Building Area Metric Geometry from Entanglement

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funded by
Narodowe Centrum Nauki
grant 2021/42/E/ST2/00234

New Insights in BH Physics from Holography

Madrid

16.06.2025

Area Metrics

- What are Area Metrics?
- Motivation(s)
- Previous Approaches
- Holographic Approach
- Holographic Dictionary
- Bulk Equations
- Area metric equations
- Next Steps

What are Area Metrics?

The metric tensor $g_{\mu\nu}$, defines a line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu .$$

From this we can calculate lengths and distances ($L[\gamma] = \int_\gamma ds$), areas, ...

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Area metrics describe a concept of *geometry of area without length*, where area is directly computed from an *area element* [Schuller and Wohlfarth 2006b]

$$dA^2 = G_{\mu\nu\rho\sigma} \left(dx^\mu \wedge dx^\nu\right) \cdot \left(dx^\rho \wedge dx^\sigma\right)$$

defined by an *area metric* $G_{\mu\nu\rho\sigma}$. The area of a surface Σ is defined as

$$\mathcal{A}[\Sigma] = \int_\Sigma dA.$$

What are Area Metrics?

$$dA^2 = G_{\mu\nu\rho\sigma}\,(dx^\mu \wedge dx^\nu) \cdot (dx^\rho \wedge dx^\sigma)\,, \quad \mathcal{A}[\Sigma] = \int_\Sigma dA.$$

Basic properties:

- Index symmetries like Riemann tensor ("algebraic curvature tensor").
- *Stick to 4d in this talk*: 20 components (metric: 10).
- **Every metric implies an area metric, but not every area metric is implied by a regular metric → *generalised notion of geometry*.**
- Metric case:

$$G_{\mu\nu\alpha\beta} = g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta}.$$

Motivation(s)

- String Theory: Area metrics are sufficient for Nambu-Goto action [Schuller and Wohlfarth 2006a].
- Loop Quantum Gravity may lead to area metrics [Borissova et al. 2024].
- Area metrics emerge in "second order geometry" [Kuipers 2025a] [Kuipers 2025b].
- Holography: Is area more fundamental than length? *Ryu-Takayanagi formula can be formulated with area metric bulk:*

$$S = \frac{\mathcal{A}(\Sigma)}{4G_N}.$$

Motivation(s)

d	3	4	5	6	7	8	9	10	26
$\binom{d}{2}$	3	6	10	15	21	28	36	45	325
$\mathrm{dof}(g_{\mu\nu})$	6	10	15	21	28	36	45	55	351
$\mathrm{dof}(G_{\alpha\beta\gamma\delta})$	6	21	55	120	231	406	666	1035	52975
$\mathrm{dof}(G_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3})$	1	10	55	210	630	1596	3570	7260	3381300
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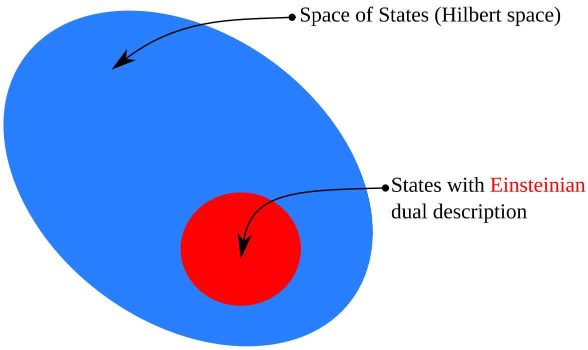
Table 1: Naive degrees of freedom for metrics, area metrics, and volume metrics in various dimensions.

Motivation(s)

In AdS/CFT, states with a semiclassical holographic dual description are *special*.

→ Not every state in the theory has an Einsteinian dual geometry:



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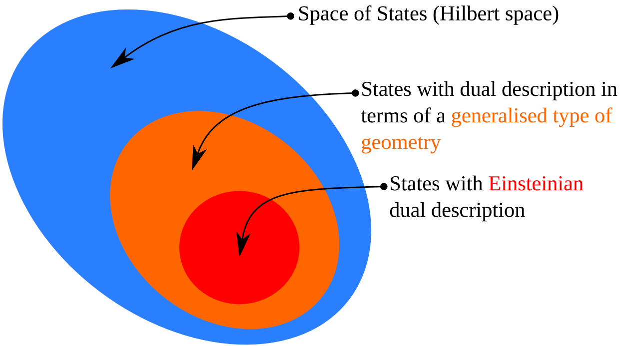

Can holography lead to a *generalised type of bulk geometry* beyond the Einsteinian paradigm?

Previous approaches

"Purist approach" [Schuller and Wohlfarth 2006a] [Schuller and Wohlfarth 2006b] [Ho and Inami 2016]:

- Construct a *background independent, covariant* theory for area metrics $G_{\alpha\beta\gamma\delta}$, which ideally recovers Einstein gravity in some limit.
- Challenges: Proliferation of indices ($\Gamma^{[\gamma\delta]}_{[\mu\nu]\alpha}, \ldots$). But Lagrangian should be a scalar, and indices can only be contracted with $G^{\alpha\beta\gamma\delta}$.

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"Linearised approach" [Borissova et al. 2024] :

- Look at linearised area metric fluctuations around fixed metric background.
- Decomposition:

$$G_{\mu\nu\alpha\beta} = g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta} + a_{\mu\nu\alpha\beta}, \quad (a \ll 1)$$

$$a_{\mu\nu\alpha\beta} = hg_{\alpha[\mu}g_{\nu]\beta} + 2\left(\tilde{h}_{\alpha[\mu}g_{\nu]\beta} - \tilde{h}_{\beta[\mu}g_{\nu]\alpha}\right) + w_{\mu\nu\alpha\beta}.$$

Metric like fluctuation $(h\eta_{\alpha\beta}, \tilde{h}_{\alpha\beta}$, 10 dof.), purely area metric like fluctuation $w_{\mu\nu\alpha\beta}$ (Weyl like, 10 dof.).

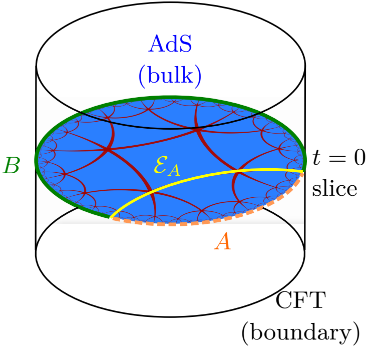
Holographic Approach

Ryu and Takayanagi [2006]: Generalisation of the black hole entropy formula, to the calculation of *entanglement entropy*:

$$\mathcal{S}_{EE}(A) = min_{\mathcal{E}_A} \frac{\text{Area}\,(\mathcal{E}_A)}{4G_N},$$

where $\mathcal{E}_A$ is a bulk surface which is

- extremal,
- codimension-2,
- spacelike,
- and anchored on the asymptotic boundary such that $\partial\mathcal{E}_A = \partial A$.



What happens if bulk geometry is given by an area metric?

Holographic Approach

For a subregion A with reduced density matrix ρ, $S_{EE}(A) = -\mathrm{Tr}_A[\rho \log(\rho)]$. Let us now look at small variations parametrized by λ: $\rho \to \rho(\lambda) = \rho_0 + \lambda\delta\rho + \dots$. We find the *first law of entanglement*

$$\frac{d}{d\lambda} S_{EE}(A)\bigg|_{\lambda=0} = \frac{d}{d\lambda}\langle H_A \rangle\bigg|_{\lambda=0}$$

with the *modular Hamiltonian* $H_A \equiv -\log(\rho_0)$.

Assuming the validity of the Ryu-Takayanagi formula in the bulk, the first law can be used to derive the holographic dictionary for the boundary stress tensor T_{ab} as well as Einstein's equations in the bulk, linearised around AdS [Blanco et al. 2013] [Lashkari et al. 2014] [Faulkner et al. 2014].

Our goal is to do the same with (linearised) area metrics, in order to derive their EOMs from first principles!

Holographic Approach

$$\underbrace{\delta S_{EE}(A)}_{\text{Change of bulk geometry}} = \underbrace{\delta\langle H_A\rangle}_{\propto\int d^2x\,\frac{R^2-\vec{x}^2}{R}\langle T_{tt}(\vec{x})\rangle}$$

Step by step process followed in [Blanco et al. 2013] [Lashkari et al. 2014] [Faulkner et al. 2014] :

- Focus on *Ball-shaped boundary regions*, closed form expression for $\delta\langle H_A\rangle$ in terms of $\langle T_{ab}(\vec{x})\rangle$.
- Take *limit of infinitesimally small balls*: RHS $\sim \langle T_{tt}(\vec{x})\rangle$; LHS depends on change of bulk geometry close to the boundary

  $\Rightarrow$ Derive holographic dictionary, leading order in z.

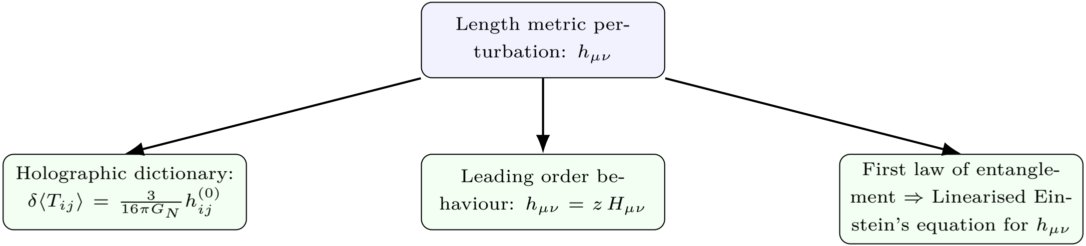
$$\delta\langle H_A\rangle_{R\to 0} \approx 2\pi\int_{B(R,x_0)} d^{d-1}x\,\frac{R^2-|\vec{x}-\vec{x}_0|^2}{2R}\,\delta\langle T_{tt}(t_0,\vec{x}_0)\rangle$$
$$= \frac{2\pi R^d\Omega_{d-2}}{d^2-1}\,\delta\langle T_{tt}(t_0,\vec{x}_0)\rangle.$$

$$\delta\langle T_{tt}(x_0)\rangle = \frac{d^2-1}{2\pi\Omega_{d-2}}\lim_{R\to 0}\Big(\frac{\delta S_{B(R,x_0)}}{R^d}\Big),$$

Holographic Approach

- Figure out the leading order z dependence in metric perturbation (AdS_4/CFT_3). In case of Length metric, $\boxed{h_{\mu\nu} = zH_{\mu\nu}}$. In case of pure area metric like perturbation (no metric like dof), $\boxed{w_{\mu\nu\rho\lambda} = \frac{1}{z}W_{\mu\nu\rho\sigma}}$.
- Impose *conditions on T_{ab}:* $T^a_a = 0; \; \partial_a T^{ab} = 0$
  $\Rightarrow$ Additional constraints for the fluctuations of the bulk geometry near the boundary.
- Look at *balls of finite radius*. Corresponding extremal surfaces reach further into the bulk
  $\Rightarrow$ Equations for the fluctuations of the bulk geometry.

Summary of Results (primary)



Holographic Dictionary

Metric fluctuation:

$$ds^2 = \underbrace{\frac{1}{z^2}\left(\eta_{mn}dx^m dx^n + dz^2\right)}_{\equiv g_{\mu\nu}dx^\mu dx^\nu,\ \text{Poincaré -AdS.}} + zH_{mn}(z,\vec{x})dx^m dx^n, \quad H \ll 1, H_{mn}(z,\vec{x}) = \sum_n z^n H^{(n)}_{mn}(\vec{x}).$$

$$\Rightarrow \boxed{\langle T_{tt}(\vec{x})\rangle \propto H_{xx}(\vec{x}, z=0) + H_{yy}(\vec{x}, z=0) \propto H^{(0)}_{ii}(\vec{x}).}$$

Pure area metric fluctuation (no metric like part):

$$G_{\mu\nu\alpha\beta} = g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta} + \frac{1}{z}W_{\mu\nu\alpha\beta}(z,\vec{x}), W_{\mu\nu\alpha\beta}(z,\vec{x}) = \sum_n z^n W^{(n)}_{\mu\nu\alpha\beta}(\vec{x}),$$

$$\Rightarrow \boxed{\langle T_{tt}(\vec{x})\rangle \propto W_{txtx}(\vec{x}, z=0) + W_{tyty}(\vec{x}, z=0) \propto \eta^{ij}W^{(0)}_{titj}(\vec{x}).}$$

Holographic Dictionary

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$$\Rightarrow \boxed{\langle T_{mn}(\vec{x})\rangle \propto H_{mn}(\vec{x}, z=0), \qquad \langle T^m_m(\vec{x})\rangle = 0 \Rightarrow H^m_m(\vec{x}, z=0) \equiv 0.}$$

Pure area metric fluctuation (no metric like part):

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$$\Rightarrow \langle T_{mn}(\vec{x})\rangle \propto W_{mznz}(\vec{x}, z=0), \quad W^{\alpha}{}_{\nu\alpha\beta} = 0 \Rightarrow \langle T^m_m(\vec{x})\rangle = 0.$$

Bulk Equations

- Linearized Einstein's equation :

$$\boxed{\frac{1}{2}\left(-\Box h_{\mu\nu} - \nabla_\mu \nabla_\nu h + \nabla^\rho \nabla_\mu h_{\nu\rho} + \nabla^\rho \nabla_\nu h_{\mu\rho}\right) - \frac{1}{2} g_{\mu\nu}\left(\nabla_\mu \nabla_\nu h^{\mu\nu} - \Box h - R_{\mu\nu} h^{\mu\nu}\right) + \Lambda h_{\mu\nu} = 0}$$

- Now, we write the Taylor series expansion ($h_{ij} = zH_{ij} = z\sum_n z^n H_{ij}^{(n)}(\vec{x})$) of the length metric perturbation function around $\vec{x} = \vec{x}_0$,

$$H_{ij}^{(n)}(t, \vec{x} + \vec{x}_0) = \sum_{m_x, m_y} \frac{1}{2m_x!} \frac{1}{2m_y!} x^{2m_x} y^{2m_y} \partial_x^{2m_x} \partial_y^{2m_y} H_{ij}^{(n)}(t, x_0, y_0)\,, \quad m_{x(y)} = 0, \frac{1}{2}, 1, \cdots$$

- Generally

$$\delta S - \delta\langle H_A\rangle = 0 \implies 2\partial_x \partial_y H_{xy}^{(n-2)} = \partial_x^2 H_{yy}^{(n-2)} + \partial_y^2 H_{xx}^{(n-2)} + (n)(n+3)(H_{xx}^{(n)} + H_{yy}^{(n)})\,, \quad n \geq 2\,.$$

$$\Rightarrow \boxed{\partial_x^2 H_{yy} + \partial_y^2 H_{xx} + \frac{1}{z^4}\partial_z\left(z^4 \partial_z H_{tt}\right) - 2\partial_x \partial_y H_{xy} = 0}$$

Bulk equations

Metric fluctuation:

$$ds^2 = \underbrace{\frac{1}{z^2}\left(\eta_{mn}dx^m dx^n + dz^2\right)}_{\equiv g_{\mu\nu}dx^\mu dx^\nu,\ \text{Poincaré -AdS.}} + zH_{mn}(z,\vec{x})dx^m dx^n, \quad H \ll 1.$$

$$\Rightarrow \boxed{\partial_x^2 H_{yy} + \partial_y^2 H_{xx} + \frac{1}{z^4}\partial_z\left(z^4\partial_z H_{tt}\right) - 2\partial_x\partial_y H_{xy} = 0 \quad \text{(tt-component of Einstein's equations).}}$$

Furthermore, $\{z,\mu\}$ components $\Rightarrow H^\mu_\mu = 0$, and $\Rightarrow \partial^\mu H_{\mu\nu} = 0$. (Valid at all orders with radial gauge $h_{zz} = 0 = h_{z,i}$)

General implication on $\{\mu,\nu\}$ coordinates

$$\Rightarrow \boxed{\delta R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\delta R - 3g_{\mu\nu} = 0.}$$

Area metric equations

Pure area metric fluctuation (no metric like part):

$$G_{\mu\nu\alpha\beta} = g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta} + \frac{1}{z}W_{\mu\nu\alpha\beta}(z,\vec{x}),$$

$$2\partial_x\partial_y W^{(n-2)}_{txty} = -\left(\partial_x^2 W^{(n-2)}_{txtx} + \partial_y^2 W^{(n-2)}_{tyty} + 2n(\partial_x W^{(n-1)}_{txtz} + \partial_y W^{(n-1)}_{tytz}) + (n)(n+1)W^{(n)}_{tztz}\right)$$

$$\Rightarrow -2\partial_z\left(\frac{1}{z^2}\partial^m W_{tmtz}\right) = \frac{1}{z^4}\partial^m\partial^n W_{tmtn} + \partial_z^2 W_{tztz} + \frac{2}{z}\left(\frac{1}{z^2}\partial^m W_{tmtz} + \partial_z W_{tztz}\right).$$

$$\Rightarrow \nabla^\mu\nabla^\nu w_{t\mu t\nu} = 0. \quad (\mu,\nu = t,x,y,z).$$

Possible full covariantization:

$$\Rightarrow \nabla^\mu\nabla^\nu w_{\rho\mu\lambda\nu} = 0. \quad \text{(conformally coupled massless spin-2 field in AdS}_4\text{)}.$$

Area metric full EOM

- $\{z,\mu\}$ component$=0$, if $\Rightarrow \partial^a\left(\eta^{ij}W_{aibj}\right)=0$ (stress tensor conservation valid at all orders). Note that the cconformally coupled EOM is not the one arising from Weyl tensor of $h_{\mu\nu}$, but fully area metric dof. $w_{\rho\mu\lambda\nu}$.
- Turn on $h_{\mu\nu}$: $a_{\mu\nu\alpha\beta}=hg_{\alpha[\mu}g_{\nu]\beta}+2\left(\tilde{h}_{\alpha[\mu}g_{\nu]\beta}-\tilde{h}_{\beta[\mu}g_{\nu]\alpha}\right)+w_{\mu\nu\alpha\beta}$. (2-indexed analogue of Linearised Einstein's equation)

$$\delta R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}\delta R-3g_{\mu\nu}+\nabla^\rho\nabla^\lambda w_{\rho\mu\lambda\nu}=0.$$

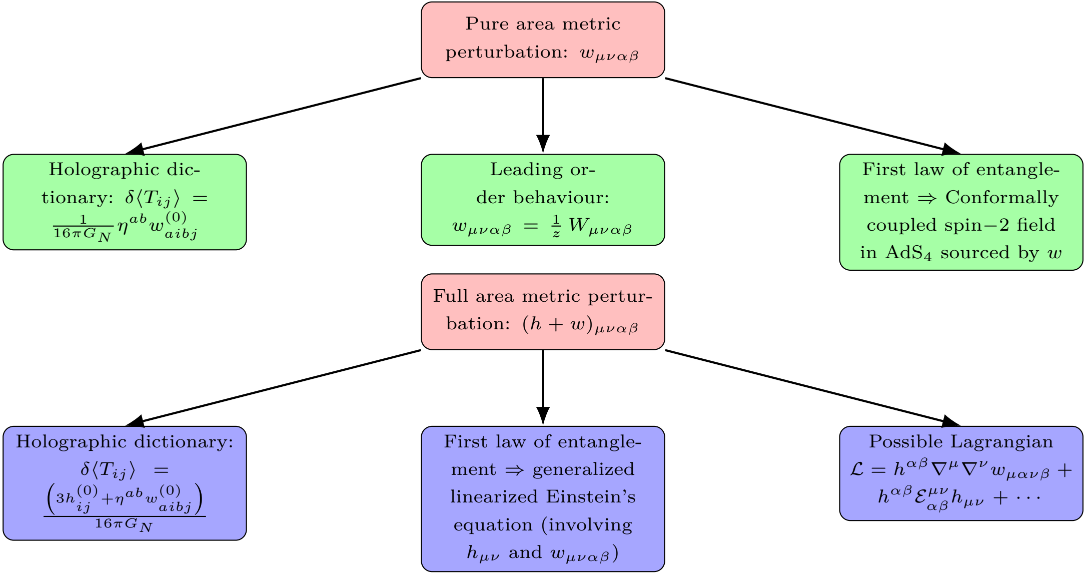
- Possibly appears from variation of length metric dof of a candidate Lagrangian

$$\mathcal{L}=h^{\alpha\beta}\nabla^\mu\nabla^\nu w_{\mu\alpha\nu\beta}+h^{\alpha\beta}\mathcal{E}^{\mu\nu}_{\alpha\beta}h_{\mu\nu}+\cdots(\text{pure areametric part of }\mathcal{L}),$$

  where $\mathcal{E}^{\mu\nu}_{\alpha\beta}$ is the standard Lichnerowicz operator for linearized Einstein gravity in AdS_4. Possible second-order gauge-invariant action in AdS_4 of the form

$$S[h,w]=\int d^4x\sqrt{-g}\left[h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}+h^{\mu\nu}\nabla^\alpha\nabla^\beta w_{\alpha\mu\beta\nu}+\cdots\right]$$

Summary of Results (NEW)



Back to Motivations

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Table 2: Naive degrees of freedom for metrics, area metrics, and volume metrics in various dimensions.

Next Steps

- *Covariantisation:* Understand the covariant expression to its full glory *a) Killing vector treament, and b) in terms of* $a_{\mu\alpha\nu\beta}$.
- Study *solutions* to the area metric bulk equations – especially black holes and their thermodynamics!?
- Go from linearized to *non-linear equations*?
- Higher dimensional generalizations: two ways, foliate codimension-2 areas through *2-dimensional area metric* (simpler) or *codimension-2 dimensional volume metrics* (harder and more general version of Einstein equations).


Thank you very much
for your attention

Back-up slides...

Formulas

$$\delta A = \int |d^2\sigma| \frac{1}{8} \frac{a_{ijkl}\epsilon^{ab}\epsilon^{cd}\partial_a X^i \partial_b X^j \partial_c X^k \partial_d X^l}{\sqrt{h_0}} . \tag{1}$$

Utilize the basic integrals,

$$\begin{aligned} \int_{\mathcal{D}_R} dx\, dy (R^2 - x^2 - y^2)^{\frac{n}{2}} x^{2m_x} y^{2m_y} &= R^{n+2m_x+2m_y+2} I_{n,m_x,m_y}, \quad m_{x(y)} = 0, 1, 2, \cdots , \\ \int_{\mathcal{D}_R} dx\, dy (R^2 - x^2 - y^2)^{\frac{n}{2}} x^{2m_x+1} y^{2m_y+1} &= R^{n+2m_x+2m_y+4} I_{n,m_x+\frac{1}{2},m_y+\frac{1}{2}} , \quad m_{x(y)} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \cdots \end{aligned} \tag{2}$$

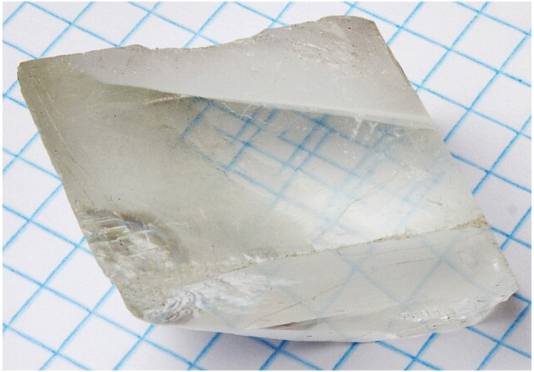
where

$$I_{n,m_x,m_y} = \frac{\Gamma(\frac{n}{2}+1)\Gamma(m_x+\frac{1}{2})\Gamma(m_y+\frac{1}{2})}{\Gamma(\frac{n}{2}+m_x+m_y+2)} . \tag{3}$$

Birefringence

Formulating electromagnetism on an area metric spacetime leads to the prediction of a peculiar effect [Grosse-Holz et al. 2017] [Werner 2019] :

Vacuum *birefringence* (propagation of light is dependent on polarisation).



Why is this noteworthy?

Cosmic birefringence

Indications of birefringence in *Cosmic Microwave Background* data:

Improved Constraints on Cosmic Birefringence from the *WMAP* and *Planck* Cosmic Microwave Background Polarization Data

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(Dated: September 13, 2022)

The observed pattern of linear polarization of the cosmic microwave background (CMB) photons is a sensitive probe of physics violating parity symmetry under inversion of spatial coordinates. A new parity-violating interaction might have rotated the plane of linear polarization by an angle β as the CMB photons have been traveling for more than 13 billion years. This effect is known as "cosmic birefringence." In this paper, we present new measurements of cosmic birefringence from a joint analysis of polarization data from two space missions, *Planck* and *WMAP*. This dataset covers a wide range of frequencies from 23 to 353 GHz. We measure $\beta = 0.342^\circ{}^{+0.094^\circ}_{-0.091^\circ}$ (68% C.L.) for nearly full-sky data, which excludes $\beta = 0$ at 99.987% C.L. This corresponds to the statistical significance of 3.6σ. There is no evidence for frequency dependence of β. We find a similar result, albeit with a larger uncertainty, when removing the Galactic plane from the analysis.

- 2020 claim: 2.4σ [Minami and Komatsu 2020]
- 2022 claim: 3.6σ [Eskilt and Komatsu 2022]
- Official discovery threshold: 5σ!

Schrödinger's Black Hole

Schrödinger's cat is a *superposition between two macroscopic states*:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|{\uparrow}\rangle + |{\downarrow}\rangle\right) \quad (\uparrow\text{: Alive}, \downarrow\text{: Dead})$$

Since we don't have a complete understanding of quantum gravity, there is an important open question:

Can two curved spacetimes be put in a quantum superposition? (And how would that look like?)

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Can we use holography to approach this problem?

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Idea: Consider the state of a rotating black hole, with two parameters: *Mass M, Angular momentum J*.

$$|\Psi\rangle \equiv |\Psi(M,J)\rangle$$

Now we define *Schrödinger's black hole state:*

$$|\Phi(M,J)\rangle \equiv \frac{\mathcal{N}_\Phi(M,J)}{2}\left(|\Psi(M,+J)\rangle + |\Psi(M,-J)\rangle\right)$$

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$$|\Psi(M,J)\rangle = \mathcal{N}_0(J)\,|0\rangle + \mathcal{N}_1(J)J\,|1\rangle + \mathcal{N}_2(J)J^2\,|2\rangle + \ldots$$

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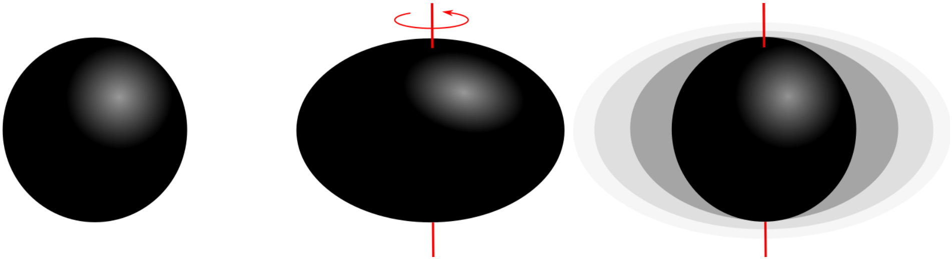
$$|\Phi(M,J)\rangle = |0\rangle + \mathcal{N}_2(J)J^2\,|2\rangle + \ldots$$

To simplify calculations, we may work perturbatively in J.

Schrödinger's Black Hole

Holographic descriptions:

- $|\Psi(M,0)\rangle$: *Non-rotating* (static) black hole, spherically symmetric.
- $|\Psi(M,J)\rangle$: *Rotating* (stationary) black hole for $J \neq 0$, axisymmetric.
- $|\Phi(M,J)\rangle$: *Superposition of spacetimes* ????????



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