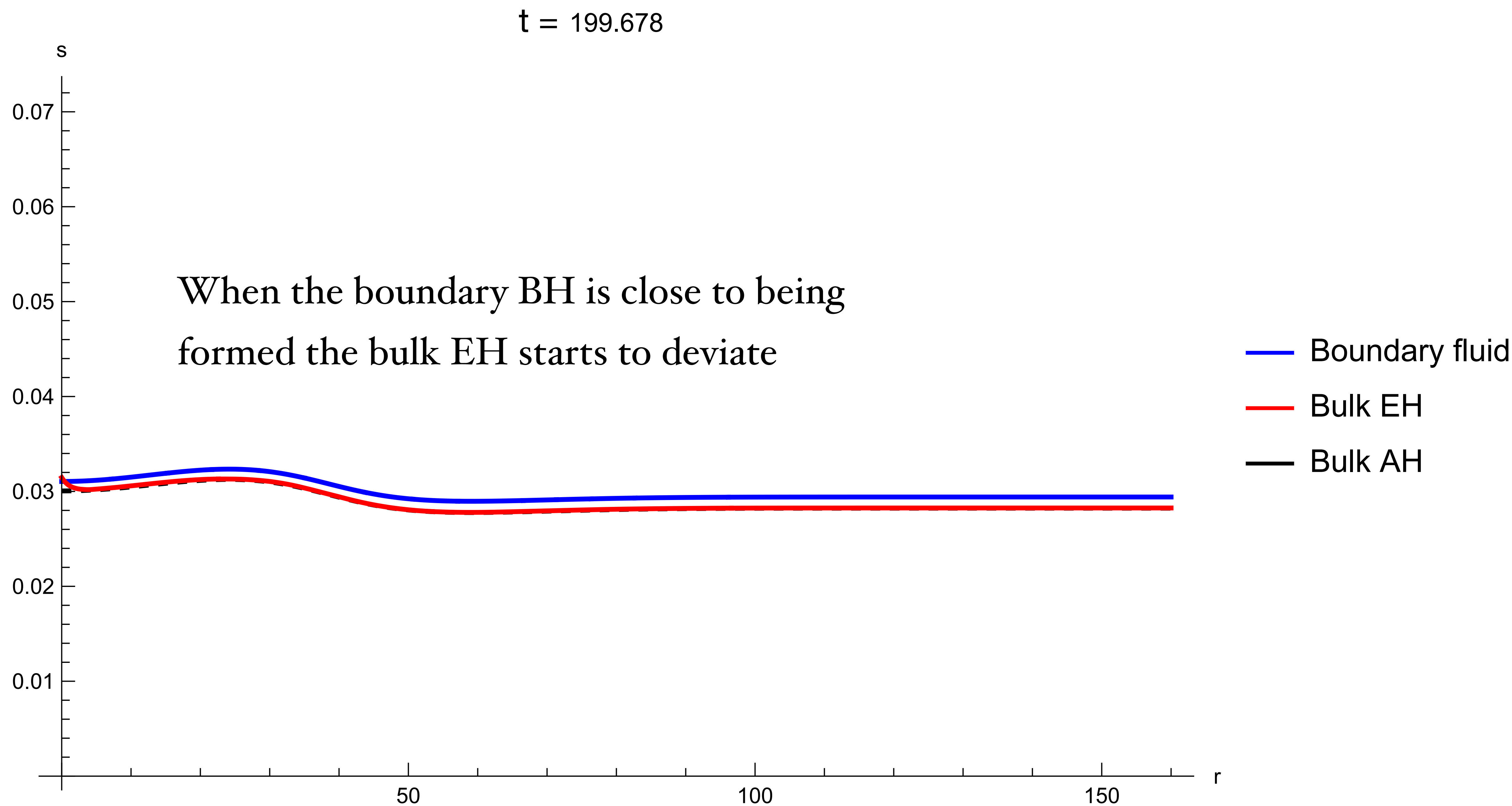
Entropy Density



Entropy Density

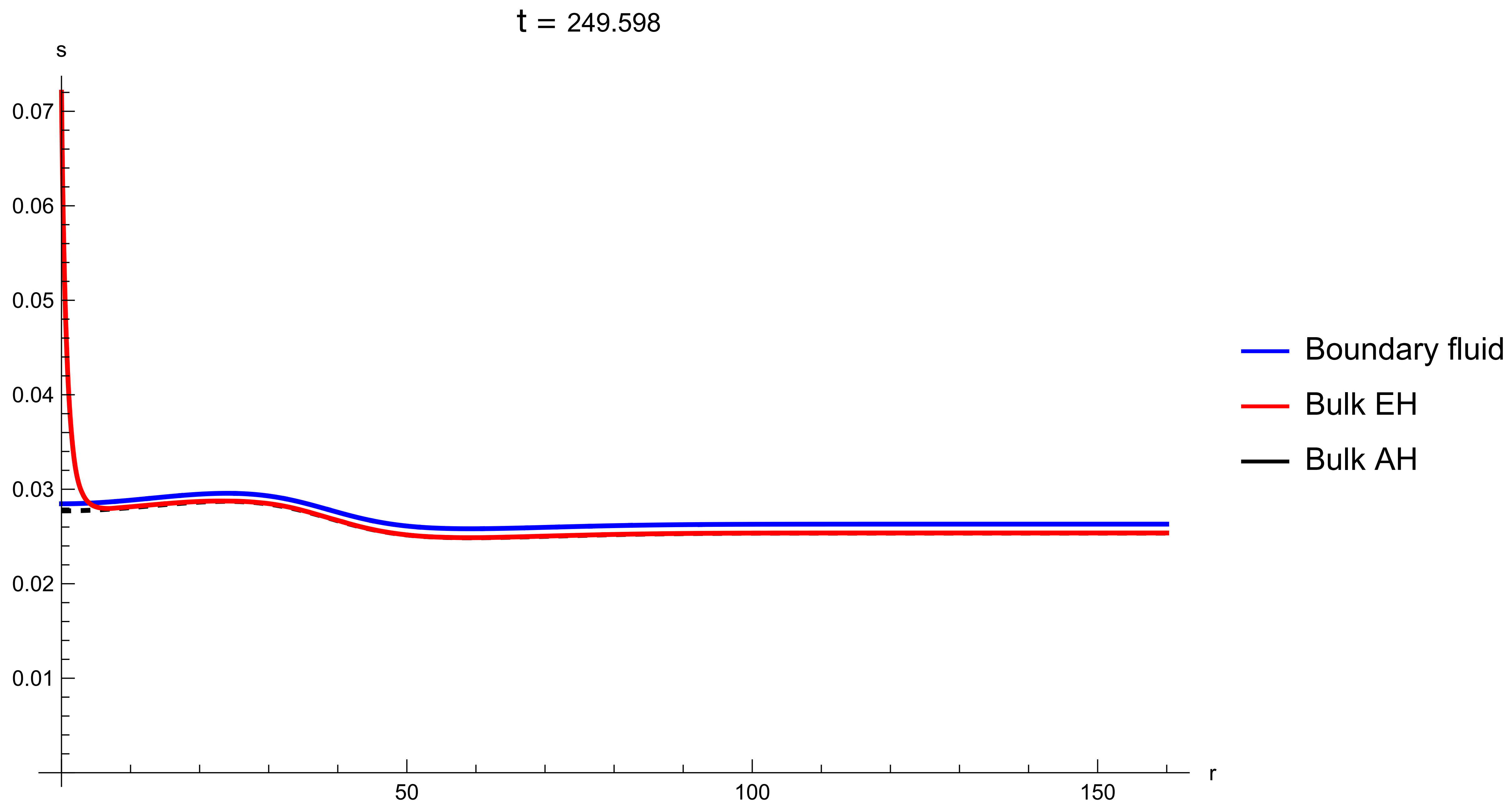


Entropy Density

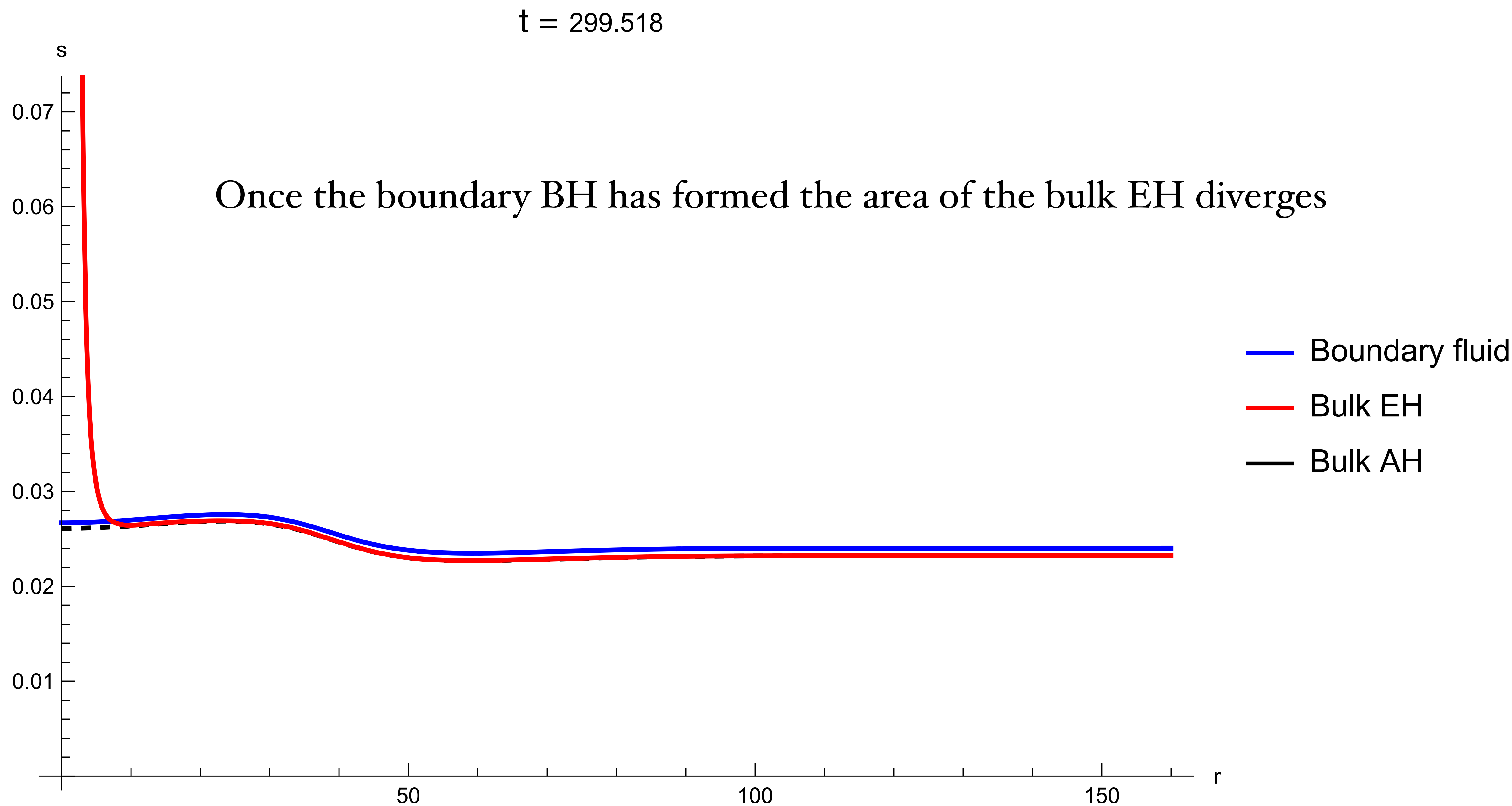


When the boundary BH is close to being formed the bulk EH starts to deviate

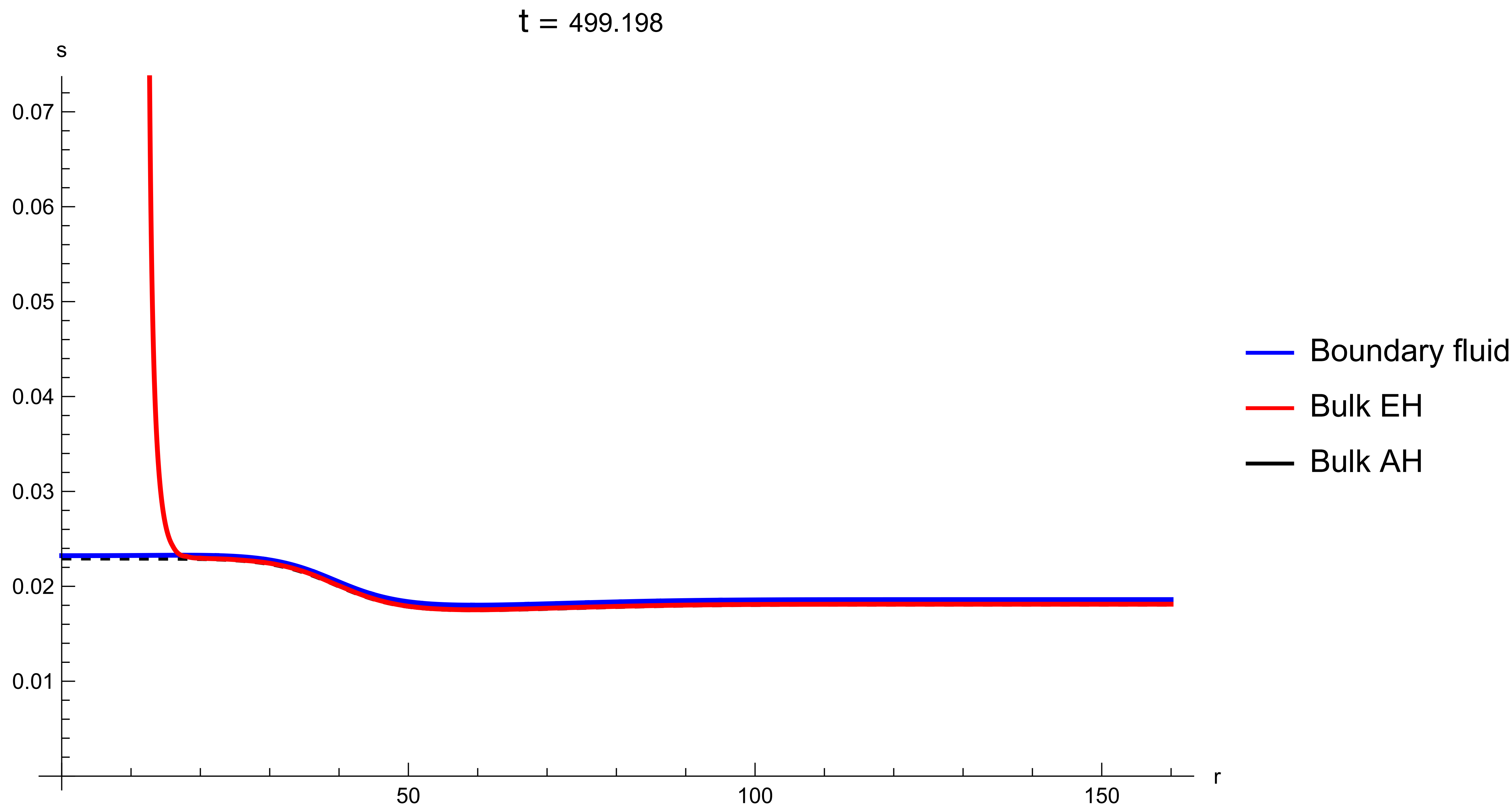
Entropy Density



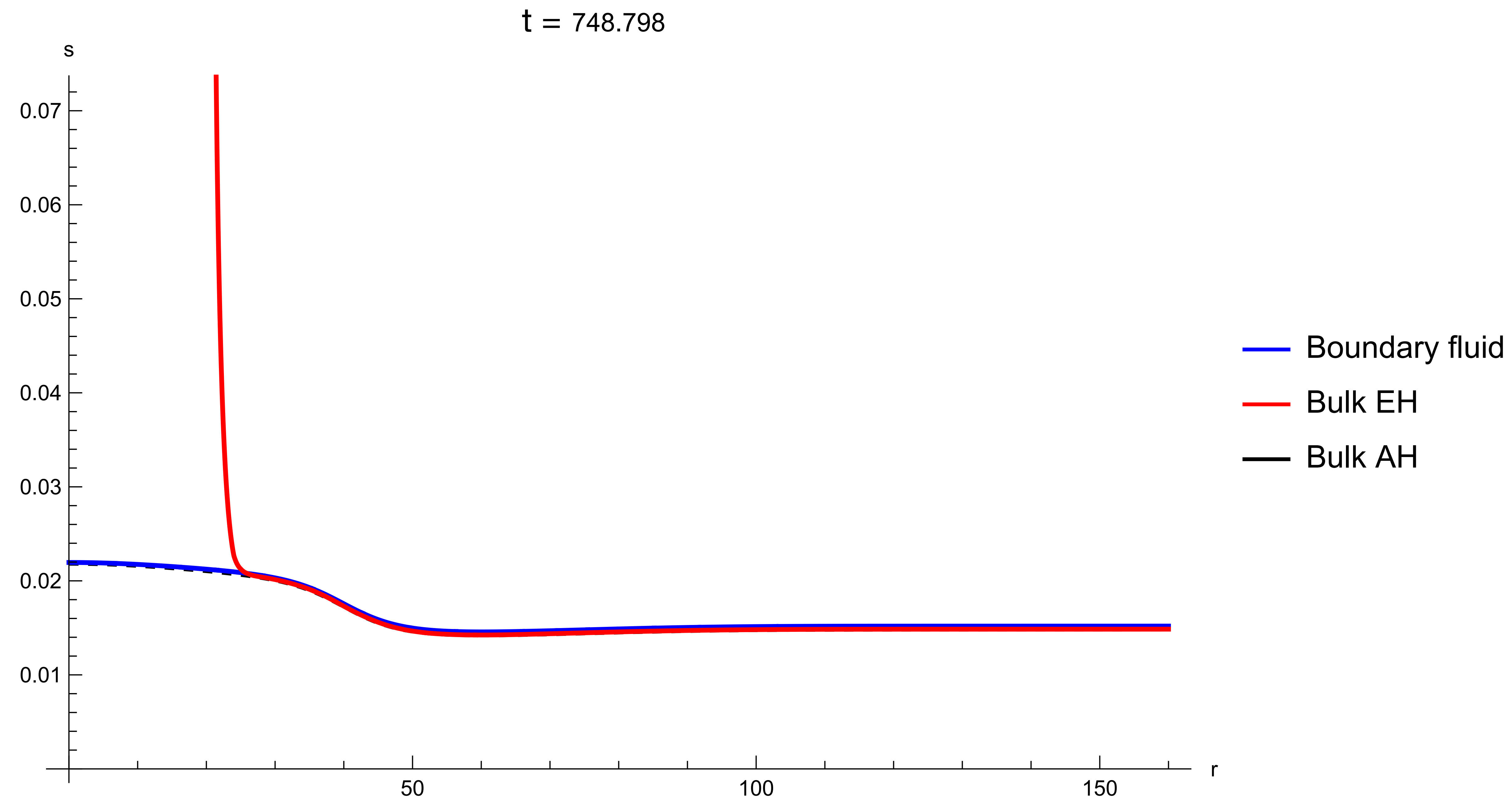
Entropy Density



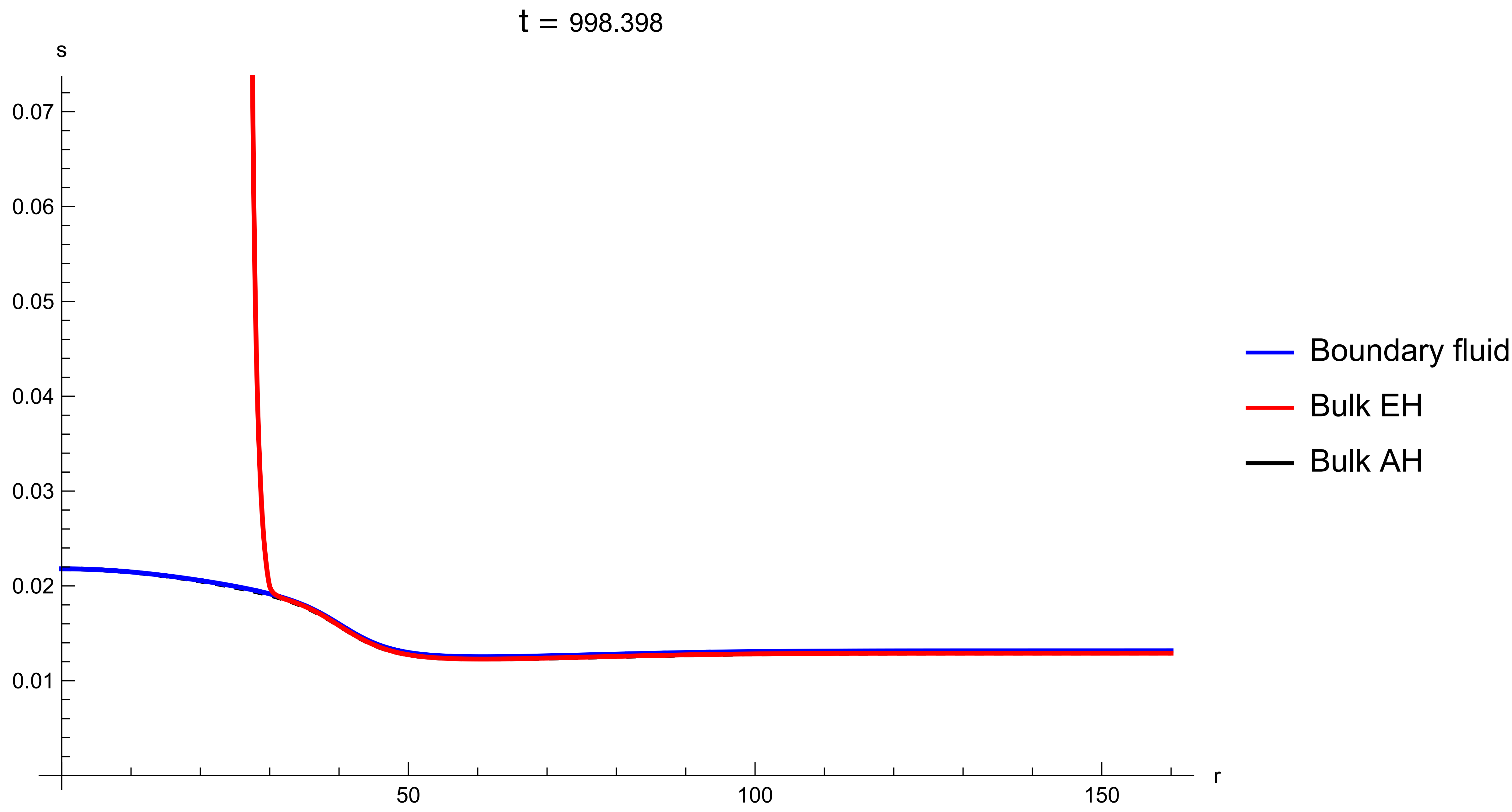
Entropy Density



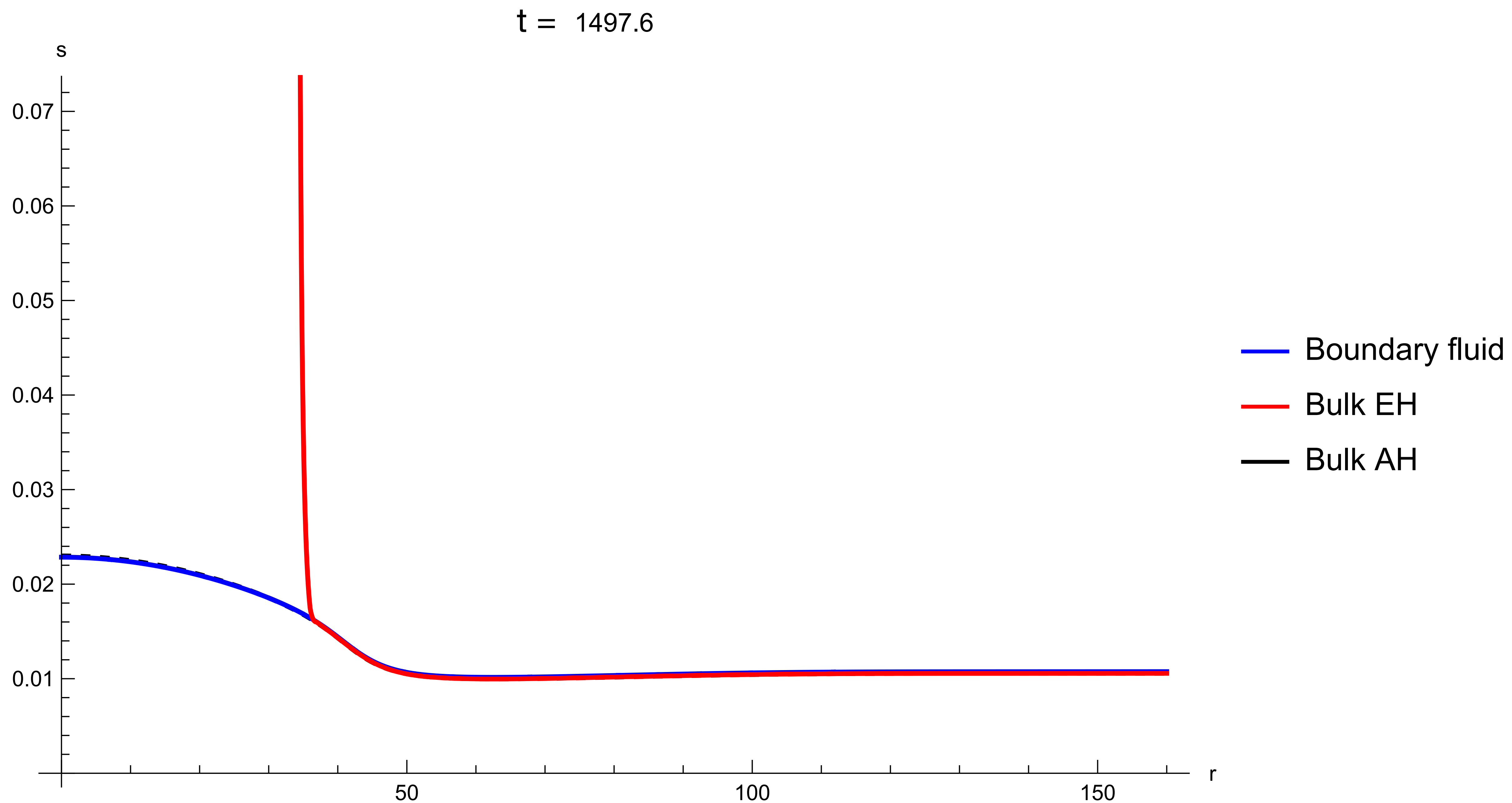
Entropy Density



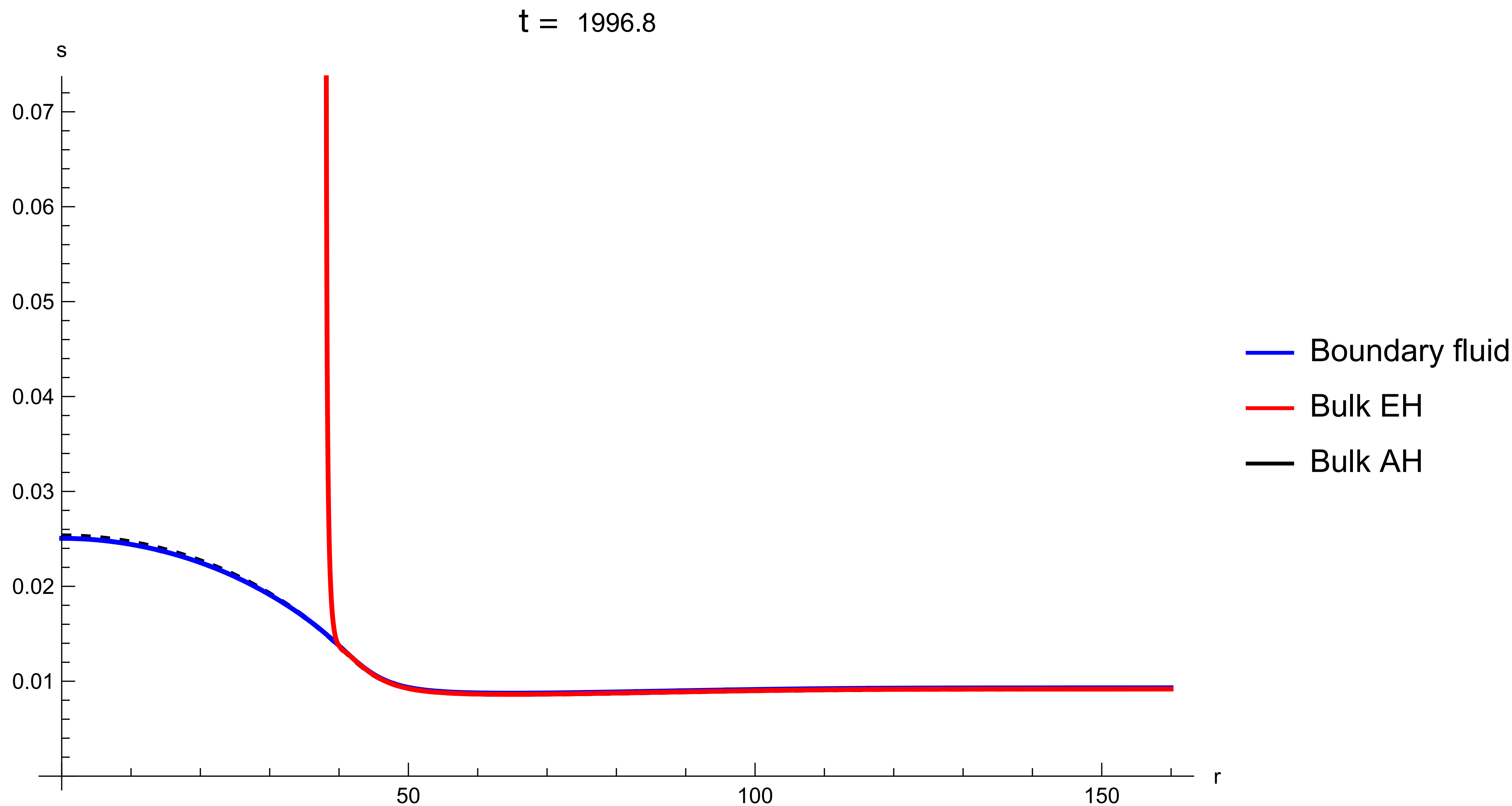
Entropy Density



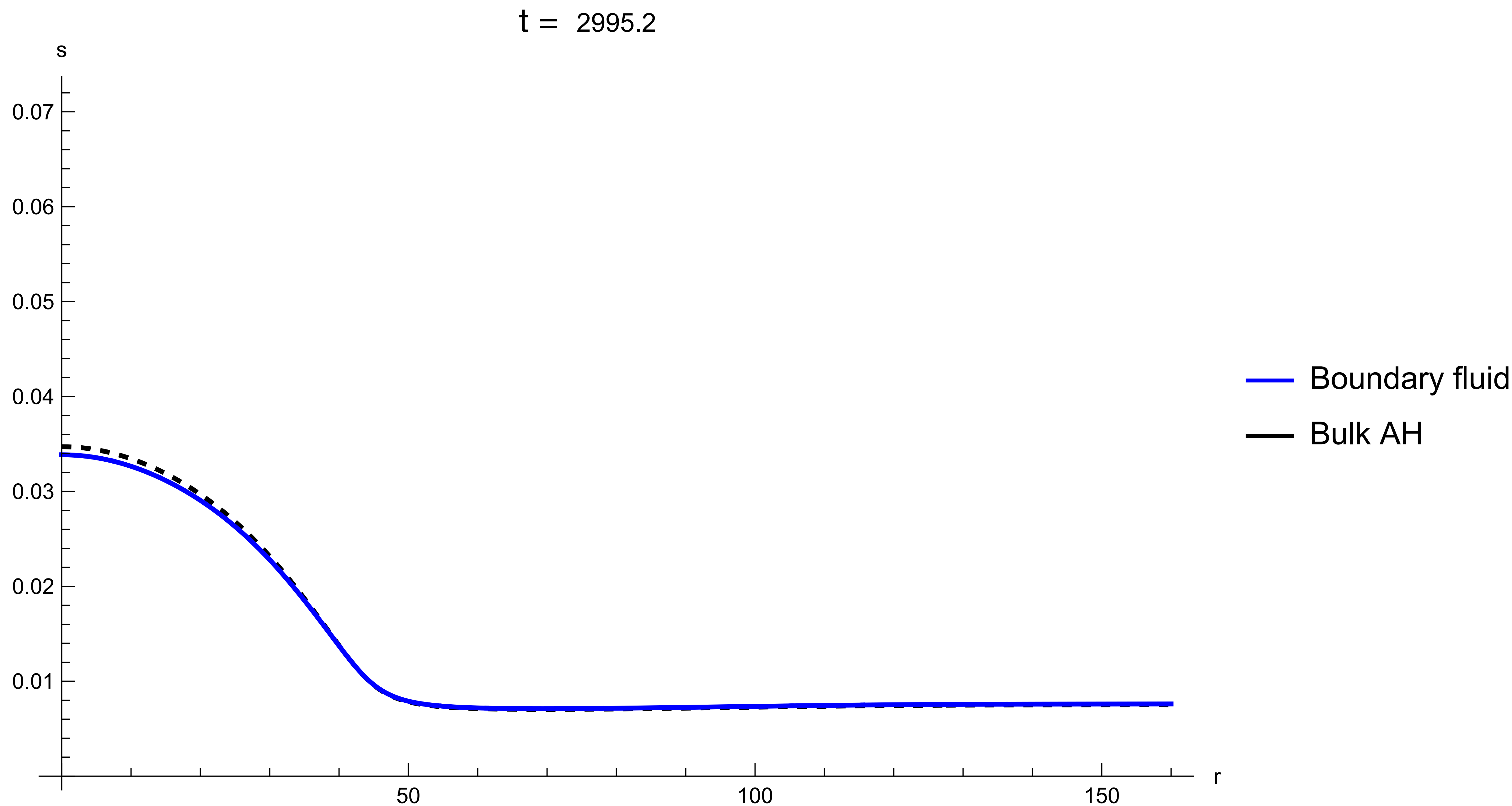
Entropy Density



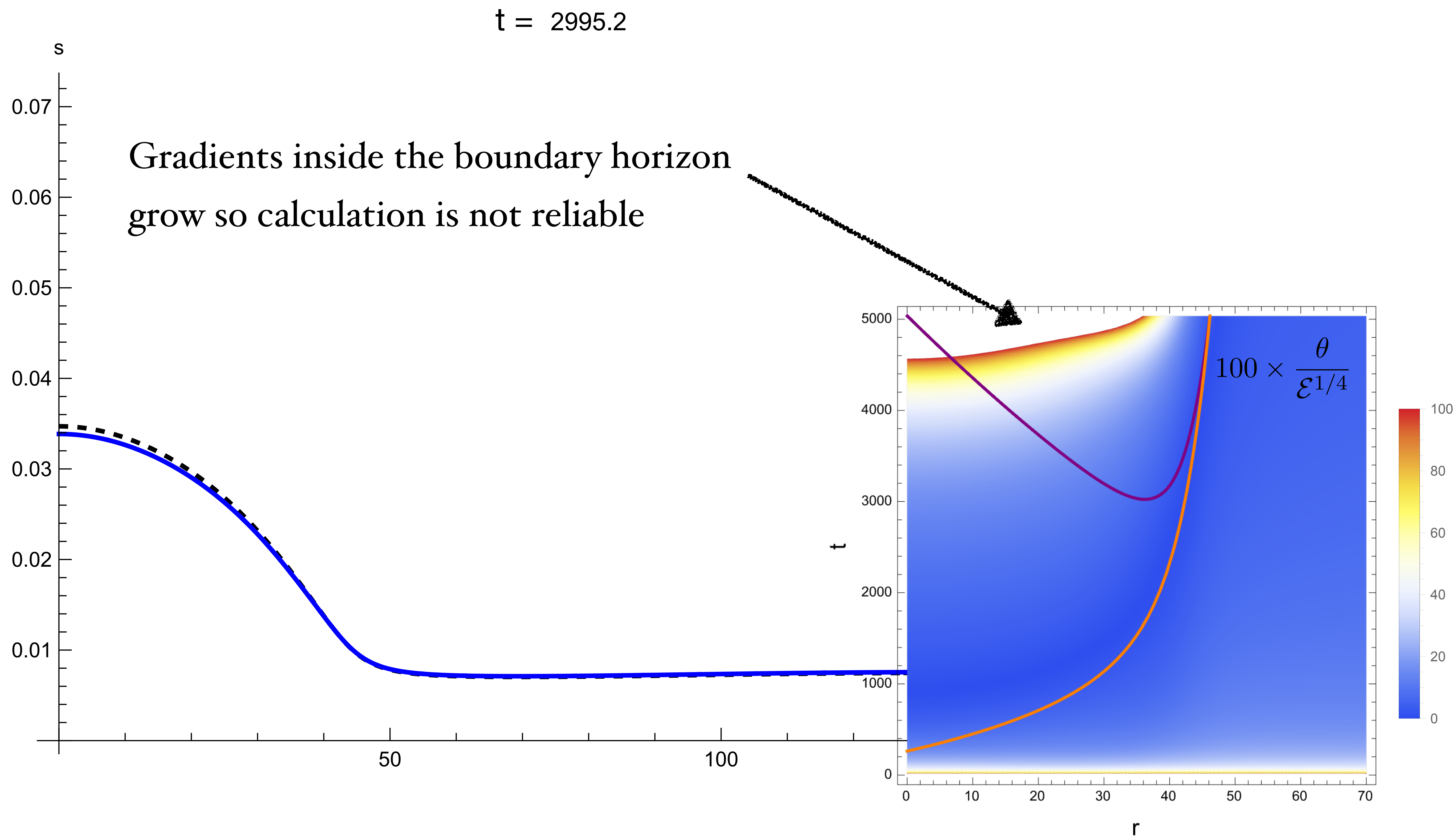
Entropy Density



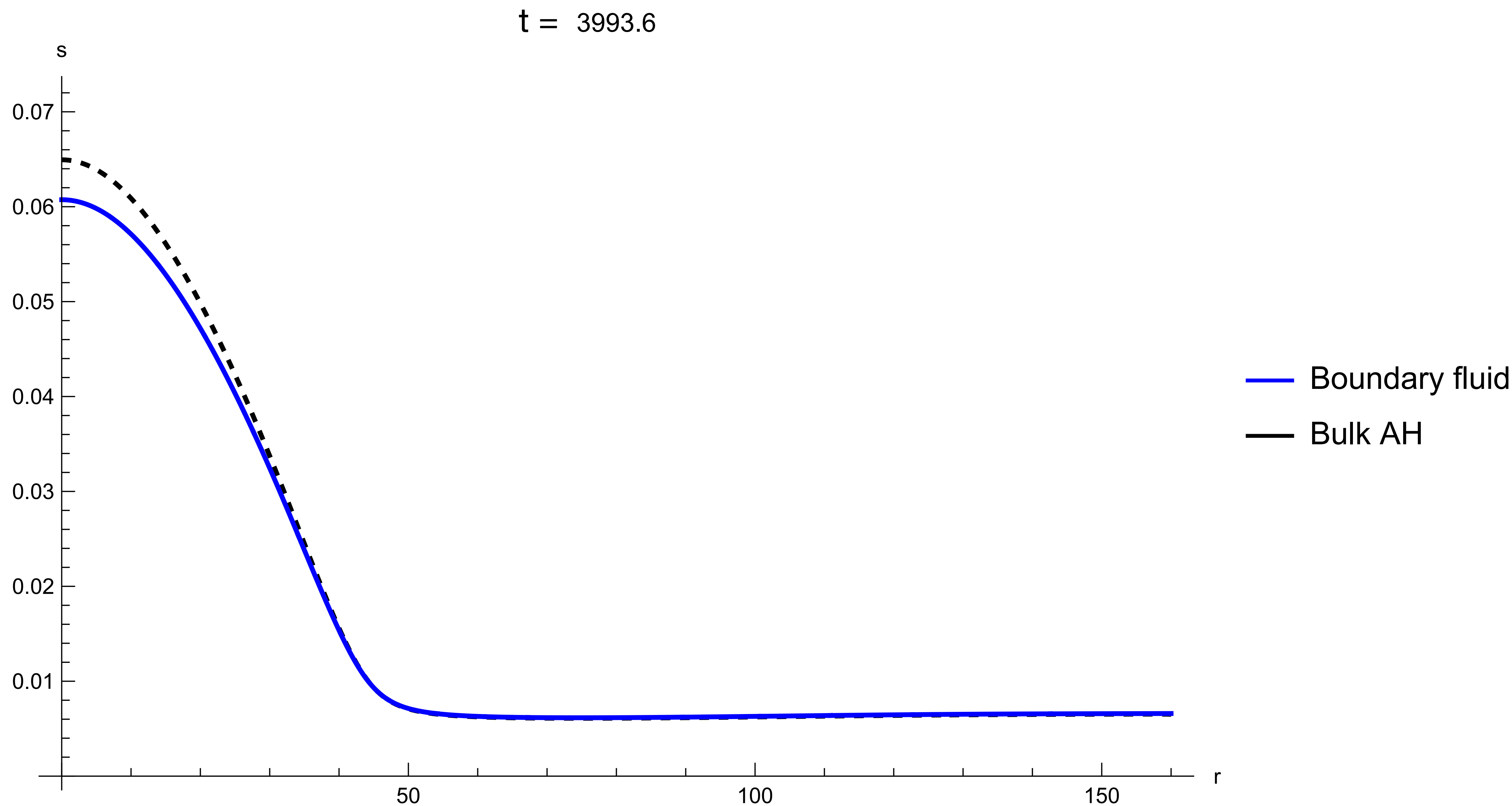
Entropy Density



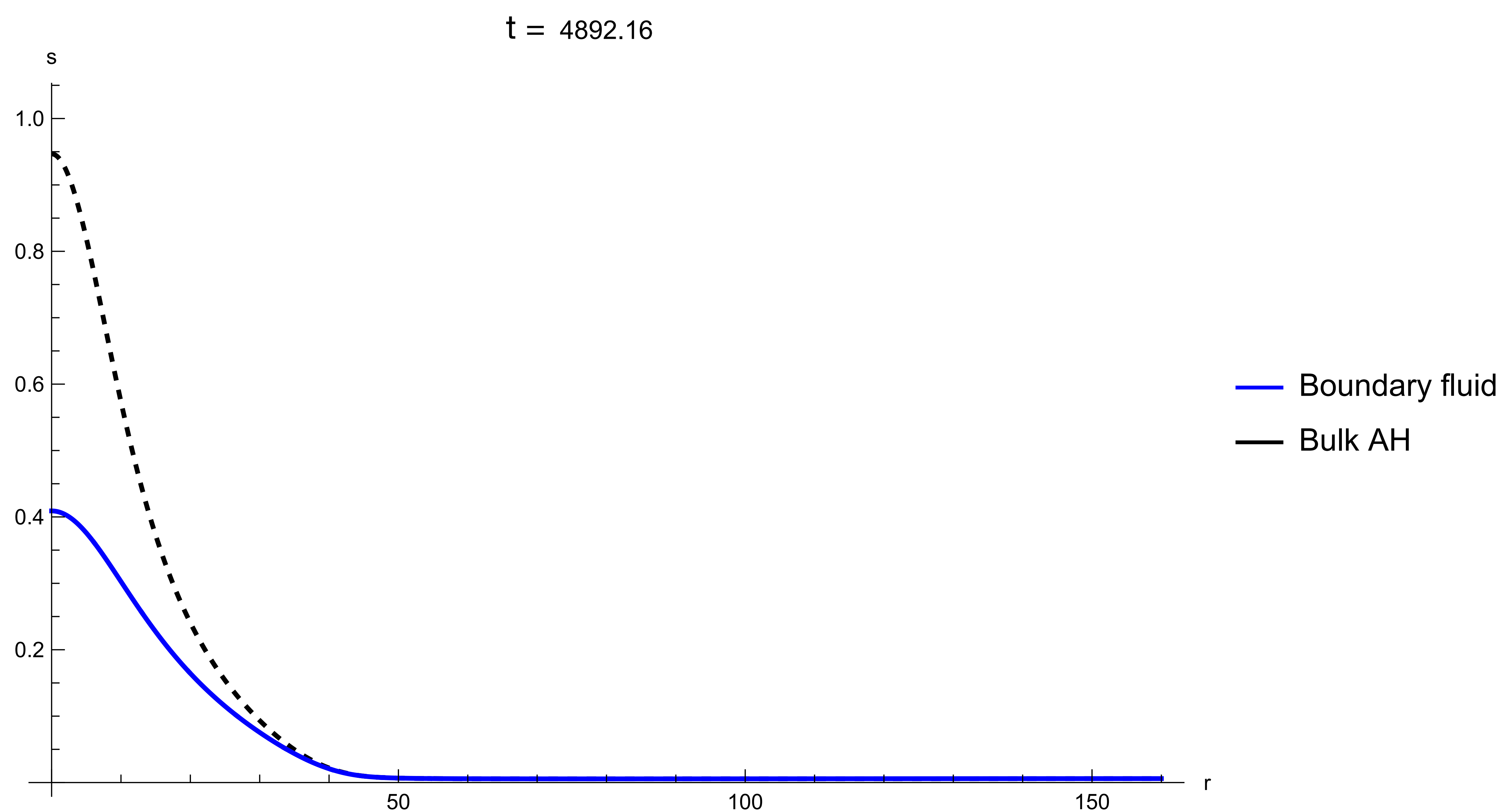
Entropy Density



Entropy Density

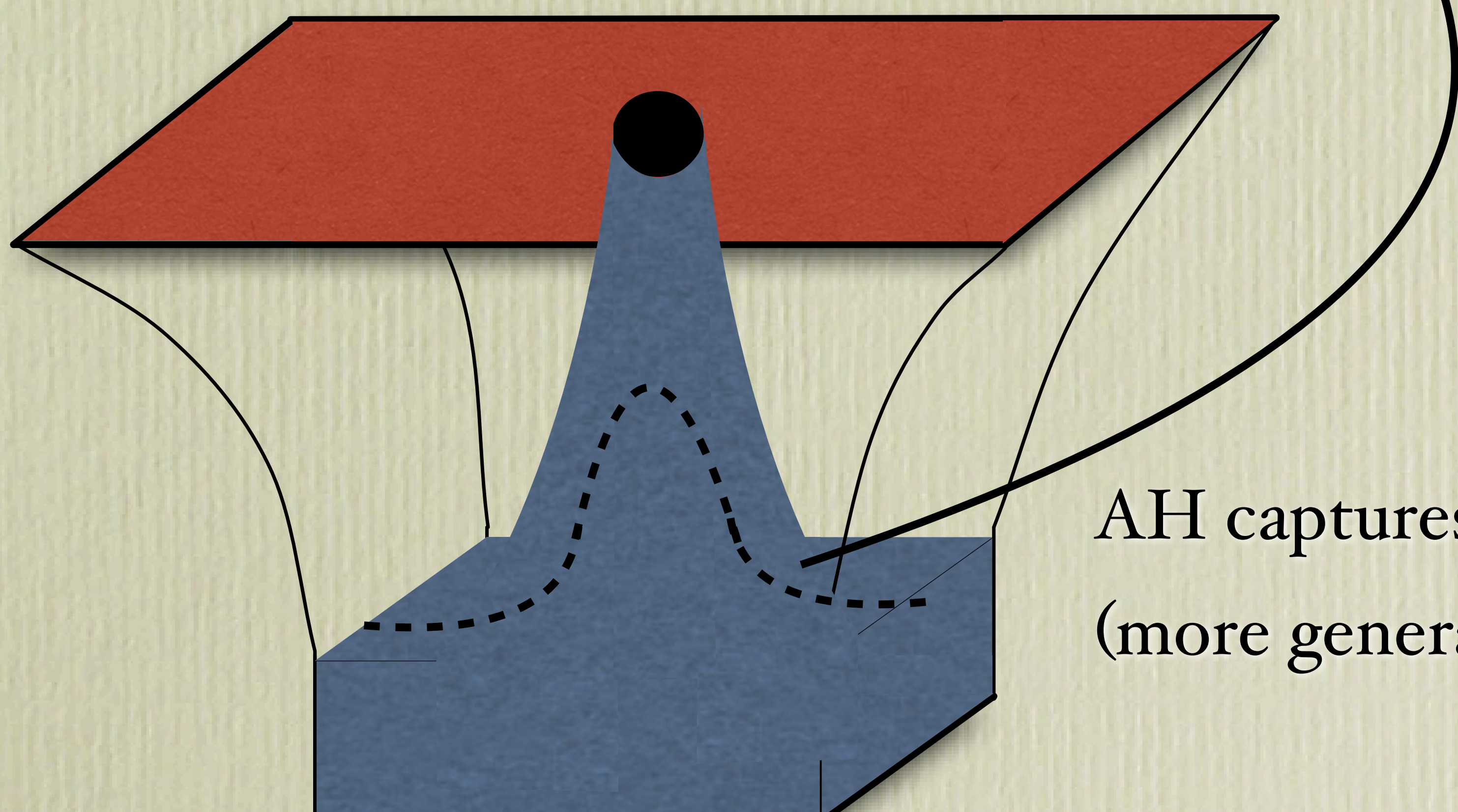


Entropy Density



Conclusion

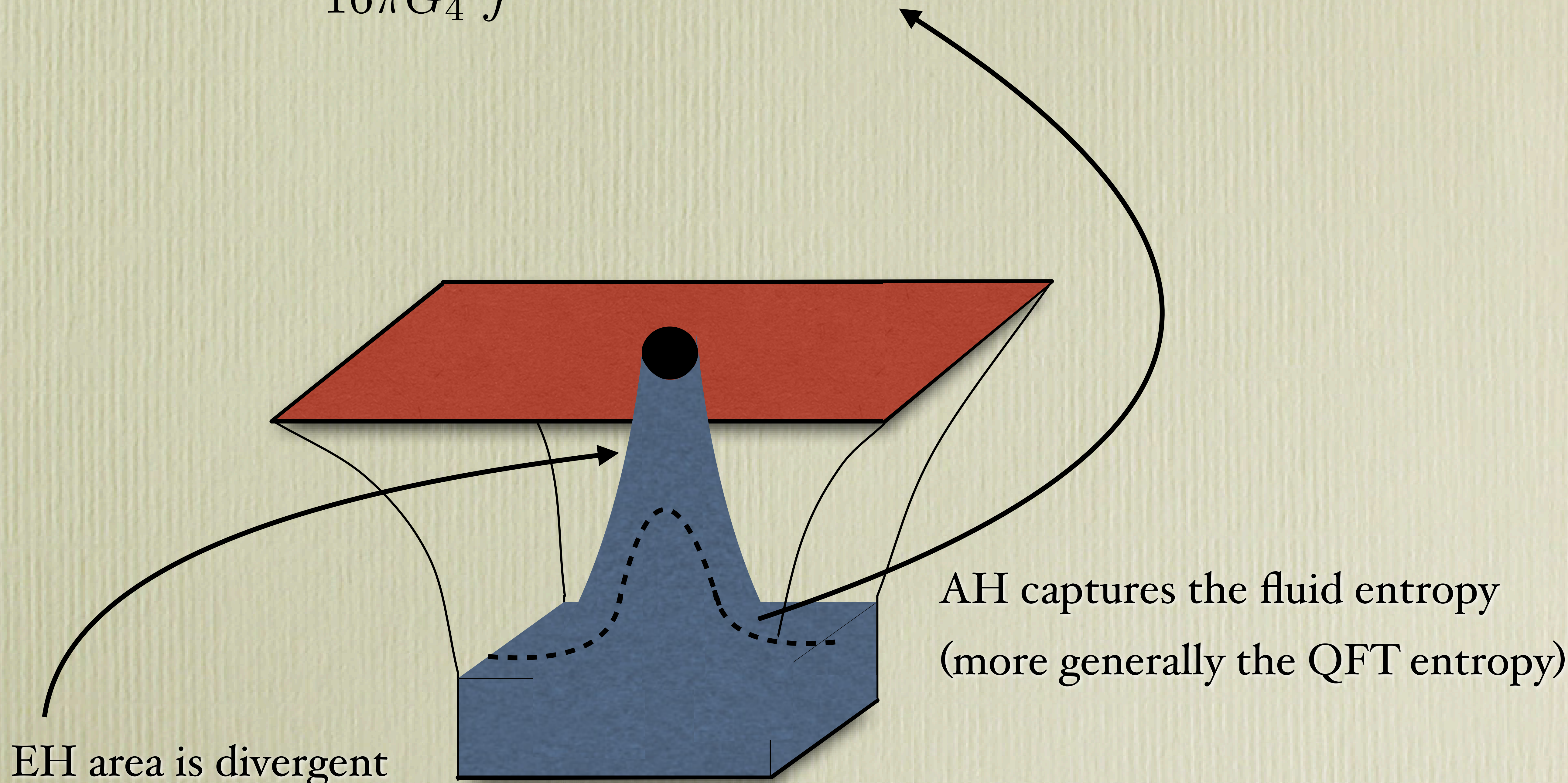
$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\rm GH} - \log Z_{\rm QFT}[g]$$



AH captures the fluid entropy
(more generally the QFT entropy)

Conclusion

$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\mathrm{GH}} - \log Z_{\mathrm{QFT}}[g]$$

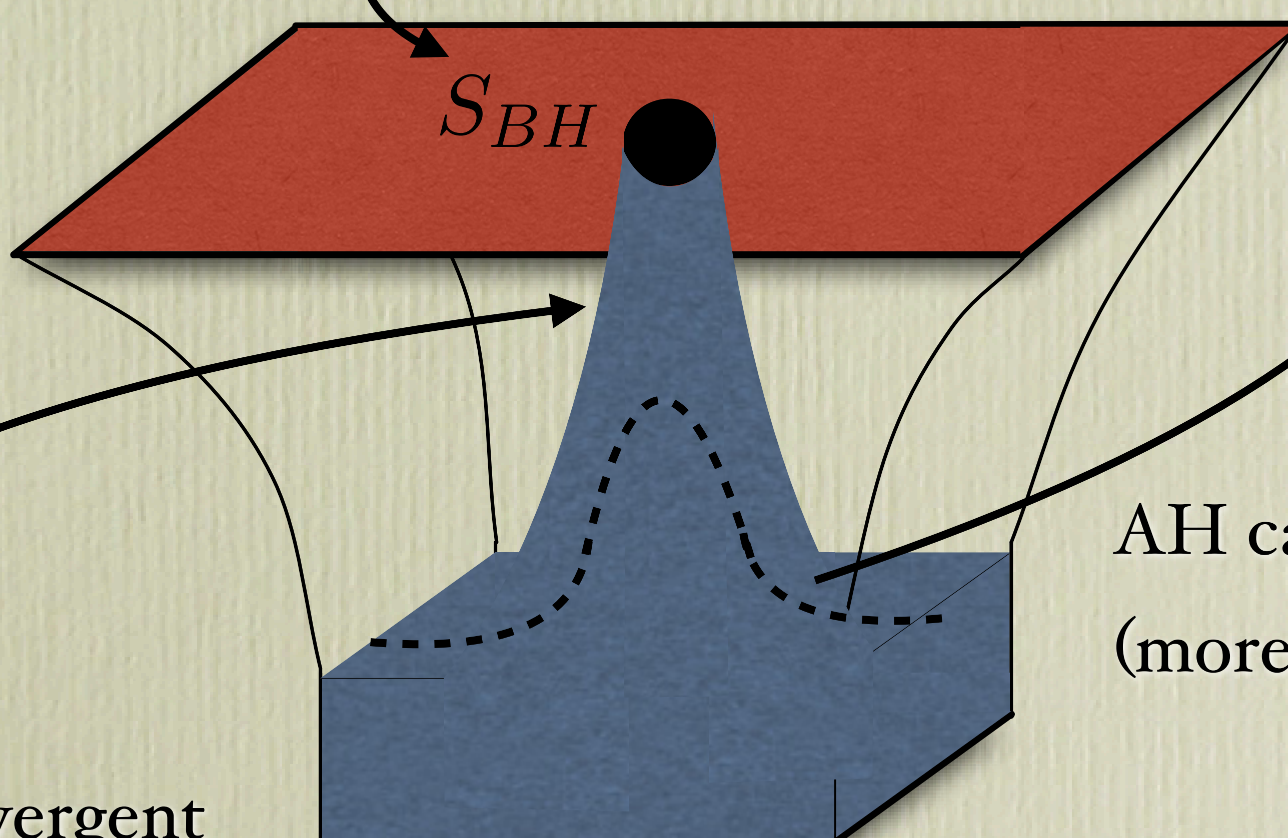


Conclusion

Agrees with Figueras, Hubeny, Rangamani & Ross '09

$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\mathrm{GH}} - \log Z_{\mathrm{QFT}}[g]$$

Bekenstein-Hawking entropy
not captured by the bulk



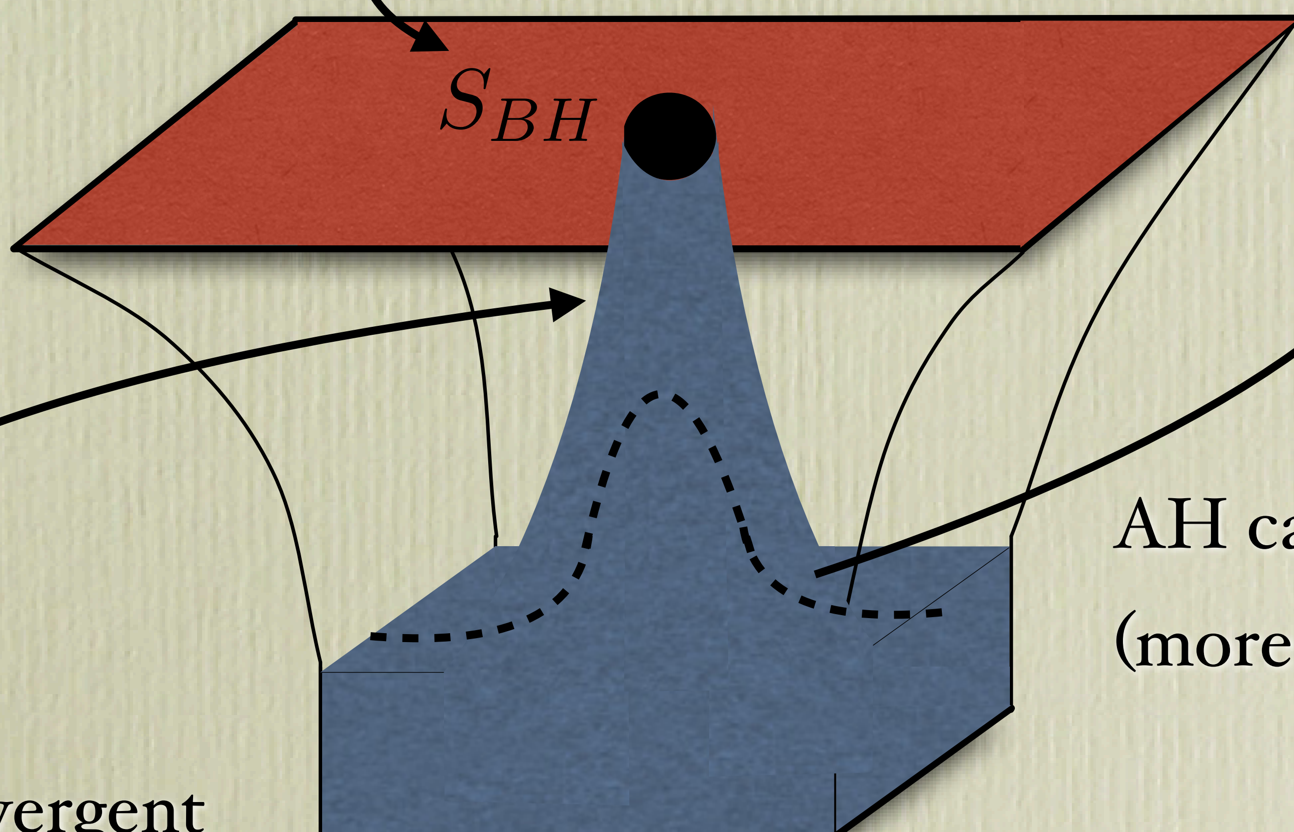
AH captures the fluid entropy
(more generally the QFT entropy)

EH area is divergent

Conclusion

$$S = \frac{1}{16\pi G_4}\int \sqrt{g}R + S_{\mathrm{GH}} - \log Z_{\mathrm{QFT}}[g]$$

Bekenstein-Hawking entropy
comes from bdry action



AH captures the fluid entropy
(more generally the QFT entropy)

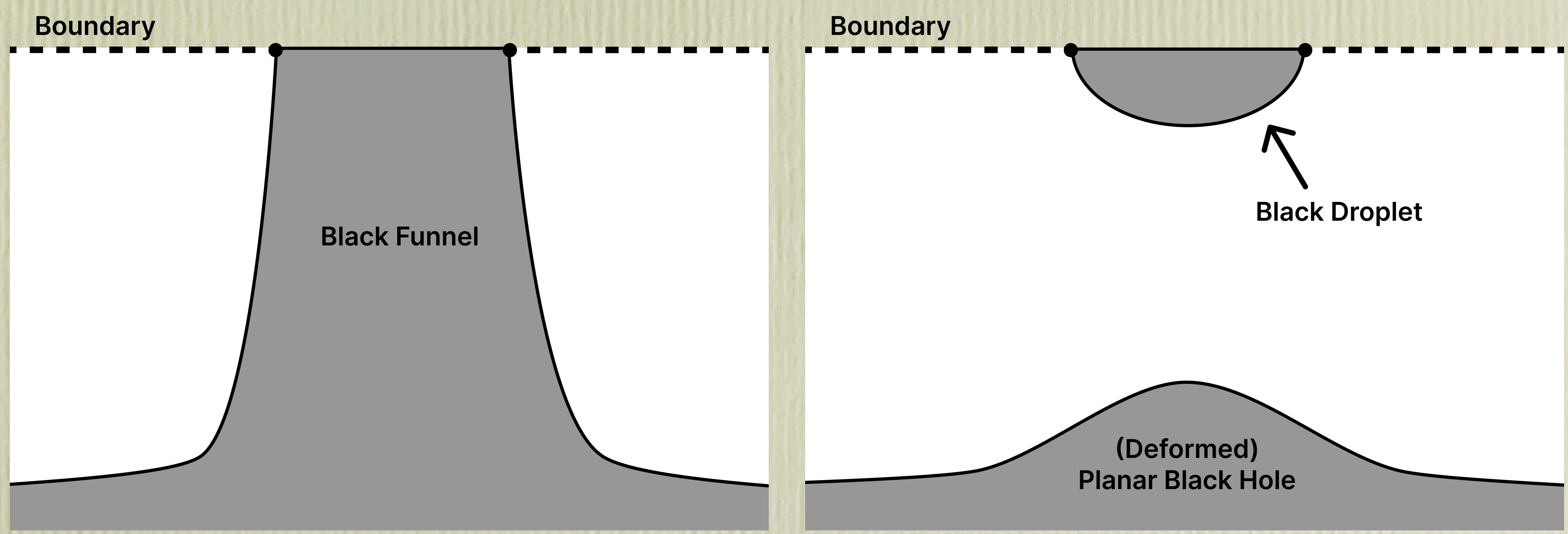
EH area is divergent

Similar phenomenon observed by Figueras, Hubeny, Rangamani & Ross '09

What about droplets?

DM, Serantes & Sole (work in progress)

- They may or may not form if the boundary BH is small enough.



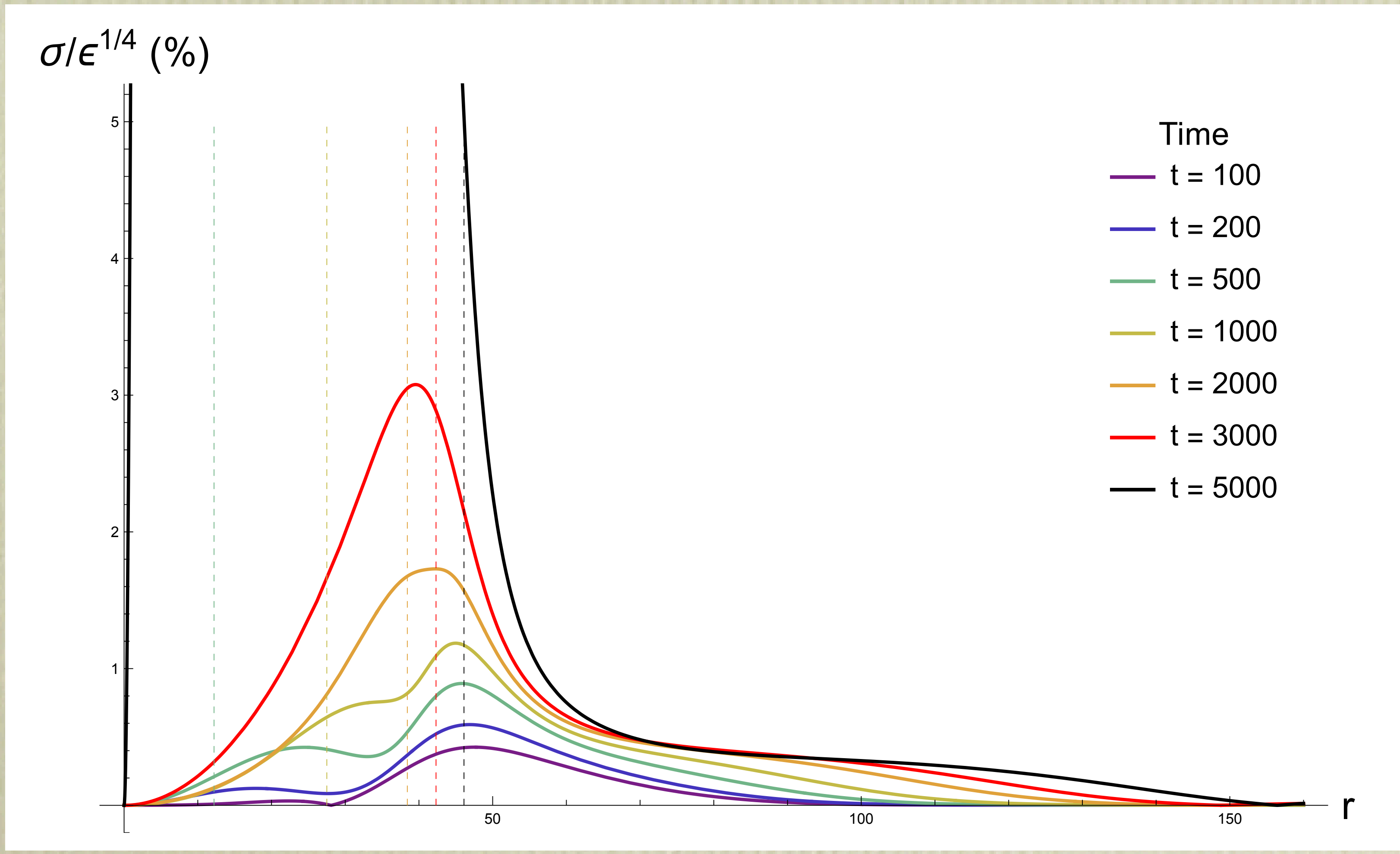
Hubeny, Marolf & Rangamani '09

Can we see evaporation?

Can we see evaporation?

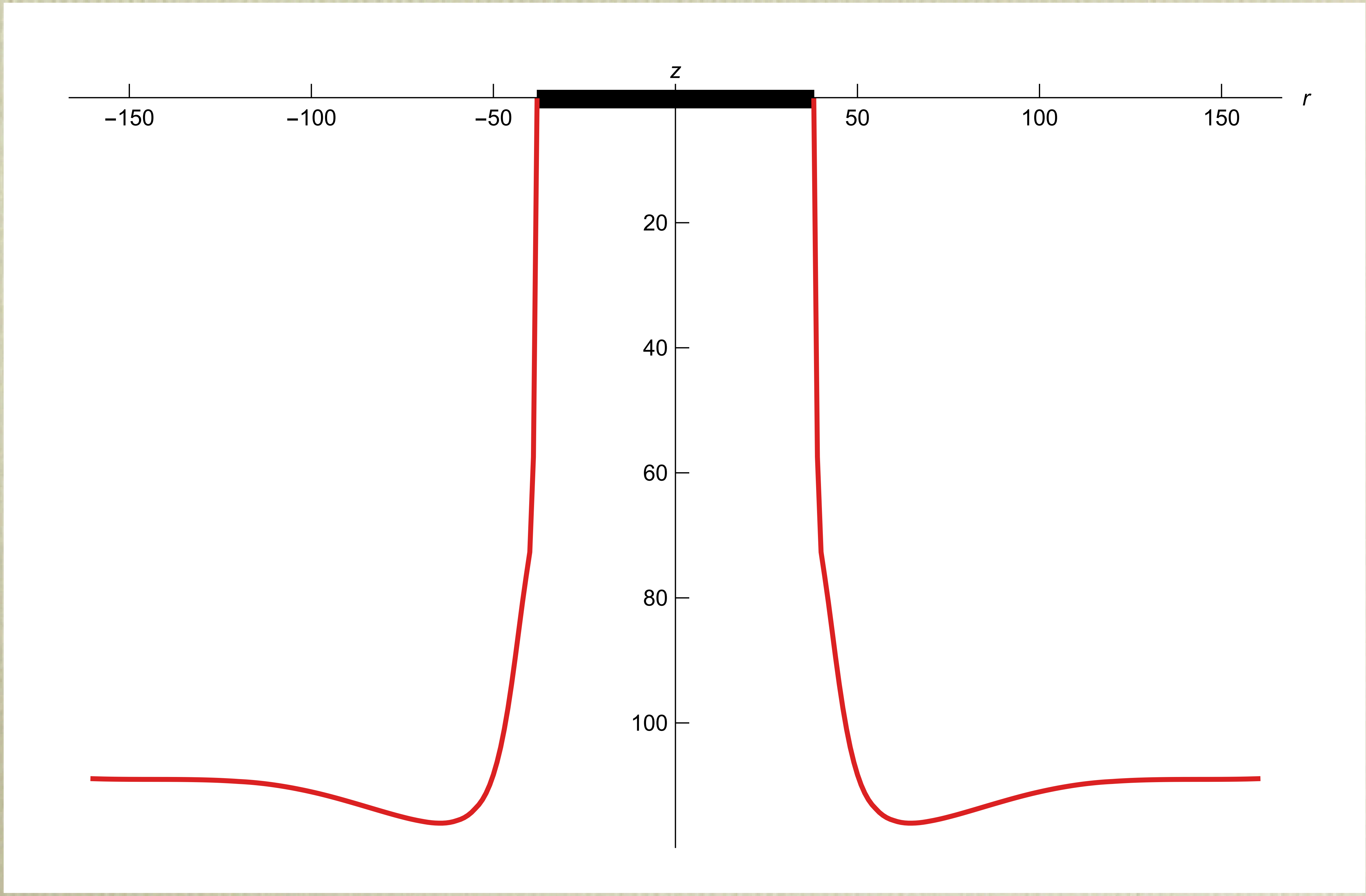
(work in progress)

- Should be possible because N^2 radiation channels.
- But then "classical GR+ fluid" description must break down.
- Boundary: Suggested by growth of corrections near horizon.



Can we see evaporation?

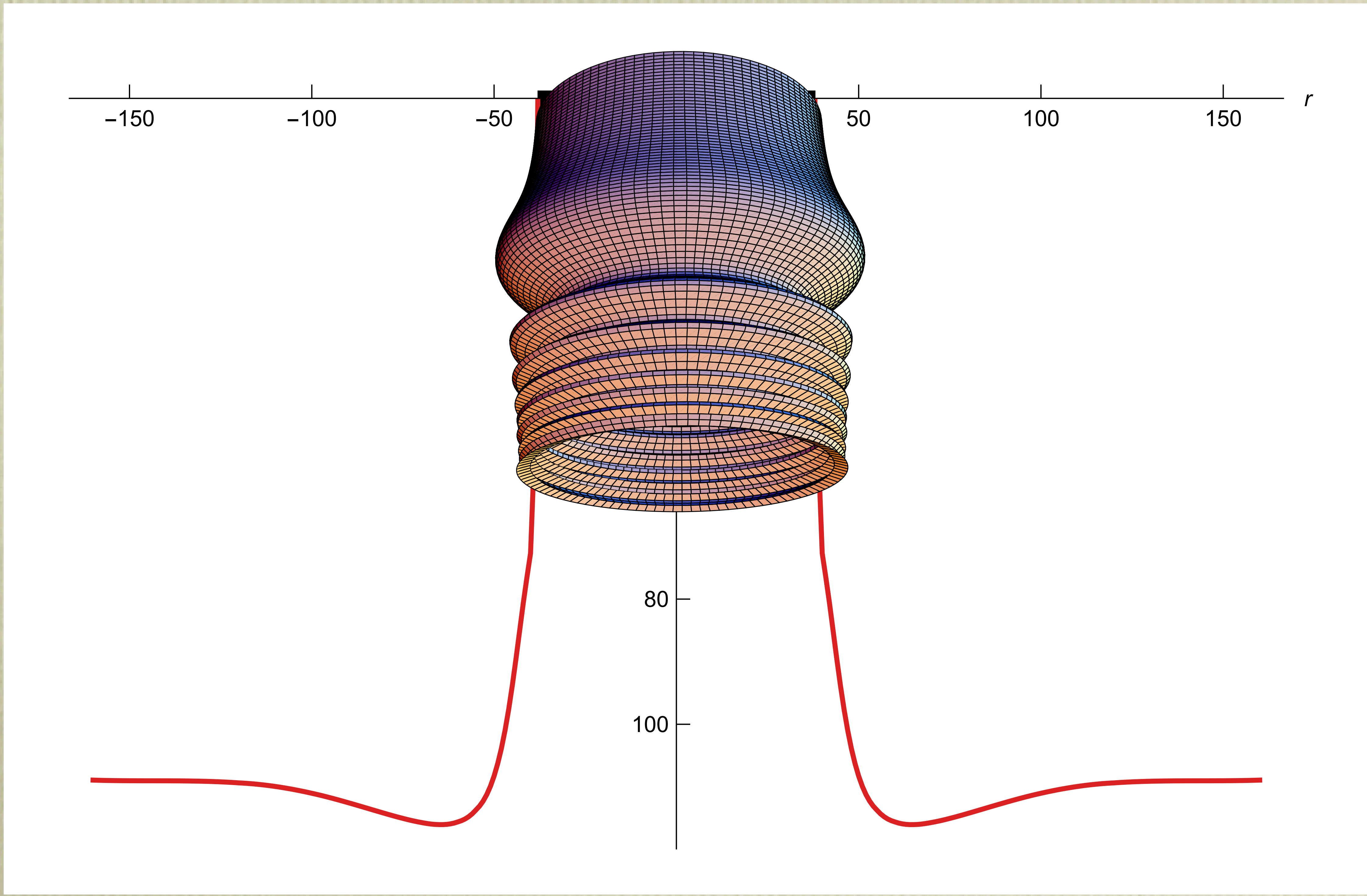
- Bulk: Suggested by Gregory-Laflamme instability.



Can we see evaporation?

Gregory '00

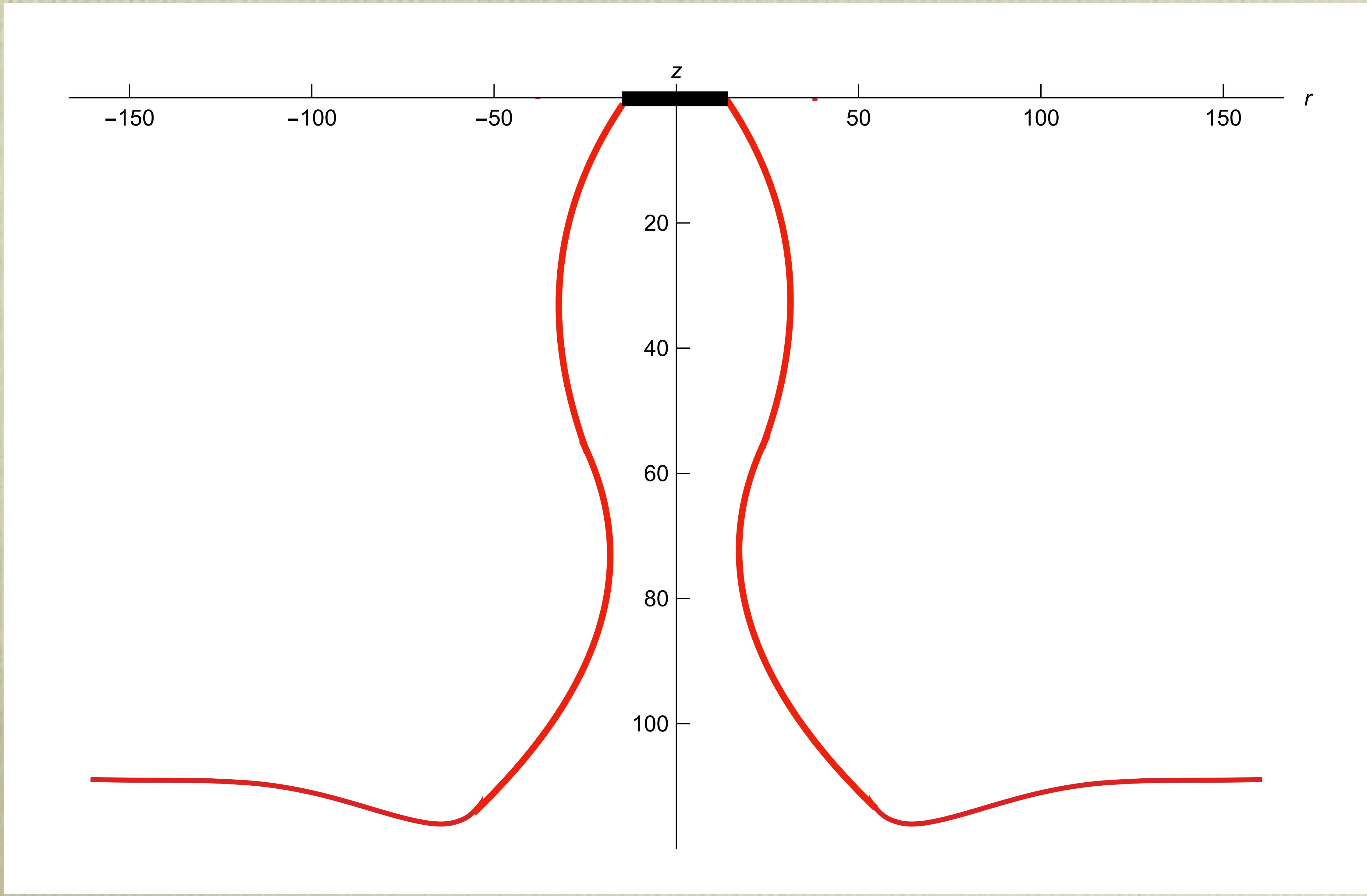
- Studied by Gregory for a fixed boundary metric.



Can we see evaporation?

(work in progres)

- In our case the boundary metric can change.

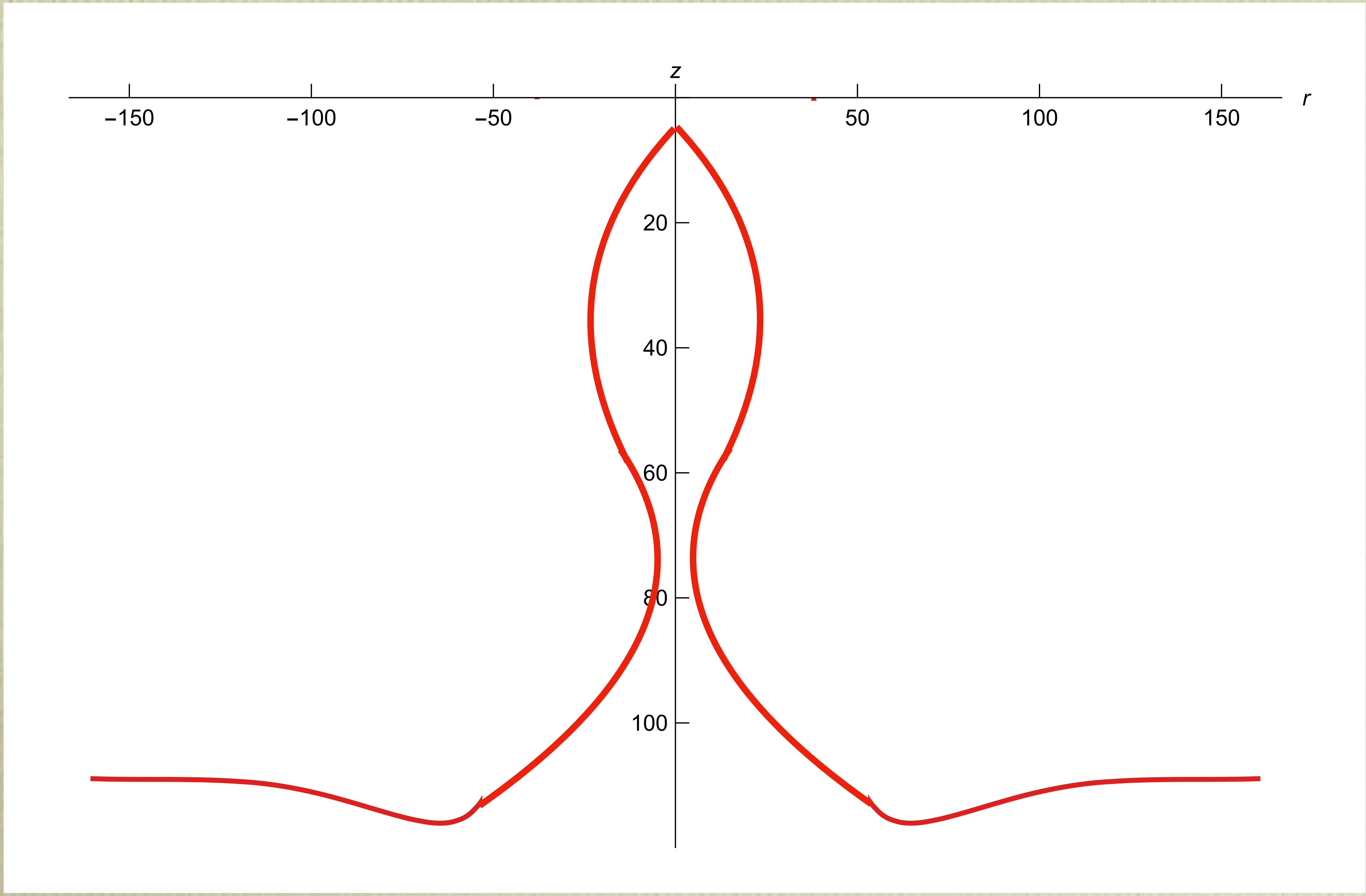


Can we see evaporation?

(work in progres)

(cf Emparan, Luna, Suzuki, Tomasevic & Way '23)

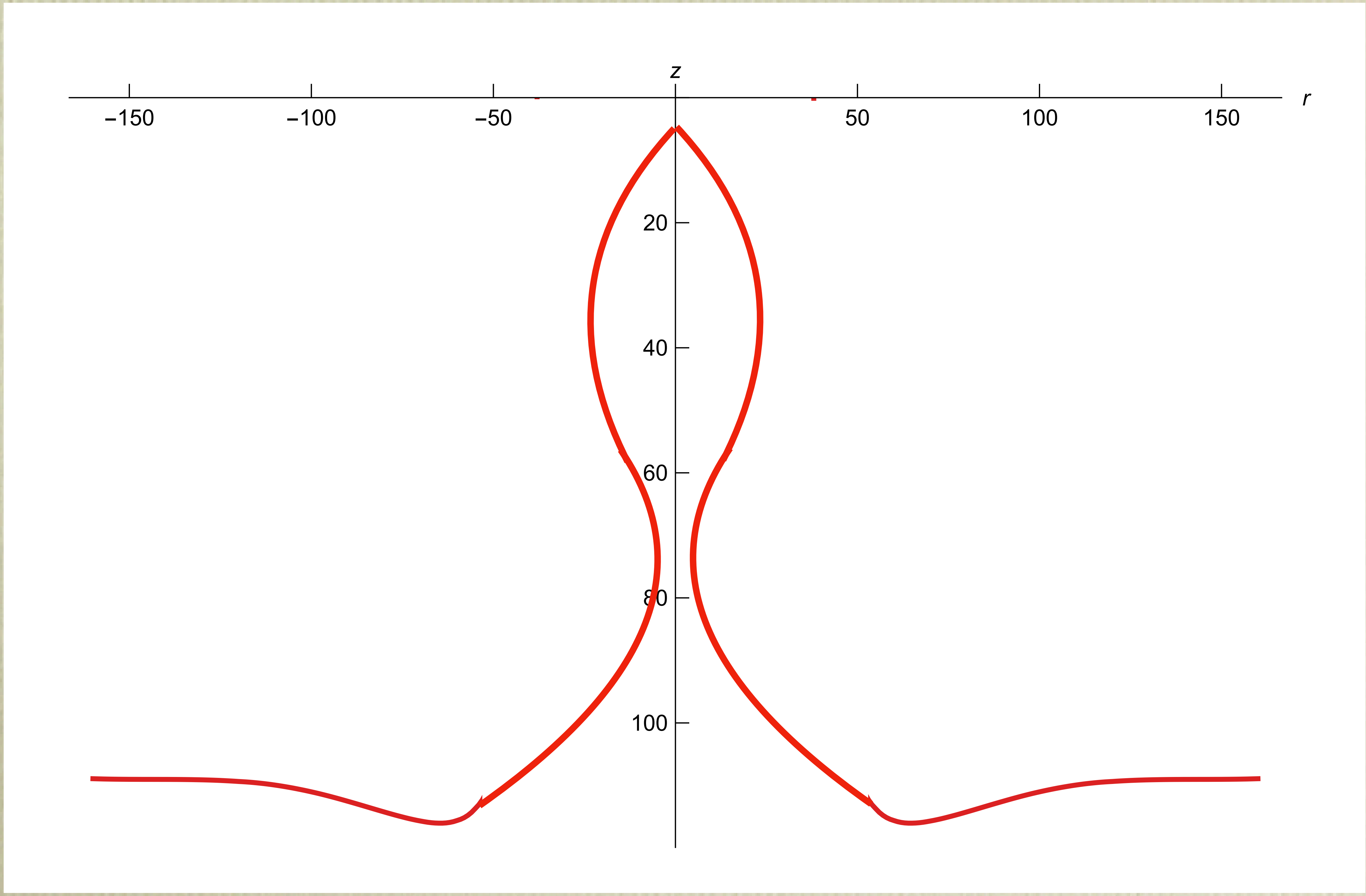
- So horizon can "slip off" the boundary.



Can we see evaporation?

(work in progres)

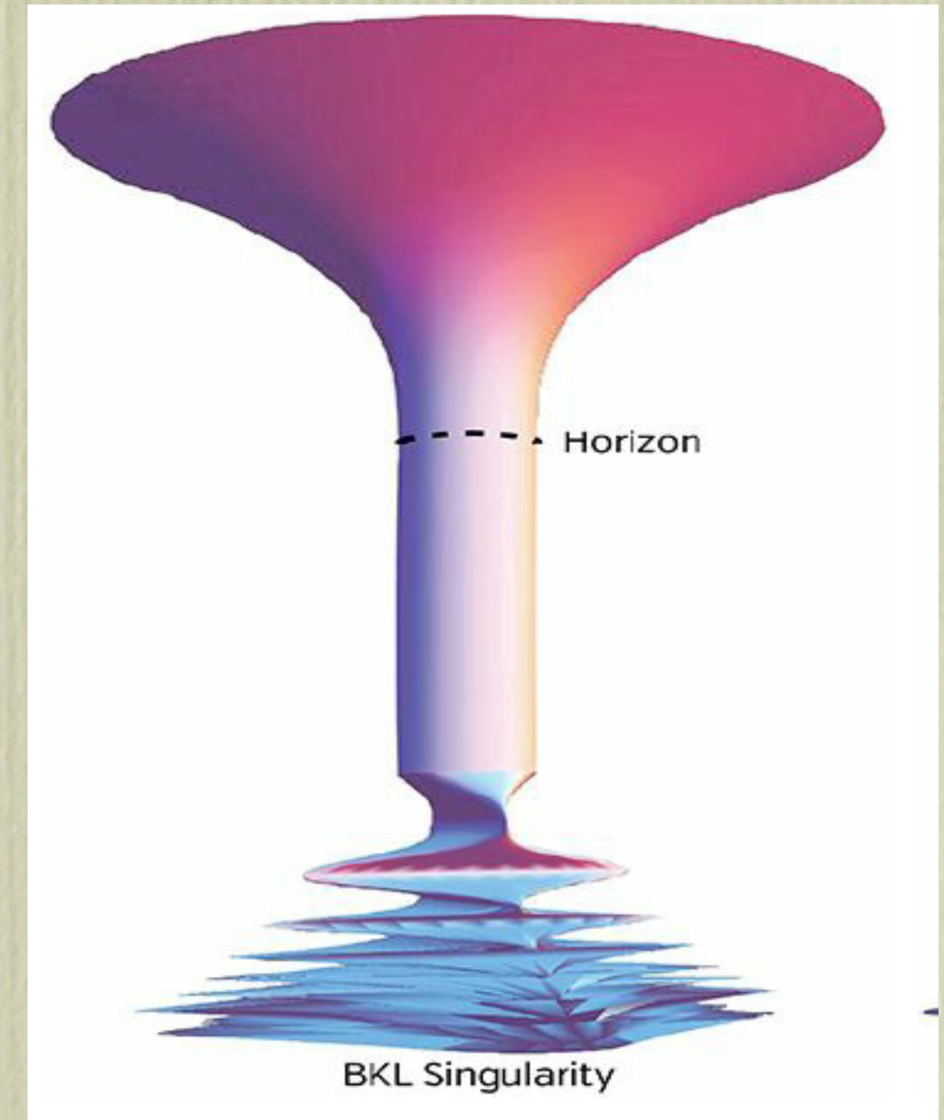
- Making this concrete requires solving coupled 4D+5D evolution.



Approaching a Cosmological Singularity

Application to cosmological singularities

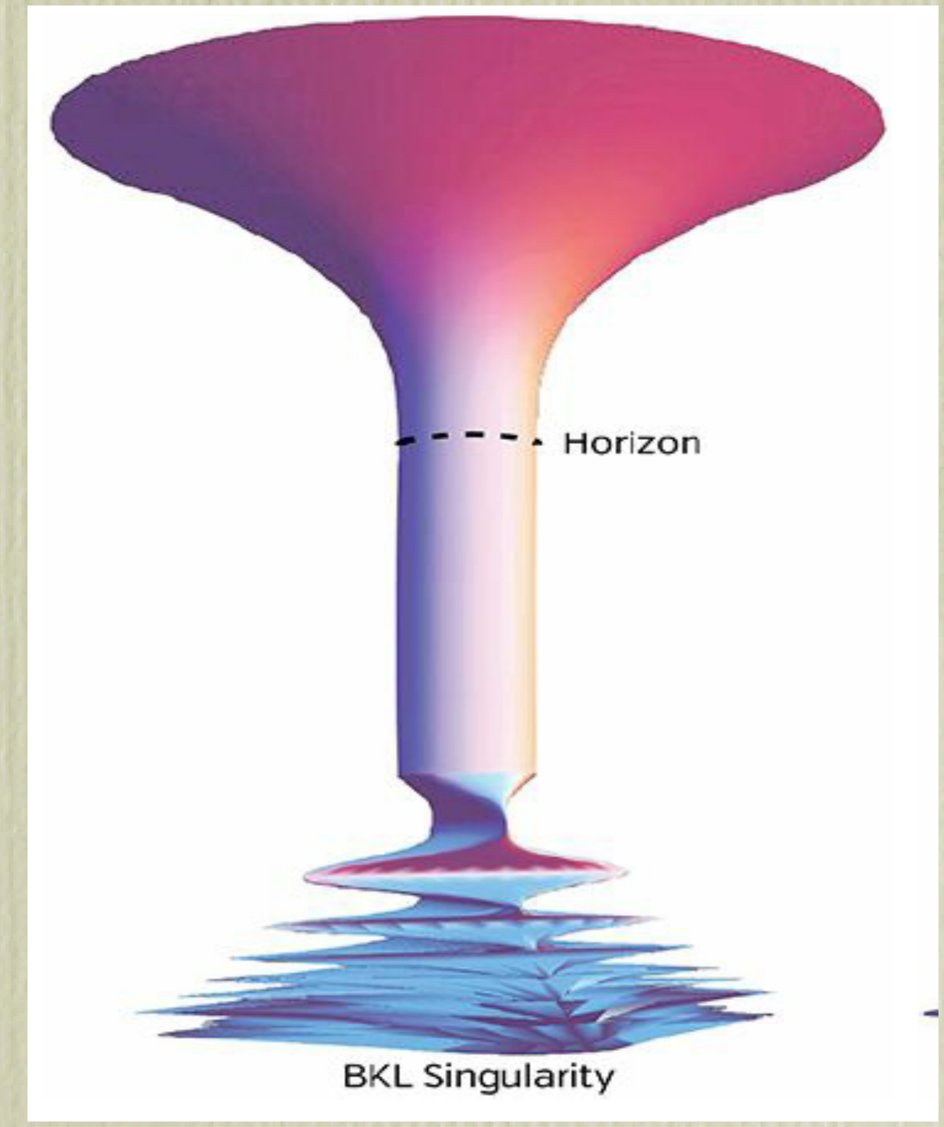
- Classical GR predicts spacetime singularities.
- Most mysterious ones are spacelike singularities (Big Bang, interior of BH, etc).



Application to cosmological singularities

- What happens to spacetime as we approach the singularity.
- Classically: Dynamics is oscillatory and chaotic (BKL behaviour).

Belinsky, Khalatnikov & Lifshitz '70



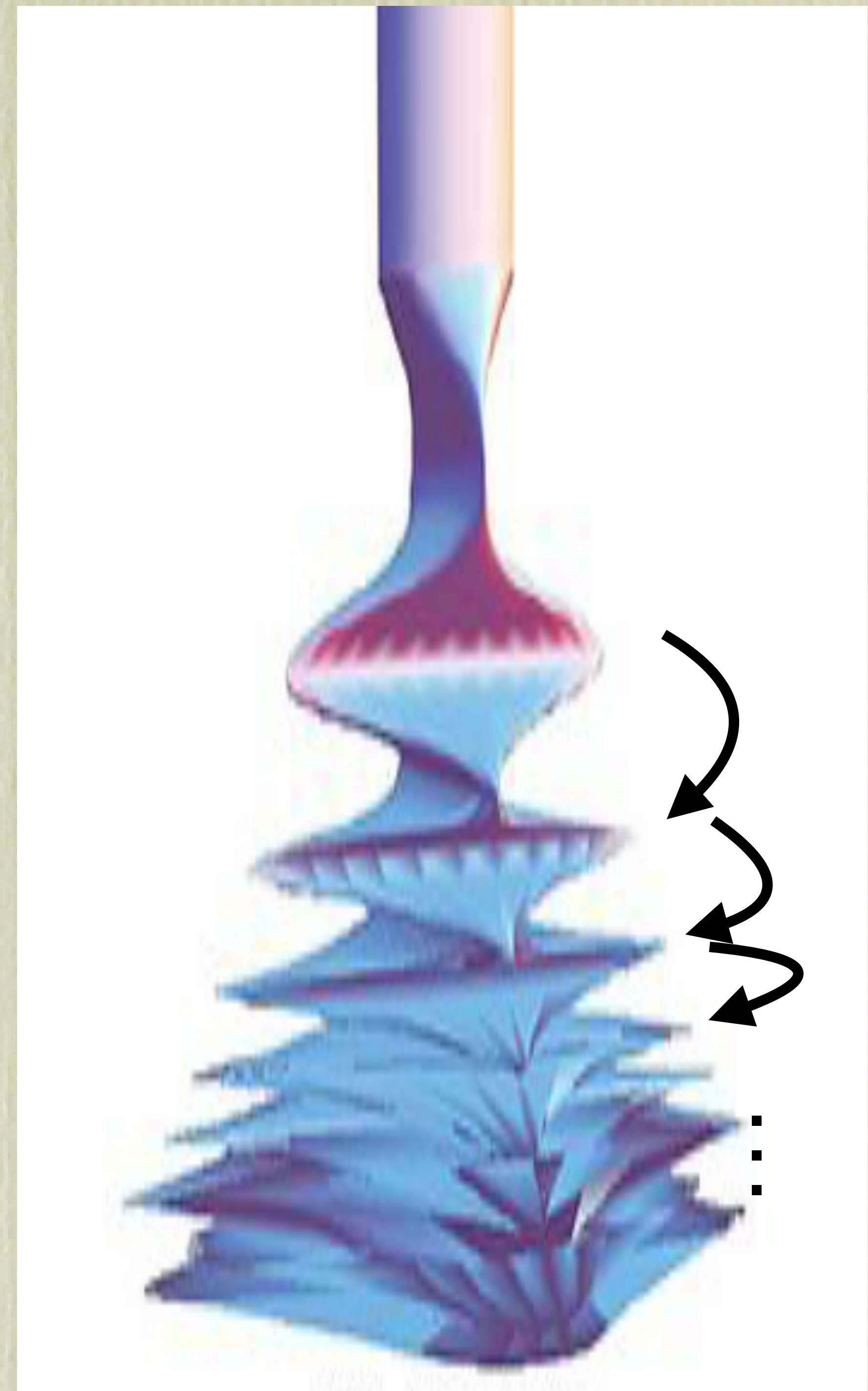
Application to cosmological singularities

Belinsky, Khalatnikov & Lifshitz '70

- BKL = Infinite sequence of Kasner epochs with rapid transitions between them:

$$ds^2 = -dt^2 + \sum_{i=1}^{3} |t|^{2p_i}\, dx_i^2$$

$$\sum_{i=1}^{3} p_i = 1\,, \qquad \sum_{i=1}^{3} p_i^2 = 1$$



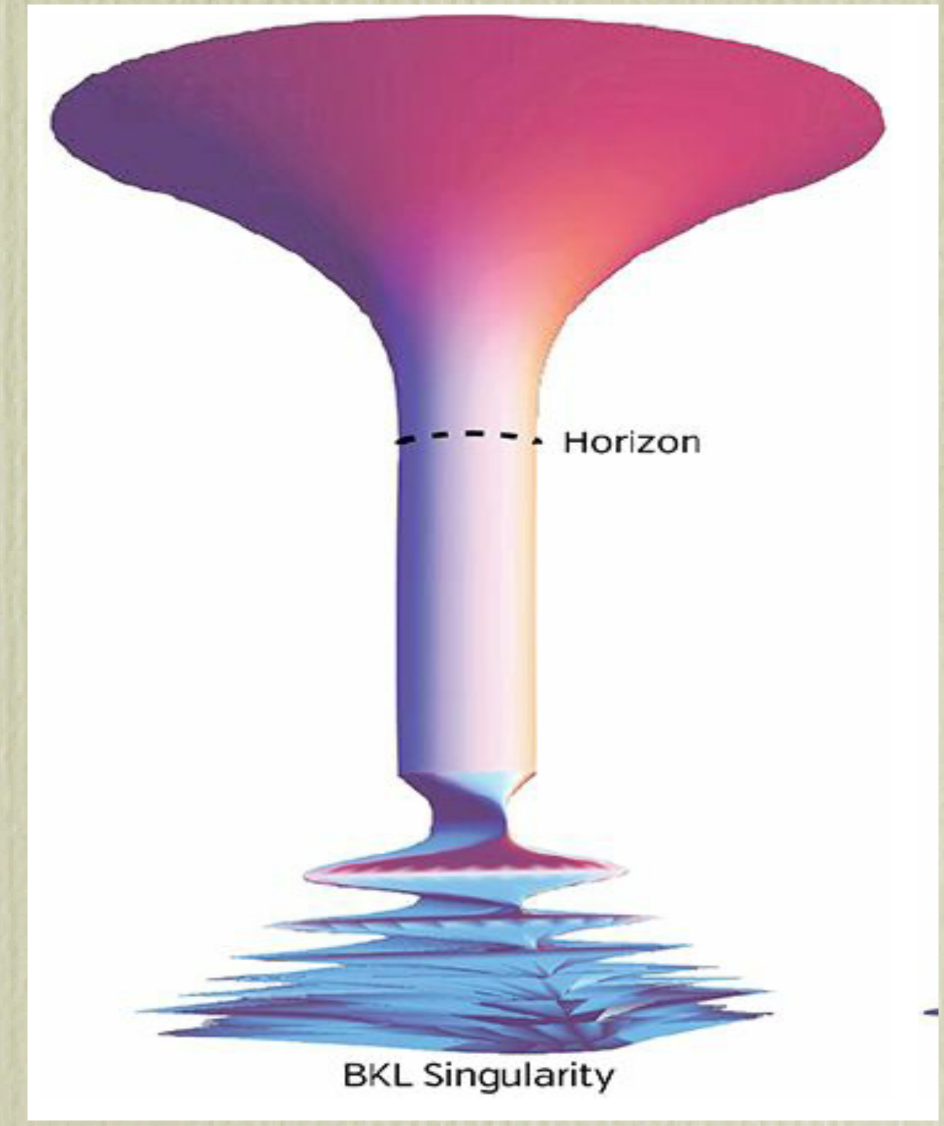
- This implies 2 p's are + and one is -.

- This makes Kasner unstable and leads to transitions:

$$p_i \;\to\; p_i' \;\to\; p_i'' \;\to\; \cdots$$

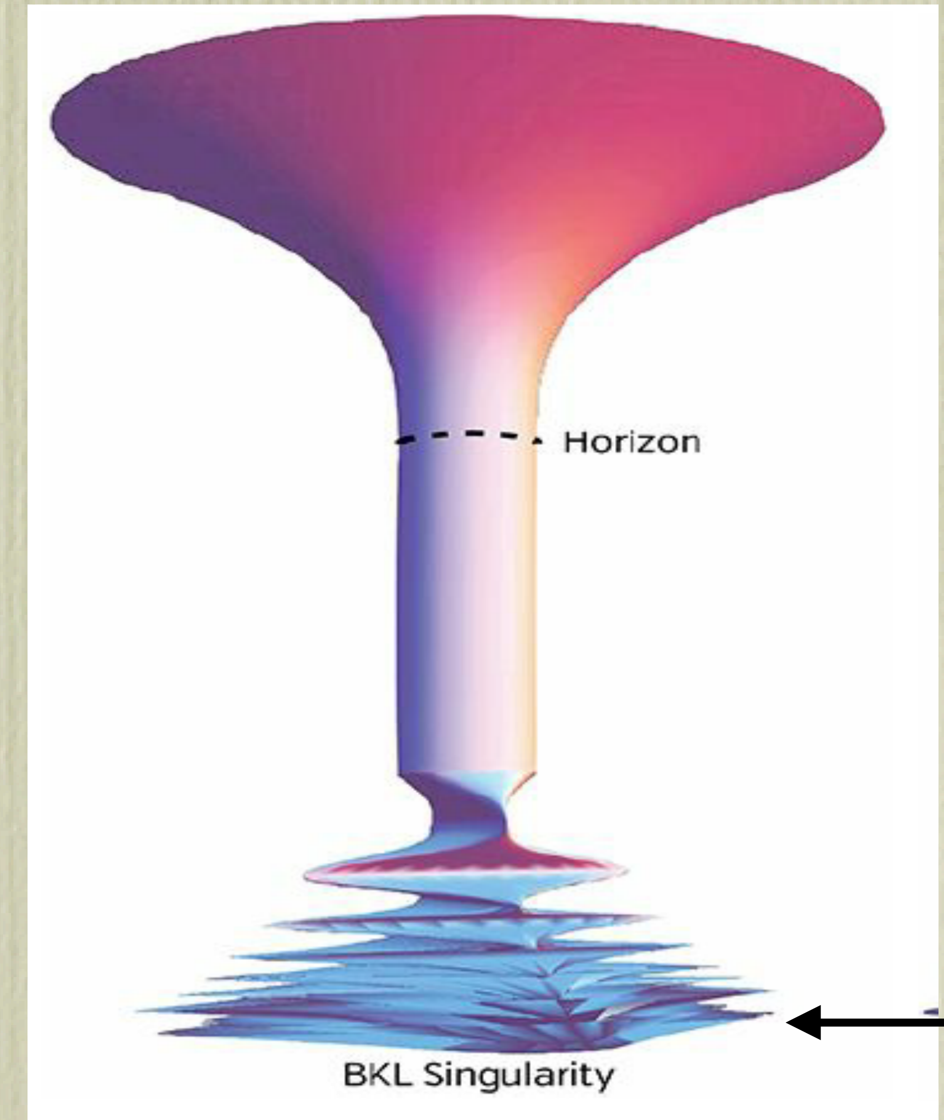
Application to cosmological singularities

- Considered a fundamental achievement in GR.
- But what about quantum effects?



Application to cosmological singularities

- Ultimate description may involve quantum gravity.

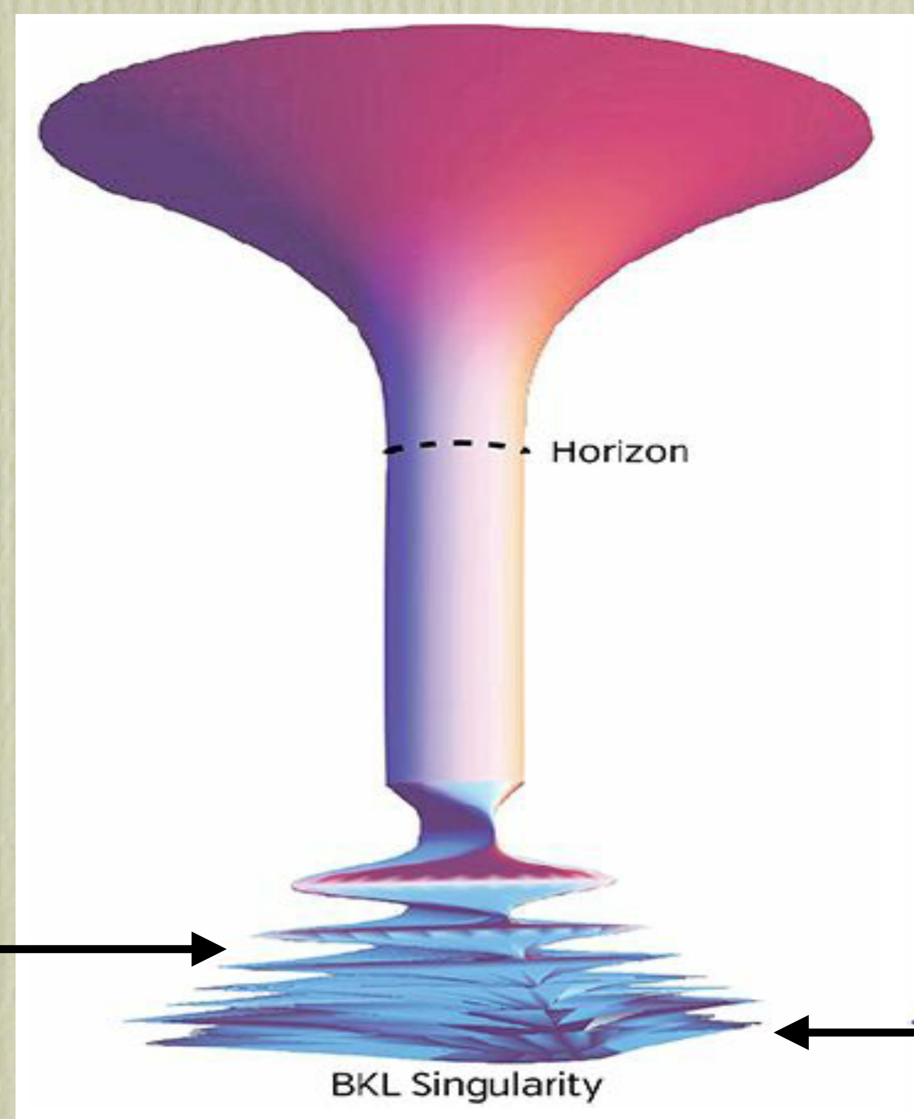


Quantum Gravity?

Application to cosmological singularities

- Ultimate description may involve quantum gravity.
- But interesting intermediate regime:

Quantum Matter +
Classical Gravity

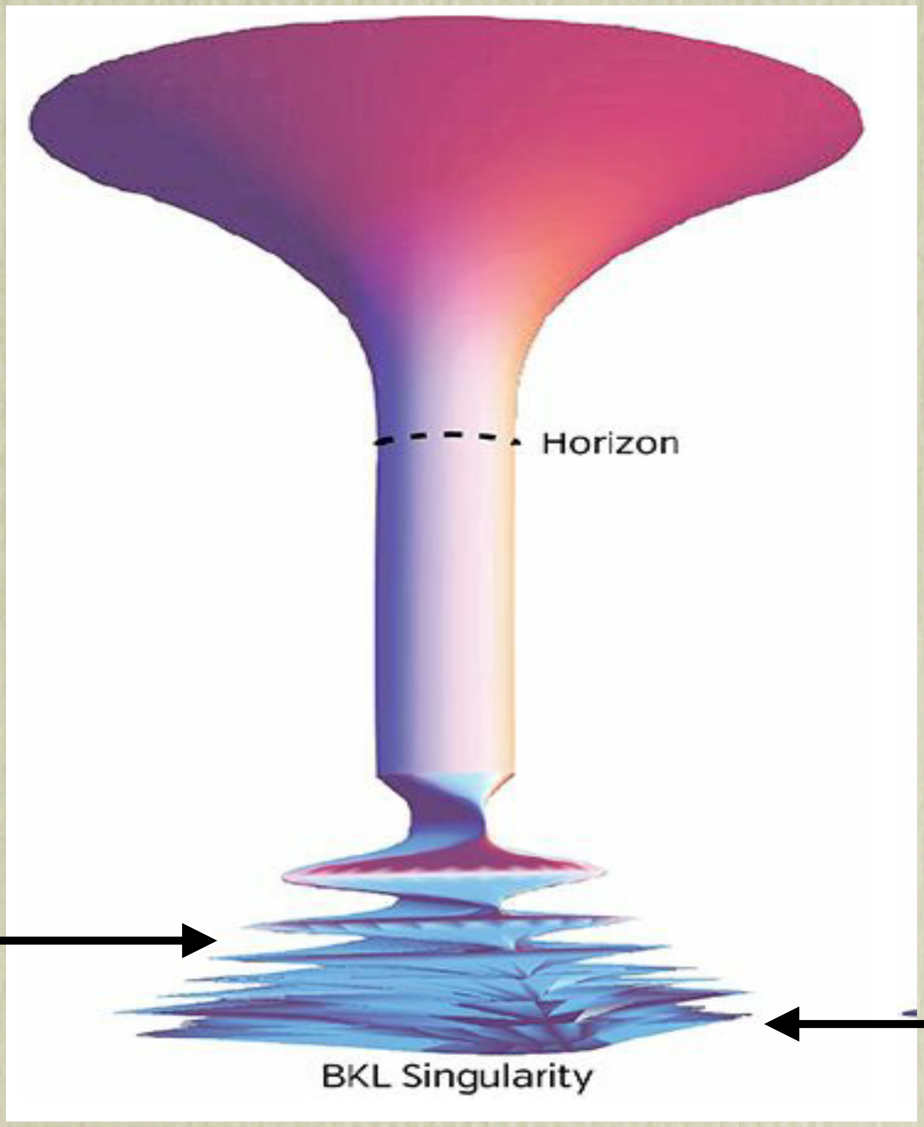


Quantum Gravity?

Application to cosmological singularities

- Main challenge: Matter is pushed far from equilibrium.
- But this regime can be described holographically.

Quantum Matter +
Classical Gravity

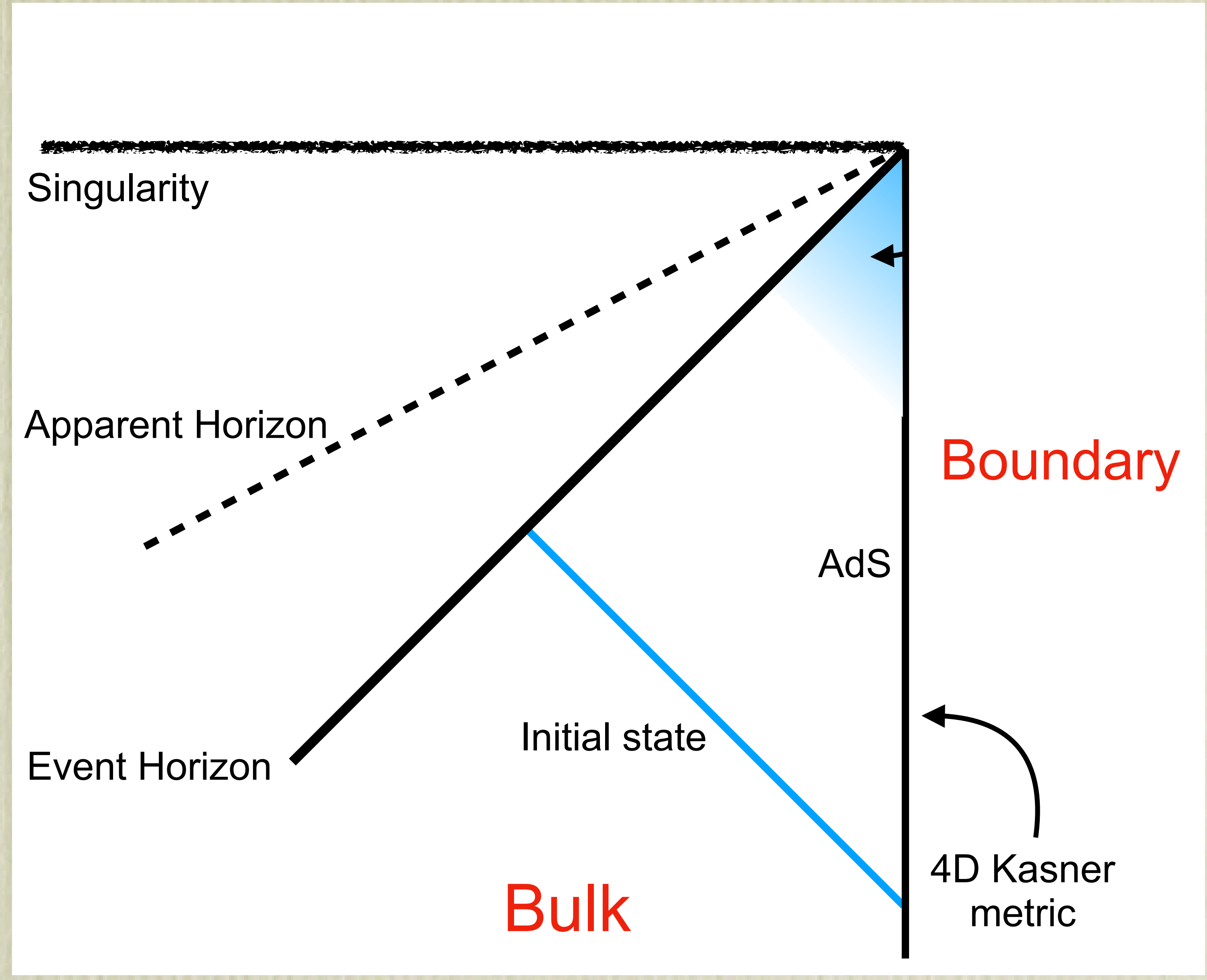


Quantum Gravity?

Application to cosmological singularities

Casalderrey, DM & Serantes '23

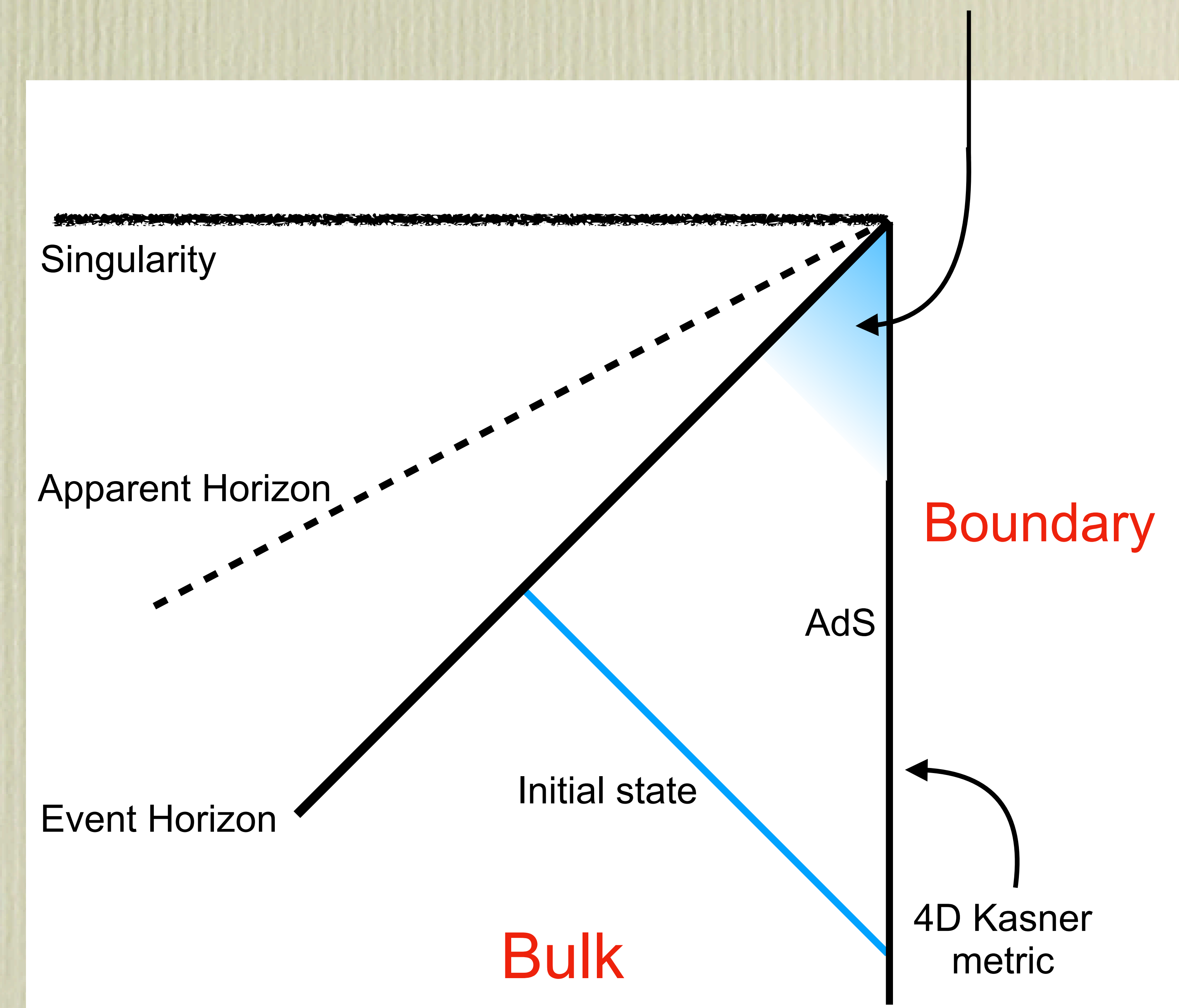
- Consider Kasner metric at the boundary:



Application to cosmological singularities

Casalderrey, DM & Serantes '23

- Consider Kasner metric at the boundary:
- Universal stress tensor near singularity: $\mathcal{E} = \dfrac{\Lambda^2}{t^2}\,, \qquad \mathcal{P}_i = p_i\,\mathcal{E}$



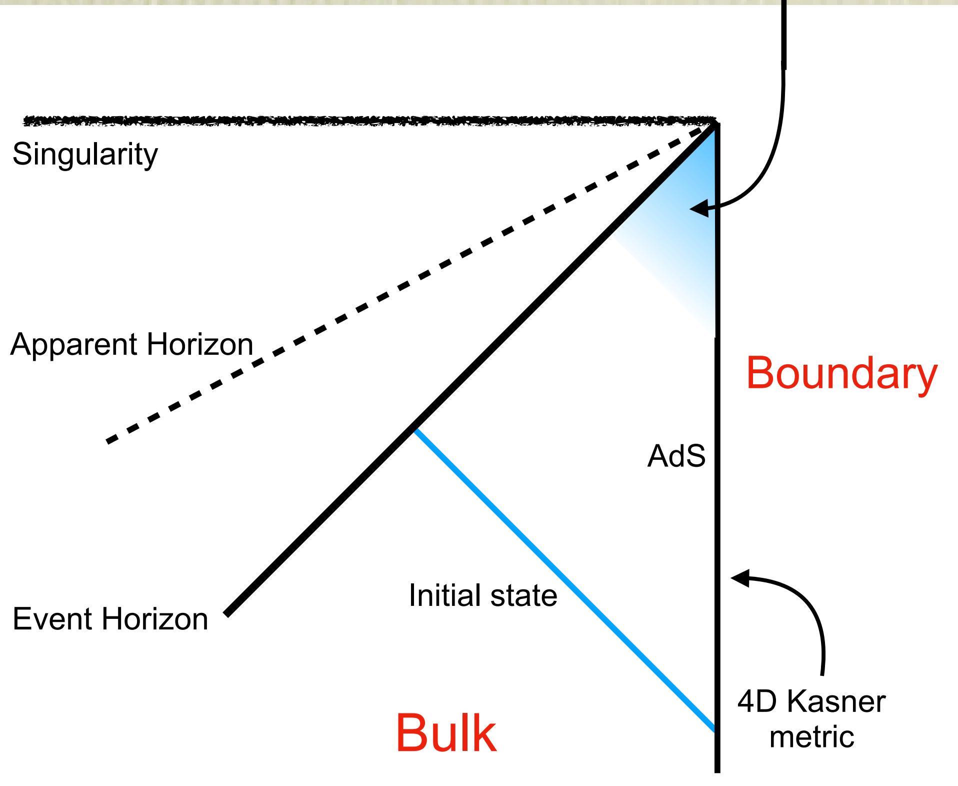
Application to cosmological singularities

Casalderrey, DM & Serantes '23

- Consider Kasner metric at the boundary:
- Universal stress tensor near singularity: $\mathcal{E} = \dfrac{\Lambda^2}{t^2}\,, \qquad \mathcal{P}_i = p_i\,\mathcal{E}$
- Backreaction at leading order:

$$\sum_{i=1}^{3} p_i = 1 + 8\pi G N^2 \Lambda^2\,, \qquad \sum_{i=1}^{3} p_i^2 = 1$$

- This allows for 3 +ve p's and makes Kasner stable.
- Suggests chaotic behaviour may be avoided.

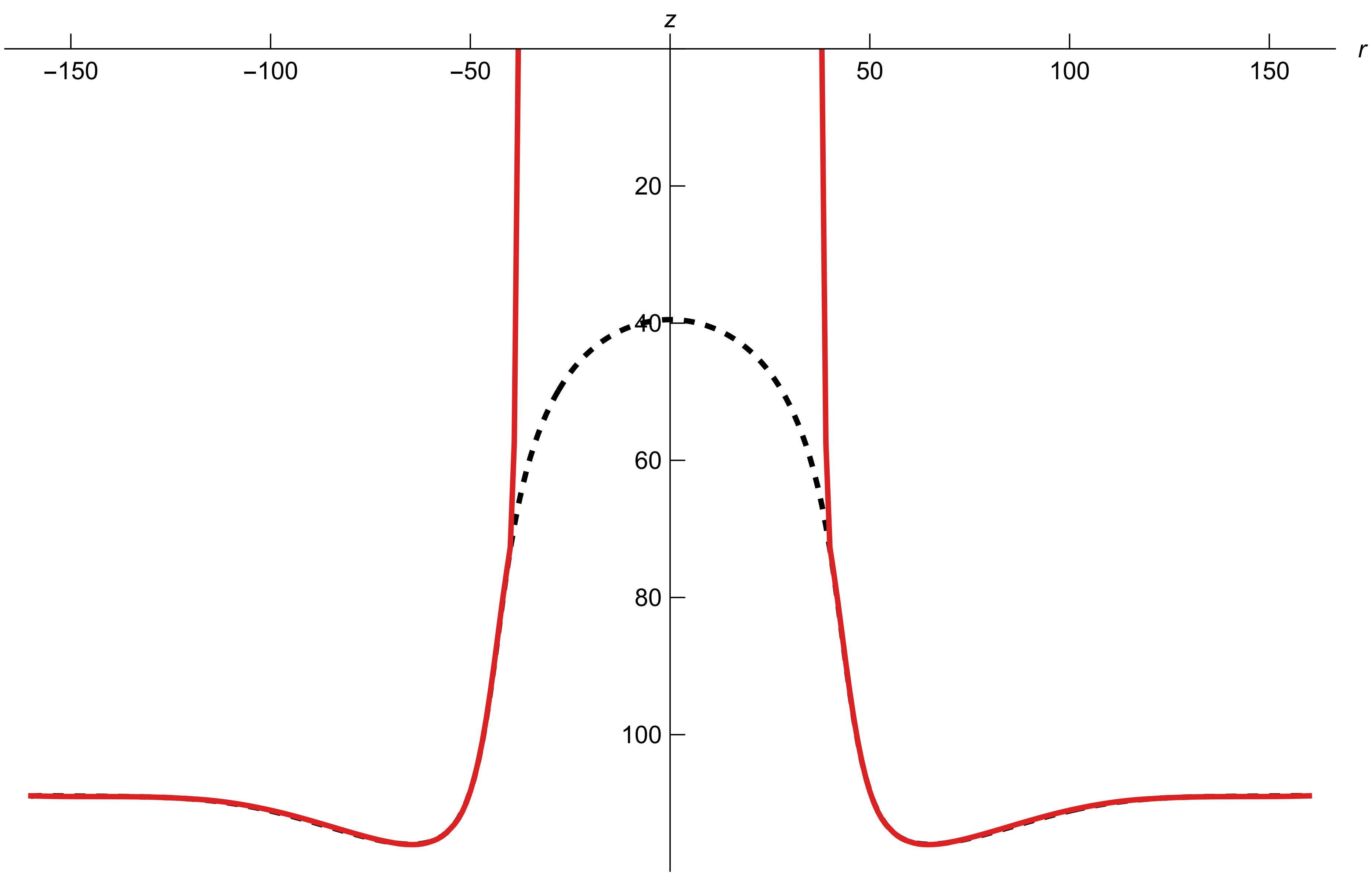


Thank you!

Event Horizon (EH) vs Apparent Horizon (AH)

EH: Solid curves

AH: Dashed curves



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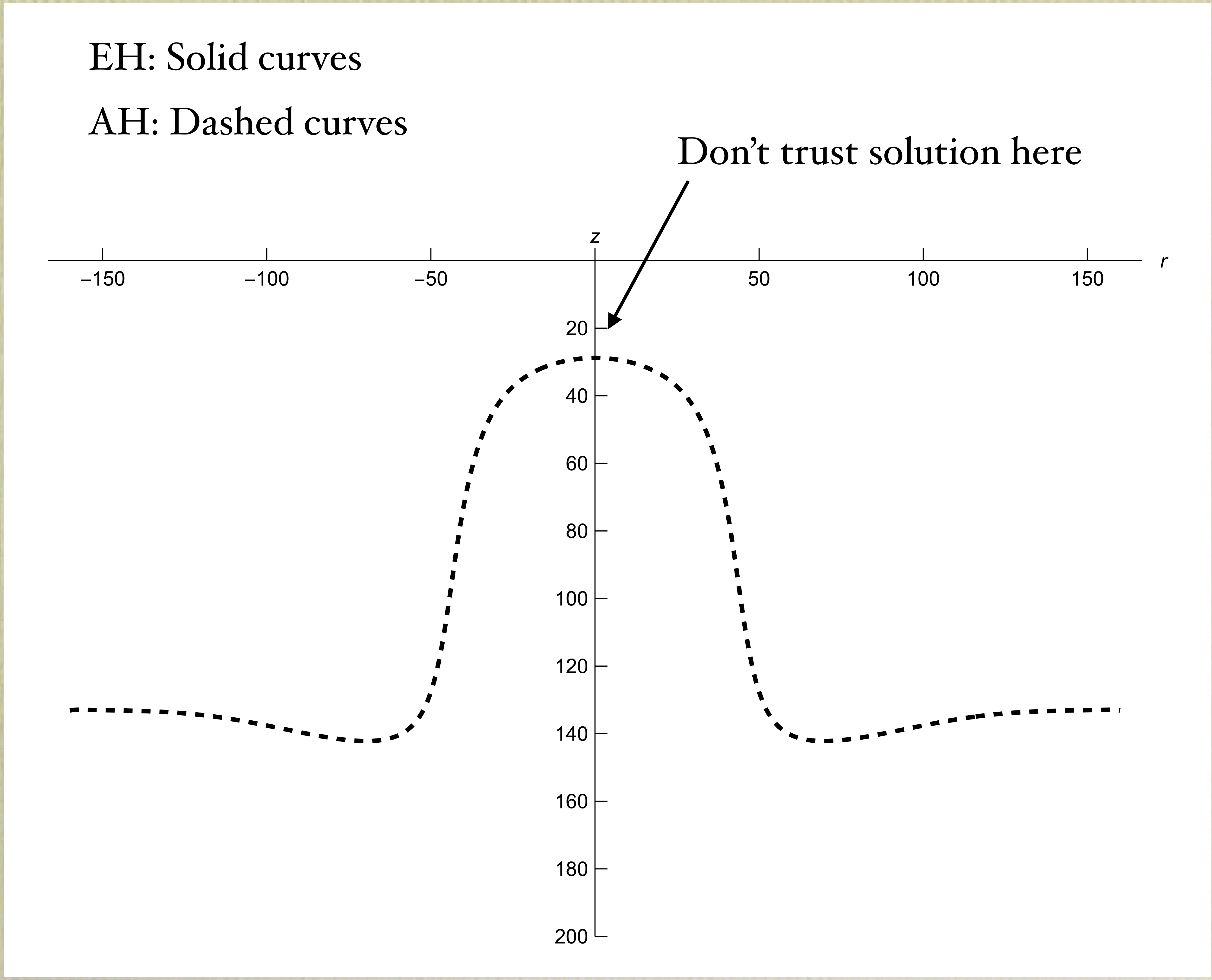

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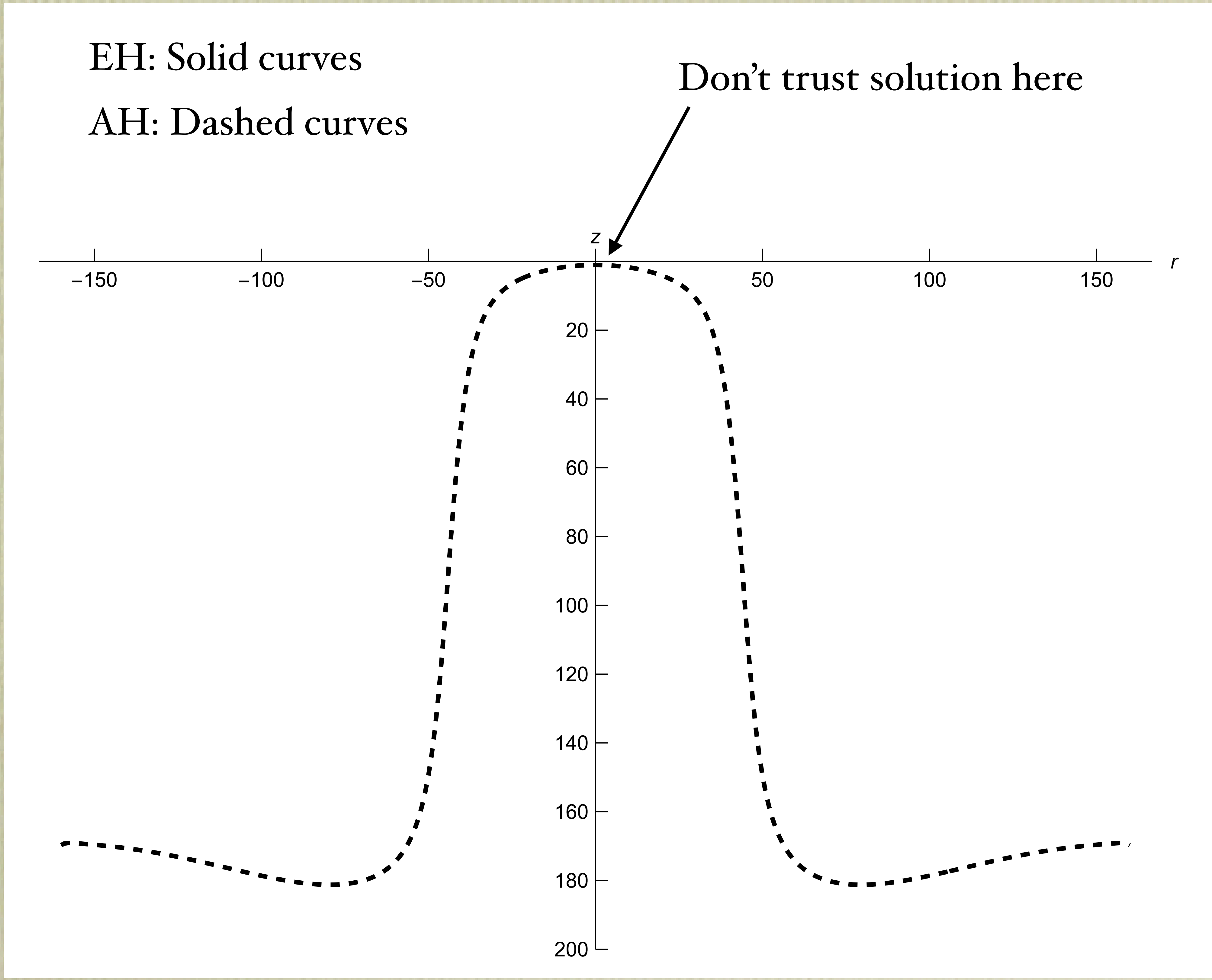
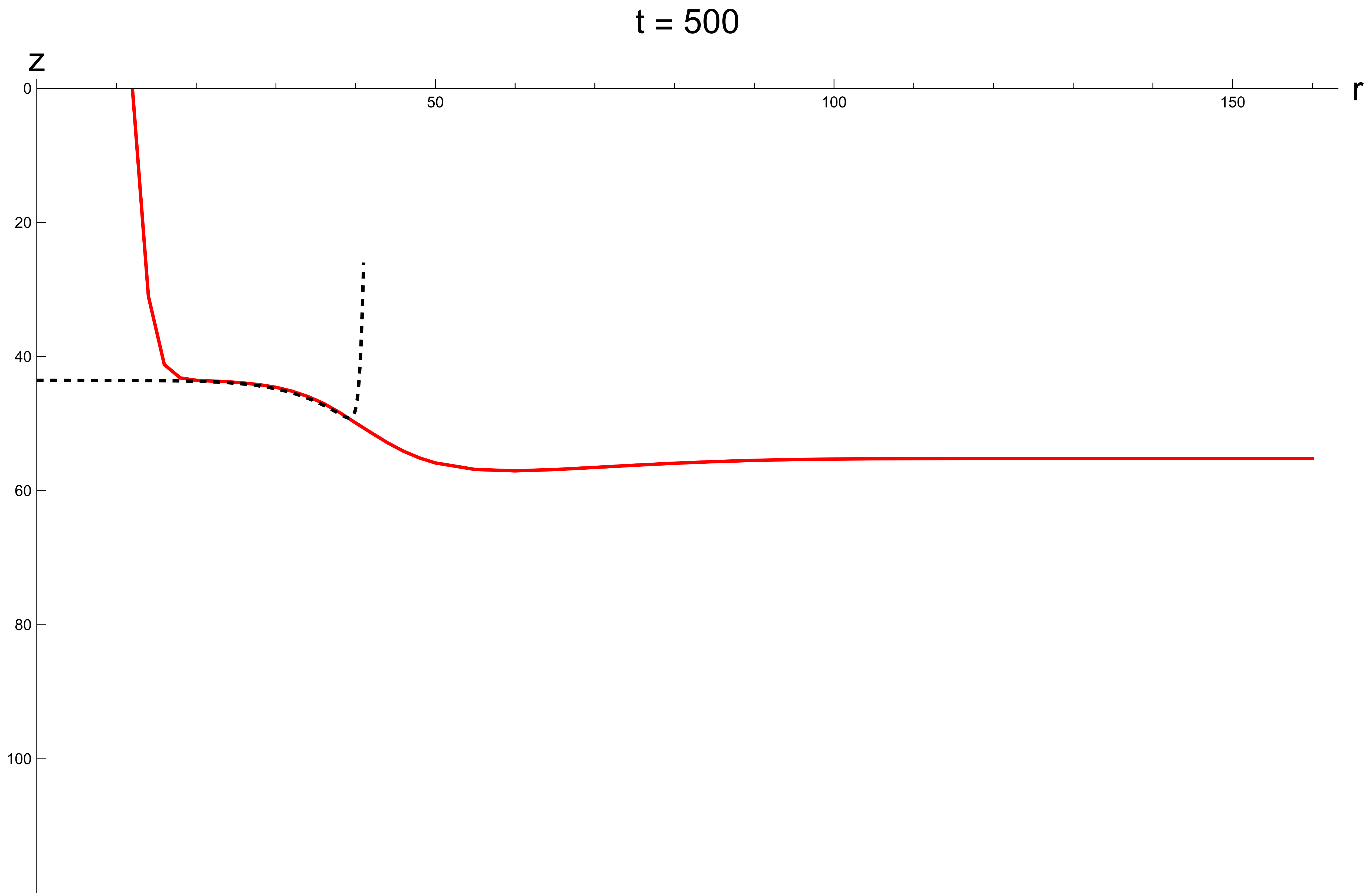
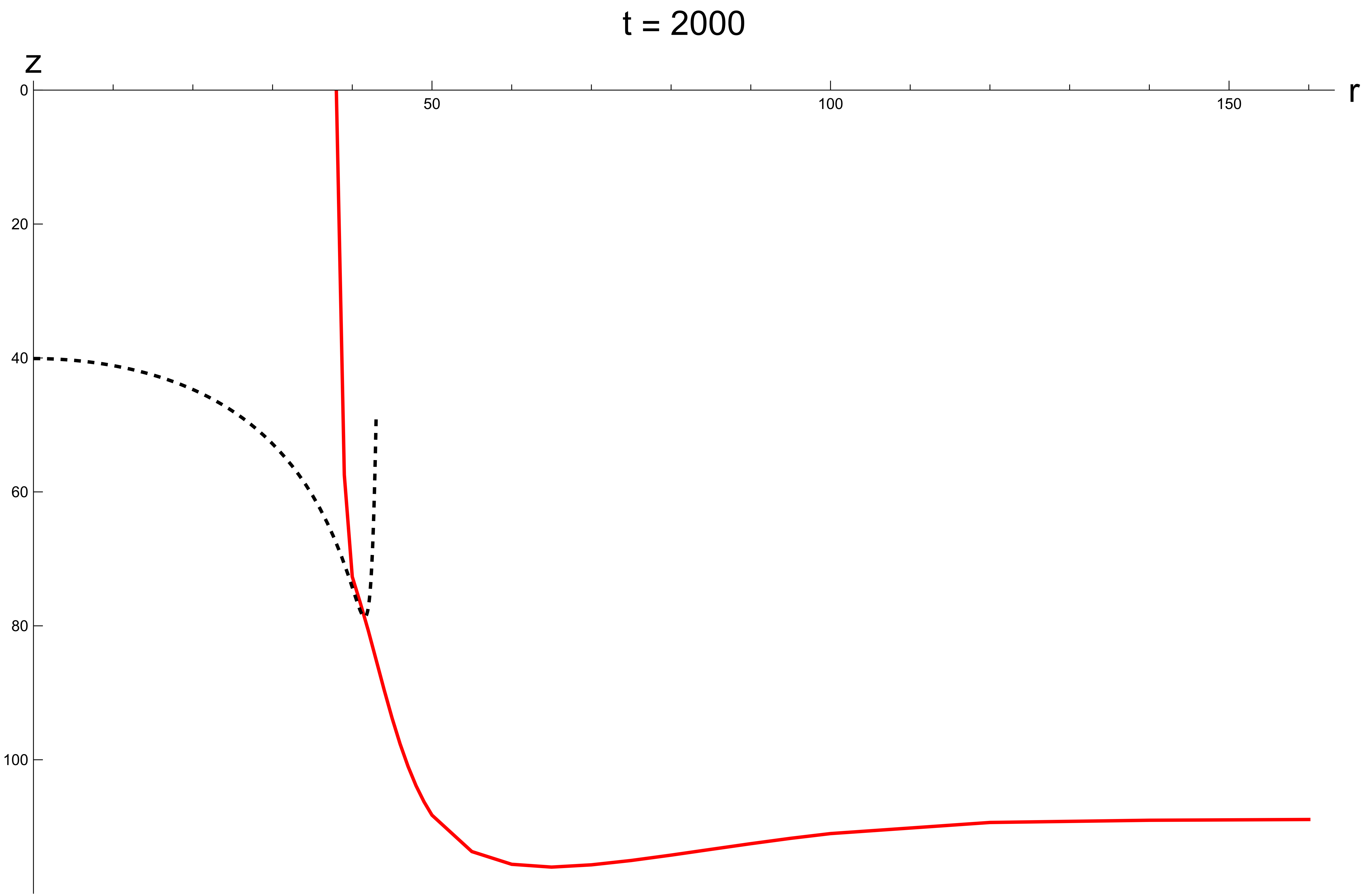



t = 500
z
r
0
20
40
60
80
100
50
100
150



t = 2000
z
r
0
20
40
60
80
100
50
100
150

Corrections to ideal fluid

DM, Serantes & Sole (work in progress)