Testing g-2 with neutrino cooling in White Dwarfs

Based on [arXiv: 2405.00094] in collaboration with **Jaime Hoefken Zink**

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WHERE TO LOOK FOR BSM

- Many UV theories predict heavy new states with sizeable couplings (e.g. SUSY, GUTs, String Models, …)

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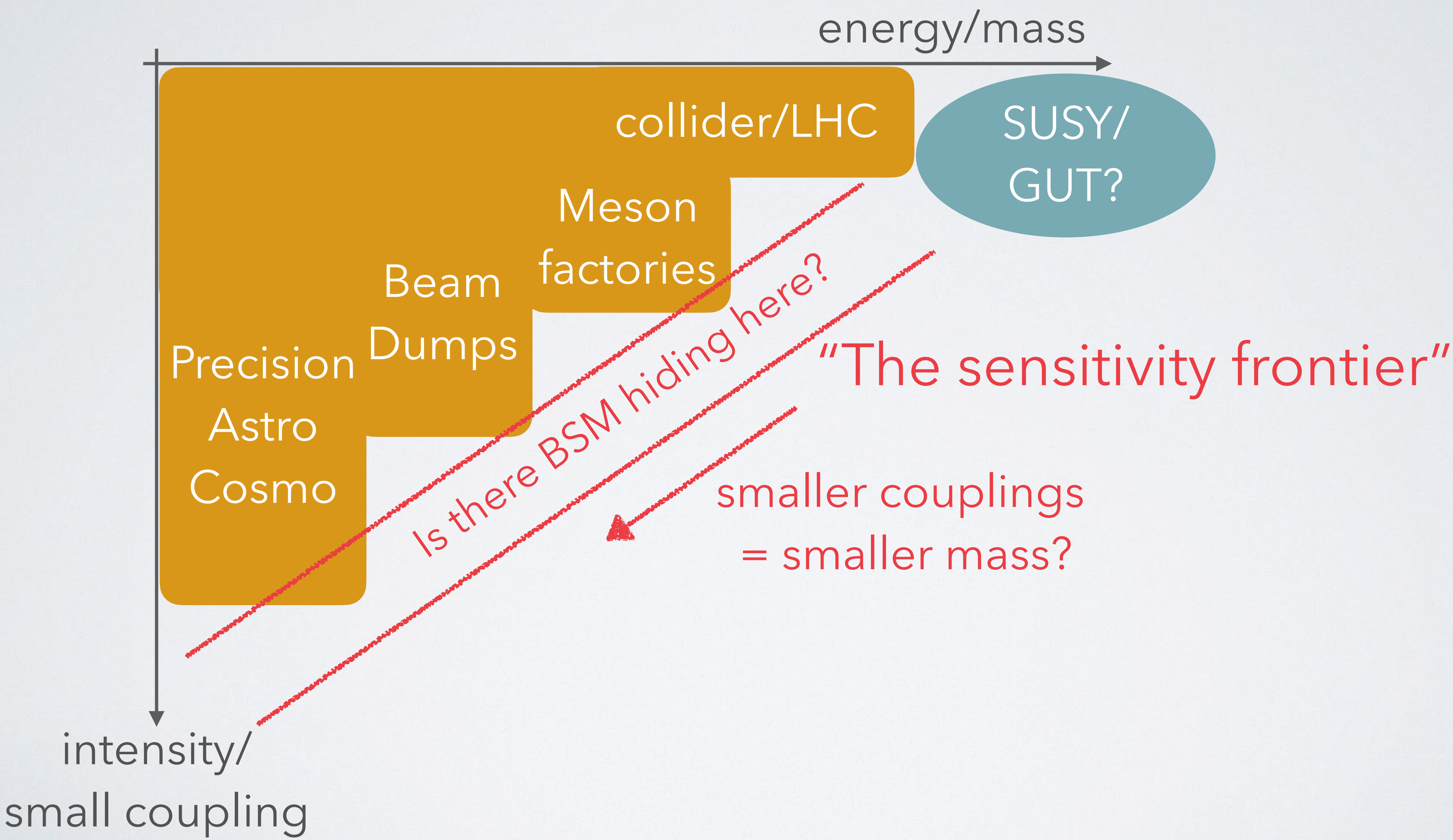

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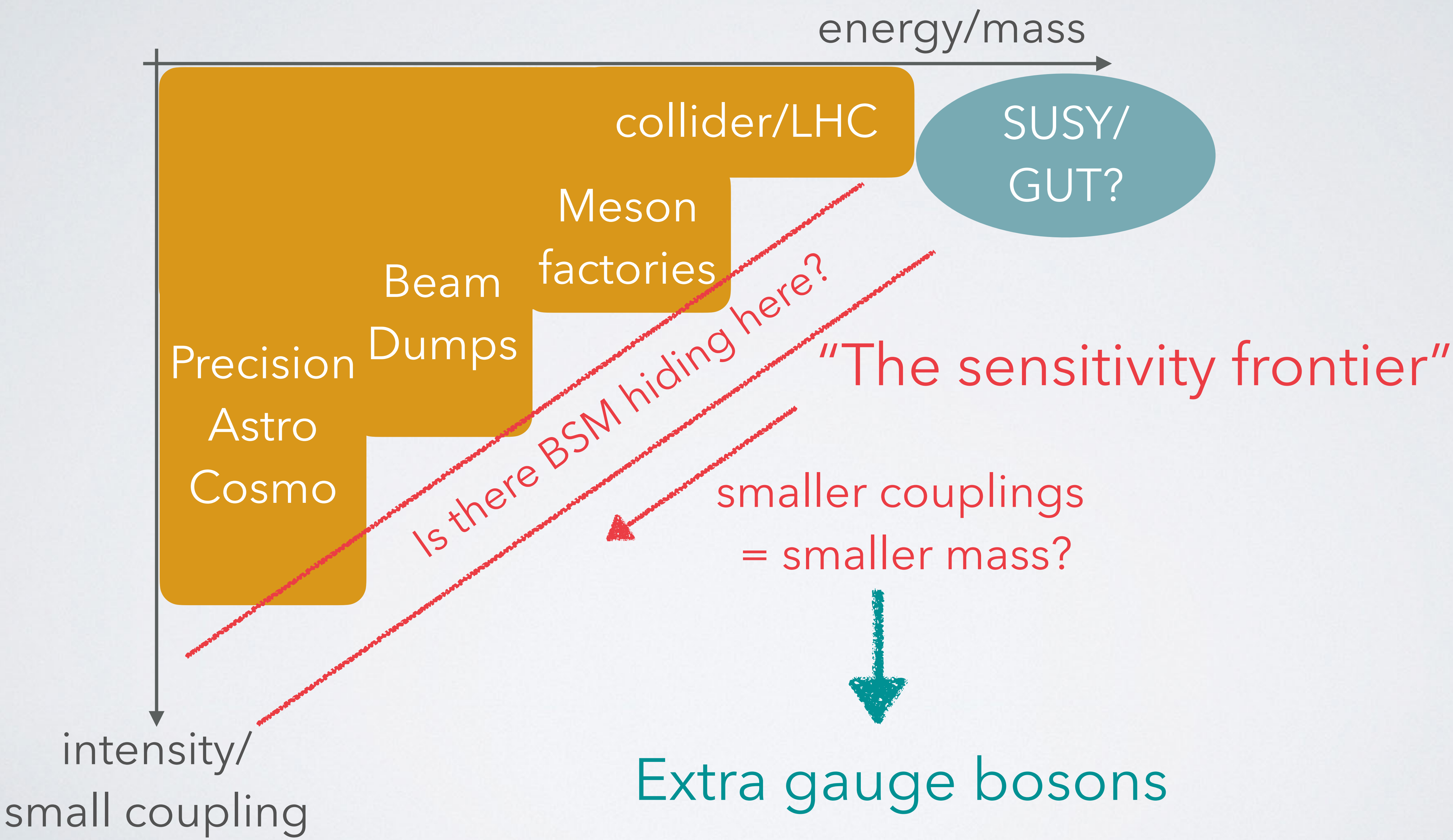


Extra gauge bosons

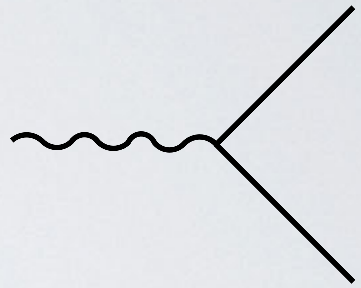
DARK PHOTONS

$$\mathcal{L} \supset -\frac{\epsilon_A}{2} F_{\mu\nu} X^{\mu\nu}$$

[Okun '82; Holdom '86]

- For light mediators $M_X \ll M_Z$ kinetic terms can be diagonalised by simple field redefinition:

$$A^\mu \to A^\mu - \epsilon_A X^\mu \quad \rightarrow \quad e A_\mu J^\mu_{\rm EM} - \epsilon_A \, e X_\mu J^\mu_{\rm EM}$$



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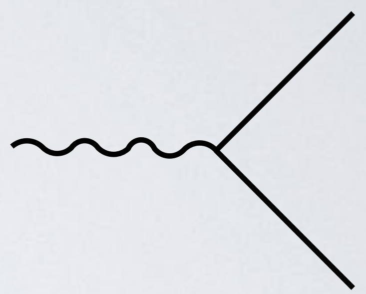

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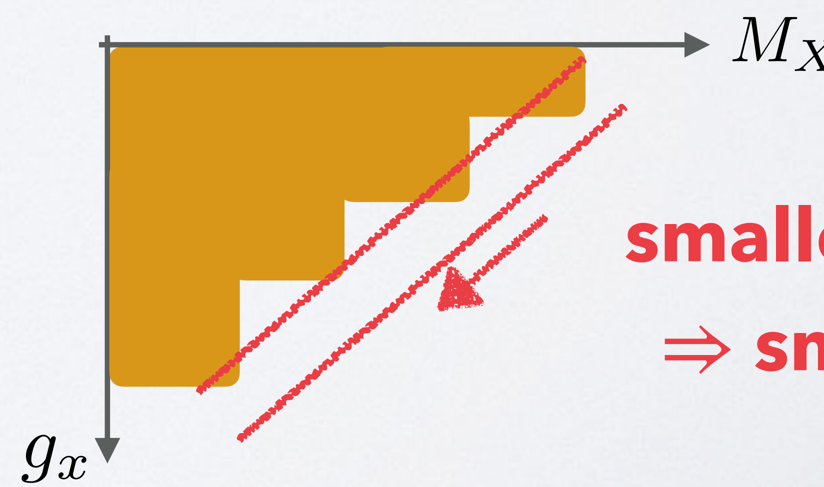
- If $U(1)_X$ is broken by VEV f of scalar, mass is related to coupling:

$$\mathcal{L} = (D_\mu S)^\dagger D^\mu S \supset g_x^2 f^2 \, X_\mu X^\mu$$

$$\Rightarrow \boxed{M_X = g_x f}$$



smaller couplings
⇒ smaller mass!

BEYOND THE MINIMAL

- SM fields can be charged under new $U(1)_X$

$$\mathcal{L}_{\rm int} = -g_x \, J^\mu_X X_\mu \qquad J^\mu_X = \sum_\psi \bar\psi \, Q_\psi \, \gamma^\mu \psi \qquad \psi = Q, L, u, d, \ell, \nu$$

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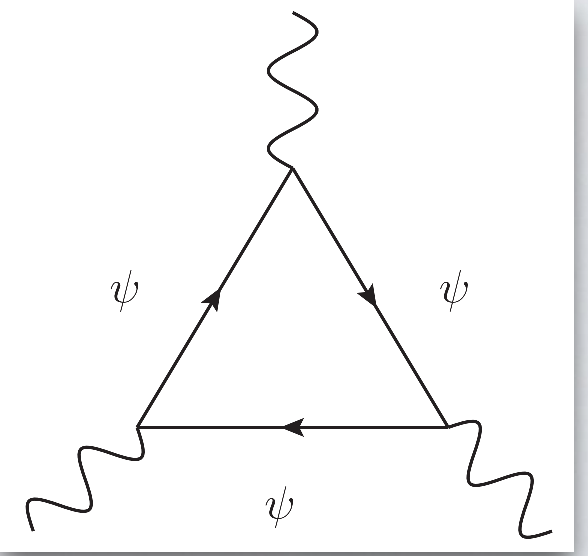
- SM Lagrangian has accidental global symmetries $U(1)_B, U(1)_{L_e}, U(1)_{L_\mu}, U(1)_{L_\tau}$.

- Four independent anomaly-free combinations:

$B-L$	$L_\mu - L_e$	$L_e - L_\tau$	$L_\mu - L_\tau$
charging quarks & leptons	charging 1st & 2nd generation leptons	charging 1st & 3rd generation leptons	charging 2nd & 3rd generation leptons



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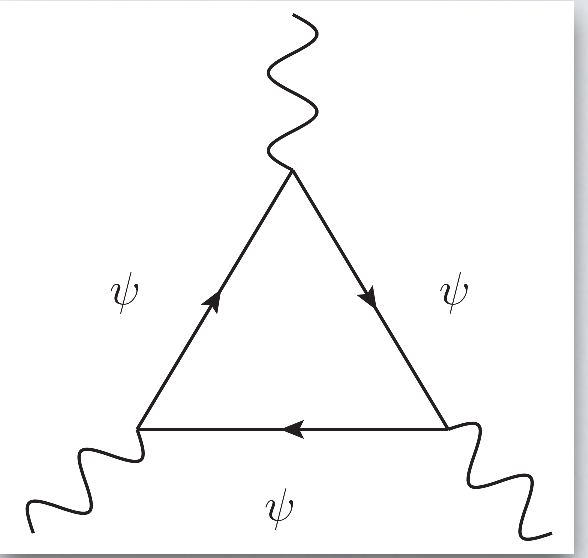
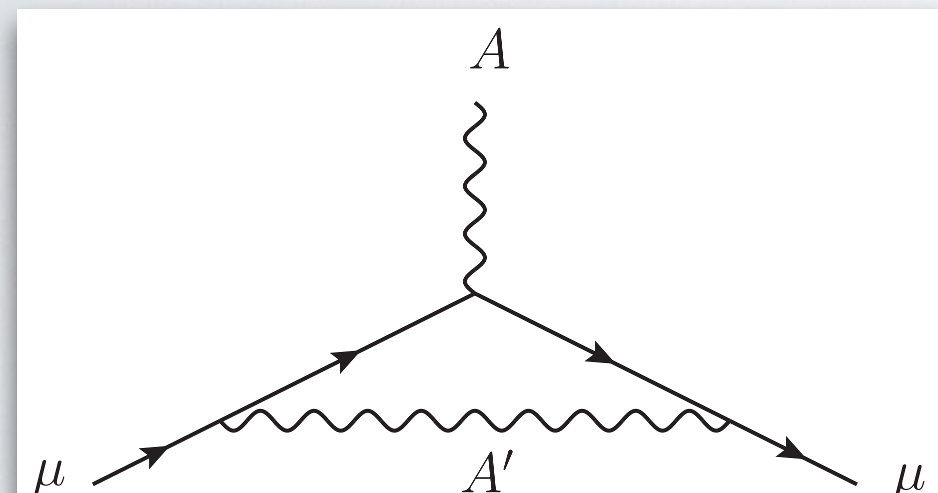


What can these do for us?

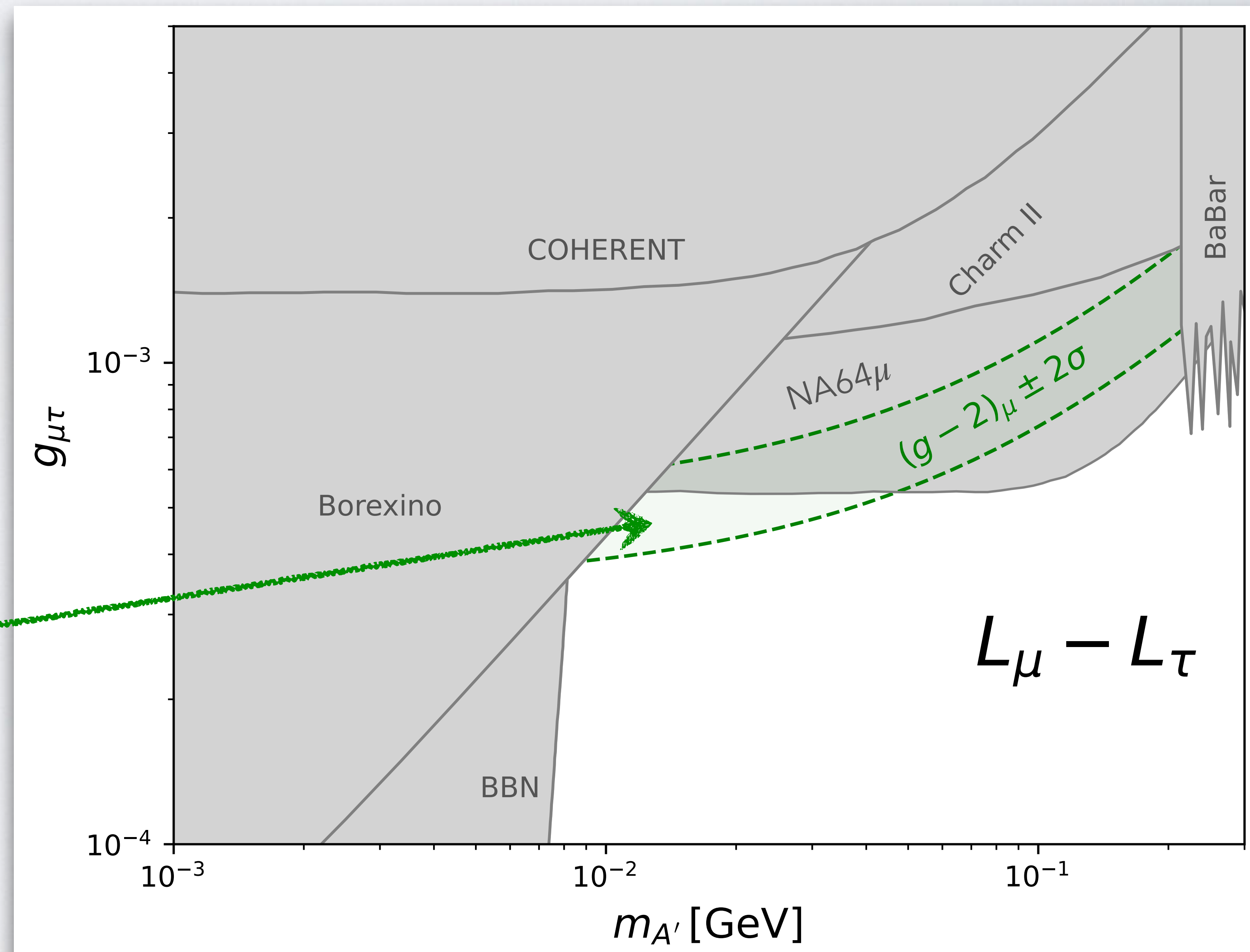
ANOMALOUS MAGNETIC MOMENT

- Muon-philic vectors contribute to $(g-2)_\mu$ at one-loop level

$$\Delta a_\mu = \frac{g_\mu^2}{4\pi^2}\int_0^1 \mathrm{d}u\ \frac{u^2(1-u)}{u^2+\frac{(1-u)}{x_\mu^2}}\,, \qquad \text{where} \quad x_\mu = m_\mu/M_{A'}$$



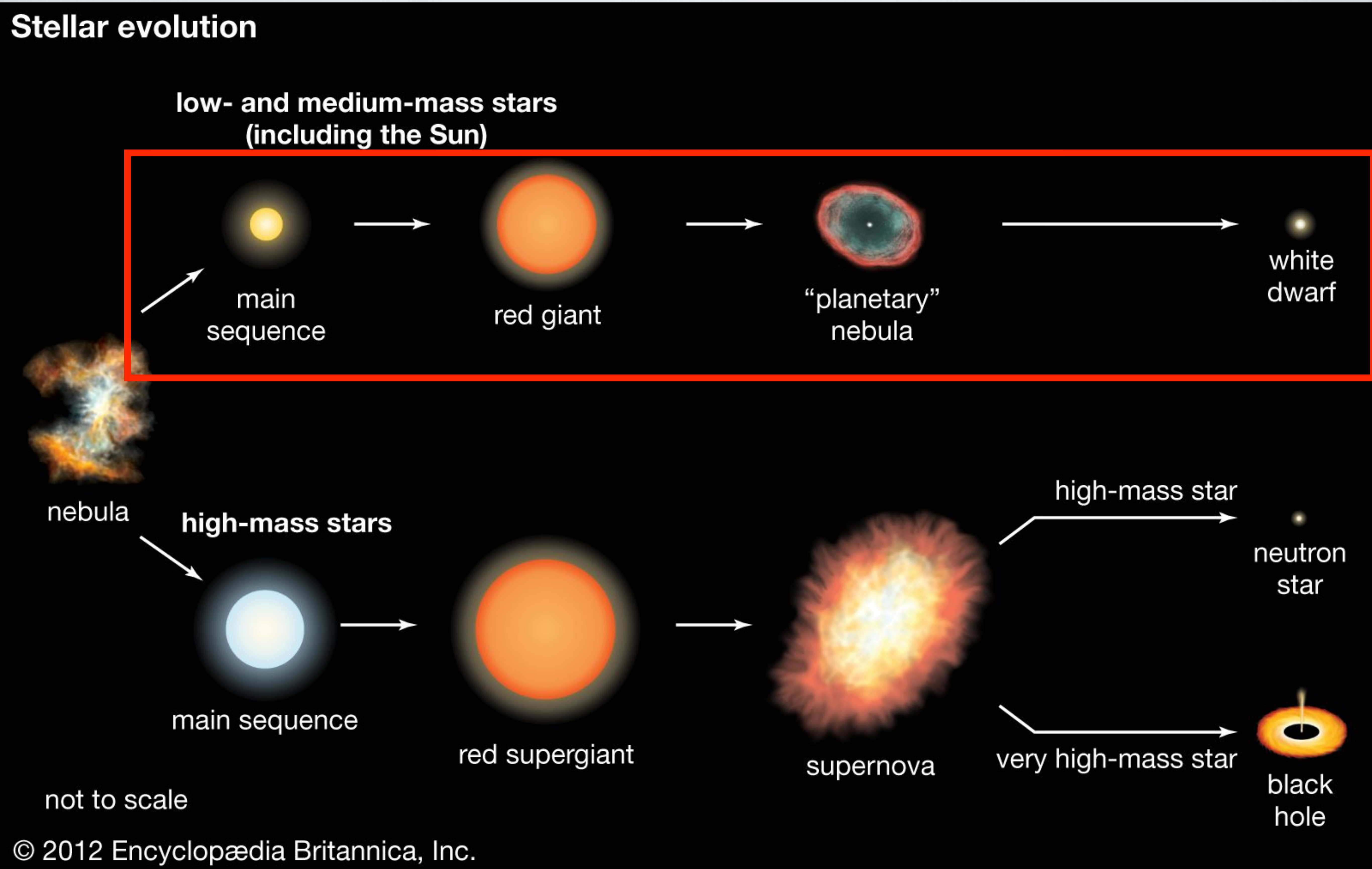
- In $U(1)_{L_\mu - L_\tau}$ this can still explain anomaly



Can we test the remaining $(g-2)_\mu$ solution?

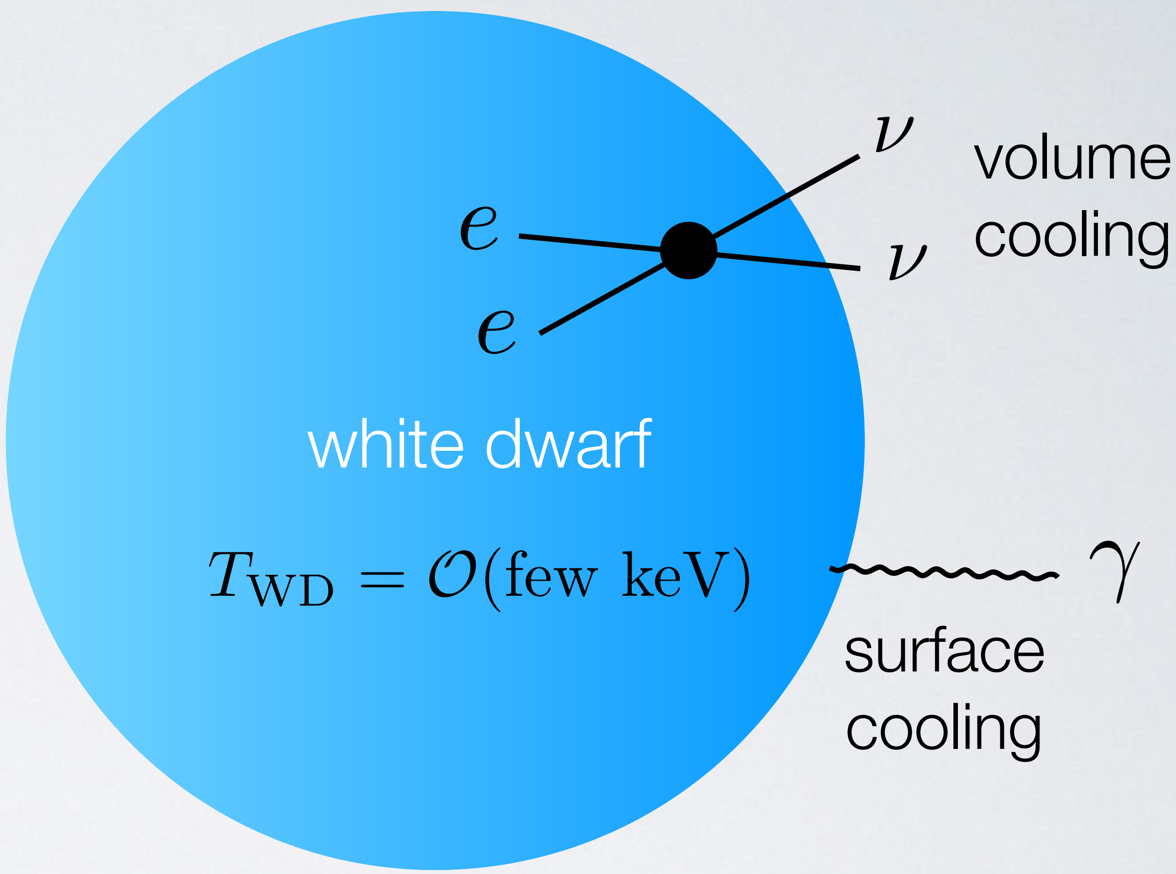
WHITE DWARF COOLING

NEXT–TO–TOPIC



WHITE DWARFS

- WDs formed after "low-mass" star has exhausted fuel
- Hot dense core of C and O
- Core **supported by electron degeneracy pressure**
  Mass of the sun, radius of the earth!
  → Very dense: $\sim 10^6$ kg/m^3
  (solar core $\sim 10^5$ kg/m^3)



- Star cools down over billions of years via photons and neutrinos:

$$\frac{dT_{WD}}{dt} = -\frac{L_\gamma}{4\pi R_{WD}\sigma_{SB}T_{WD}} - \frac{L_\nu}{4\pi R_{WD}\sigma_{SB}T_{WD}}$$

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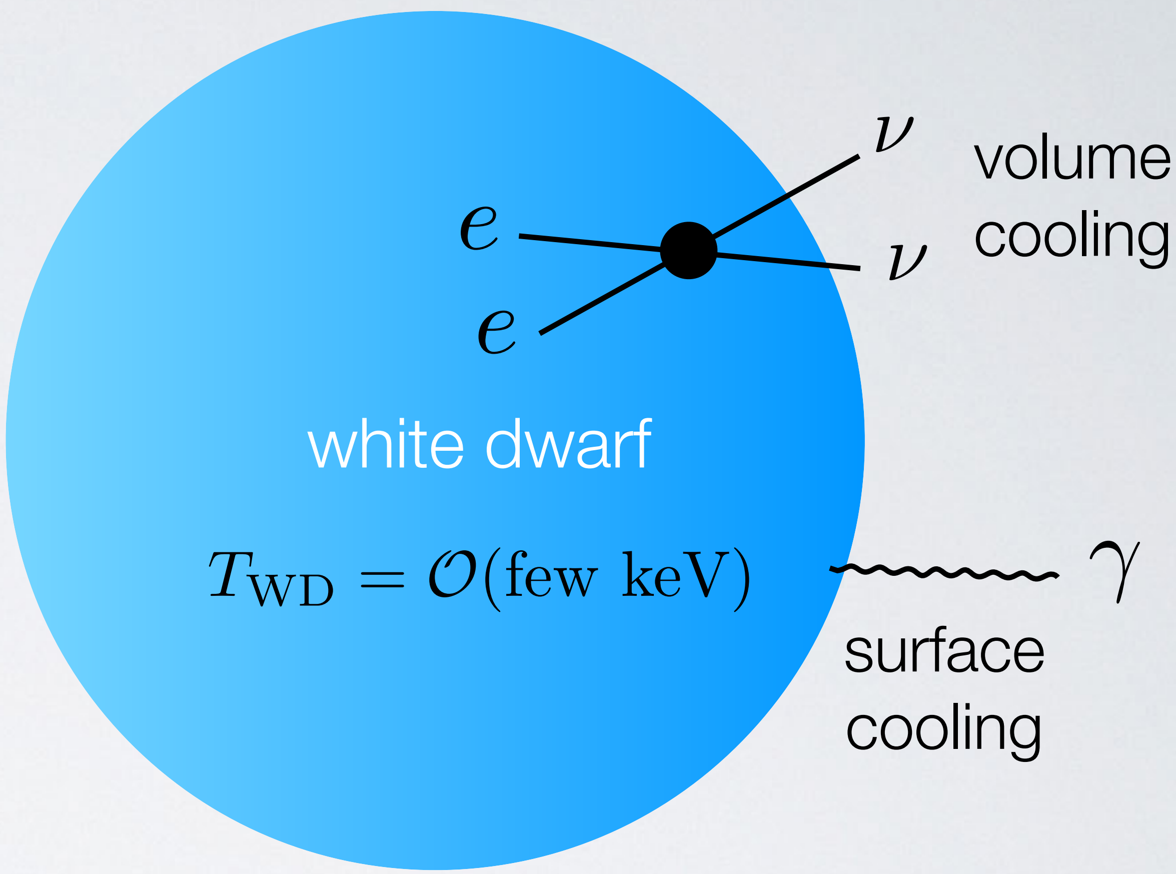

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COLD HOT

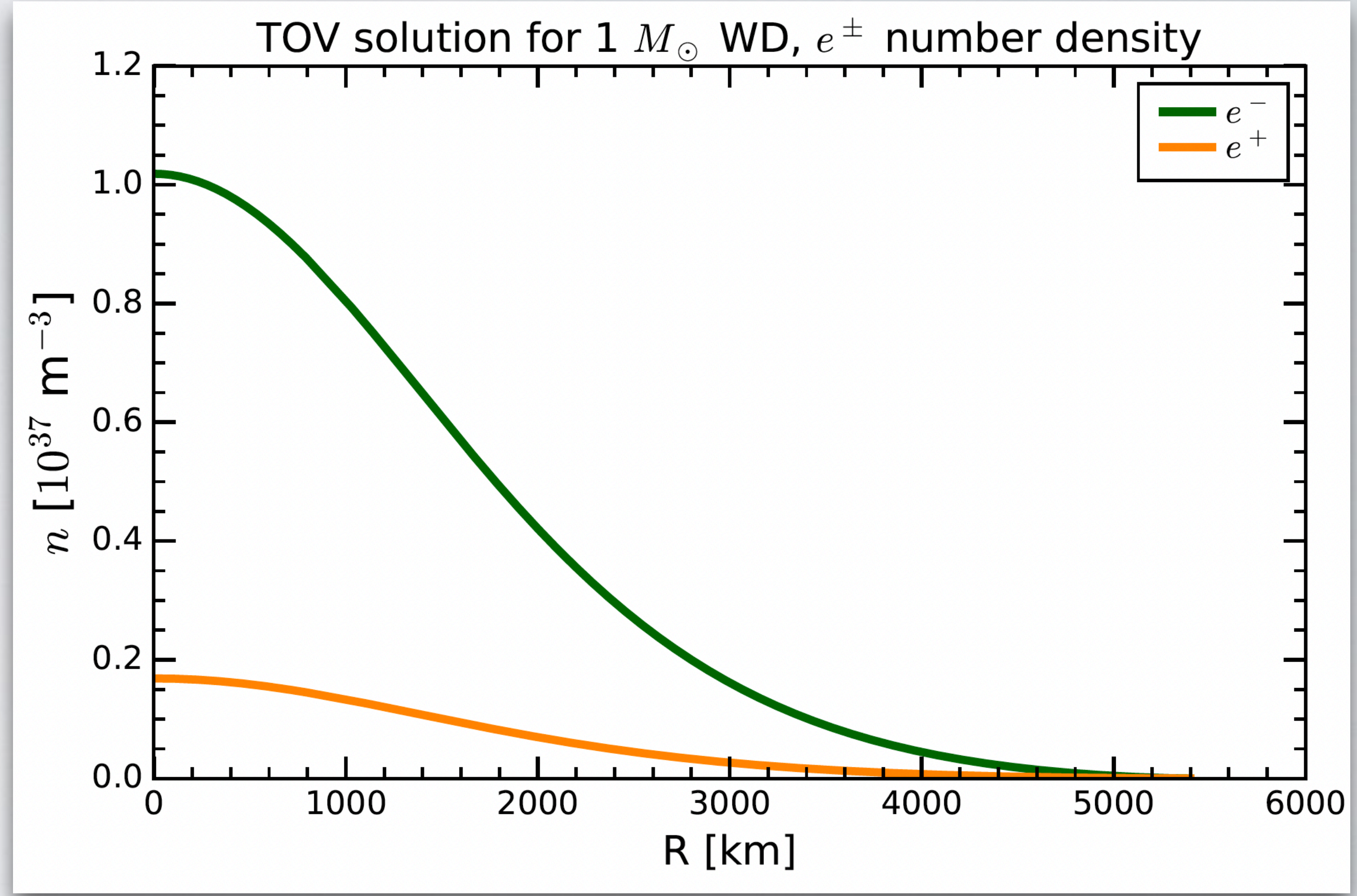
EQUATION OF STATE

- **EoS of White Dwarfs well-known!**

- Salpeter EoS:
  degenerate ideal gas
  + corrections (non-uniformity, Coulomb potential, …)

  [Salpeter; *Astrophys. J. 134, 669 (1961)*]

- **Tolman-Oppenheimer-Volkoff (TOV) equations:** solving the Einstein field equations in Schwarzschild metric with fluid



Extract density profiles

$$\frac{dp(r)}{dr} = -G\frac{\epsilon(r) + p(r)}{r(r - 2G\,m(r))}\left[m(r) + 4\pi\,p(r)\,r^3\right]$$

$$\frac{dm(r)}{dr} = 4\pi\epsilon(r)\,r^2$$

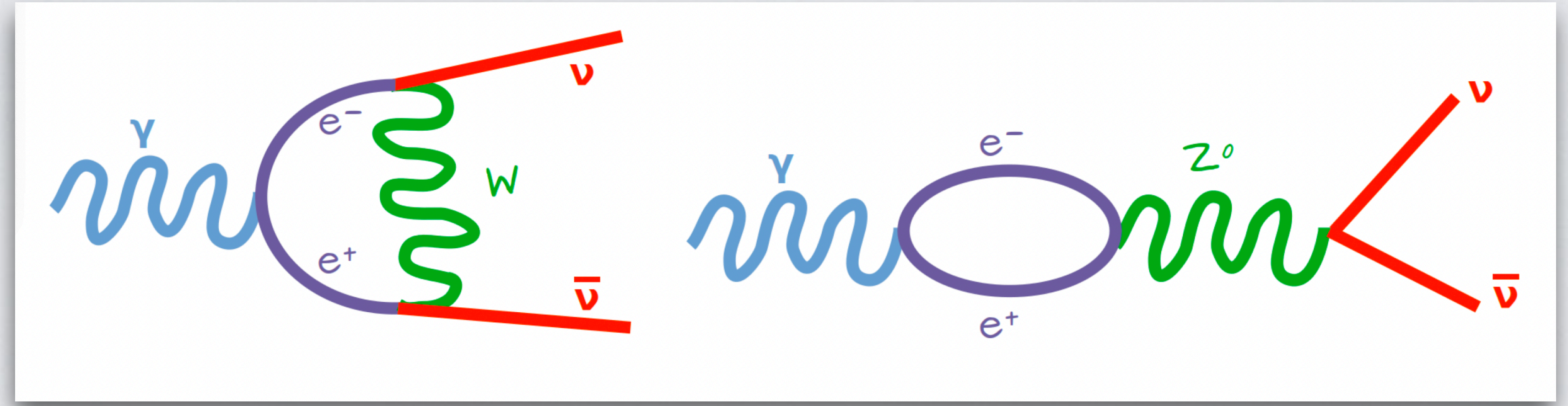
[Tolman, *Phys. Rev., 55, 364 (1939)*]

[Oppenheimer & Volkoff, *Phys. Rev., 55, 374 (1939)*]

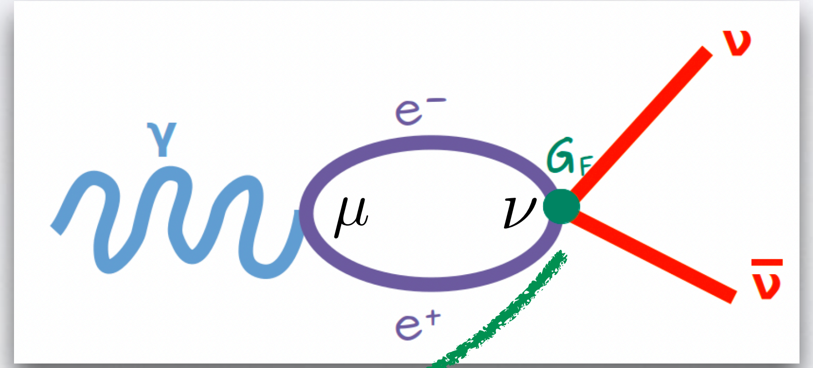
[Mathew & Nandy, *Res. Astron. Astrophys.* **17** 061]

COOLING: PLASMON DECAY

- Early WD cooling via ``on-shell" photon decay in plasma into neutrinos



- Since in WDs the typical $q^2 \ll M_W^2, M_Z^2$ we can compute this as



$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\Gamma_\lambda^{\mu\nu} \, \varepsilon_\mu(\boldsymbol{q}, \lambda) \right) \, \bar{u}(p_1)\gamma_\nu(1-\gamma_5)u(p_2)$$

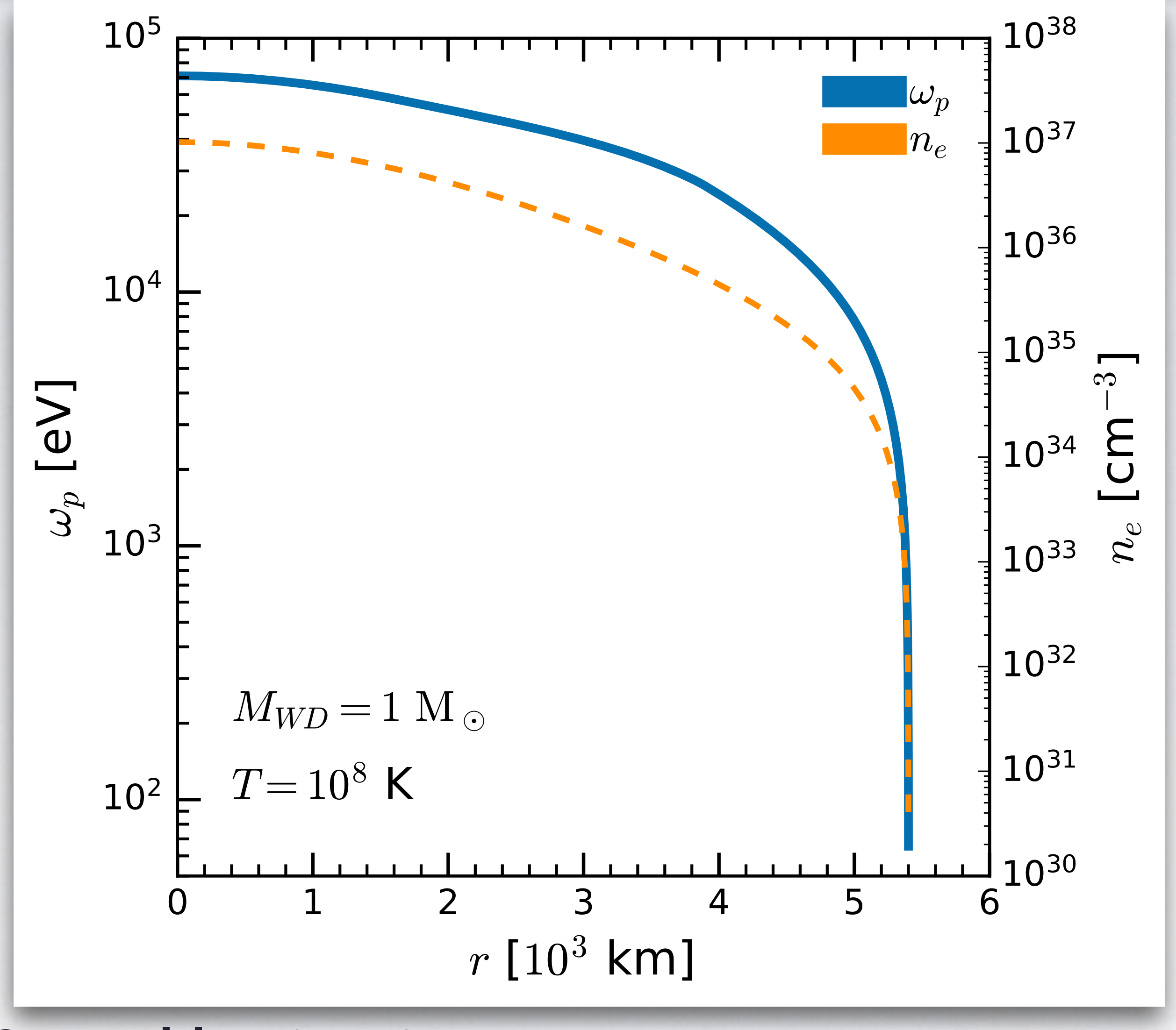
with **effective vertex** $\Gamma_\lambda^{\mu\nu}$ for each photon polarization with couplings C_V^{SM}, C_A^{SM}

[Braaten & Segel; *Phys.Rev.D* 48 (1993) 1478]

WD NEUTRINO LUMINOSITY

- **Plasmon decay width** in terms of effective vertex $\Gamma_\lambda^{\mu\nu}$ and **plasmon frequencies** $\omega_\lambda(q)$.

$$\Gamma_\lambda(q) = -\frac{G_F^2}{12\pi}\frac{\omega_\lambda(q)^2 - q^2}{\omega_\lambda(q)}\left(\Gamma_\lambda^{\alpha\mu}\varepsilon_\mu(q,\lambda)\right)\left(\Gamma^\lambda_{\alpha\rho}\varepsilon^\rho(q,\lambda)\right)^*$$



- **Neutrino emissivity & total luminosity:**

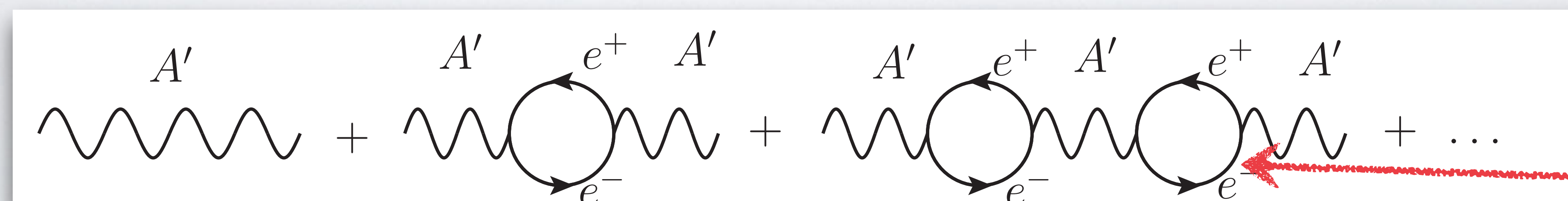
$$\mathcal{Q}_\lambda \equiv \int d^3\vec{q}\ \ \Gamma_\lambda(q)\ \omega_\lambda(q)\ n_B\left(\omega_\lambda(q), T\right) \qquad L_\nu = 4\pi\int_0^{R_{\mathrm{WD}}}\sum_\lambda \mathcal{Q}_\lambda(r) r^2 dr$$

PLASMON DECAY - DARK PHOTONS

- Leptophilic dark photons contribute

- Since **dark photon couples to plasma electrons** have to compute full thermal propagtor (Dyson sum)

$D^{\mu\nu}_{A'} =$



Thermal loops!

$$= \frac{-i\,(g^{\mu\nu} - q^\mu q^\nu / m^2_{A'})}{q^2 - m^2_{A'}} + \frac{-i\,(g^\mu_\lambda - q^\mu q_\lambda / m^2_{A'})}{q^2 - m^2_{A'}} \left(i\,\Pi^{\lambda\sigma}_{A'}\right) \frac{-i\,(g^\nu_\sigma - q_\sigma q^\nu / m^2_{A'})}{q^2 - m^2_{A'}} + \ldots$$

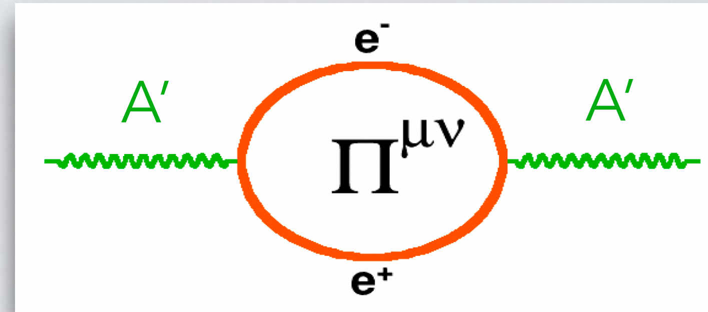
$$= \frac{-i\,g^{\mu\lambda}}{q^2 - m^2_{A'} - F_{A'}} P^\nu_{L\lambda} + \frac{-i\,g^{\mu\lambda}}{q^2 - m^2_{A'} - G_{A'}} P^\nu_{T\lambda}$$

Longitudinal & transverse components

with

$$F_{A'} = \frac{q^2}{\boldsymbol{q}^2}\,\Pi^{00}_{A'} \qquad G_{A'} = \Pi^{xx}_{A'}$$

DARK PHOTON SELF ENERGY



- Evaluate A' self-energy in **thermal background — a beast!**

$$\Pi^{\mu\nu}_{A'}(q) = -\epsilon_A^2 \int \frac{d^4k}{(2\pi)^4}\, \mathrm{tr}\left[\gamma^\mu(\not{k} + m_e)\gamma^\nu(\not{q} - \not{k} - m_e)\right]$$
$$\times \left\{ \frac{i}{k^2 - m_e^2} - 2\pi\left[\theta(-k^0) + \mathrm{sign}(k^0)\,\tilde{f}(k^0 - \mu)\right]\delta\left(k^2 - m_e^2\right) \right\}$$
$$\times \left\{ \frac{i}{(q-k)^2 - m_e^2} - 2\pi\left[\theta(-q^0 + k^0) + \mathrm{sign}(q^0 - k^0)\,\tilde{f}(q^0 - k^0 + \mu)\right]\delta\left((q-k)^2 - m_e^2\right) \right\}$$

with $\tilde{f}(x) = (1 + e^{\beta x})^{-1}$

Thermal fermion propagators

→ Finite temperature QFT

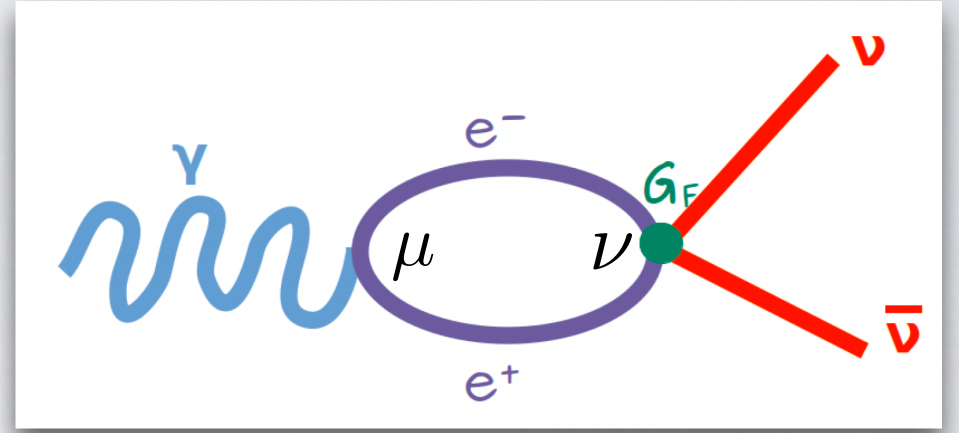
- **But, this is essentially the plasmon self-energy**: $\epsilon_A^2 \times \Pi^{\mu\nu}_\gamma$!

Identify $\boxed{F_{A'} = \frac{q^2}{\boldsymbol{q}^2}\,\Pi^{00}_{A'} = \epsilon_A^2\,\frac{q^2}{\boldsymbol{q}^2}\,\Pi^\gamma_L \qquad G_{A'} = \Pi^{xx}_{A'} = \epsilon_A^2\,\Pi^\gamma_T}$ with known results!

[Hoefken Zink, Ramirez-Quezada; *PRD* 108 (2023) 4, 043014]

PLASMON DECAY - DARK PHOTONS

- Mimic SM-like computation



but shifting the SM couplings by the A' coupling and full propagator:

$$C_{V,L}(q) \to C_V^{SM} + \frac{\sqrt{2}}{2\,G_F}\frac{e\,\epsilon_A\,g_x\,Q_{\nu_\alpha}}{q^2 - m_{A'}^2 - F_{A'}} \qquad \Pi_L^\gamma$$

$$C_{V,T}(q) \to C_V^{SM} + \frac{\sqrt{2}}{2\,G_F}\frac{e\,\epsilon_A\,g_x\,Q_{\nu_\alpha}}{q^2 - m_{A'}^2 - G_{A'}}$$

$$\Pi_T^\gamma$$

$$C_A(q) \to C_A^{SM} - \frac{\sqrt{2}}{16\,G_F}\frac{\tan^2\theta_W\;e\,\epsilon_A\,g_x\,Q_{\nu_\alpha}}{q^2 - m_{A'}^2 - G_{A'}}$$

THREE REGIMES

- **Heavy regime** $(m_{A'} \gg T, \omega_P)$

$$\frac{1}{q^2 - m_{A'}^2} \sim \frac{-1}{m_{A'}^2}$$

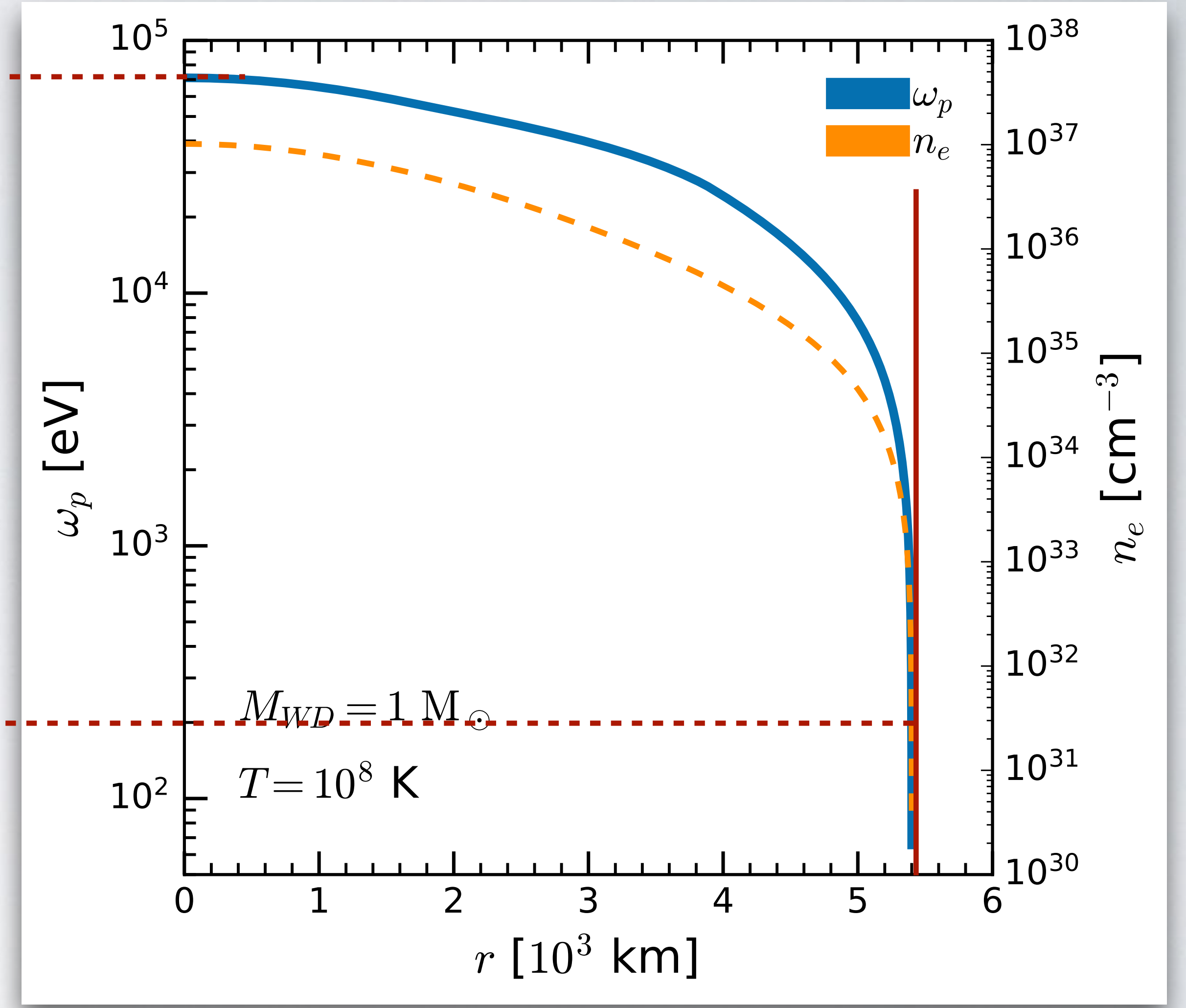
- **Ultra-light regime** $(m_{A'} \ll T, \omega_P)$:

$$\frac{1}{q^2 - m_{A'}^2} \sim \frac{1}{q^2}$$

Mass independent!

100 keV

200 eV



- **Resonant regime** $(m_{A'} \sim T, \omega_P)$:

  - dark photon goes on resonance w/ plasma frequency $\omega_P(r)$!
  - *regulate pole via* ***Breit-Wigner propagator:***

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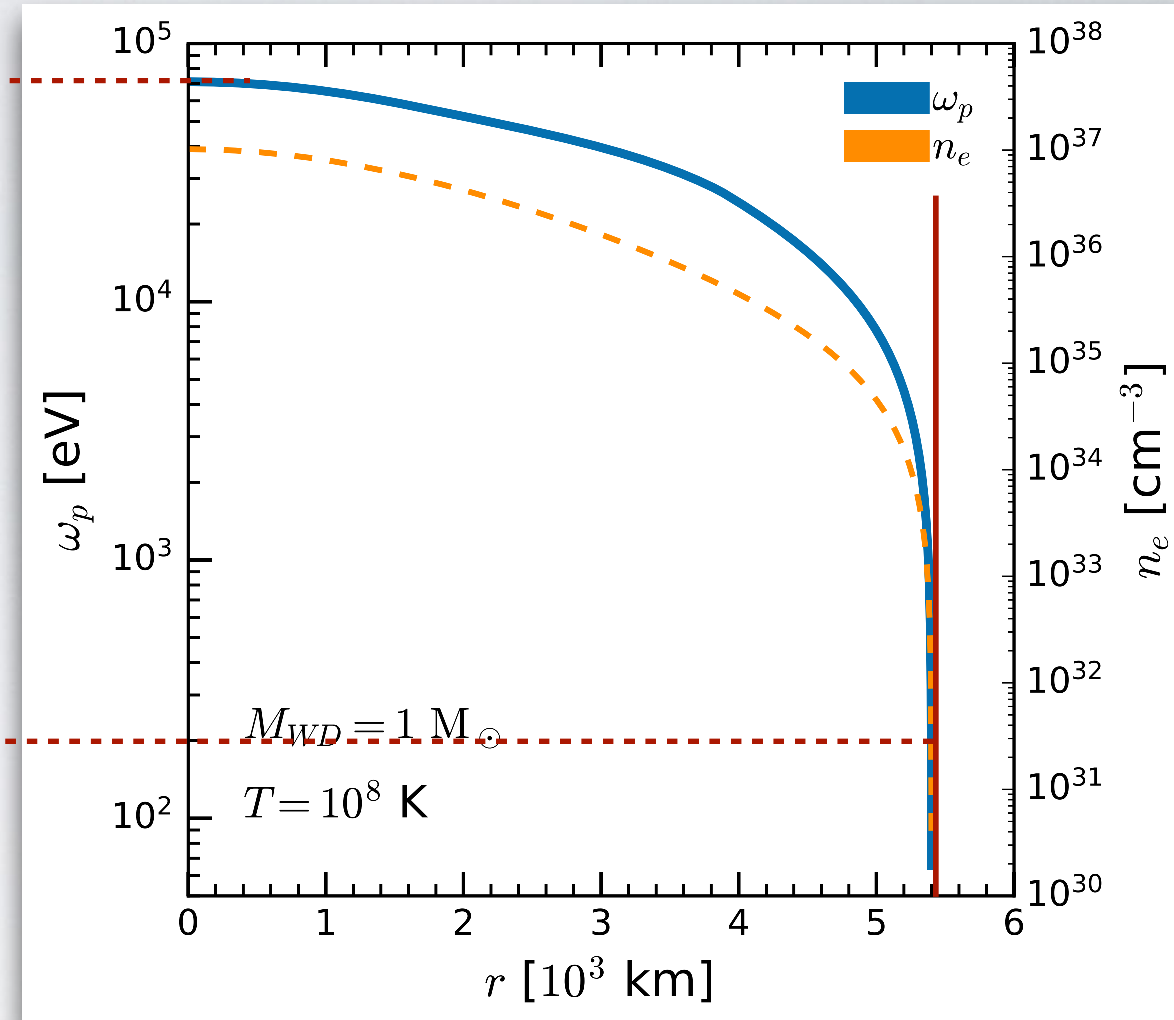

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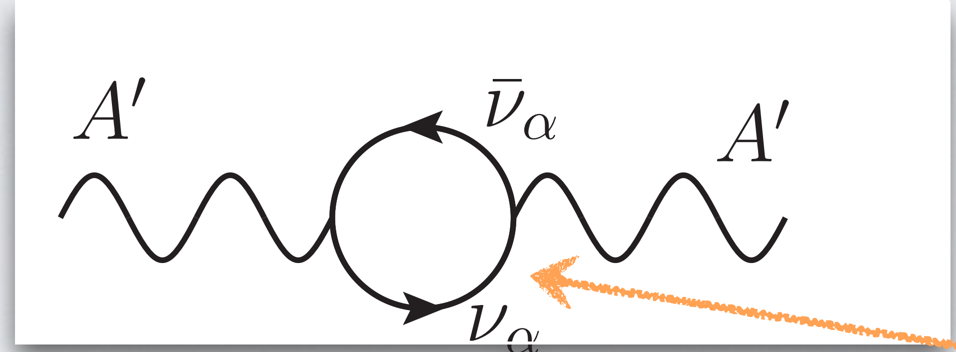
$$G^{\mu\nu}_{\mathrm{BW}}(q^2) = \frac{-i(g^{\mu\lambda} - q^\mu q^\lambda/m^2)}{q^2 - m^2 - \mathrm{Re}(F) - i\,\mathrm{Im}(F)} P^\nu_{L\lambda} + \frac{-i(g^{\mu\lambda} - q^\mu q^\lambda/m^2)}{q^2 - m^2 - \mathrm{Re}(G) - i\,\mathrm{Im}(G)} P^\nu_{T\lambda}$$

BREIT WIGNER REGULATOR

- Compute the imaginary part of dark photon self-energy
  → In resonant region only due to neutrinos



Neutrinos are non-thermal!

- We find the typical relation

$$\bar{\Pi}^{\mu\nu}_{A'}(q^2) = -\frac{\left(k^\alpha_\nu\right)^2}{4\pi^2} q^2 g^{\mu\nu} \int_0^1 dx\, x\,(1-x)\ \log\left(\frac{m_\alpha^2}{m_\alpha^2 - x(1-x)q^2}\right)$$

- So the regulators

$$\mathrm{Im}(\bar{\Pi}^{\mu\nu}_{A'})(q^2) = \frac{\left(k^\alpha_\nu\right)^2}{24\pi}\frac{(\omega_l^2 - \boldsymbol{q}^2)^2}{\boldsymbol{q}^2} P_L^{\mu\nu} - \frac{\left(k^\alpha_\nu\right)^2}{24\pi}(\omega_t^2 - \boldsymbol{q}^2) P_T^{\mu\nu}$$

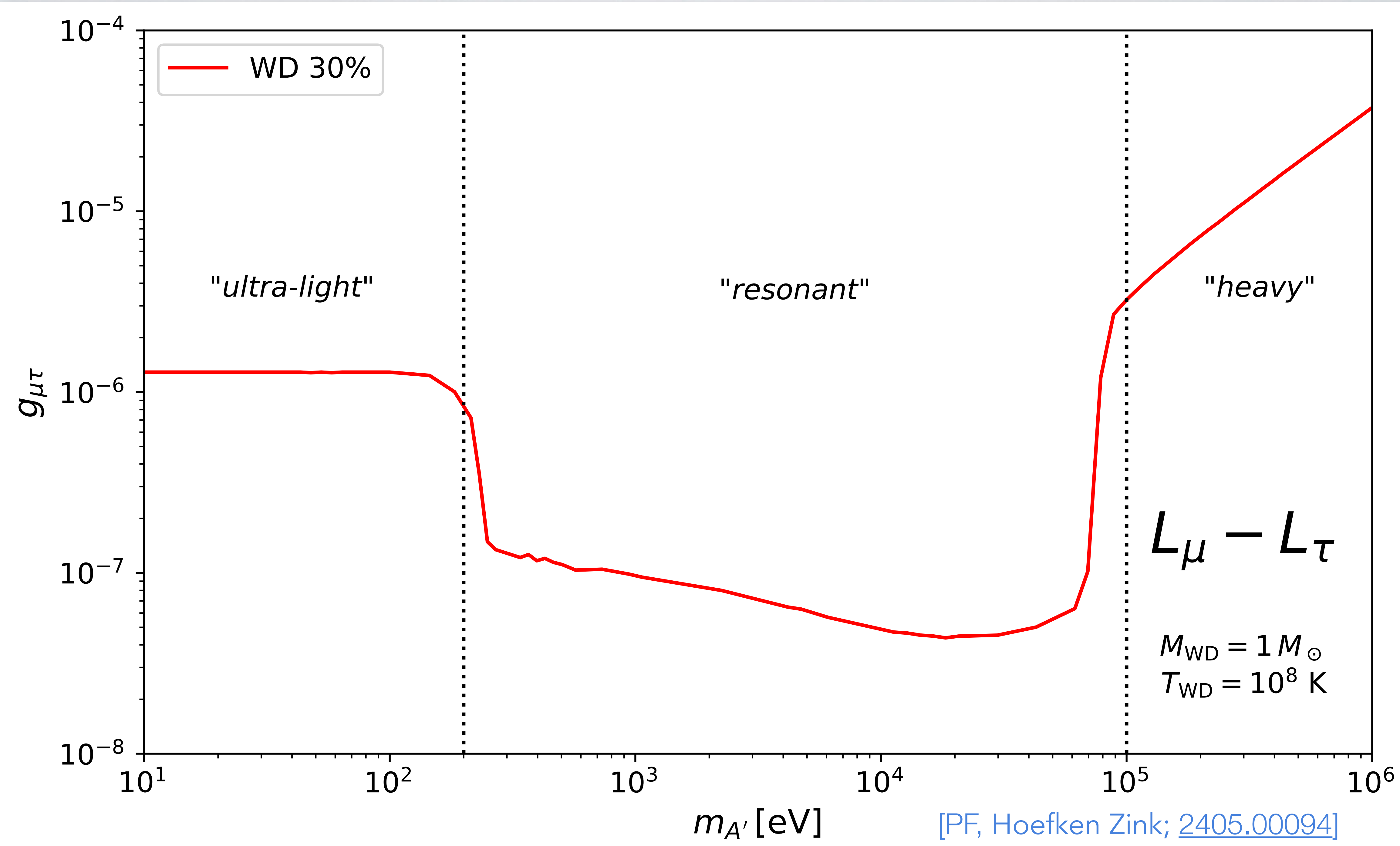
$\mathrm{Im}(F)$ $\mathrm{Im}(G)$

WD COOLING SENSITIVITIES

- Fraction of extra cooling $\varepsilon^{\mathrm{BSM}} = L_\nu^{\mathrm{BSM}}/L_\nu^{\mathrm{SM}} - 1$
- Existing bounds at 30% extra cooling @ 90% CL from Hubble

[Hansen et al., *Astrophys. J. 809 (2015) no. 2, 141*]

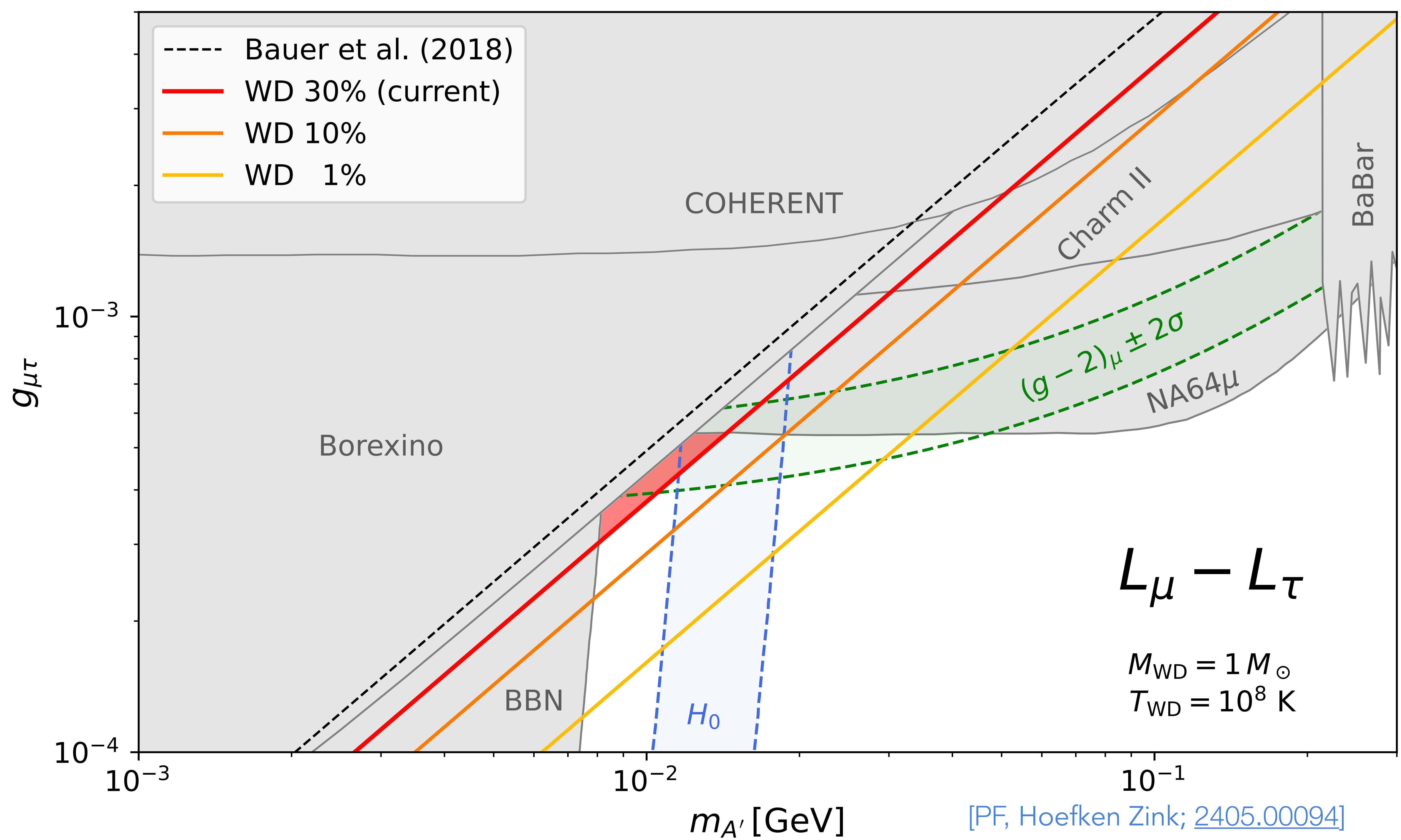
Finally some plots!



[PF, Hoefken Zink; 2405.00094]

WD COOLING & $(g-2)_\mu$

- Current bounds exclude 30% extra cooling ⟹ **leading limit on g-2**



[PF, Hoefken Zink; 2405.00094]

CONCLUSIONS

- Neutrino cooling of White Dwarfs is sensitive laboratory for (light) leptophilic mediators
- **First full computation of A' induced plasmon decay in resonant domain.**
- Already at current sensitivities WD cooling excludes unconstraint parameter space of $U(1)_{L_\mu - L_\tau}$
- **Measurements of hot WD neutrino luminosity function is testing** $(g-2)_\mu$ **explanation within** $U(1)_{L_\mu - L_\tau}$**!**
- For all the fun details ask me and check out our paper :)

[arXiv:2405.00094]

THANK YOU!

BACKUP

PLASMON PROPAGATOR

- Photon in plasma is on-shell with plasmon frequencies $\omega_\lambda(q)$
- Can extract field strength normalisations $Z_l(q)$ & $Z_t(q)$

Longitudinal: $D^{00} = \frac{1}{q^2 - \Pi_L(Q)}$

$$\lim_{q_0 \to \omega_l(q)} D^{00} = \frac{\omega_l^2(q)}{q^2} \frac{Z_l(q)}{q_0^2 - \omega_l(q)^2}$$

Transverse: $D^{xx} = \frac{1}{q_0^2 - q^2 - \Pi_T(Q)}$

$$\lim_{q_0 \to \omega_t(q)} D^{xx} = \frac{Z_t(q)}{q_0^2 - \omega_t(q)^2}$$

Solution

$$Z_l(q) = \frac{q^2}{\omega_l(q)^2} \left[-\frac{\partial \Pi_L}{\partial q_0^2} (\omega_l(q), q) \right]^{-1}$$

$$Z_t(q) = \left[1 - \frac{\partial \Pi_T}{\partial q_0^2} (\omega_t(q), q) \right]^{-1}$$

[J. Hoefken Zink]

PLASMON PROPAGATOR

The residue of a pole in q_0^2 of $D^{\mu\nu}(q_0, q)$ can be identified with $\varepsilon^\mu(q)\varepsilon^\nu(q)^*$. So we have:

$$\mathrm{Res}\, D^{00} = \mathrm{Res}\left(\frac{\omega_l(q)^2}{q^2}\frac{Z_l(q)}{q_0^2 - \omega_l(q)^2}\right) = \frac{\omega_l(q)^2}{q^2} Z_l(q)$$

$$\mathrm{Res}\, D^{xx} = \mathrm{Res}\left(\frac{Z_t(q)}{q_0^2 - \omega_t(q)^2}\right) = Z_t(q)$$

From these expressions, we can find the polarization 4-vectors:

$$\varepsilon^\mu(q, \lambda = 0) = \frac{\omega_l(q)}{q}\sqrt{Z_l(q)}(1, 0)^\mu$$

$$\varepsilon^\mu(q, \lambda = \pm 1) = \sqrt{Z_t(q)}(0, \varepsilon_\pm(q))^\mu$$

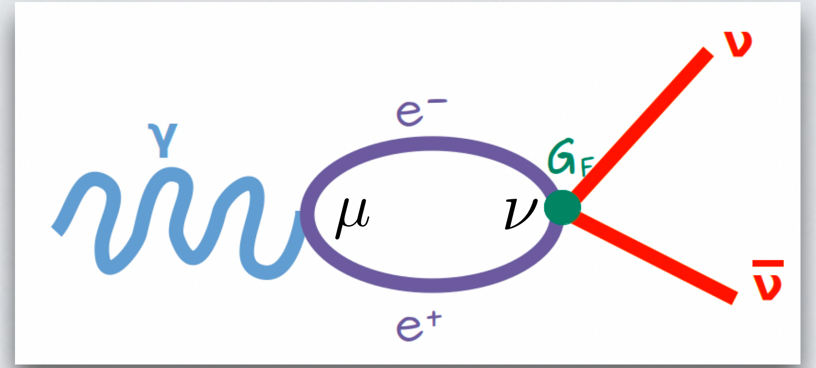
[J. Hoefken Zink]

- Obtain dispersion relations

$$\omega_l(q)^2 = \frac{\omega_l(q)^2}{q^2}\Pi_L(\omega_l(q), q)$$

$$\omega_t(q)^2 = q^2 + \Pi_T(\omega_t(q), q)$$

PLASMON DECAY AMPLITUDE



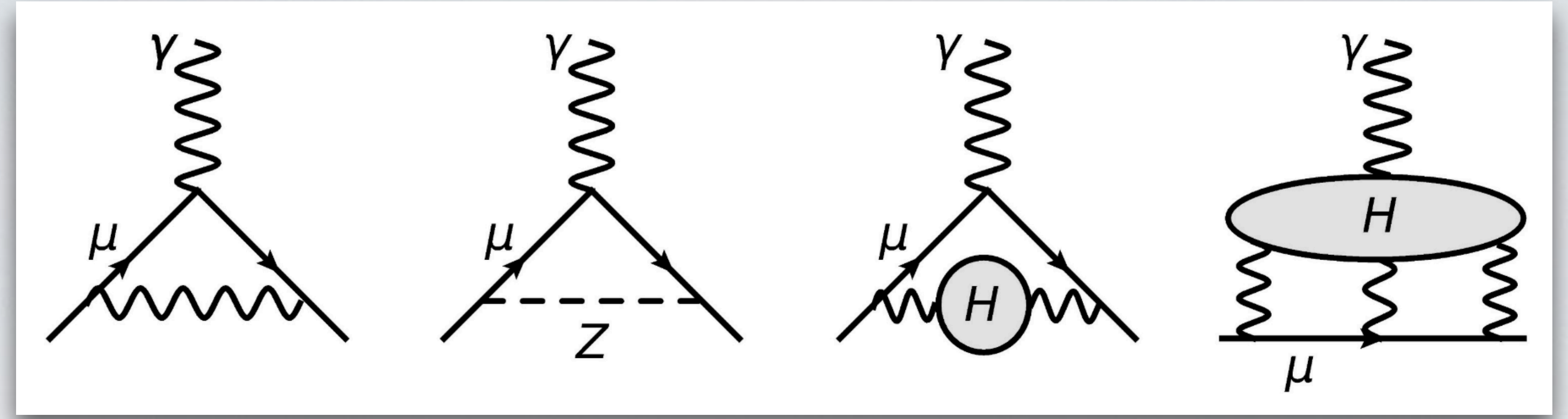
- In effective theory can write the plasmon decay amplitude in the SM as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{4\pi\alpha}} \Bigg[\varepsilon_\mu(\omega_l, q) C_V \left(\Pi_L(\omega_l, q) \left(1, \frac{\omega_l}{q}\hat{q}\right)^\mu \left(1, \frac{\omega_l}{q}\hat{q}\right)^\nu \right)$$
$$+ \varepsilon_\mu(\omega_t, q) g^{\mu i} \Big(C_V \Pi_T(\omega_t, q) \left(\delta^{ij} - \hat{q}^i \hat{q}^j\right)$$
$$+ C_A \Pi_A(\omega_t, q)(i\varepsilon^{ijm}\hat{q}^m) \Big) g^{\nu j} \Bigg] \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2)$$

- Write this in terms of effective vertex $\Gamma^{\mu\nu}_\lambda$ as function of couplings C_V & C_A

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\Gamma^{\mu\nu}_\lambda \, \varepsilon_\mu(\boldsymbol{q}, \lambda) \right) \bar{u}(p_1)\gamma_\nu(1-\gamma_5)u(p_2)$$

MUON MAGNETIC MOMENT



- The theoretical prediction for $(g-2)_\mu$ within the SM has been determined to

$$a_\mu^{\mathrm{SM}} = 116\ 591\ 810(43) \times 10^{-11}$$

[Aoyama et al; *Phys.Rept.* 887 (2020) 1-166]

- The recent Fermilab E989 result

$$a_\mu^{\mathrm{FNAL}} = 116\ 592\ 059(22) \times 10^{-11}$$

[MUON g-2; *PRL* 131 (2023), 161802]

when combined with the previous BNL results leads to the **5.2σ excess** of

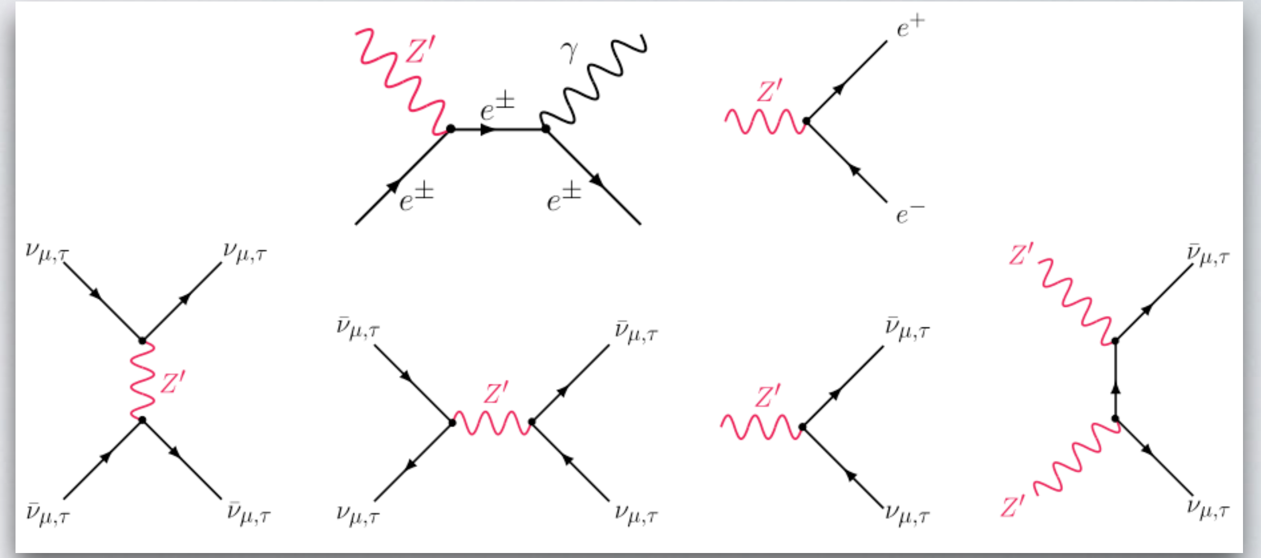
$$\Delta a_\mu = 249(48) \times 10^{-11}$$

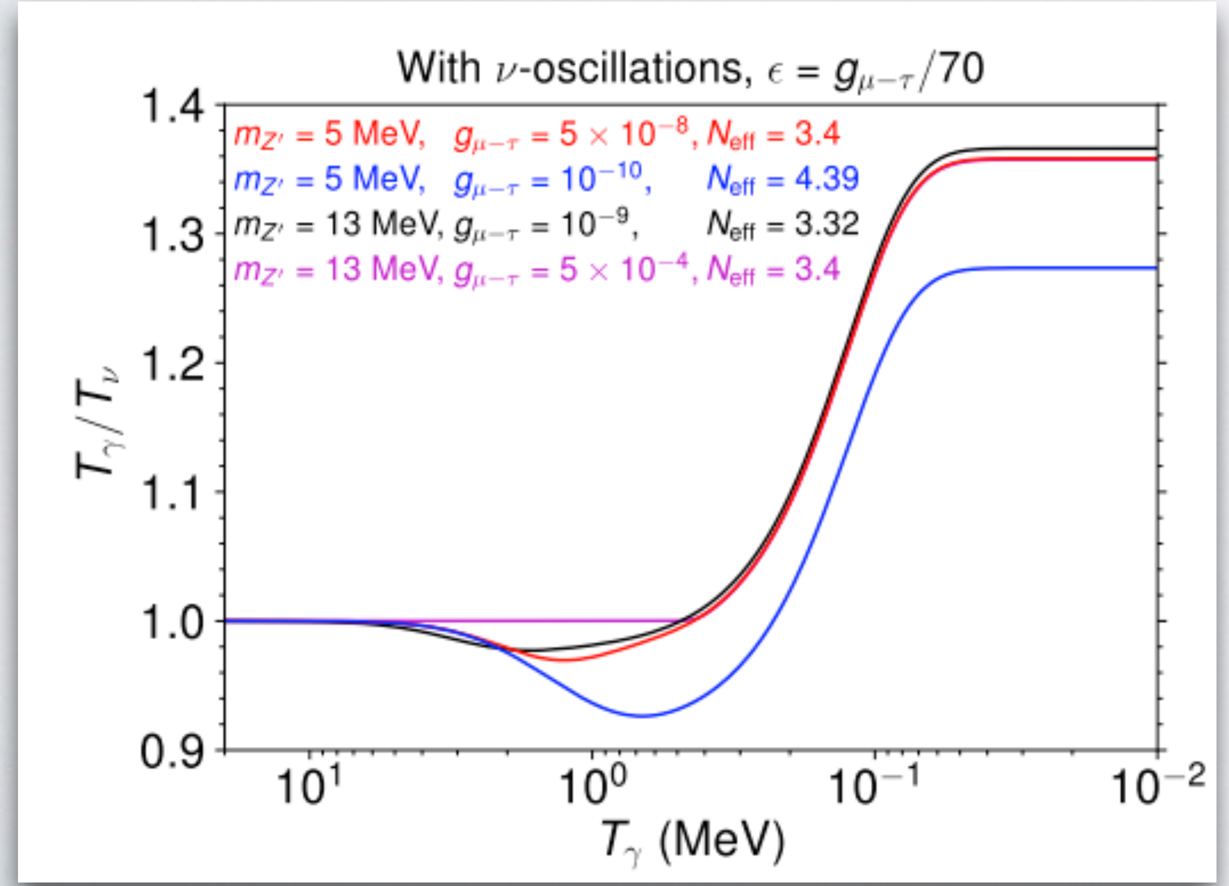
NEUTRINOS AND HUBBLE

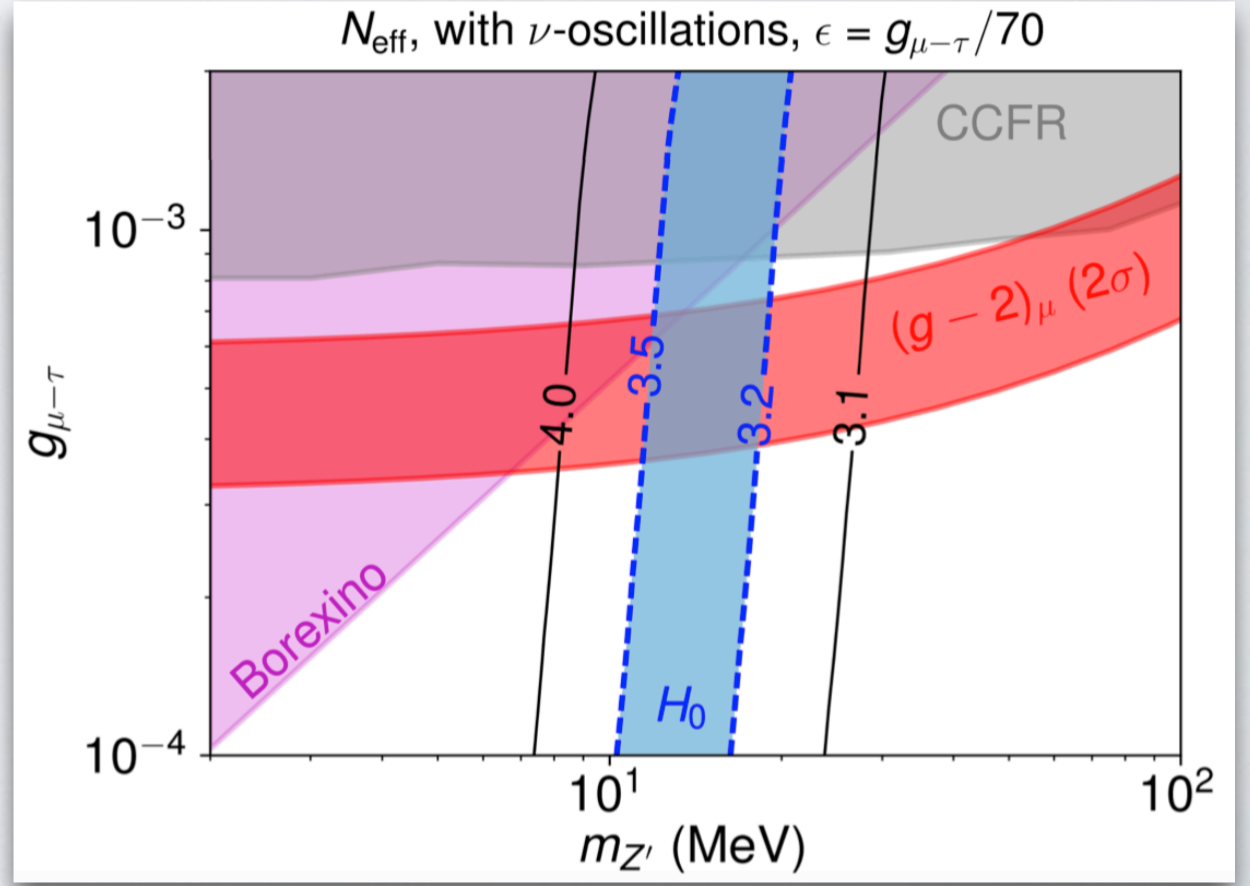
- Decay of A' heats neutrino gas and delays the decoupling

  $\Rightarrow$ increase of $N_{\rm eff}$ at early times

- Leads to larger H_0







[Escudero, Hooper, Krnjaic, Pierre; JHEP 1903 (2019) 071]