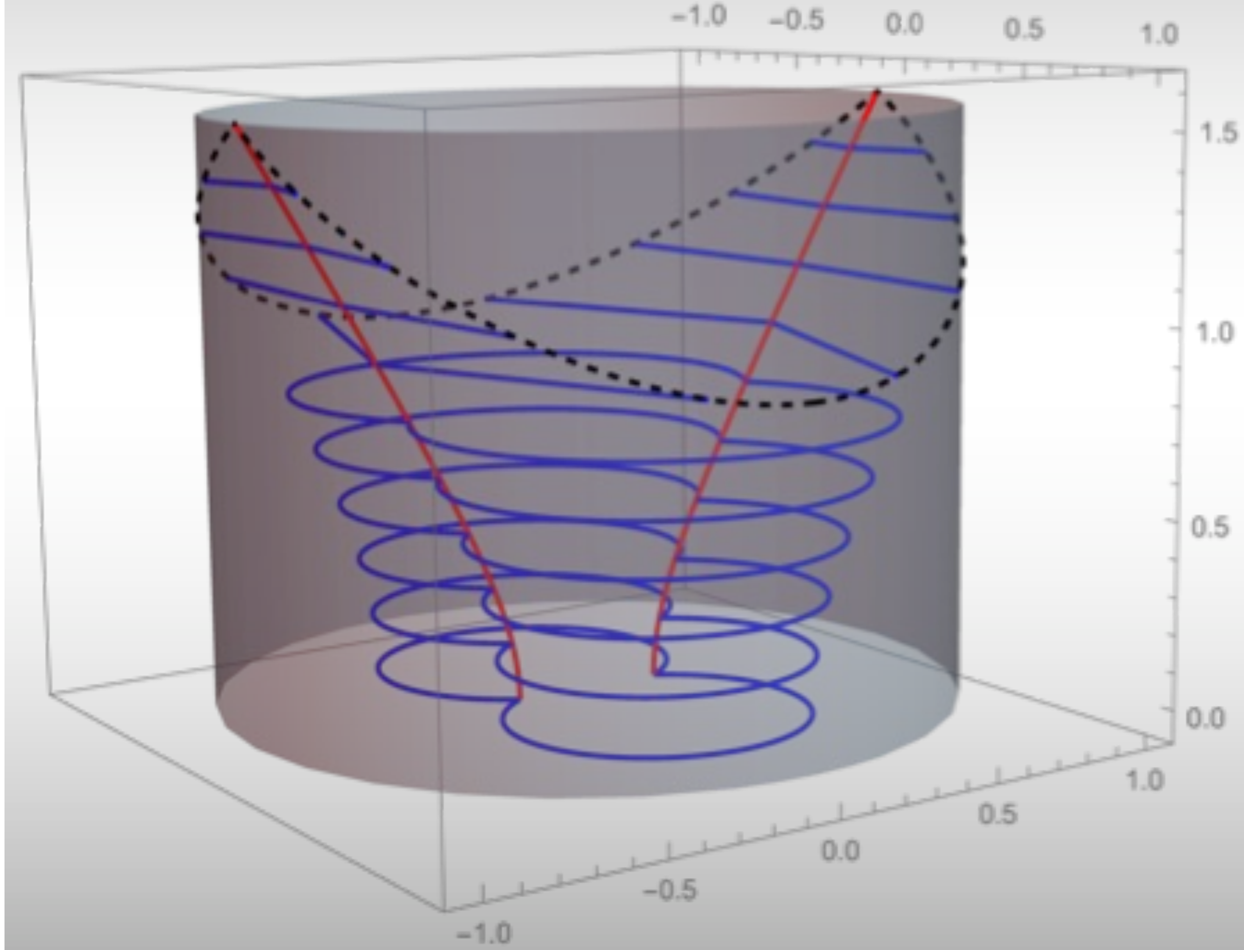
The Exotic Life of *Accelerating* Black Holes



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New Insights in Black Hole Physics from Holography 16/6/25

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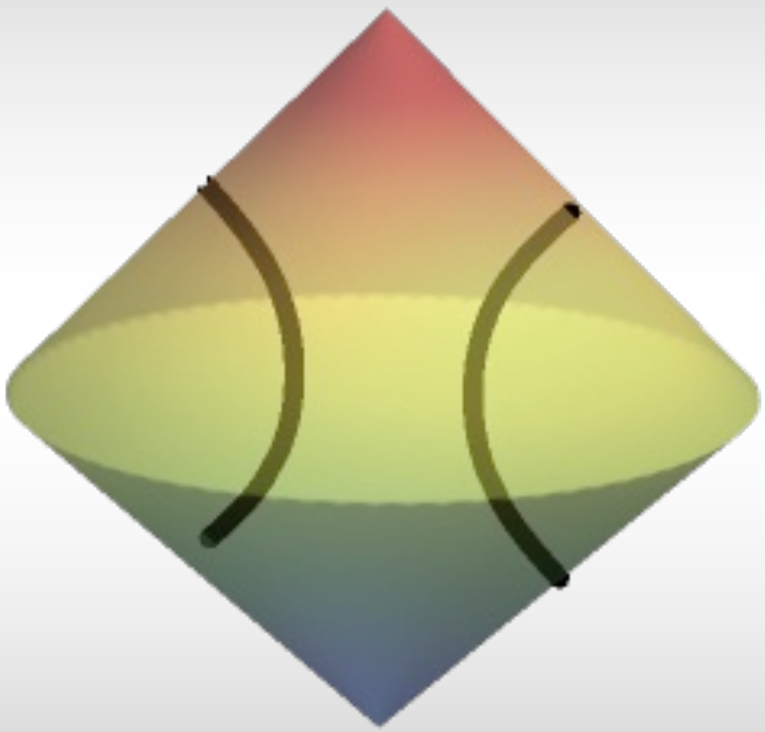
Outline



- On acceleration
- C-thermodynamics
- Acceleration in 3D

On Acceleration

Acceleration is when an object is not travelling on a geodesic.

$$\nabla_T T \not\propto T$$

… but this does not mean time dependence!

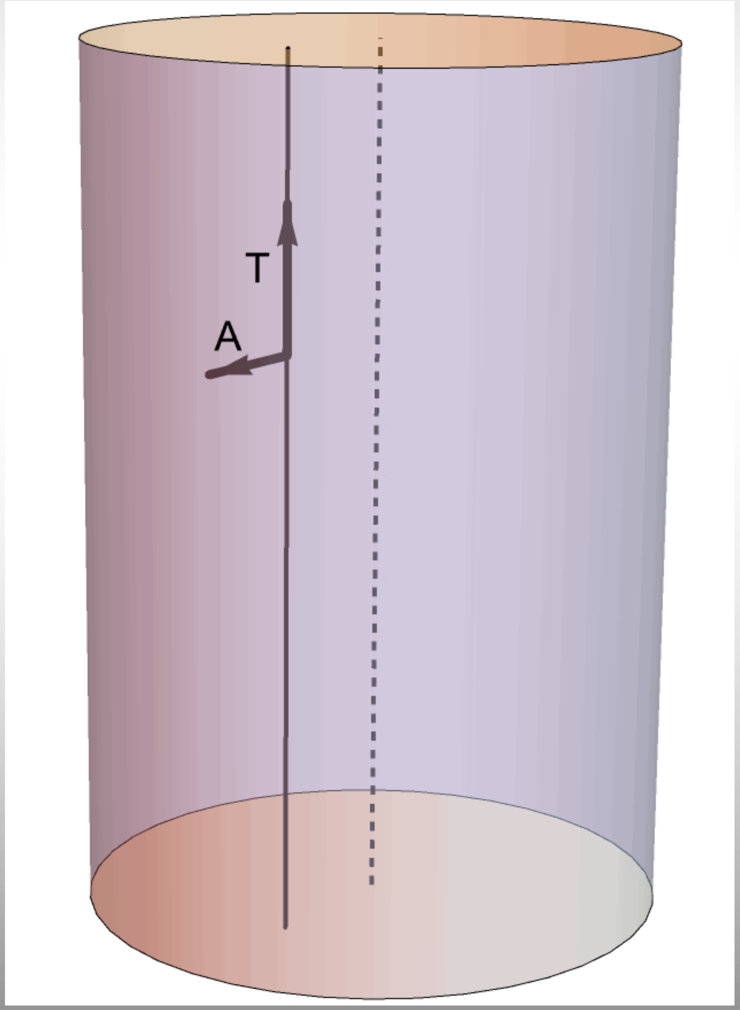
Start in 4D AdS to build a picture:

$$ds^2_{AdS} = \left(1 + \frac{R^2}{\ell^2}\right)dt^2 - \frac{dR^2}{1 + \frac{R^2}{\ell^2}} - R^2\left(d\Theta^2 + \sin^2\Theta d\phi^2\right)$$

On Acceleration

For an observer at $R=R_0$ in AdS, the tangent vector is purely timelike:

$$\mathbf{T} = \frac{1}{\sqrt{1+\frac{R_0^2}{\ell^2}}}\frac{\partial}{\partial t}$$



but the acceleration is radial:

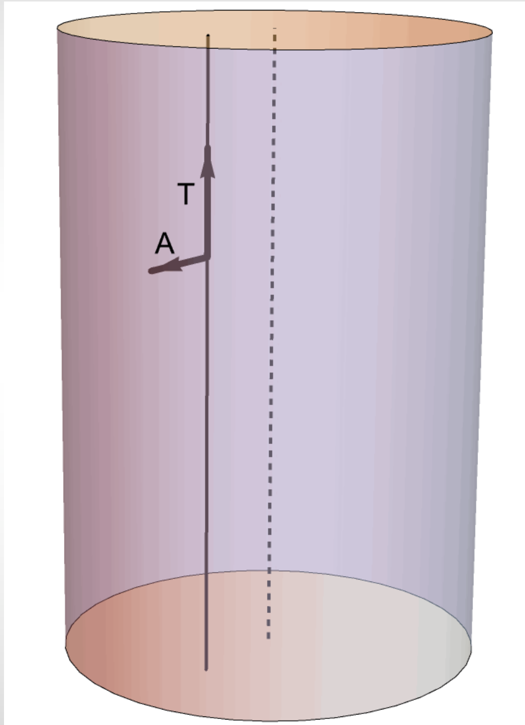
$$\mathbf{A} = \nabla_T T = \frac{R_0}{\ell^2}\frac{\partial}{\partial r}$$

Rindler with no Horizon

The magnitude of the acceleration is related to R_0 :

$$|\mathbf{A}|^2 = \frac{R_0^2/\ell^4}{1 + R_0^2/\ell^2}$$

$$R_0 = \frac{A\ell^2}{\sqrt{1 - A^2\ell^2}}$$

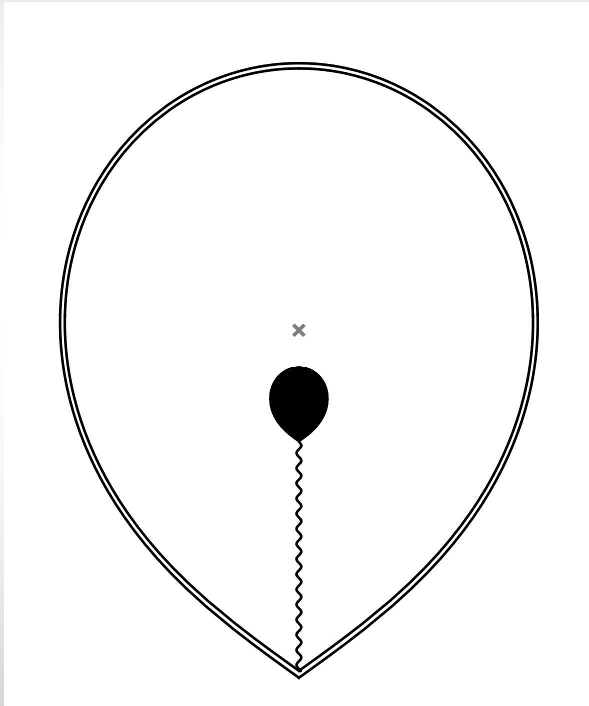


So,

$$R_0 \to \infty \quad \Rightarrow \quad A\ell \to 1$$

The Slowly Accelerating Black Hole

The slowly accelerating black hole in AdS is displaced from centre. It has a conical deficit running from the horizon to the boundary. The string tension provides the force that hold the black hole off-centre.



Acceleration & Black Holes

This accelerating black hole in 4D is described by the C-metric

$$ds^2 = \Omega^{-2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

Where

$$f = \left(1 - \frac{2m}{r}\right)\left(1 - A^2r^2\right) + \frac{r^2}{\ell^2}$$

$$g = 1 + 2mA\cos\theta$$

$$\Omega = 1 + Ar\cos\theta$$

f determines horizon structure – black hole / acceleration / cosmological constant

Hong & Teo, CQG20 3629 (2003)

Slowly Accelerating Rindler

As a check, set m to zero:

$$ds^2 = \frac{\left[f(r)\frac{dt^2}{\alpha^2} - \frac{dr^2}{f(r)} - r^2 d\Omega_{II}^2 \right]}{(1 + Ar\cos\theta)^2}$$

$$f(r) = 1 + \frac{r^2(1 - A^2\ell^2)}{\ell^2}$$

& using the coordinate transformation

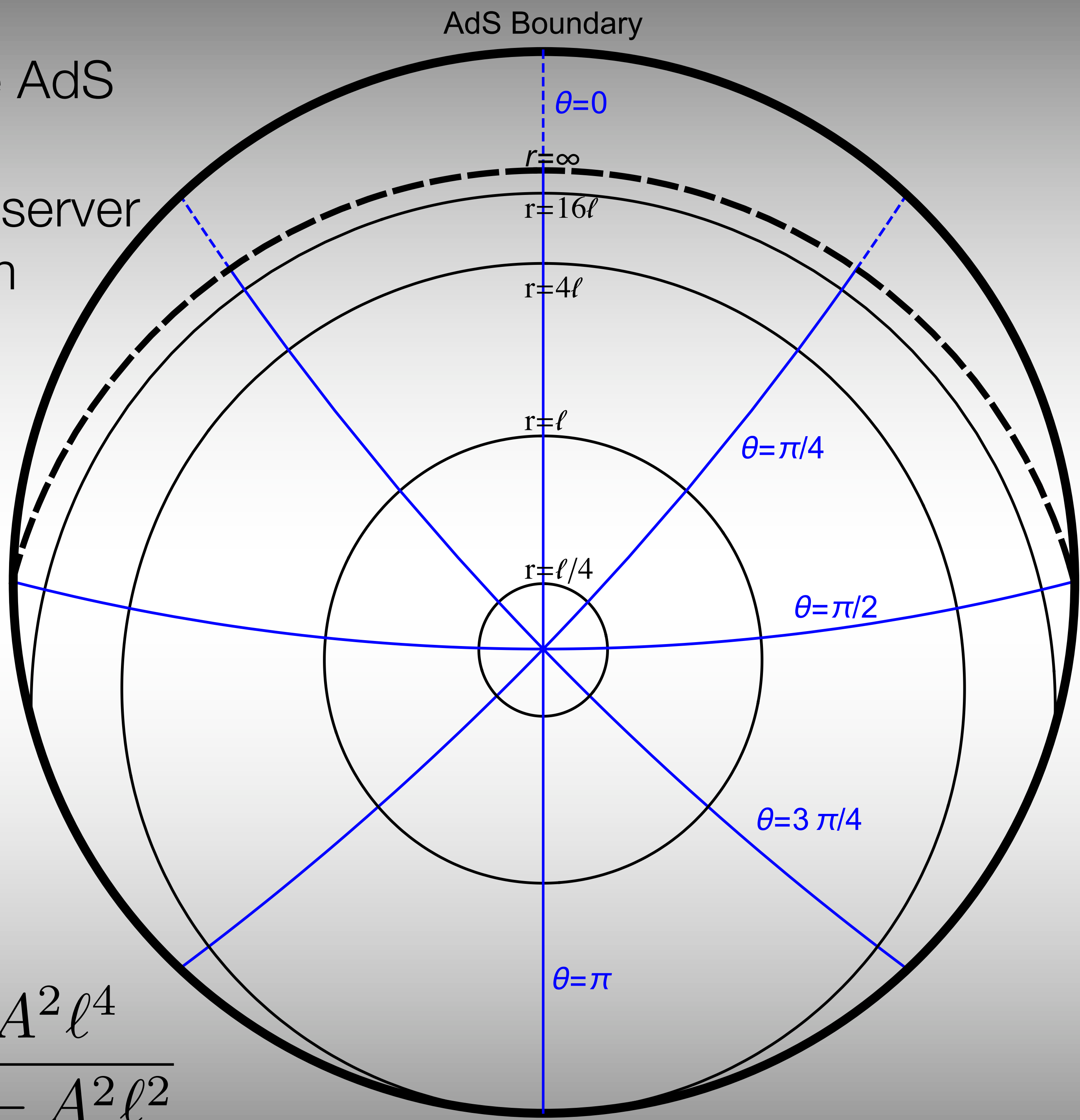
$$1 + \frac{R^2}{\ell^2} = \frac{1 + (1 - A^2\ell^2)r^2/\ell^2}{(1 - A^2\ell^2)\Omega^2}, \qquad R\sin\Theta = \frac{r\sin\theta}{\Omega}$$

we get back to global AdS

$$ds^2_{AdS} = \left(1 + \frac{R^2}{\ell^2}\right)dt^2 - \frac{dR^2}{1 + \frac{R^2}{\ell^2}} - R^2\left(d\Theta^2 + \sin^2\Theta\frac{d\phi^2}{K^2}\right)$$

$$\alpha^2 = 1 - A^2\ell^2$$

C-coordinates give AdS from an off-centre perspective. An observer hovering away from centre of AdS is accelerating.



$$r = 0 \leftrightarrow R = \frac{A^2 \ell^4}{1 - A^2 \ell^2}$$

Acceleration & Strings

Once *mA* is nonzero, $g = 1 + 2mA\cos\theta$ distorts the 2-sphere:

- $\theta \to 0$ $$ds^2_{\theta,\phi} \propto d\theta^2 + \frac{(1+2mA)^2}{K^2}\theta^2 d\phi^2$$
- $\theta \to \pi$ $$ds^2_{\theta,\phi} \propto d\theta^2 + \frac{(1-2mA)^2}{K^2}(\pi-\theta)^2 d\phi^2$$

This is a conical deficit, which we associate to a cosmic string with tension

$$\delta_\pm = 2\pi\left(1 - \frac{g(0)}{K}\right) = 2\pi\left(1 - \frac{1 \pm 2mA}{K}\right) = \text{“}8\pi\mu_\pm\text{”}$$

Introduce K so that ϕ has a uniform periodicity. The tension of the string now relates to a parameter in the metric.

The Black Hole

$$R_0 = \frac{A\ell^2}{\sqrt{1 - A^2\ell^2}}$$

Suppose K = 1+2mA, then

$$\mu_+ = 0\,, \quad \mu_- = mA/K$$

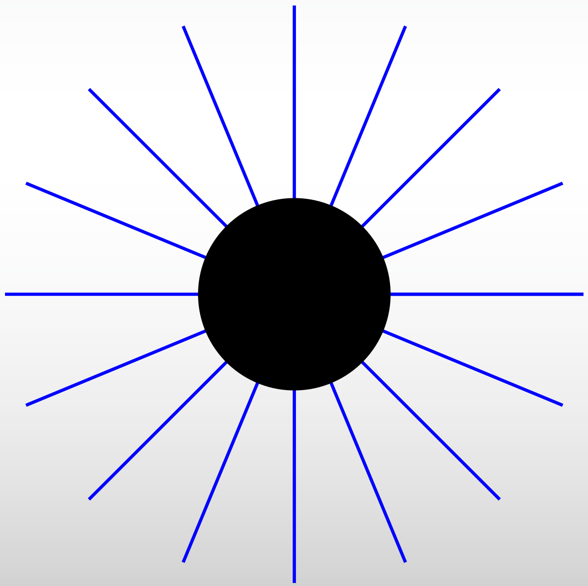
So as m increases, A decreases for the same tension.

Recall that the lower the acceleration, the closer to the centre of global AdS.

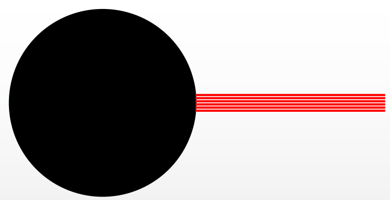
If an accelerating black hole accretes, it drops further down, if it evaporates, it moves to the boundary.

String Formation?

A string could form emerging from a black hole when a phase transition gives mass to a previously massless U(1) gauge boson.



Monopole



Confined flux.

Thermodynamics with Strings

To illustrate the process of finding a first law, simplify to Schwarzschild-AdS with a deficit:

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left[d\theta^2 + \sin^2\theta\frac{d\phi^2}{K^2}\right]$$

Black hole horizon defined by f=0, look at small changes in f. Horizon still defined by f(r) = 0.

$$f(r_+ + \delta r_+) = f'(r_+)\delta r_+ - \frac{2\delta m}{r_+} - \frac{r_+^2}{\ell^3}\delta\ell = 0$$

Changes r+, hence S

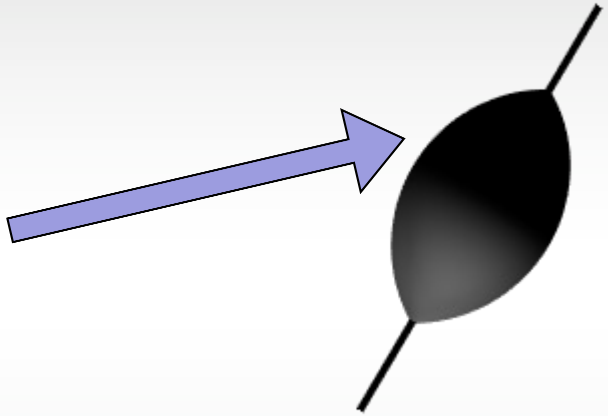
Changes m, hence M

Changes ℓ, hence Λ

Entropy with Strings

The temperature has its usual expression from a Euclidean calculation, the entropy is given by the area of the horizon, which is affected by K:

$$T = f'(r_+)/4\pi \qquad S = \frac{\pi r_+^2}{K}$$



So

$$f'(r_+)\delta r_+ = \frac{2K}{r_+}\left(T\delta S + \frac{r_+^2 f'(r_+)}{4}\frac{\delta K}{K^2}\right)$$

And we have to identify changes in K

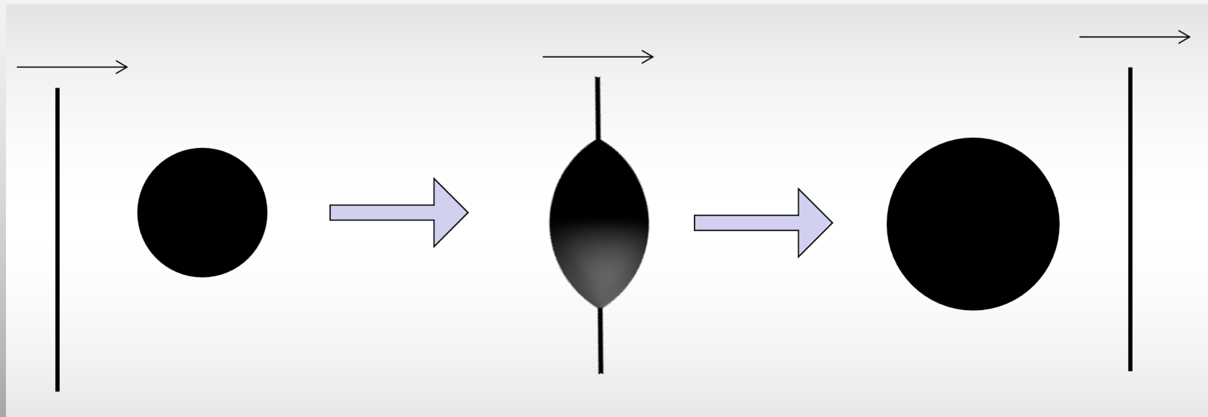
(S): Herdeiro, Kleihaus,Kunz,Radu: 0912:3386 [gr-qc]

Changing Tension

Tension is related to K: $$\mu = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

So easily get $$\delta\mu = \frac{\delta K}{4K^2}$$

Finally $$P = -\Lambda = \frac{3}{8\pi\ell^2} \qquad V = \frac{4\pi r_+^3}{3K}$$



First Law with Tension

Putting together:

$$0 = \frac{2K}{r_+}\left(T\delta S + 2(m - r_+)\delta\mu + V\delta P - \delta(\frac{m}{K})\right)$$

So identify $M = \frac{m}{K}$

Then also get Smarr relation:

$$M = 2TS - 2PV$$

Thermodynamic Length

The term multiplying the variation in tension is a "thermodynamic length"

$$\lambda = r_+ - m$$

Reinforces interpretation of M as **enthalpy**, if black hole grows, it swallows some string, but has also displaced the same amount of energy from environment.

Kastor & Traschen: 1207:5415 [hep-th]

Back to the acc black hole:

The accelerating, rotating, charged Kerr-AdS black hole is obviously more algebraically complex, but the principle is the same (we need to introduce a rescaling of the time coordinate)

$$ds^2 = \frac{1}{H^2}\left\{\frac{f(r)}{\Sigma}\left[\frac{dt}{\alpha} - a\sin^2\theta\frac{d\varphi}{K}\right]^2 - \frac{\Sigma}{f(r)}dr^2 - \frac{\Sigma r^2}{h(\theta)}d\theta^2 - \frac{h(\theta)\sin^2\theta}{\Sigma r^2}\left[\frac{adt}{\alpha} - (r^2+a^2)\frac{d\varphi}{K}\right]^2\right\}$$

This will rescale temperature, and also changes computations of the mass.

$$f(r) = (1-A^2r^2)\left[1 - \frac{2m}{r} + \frac{a^2+e^2}{r^2}\right] + \frac{r^2+a^2}{\ell^2},$$

$$h(\theta) = 1 + 2mA\cos\theta + \left[A^2(a^2+e^2) - \frac{a^2}{\ell^2}\right]\cos^2\theta,$$

$$\Sigma = 1 + \frac{a^2}{r^2}\cos^2\theta, \qquad H = 1 + Ar\cos\theta.$$

Holographic M

Expand the metric near the boundary (Fefferman-Graham):

$$\frac{1}{r} = -A\xi - \sum F_n\left(\xi\right) z^n$$

$$\cos\theta = \xi + \sum G_n\left(\xi\right) z^n$$

F_n and G_n determined by the requirement that

$$ds^2 = -\ell^2 dz^2 + \frac{1}{z^2}\left[\gamma_{\mu\nu} + z^2\Psi_{\mu\nu} + z^3 M_{\mu\nu}\right] dx^\mu dx^\nu + \mathcal{O}(z^2)$$

Fefferman-Graham

For the boundary metric, get:

$$\frac{\left(1 - A^2\ell^2 g(\xi)\right)^3}{\alpha^2\ell^2 F_1^2(\xi)} d\tau^2 - \frac{\left(1 - A^2\ell^2 g(\xi)\right)}{F_1^2(\xi) g(\xi)} d\xi^2 - \frac{g(\xi)\left(1 - A^2\ell^2 g(\xi)\right)^2}{K^2 F_1^2(\xi)} d\phi^2$$

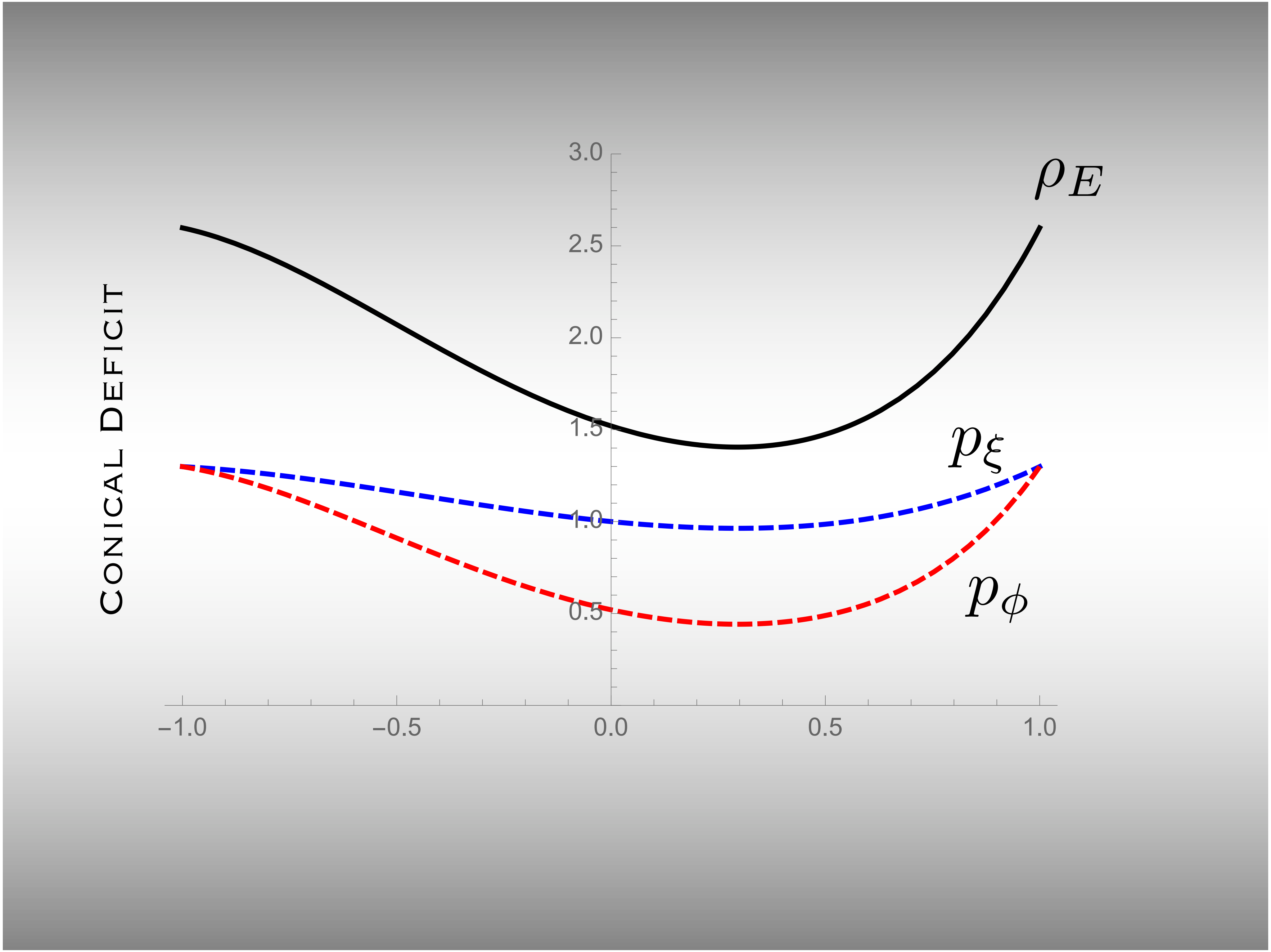
And for the boundary fluid stress tensor:

$$\langle \mathcal{T}^\mu_\nu \rangle = \mathrm{diag}\left\{\rho_E, -\rho_E/2 + \Pi, \rho_E/2 - \Pi\right\}$$

where

$$\rho_E = \frac{m}{\alpha}(1 - A^2\ell^2 g)^{3/2}(2 - 3A^2\ell^2 g)$$

$$\Pi = \frac{3A^2\ell^2 gm}{2\alpha}(1 - A^2\ell^2 g)^{3/2}$$



Conical Deficit
ρ_E
p_ξ
p_ϕ
3.0
2.5
2.0
1.5
1.0
0.5
−1.0
−0.5
0.0
0.5
1.0

Accelerating Thermodynamics

Integrate up the boundary stress-energy to get the mass:

$$M = \int \rho_E \sqrt{\gamma} = \frac{\alpha m}{K}$$

What is alpha? Setting m (and other charges) to zero, and demanding that the boundary is a round 2-sphere gives

$$\alpha = \sqrt{1 - A^2 \ell^2}$$

Get a consistent first law with corrections to V and TD length, and – can generalise to rotation

General Thermo Parameters

$$M = \frac{m(\Xi + a^2/\ell^2)(1 - A^2\ell^2\Xi)}{K\Xi\alpha(1 + a^2A^2)}$$

$$T = \frac{f'_+ r_+^2}{4\pi\alpha(r_+^2 + a^2)}\,, \quad S = \frac{\pi(r_+^2 + a^2)}{K(1 - A^2r_+^2)}\,,$$

$$Q = \frac{e}{K}\,, \quad \Phi = \Phi_t = \frac{er_+}{(r_+^2 + a^2)\alpha}\,,$$

$$J = \frac{ma}{K^2}\,, \quad \Omega = \Omega_H - \Omega_\infty\,, \quad \Omega_H = \frac{Ka}{\alpha(r_+^2 + a^2)}$$

$$P = \frac{3}{8\pi\ell^2}\,, \quad V = \frac{4\pi}{3K\alpha}\left[\frac{r_+(r_+^2 + a^2)}{(1 - A^2r_+^2)} + \frac{m[a^2(1 - A^2\ell^2\Xi) + A^2\ell^4\Xi(\Xi + a^2/\ell^2)]}{(1 + a^2A^2)\Xi}\right]$$

$$\lambda_\pm = \frac{r_+}{\alpha(1 \pm Ar_+)} - \frac{m}{\alpha}\frac{[\Xi + a^2/\ell^2 + \frac{a^2}{\ell^2}(1 - A^2\ell^2\Xi)]}{(1 + a^2A^2)\Xi^2} \mp \frac{A\ell^2(\Xi + a^2/\ell^2)}{\alpha(1 + a^2A^2)}$$

$$\Xi = 1 - \frac{a^2}{l^2} + A^2(e^2 + a^2) \qquad \alpha = \frac{\sqrt{(\Xi + a^2/\ell^2)(1 - A^2\ell^2\Xi)}}{1 + a^2A^2}$$

"Chemical" Expressions

The expressions for the thermodynamic parameters are not particularly illuminating & analysis is facilitated by expressing the chemical potentials in terms of thermodynamical charges, rather than geometrical quantities like r_+.

This reveals that it is more natural to think in terms of an overall average deficit, and the differential deficit that produces acceleration. We therefore encode these as:

$$\Delta = 1 - 2(\mu_+ + \mu_-) = \frac{\Xi}{K}$$

$$C = \frac{(\mu_- - \mu_+)}{\Delta} = \frac{mA}{K\Delta} = \frac{mA}{\Xi}$$

$$\left(\Xi = 1 + e^2 A^2 - \frac{a^2}{\ell^2}(1 - A^2 \ell^2) \right)$$

$$M^2 = \frac{\Delta S}{4\pi}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)^2+\left(1+\frac{8PS}{3\Delta}\right)\left(\frac{4\pi^2 J^2}{(\Delta S)^2}-\frac{3C^2\Delta}{2PS}\right)\right]$$

$$V = \frac{2S^2}{3\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)+\frac{2\pi^2 J^2}{(\Delta S)^2}+\frac{9C^2\Delta^2}{32P^2S^2}\right],$$

$$T = \frac{\Delta}{8\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)\left(1-\frac{\pi Q^2}{\Delta S}+\frac{8PS}{\Delta}\right)-\frac{4\pi^2 J^2}{(\Delta S)^2}-4C^2\right],$$

$$\Omega = \frac{\pi J}{SM\Delta}\left(1+\frac{8PS}{3\Delta}\right),$$

$$\Phi = \frac{Q}{2M}\left(1+\frac{\pi Q^2}{S\Delta}+\frac{8PS}{3\Delta}\right),$$

$$\lambda_{\pm} = \frac{S}{4\pi M}\left[\left(\frac{8PS}{3\Delta}+\frac{\pi Q^2}{\Delta S}\right)^2+\frac{4\pi^2 J^2}{(\Delta S)^2}\left(1+\frac{16PS}{3\Delta}\right)-(1\mp 2C)^2\pm\frac{3C\Delta}{2PS}\right]$$

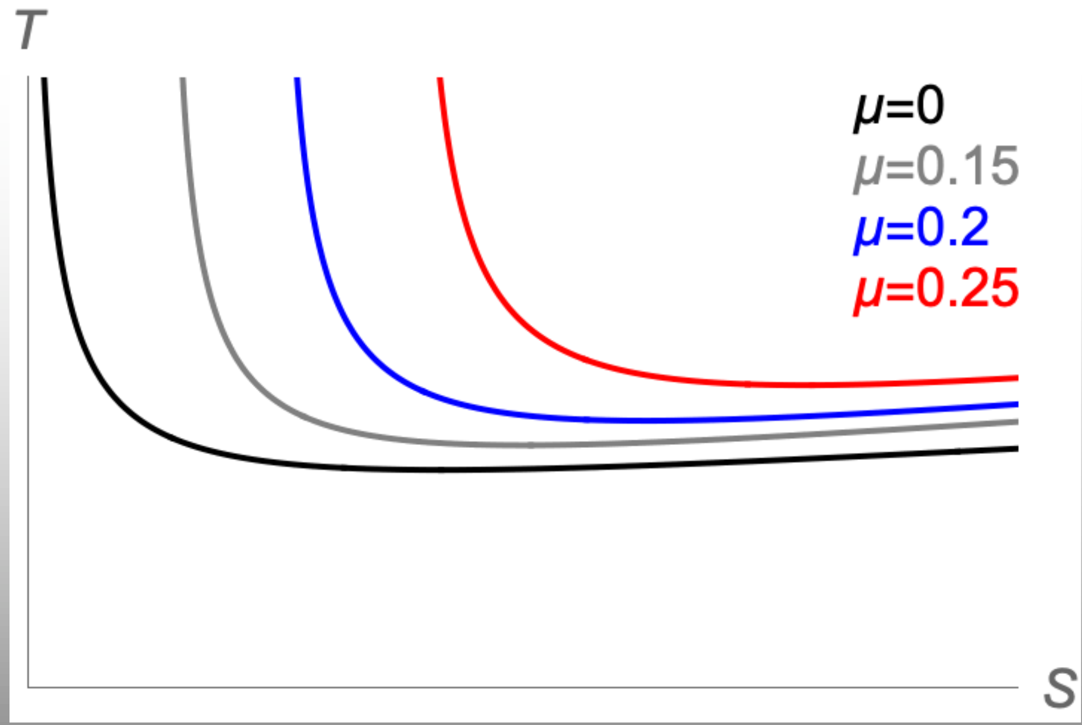
RG & Scoins, 1904.09660

To show the impact of acceleration, focus on just the deficits

$$M^2 = \frac{\Delta S}{4\pi}\left[\left(1+\frac{8PS}{3\Delta}\right)\left(1+\frac{8PS}{3\Delta}-\frac{3C^2\Delta}{2PS}\right)\right]$$

$$T = \frac{\Delta S}{8\pi M}\left[\left(1+\frac{8PS}{3\Delta}\right)\left(1+\frac{8PS}{\Delta}\right)-4C^2\right]$$

Keeping NP regular, looks like acceleration increases T, although have to bear in mind that tension reduces entropy for a given metric "m" parameter.

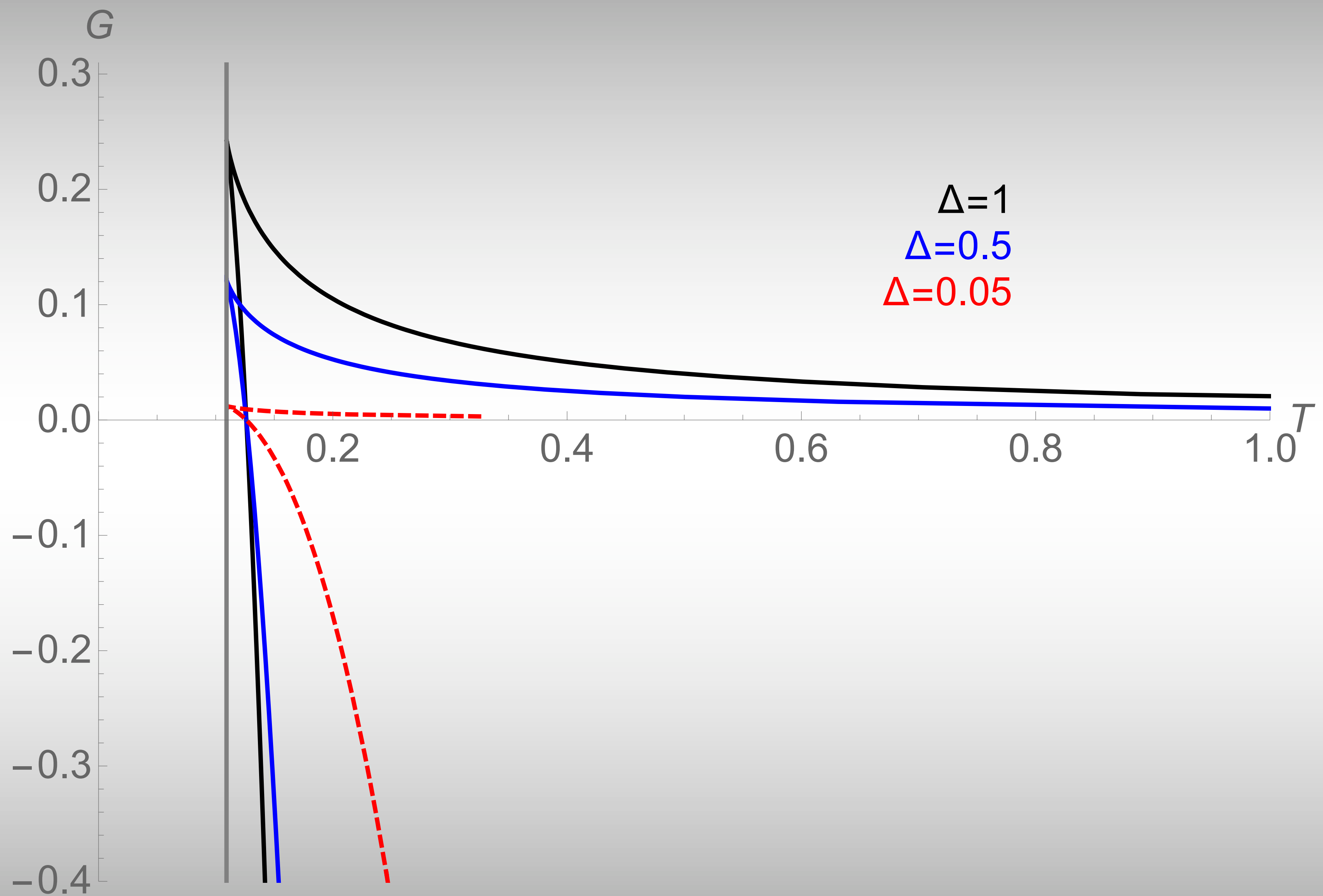


Hawking-Page

With a simple deficit (uncharged nonrotating black hole) it looks like Δ is almost irrelevant; the Gibbs free energy: G = M-TS is

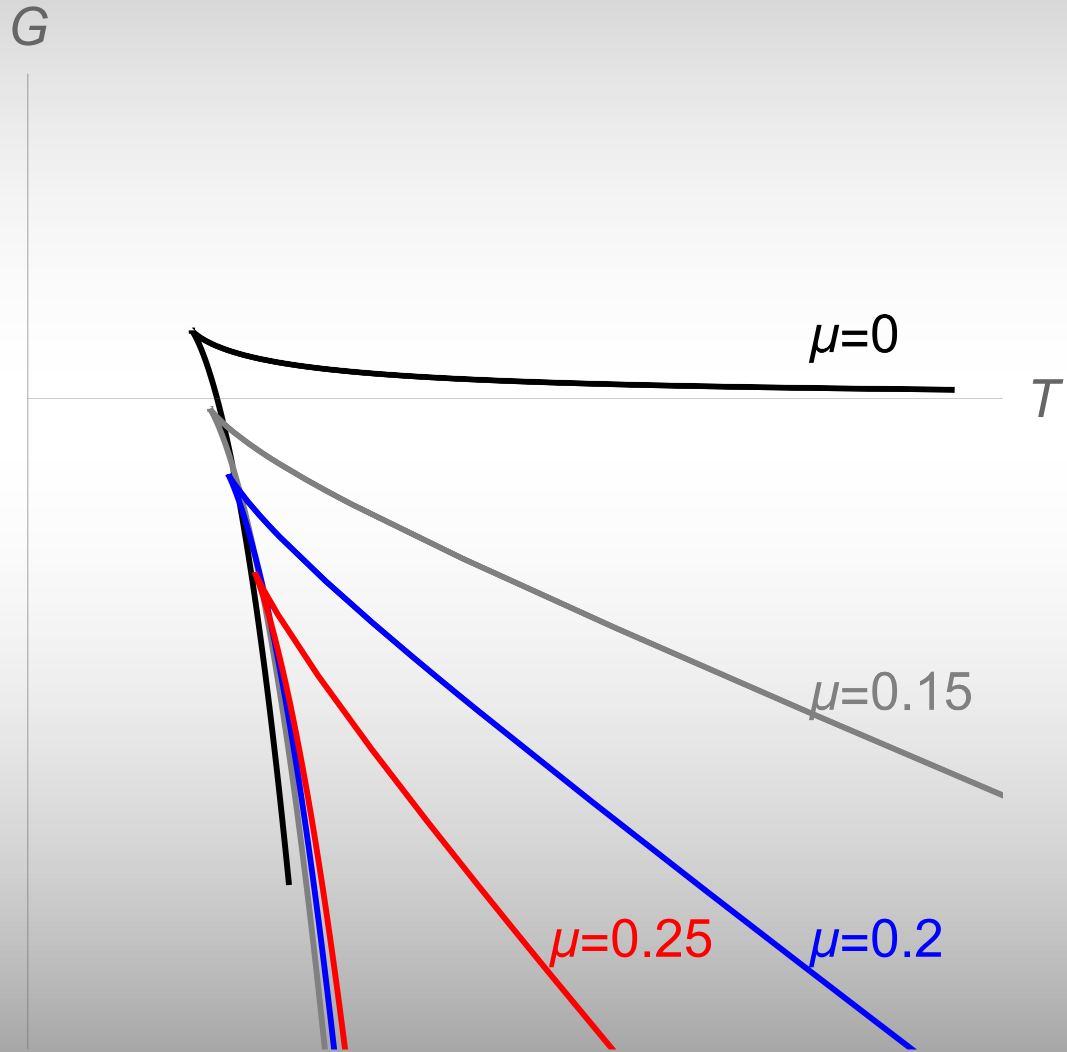
$$G = \frac{\Delta S}{8\pi M}\left(1 + \frac{8PS}{3\Delta}\right)\left(1 - \frac{8PS}{3\Delta}\right)$$

Δ decreases the free energy – though the HP transition remains at the same T.



G
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.2
0.4
0.6
0.8
1.0
T
Δ=1
Δ=0.5
Δ=0.05

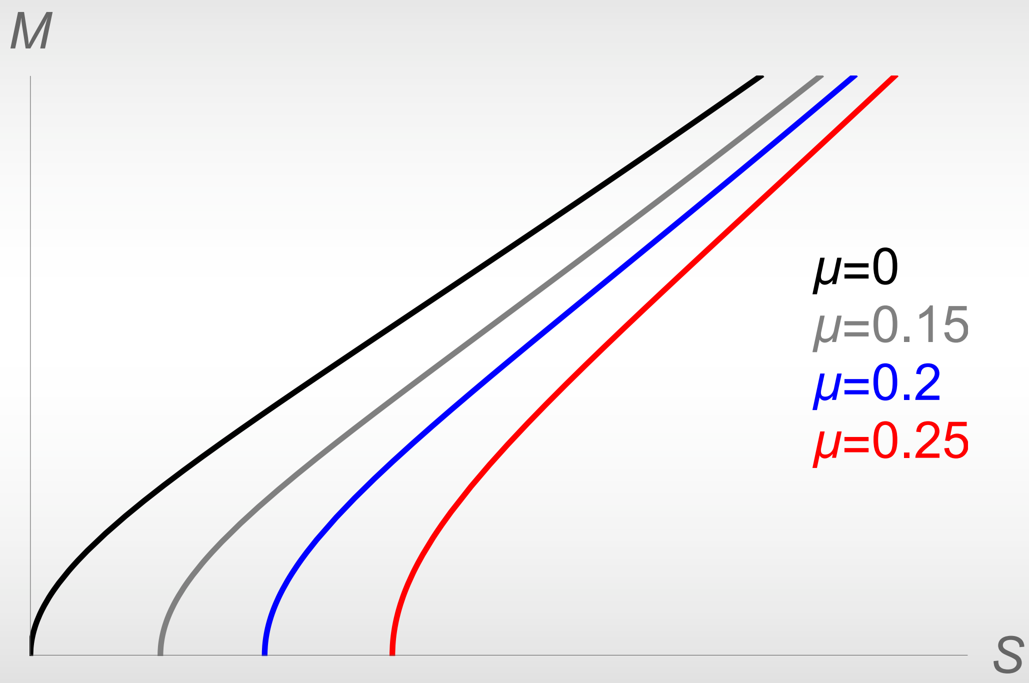
Hawking-Page with A?



However, once we have acceleration the picture changes.

What does Hawking-Page mean with acceleration?

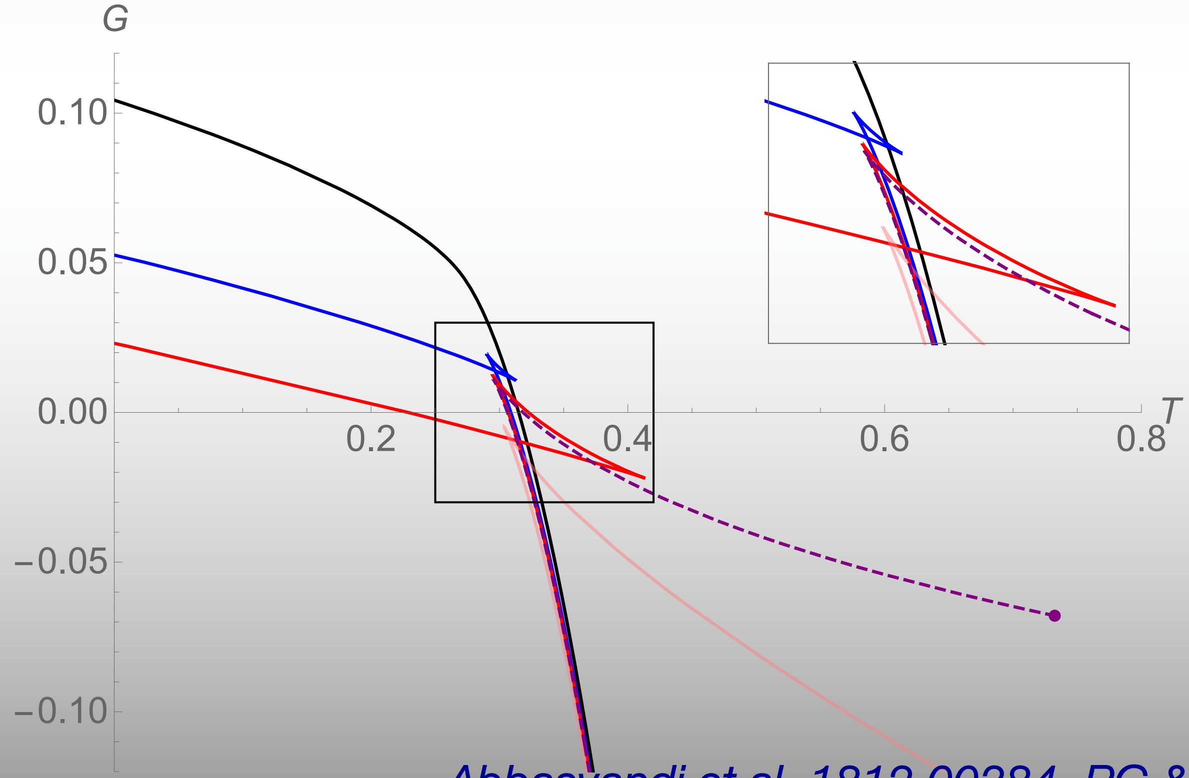
Zero Enthalpy



For small/low P black holes, the C-term becomes significant, and zeros the enthalpy (here shown for pure acceleration).

Snapped Swallowtails

This same phenomenon leads to snapping swallowtails: as the charge of an accelerating charged black hole drops, a swallowtail forms, then snaps at a critical Q.



Abbasvandi et al. 1812.00384, RG & Scoins 1904.09660

Reverse Isoperimetric Inequality

The Isoperimetric Inequality in Mathematics says that the minimal boundary length enclosing a given area is a circle (or suitable higher dimensional generalisation). This is a problem if true for thermodynamic volume and black holes, since it would say that a round black hole would *minimize* area for a given volume – but entropy should be maximized! Cvetic et al conjectured that black hole satisfied a reverse of this mathematical inequality, and demonstrated its validity for known examples.

Reverse Isoperimetric Inequality

Focus on uncharged case:

$$\left(x = \frac{8PS}{3\Delta}\right)$$

$$\left(y = \frac{\pi^2 J^2}{\Delta^2 S^2}\right)$$

$$M^2 = \frac{\Delta S}{4\pi}(1+x)\left[1 + x + 4y - 4\frac{C^2}{x}\right]$$

$$V = \frac{2S^2}{3\pi M}\left[1 + x + 2y + \frac{2C^2}{x^2}\right]$$

& combine the expressions

$$\frac{4\pi M^2}{\Delta S} = \left(\frac{3\pi M V}{2S^2} - \frac{2C^2}{x^2}\right)^2 - 4y^2 - 4(1+x)\frac{C^2}{x}$$

New Reverse Isoperimetric Inequality

$$\frac{4\pi M^2}{\Delta S} = \left(\frac{3\pi MV}{2S^2} - \frac{2C^2}{x^2}\right)^2 - 4y^2 - 4\left(1+x\right)\frac{C^2}{x}$$

This expression implies

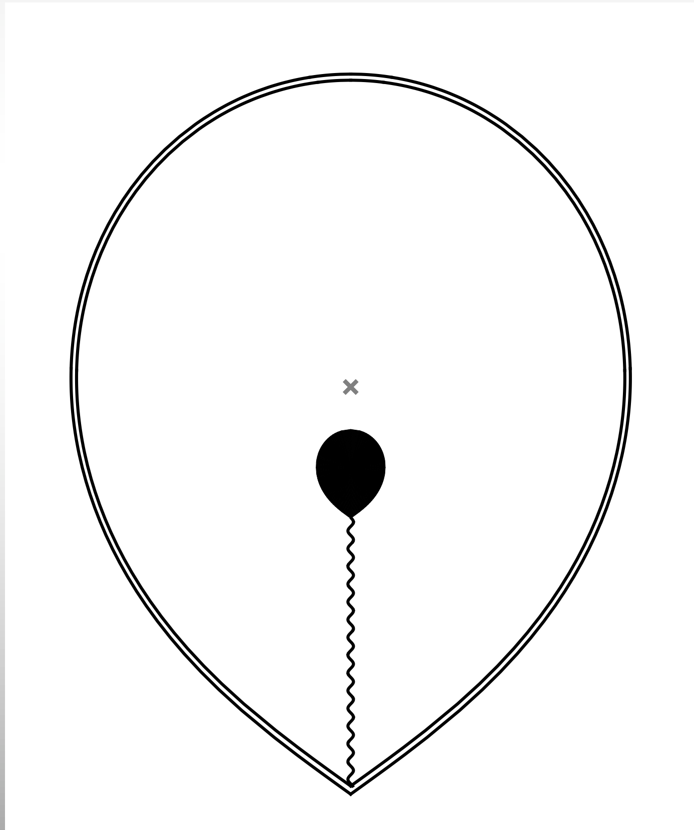
$$\frac{4\pi M^2}{\Delta S} \le \left(\frac{3\pi MV}{2S^2} - \frac{2C^2}{x^2}\right)^2 \le \left(\frac{3\pi MV}{2S^2}\right)^2 = 4M^2\left(\frac{3V}{4\pi}\right)^2\left(\frac{\pi}{S}\right)^4$$

hence a new inequality appropriate for conical deficit black holes:

$$\left(\frac{3V}{4\pi}\right)^2 \ge \frac{1}{\Delta}\left(\frac{\mathcal{A}}{4\pi}\right)^3$$

Acceleration in 3D

How far can we push these ideas in 3 dimensions?



In 3D we have two different types of solution: a point mass (conical deficit) and the BTZ ”black hole”, i.e. 3D AdS with identifications.

The equivalent of a string one dimension down is a WALL.

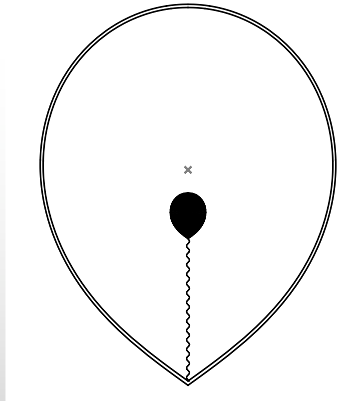
Acceleration in 3D

From the geometry perspective

$$ds^2_{AdS} = \left(1 + \frac{R^2}{\ell^2}\right)dt^2 - \frac{dR^2}{1 + \frac{R^2}{\ell^2}} - R^2\left(d\Theta^2 + \sin^2\Theta d\phi^2\right)$$

3D AdS drops ϕ – can we do the same here?

$$ds^2 = \Omega^{-2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$



Turns out we can!

"C" IN 3

We can look for an exact solution in 3D with the same type of structure:

$$ds^2 = \frac{1}{A^2(x-y)^2}\left[P(y)d\tau^2 - \frac{dy^2}{P(y)} - \frac{dx^2}{Q(x)}\right]$$

With general solution: $Q(x) = c + bx + ax^2\,, \qquad P(y) = \frac{1}{A^2\ell^2} - Q(y)$

Which, after coordinate rescaling/shifts reduces to:

Class	$Q(x)$	$P(y)$	Maximal range of x
I	$1-x^2$	$\frac{1}{A^2\ell^2} + (y^2-1)$	$\lvert x\rvert < 1$
II	x^2-1	$\frac{1}{A^2\ell^2} + (1-y^2)$	$x > 1$ or $x < -1$
III	$1+x^2$	$\frac{1}{A^2\ell^2} - (1+y^2)$	$\mathbb{R}$

Arenas-Henriquez, RG, Scoins: 2202:08823; Anber 0809:2789; Astorino 1101.2616

Accelerating Particle

Take each in turn. The first class looks very similar to the 4D C-metric (r=-1/Ay, t = ατ/A, x = cos(ϕ/K))

$$ds^2 = \frac{1}{\left[1 + Ar\cos\left(\phi/K\right)\right]^2}\left[f(r)\frac{dt^2}{\alpha^2} - \frac{dr^2}{f(r)} - r^2\frac{d\phi^2}{K^2}\right]$$

$$f(r) = 1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}$$

Slow Acceleration $A\ell < 1$ No horizon

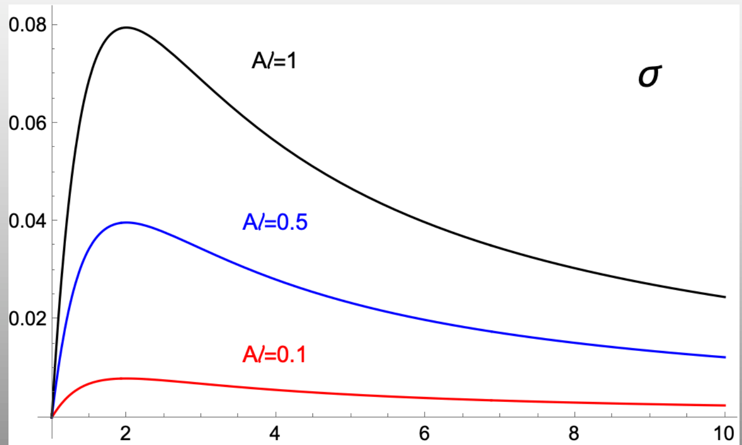
Rapid Acceleration $A\ell > 1$ Acc. horizon

Slow Acceleration

The presence of K now indicates both a conical deficit (the particle) and a *domain wall* at $\phi = \pm\pi$, i.e. codimension 1 defect. The conical deficit at r=0 has a natural mass:

$$m_c = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

Because of the nonzero extrinsic curvature along $\phi = \pm\pi$, (thanks to A) there is a wall of tension



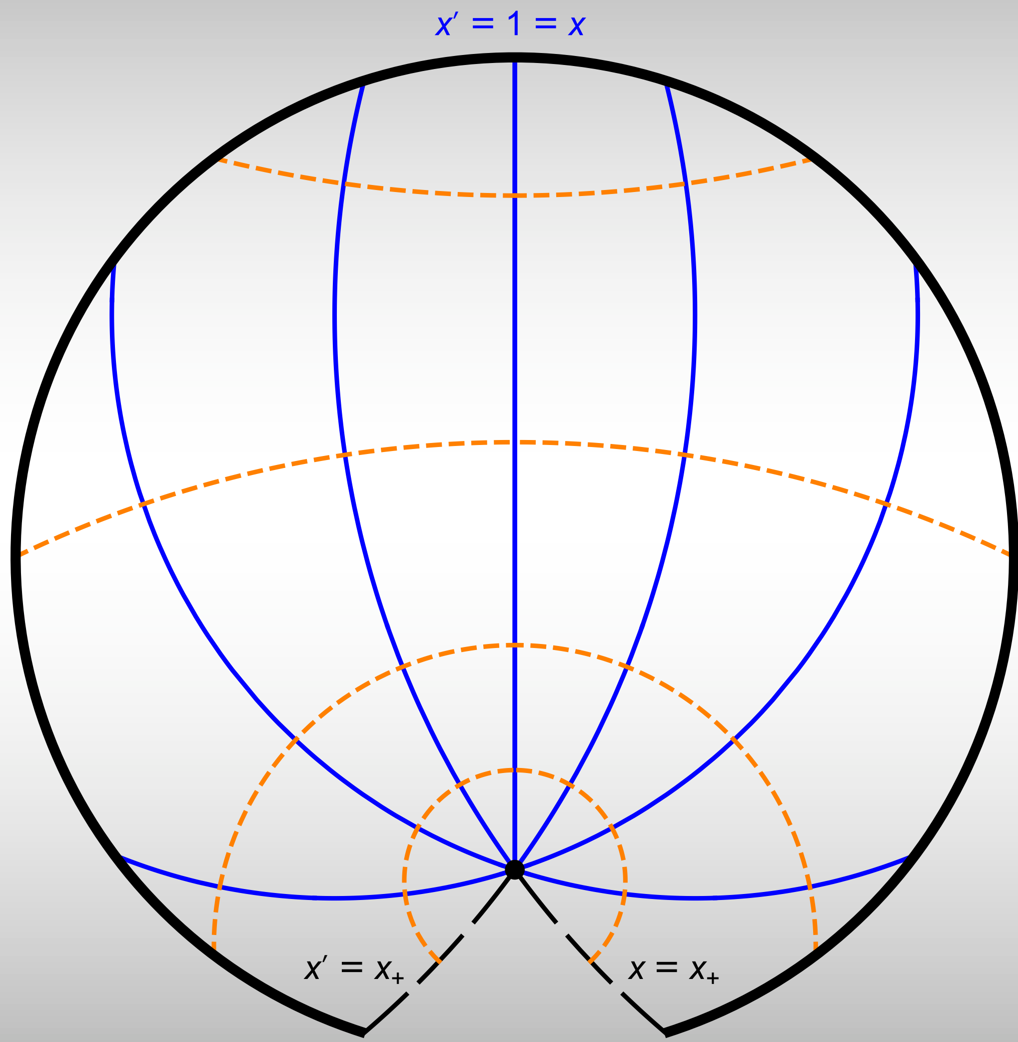
$$\sigma = \frac{A}{4\pi}\sin\left(\frac{\pi}{K}\right)$$

Slow Acceleration

We can do the same coord transformation as in the 4D slow acceleration case, to get the same sort of picture:

$$R_0 = \frac{A\ell^2}{\alpha}$$

A determines the displacement from origin, and x_+ both the particle "mass" and wall tension.



Particle Mass?

Can follow the same Fefferman-Graham prescription as for 4D, giving the expected boundary metric:

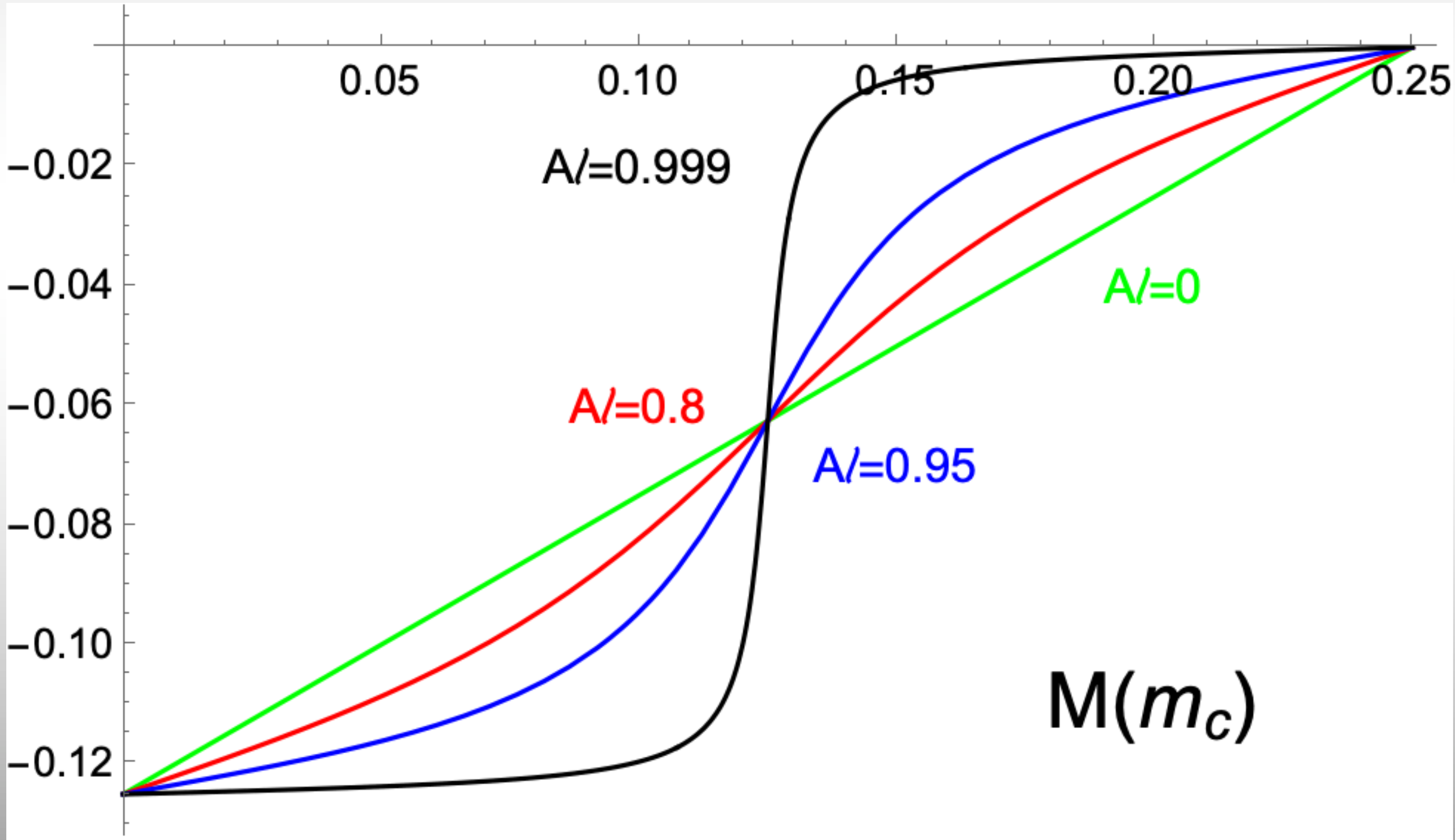
$$\gamma_0 = \frac{\omega(\xi)^2}{A^2}\left[d\tau^2 - A^2\ell^2\frac{d\xi^2}{1-\xi^2}\right]$$

And, integrating the holographic energy, get the mass:

$$M = -\frac{1}{8\pi}\left(\frac{\pi}{2} - \arctan\left[\frac{\cot\left(\frac{\pi}{K}\right)}{\sqrt{1-A^2\ell^2}}\right]\right)$$

Holographic Mass

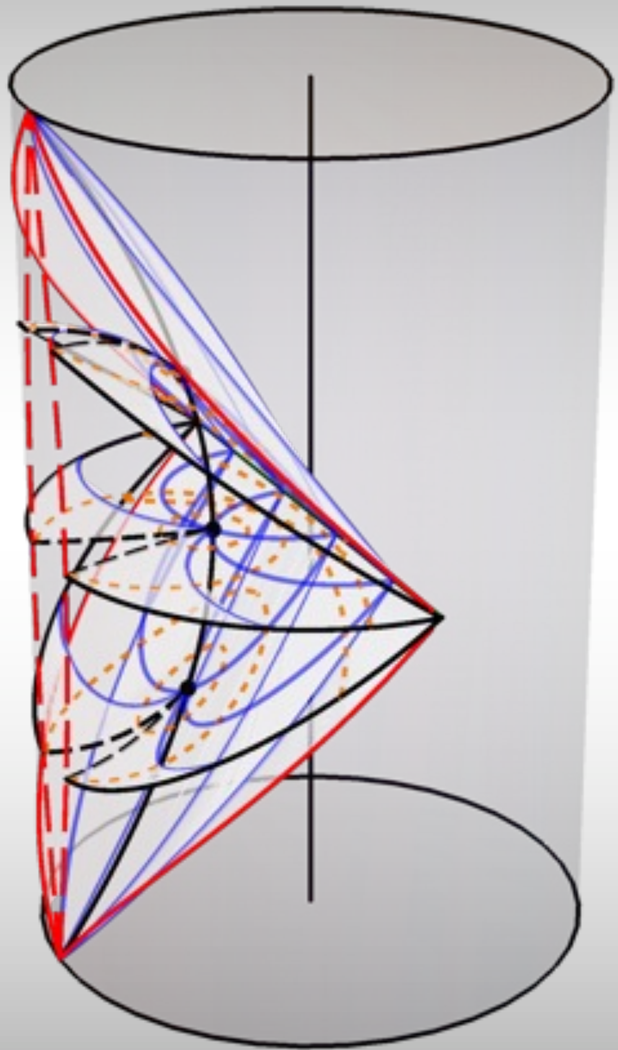
Compare to "particle" mass from conical deficit:

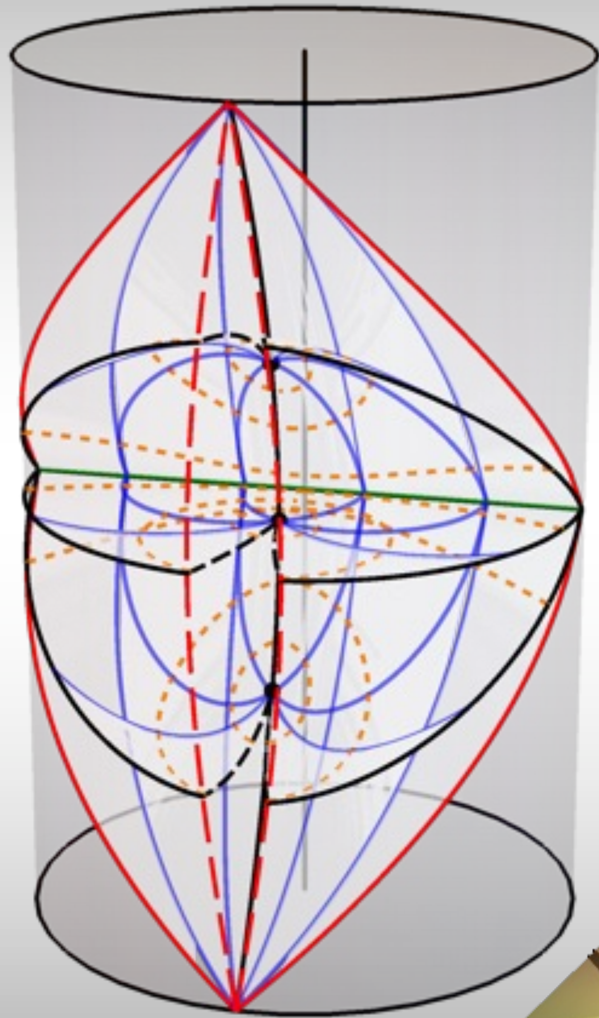


New Solutions?

Although these have been derived as, and look like, “new” solutions, we know that in 3D, gravity does not propagate, so any ”vacuum” solution has to be locally equivalent to AdS. The transformation formulae for the various solutions are quite lengthy, but give an interesting alternative viewpoint, and help with understanding the “BTZ” family of solutions.

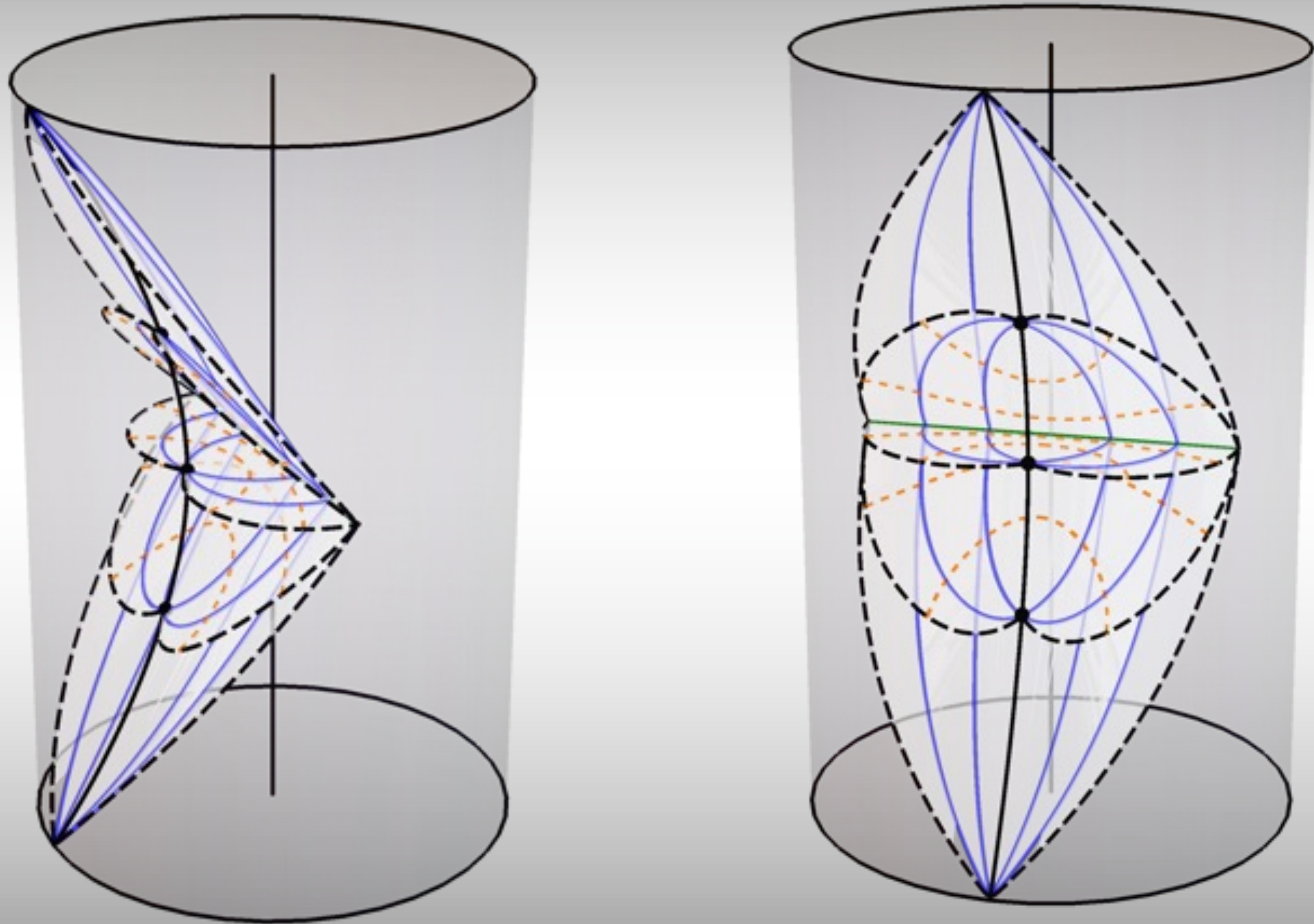
Rapidly accelerating light Particle



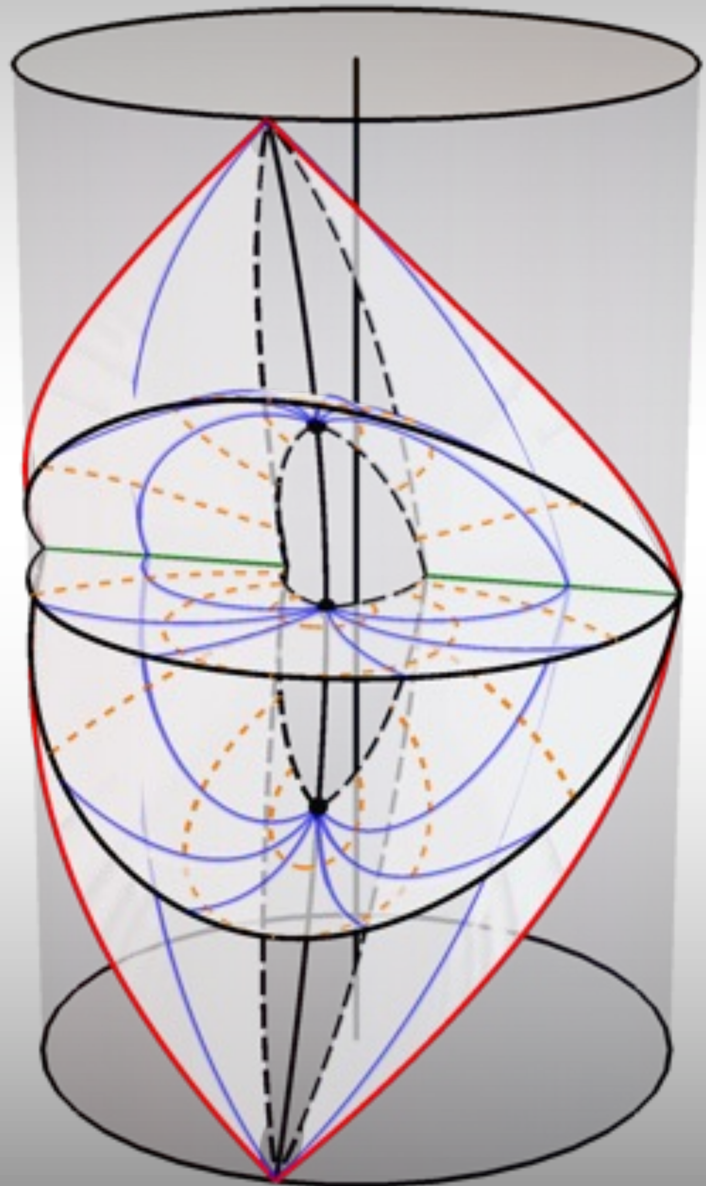


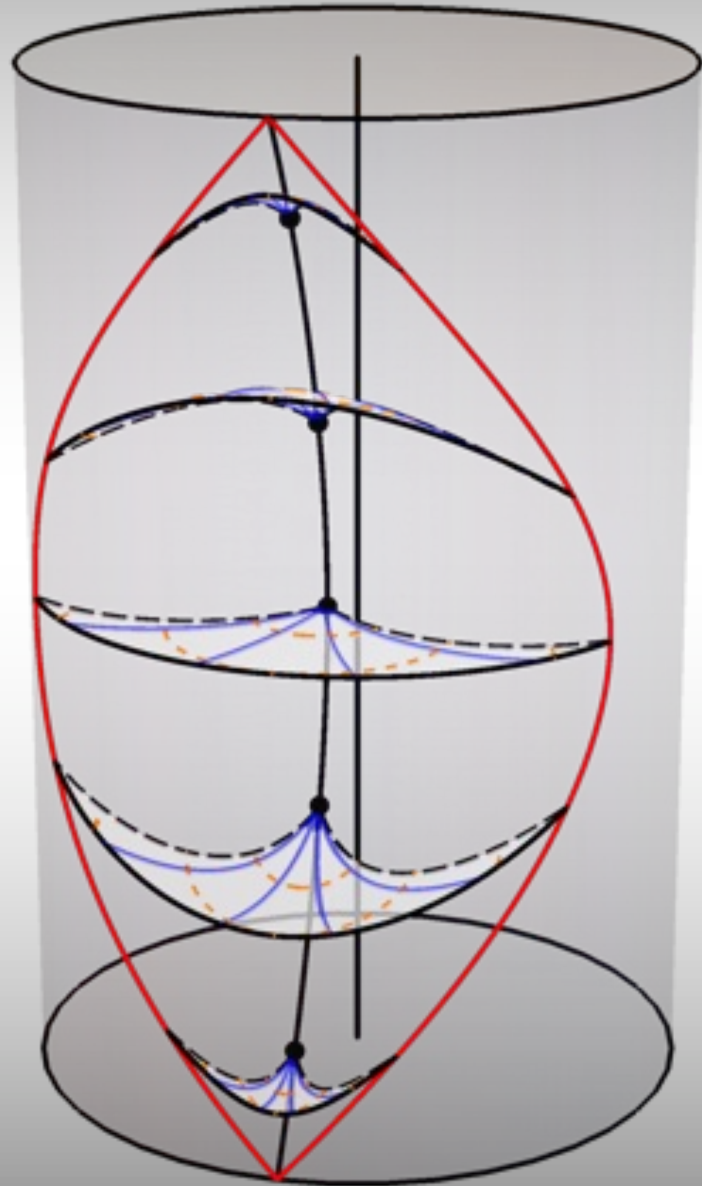
Main difference to 4D: no accelerating partner needed!

Rapidly accelerating heavy Particle



Acceleration with Struts

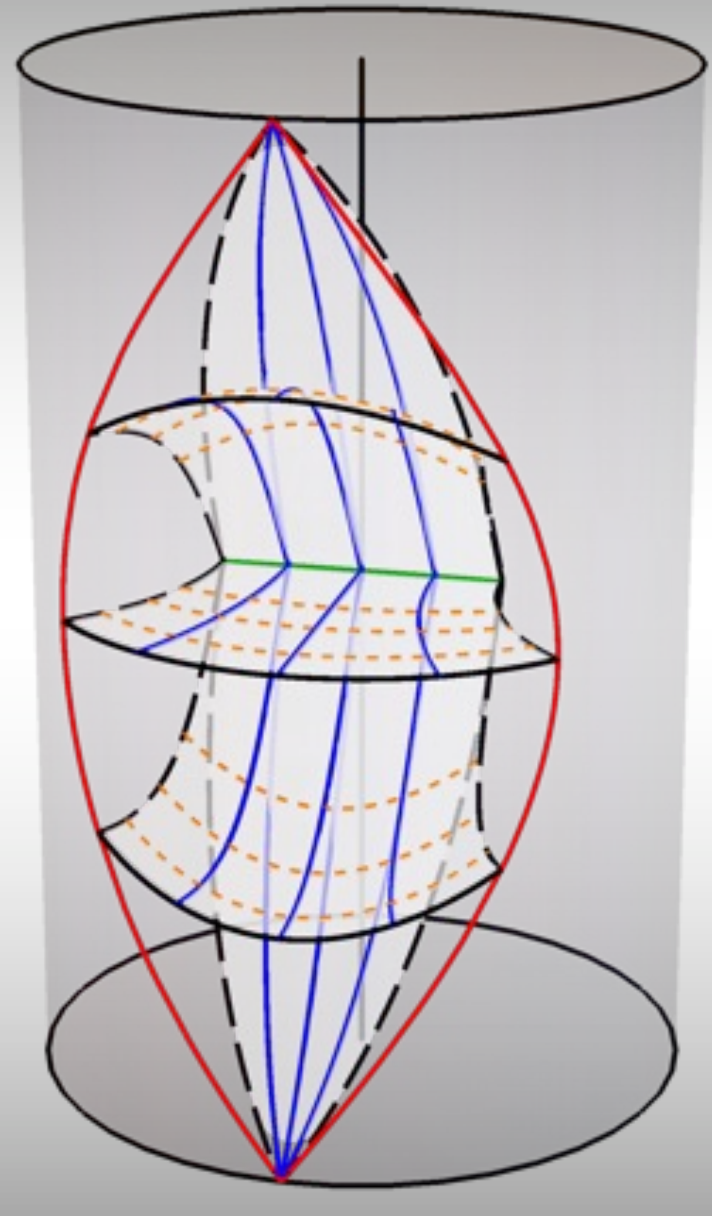




BTZ

Recall the BTZ black hole is an identification of the Rindler wedge:

Blue lines are constant ϕ, and have zero extrinsic curvature, so can cut and paste along ϕ–lines to form the BTZ black hole.



BTZ's as class II

Looking at BTZ from the exact solution perspective:

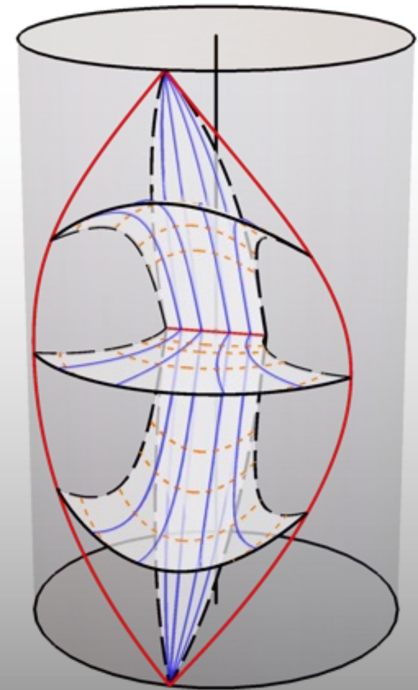
$$ds^2 = \frac{1}{\Omega(r,\psi)^2}\left[F(r)\frac{d\tilde{t}^2}{\alpha^2} - \frac{dr^2}{F(r)} - r^2 d\psi^2\right],$$

$$\begin{pmatrix} K = 1/m \\ A = m\mathcal{A} \end{pmatrix}$$

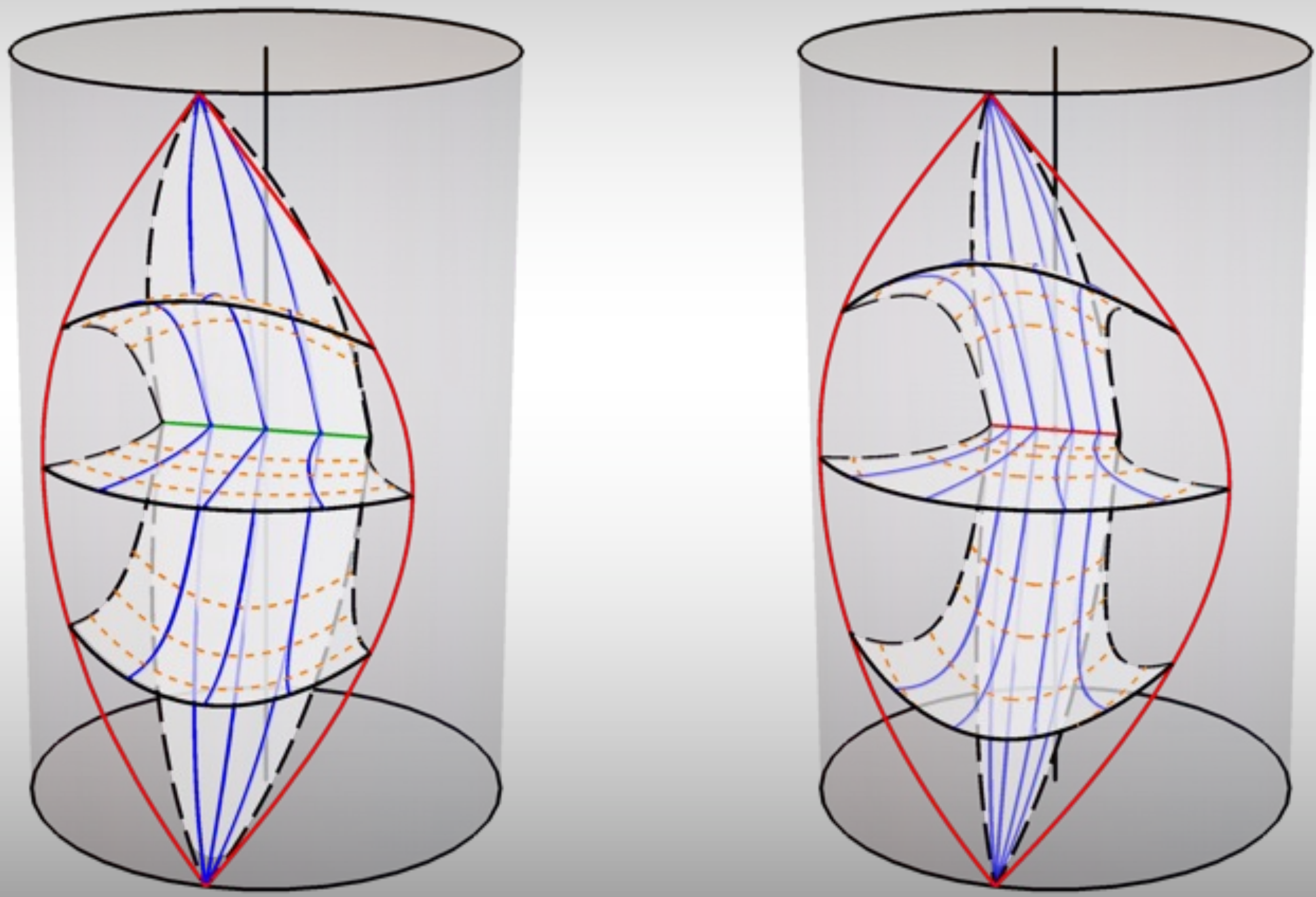
$$F(r) = -m^2(1-\mathcal{A}^2 r^2) + \frac{r^2}{\ell^2},$$

$$\Omega(r,\psi) = 1 + \mathcal{A} r \cosh(m\psi)$$

A "skews" the constant ϕ lines, changing the way AdS is sliced and adding extrinsic curvature, hence a wall and "acceleration"

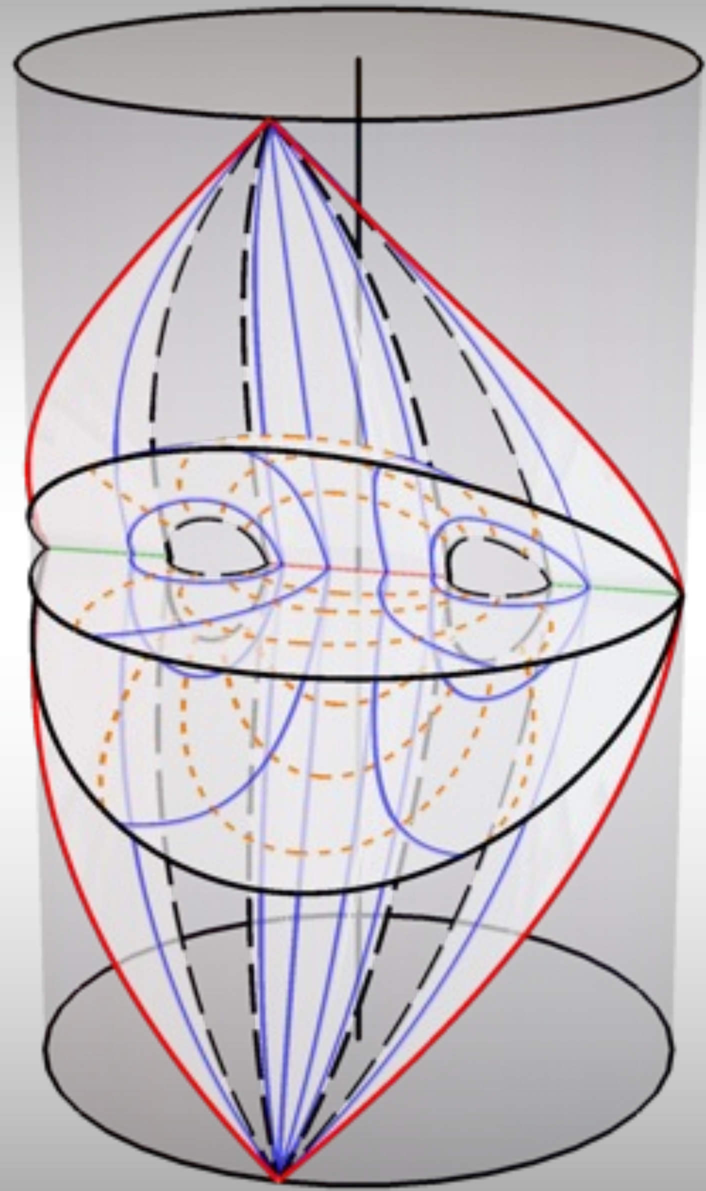


Comparing the global picture to BTZ (Left) shows this distortion (here for slow acceleration)



Rapid BTZ's

Because the distorted ϕ–lines now wrap back to the Rindler wedge horizon, for some values of ϕ we get an "additional" horizon (different portions of the bulk Rindler horizon).



Novel BTZ

Hiding within class I is a new BTZ-like solution. If Al>1, have a horizon at

$$y_h^2 = 1 - \frac{1}{A^2\ell^2}$$

For the accelerating particle, we usually take $y<-y_h$ with y ~ -1/Ar, but can also have $y \in (y_h, x) \qquad x \in (x_+, 1)$

To make this look more familiar, take

$$\tau = \frac{At}{\alpha}, \qquad y = \frac{1}{A\rho}, \qquad x = \cos(\phi/K)$$

where

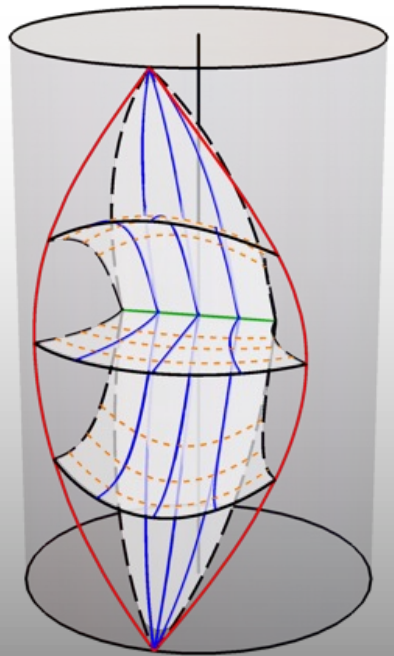
$$K = \pi/\arccos(x_+) > \pi/\arccos(y_h) > 2$$

So that this solution is clearly disconnected from the non-accelerating solutions.

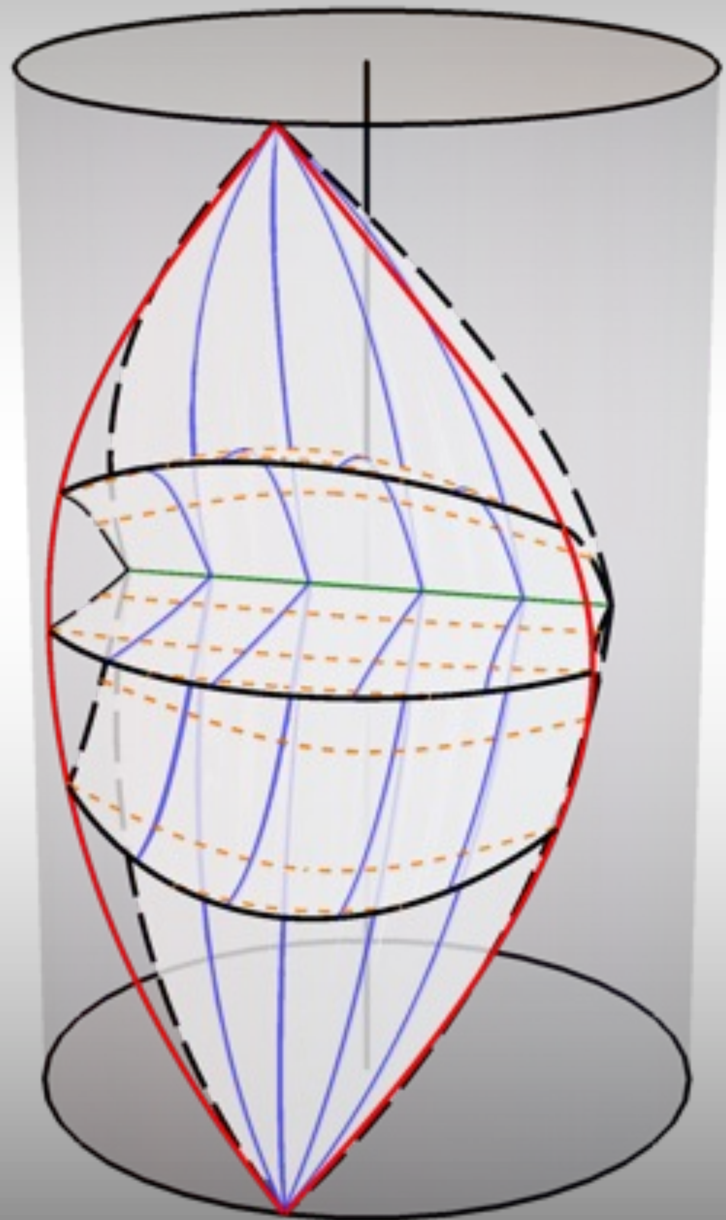
$$P(y) = \frac{1}{A^2\ell^2} - 1 + y^2$$

$$ds^2 = \frac{1}{\left[A\rho\cos\left(\frac{\phi}{K}\right) - 1\right]^2}\left(f(\rho)\frac{dt^2}{\alpha^2} - \frac{d\rho^2}{f(\rho)} - \rho^2\frac{d\phi^2}{K^2}\right)$$

$$f(\rho) = 1 - (A^2\ell^2 - 1)\rho^2/\ell^2$$

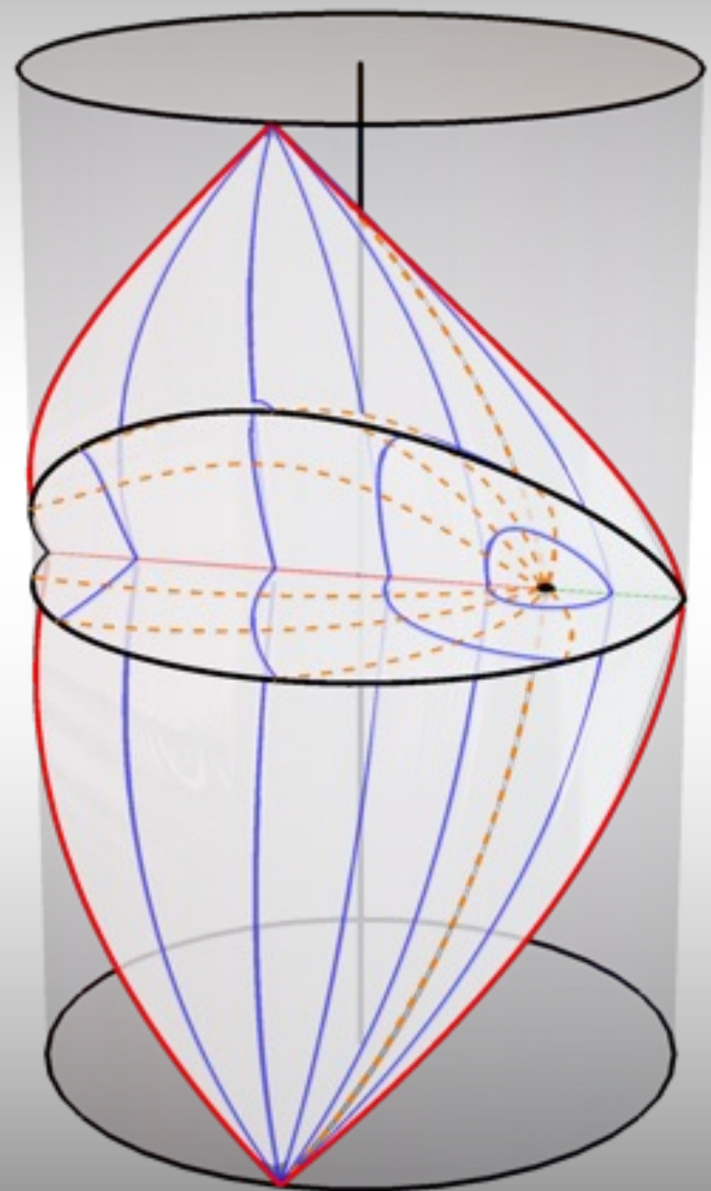


Plotting this solution in global coordinates shows a clear parallel with BTZ. This time however, there is no continuous link to the BTZ metric.

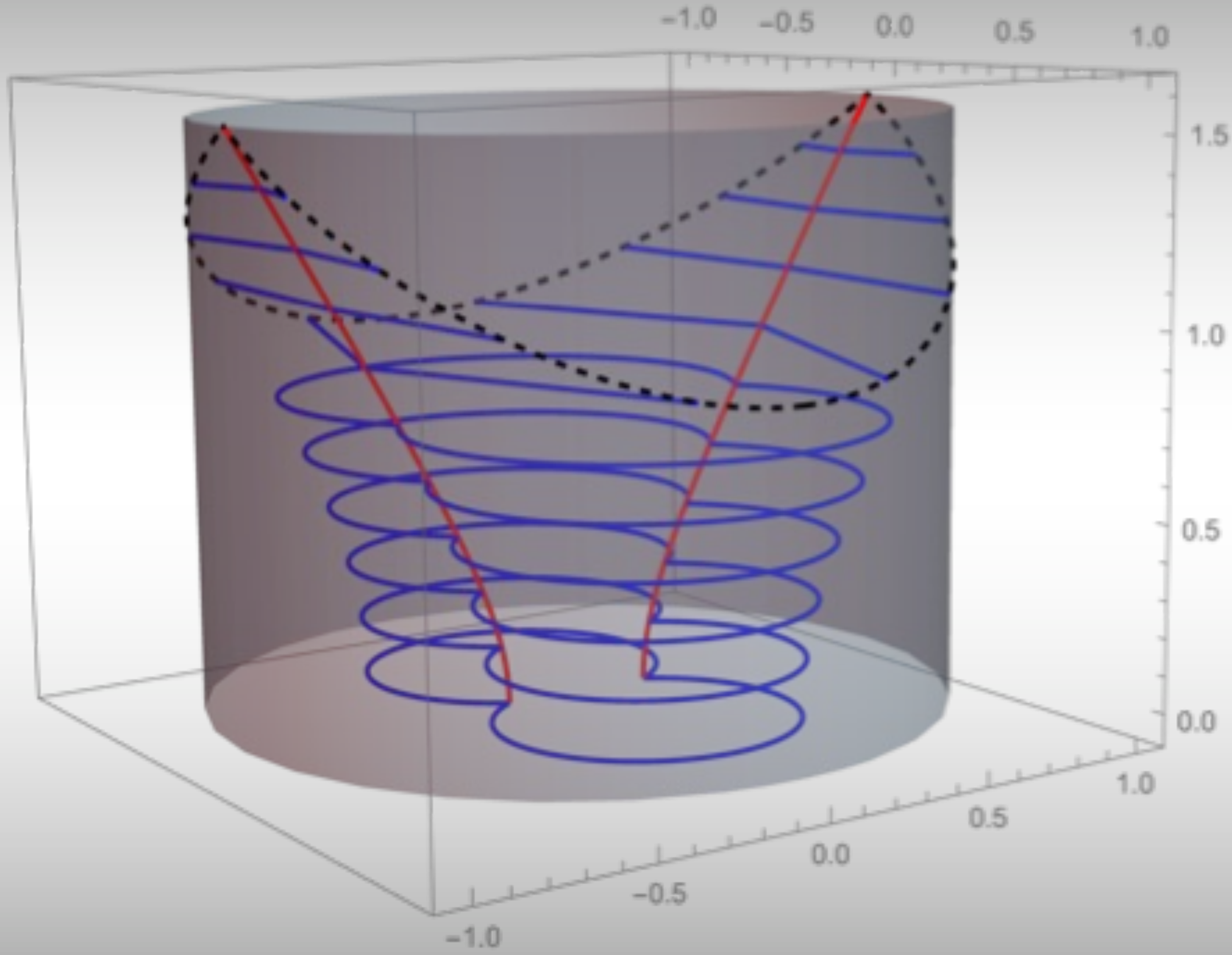


Class III - braneworld

Finally, the class 3 solutions don’t allow for an identification of a single bulk with a wall, instead we take 2 bulk copies a la Randall-Sundrum to form a braneworld.



Rapidly accelerating heavy particle – full bulk.



Summary

- Have shown how to derive black hole thermodynamics with conical deficits, including acceleration.
- Conjugate variable for tension is *Thermodynamic Length*
- Classified all possible 3D AdS solutions with “acceleration”.
- Class I are (mostly) accelerating “particles”; Class II are generalized accelerating BTZ; Class III are braneworld-type solutions.
- Have found a new solution (in Class I) that is BTZ in nature but disconnected parametrically from BTZ.