Matching CP-odd operators to the gradient-flow scheme

Peter Stoffer

Physik-Institut, University of Zurich
and Paul Scherrer Institut

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IFT UAM/CSIC Madrid

University of Zurich[UZH]

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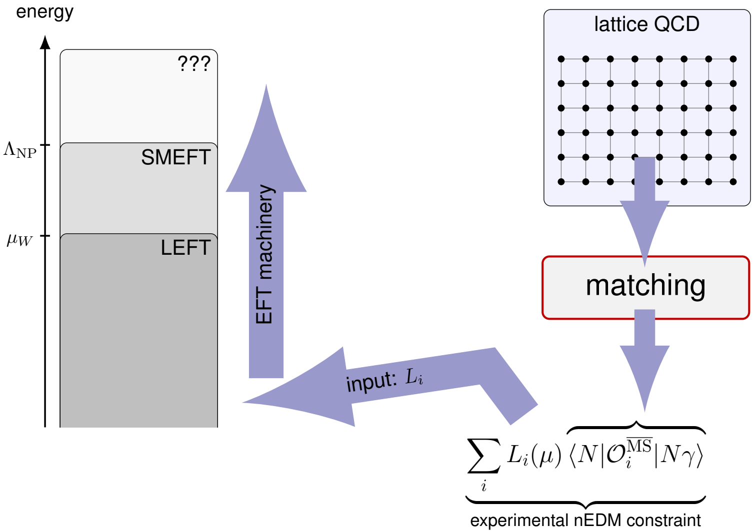
Neutron EDM in the LEFT



Neutron EDM in the LEFT

- current bound from PSI: $|d_n| < 1.8 \times 10^{-13}\, e\,\mathrm{fm}$

  → C. Abel et al. (nEDM collaboration), PRL **124** (2020) 081803

- theoretical status:

$$
\begin{aligned}
d_N = &- (1.5 \pm 0.7) \times 10^{-3}\, \bar{\theta}\; e\;\mathrm{fm} \\
&- (0.20 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s \\
&- (0.55 \pm 0.28) e\, \tilde{d}_u - (1.1 \pm 0.55) e\, \tilde{d}_d + (??) e\, \tilde{d}_s \\
&+ (50 \pm 40) \mathrm{MeV}\; e\, \tilde{d}_G + (??)\ \text{four-quark}
\end{aligned}
$$

→ Alarcon et al., arXiv:2203.08103 [hep-ph]

General procedure

- **non-perturbative definition** of renormalized operators in a scheme amenable to lattice computations
- define non-perturbative subtraction scheme of **power divergences** $\rightarrow$ Maiani, Martinelli, Sachrajda, NPB **368** (1992) 281
- compute matrix elements in lattice QCD
- for operators without power divergences: calculate relation between $\overline{\mathrm{MS}}$ and lattice scheme at $\mu \sim 2 \ldots 3$ GeV
- use this matching to derive matrix elements of $\overline{\mathrm{MS}}$ operators

RI-(S)MOM

previous work on flavor-conserving CP-odd operators:

- dimension 5 electric & chromo-electric dipoles:

  $\rightarrow$ Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon, PRD **92** (2015) 11, 114026

- dimension 6 three-gluon operator:

  $\rightarrow$ Cirigliano, Mereghetti, Stoffer, JHEP **09** (2020) 094

- complications: power divergences, large number of nuisance operators (without BFM even gauge-variant ones)

A promising scheme: gradient flow

→ Lüscher, JHEP **08** (2010) 071, JHEP **04** (2013) 123

- **gradient flow**: introduce new artificial dimension: flow time t (not related to ordinary time)
- boundary condition: ordinary QCD at $t = 0$, $B_\mu(t = 0) = G_\mu$, $\chi(t = 0) = \psi$
- **flow equations**:

$$\partial_t B_\mu = D_\nu G_{\nu\mu} + \alpha_0 D_\mu \partial_\nu B_\nu \, ,$$
$$\partial_t \chi = D^2 \chi - \alpha_0 (\partial_\mu B_\mu) \chi$$

- flow acts as a UV regulator

Gradient flow: advantages

- “flowed operators” are automatically **UV finite**, apart from quark-field (+ coupling & mass) renormalization
- non-perturbative RI renormalization condition for quark fields: → Makino, Suzuki, PTEP **2014**, 063B02 (2014)

$$\langle 0|\mathring{\bar{\chi}}(x;t)\overleftrightarrow{\not{D}}\mathring{\chi}(x;t)|0\rangle = -\frac{2N_c}{(4\pi)^2 t^2}$$

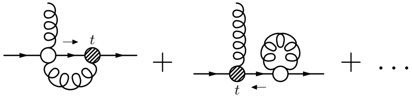
- on the lattice: **continuum limit** $a \to 0$ for fixed t possible
- **power divergences** no longer in $1/a$, but in $1/t$
  $\Rightarrow$ **disentangled from continuum limit**

Matching calculations

- perform **operator-product expansion** at short flow times

$$\mathcal{O}_i^R(t) = \sum_j c_{ij}(t,\mu)\mathcal{O}_j^{\overline{\mathrm{MS}}}(\mu)$$

- structure of mixing and power divergences can be analyzed using **chiral symmetry**
- after subtraction of power divergences: connect flowed operators with $\overline{\mathrm{MS}}$ in perturbation theory, e.g., for dipole:



Matching calculations

- a priori complicated flow-time integrals simplified by **method of regions**, flow-time t is the only hard scale
- CP-odd flavor-conserving sector: one-loop matching close to completion; effects $\sim 30\% - 60\%$
- $\mathcal{O}(40\%)$ relative perturbative uncertainty on one-loop coefficients motivates matching at two loops

$\rightarrow$ Harlander, Kluth, Lange, EPJC **78** (2018) 11, 944
$\rightarrow$ Harlander, Lange, PRD **105** (2022) 7, L071504

- on the $\overline{\mathrm{MS}}$ side: HV scheme breaks chiral symmetry,

$$\mathcal{L}_{\mathrm{kin}} \supset \bar{\psi}_L \hat{\not{D}} \psi_R + \bar{\psi}_R \hat{\not{D}} \psi_L \,,$$

but can be restored by **finite renormalizations**

$\rightarrow$ Naterop, Stoffer, to appear

Gradient-flow matching: status for CP-odd operators

- power divergences:

  → Rizik, Monahan, Shindler, PRD **102** (2020) 3, 034509
  → Kim, Luu, Rizik, Shindler, PRD **104** (2021) 7, 074516

- quark bilinears:

  → Hieda, Suzuki, MPLA **31** (2016) 1650214

- dimension-5 quark (C)EDM:

  → Mereghetti, Monahan, Rizik, Shindler, Stoffer, JHEP **04** (2022) 050

- four-quark operators:

  → Bühler, Stoffer, arXiv:2304.00985 [hep-lat]

- dimension-6 three-gluon operator:

  → Lara Crosas, Mereghetti, Monahan, Rizik, Shindler, Stoffer, to appear