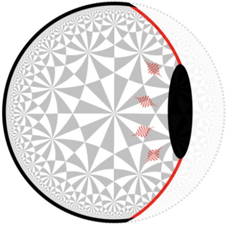
Quantum black holes at world's end

Andrew Svesko
Department of Mathematics
King's College London



New Insights in Black Hole Physics from Holography

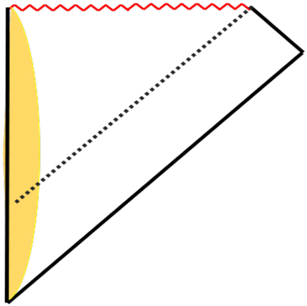
[hep-th/2501.17231] – Cartwright, Gürsoy, Pedraza, Svesko
PRL 133 (2024) 18; [hep-th/2406.17860] – Frassino, Hennigar, Pedraza, Svesko
PRD 109 (2024) 12; [hep-th/2310.12220] – Frassino, Pedraza, Svesko, Visser
PRL 130 (2023) 16; [hep-th/2212.14055] – Frassino, Pedraza, Svesko, Visser

Black hole singularity theorem
(Penrose, '65)

Generic BHs develop singularities

(Weak) Cosmic Censorship conjecture (Penrose, '68)

Naked singularities cannot form



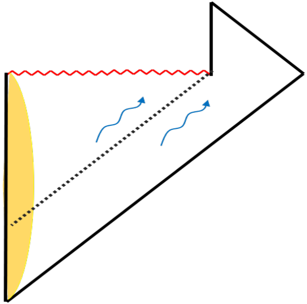
How can singularities be resolved?

What is the status of WCCC?

Black hole thermodynamics

(Bekenstein, ‘72, ’73; Hawking, ’74, ’75)

$$T_{\mathrm{H}} = \frac{\kappa\hbar}{2\pi}\,, \qquad S_{\mathrm{BH}} = \frac{\mathrm{Area}(\mathcal{H})}{4G\hbar}$$



Black holes evaporate

(Hawking, ‘75, ’76)

How to interpret S_{BH}?

Quantum black hole evolution?

Quantum effects in gravity

QUANTUM FIELDS IN CURVED SPACE

N. D. BIRRELL
Logica Pty Ltd, Australia
AND
P. C. W. DAVIES
Professor of Theoretical Physics, University of Newcastle upon Tyne

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Semi-classical gravity: proxy to study quantum effects in gravity

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \langle T^{\text{mat}}_{\mu\nu} \rangle}$$

- Classical dynamical spacetime + quantum fields
- *Backreaction problem:* solve coupled system self-consistently
- Exact ‘quantum’ black holes?
  - 2D dilaton gravity, e.g., (Russo, Susskind, Thorlacius; ’92, ’93) (see also Merten’s review [2210.10846])
  - Beyond 2D requires perturbative or numerical methods
  - 4D evolution of dynamically spherical BHs (Parentani... ’94); (Boyanov...’25)

Ex: 3D Einstein ($\Lambda < 0$) + conformally coupled scalar

- *Classical BTZ geometry:* (Bañados, Teitelboim Zanelli, '92, '93)

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\phi^2, \quad f(r) = \frac{r^2}{\ell_3^2} - 8G_3 M$$

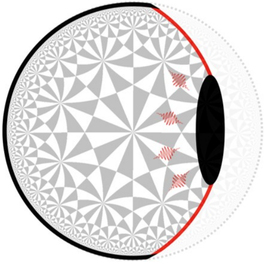
- *Renormalized stress-tensor* (Steif, '93); (Lifschytz..., '93); (Casals..., '16,'19)

$$\boxed{\langle T^a_b \rangle = \frac{\hbar F(M)}{8\pi r^3} \mathrm{diag}(1, 1, -2)}$$

- *Quantum-corrected geometry*

$$\boxed{\delta g_{tt} = \frac{2L_{\mathrm{P}} F(M)}{r} > 0}$$

- Planck-sized correction; many fields $c \gg 1$, $\delta g_{tt} \sim c L_{\mathrm{P}} \gg L_{\mathrm{P}}$



Quantum black holes via braneworld holography:

- Bulk *classical* dynamics encodes *quantum* dynamics of brane

$$\boxed{\text{Classical GR} \quad \Leftrightarrow \quad \text{Semi-classical gravity}}$$

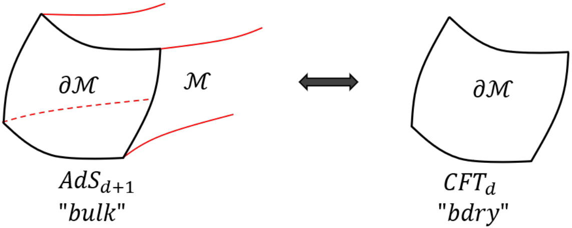
- Study semi-classical backreaction to *all* orders
- *Classical* BHs localized on end-of-the-world brane $\leftrightarrow$ *quantum* BHs

  (Emparan, Fabbri, Kaloper, '02; Tanaka, '02)

- Exact (analytic) quantum black holes in 3D
- New phenomena
  - Reentrant phase transitions (FPSV, '23)
  - Quantum cosmic censorship (Emparan...'20), (FHPS, '24, '25)
  - Quantum induced superradiance (CGPS, '25)

- **Holographic braneworlds: review**
- **Exact ‘quantum’ black holes**
- **Applications**

Holographic braneworlds

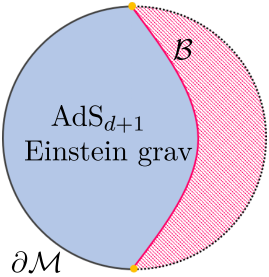


- Dynamical gravity in AdS has a *dual, holographic* description in terms of a CFT ⇒ **AdS/CFT dictionary** (GKPW, '98)

$$Z^{\text{on-shell}}_{\text{grav}}[\phi_0]|_{\mathcal{M}} = \left\langle e^{-\int_{\partial\mathcal{M}} \phi_0 \mathcal{O}} \right\rangle_{\text{CFT}}$$

- Z_{grav} has IR divergences ↔ UV divergences in CFT correlators
- *Holographic renormalization* (Balasubramanian, Kraus, '99); (de Haro,...'00)

- Instead of renormalization, place 'brane' at $\rho = \epsilon$ (de Haro, et. al., '00)



- 'Bulk' governed by GR+ brane

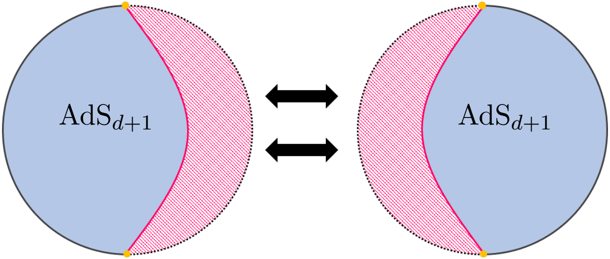
$$I_{\text{bulk}} = I_{\text{GR}} + I_{\text{brane}} \,, \quad I_{\text{brane}} = -\tau \int_{\mathcal{B}} d^d x \sqrt{-h}$$

- Integrate out bulk from $\partial\mathcal{M}$ to $\mathcal{B}$

$$\boxed{I_{\text{eff}}^{\mathcal{B}} = I_{\text{grav}}[\mathcal{B}] + I_{\text{CFT}}^{\text{cut-off}}[\mathcal{B}]}$$

Holographically induced gravity

- Karch-Randall braneworld (Karch, Randall, '00)



- Surgery to complete space; junction conditions fix location of brane
- Induced theory of gravity on brane (Emparan, Johnson, Myers, '99); (Bueno, Emparan, Llorens, '22)

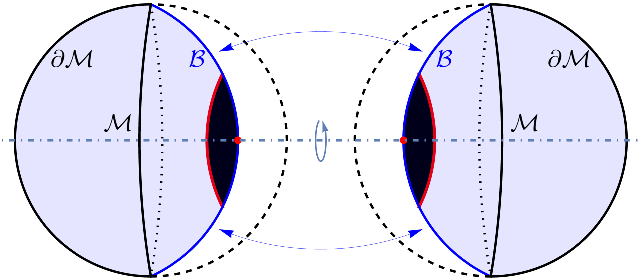
$$I^{\mathcal{B}}_{\text{grav}} = \frac{1}{16\pi G_d}\int_{\mathcal{B}} d^dx\sqrt{-h}\left[R - 2\Lambda_d + \frac{L^2_{d+1}}{(d-4)(d-2)}(R^2 - \text{terms}) + \dots\right]$$

Brane-power!

- *Classical* dynamics of AdS bulk encodes *quantum* dynamics of brane (Gubser, '99); (Duff, Liu, '00); (Emparan, Fabbri, Kaloper, '02); (Tanaka,'02)

Classical GR $\Leftrightarrow$ Semi-classical gravity

- *Classical* BHs localized on braneworld $\leftrightarrow$ *quantum* BHs (EFK '02)



- Study semi-classical backreaction to *all* orders

Exact descriptions of quantum black holes

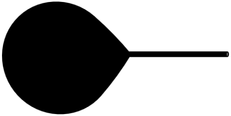
Accelerating black holes: geometry

AdS$_4$ C-metric with Karch-Randall brane (Emparan, Horowitz, Myers, '99)

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + H^{-1}(r)dr^2 + r^2\left(G^{-1}(x)dx^2 + G(x)d\phi^2\right)\right]$$

$$H(r) = \kappa + \frac{r^2}{\ell_3^2} - \frac{\mu\ell}{r}\,, \quad G(x) = 1 - \kappa x^2 - \mu x^3$$

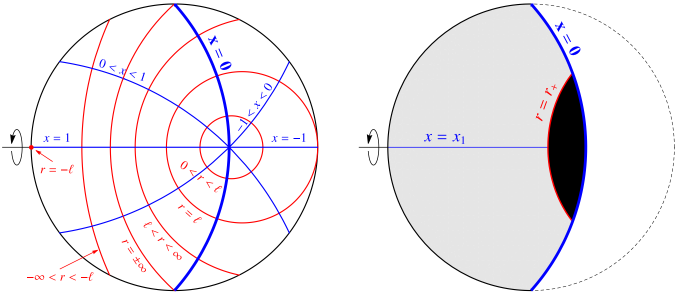
- Accelerating due to cosmic string, acceleration ℓ^{-1}



Brane:

- *Umbilic* surface at $x = 0$: $K_{ij} = \ell^{-1} h_{ij}$
- Brane at $x = 0$, where Israel-junction conditions are satisfied

$$\boxed{\tau = \frac{1}{2\pi G_4 \ell}}$$



($\mu = 0$) ($\mu \neq 0$)

Brane geometry:

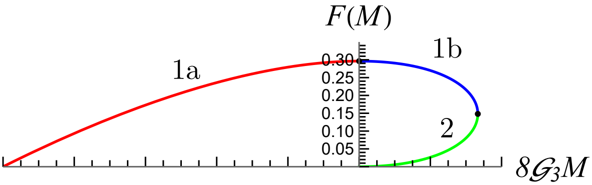
$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\phi^2 \ , \quad f(r) = \frac{r^2}{\ell_3^2} - 8G_3 M - \frac{\ell F(M)}{r}$$

Quantum BTZ black hole: (Emparan, Frassino, Way, '20)

$$\langle T^i_{\ j} \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \text{diag}\{1, 1, -2\} + \dots$$

Strength of backreaction controlled by $\ell \sim c L_P$; *not* Planck-sized

Classical BTZ	Quantum BTZ
$R^2_{ijkl} = \frac{12}{\ell_3^4}$	$R^2_{ijkl} = \frac{12}{\ell_3^4} + \frac{6F^2\ell^2}{r^6}$
BH or naked conical singularities	Family of black holes



- **Branch 1a:** $M \leq 0$ quantum-corrected conical defects
- **Branch 1b:** $M \geq 0$ quantum bh; Casimir dominated
- **Branch 2:** $M \geq 0$ quantum bh; Casimir subtracted

Mass gap removed; finite mass range

Given appropriate AdS_4 C-metric, known quantum BHs include:

Quantum AdS_3 black holes:

- Rotating qBTZ (Emparan, Frassino, Way, '20)
- Charged qBTZ (Climent, Emparan, Hennigar,'24); (Feng, Ma, Mann,...,'24)

Quantum dS_3 black holes:

- Quantum SdS_3 (Emparan, Pedraza, Svesko, Tomasevic, Visser, '22)
- Quantum Kerr-dS_3 (Panella, Svesko, '23)
- Charged quantum SdS_3 (Climent, Hennigar, Panella, Svesko, '24)

Three-dimensional quantum black holes: a primer

[2407.03410]

Emanuele Panella,[a] Juan F. Pedraza[b] and Andrew Svesko[c]

[a] *Department of Physics and Astronomy, University College London, Gower Street, London, WC1E 6BT, United Kingdom*

[b] *Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, 28049 Madrid, Spain*

[c] *Department of Mathematics, King's College London, Strand, London, WC2R 2LS, United Kingdom*

Quantum black hole thermodynamics

Bulk BH thermodynamics ↔ thermodynamics of qBTZ:

$$S_{\rm BH}^{\rm 4D} = \frac{A_4}{4G_4} = \frac{A_3}{4G_3} + \ell S_{\rm CFT} + \ell^2 S_{\rm Wald} + \ldots \equiv S_{\rm gen}^{\rm 3D}$$

First law of quantum black holes (Emparan, Frassino, Way, ‘20)

$$\boxed{dM = TdS_{\rm gen}} + \Omega dJ + \Phi dQ$$

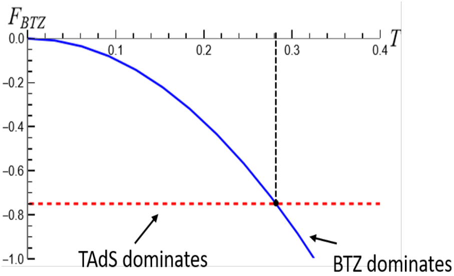
- Consistent with 2D quantum BHs

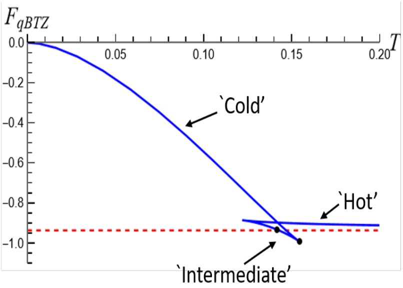
Thermal stability of quantum BHs

- Static qBTZ (Frassino, Pedraza, Svesko, Visser, ‘23); (Johnson, Nazario, ’23)
- Rotating qBTZ (Frassino, Hennigar, Pedraza, Svesko, ‘24)

$$C_{V,J,c_3} \equiv T\left(\frac{\partial S_{\rm gen}}{\partial T}\right)_{V,J,c_3} < 0\,, \qquad (a > a_{\rm ext}, \kappa = +1)$$

- $\text{Hess}_{S_{\rm gen}}$ has positive eigenvalue for ‘quantum cones’ (with $M < 0$) thermally unstable (Cartwright, Gürsoy, Pedraza, Svesko, ‘25)





$$F_{\mathrm{qBTZ}} = M - TS_{\mathrm{gen}}$$

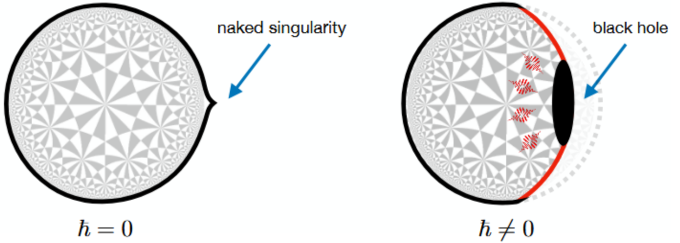
- Large backreaction $\Rightarrow$ 'reentrant' phase transitions (FPSV, '23)

$$\text{As } T \text{ increases, } \mathrm{TAdS} \overset{\mathrm{1st}}{\to} \mathrm{qBTZ} \overset{\mathrm{0th}}{\to} \mathrm{TAdS}$$

- Intermediate BH always thermodynamically stable, $C_{P_3} > 0$

Quantum cosmic censorship

The need for (weak) quantum cosmic censorship



Backreaction builds horizons (FHPS, ‘25)

Naked (conical) singularities ‘dressed’ (Emparan,...‘02; Casals...’17; ’19)

- Yet, qBHs have curvature singularities
- **Quantum** singularity theorems (Wall, ’10; Fewster, Kontou, ’21)

> When spacetime has a reliable semi-classical description, singularities are inevitable

$$\boxed{G_4 M_{\mathrm{ADM}} \geq \sqrt{\frac{A[\sigma]}{16\pi}}}$$

Assume:

- flat initial data
- (weak) cosmic censorship
- BH settles to Kerr

Penrose inequality in AdS ($D \geq 4$): (Itzkin, Oz, '12); (Folkestad, '22)

$$\boxed{\frac{16\pi G_D M_{\mathrm{AMD}}}{(D-2)\Omega_{D-2}} \geq \left(\frac{A[\sigma]}{\Omega_{D-2}}\right)^{\frac{D-3}{D-2}} + \ell_D^{-2}\left(\frac{A[\sigma]}{\Omega_{D-2}}\right)^{\frac{D-1}{D-2}}}$$

Many formulations; no generic proof – see review (Mars, '09)

Quantum violation and a proposal

Counterexample: (Bousso, Shahbazi-Moghaddam, Tomasevic, '19)

Massless scalar in Boulware state on Schwarzschild BH

Quantum Penrose inequality (Bousso..., '19)

- Replace classical area with generalized entropy, $\Delta S_{\text{gen}} \geq 0$

$$\boxed{\frac{16\pi G_D M_{\text{AMD}}}{(D-2)\Omega_{D-2}} \geq \left(\frac{4G_D S_{\text{gen}}}{\Omega_{D-2}}\right)^{\frac{D-3}{D-2}} + \ell_D^{-2}\left(\frac{4G_D S_{\text{gen}}}{\Omega_{D-2}}\right)^{\frac{D-1}{D-2}}}$$

Evidence and open question:

- QPI evades counterexample; states with *small* backreaction
- *What if backreaction is large?*
- Naive application for $D=3$: violation for *large* backreaction

Quantum Penrose inequality in 3D: (FHPS, '24)

$$8\pi\mathcal{G}_3 M_{\text{AMD}} \geq \ell_3^{-2}\left(\frac{4\mathcal{G}_3 S_{\text{gen}}}{2\pi}\right)^2$$

- *Valid for all AdS_3 quantum BHs, at all orders of backreaction!*

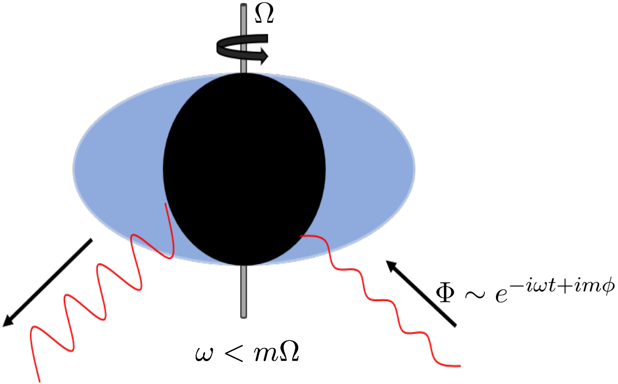
Classical Penrose inequality in 3D:

- No known derivation of naive $D = 3$
- $\ell = 0$ (no backreaction) limit is *not* saturated by classical BTZ
- Saturation linked to mass gap:

> Quantum effects allow for formation of black holes with classically unallowed masses

Quantum induced superradiance

Classical black hole superradiance



Low frequency wave amplification (Zeldovich, '66)

Superradiance: Radiation amplification (Dicke, '54)

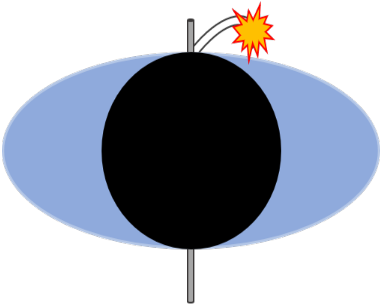
Allows for energy extraction; analogous to Penrose process

Black hole "bombs"

(Press-Teukolsky, '72)

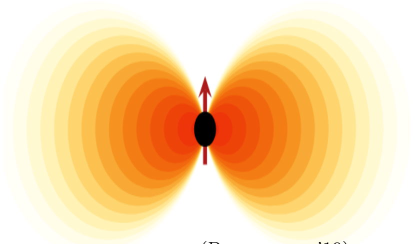
Repeated amplification ⇒ *superradiant instability*

- Massive bosons confined by BH gravitational potential
- Massless bosons in (small) Kerr-AdS (Cardoso... '04, '06)



Superradiance in astrophysics

- Gravitational 'atoms'
- Ultralight bosons?
- SR puts upper bound on BH spin (below Kerr bound)



(Baumann,...,'19)

Fundamental BH physics

- Final state of Kerr-AdS?
- Cosmic censorship?

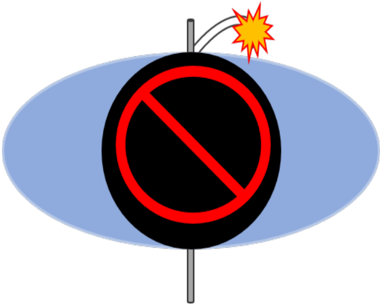
Superradiance is not universal

Notable exceptions

- $D \geq 4$ *Large* Kerr-AdS $(r_+ \gg L)$ (Hawking-Reall, '99; Cardoso...'04)

$$\Omega = \frac{a}{r_+^2 + a^2}\left(1 - \frac{a^2}{L^2}\right) \to 0$$

$$\boxed{\omega \not< m\Omega}$$



- 3D: BTZ (Ortiz, '12)

Superradiance is a *classical* process

What happens in semi-classical gravity?

Does backreaction induce superradiance?

Strategy: Linearly perturb BH with test matter $\Phi \sim e^{-i\omega t + im\phi}$

$$\boxed{\omega_{\rm QNM} = \mathrm{Re}(\omega) + i\mathrm{Im}(\omega)}$$

- Dissipative QNMs: $\mathrm{Im}(\omega) < 0$ – amplitude decays in time
- Onset of instability: $\mathrm{Im}(\omega) = 0$ ($\mathrm{Re}(\omega) = m\Omega$)
- Superradiant QNMs: $\mathrm{Im}(\omega) > 0$ ($\mathrm{Re}(\omega) < m\Omega$); exponentially growing mode; linearly unstable

Massless scalar probe:

$$\Box\Phi = 0\,, \quad \Phi(r,t,\phi) = \sum_{m\in\mathbb{Z}} \int d\omega e^{-i\omega t + im\phi} \tilde{\Phi}_m(\omega, r)$$

Boundary conditions:

- vanish at boundary: $\Phi \sim \Phi_\infty r^{-2}$, $(r \to \infty)$
- Ingoing at horizon: $\Phi \sim \Xi_+ e^{-ir_*(\omega - m\Omega)}$, $(r \to r_+)$

Static qBTZ: (Cartwright, Gürsoy, Pedraza, Planas, '24)

$$r[rf\tilde{\Phi}''_m + \tilde{\Phi}'_m(rf' + f - 2ir\omega)] - \tilde{\Phi}_m(m^2 + i\omega r) = 0$$

Classical BTZ ($\ell = 0$) (Cardoso, Lemos, '01)

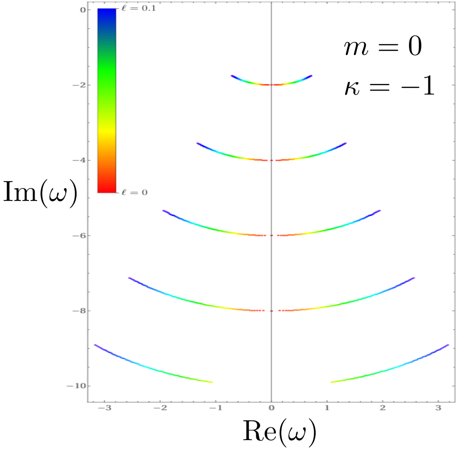
$$\boxed{\frac{\omega}{\ell_3} = \pm m - 2iM^{1/2}(n+1)}$$

Overtones $n \in \mathbb{Z}_+$

Highlights:

- QNMs: purely dissipative to propagating modes
- Rotating qBTZ (Cartwright, Gürsoy, Pedraza, Svesko, '25); non-monotonic a-dependence

Quantum BTZ



Classical BTZ (Ortiz,...'12)

No superradiant modes

Classical Kerr-AdS$_4$

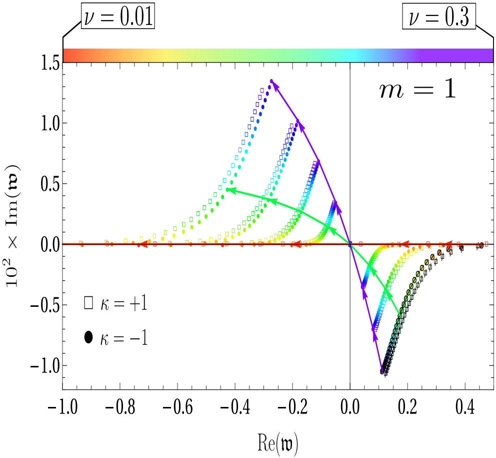
(Cardoso,...'04)

Superradiance for small BHs $(r_+/L \ll 1)$

Highlights:

- Not 'small' $(r_+/\ell_3 \sim .75)$
- $\kappa = \pm 1$ distinction

Quantum BTZ



Classically stable black holes become unstable via quantum backreaction

Summary:

- *Exact* constructions of quantum BHs
- New thermal phase transitions
- Backreaction builds horizons; weak (quantum) CC is viable
- Quantum backreaction can induce superradiance
- Classically stable black holes can become unstable semiclassically

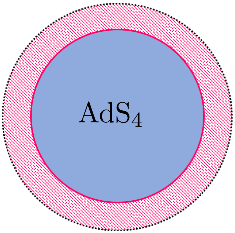
Open problems:

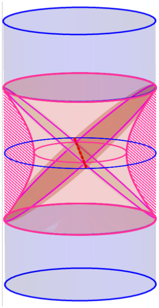
- More exact 3D constructions?
- Cosmic censorship and superradiance beyond AdS_3?
- Quantum black hole (modal) stability?
- Quantum black holes in higher dimensions?

Thank you!

(Quantum) dS_3 black holes

AdS_4 C-metric with Randall-Sundrum brane



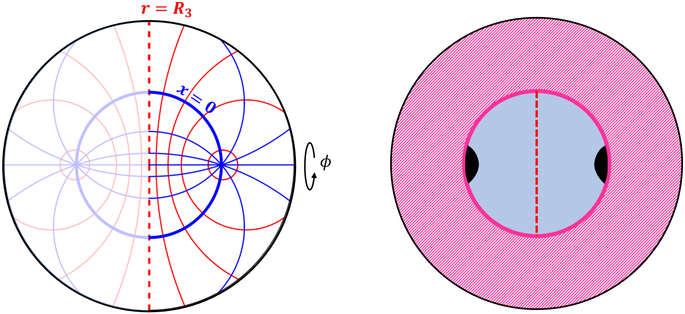


- Bulk acceleration horizon $\leftrightarrow$ cosmological horizon

Quantum dS_3 black holes:

- Quantum SdS_3 (Emparan, Pedraza, Svesko, Tomasevic, Visser, '22)
- Quantum Kerr-dS_3 (Panella, Svesko, '23)
- Charged quantum dS_3 (Climent, Hennigar, Panella, Svesko, '24

See review (Panella, Pedraza, Svesko, '24)



$(\mu = 0)$ $(\mu \neq 0)$

Brane geometry:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\phi^2 \ , \quad f(r) = \frac{-r^2}{R_3^2} - 8\mathcal{G}_3 M - \frac{\ell F(M)}{r}$$

Quantum dS_3 black hole: (EPSTV, '22)

- Nariai black hole
- Quantum Kerr-dS_3 (PS, '23)

For the set-up with bulk AdS_4,

$$I = \frac{1}{16\pi G_3}\int d^3x\sqrt{h}\left[\frac{2}{L_3^2} + R + \ell^2\left(\frac{3}{8}R^2 - R_{ij}^2\right) + \ldots\right] + I_{\mathrm{CFT}}$$

with scales

$$G_3 = \frac{1}{2L_4}G_4\,, \quad \frac{1}{L_3^2} = \frac{2}{L_4^2}\left(1 - \frac{L_4}{\ell}\right) \approx \frac{1}{\ell_3^2}\left(1 + \frac{\ell^2}{4\ell_3^2}\right)$$

- As $\ell \to 0$ the brane approaches AdS_4 bdry, where $G_3 \to 0$, implies ℓ controls strength of backreaction
- Higher derivative terms suppressed when curvature scale $L_3 > L_4 \sim \ell \sim c_3 G_3$; ℓ is *cutoff* scale of effective theory

Energy extraction

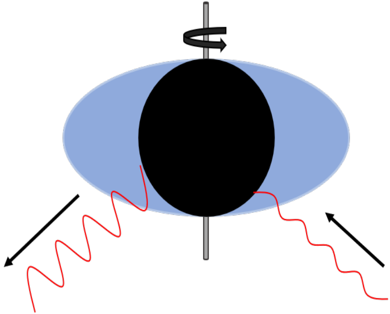
Assume generalized 2nd law:

$dS_{\text{gen}} \geq 0$

$$dM = \frac{\omega T}{(\omega - m\Omega)} dS_{\text{gen}} < 0$$

$$\boxed{\omega < m\Omega}$$

Unstable *quantum* BH?



Faster than light surfaces (Hawking, Reall, '99)

Horizon generator $\chi = \partial_t + \frac{a}{r_+^2}\partial_\phi$ has norm

$$\chi^2_{\text{qBTZ}}|_{r\to\infty} = \frac{r^2}{\ell_3^2}(\Omega^2\ell_3^2 - 1) + \ldots$$

- Spacelike for $|\Omega|\ell_3 > 1$; energy escaping $E_{\text{esc}} < 0$
- $\chi^2_{\text{cBTZ}} < 0$ everywhere $r > r_+$; no FTL surfaces

Classical BTZ ($\nu = 0$)

(Cardoso,...'01); (Birmingham,...'02)

Ingoing at horizon

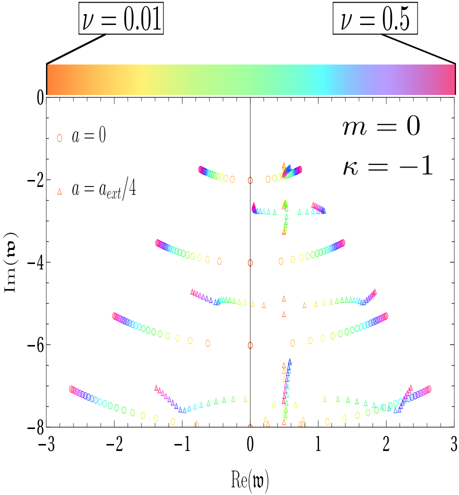
$$\frac{\omega}{\ell_3} = \pm m - 4\pi i T_{R/L}(n + h_{R,L})$$

$n \in \mathbb{Z}_+$, conformal weights (h_L, h_R), L/R temperature $T_{L/R}$

Highlights:

- QNMs: purely dissipative to propagating modes
- non-monotonic dependence on a for small ν
- Distinct from quantum-dressed conical singularities

Quantum BTZ



Quantum black hole stability

Classical BH stability

- Superradiance = *dynamical* instability
- Thermal stability? Small Kerr-AdS BHs thermally *unstable*
- Classical (stationary) BHs *dynamically* stable (Hollands-Wald, '13; Green,...'16)

$$\mathcal{E} = \delta^2 M - \Omega\delta^2 J - \frac{\kappa}{8\pi G}\delta^2 A \geq 0$$

Stationary BHs that locally extremize the entropy for fixed conserved charges are linearly stable under perturbations that preserve said conserved charges

Quantum BH stability

- Quantum-corrected BTZ ($\kappa = -1$) thermally *stable*
- Quantum cones ($\kappa = +1, M < 0$) thermally *unstable*
- No semi-classical counterexample to HW

$$\boxed{\mathcal{E} = \delta^2 M - \Omega\delta^2 J - T\delta^2 S_{\text{gen}} \geq 0}$$