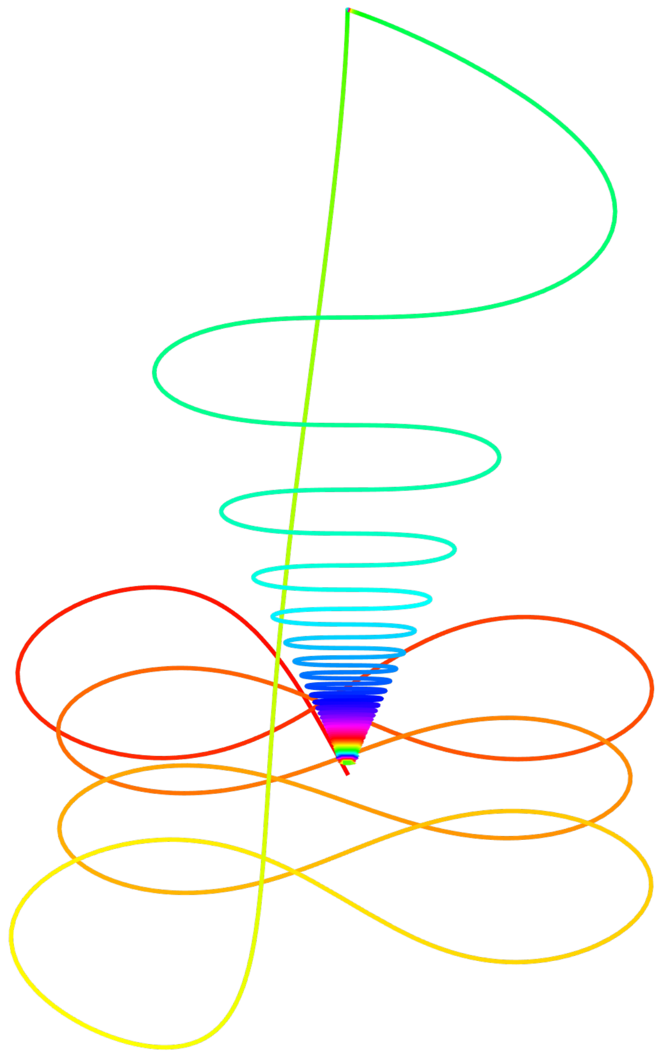
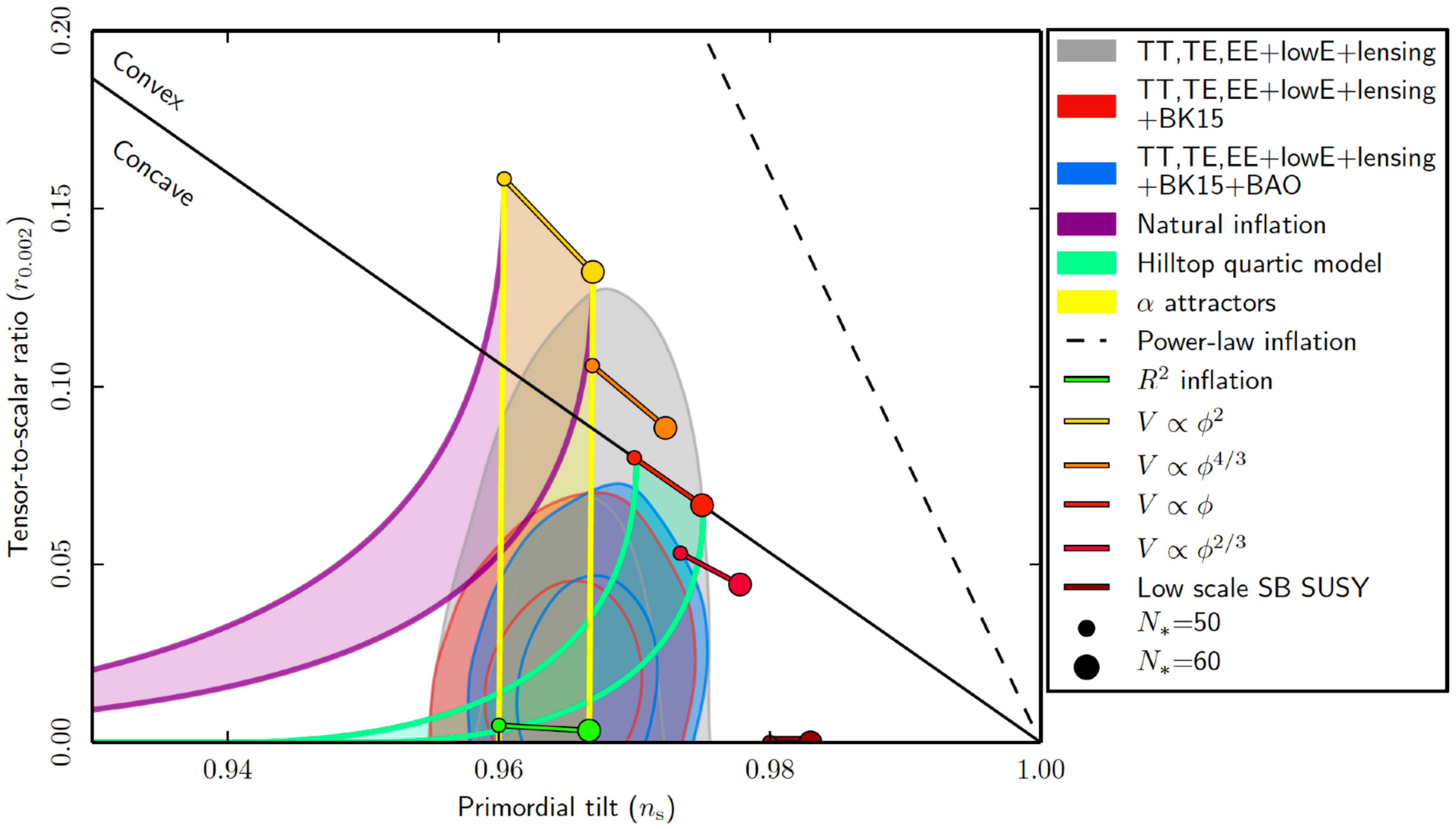


Dissipative Genesis of the Inflationary Universe

Takahiro Terada
(PTC, CTPU, IBS)

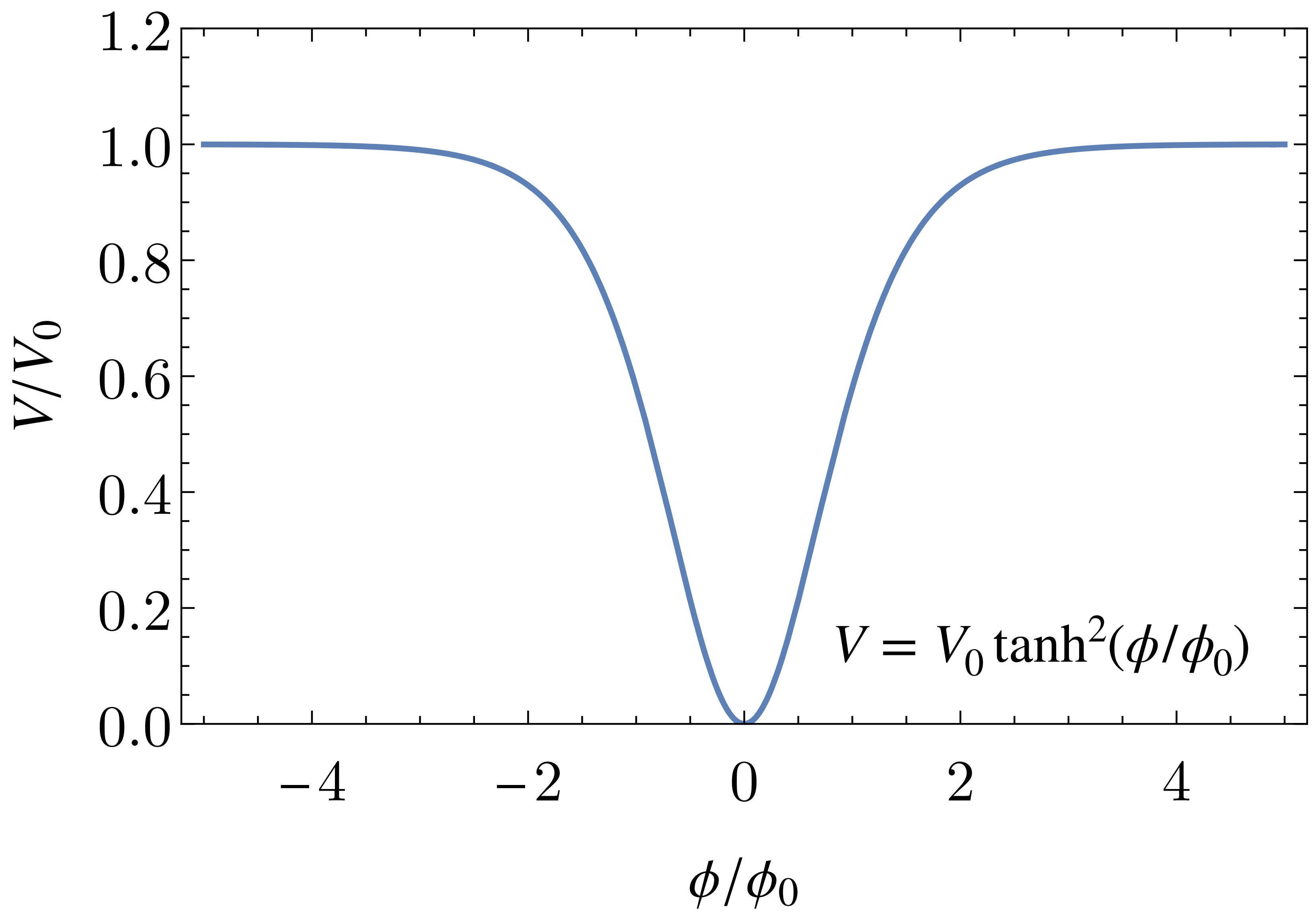
Hiroki Matsui, Alexandros Papageorgiou, Fuminobu Takahashi, and Takahiro Terada, 2305.02366; 2305.02367

Inflation



[Akrami et al. (Planck 2018), 1807.06211]. See also [Ade et al. (BICEP/Keck), 2110.00483], [Tristram et al., 2112.7961], and [Paoletti et al., 2208.10482].

A **flat** potential fits the data well.



However, the Big Bang singularity is NOT resolved by inflation.

[Borde, Guth, Vilenkin, gr-qc/0110012]. See also [Lesnefsky Easson, Davies, 2207.00955] for critical discussions.

Creation of the Universe from Nothing

Path integral, No-boundary proposal

[Hawking, Pontif. Acad. Sci. Scr. Varia 48 (1982) 563]
[Hartle, Hawking, PRD28 (1983) 2960]
[Hawking, NPB239 (1984) 257]

[Linde, Sov. Phys. JETP 60 (1984) 211]
[Linde, Rept. Prog. Phys. 47 (1984) 925]

Wheeler-DeWitt eq., Tunneling effect

[Vilenkin, PLB117 (1982)]
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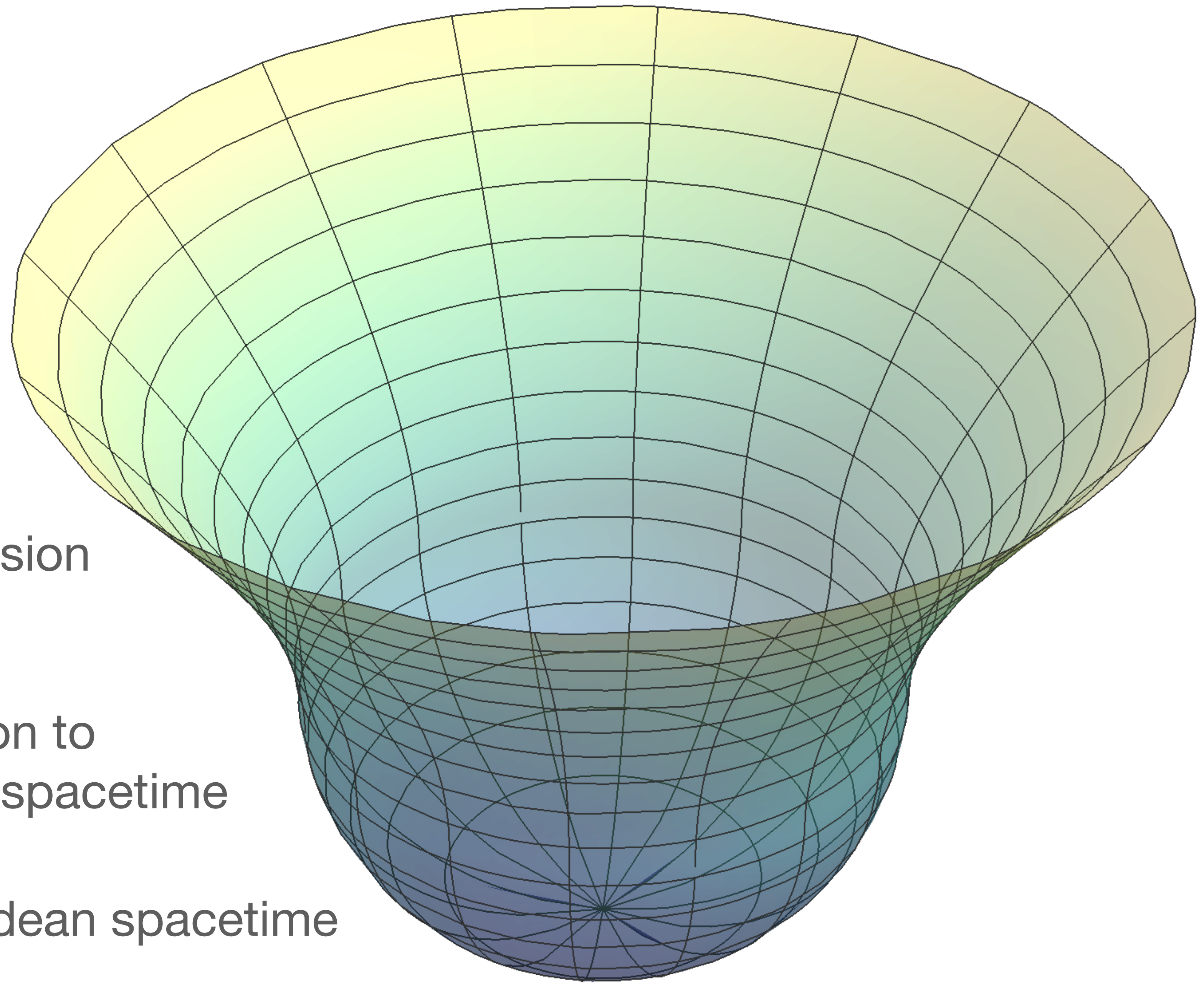
[Vilenkin, PRD33 (1986) 3560]
[Vilenkin, PRD37 (1988) 888]

Closed Universe (**Positive** spatial curvature)



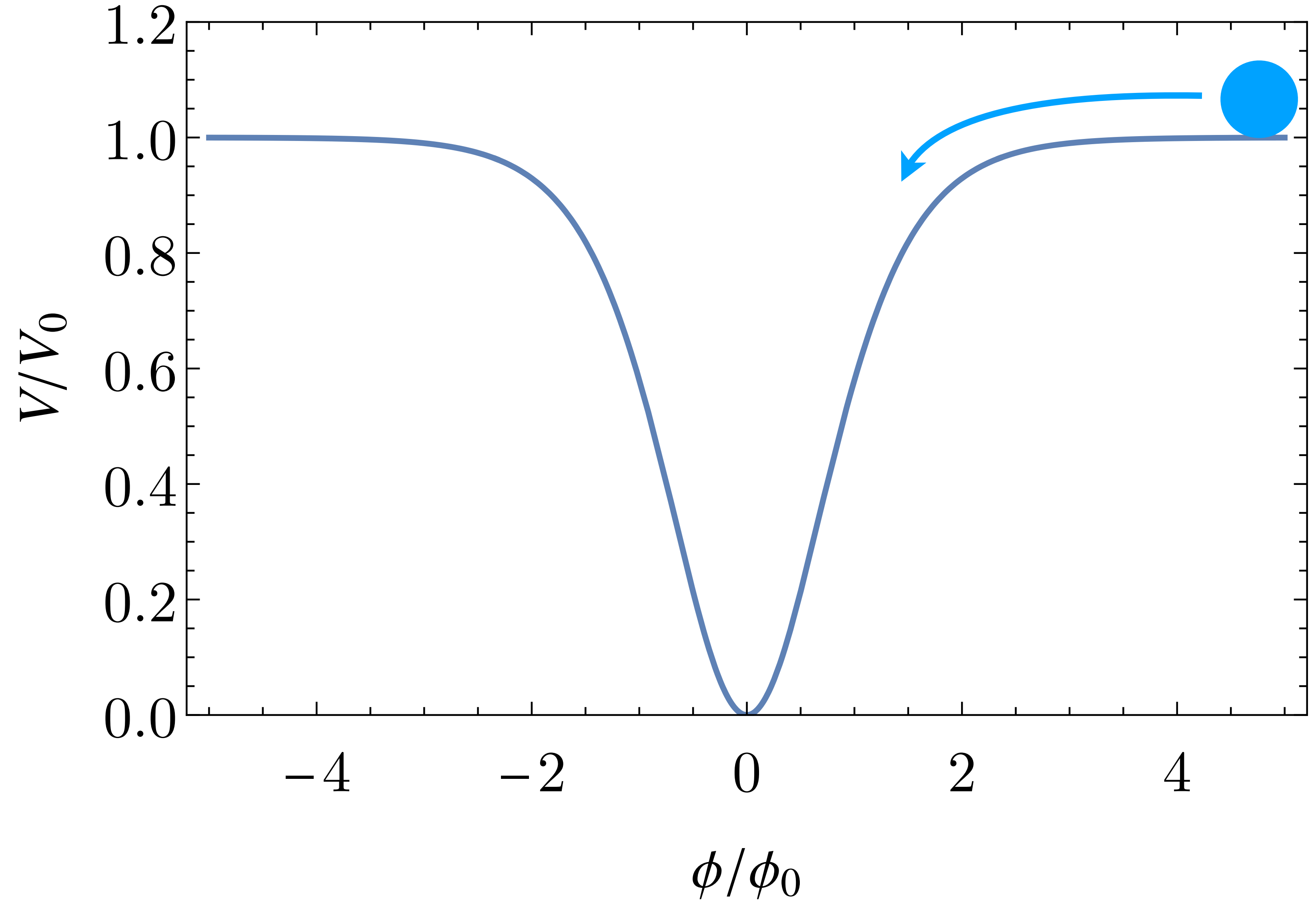
(Quasi) de Sitter expansion

Analytic continuation to Lorentzian spacetime

Compact Euclidean spacetime

Non-singular creation from "Nothing"

Sufficiently long ($N_e \gtrsim 50$) inflation



Creation of the Universe from Nothing

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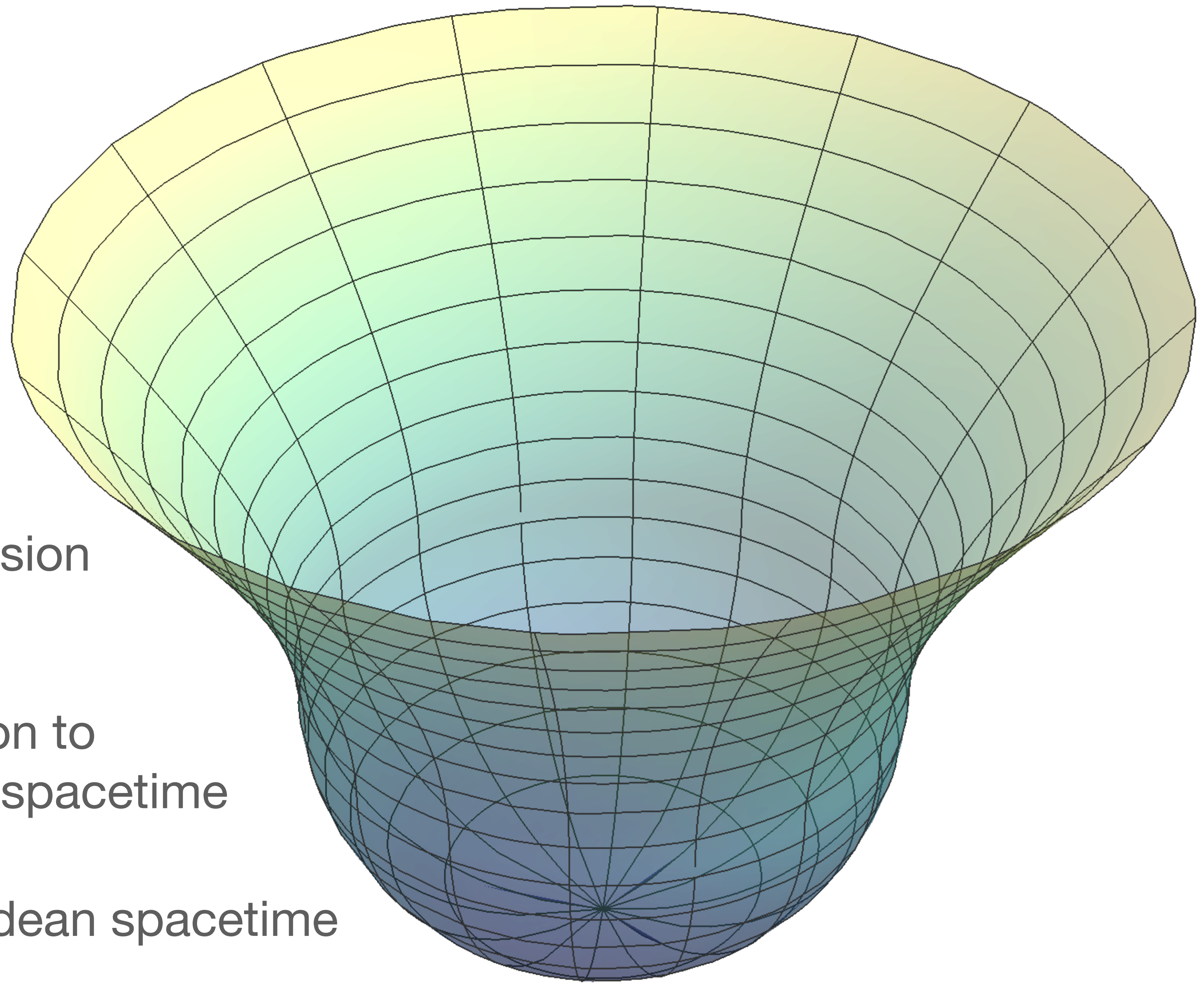


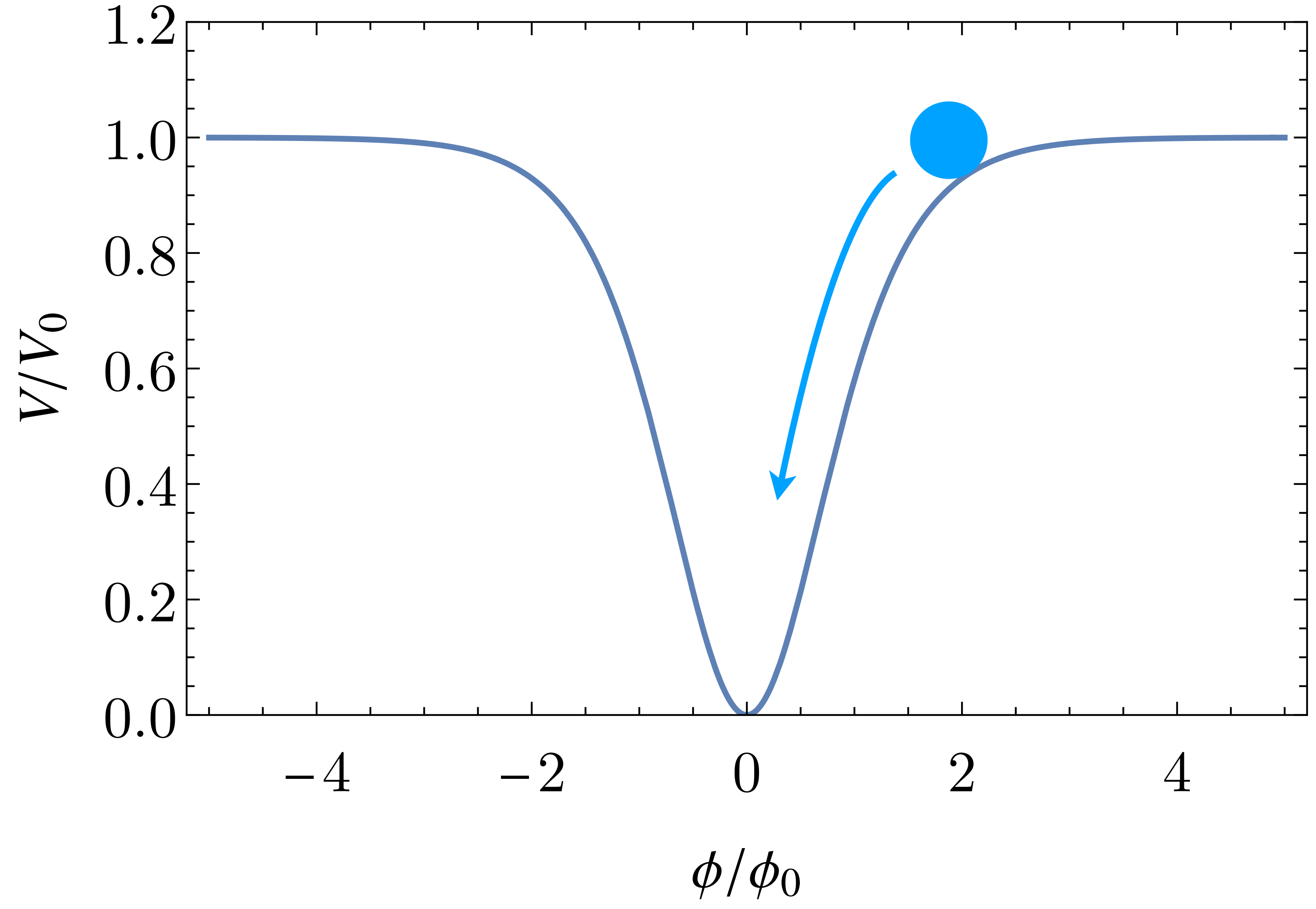
(Quasi) de Sitter expansion

Analytic continuation to Lorentzian spacetime

Compact Euclidean spacetime

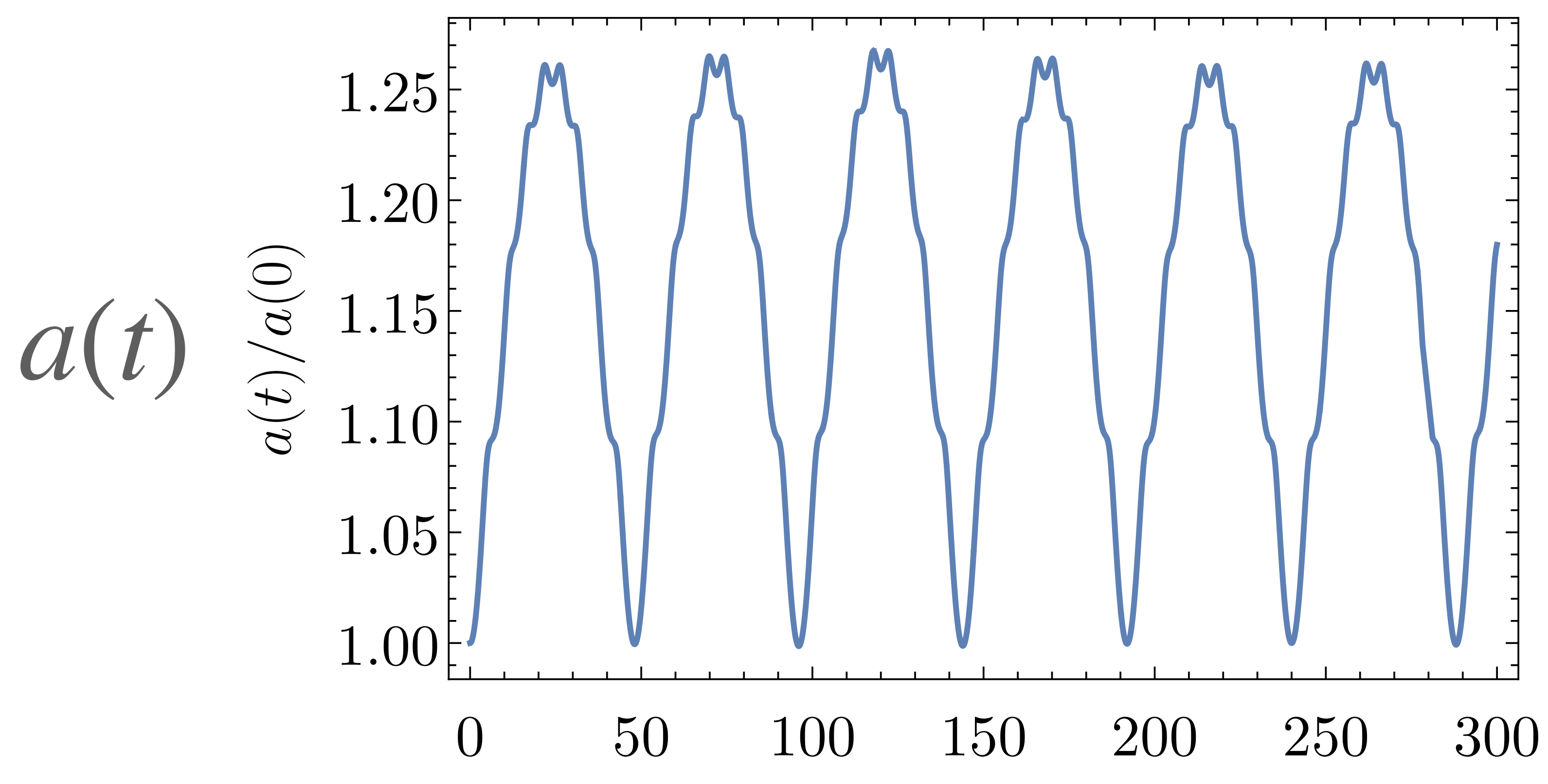
Non-singular creation from “Nothing”

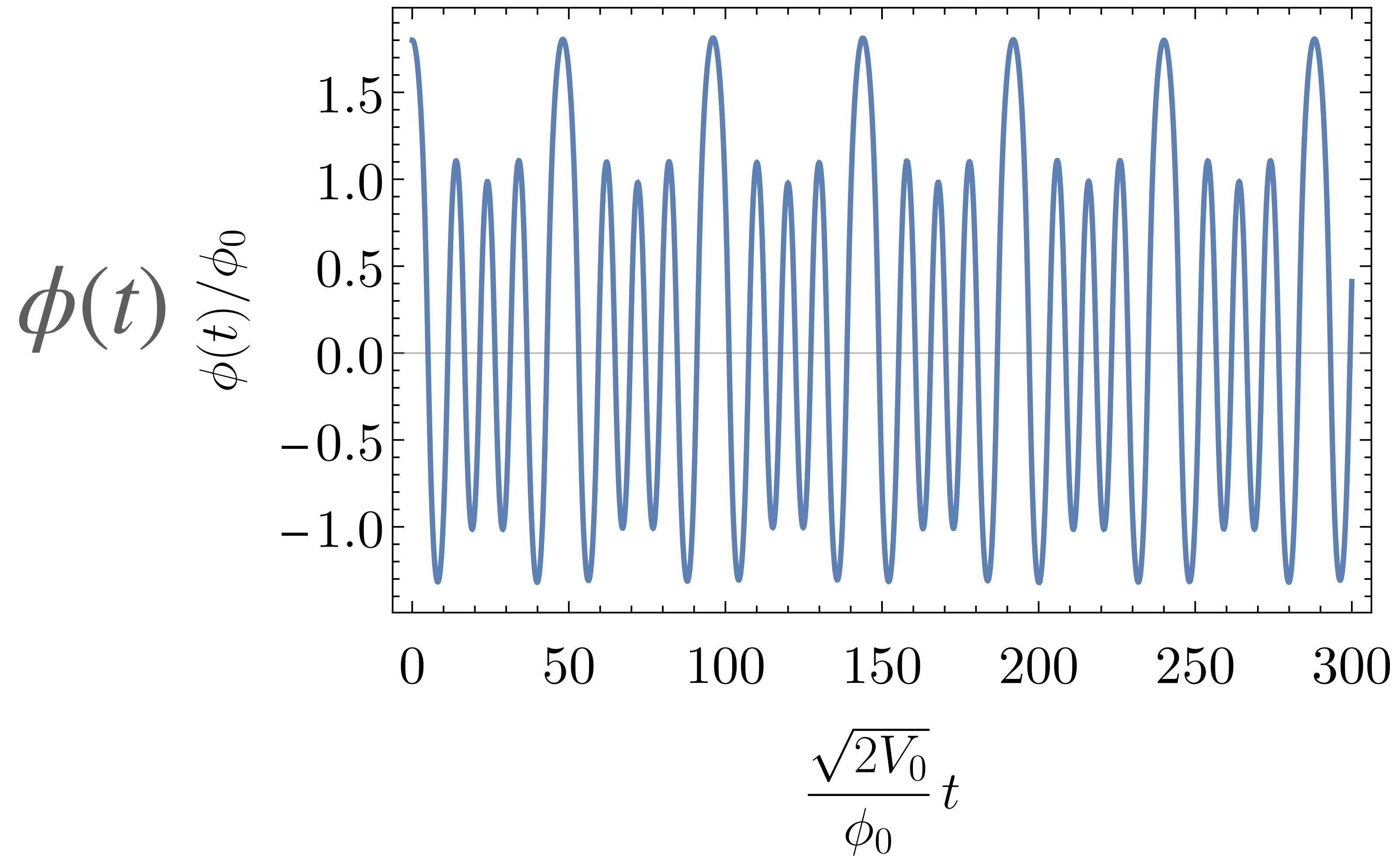
How about this case?



Big Crunch!?

Cyclic Universe





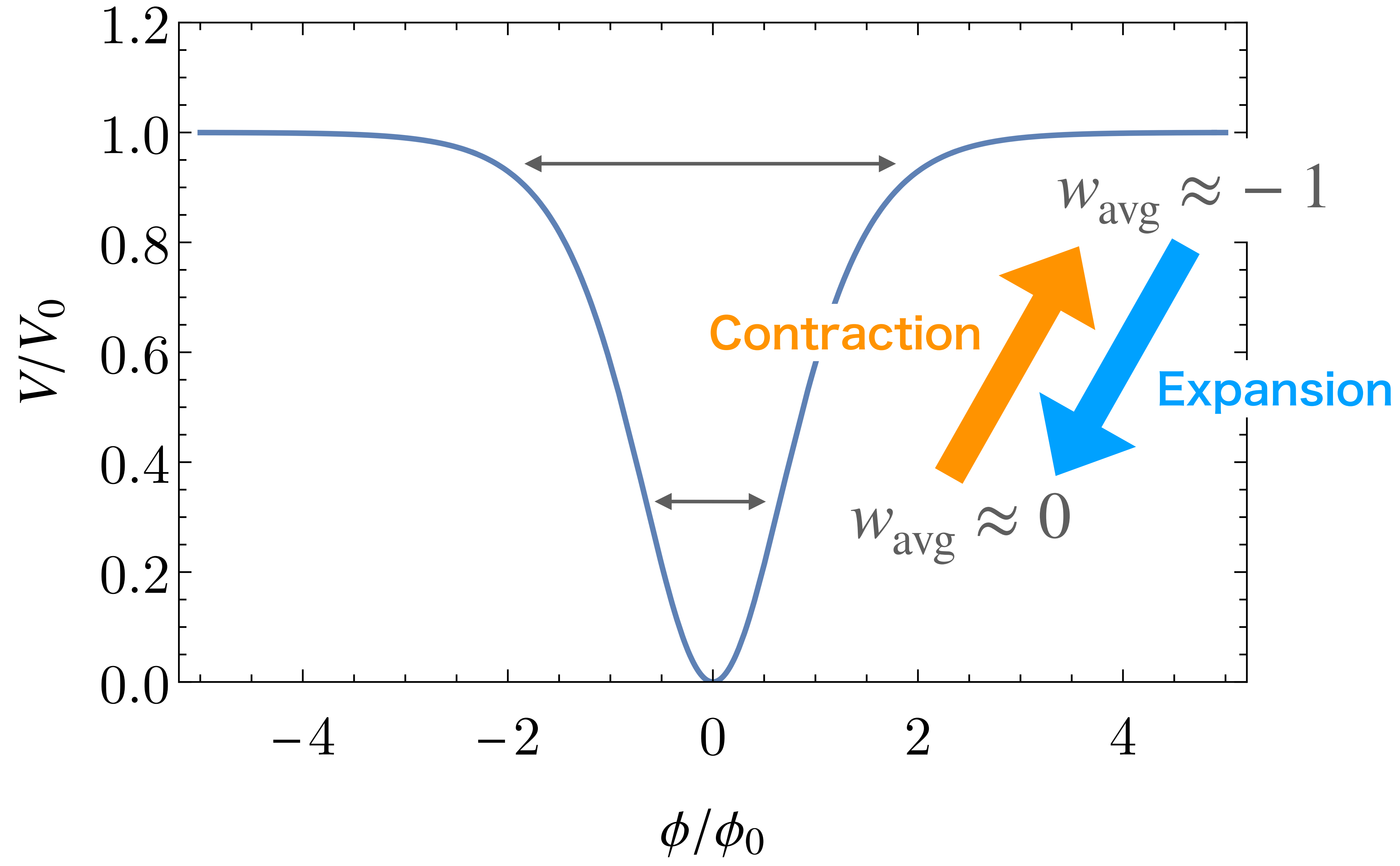
Conditions for bounce/turn-around

1. $H = 0$ is satisfied.
2. $\ddot{a}/a = -(\rho + 3P)/6$ is positive (bounce) or negative (turn-around) at the moment of $H = 0$.

$$H^2 = \frac{\rho}{3M_P^2} - \frac{K}{a^2}$$

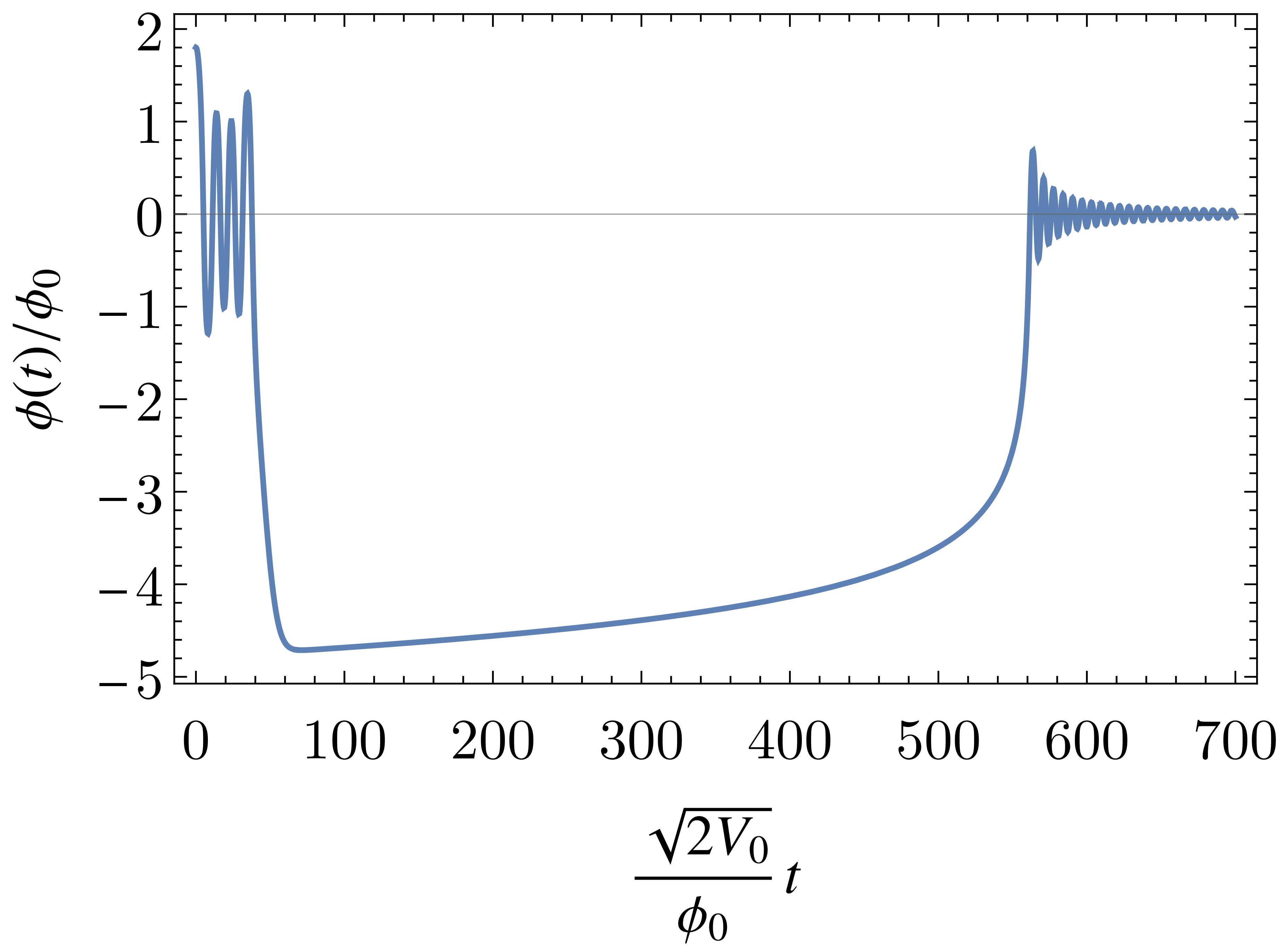
$$w = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$

$$w_{avg} = \langle w \rangle_{osc}$$



[Matsui, Takahashi, Terada, 1904.12312]
[Matsui, Papageorgiou, Takahashi, Terada, 2305.02366; 2305.02367]

The full picture of the dynamics of ϕ



What we use

✓ Classical General Relativity

✓ Canonical scalar field

✓ Interactions to reheat the Universe

What we do not use

✗ Modification to General Relativity

✗ Negative Cosmological Constant

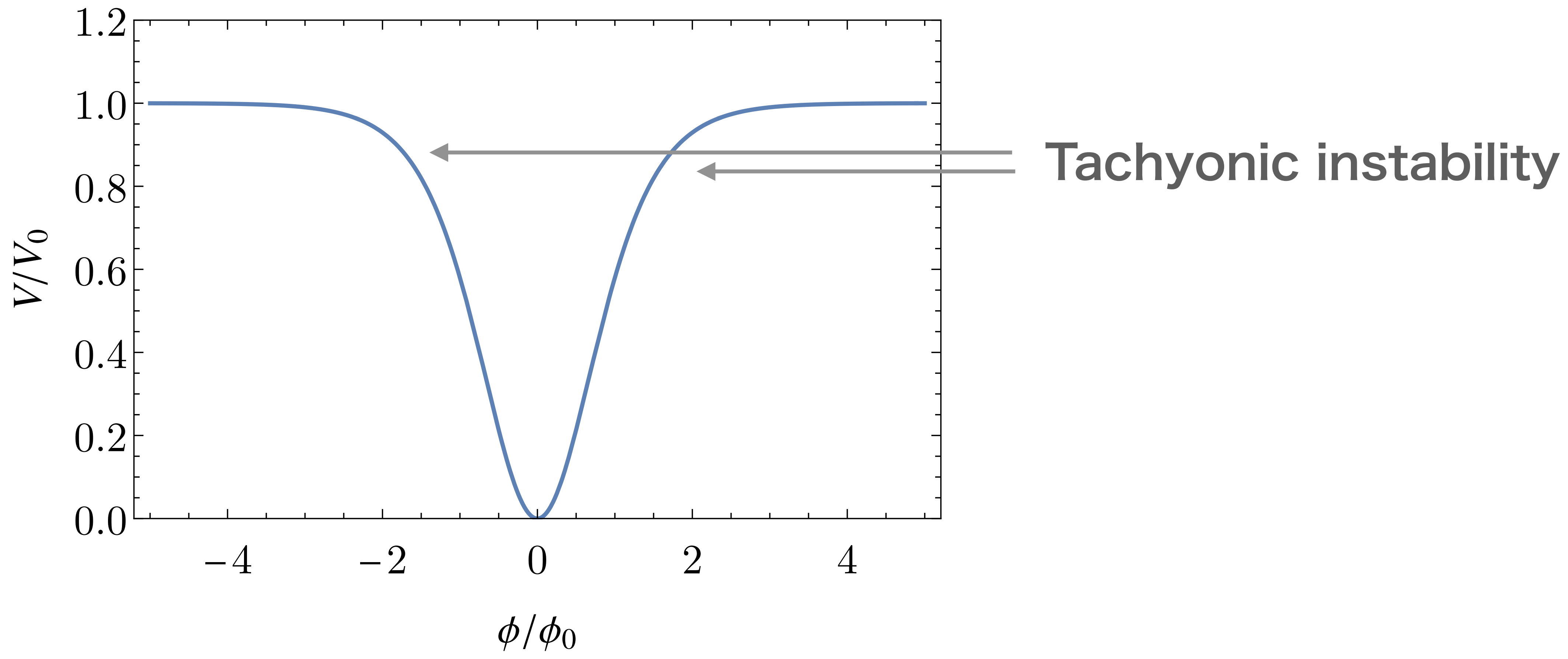
✗ Negative Casimir energy

✗ Singularity at the bounce

✗ Violation of Null Energy Condition

✗ Negative-norm state (“ghost”)

Tachyonic Instability



Analyses in the context of preheating [Tomberg, Veermäe, 2108.10767]

Energy density fluctuation

$$\delta\rho \approx \frac{(k_{\rm peak}/a)^4}{4\pi^{3/2}\sqrt{2\mu_{\rm peak}t}} e^{2\mu_{\rm peak}t}$$

$k_{\rm peak} \approx \pi a/\Delta t_\phi$: wavenumber of the most unstable mode

$\Delta t_\phi \simeq \pi\phi_0/\sqrt{2(V_0-\rho)}$: half-period of ϕ oscillations

Fragmentation time

$$t_{\rm frag} \approx \frac{1}{2\mu_{\rm peak}}\left(\ln\left(\frac{4\pi^{3/2}c\rho}{(k_{\rm peak}/a)^4}\right) + \frac{1}{2}\ln\ln\left(\frac{4\pi^{3/2}c\rho}{(k_{\rm peak}/a)^4}\right)\right)$$

$c = \mathcal{O}(1)$: the energy-density fraction of fluctuations and the background when fragmentation happens

Tachyonic Instability

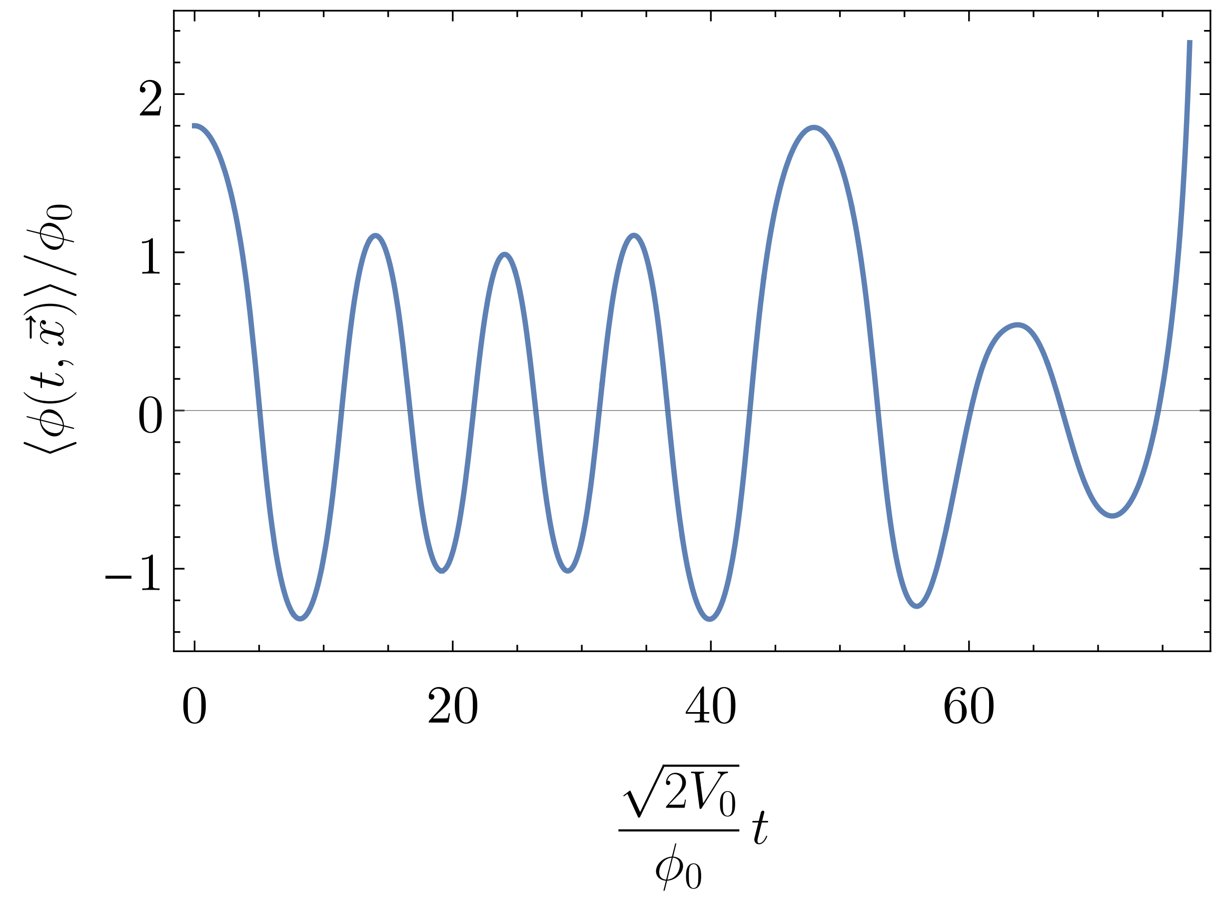
Public classical lattice calculation tool

CosmoLattice

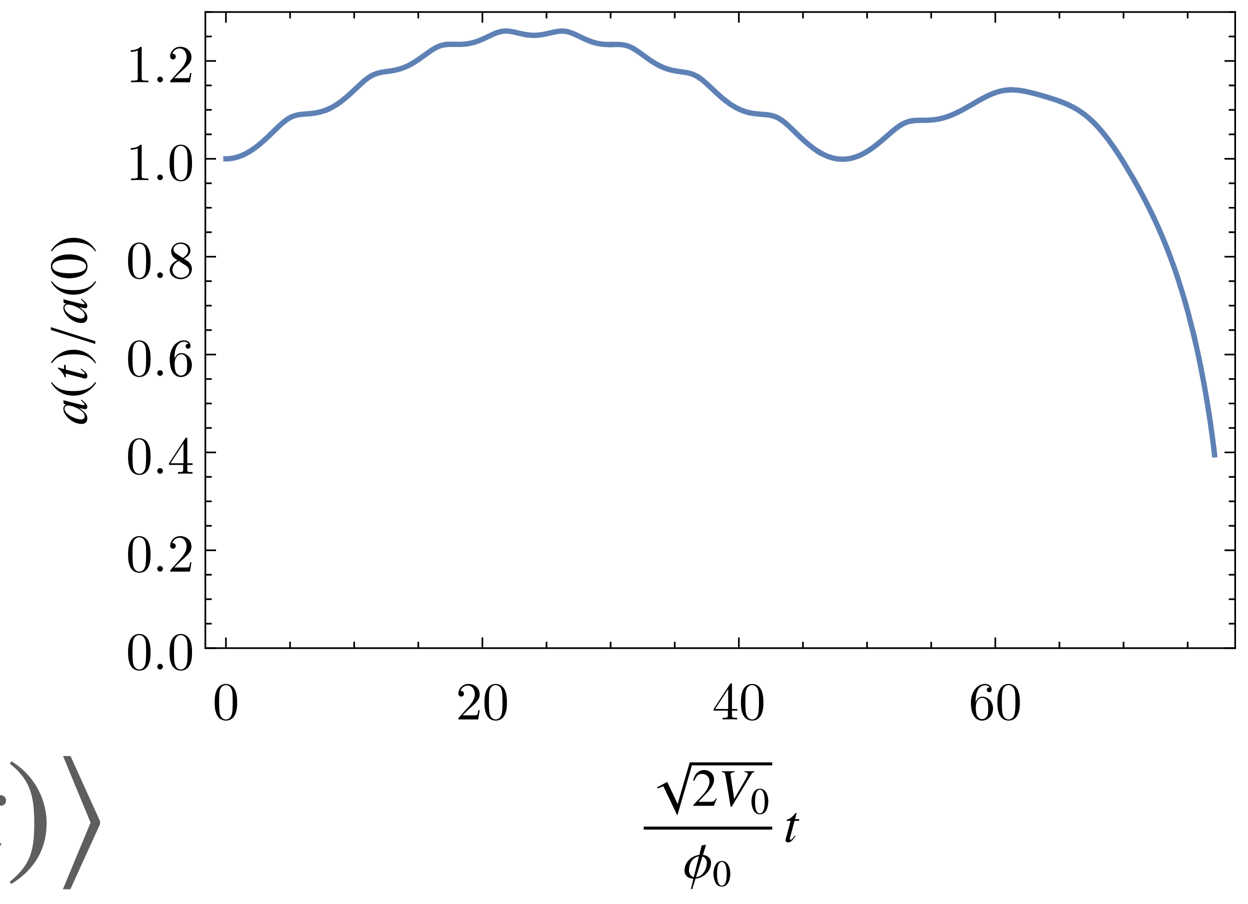
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

[Figueroa, Florio, Torrenti, Valkenburg, 2006.15122; 2102.01031]
https://cosmolattice.net/

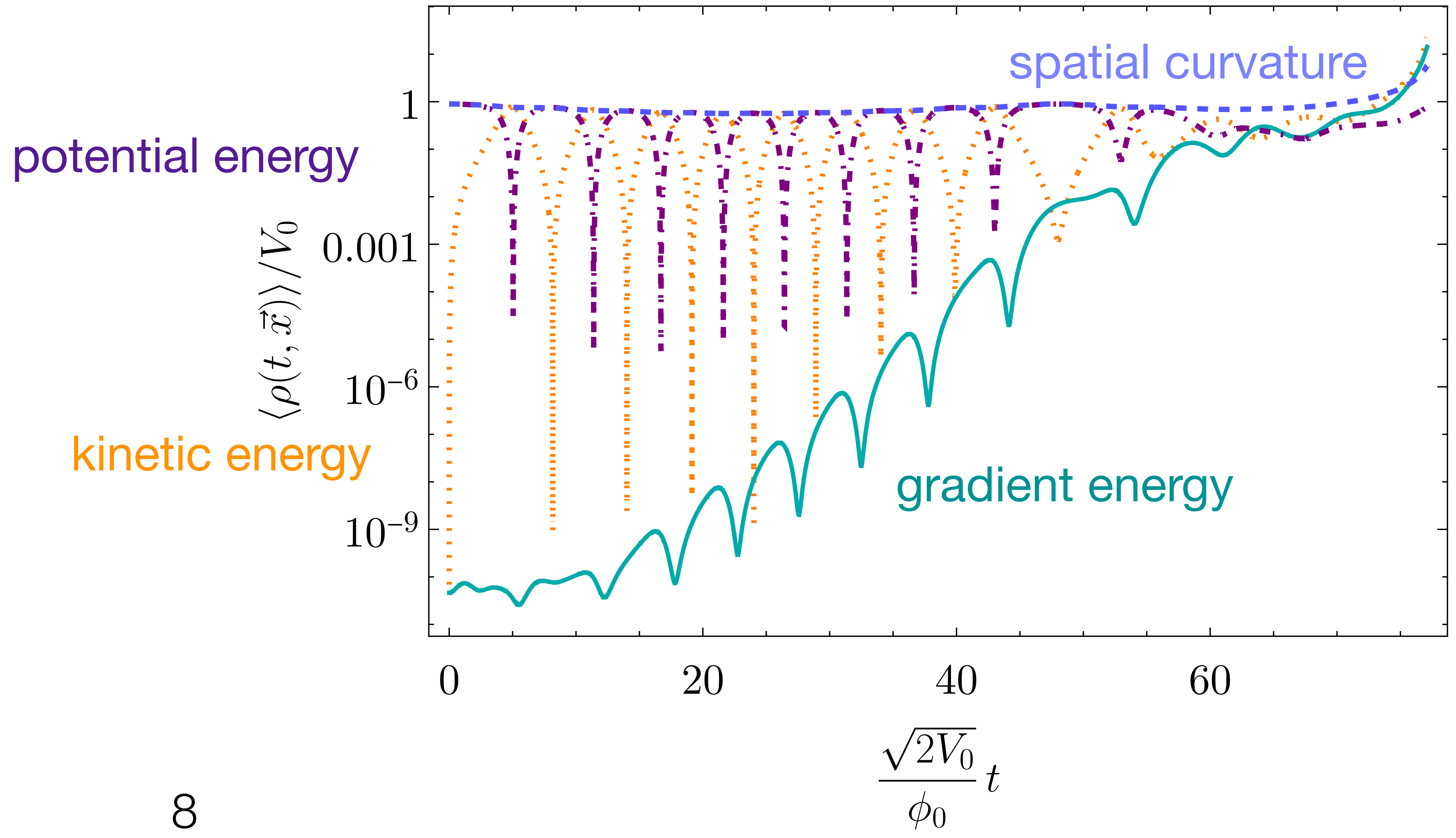
$\langle \phi(t,x) \rangle$



$a(t)$



$\langle \rho_X(t,x) \rangle$



The Question

Is it possible to turn a quasi-cyclic universe into an inflationary universe by dissipative effects *before the fragmentation time*?

Setup

Lagrangian density

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_{\mathrm{int}} + \mathcal{L}_{\mathrm{matter}}$$

Scalar potential

$$V(\phi) = V_0 \tanh^2\left(\phi/\phi_0\right)$$

Metric Ansatz

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2\left(\frac{\mathrm{d}r^2}{1 - Kr^2} + r^2\mathrm{d}\Omega_2^2\right)$$

with the positive spatial curvature $K > 0$.

Equations of motion

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 - \frac{2}{3}\rho_{\mathrm{r}} + \frac{K}{a^2},$$

$$\ddot{\phi} + (3H + \Gamma)\,\dot{\phi} + V' = 0,$$

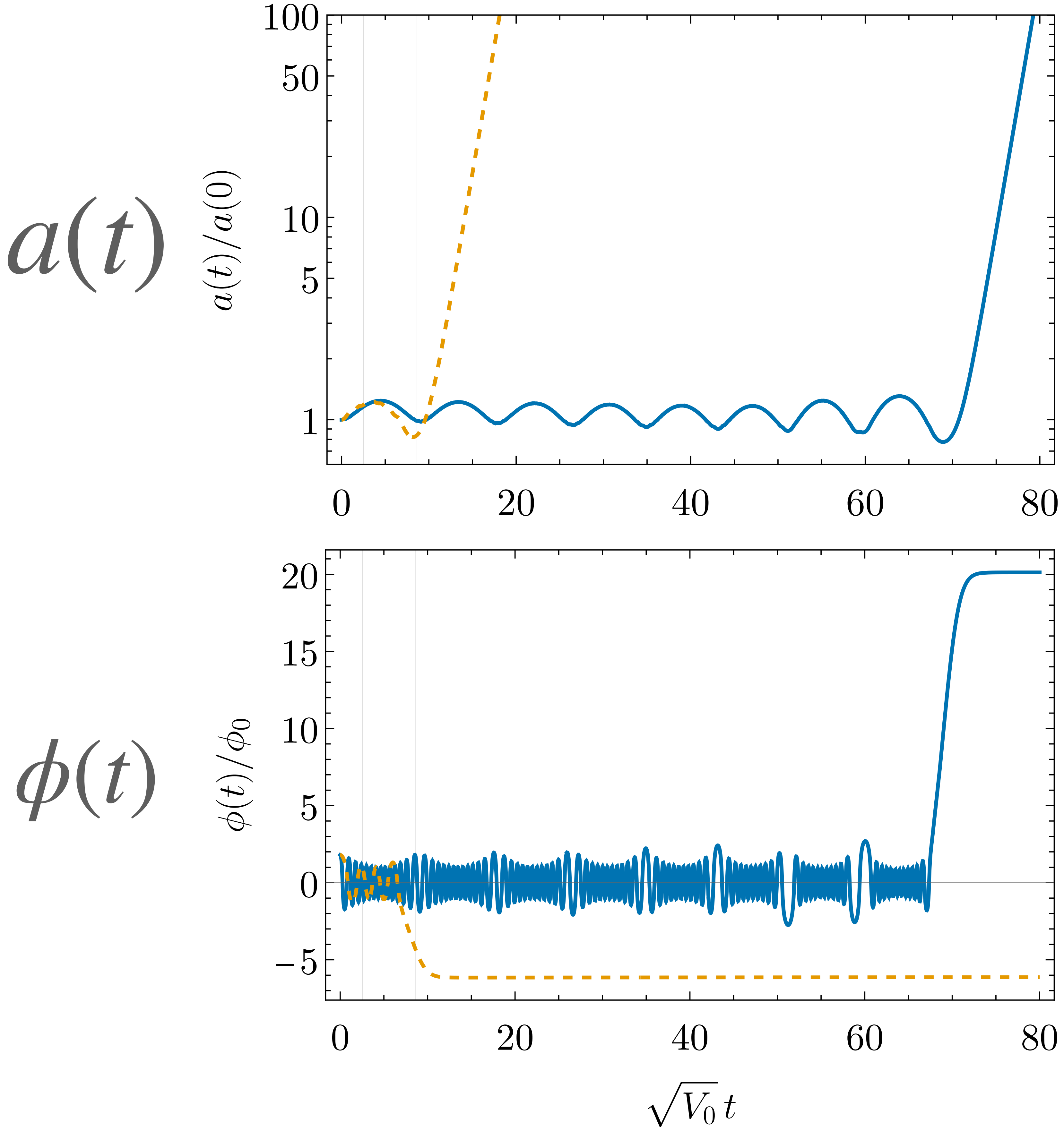
$$\dot{\rho}_{\mathrm{r}} + 4H\rho_{\mathrm{r}} = \Gamma\dot{\phi}^2.$$

For a more concrete model,
see our longer paper: 2305.02367.

Initial conditions

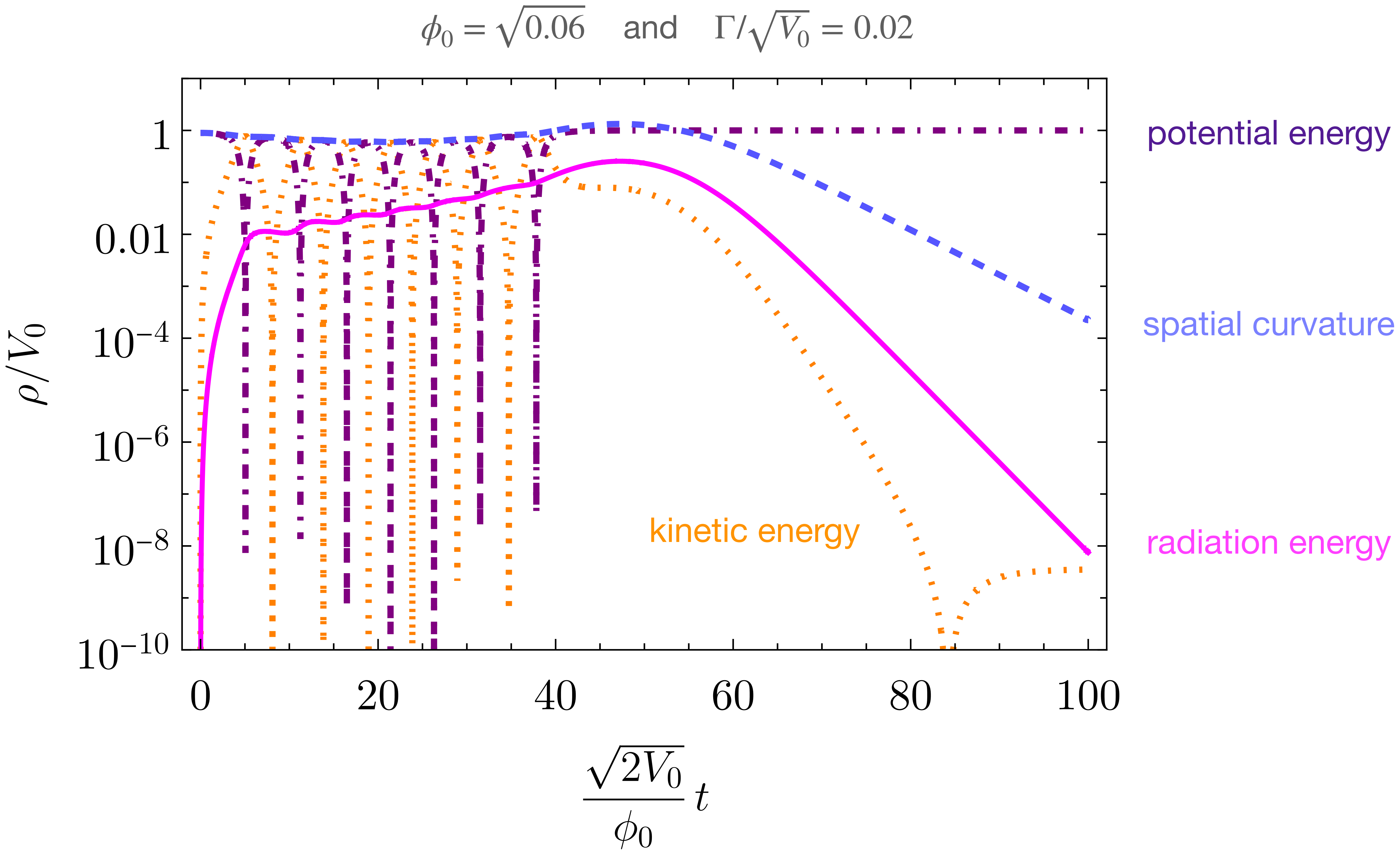
$$a(0) = \sqrt{\frac{3K}{V(\phi(0))}}, \quad \dot{a}(0) = 0, \quad \dot{\phi}(0) = 0, \quad \text{and} \quad \rho_{\mathrm{r}}(0) = 0.$$

Emergence of Inflation



Solid lines: $\phi_0 = \sqrt{0.006}, \quad \Gamma/\sqrt{V_0} = 0.003.$

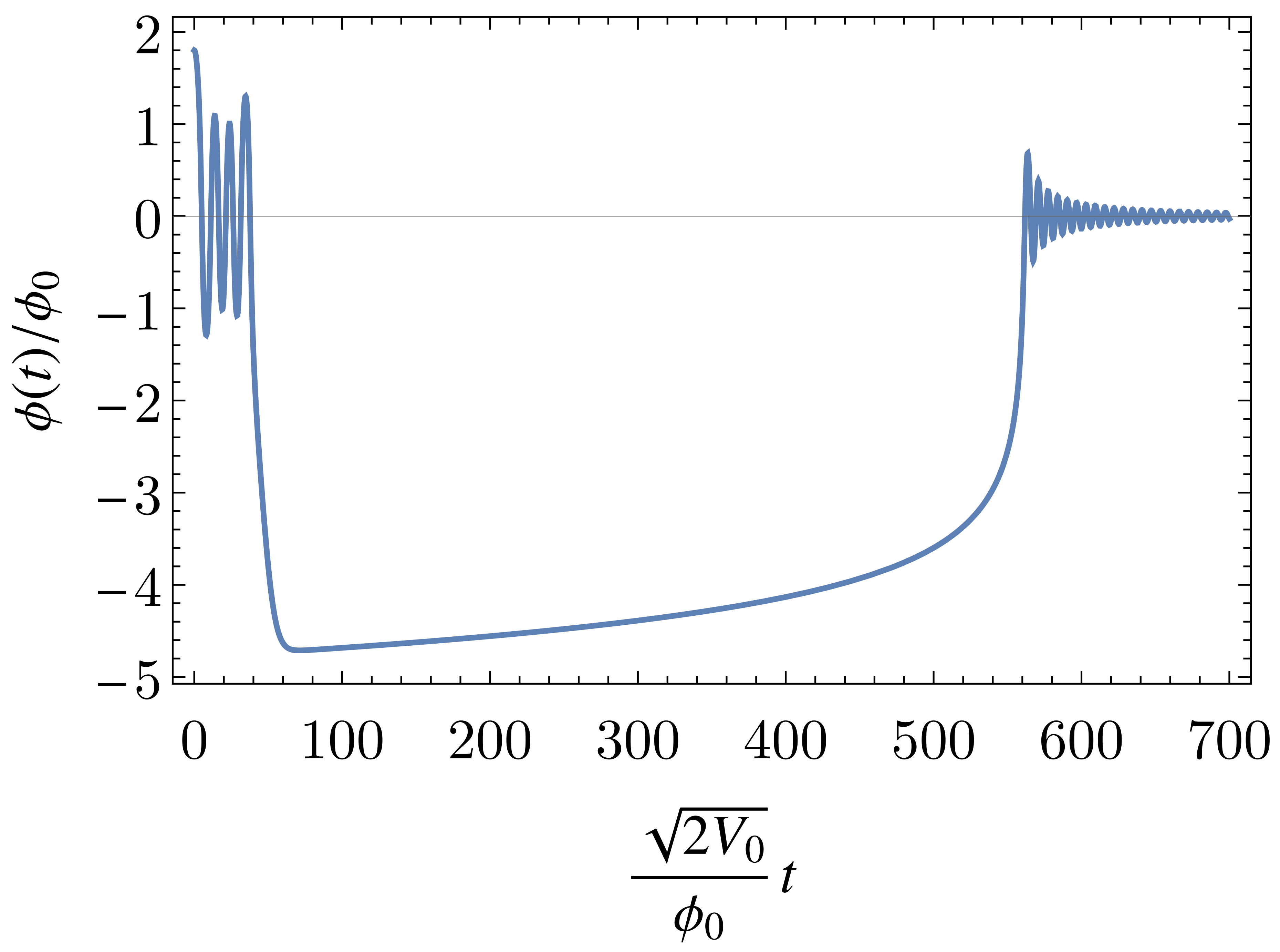
Dashed lines: $\phi_0 = \sqrt{0.06}, \quad \Gamma/\sqrt{V_0} = 0.02.$

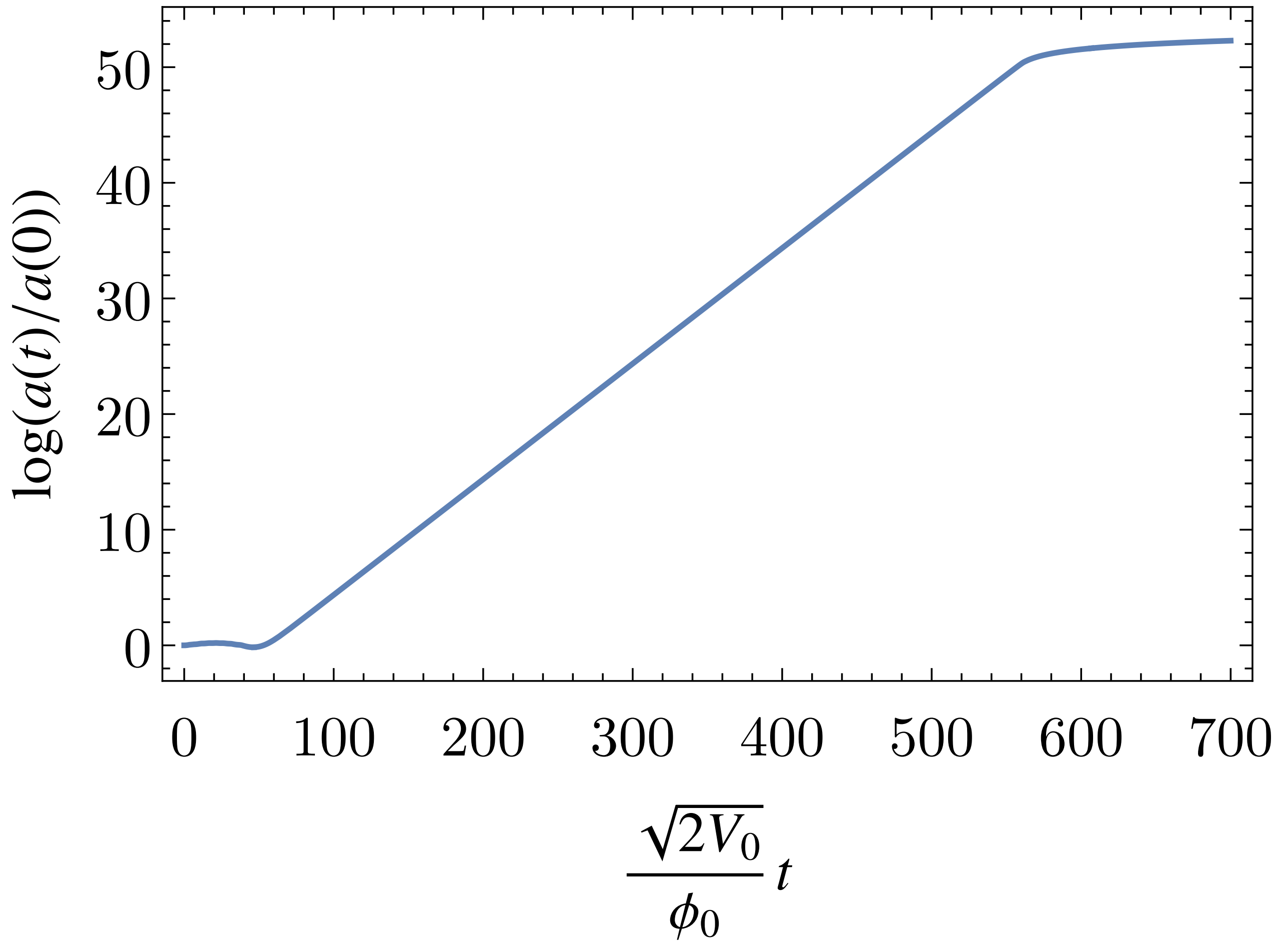


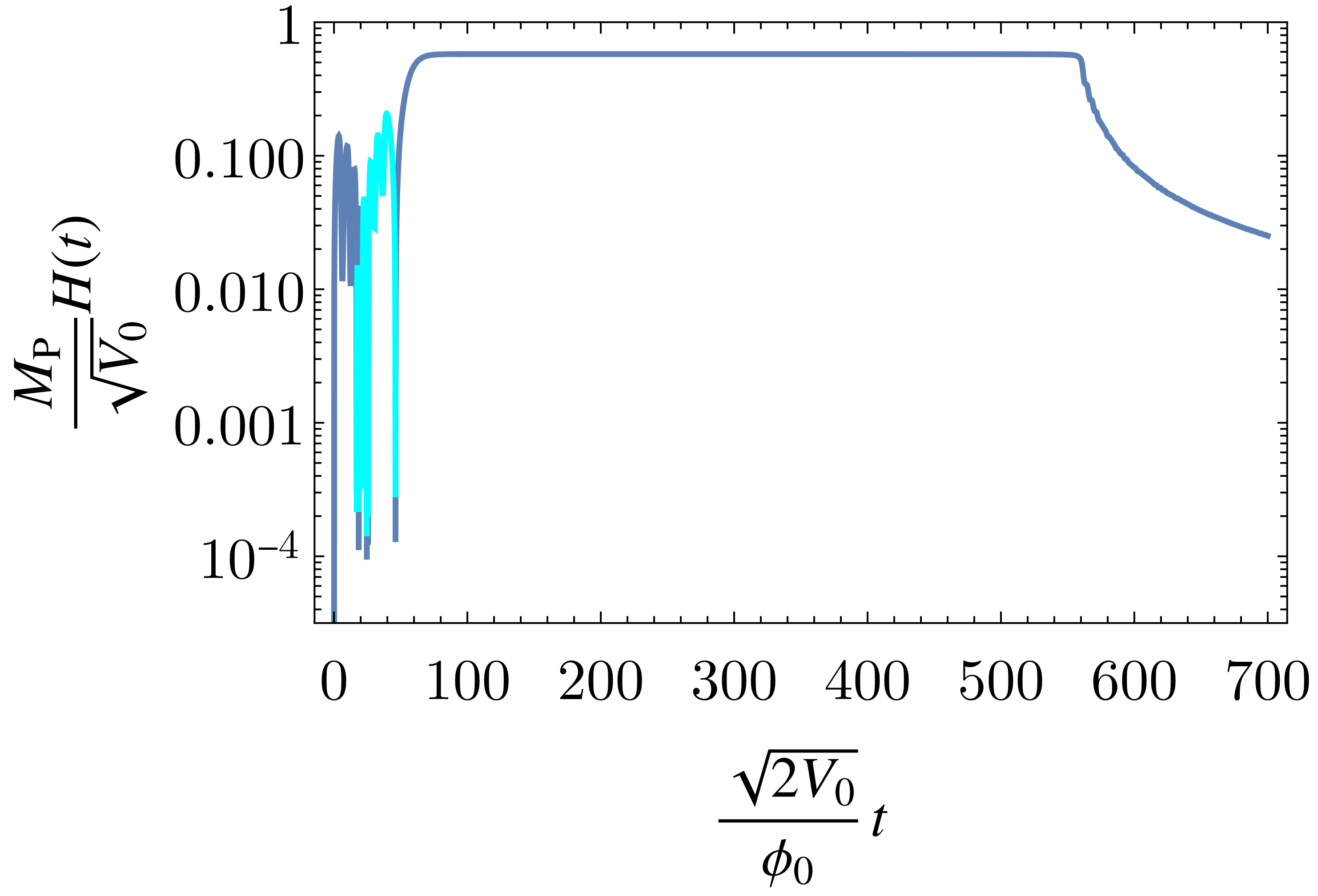
$\rho_{\mathrm{r}} > 0$ → Bounce delayed → ρ_ϕ increases → ϕ reaches the plateau

Full Picture

$\phi_0 = \sqrt{0.06}$ and $\Gamma/\sqrt{V_0} = 0.019$



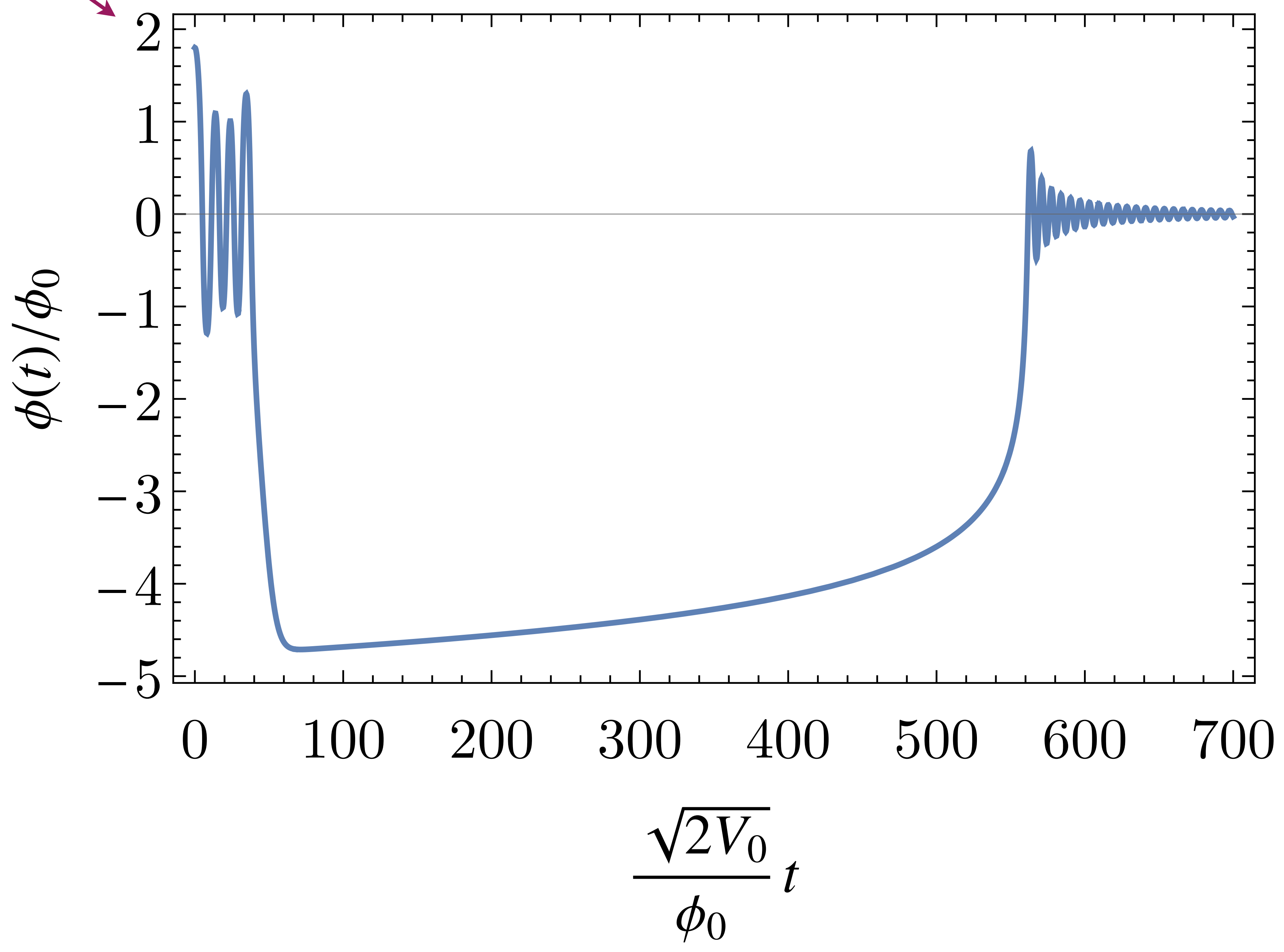


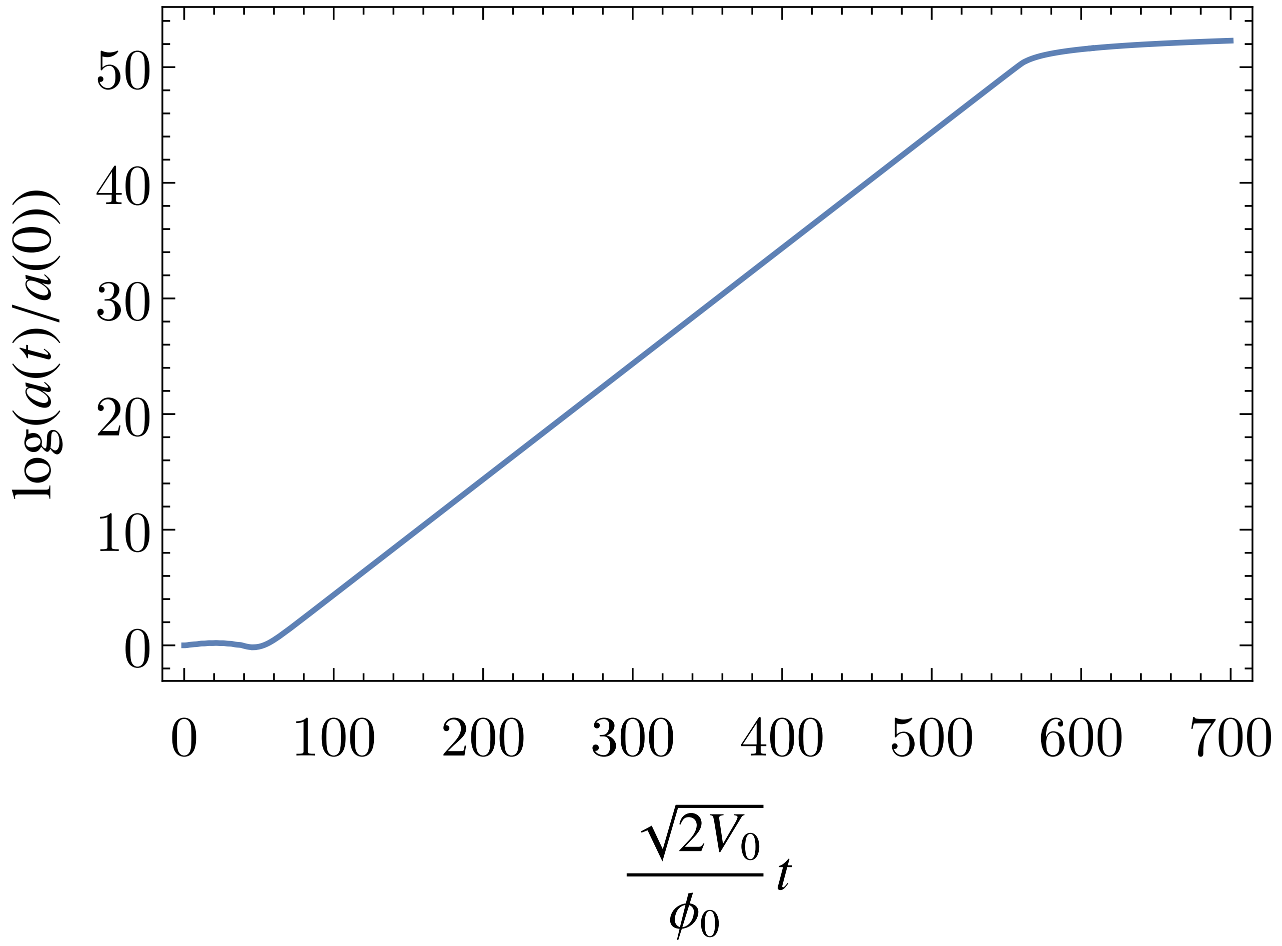


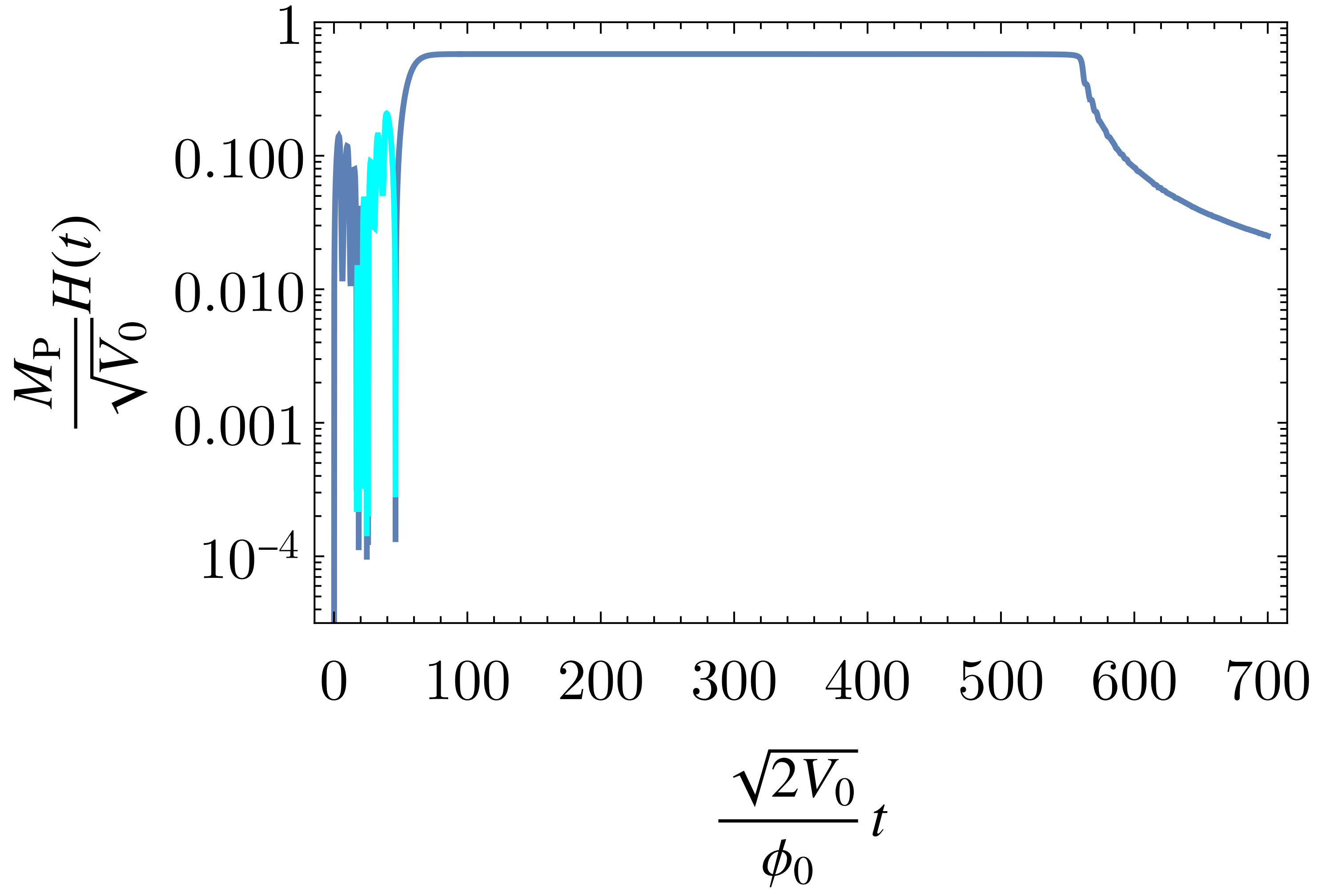
Full Picture

Quantum creation
(initial conditions)

$\phi_0 = \sqrt{0.06}$ and $\Gamma/\sqrt{V_0} = 0.019$





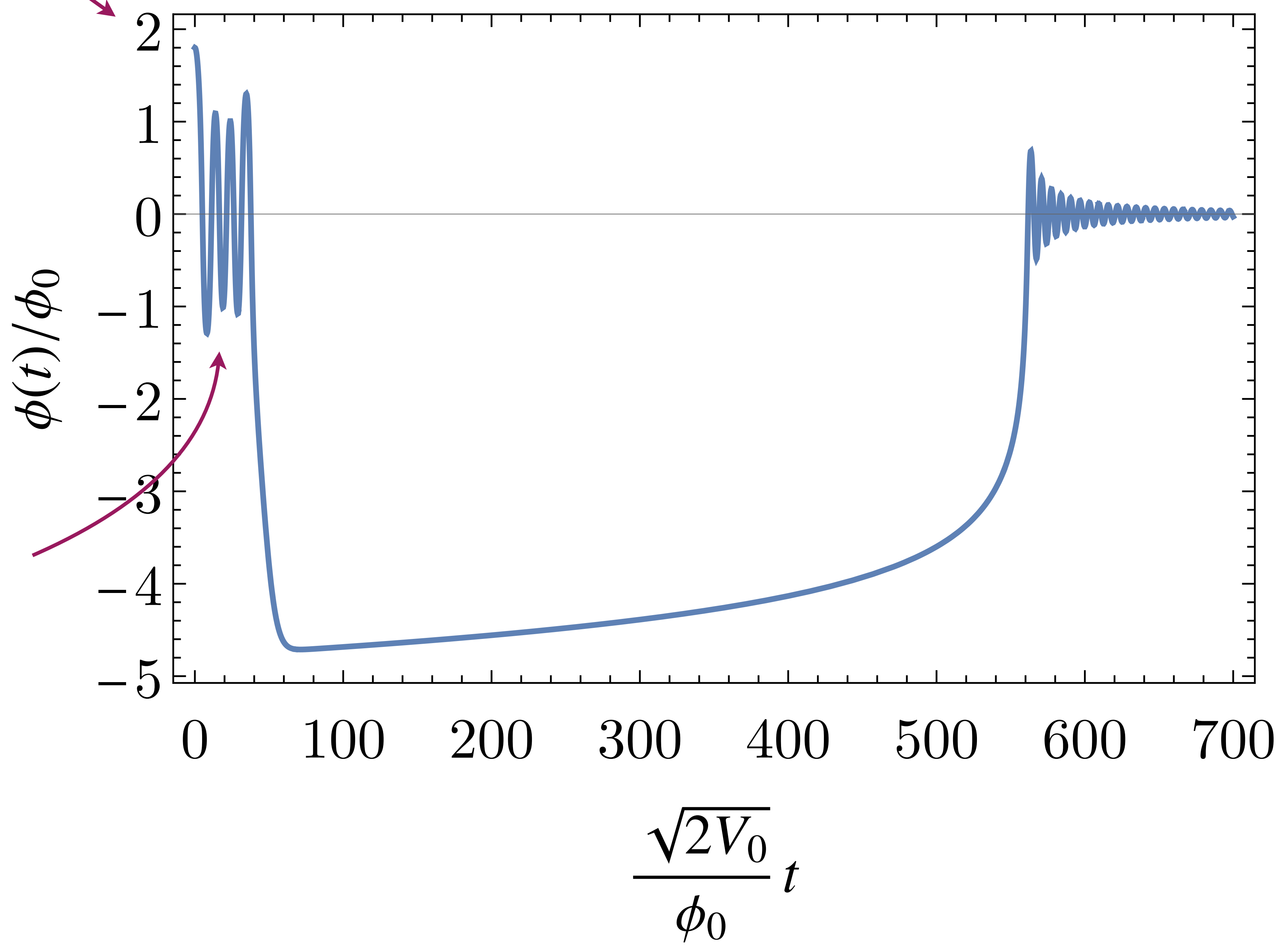


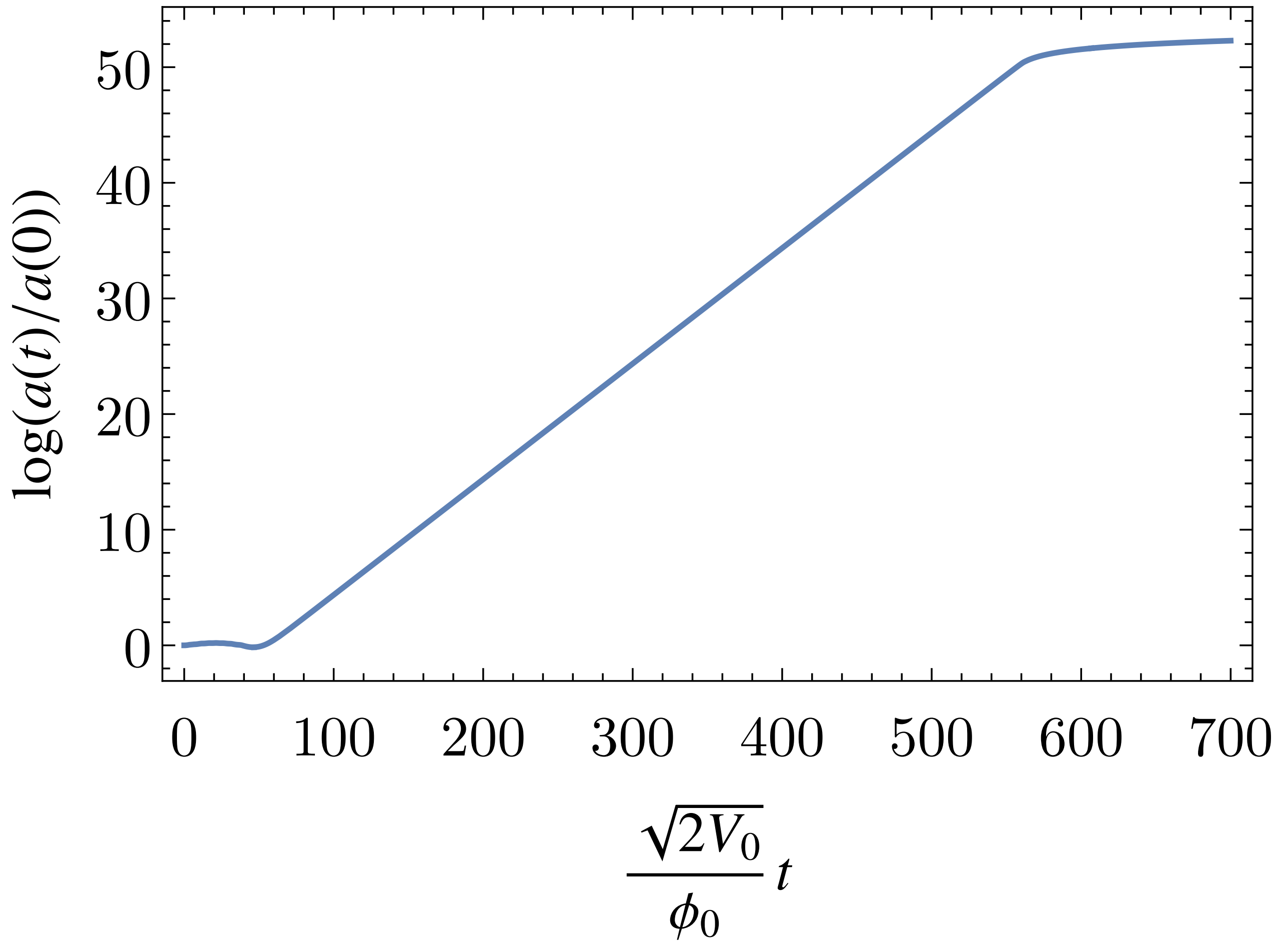
Full Picture

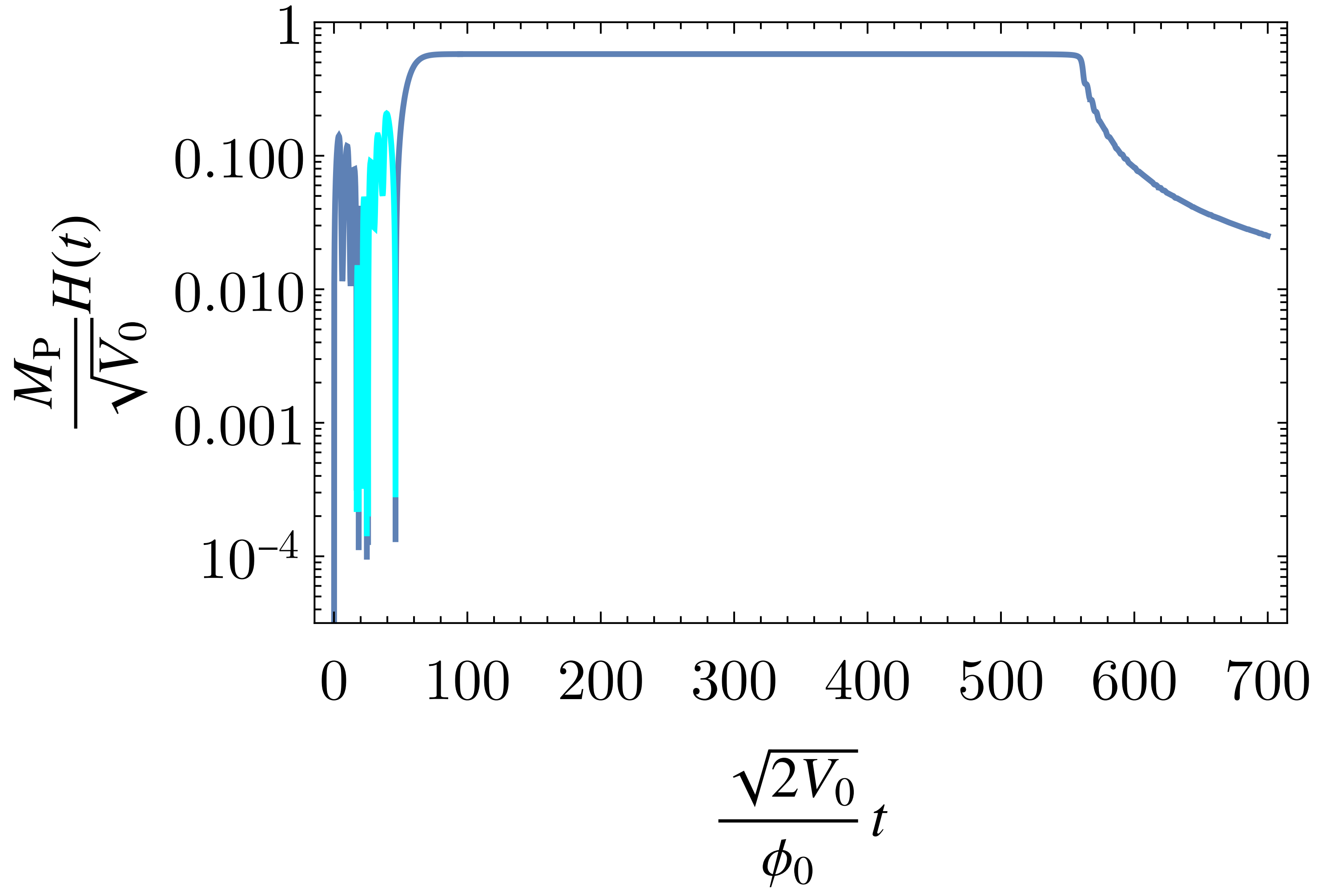
Quantum creation (initial conditions)

Quasi-cyclic period

$\phi_0 = \sqrt{0.06}$ and $\Gamma/\sqrt{V_0} = 0.019$







Full Picture

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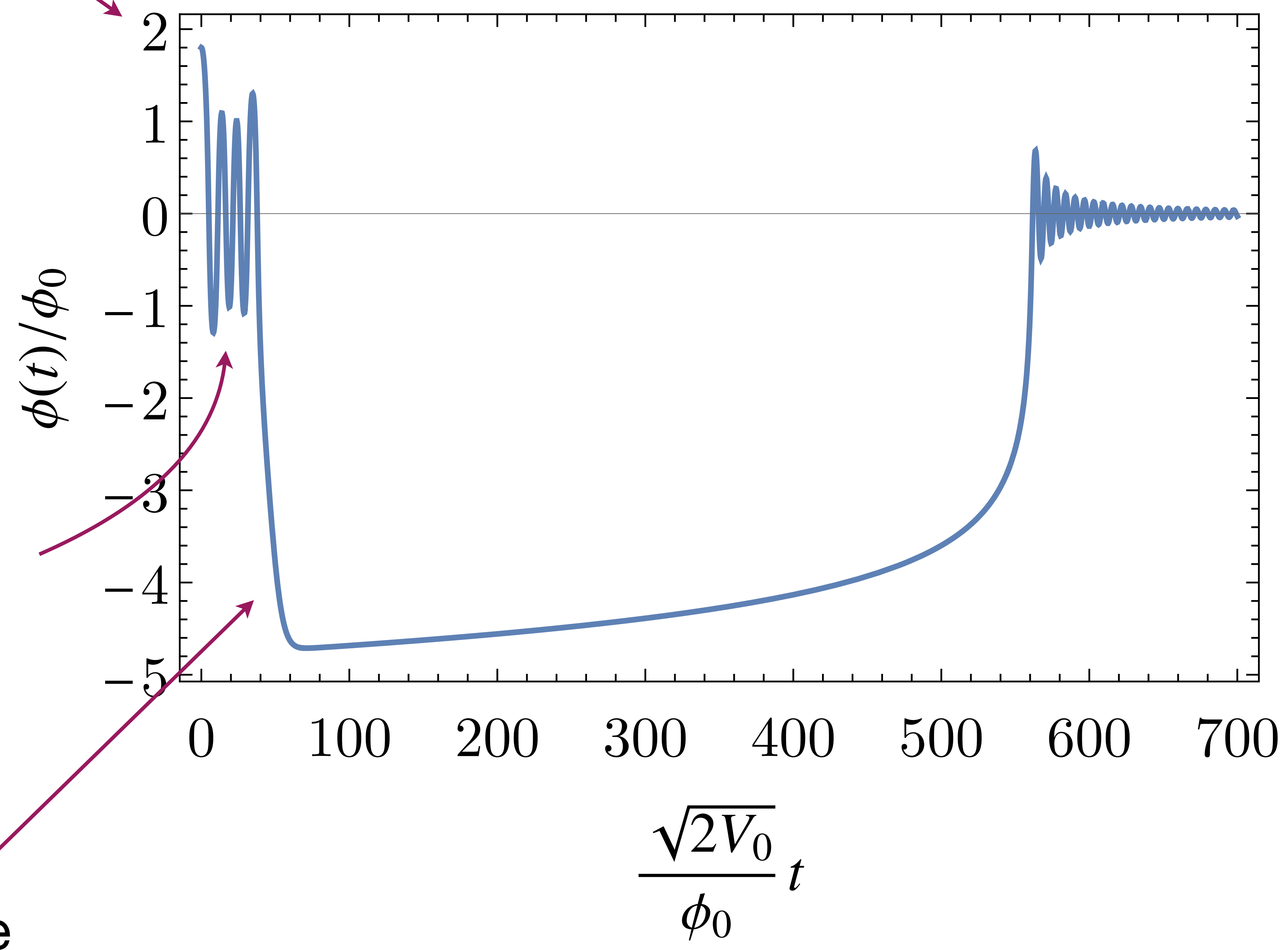
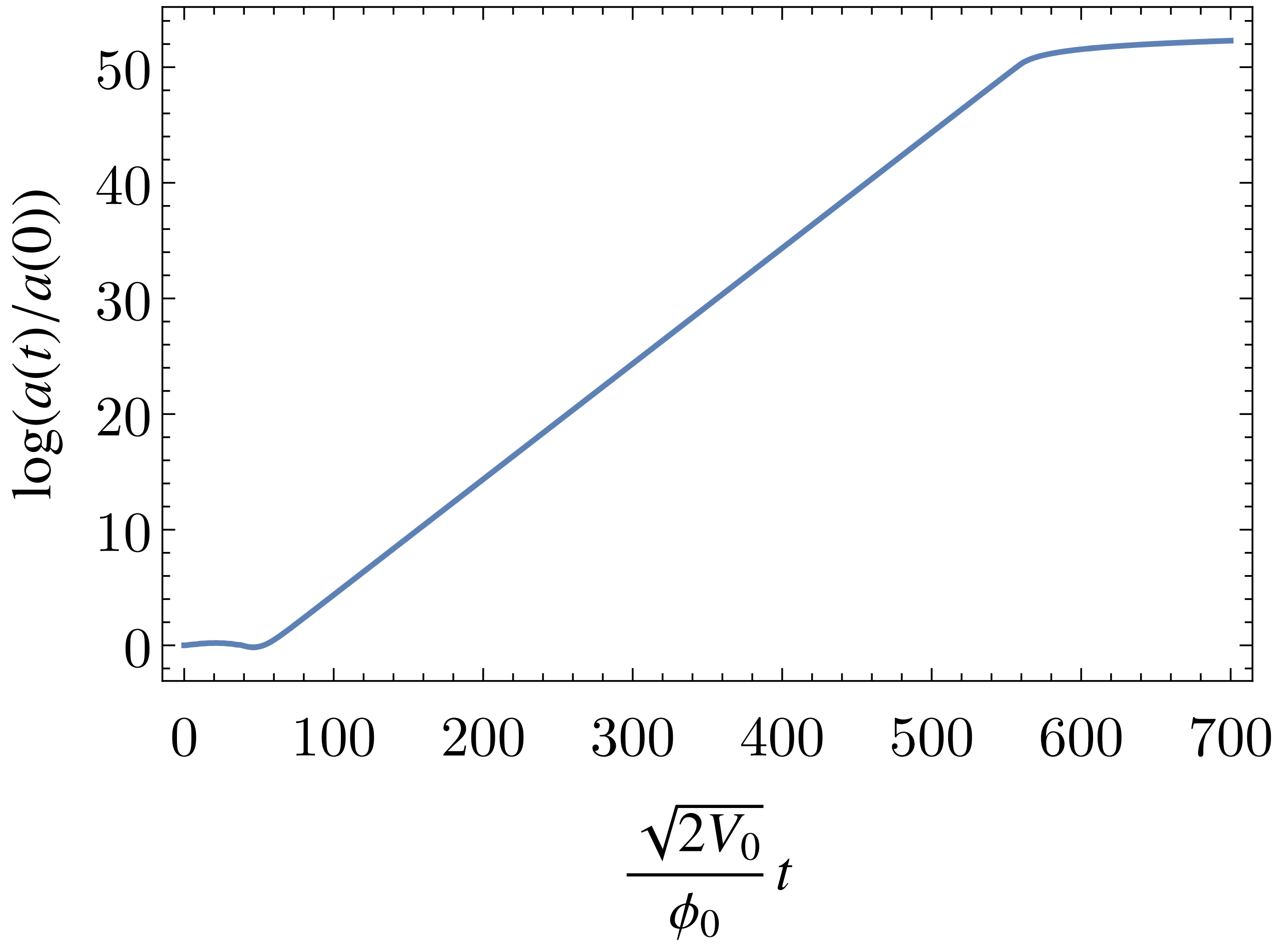
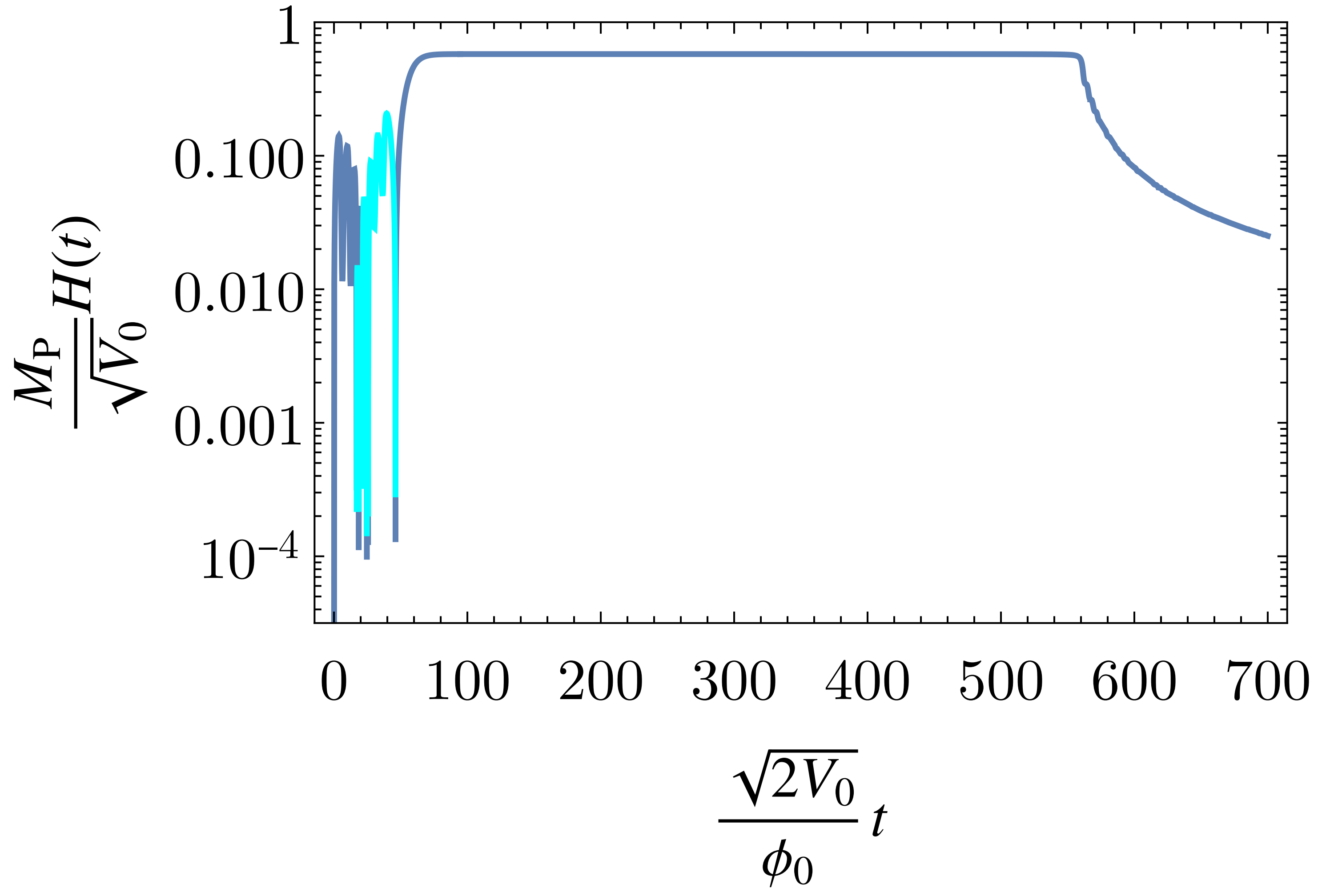


Quasi-cyclic period

Final bounce





Full Picture

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(initial conditions)

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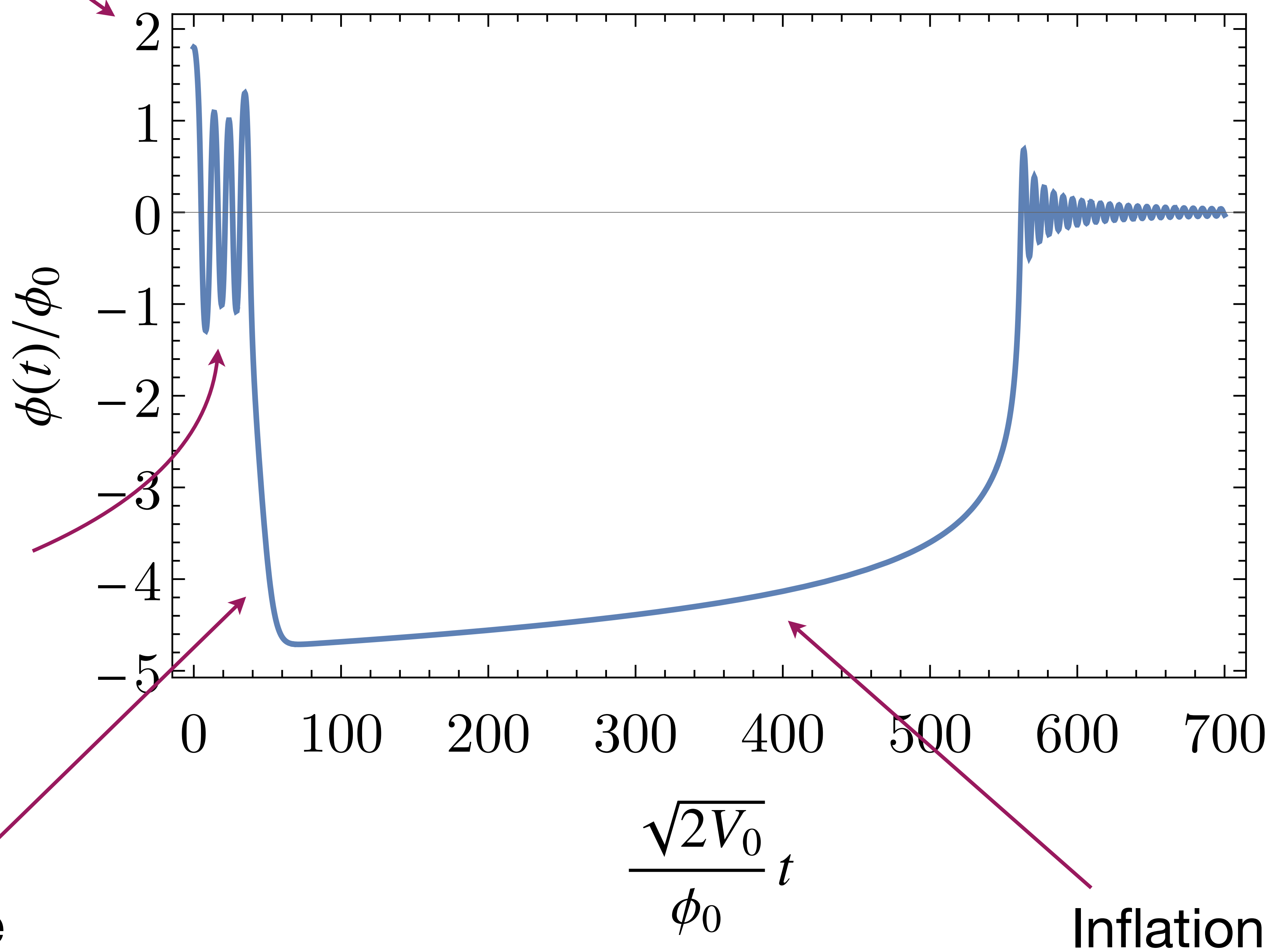
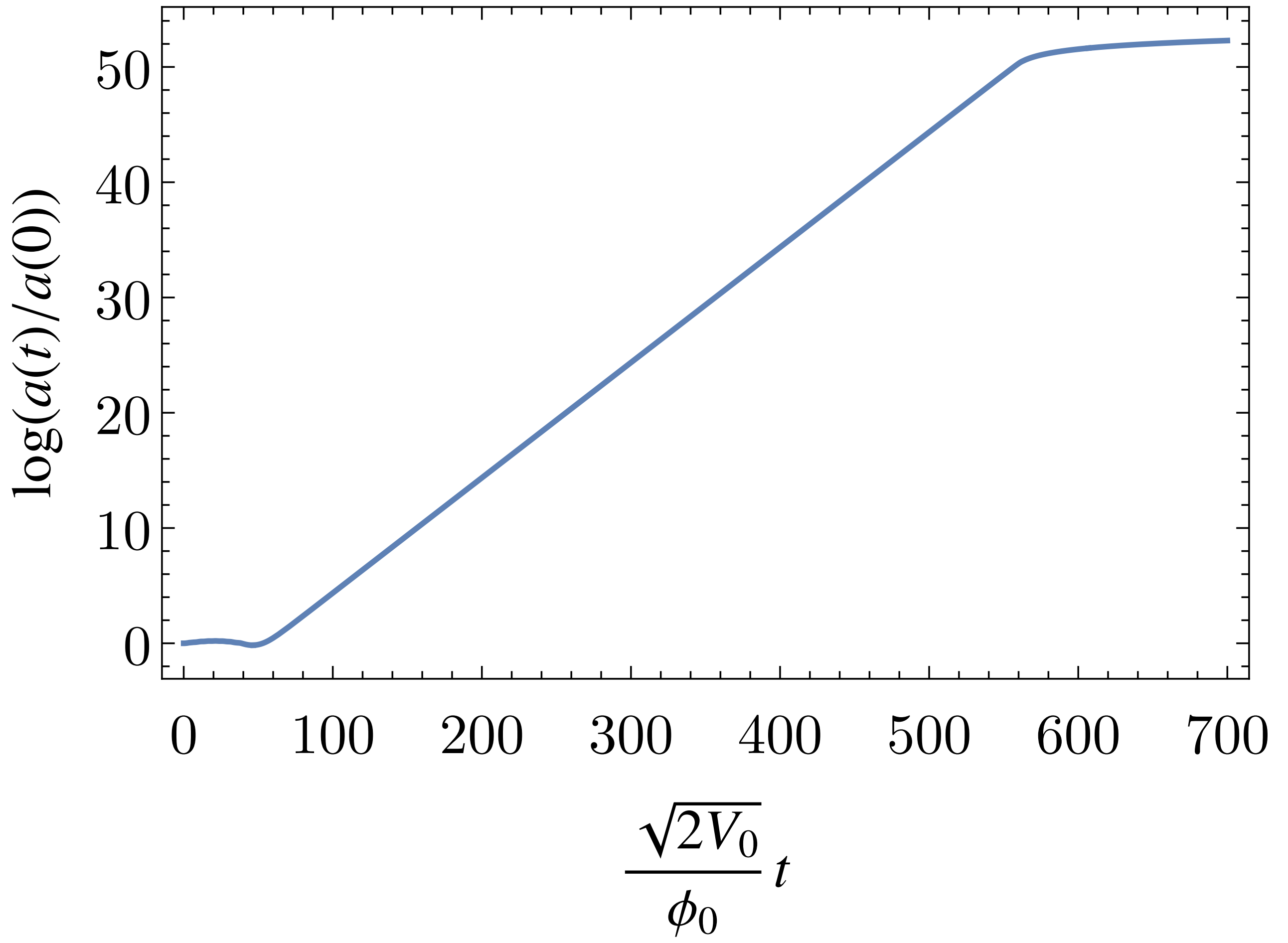


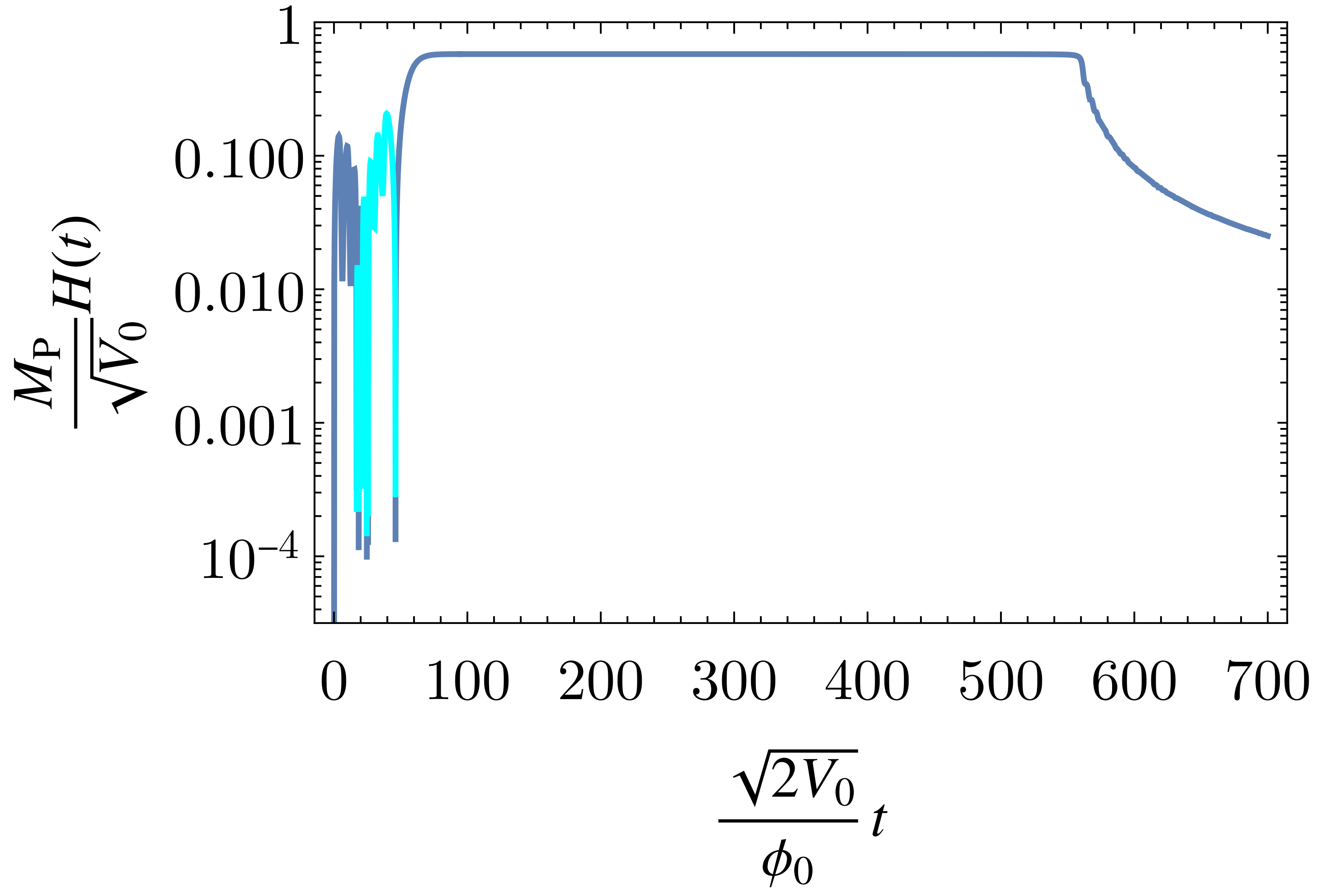
Quasi-cyclic period

Final bounce

Inflation
(consistent with CMB)

$V_0 = (2.6 \times 10^{15}\,\mathrm{GeV})^4, \quad r \approx 4 \times 10^{-5}\,(55/N_\mathrm{e})^2$





Full Picture

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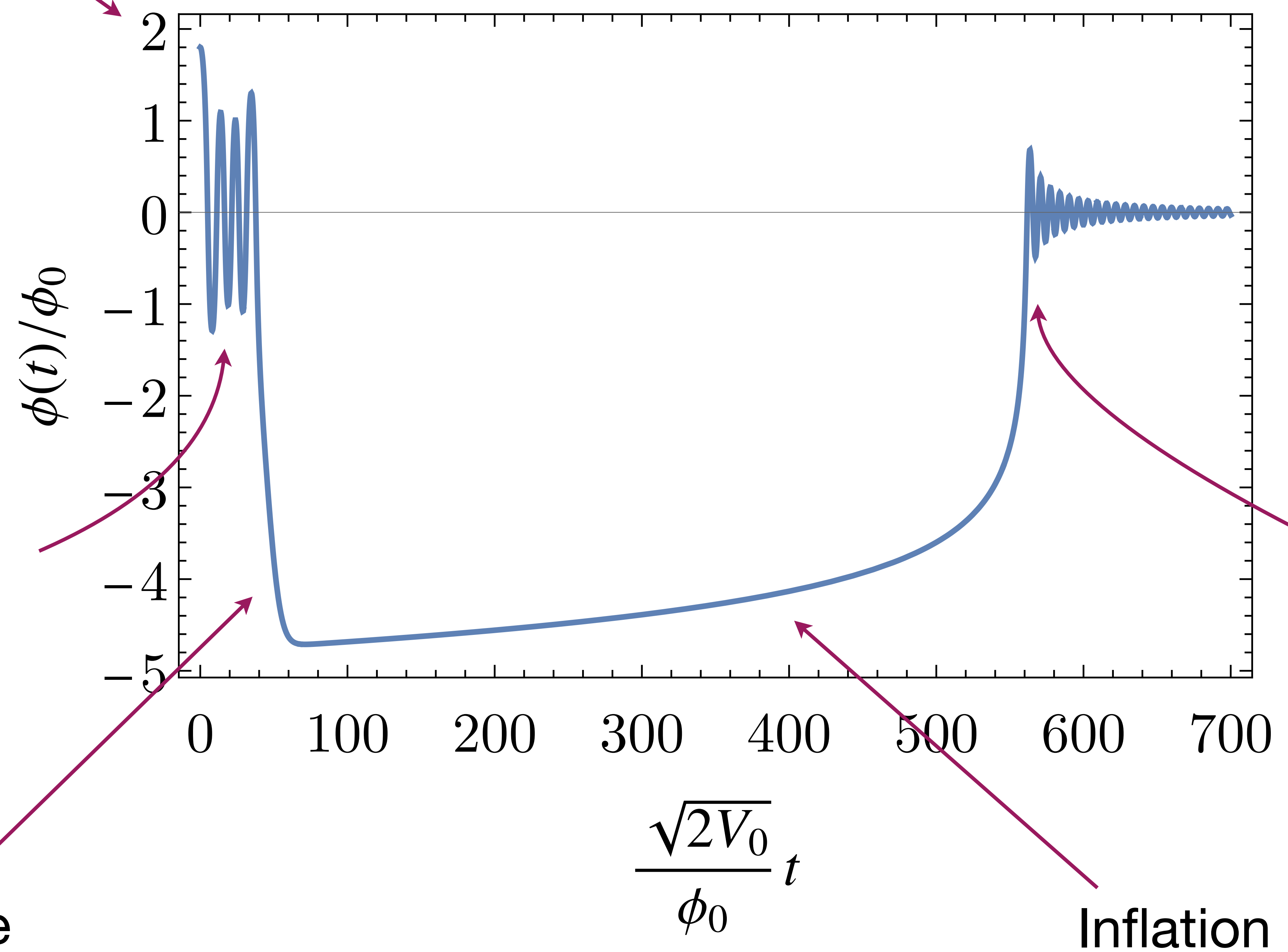
Quantum creation
(initial conditions)

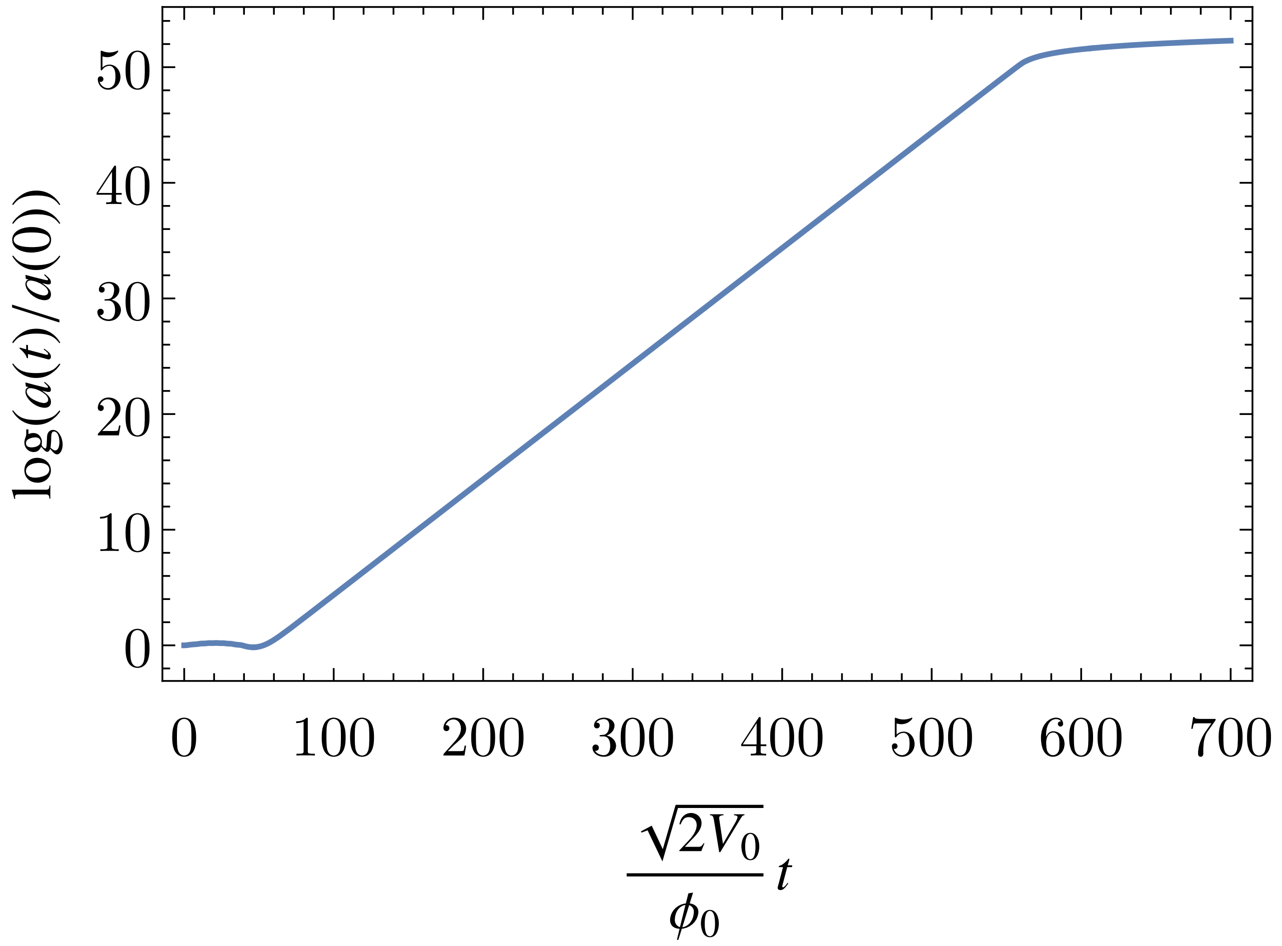
Quasi-cyclic period

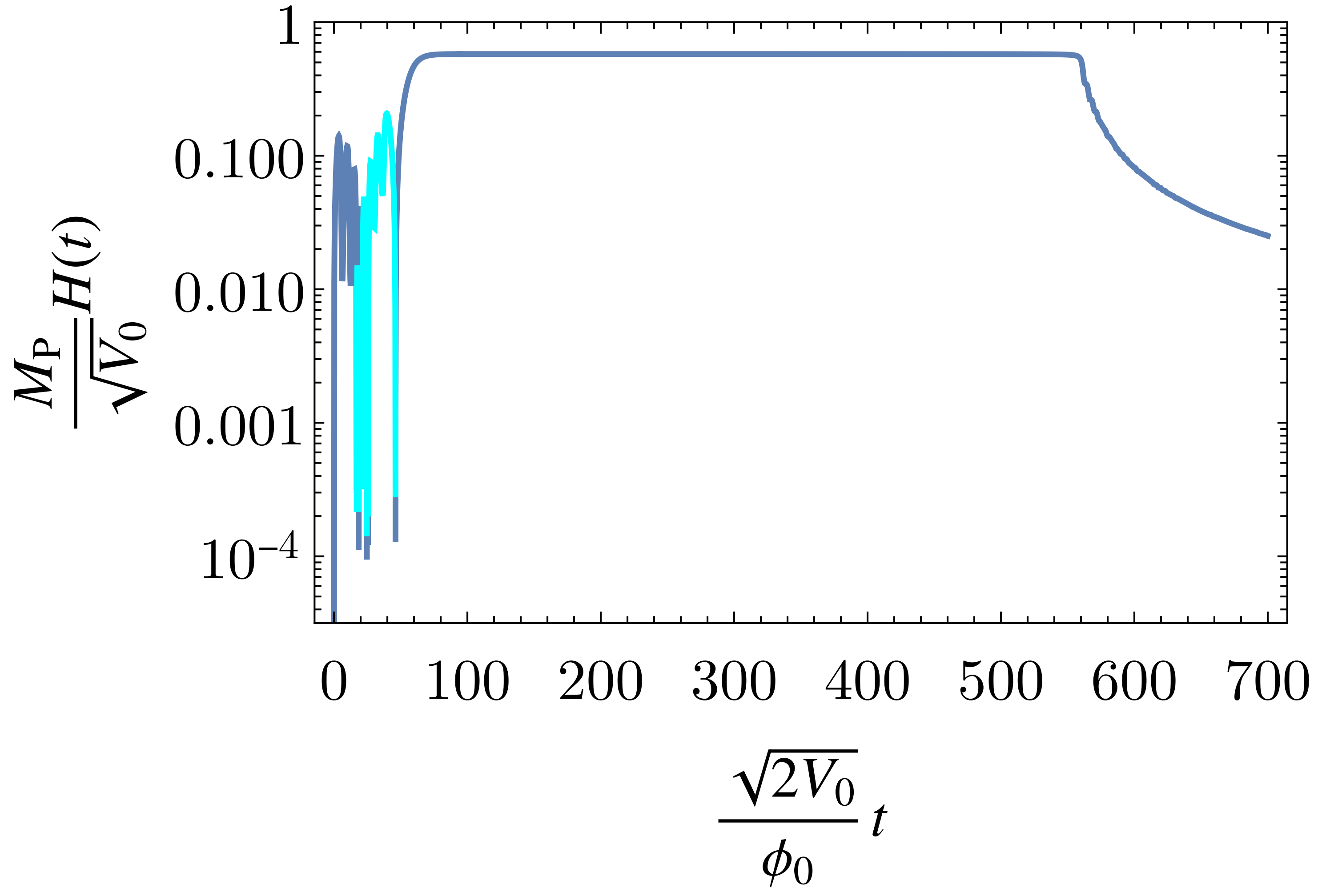
Final bounce



Inflation
(consistent with CMB)

No self-preheating



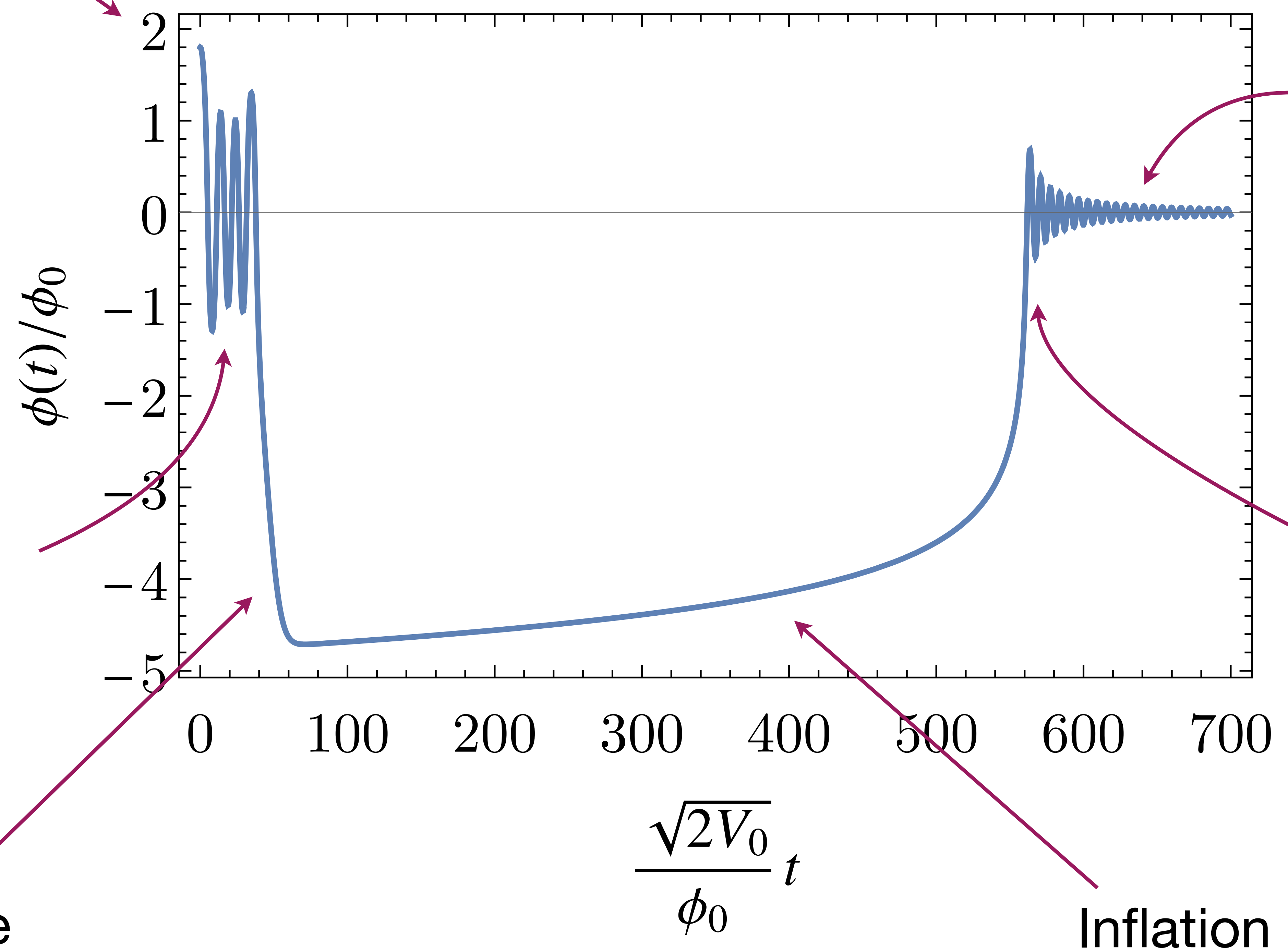


$V_0 = (2.6 \times 10^{15}\,\mathrm{GeV})^4, \quad r \approx 4 \times 10^{-5}\,(55/N_e)^2$

Full Picture

Quantum creation
(initial conditions)

$\phi_0 = \sqrt{0.06}$ and $\Gamma/\sqrt{V_0} = 0.019$



Reheating

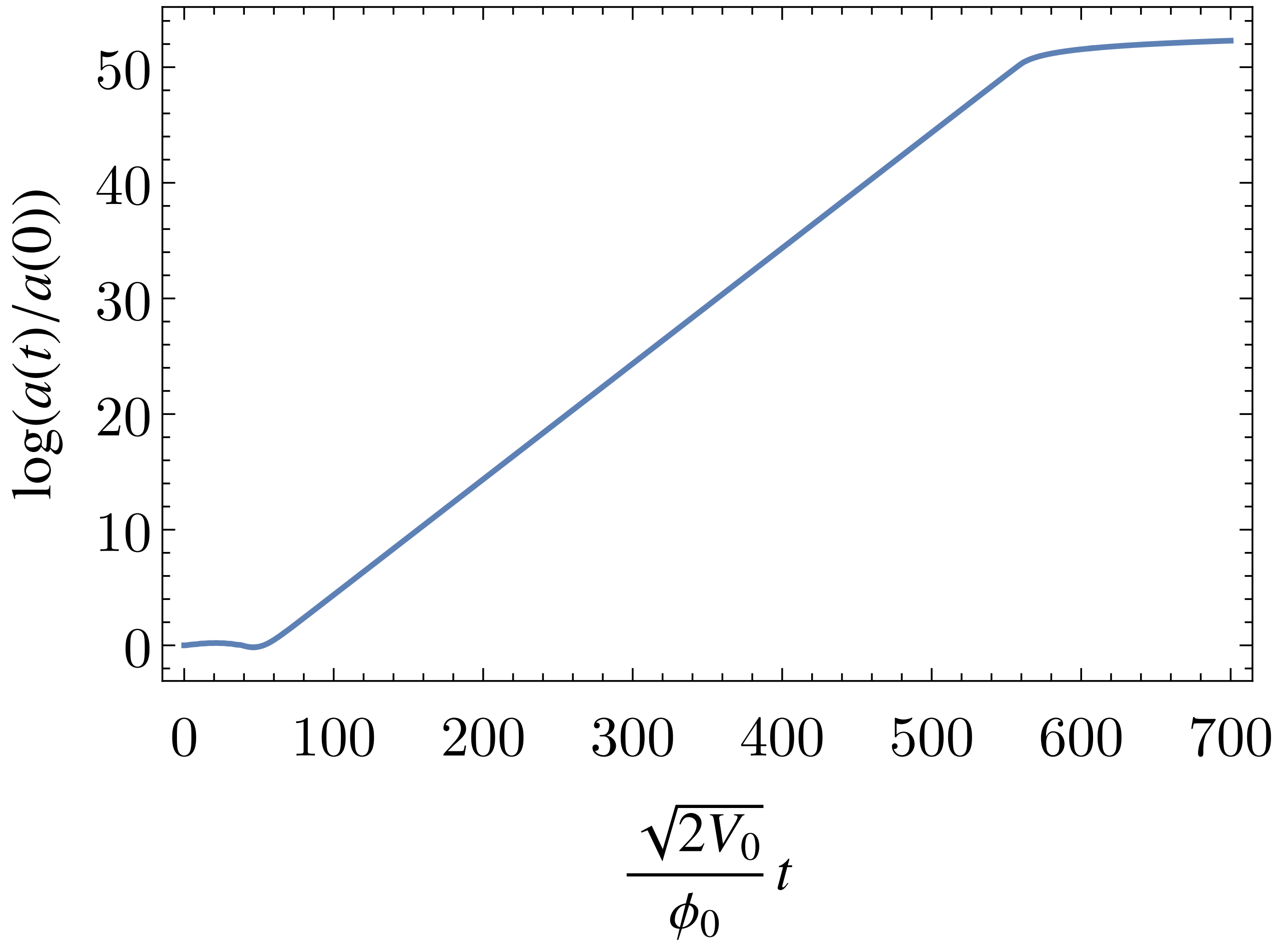
Quasi-cyclic period

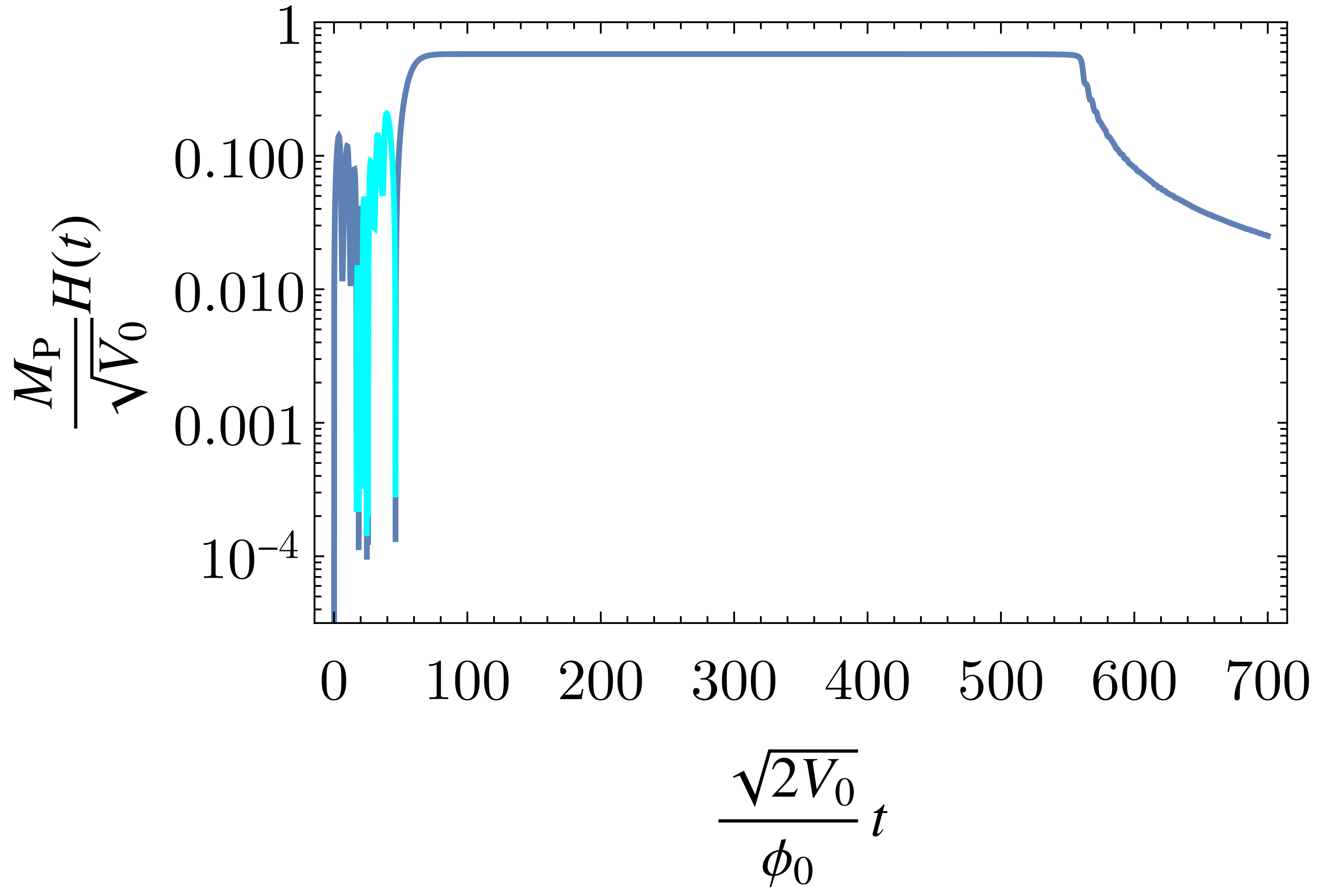
No self-preheating

Final bounce

Inflation
(consistent with CMB)

$V_0 = (2.6 \times 10^{15}\,\mathrm{GeV})^4, \quad r \approx 4 \times 10^{-5}\,(55/N_e)^2$





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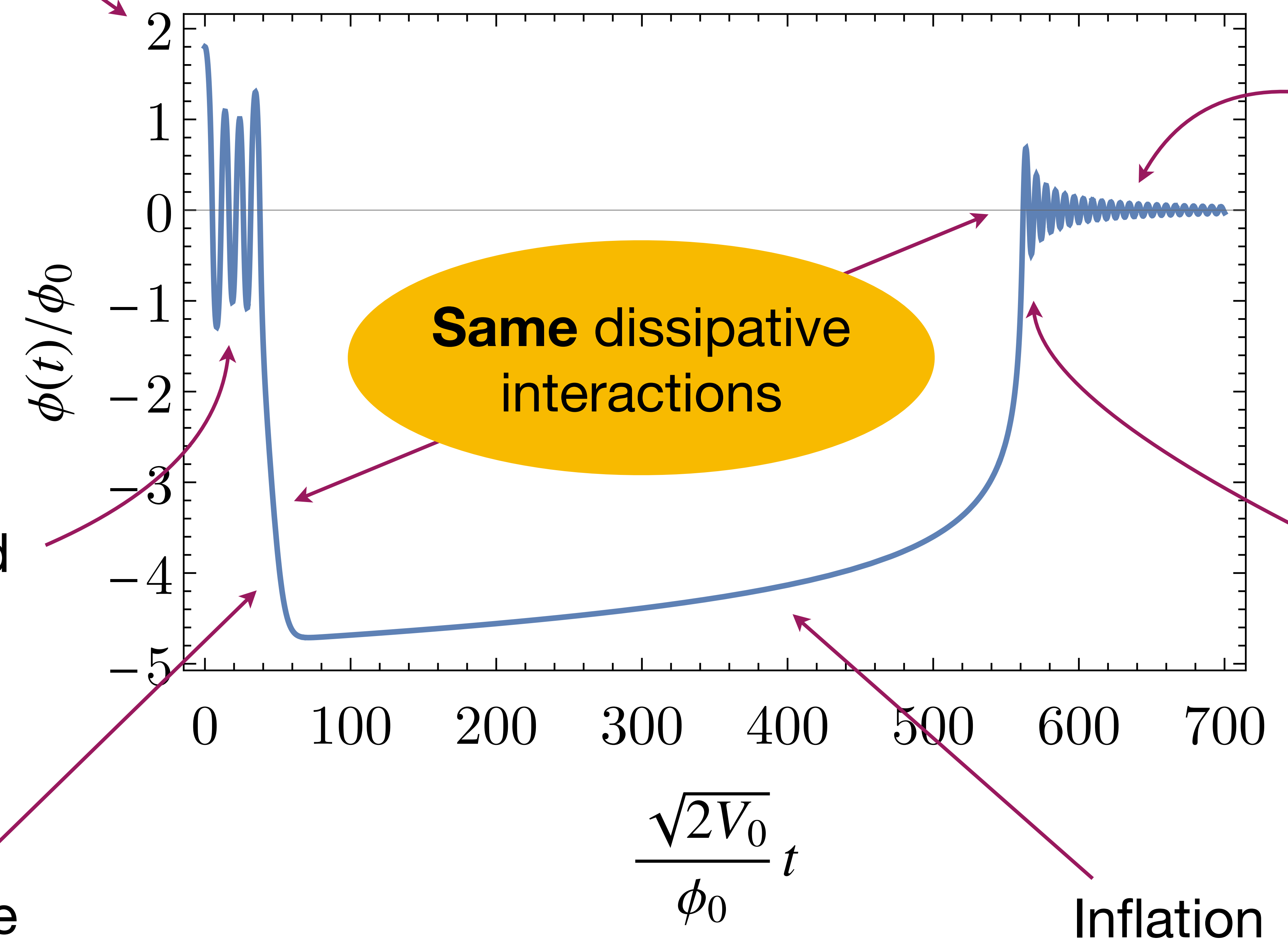
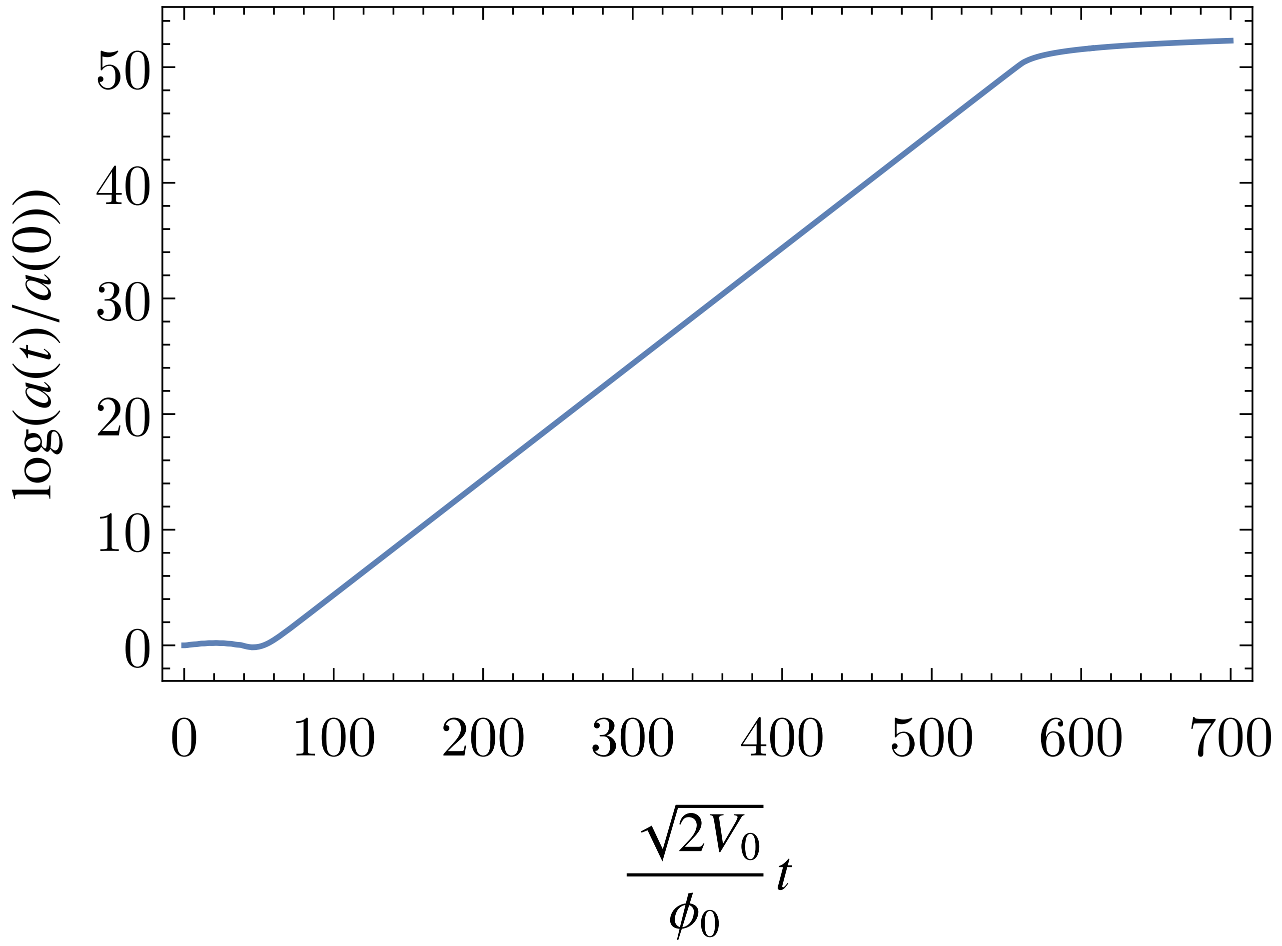
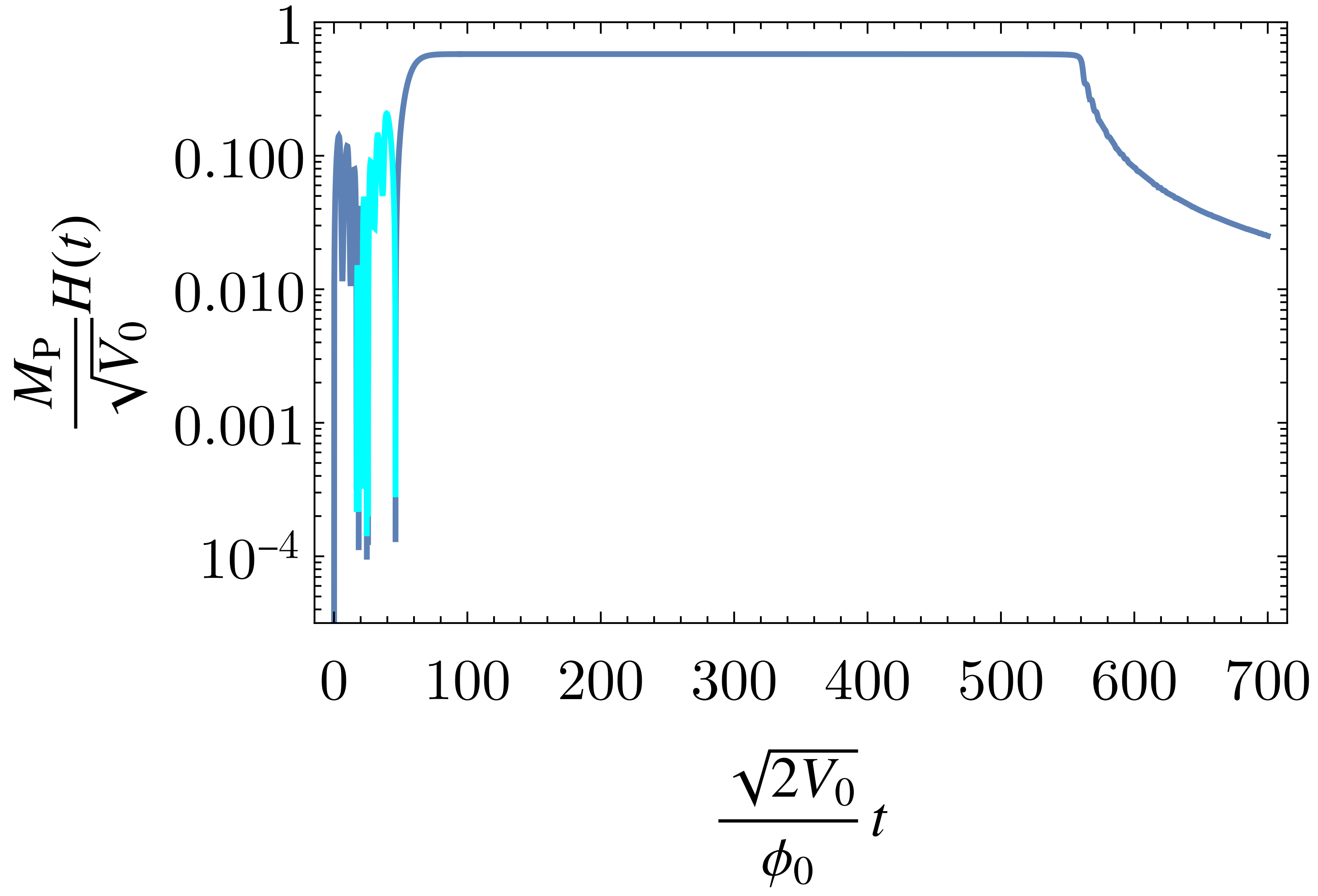
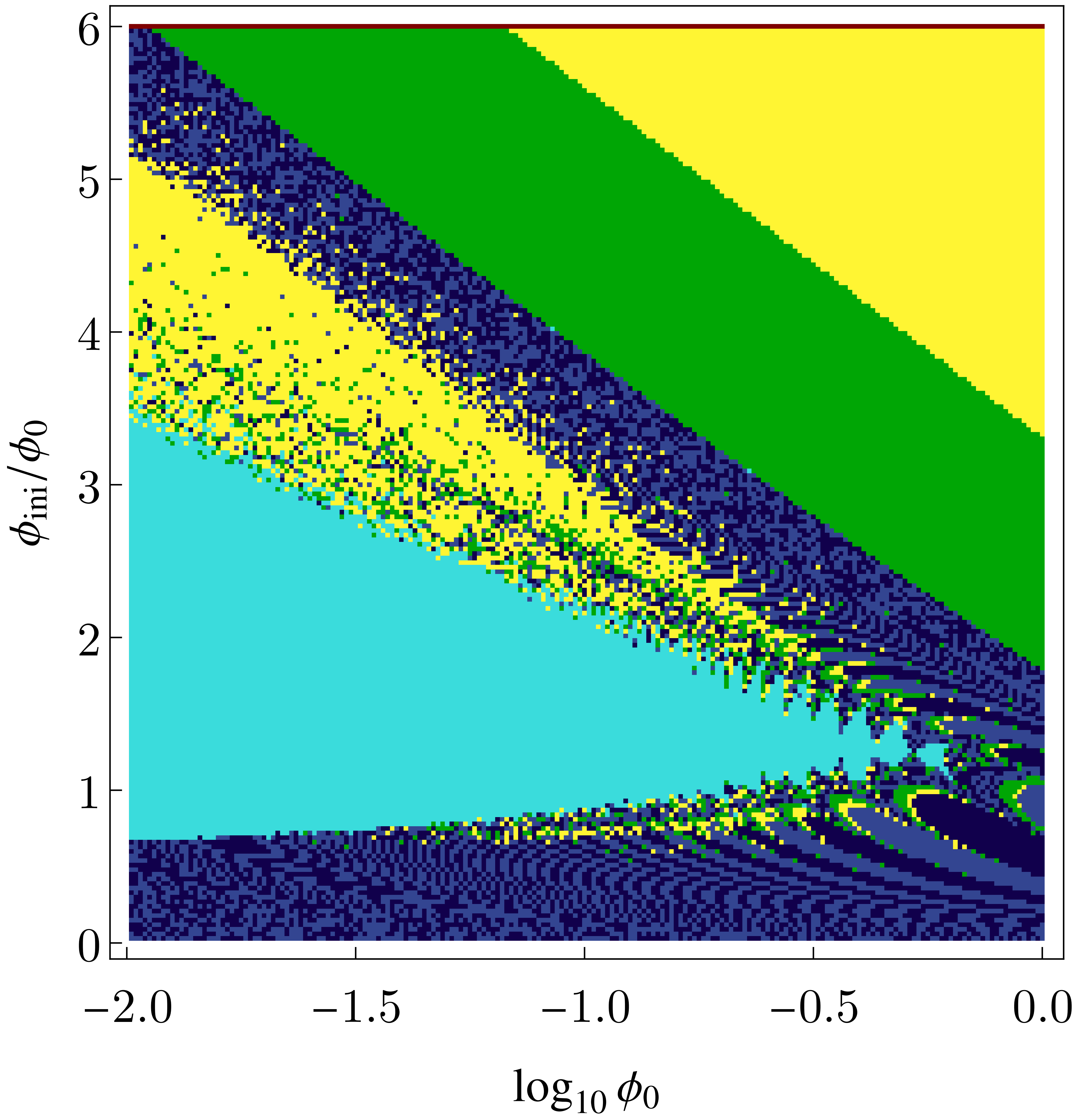






$V_0 = (2.6 \times 10^{15}\,\mathrm{GeV})^4, \quad r \approx 4 \times 10^{-5}\,(55/N_e)^2$

Generality of the Mechanism

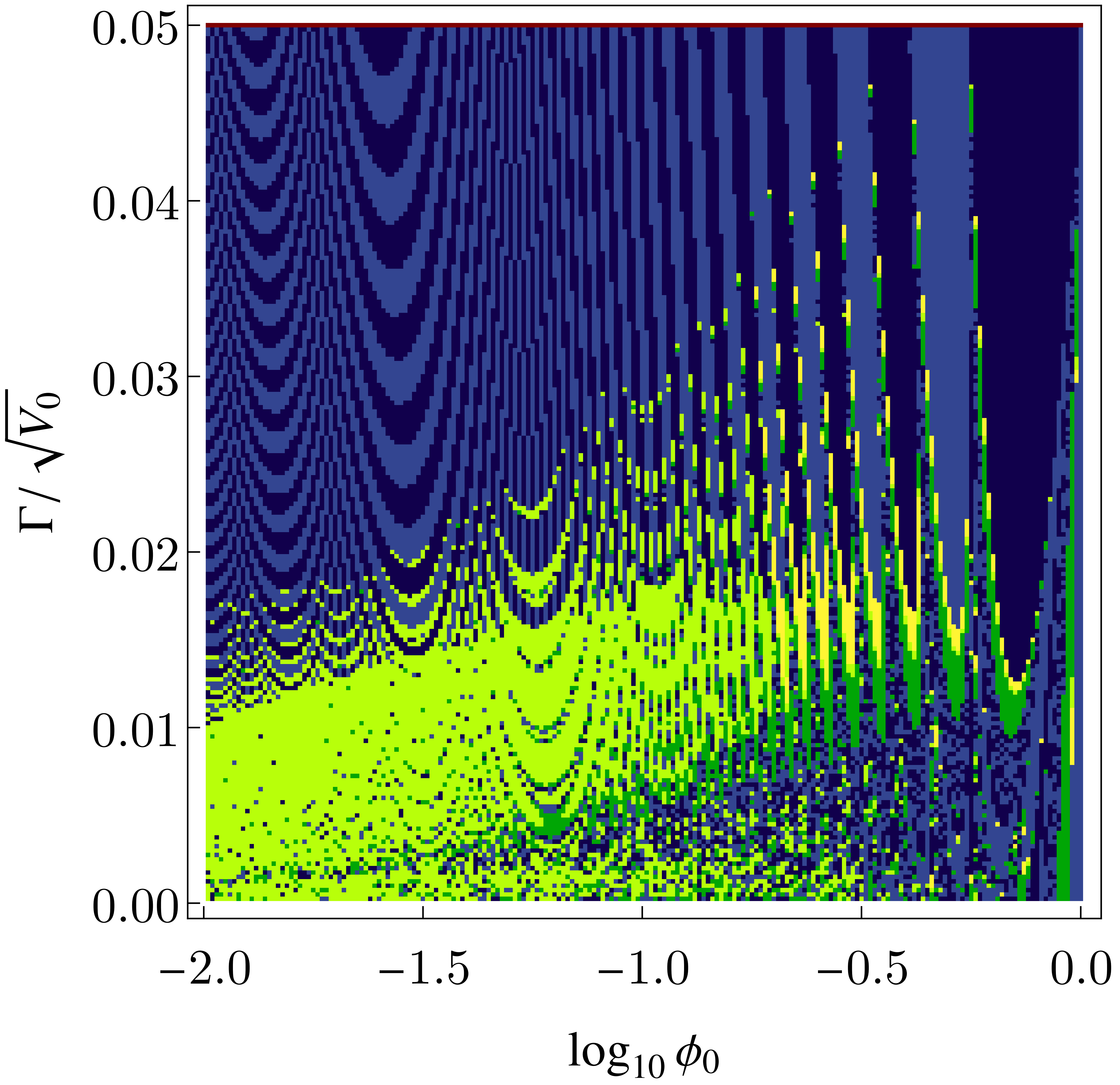


How generic are the cyclic solutions?

To answer the question,
we turn off the tachyonic instability and dissipation.

- Big Crunch with $\dot{\phi} > 0$
- Big Crunch with $\dot{\phi} < 0$
- Cyclic Universe
- Short inflation ($N < 50$)
- Long inflation ($N \geq 50$)

Generality of the Mechanism



How generic is the beginning of inflation?

We set $\phi_0/\sqrt{V_0} = 2.1 \times 10^5$ and $\phi_{\rm ini}/\phi_0 = 1 - 1.3 \log_{10} \phi_0$.

- Big Crunch with $\dot{\phi} > 0$
- Big Crunch with $\dot{\phi} < 0$
- Cyclic Universe
- Short inflation ($N < 50$)
- Long inflation ($N \geq 50$) if fragmentation is neglected
- Long inflation ($N \geq 50$) before the fragmentation time

Explicit Model

$\mathrm{SU}(N_c)$ gauge fields with $\quad \mathscr{L}_{\text{int}} = -\dfrac{\alpha}{4\pi f}\phi \tilde{F}^{\mu\nu}F_{\mu\nu}, \quad \mathscr{L}_{\text{matter}} = -\dfrac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}.$

Dissipation rate $\quad \Gamma(T) = \Upsilon(T) + \Gamma_{\text{sct}}(T) + \Gamma_{\text{dec}}$

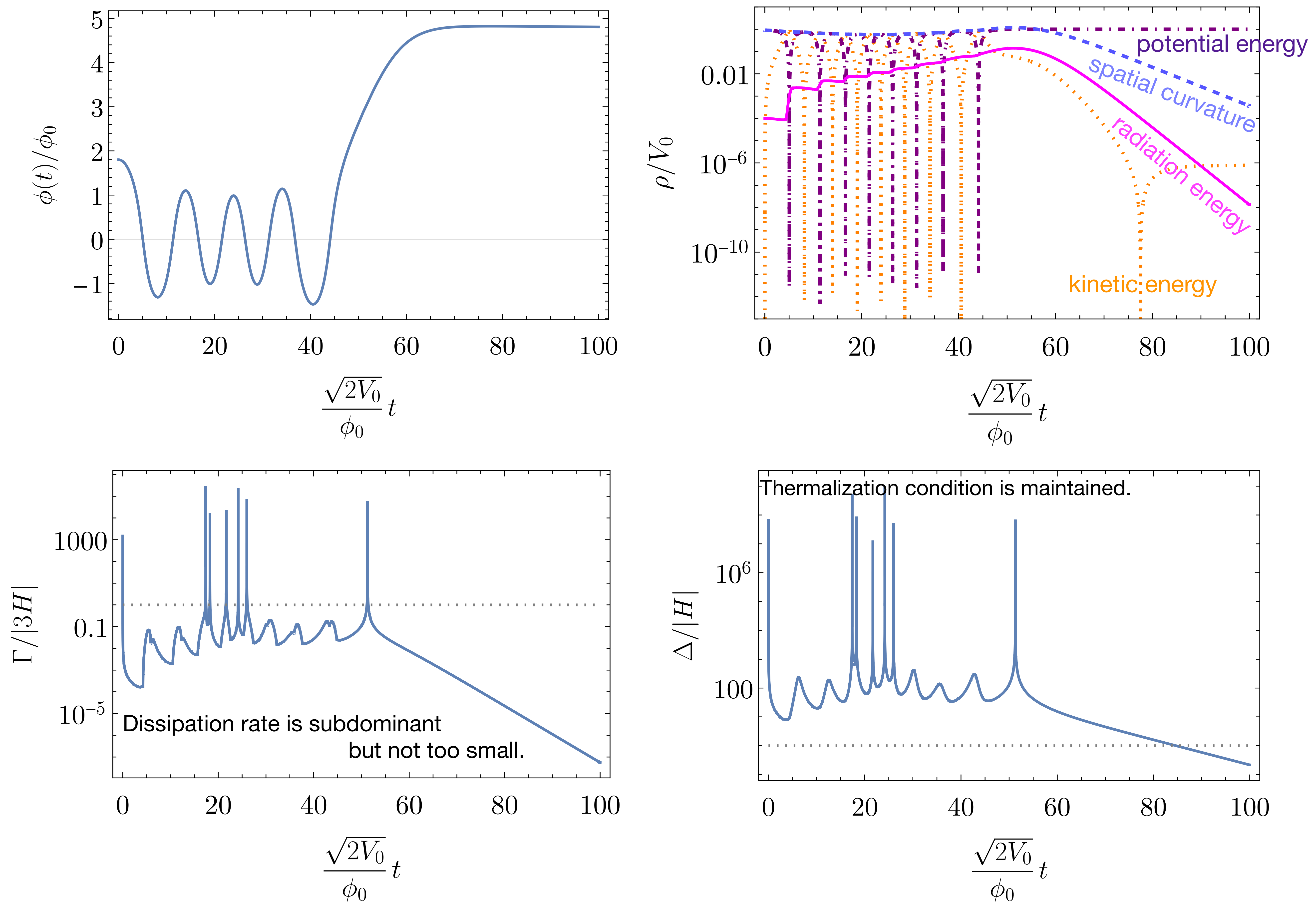
sphaleron-induced friction $\quad \Upsilon(T) \sim \dfrac{(N_c\alpha)^5 T^3}{f^2}$

scattering rate $\quad \Gamma_c \simeq \dfrac{C(N_c^2-1)T(p^0)^2}{64\pi^4 f^2}$

decay rate $\quad \Gamma_{\text{dec}} = \dfrac{(N_c^2-1)\alpha^2 m_\phi^3}{64\pi^3 f^2}$

We follow [DeRocco, Graham, Kalia, 2107.0757]
for the thermalization criteria.

- Nonlinearity develops in the relevant time scale.
- Backreaction is negligible until it becomes nonlinear.
- Thermalization rate is greater than the Hubble rate.



Explicit Model

$\mathrm{SU}(N_c)$ gauge fields with $\mathscr{L}_{\text{int}} = -\frac{\alpha}{4\pi f}\phi \tilde{F}^{\mu\nu}F_{\mu\nu}, \quad \mathscr{L}_{\text{matter}} = -\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}.$

Reheating temperature in the dark sector $T_{\mathrm{R}}^{\text{dark}} = \max\left[T_{\mathrm{R,\,dec}}, \min\left[T_{\mathrm{R,\,sct}}, T_{\max}\right]\right]$

where $T_{\mathrm{R,\,sct}} = 1.5 \times 10^{14}\,\mathrm{GeV}\left(\frac{C}{1}\right)\left(\frac{g_*}{22}\right)^{-1/2}\left(\frac{N_c^2-1}{3^2-1}\right),$

$$T_{\mathrm{R,\,dec}} = 1.3 \times 10^{12}\,\mathrm{GeV}\left(\frac{g_*}{22}\right)^{-1/4}\left(\frac{N_c^2-1}{3^2-1}\right)^{1/2}\left(\frac{\alpha}{0.015}\right)\left(\frac{m_\phi}{1.6\times 10^{13}\,\mathrm{GeV}}\right)^{3/2}\left(\frac{f}{5.8\times 10^{13}\,\mathrm{GeV}}\right)^{-1},$$

and the maximal attainable temperature after inflation $T_{\max} \leq T_{\text{inst}} \simeq 1.6\times 10^{15}\,\mathrm{GeV}\left(\frac{g_*}{22}\right)^{-1/4}\left(\frac{V_0^{1/4}}{2.6\times 10^{15}\,\mathrm{GeV}}\right).$

Some options to heat the Standard Model sector

- Dark glueball decay via dimension-6 operators.
- Dark Higgs portal.

Probability of Inflation

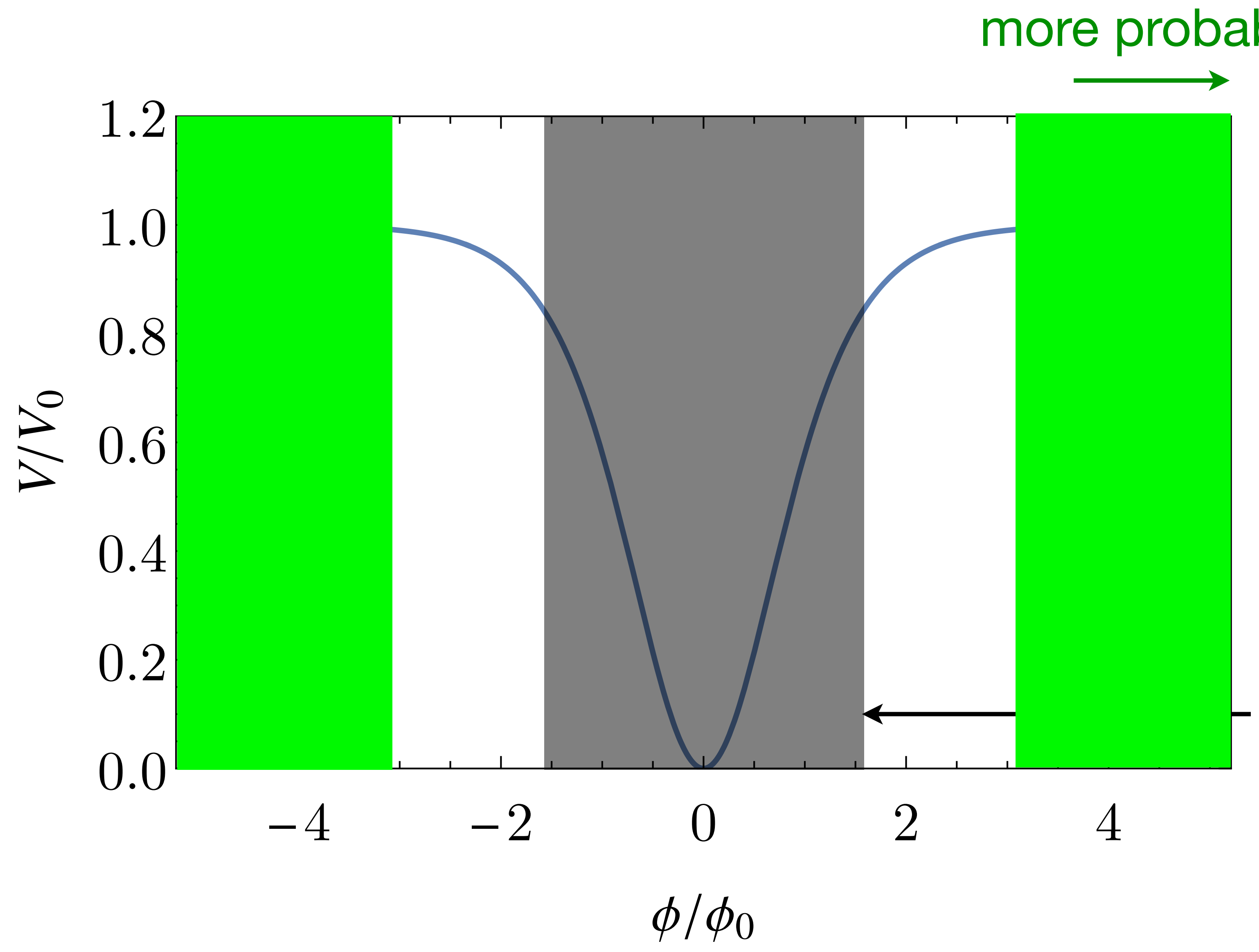
Probability distribution function of the homogeneous initial field value $\phi(0)$ in quantum cosmology

$$P[\phi(0)] \sim \exp\left(\pm\frac{24\pi^2}{V(\phi(0))}\right)$$

+: no-boundary proposal
−: tunneling proposal

Note: the exponent is $\mathcal{O}(10^{14})$ for the CMB-compatible case.

<u>tunneling proposal</u>



flat-potential assumption violated

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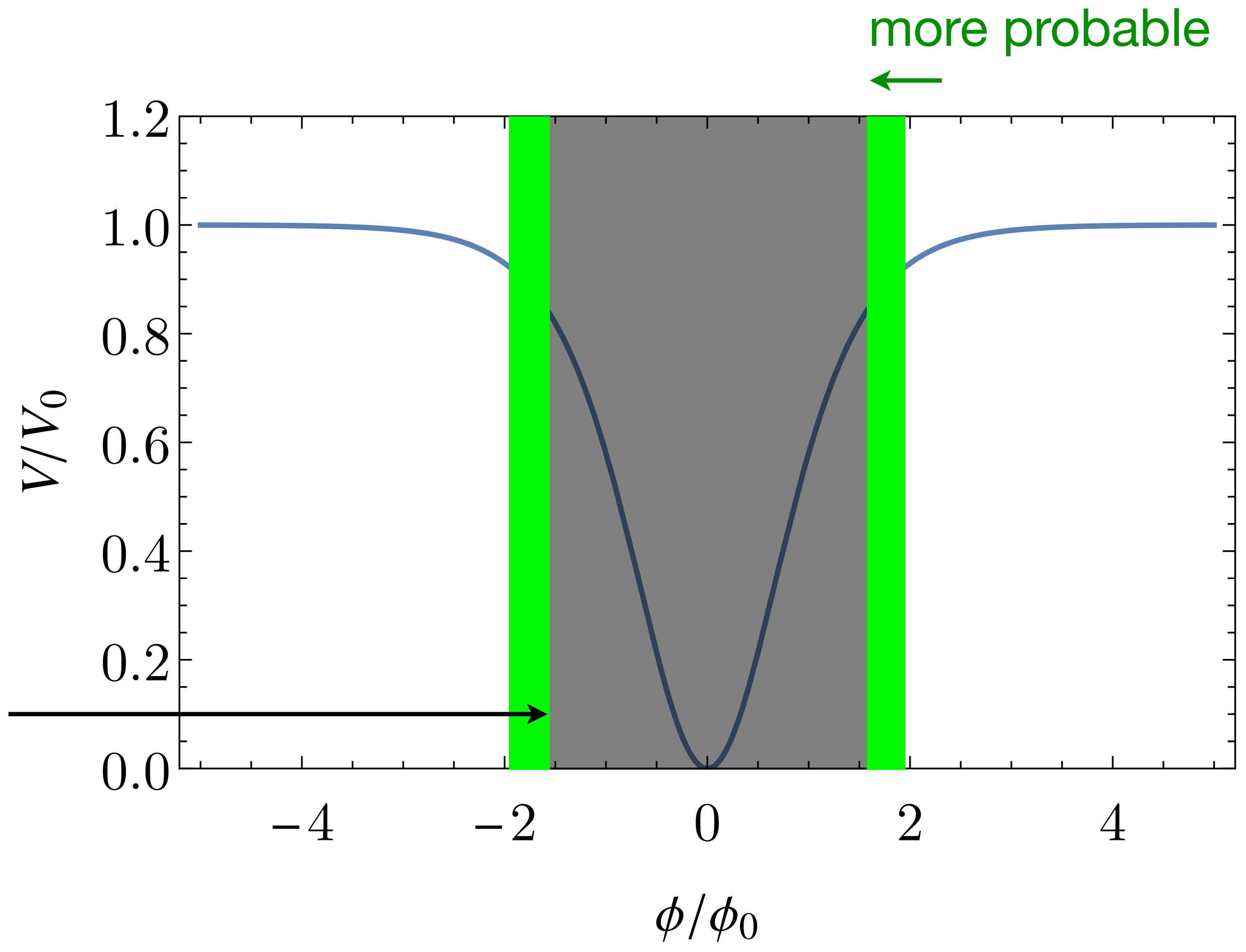

Probability of Inflation

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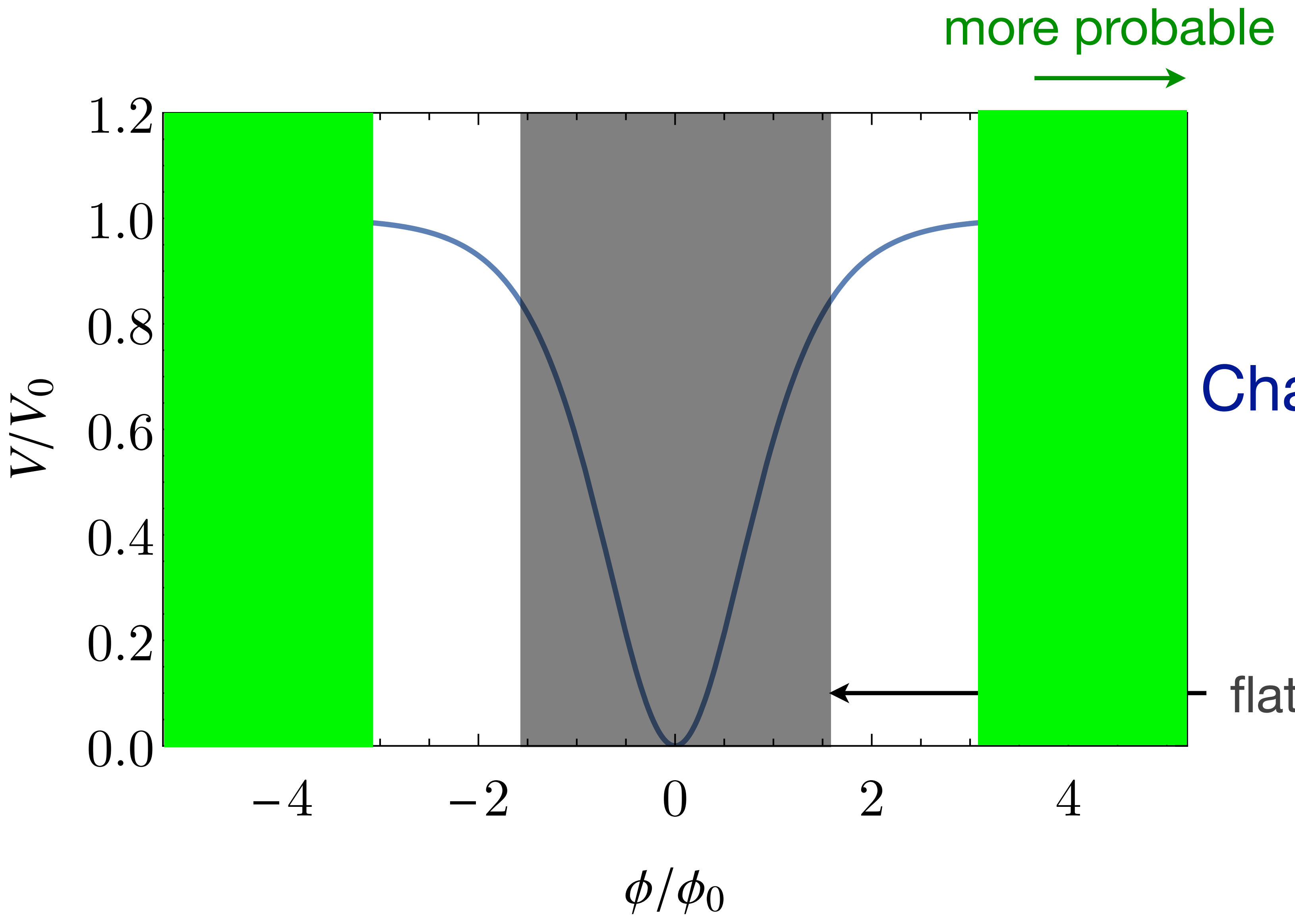

Chaotic inflation naturally occurs.

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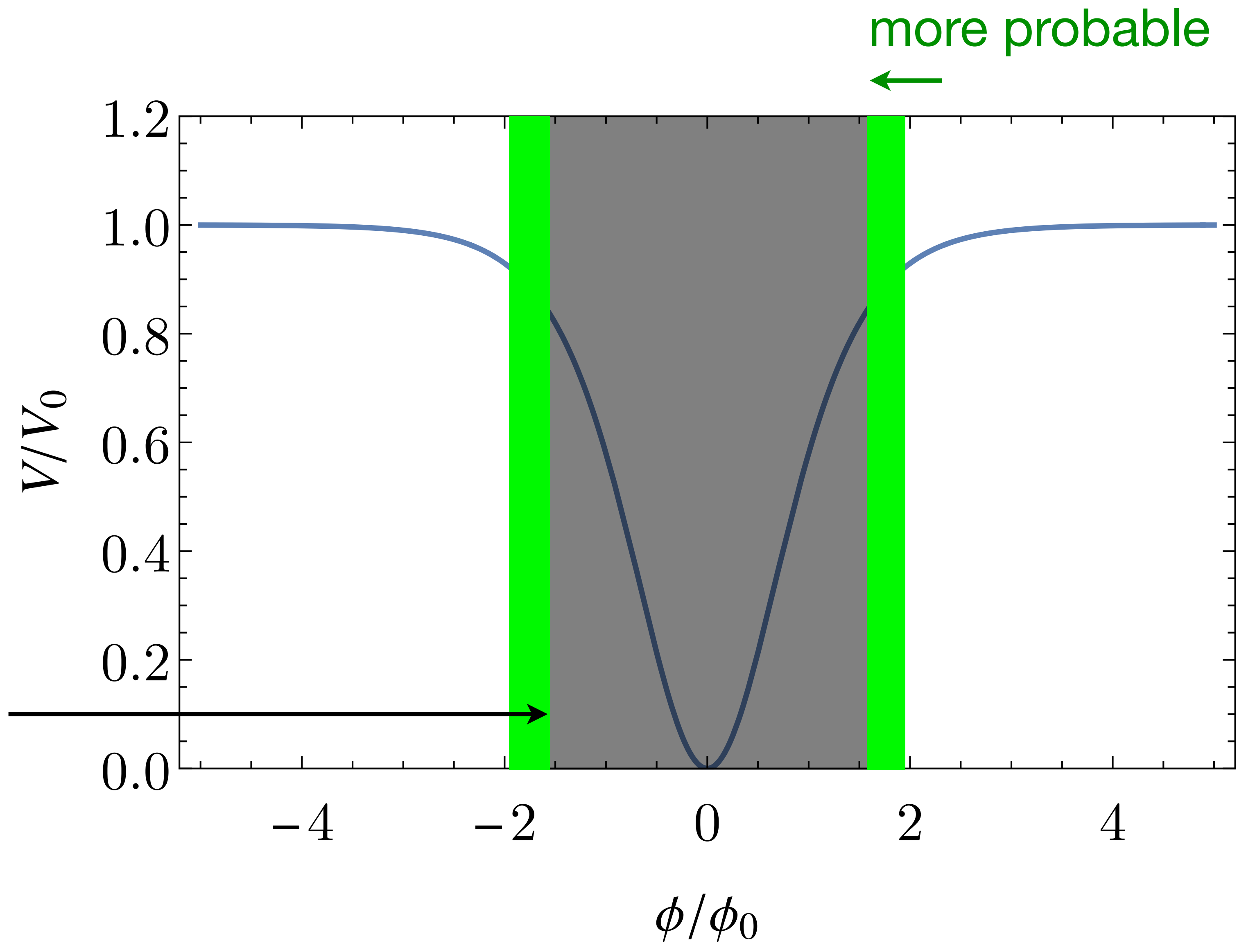

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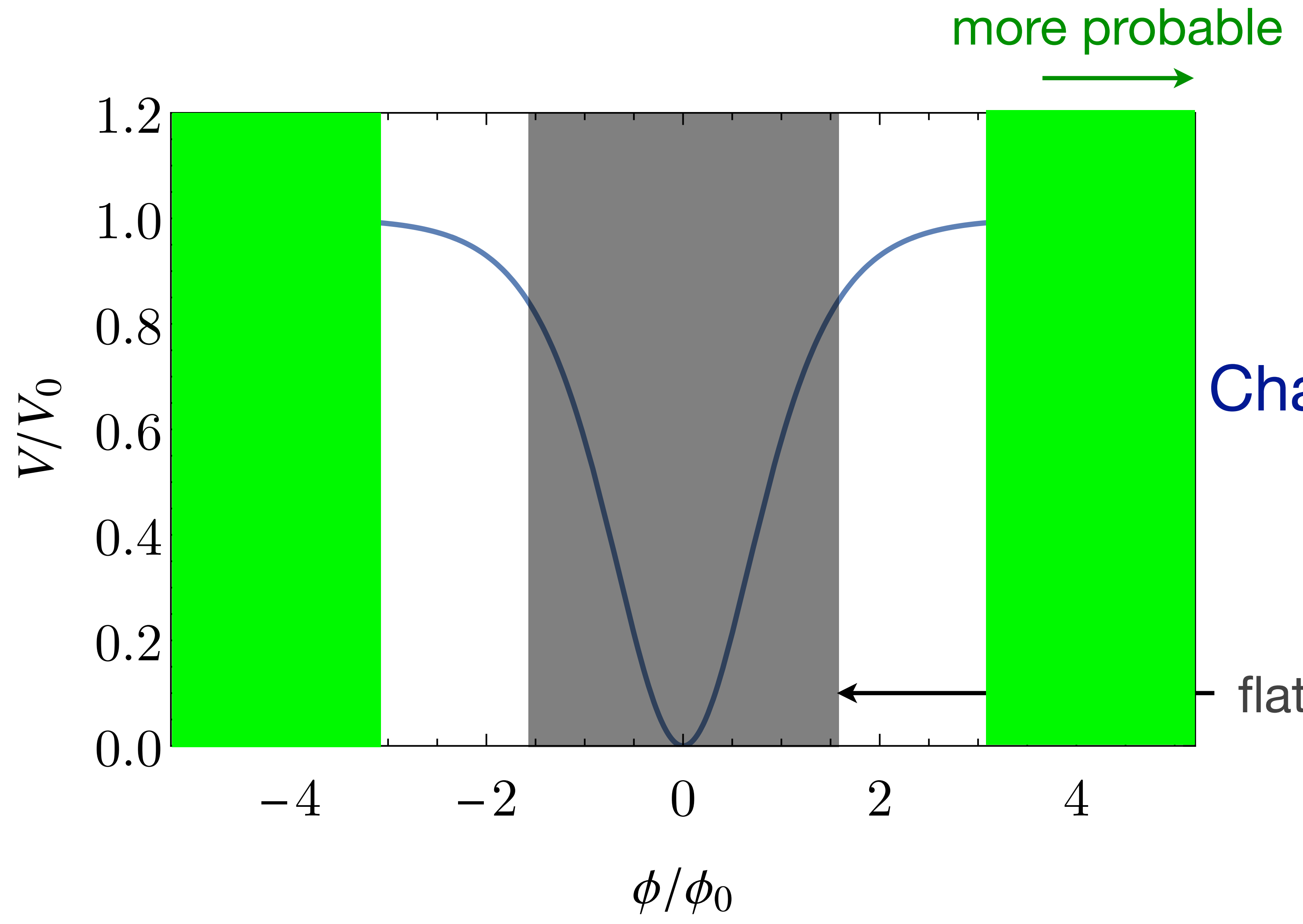

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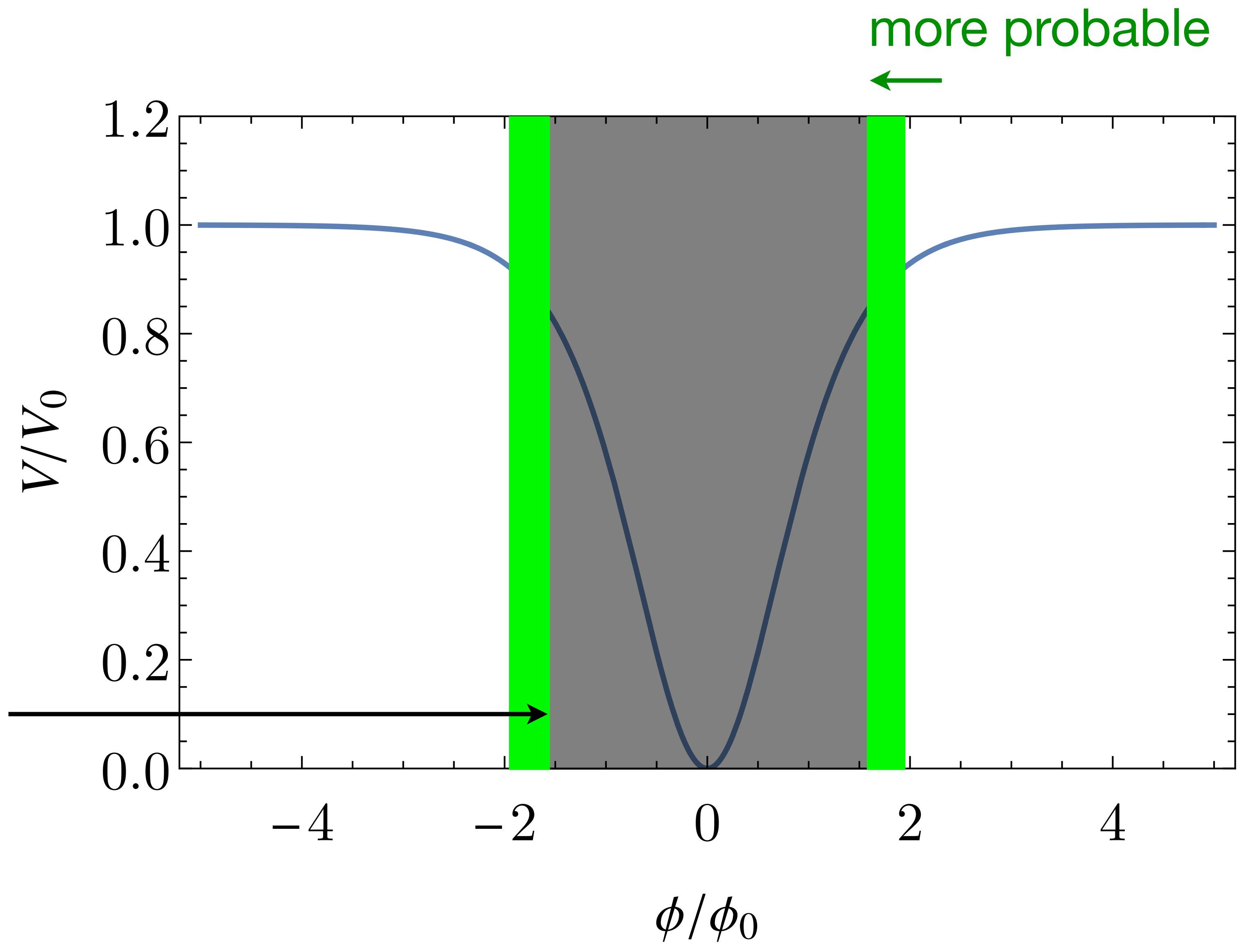

Our solutions are relevant.

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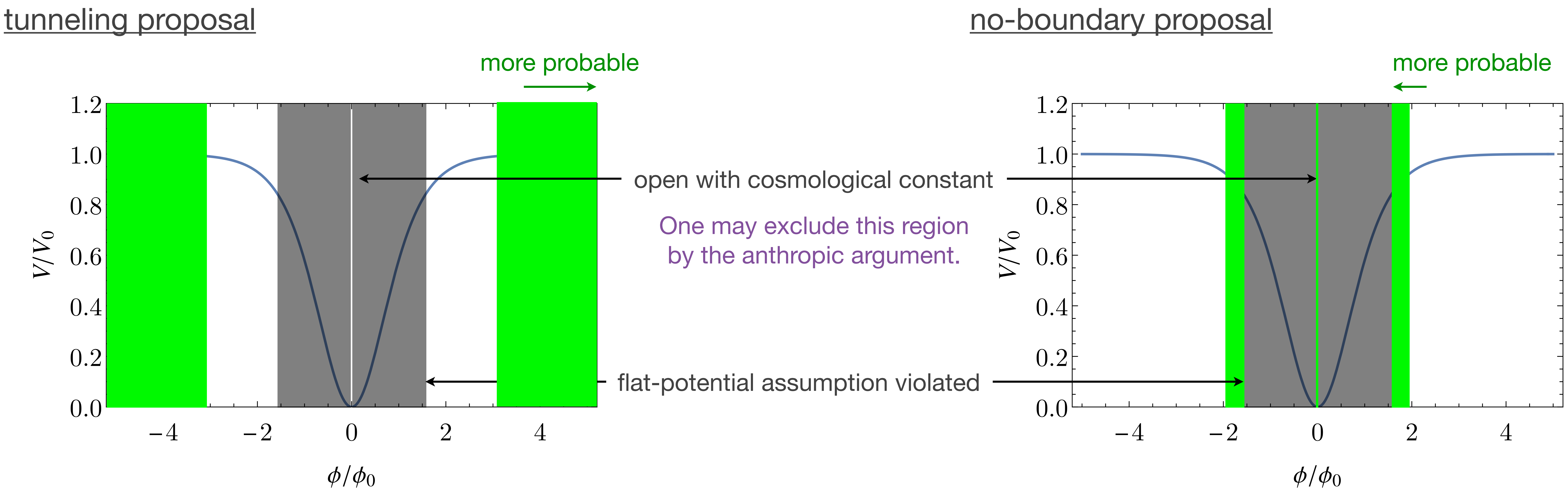


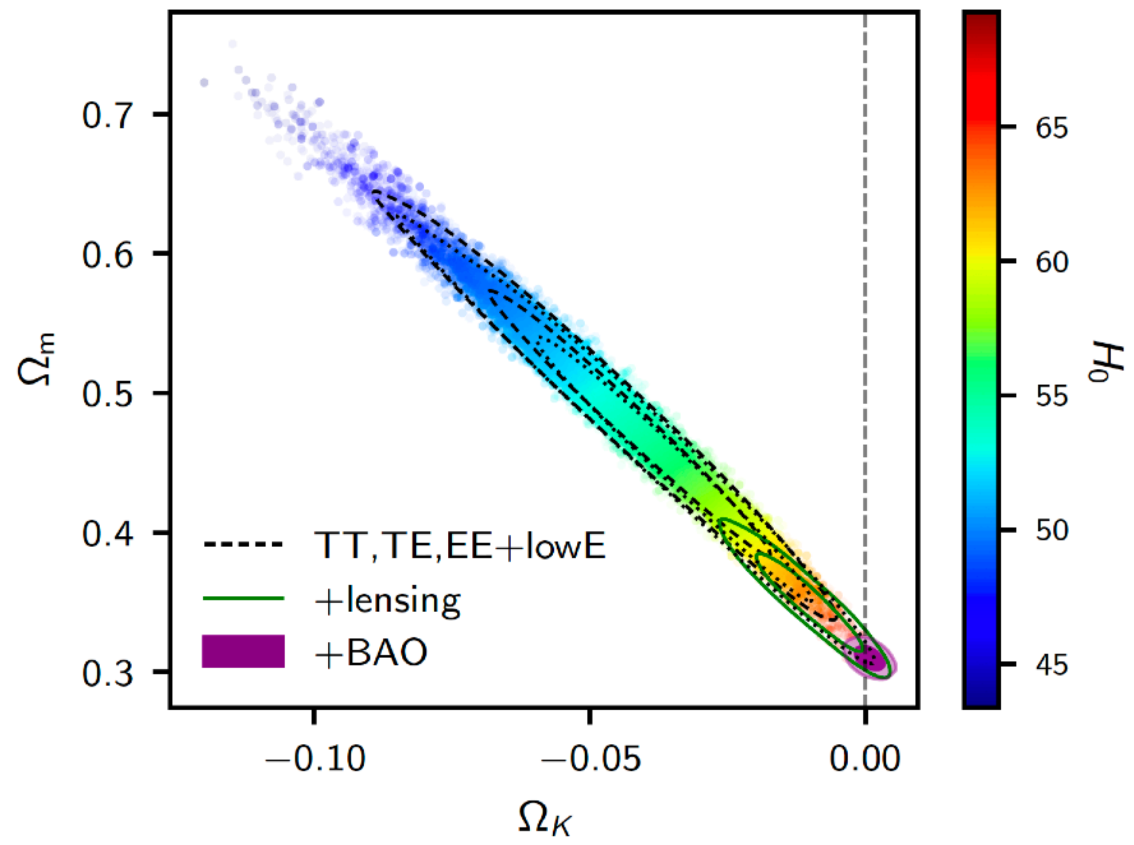
Our solutions are relevant.

Observational Tests?

Spatial curvature constraints and the Hubble tension

The Planck data prefer positive spatial curvature.

Positive spatial curvature reduces the Hubble tension.



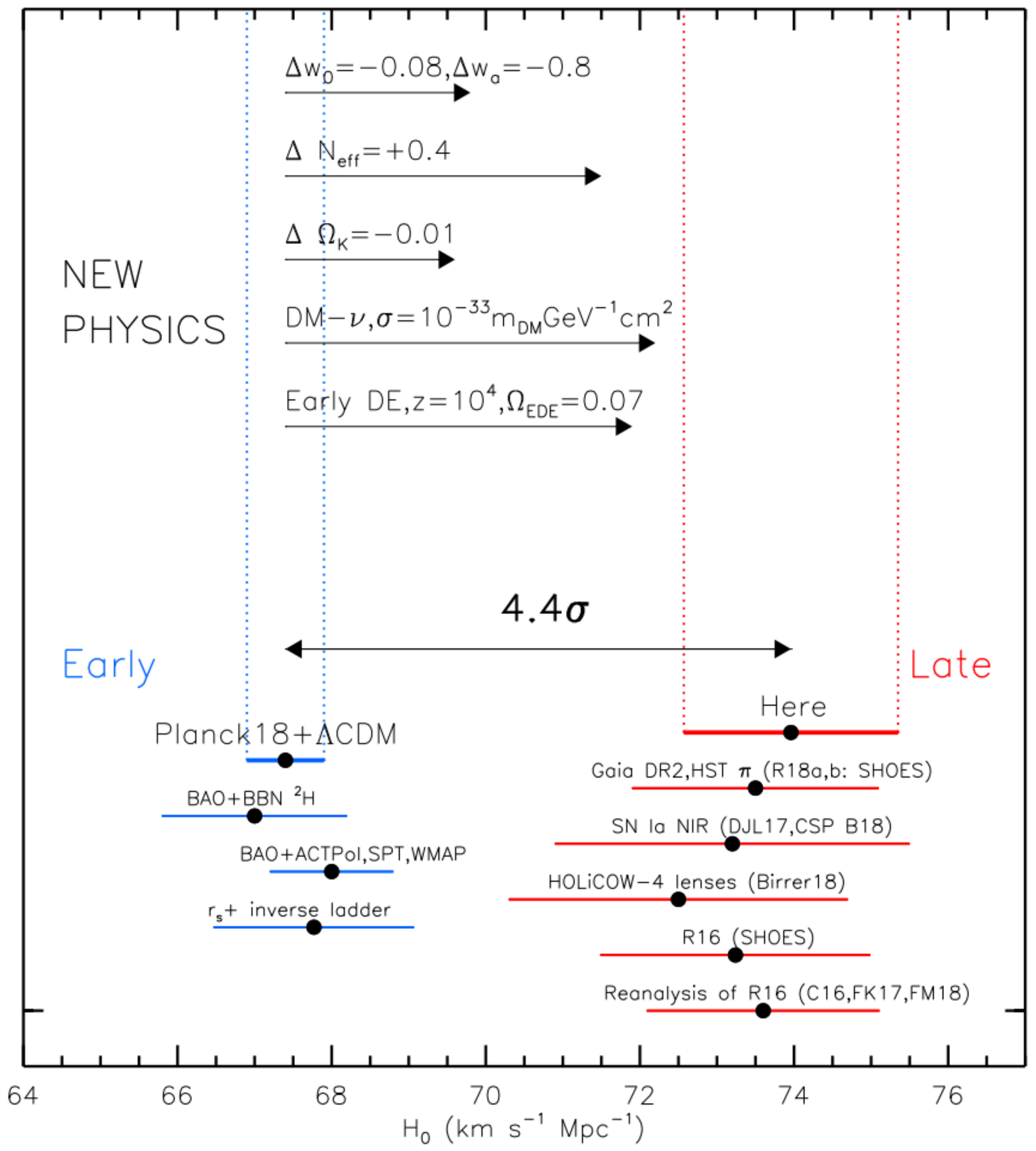
[Aghanim et al. (Planck 2018), 1807.06209]

$$\Omega_{\mathcal{K}} = -0.044^{+0.018}_{-0.015}$$

(68%, Planck TT,TE,EE+lowE)

$$\Omega_{\mathcal{K}} = 0.0007 \pm 0.0019$$

(68%, Planck TT,TE,EE+lowE+lensing+BAO)



[Riess, Casertano, Yuan, Macri, Scolnic, 1903.07603]

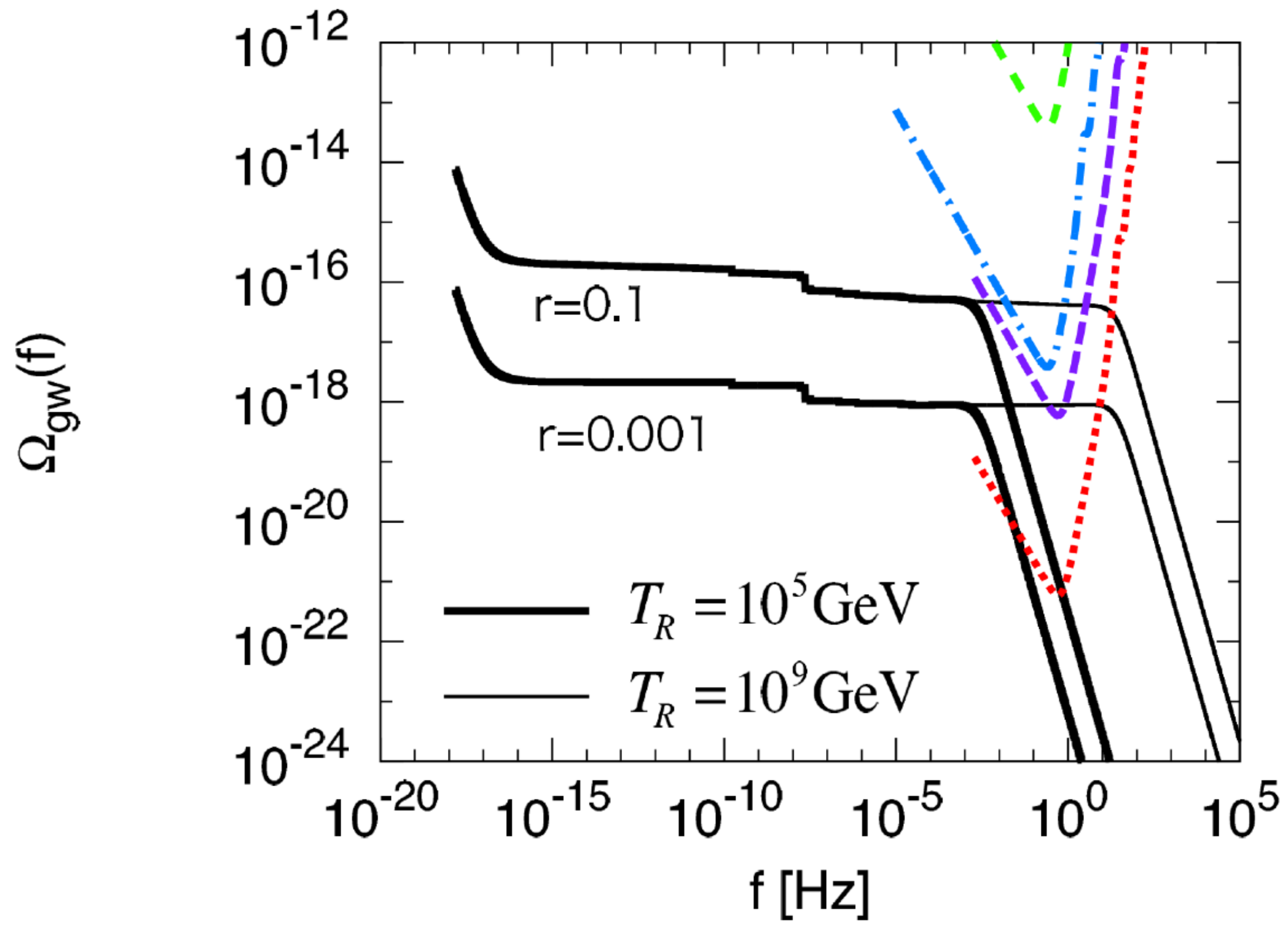
CMB low-multipole suppression

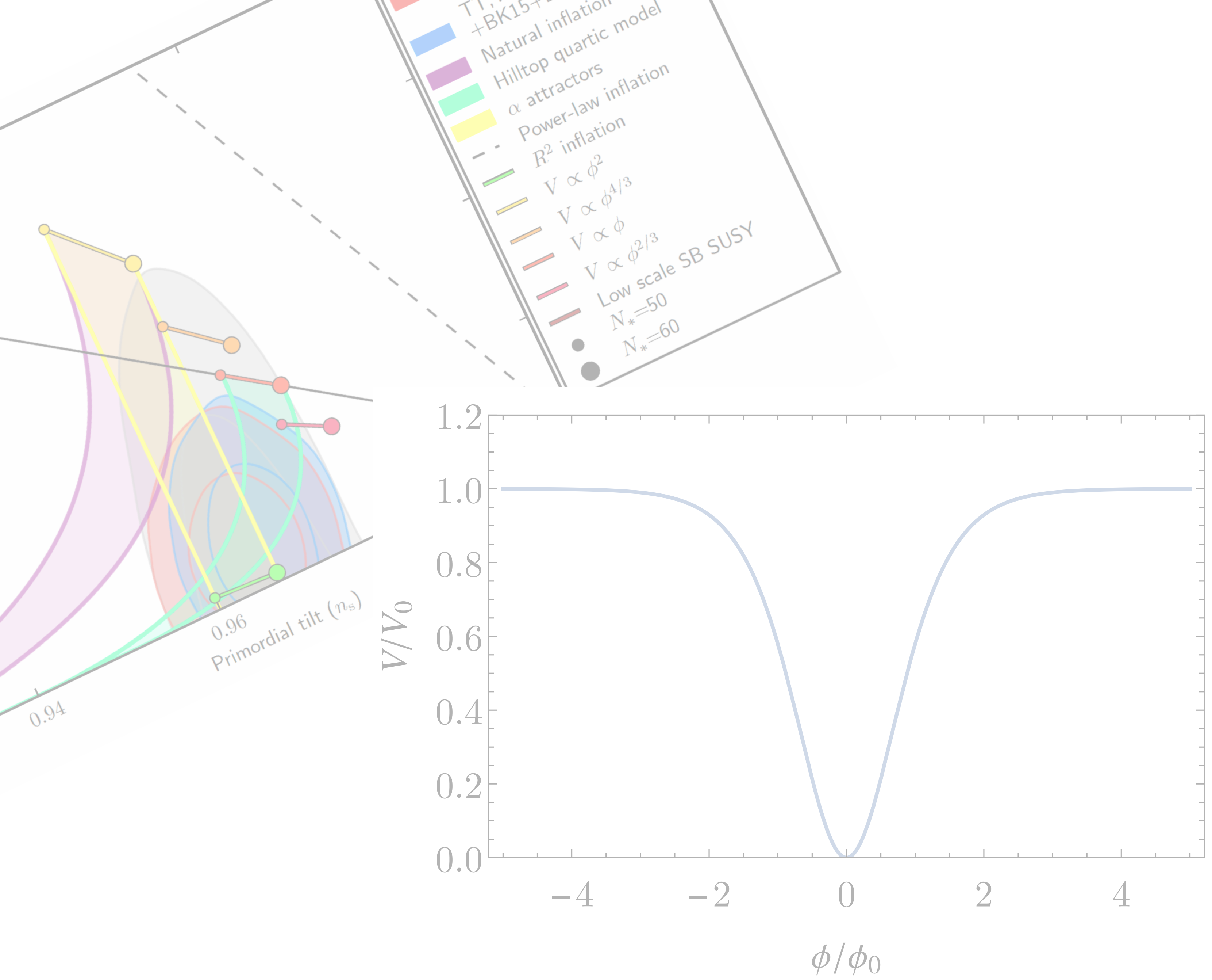
[Sloan, Dimopoulos, Karamitsos, 1912.00090] studied suppression of the power spectrum on large scales in a setup similar to ours.

Ideas to probe the reheating of the Universe

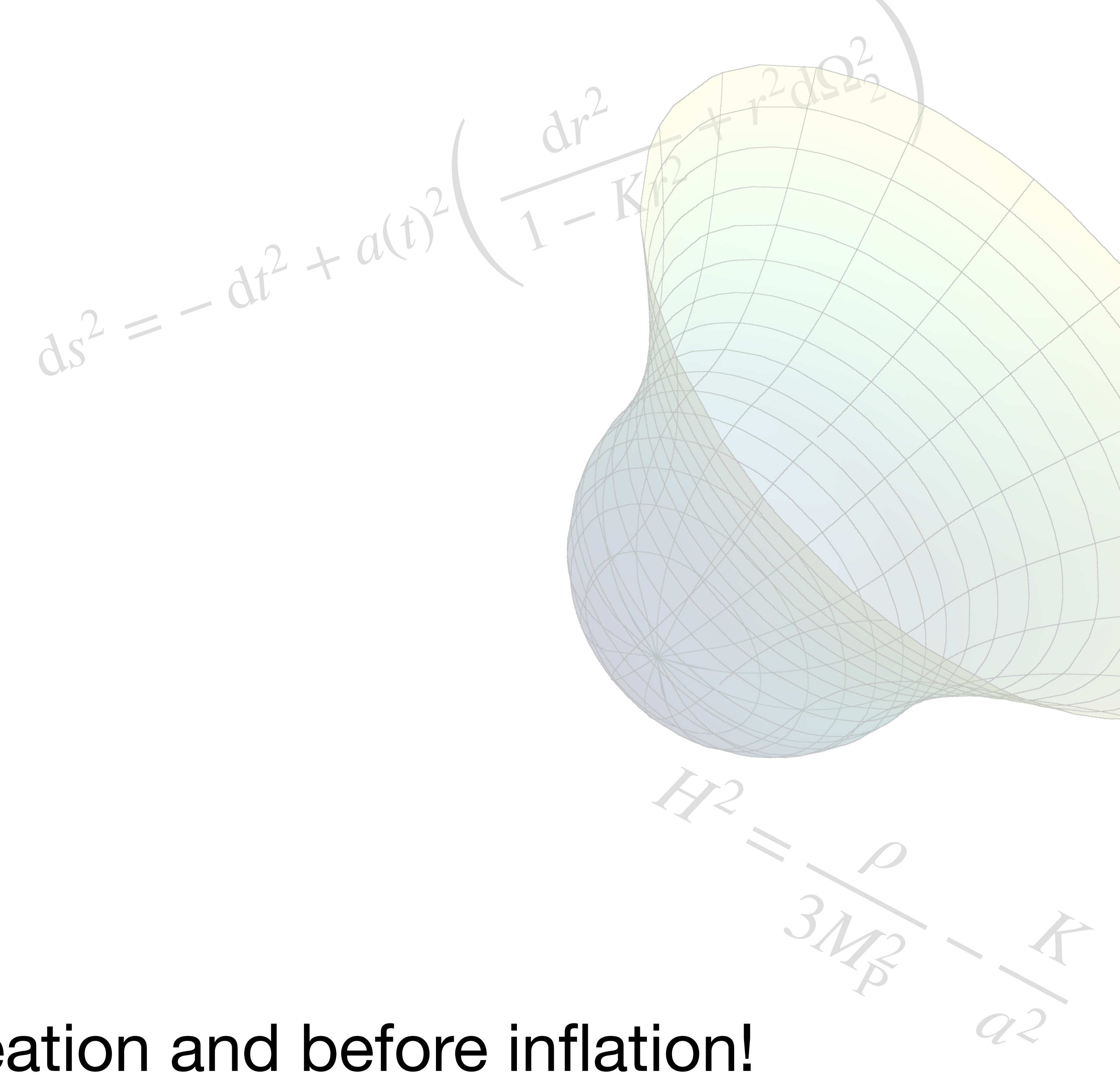
e.g.) Measuring the reheating temperature through the spectral break of the gravitational waves.

[Nakayama, Saito, Suwa, Yokoyama, 0804.1827]



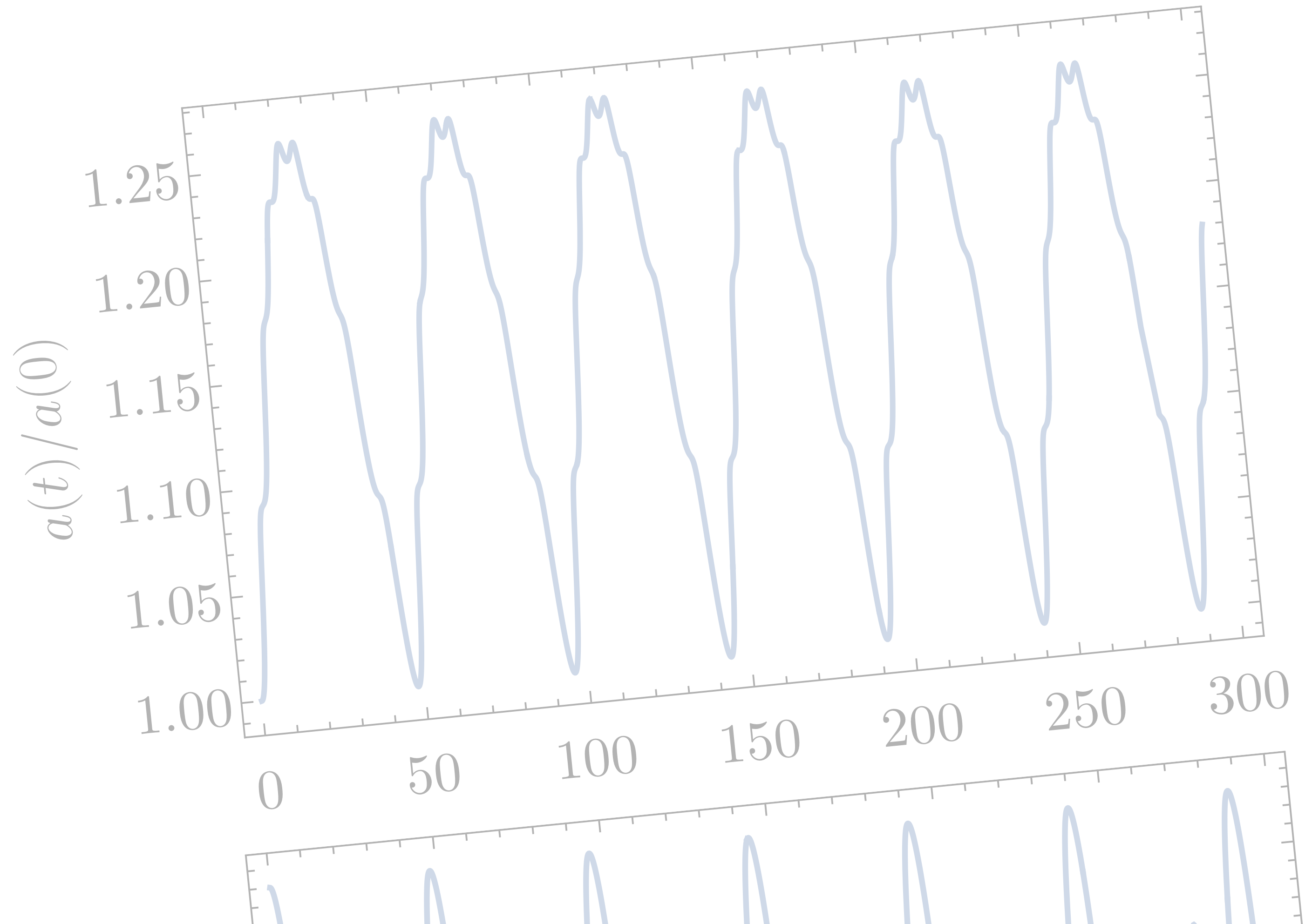


Conclusion



Our Universe may have experienced the quasi-cyclic period just after the creation and before inflation!

We hope to study the mechanism of starting inflation through observations related to the reheating of the Universe.



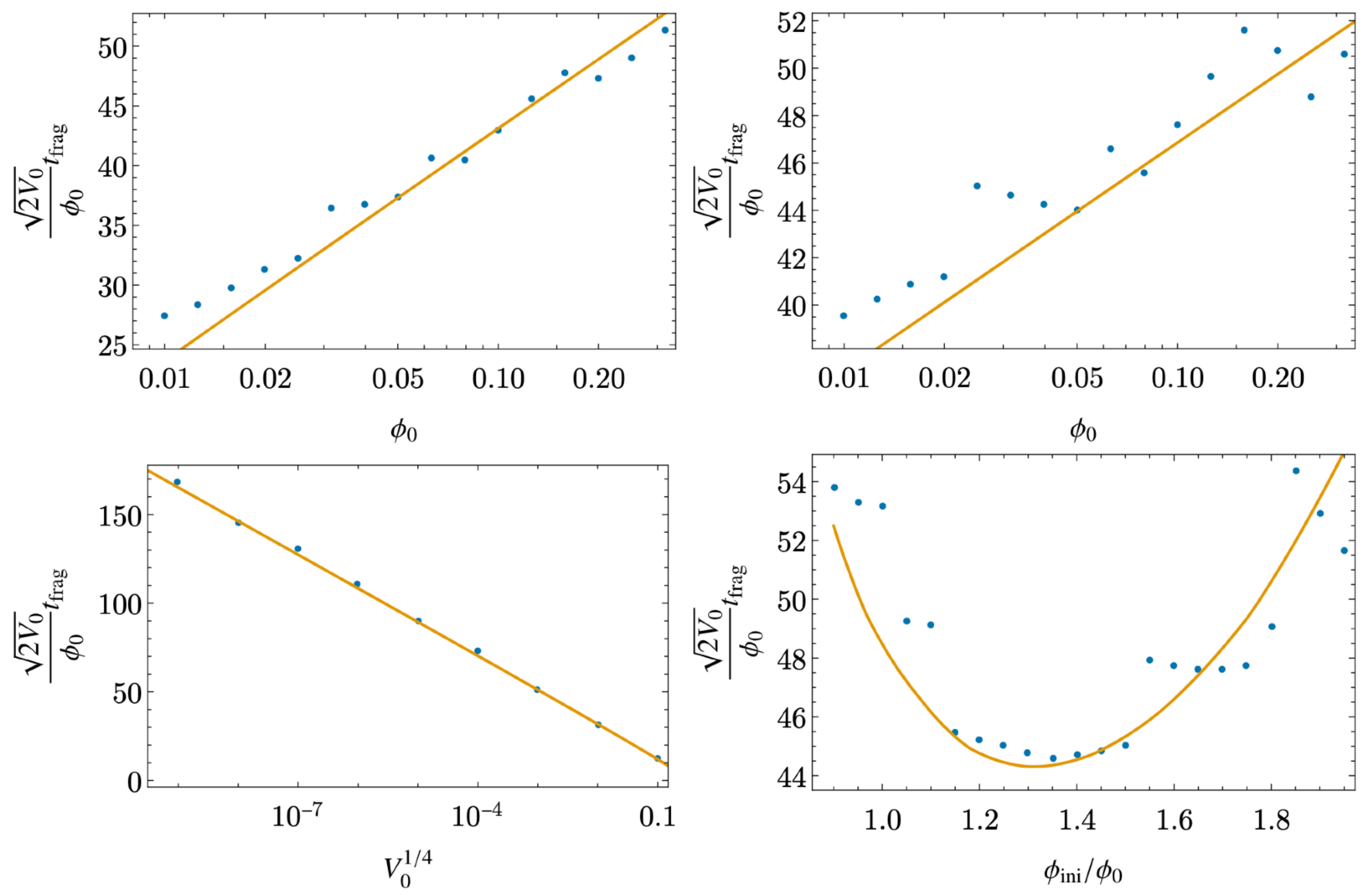
$$w = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$

Fragmentation Time

$$t_{\text{frag}} \approx \frac{1}{2\mu_{\text{peak}}}\left(\ln\left(\frac{4\pi^{3/2}c\rho}{(k_{\text{peak}}/a)^4}\right)+\frac{1}{2}\ln\ln\left(\frac{4\pi^{3/2}c\rho}{(k_{\text{peak}}/a)^4}\right)\right)$$

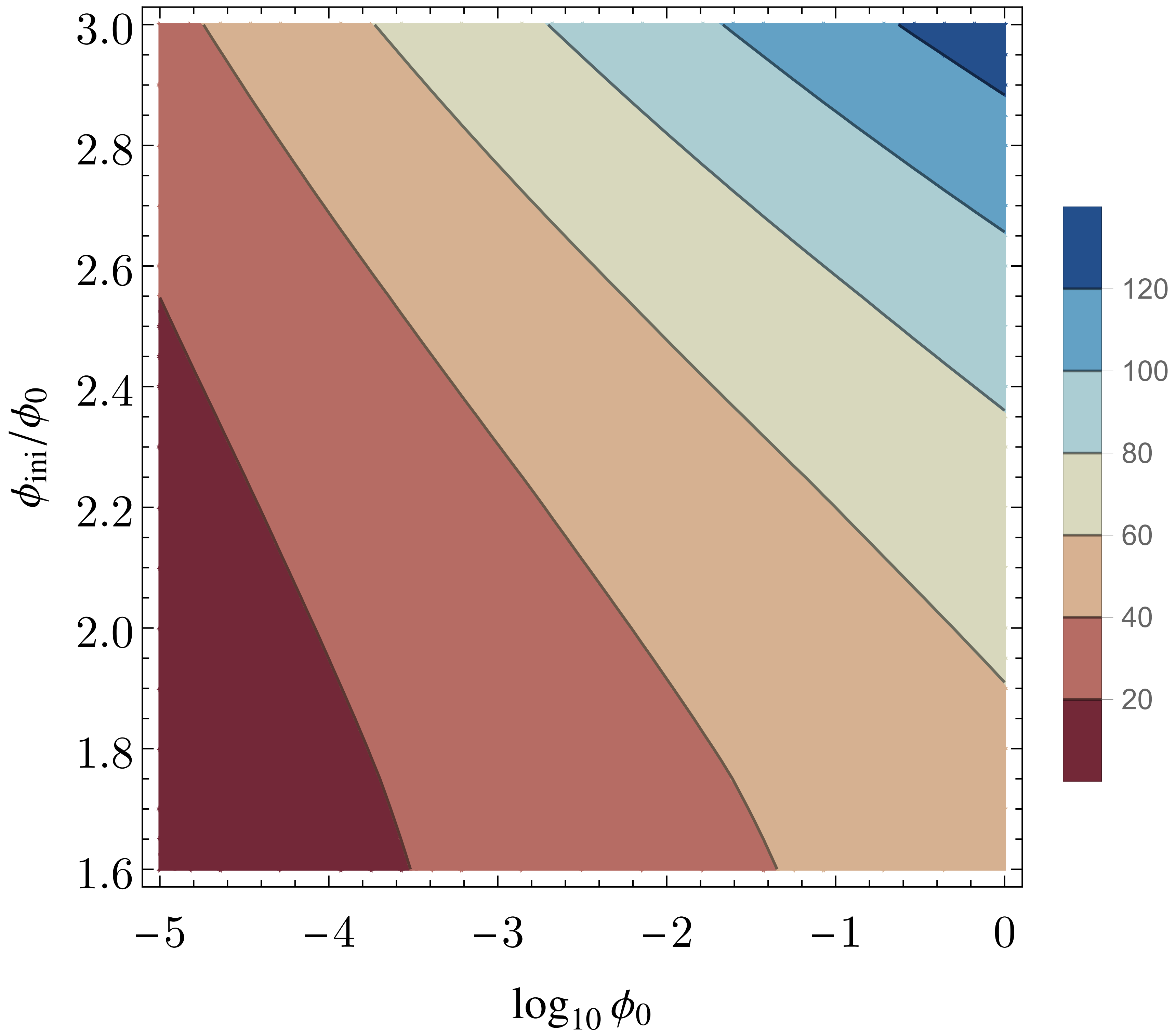
We set $c = 0.01$ and used numerical values of μ_{peak} and k_{peak} from [Tomberg, Veermäe, 2108.10767].

Validation of the approximated formula with Cosmo*L*attice calculation

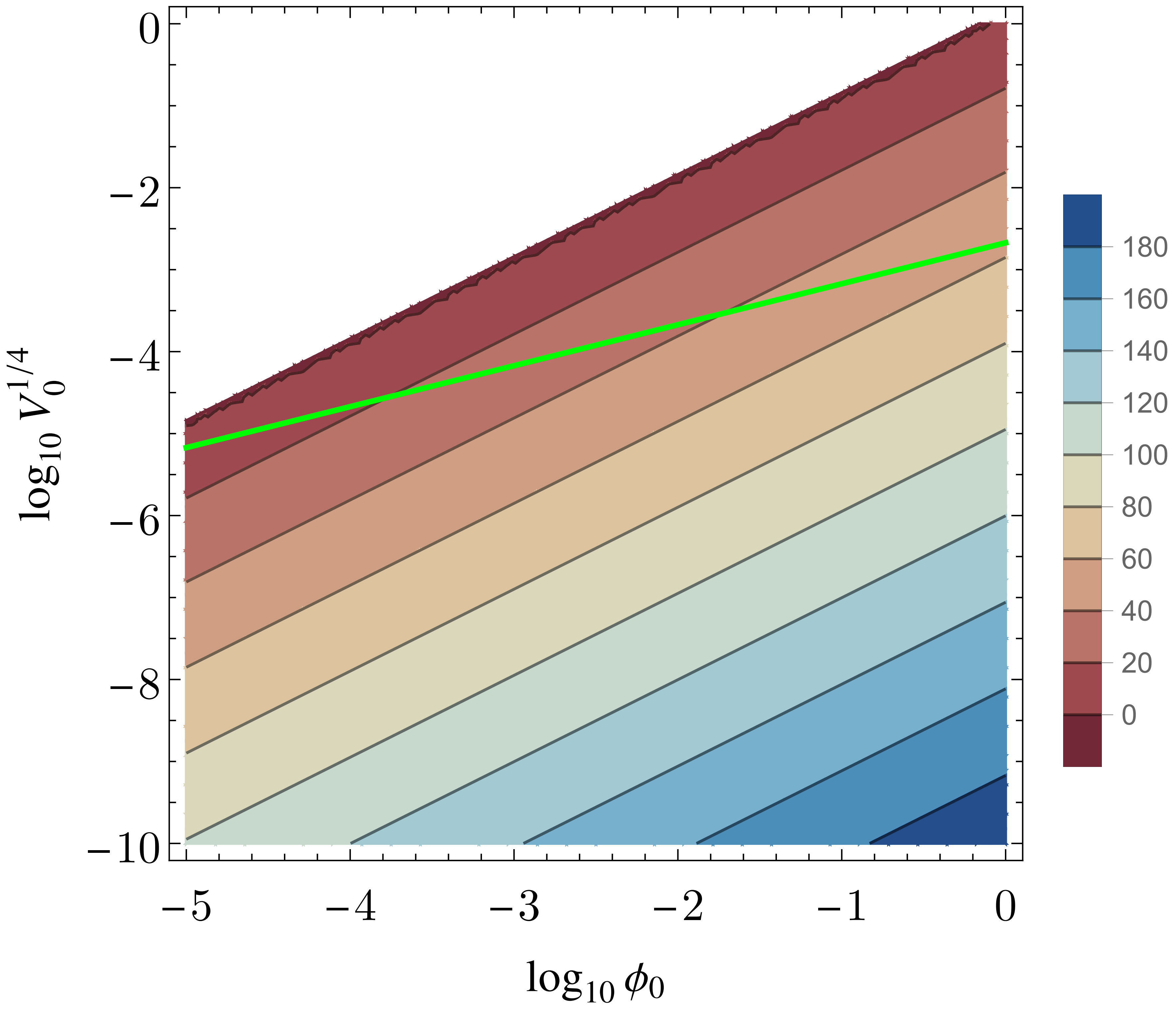


Fragmentation Time

Contour plot of $(\sqrt{2V_0}/\phi_0)\, t_{\rm frag}$ with $\phi_0/\sqrt{V_0} = 2.1 \times 10^5$



Contour plot of $(\sqrt{2V_0}/\phi_0)\, t_{\rm frag}$ with $\phi_{\rm ini}/\phi_0 = 1.8$

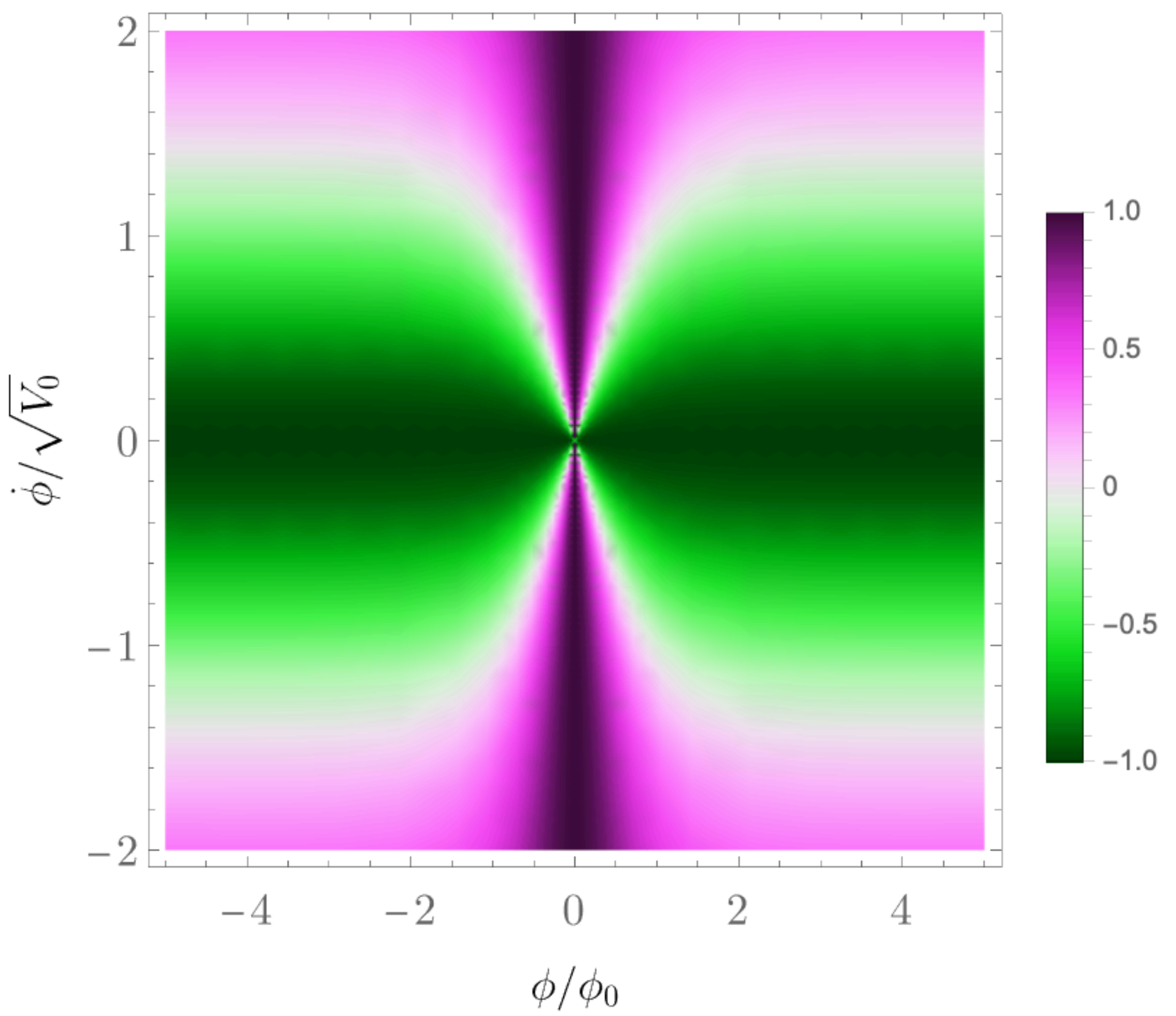


Analytic Understanding of Cyclic Solutions

Approximation based on the average over oscillations of ϕ (Note that this breaks down when the amplitude becomes too large.)

(The technique is based on [Tomberg, Veermäe, 2108.10767; Karam, Tomberg, Veermäe, 2102.02712])

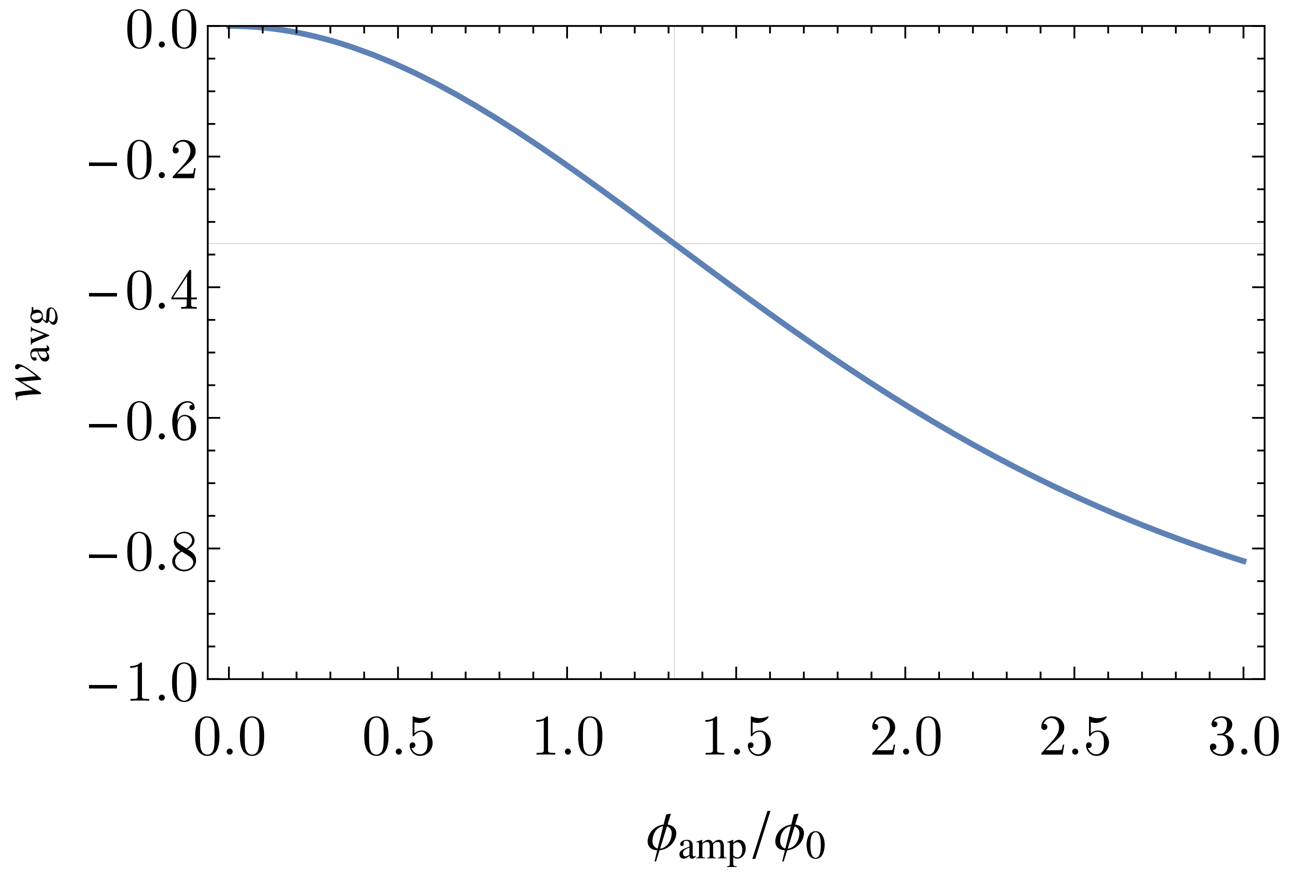
The instantaneous equation-of-state parameter



Oscillation average

The averaged equation-of-state parameter

$$w_{\rm avg} = \frac{-2 + 2\sqrt{1-\rho/V_0} + \rho/V_0}{\rho/V_0}$$



Energy density as a function of the scale factor

$$\frac{\rho_{\rm avg}}{V_0} = 2\left(1-\sqrt{1-\frac{\rho_{{\rm avg},*}}{V_0}}\right)\left(\frac{a_*}{a}\right)^3 - \left(1-\sqrt{1-\frac{\rho_{{\rm avg},*}}{V_0}}\right)^2\left(\frac{a_*}{a}\right)^6$$

Friedmann equation

$$\left(\frac{\dot a}{a}\right)^2 = \frac{2V_0X_*}{3}\left(\left(\frac{a_*}{a}\right)^3 - \frac{X_*}{2}\left(\frac{a_*}{a}\right)^6 - \left(1-\frac{X_*}{2}\right)\left(\frac{a_*}{a}\right)^2\right) \quad \text{with} \quad X_* \equiv 1-\sqrt{1-\rho_{{\rm avg},*}/V_0}.$$

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The averaged equation-of-state parameter

$$w_{\rm avg} = \frac{-2 + 2\sqrt{1-\rho/V_0} + \rho/V_0}{\rho/V_0}$$

The critical amplitude (bounce vs turn-around)

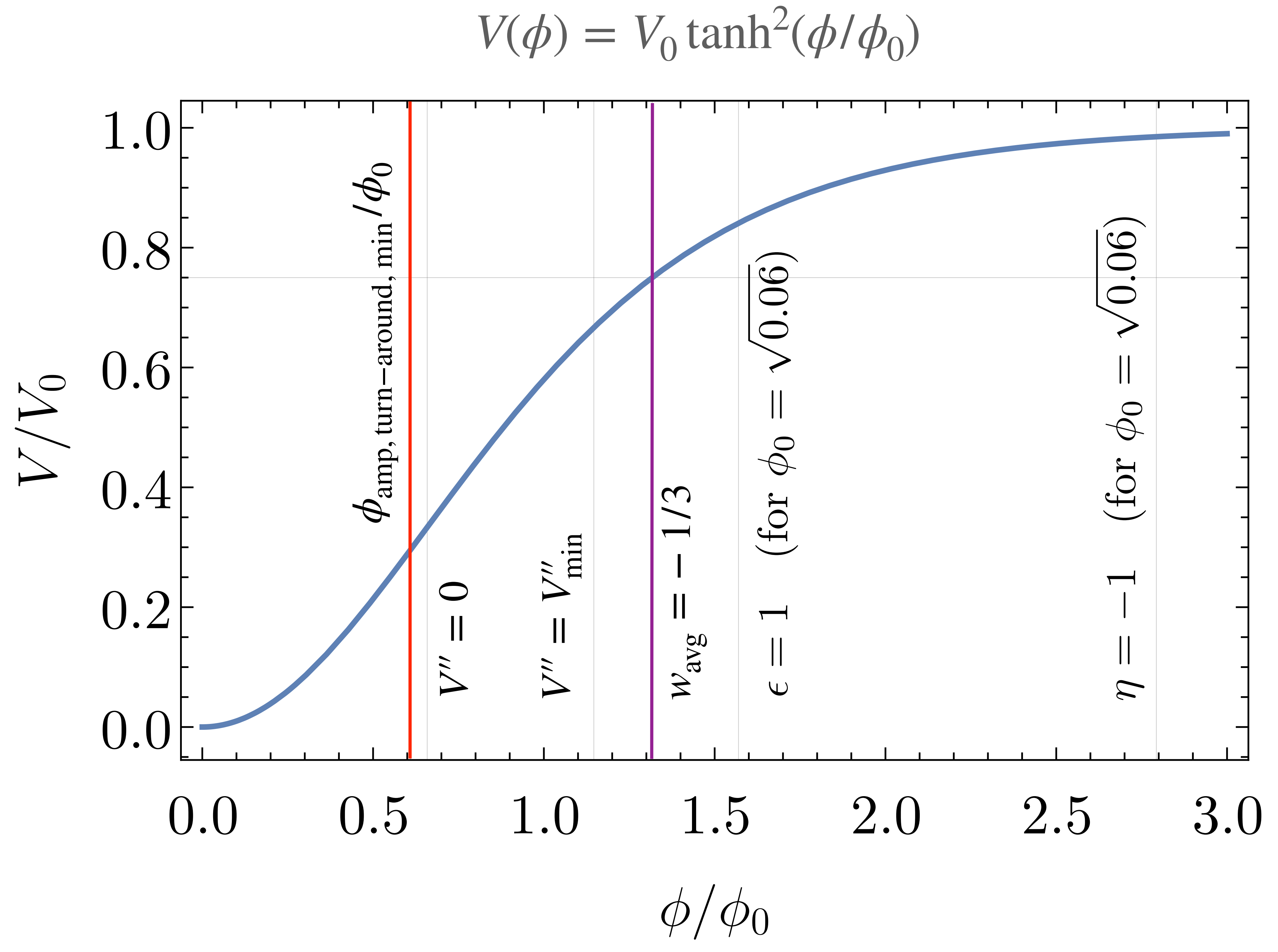
$$\phi_{{\rm amp},w_{\rm avg}=-1/3}/\phi_0 = \log(2+\sqrt{3}) \approx 1.31696$$

The critical amplitude (below which no cyclic solution)

$$\min \phi_{\rm amp,turn-around}/\phi_0 = {\rm artanh}\left(\frac{3}{1+\sqrt[3]{19-3\sqrt{33}}+\sqrt[3]{19+3\sqrt{33}}}\right) \approx 0.609378$$

The maximal ratio of the max/min of the scale factor

$$\lim_{\rho_{\rm avg,bounce}\to V_0} \frac{a_{\rm max}}{a_{\rm min}} = \frac{1+\sqrt[3]{19-3\sqrt{33}}+\sqrt[3]{19+3\sqrt{33}}}{3} \approx 1.83929$$



Characteristic field values.

Lattice Simulation

Lattice topology

For simplicity, we use $\mathbb{T}^3$ rather than $\mathbb{S}^3$.

The IR cutoff due to $\mathbb{S}^3$ should be $\dfrac{k_{\mathrm{IR},\mathbb{S}^3}}{a} = \dfrac{2\sqrt{K}}{a} \simeq 2\sqrt{\dfrac{\rho_{\mathrm{avg}}}{3M_{\mathrm{P}}^2}}$.

Spatial curvature

We modify the codes of $\mathcal{C}$osmo$\mathcal{L}$attice to include the spatial curvature.

Dimensionless variables

Quantities with a tilde are measured in units of $f_* = \phi_0$ and $\omega_* = \sqrt{2V_0}/\phi_0$.

IR cutoffs

The minimum wave number in $\mathcal{C}$osmo$\mathcal{L}$attice is defined to be $k_{\mathrm{IR,lattice}} = \dfrac{2\pi}{L} = \dfrac{2\pi}{N_{\mathscr{C}\mathscr{L}}\delta x}$.

This has to be smaller than the most tachyonic mode $\dfrac{k_{\mathrm{peak}}}{a} \approx \dfrac{3}{\Delta t_\phi}$.

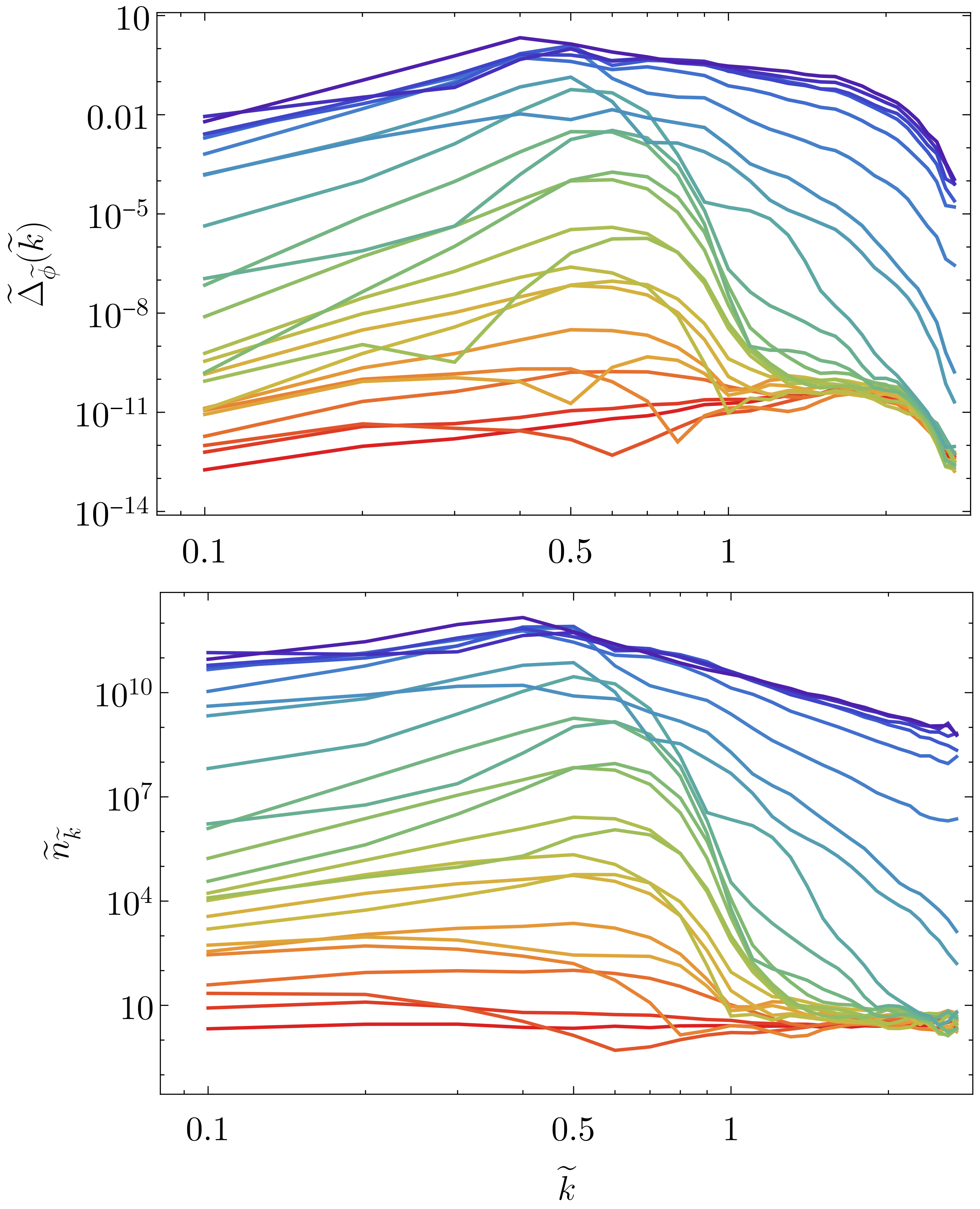
Evolver and input parameters

The default evolver (velocity Verlet algorithm: VV2).

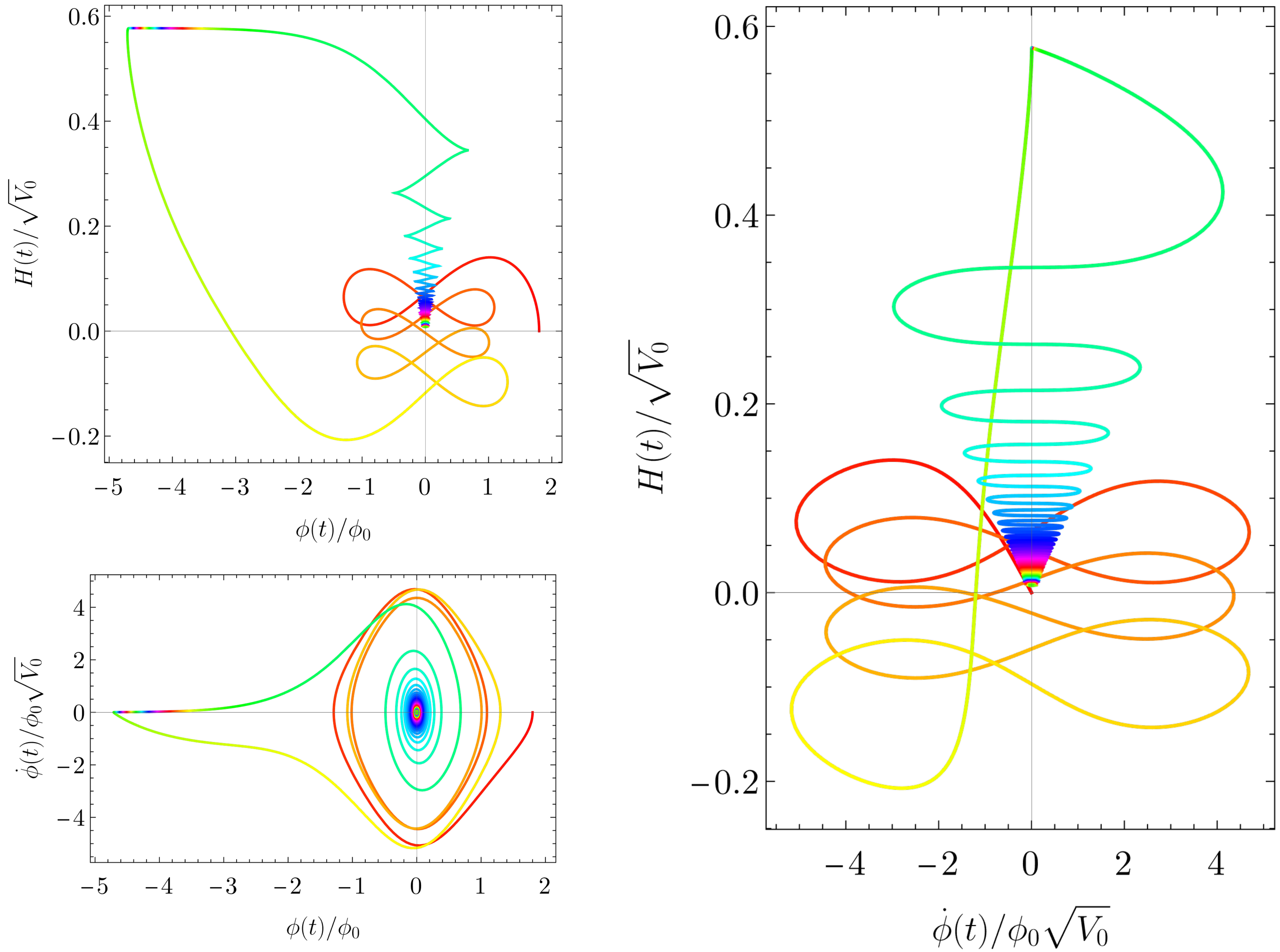
$N_{\mathscr{C}\mathscr{L}} = 32,\ \tilde{k}_{\mathrm{IR,lattice}} = 0.1,$ and $\Delta\tilde{t} = 0.01.$

Spectrum of perturbations

The tachyonic peak structure is well resolved.



Gallery of Phase Space Trajectory



$\phi_0 = \sqrt{0.06}$ and $\Gamma/\sqrt{V_0} = 0.019$