Confronting e^+e^- data and lattice QCD

Hartmut Wittig

Introductory remarks

Antonio Pich

Reflections on the Adler function and the role of τ-decays

Fedor Ignatov / Graziano Venanzoni

Experimental status and considerations

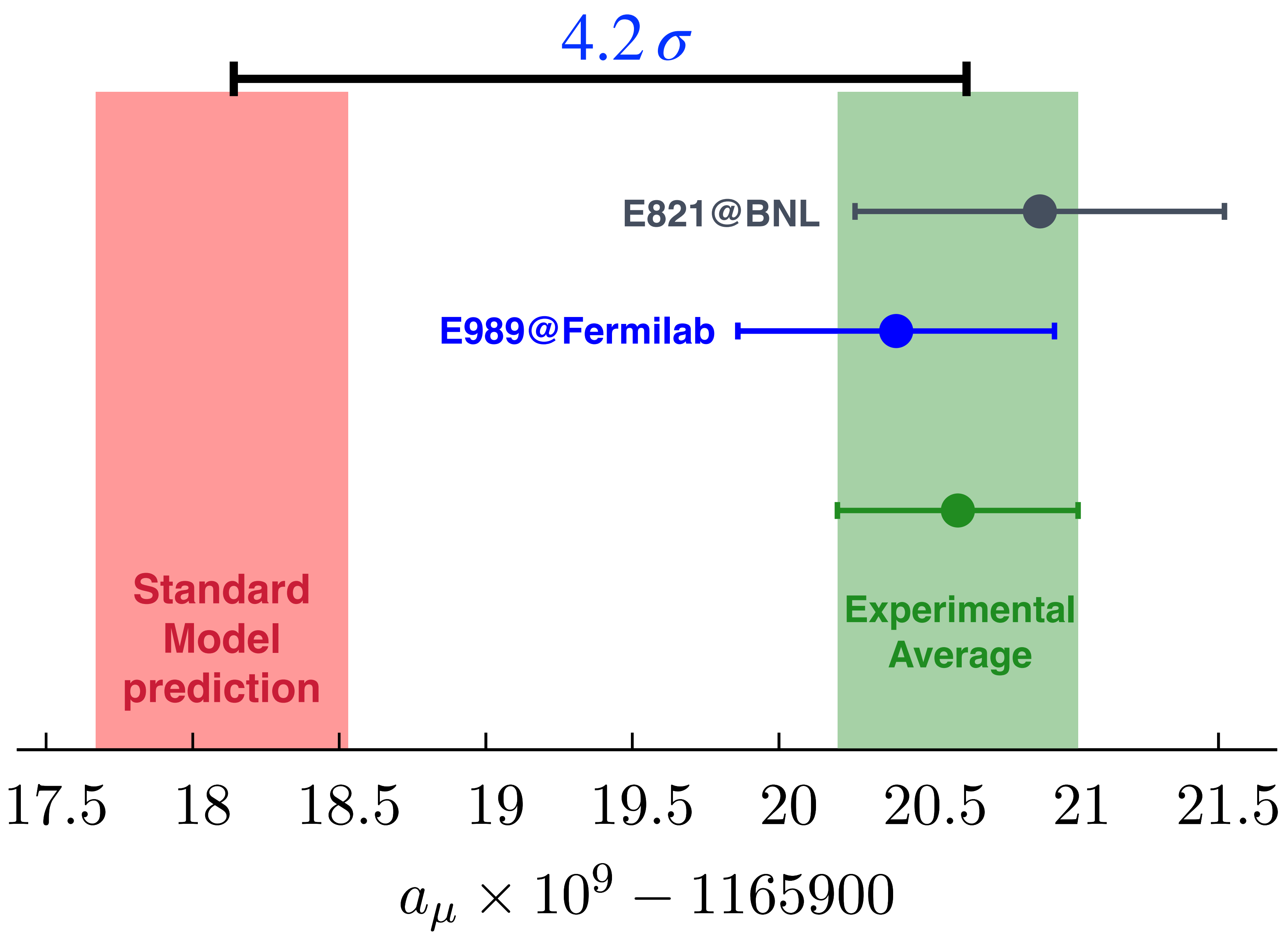
Lattice Gauge Theory Contributions to New Physics Searches
Strong 2020 Workshop, IFT UAM/CSIC Madrid, 13 June 2023

Tension, tensions,….

Hadronic vacuum polarisation

- There is a tension of 4.2σ between the experimental average for a_μ and the SM prediction, with the HVP contribution evaluated via dispersion integrals and e^+e^- cross section data:
  [Aoyama et al., Phys. Rep. 887 (2020) 1]

$$\Rightarrow \quad a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (25.1 \pm 5.9) \cdot 10^{-10} \quad [4.2\sigma]$$



Tension, tensions,….

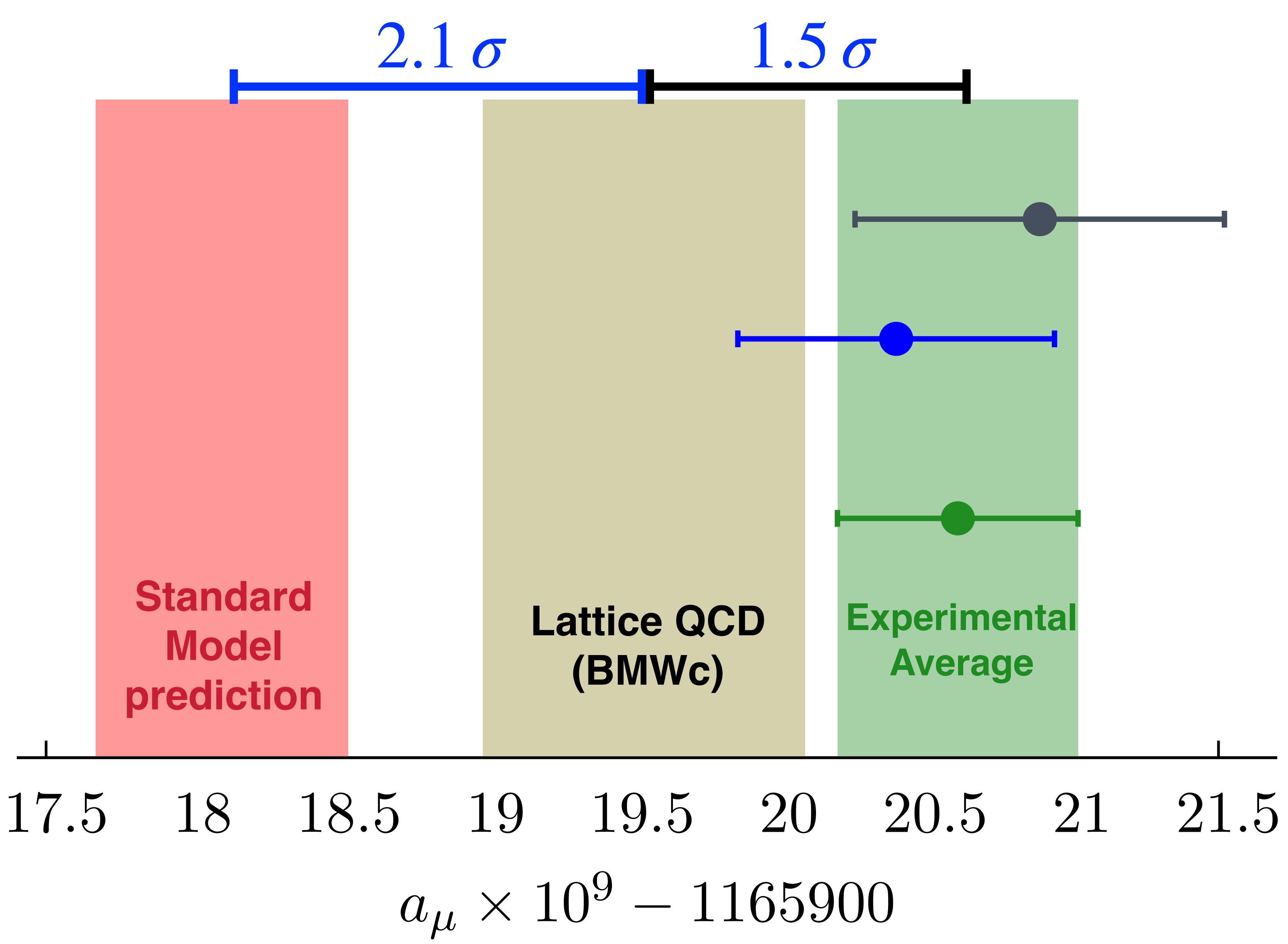
Hadronic vacuum polarisation

- There is a tension of 4.2σ between the experimental average for a_μ and the SM prediction, with the HVP contribution evaluated via dispersion integrals and e^+e^- cross section data:
  [Aoyama et al., Phys. Rep. 887 (2020) 1]

$$\Rightarrow \quad a_\mu^{\rm exp} - a_\mu^{\rm SM} = (25.1 \pm 5.9)\cdot 10^{-10} \quad [4.2\sigma]$$

- There is a tension of 2.1σ between the HVP contribution evaluated from e^+e^- cross section data and from a single lattice calculation:
  [Borsányi et al. (BMW Collab.), Nature 593 (2021) 7857]

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}\Big|_{\rm BMWc}^{\rm hvp,\,LO} = (10.7 \pm 7.0)\cdot 10^{-10} \quad [1.5\,\sigma]$$



Tensions: The Next Generation

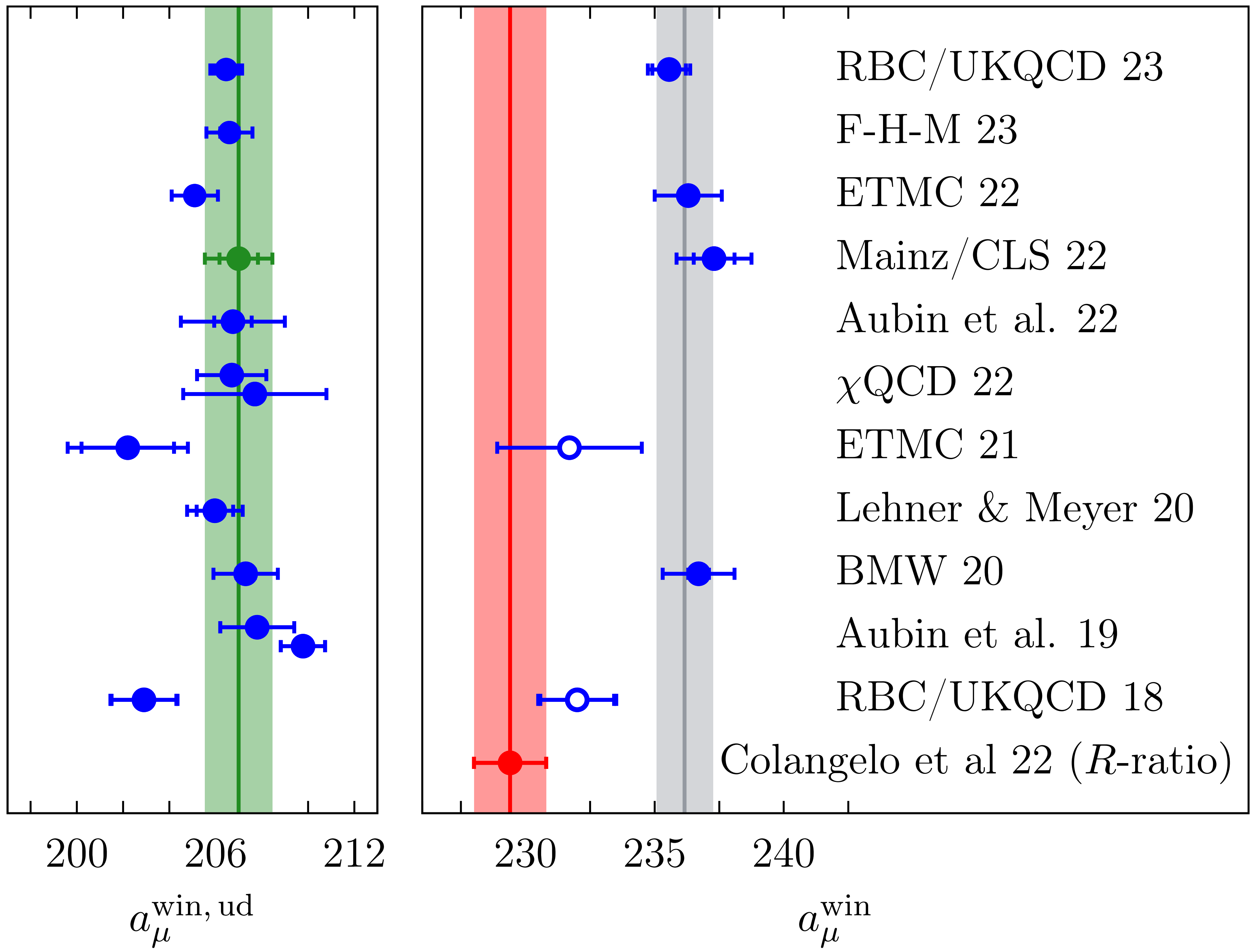
Intermediate window observable

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

Lattice average: $a_\mu^{\rm win} = (236.16 \pm 1.09) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)

[HW, arXiv:2306.04165]



Tensions: The Next Generation

Intermediate window observable

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

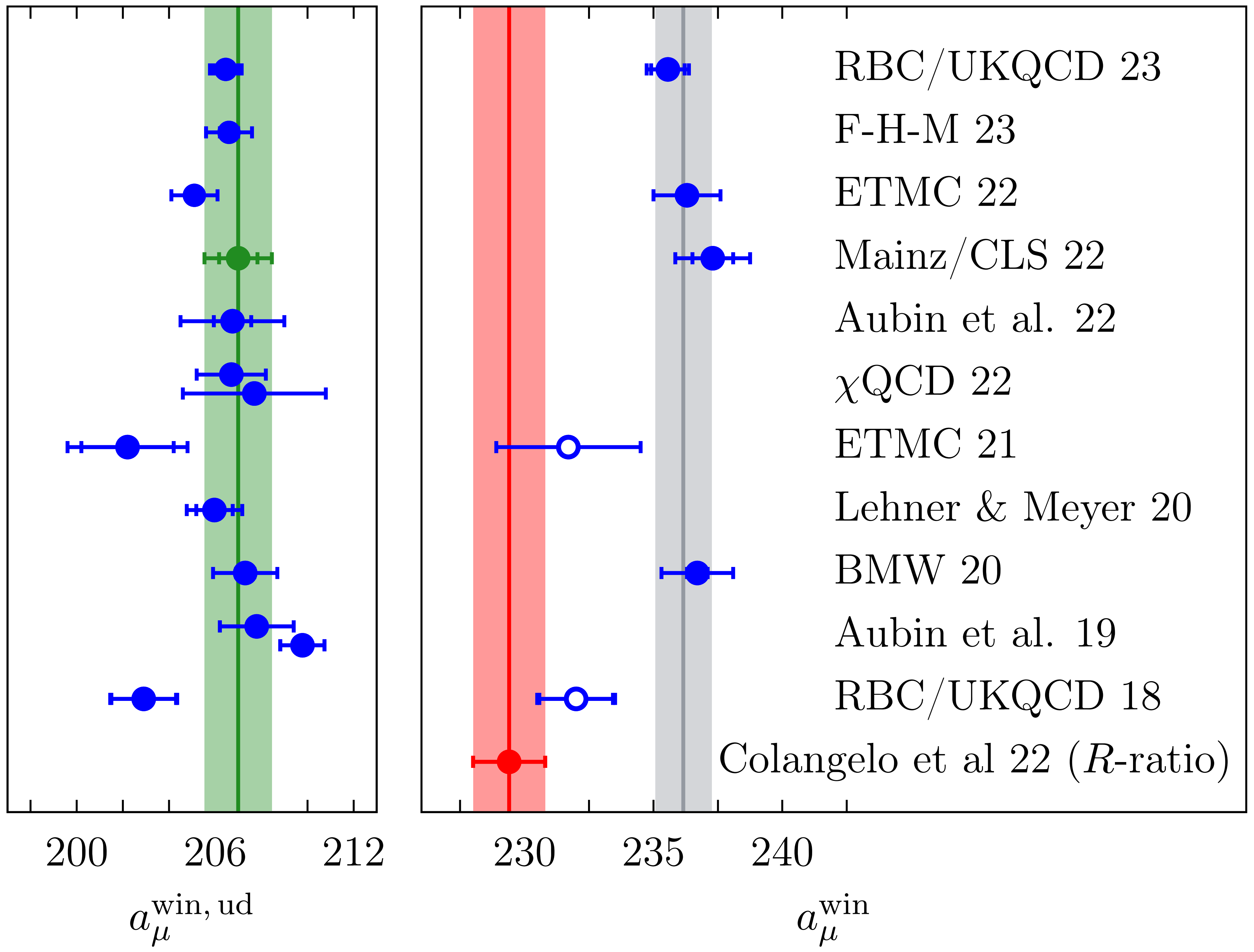
Lattice average: $a_\mu^{\rm win} = (236.16 \pm 1.09) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)

[HW, arXiv:2306.04165]

- There is a tension of 3.8σ in the window observable evaluated from e^+e^- data and four lattice calculations

$$a_\mu^{\rm win}\Big|_{\langle\rm lat\rangle} - a_\mu^{\rm win}\Big|_{e^+e^-} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$



Tensions: The Next Generation

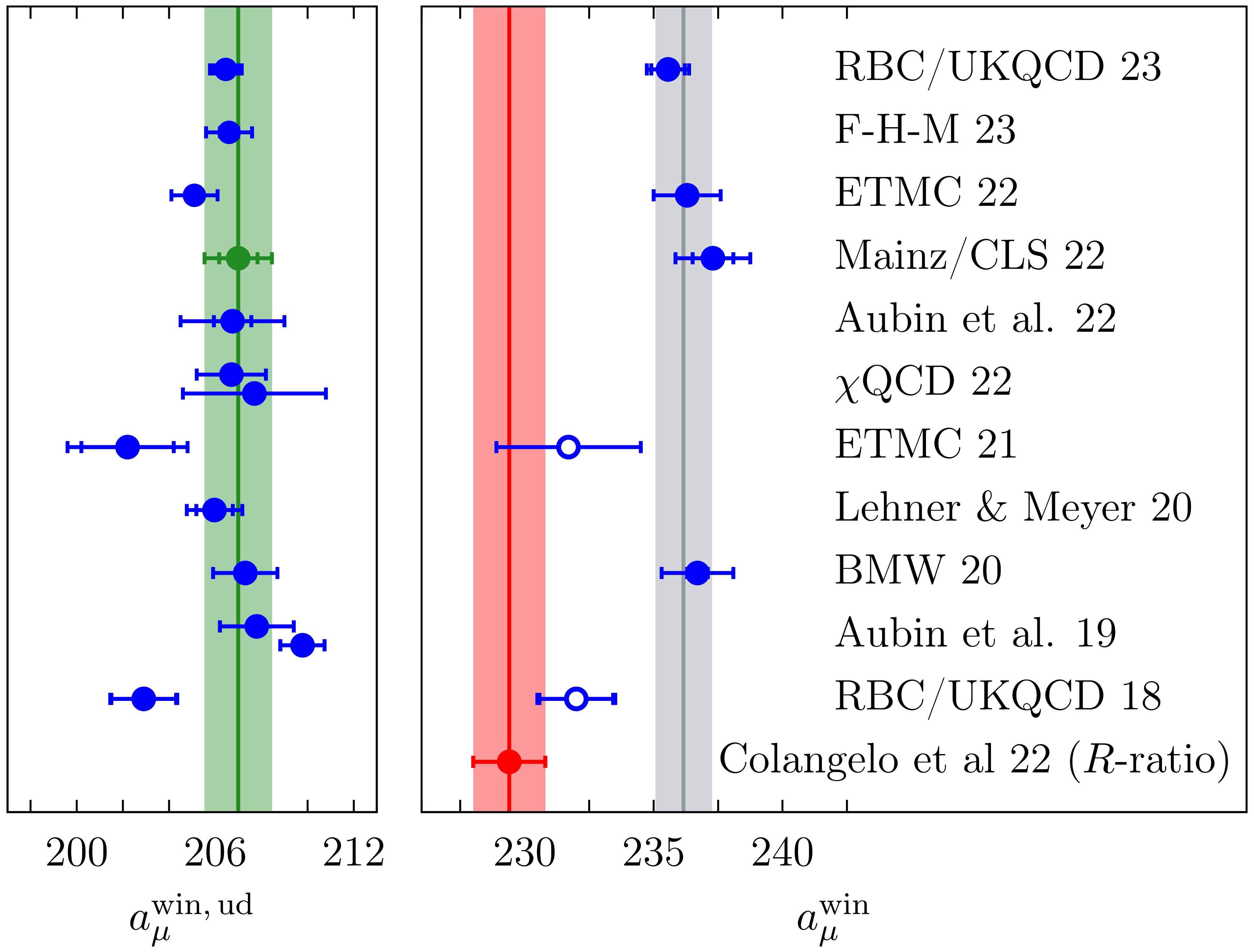
Intermediate window observable

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

Lattice average: $a_\mu^{\rm win} = (236.16 \pm 1.09) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)

[HW, arXiv:2306.04165]



- There is a tension of 3.8σ in the window observable evaluated from e^+e^- data and four lattice calculations

$$a_\mu^{\rm win}\big|_{\langle{\rm lat}\rangle} - a_\mu^{\rm win}\big|_{e^+e^-} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

- Subtract R-ratio result $a_\mu^{\rm win}\big|_{e^+e^-}$ from WP estimate and replace by lattice average $a_\mu^{\rm win}\big|_{\langle{\rm lat}\rangle}$:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}\Big|^{\rm win}_{e^+e^-\to\langle{\rm lat}\rangle} = (18.3 \pm 5.9) \cdot 10^{-10} \quad [3.1\,\sigma]$$

What else can we learn from a_μ^{win} ?

Primary observable in lattice calculations: vector correlator $G(t)$

$$G(t) \equiv -\frac{a^3}{3}\sum_k \sum_{\vec{x}} \left\langle j_k^{\mathrm{em}}(\vec{x},t)\, j_k^{\mathrm{em}}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\, R(s)^{\mathrm{lat}}\, s\, \mathrm{e}^{-\sqrt{s}t}$$

What else can we learn from a_μ^{win} ?

Primary observable in lattice calculations: vector correlator $G(t)$

$$G(t) \equiv -\frac{a^3}{3}\sum_k \sum_{\vec{x}} \left\langle j_k^{\mathrm{em}}(\vec{x},t)\, j_k^{\mathrm{em}}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\, R(s)^{\mathrm{lat}}\, s\, \mathrm{e}^{-\sqrt{s}t}$$

$a_\mu^{\mathrm{win}}\big|_{\mathrm{lat}} > a_\mu^{\mathrm{win}}\big|_{e^+e^-}$ suggests that $R(s)^{\mathrm{lat}}$ is enhanced relative to $R(s)^{e^+e^-}$

What else can we learn from $a_\mu^{\rm win}$?

Primary observable in lattice calculations: vector correlator $G(t)$

$$G(t) \equiv -\frac{a^3}{3}\sum_k \sum_{\vec{x}} \left\langle j_k^{\rm em}(\vec{x},t)\, j_k^{\rm em}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\, R(s)^{\rm lat}\, s\, {\rm e}^{-\sqrt{s}t}$$

$a_\mu^{\rm win}\big|_{\rm lat} > a_\mu^{\rm win}\big|_{e^+e^-}$ suggests that $R(s)^{\rm lat}$ is enhanced relative to $R(s)^{e^+e^-}$

WINDOW OBSERVABLE FOR THE HADRONIC VACUUM … PHYS. REV. D **106,** 114502 (2022)

TABLE IV. Fractional contributions in percent from different regions in $\sqrt{s}$ to $a_\mu^{\rm hvp}$ and the partial quantities $(a_\mu^{\rm hvp})^{\rm SD,ID,LD}$, as well as the subtracted vacuum polarization at scale $Q^2 = 1$ GeV^2, according to the R-ratio model given in Ref. [49]. Note that this model includes neither the charm nor final states containing a photon, such as $\pi^0\gamma$.

$\sqrt{s}$ interval	$a_\mu^{\rm hvp}$	$(a_\mu^{\rm hvp})^{\rm SD}$	$(a_\mu^{\rm hvp})^{\rm ID}$	$(a_\mu^{\rm hvp})^{\rm LD}$	$\bar{\Pi}(1\ {\rm GeV}^2)$
Below 0.6 GeV	15.5	1.5	5.5	23.5	8.2
0.6 to 0.9 GeV	58.3	23.1	54.9	65.4	52.6
Above 0.9 GeV	26.2	75.4	39.6	11.1	39.2
Total	100.0	100.0	100.0	100.0	100.0

What else can we learn from $a_\mu^{\rm win}$?

- Phenomenological model for R-ratio predicts *[Mainz/CLS, Cè et al., et al., PRD 106 (2022) 114502]*

$$\sqrt{s} = 600 - 900\,\mathrm{MeV}: \qquad \frac{R(s)^{\rm lat}}{R(s)^{e^+e^-}} = 1 + \epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\rm hvp})^{\rm lat}}{(a_\mu^{\rm hvp})^{e^+e^-}} \approx \frac{(a_\mu^{\rm win})^{\rm lat}}{(a_\mu^{\rm win})^{e^+e^-}} = 1 + 0.6\epsilon$$

What else can we learn from $a_\mu^{\rm win}$?

- Phenomenological model for R-ratio predicts *[Mainz/CLS, Cè et al., et al., PRD 106 (2022) 114502]*

$$\sqrt{s} = 600 - 900\,\mathrm{MeV}: \qquad \frac{R(s)^{\rm lat}}{R(s)^{e^+e^-}} = 1 + \epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\rm hvp})^{\rm lat}}{(a_\mu^{\rm hvp})^{e^+e^-}} \approx \frac{(a_\mu^{\rm win})^{\rm lat}}{(a_\mu^{\rm win})^{e^+e^-}} = 1 + 0.6\epsilon$$

- Lattice average vs. R-ratio: $(a_\mu^{\rm win})^{\rm lat}/(a_\mu^{\rm win})^{e^+e^-} = 1.030(8)$

$\Rightarrow$ $R(s)^{\rm lat}$ is enhanced by 5% relative to $R(s)^{e^+e^-}$ for $\sqrt{s} = 600 - 900\,\mathrm{MeV}$

What else can we learn from a_μ^{win} ?

- Phenomenological model for R-ratio predicts *[Mainz/CLS, Cè et al., et al., PRD 106 (2022) 114502]*

$$\sqrt{s} = 600 - 900\,\text{MeV}: \qquad \frac{R(s)^{\text{lat}}}{R(s)^{e^+e^-}} = 1 + \epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\text{hvp}})^{\text{lat}}}{(a_\mu^{\text{hvp}})^{e^+e^-}} \approx \frac{(a_\mu^{\text{win}})^{\text{lat}}}{(a_\mu^{\text{win}})^{e^+e^-}} = 1 + 0.6\epsilon$$

- Lattice average vs. R-ratio: $(a_\mu^{\text{win}})^{\text{lat}}/(a_\mu^{\text{win}})^{e^+e^-} = 1.030(8)$

$\Rightarrow$ $R(s)^{\text{lat}}$ is enhanced by 5% relative to $R(s)^{e^+e^-}$ for $\sqrt{s} = 600 - 900\,\text{MeV}$

- If confirmed, it would imply that BMW's estimate might be too low….
  (see discussion in *Colangelo, Hoferichter, Stoffer, Phys. Lett. B814 (2021) 136073*)

The Tension Returns….

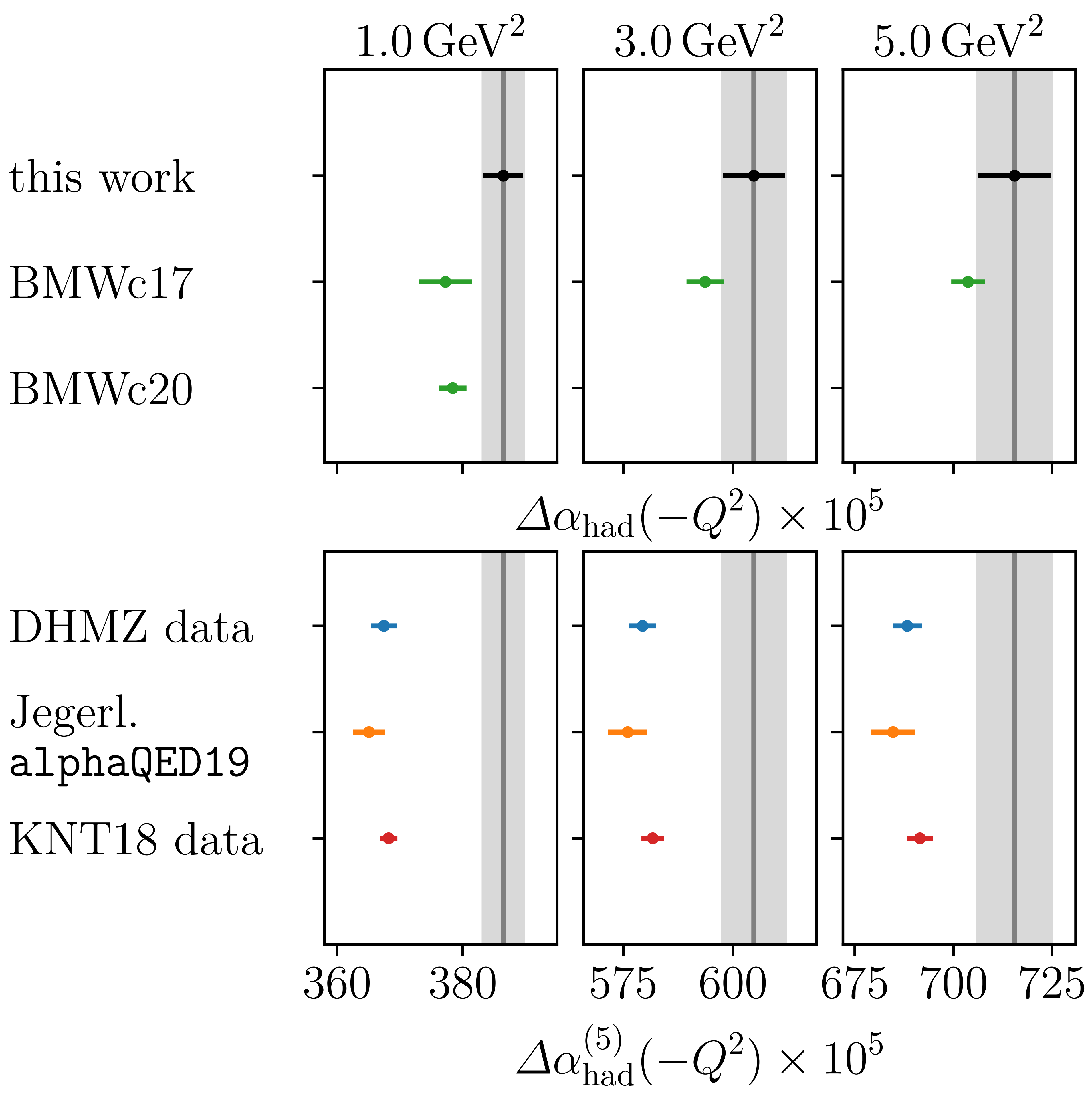
Hadronic running of α

Dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{\alpha\, q^2}{3\pi} \,⨍_{m^2_{\pi^0}}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}$$

Lattice QCD:

$$\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^\infty dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$



[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

The Tension Returns….

Hadronic running of α

Dispersion integral:

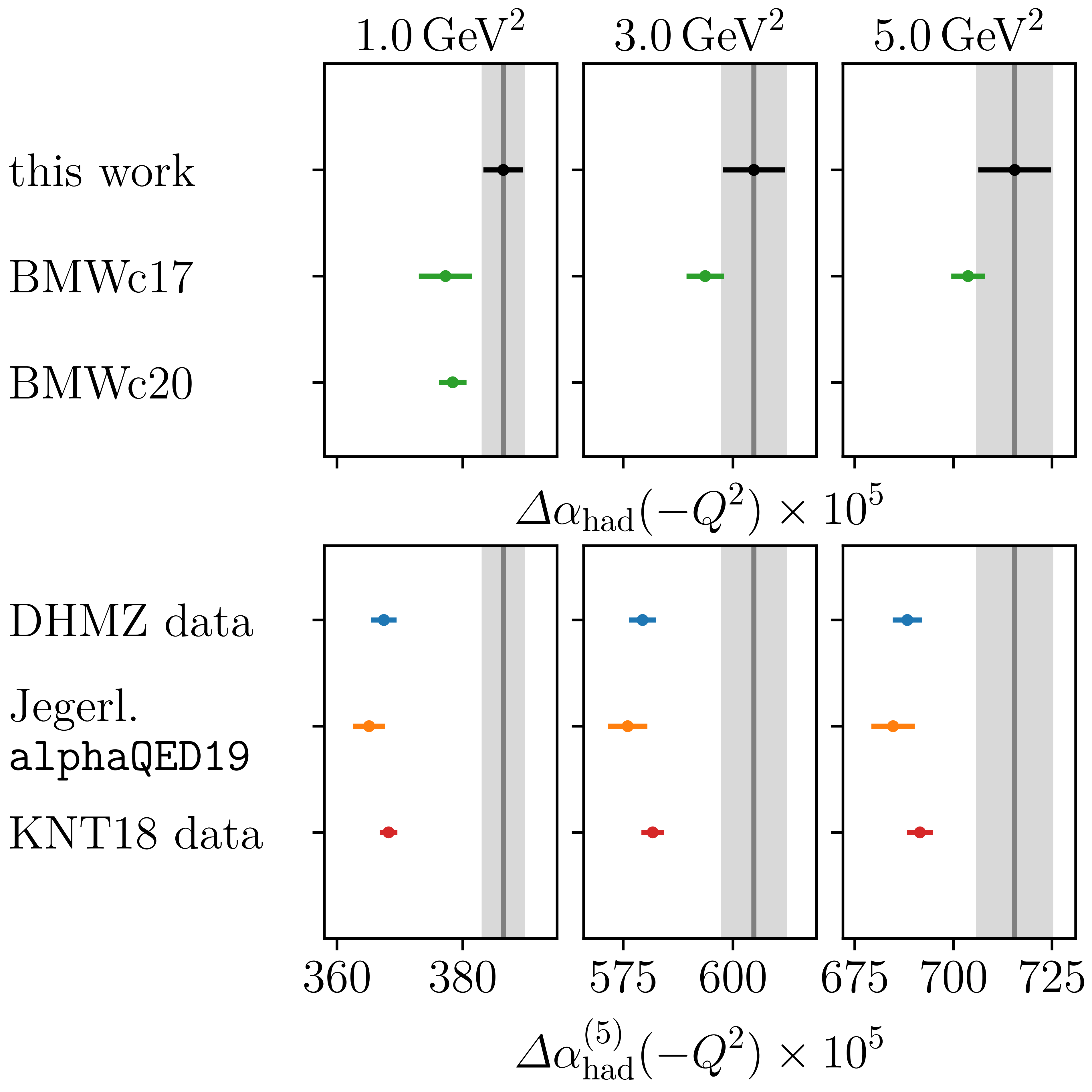
$$\Delta\alpha_{\rm had}^{(5)}(q^2) = -\frac{\alpha\, q^2}{3\pi} \oint_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}$$

Lattice QCD:

$$\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^{\infty} dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$

- Tension of $\sim 3\sigma$ observed with data-driven evaluation of $\Delta\alpha_{\rm had}(-Q^2)$ for $Q^2 \gtrsim 3\,\mathrm{GeV}^2$

  $\rightarrow$ consistent with tension for window observable



[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

But what about electroweak precision data?

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2) \quad \leftarrow \text{ lattice QCD}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)] \quad \leftarrow \text{ perturbative Adler function}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)] \quad \leftarrow \text{ pQCD}$$

But what about electroweak precision data?

Adler function approach, aka. “Euclidean split technique”

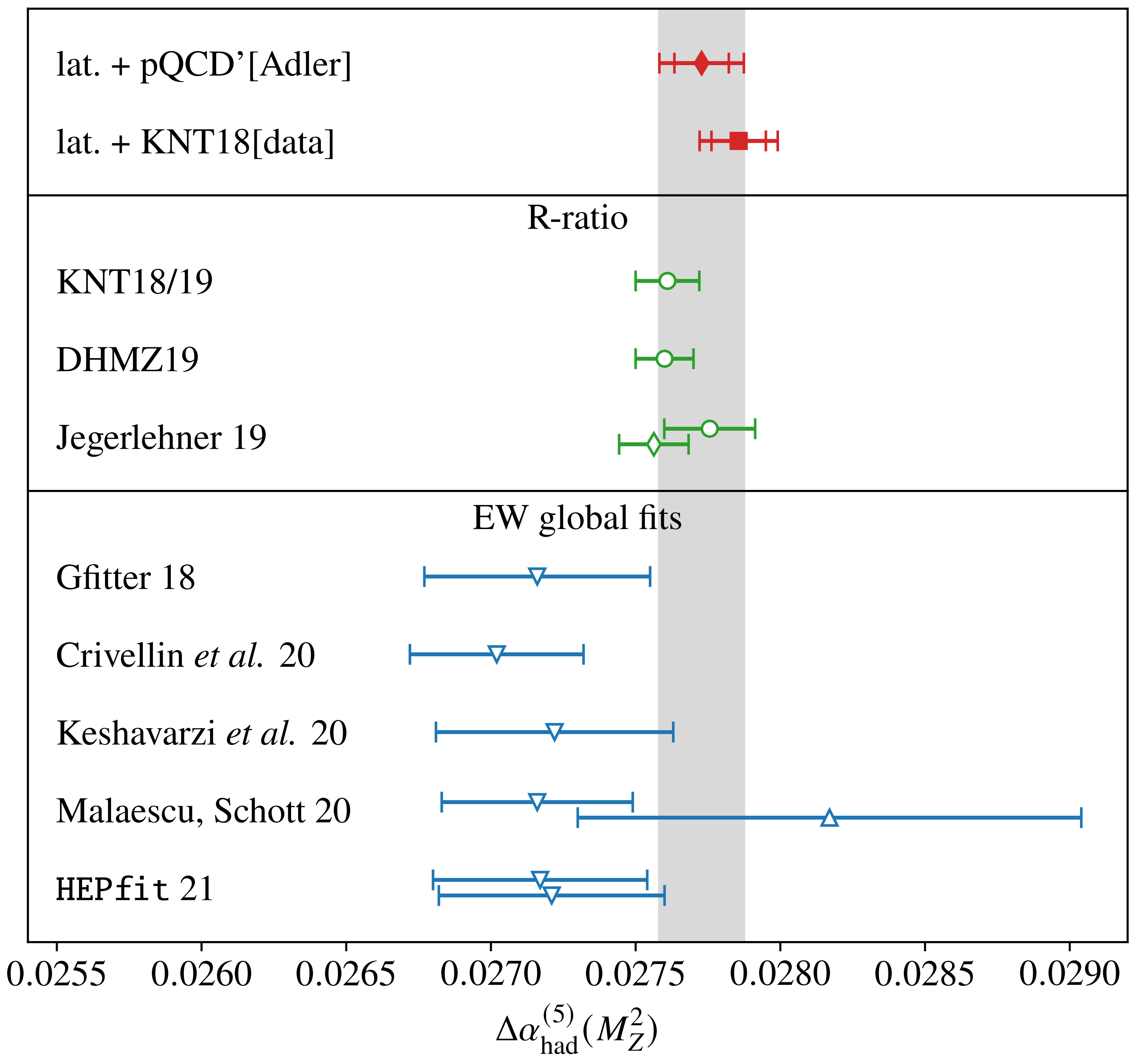
$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]



But what about electroweak precision data?

Adler function approach, aka. “Euclidean split technique”

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$

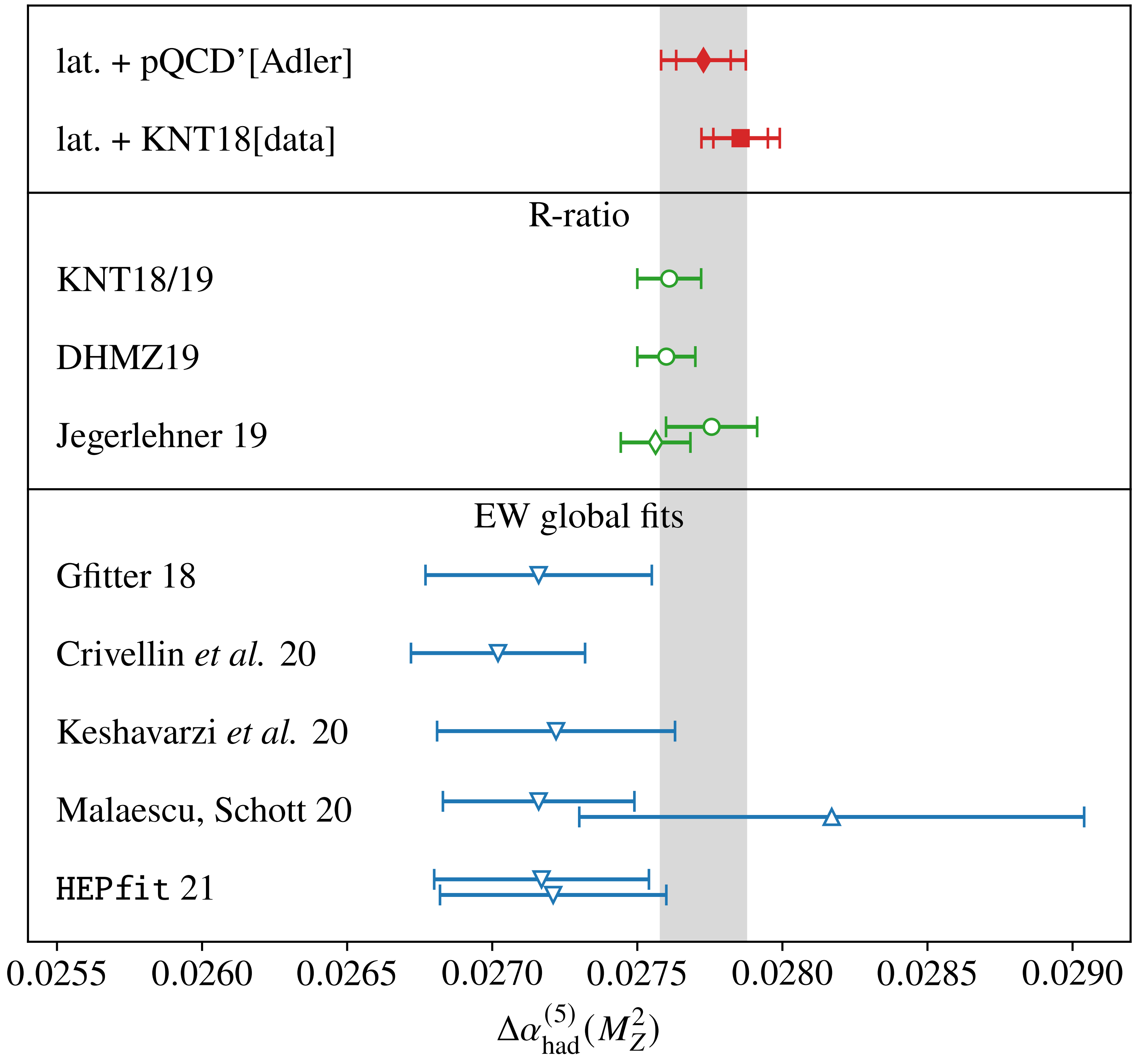
$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

- Agreement between lattice QCD and R-ratio estimate
- Contradiction with tension observed at low energies?



But what about electroweak precision data?

Adler function approach, aka. “Euclidean split technique”

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$

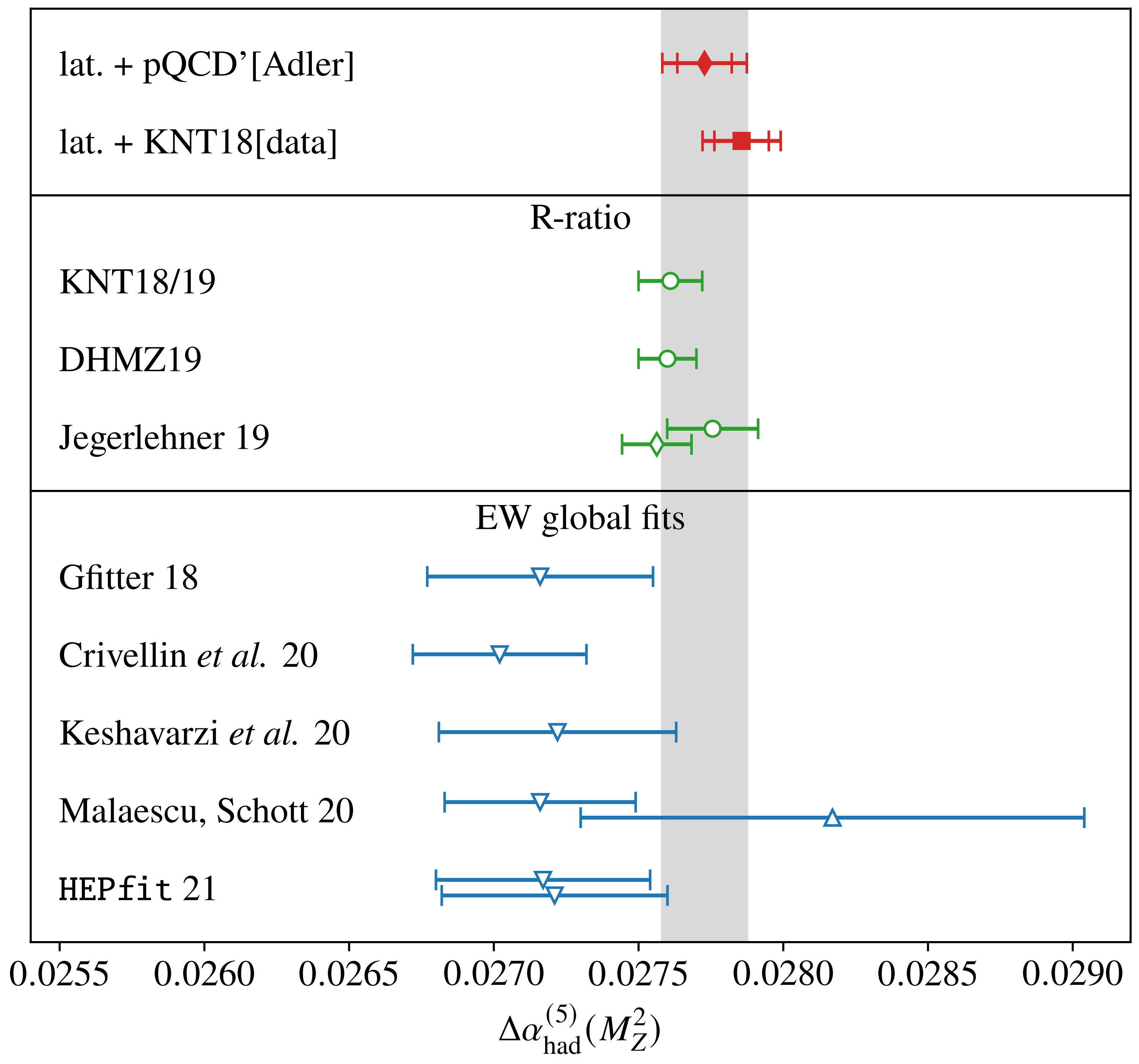
$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

- Agreement between lattice QCD and R-ratio estimate
- Contradiction with tension observed at low energies?

  Not in the correlated difference!



But what about electroweak precision data?

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

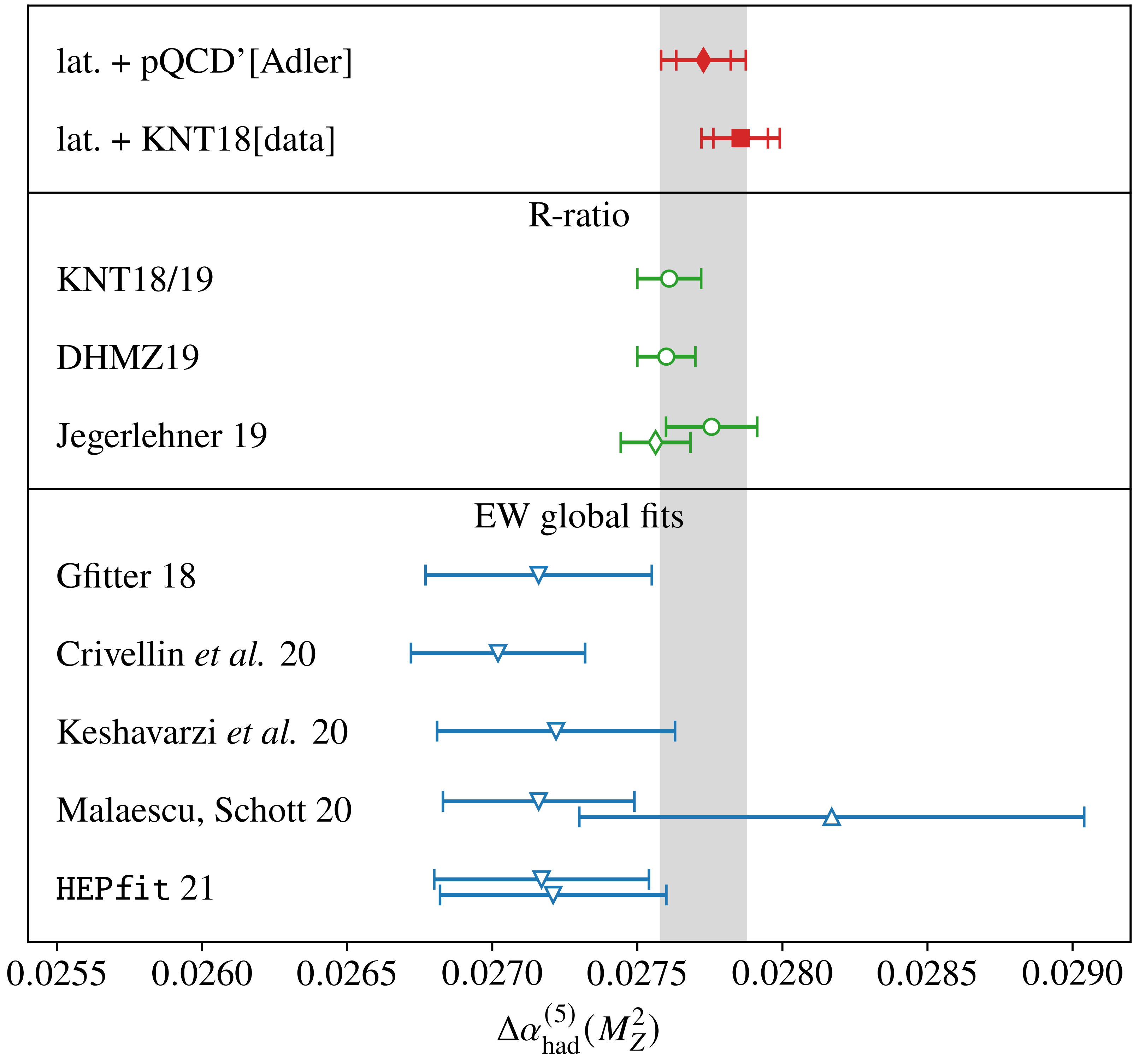
$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

- Agreement between lattice QCD and R-ratio estimate
- Contradiction with tension observed at low energies?

  Not in the correlated difference!

No inconsistency with global electroweak fit



Comparison with perturbative Adler function

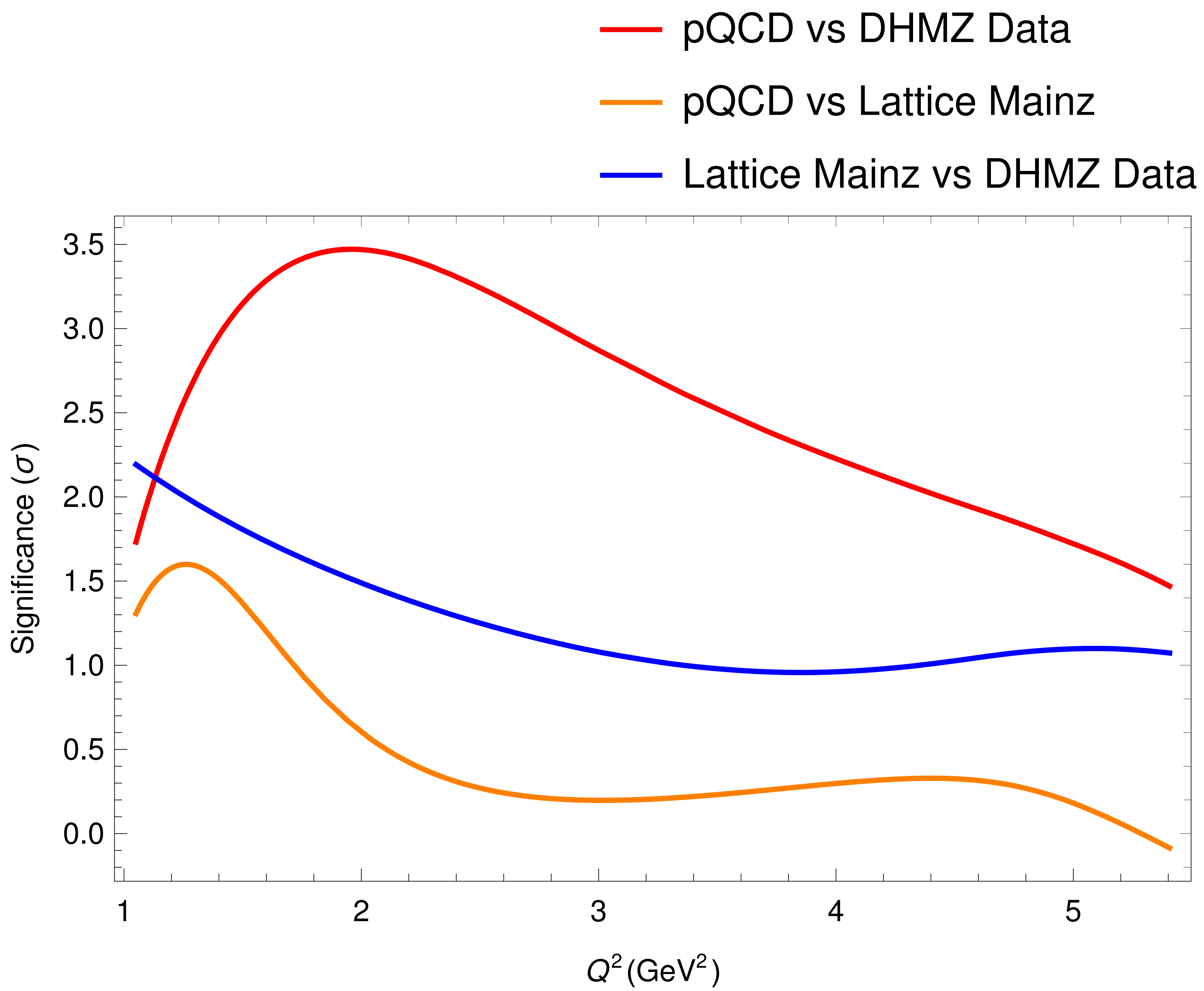
Adler function:

$$D(-s) = \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\, \Delta\alpha_{\mathrm{had}}(s)$$

- Known in massive QCD perturbation theory at four loops
- Data-driven evaluation of $D(Q^2)$ via R-ratio:

$$D(Q^2) = Q^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}$$

- Determine $D(Q^2)$ from lattice calculation of $\Delta\alpha_{\mathrm{had}}(Q^2)$

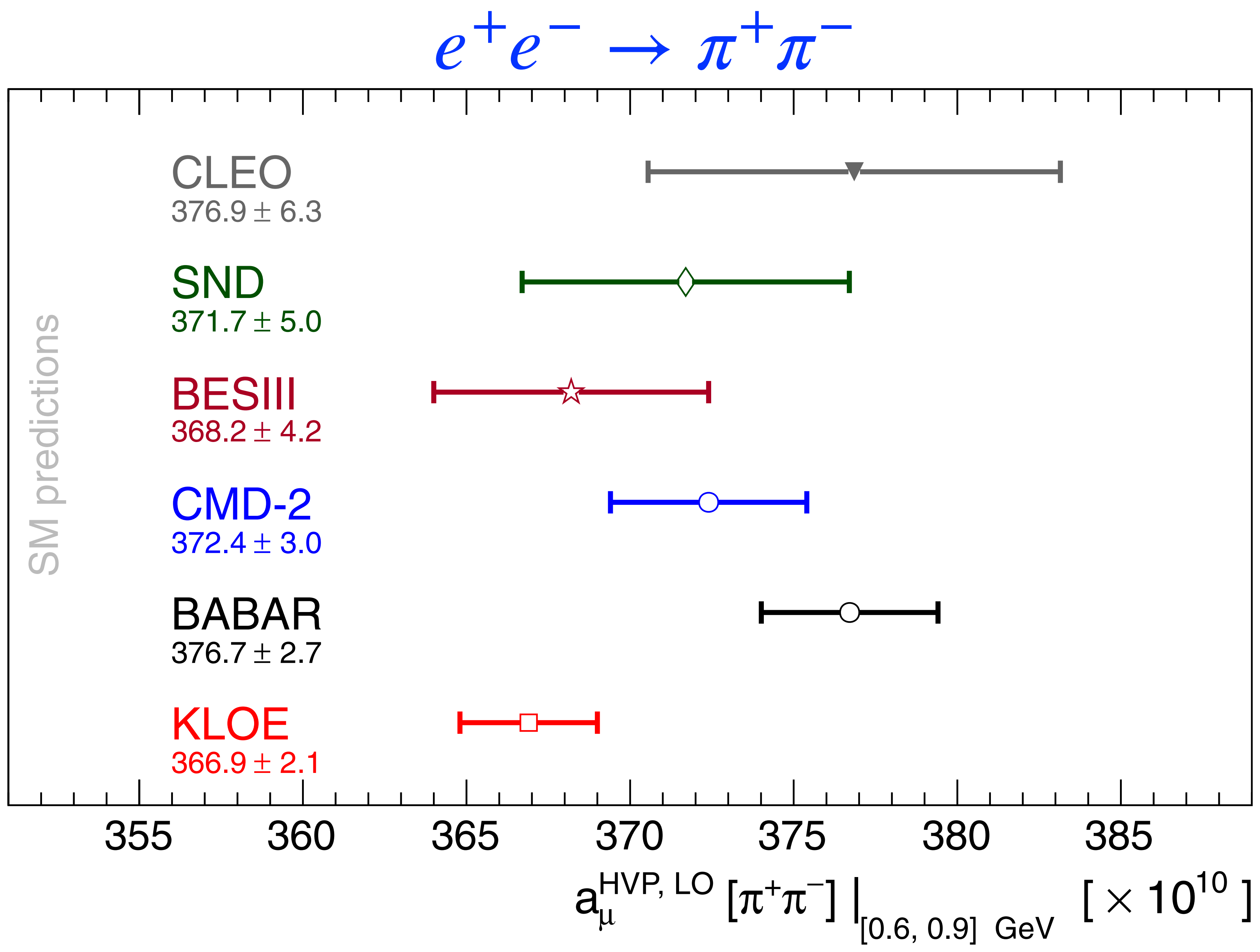


Good agreement between perturbative and lattice QCD for $Q^2 \gtrsim 2\,\mathrm{GeV}^2$

Slight tension of 1–2σ between data-driven evaluation and QCD

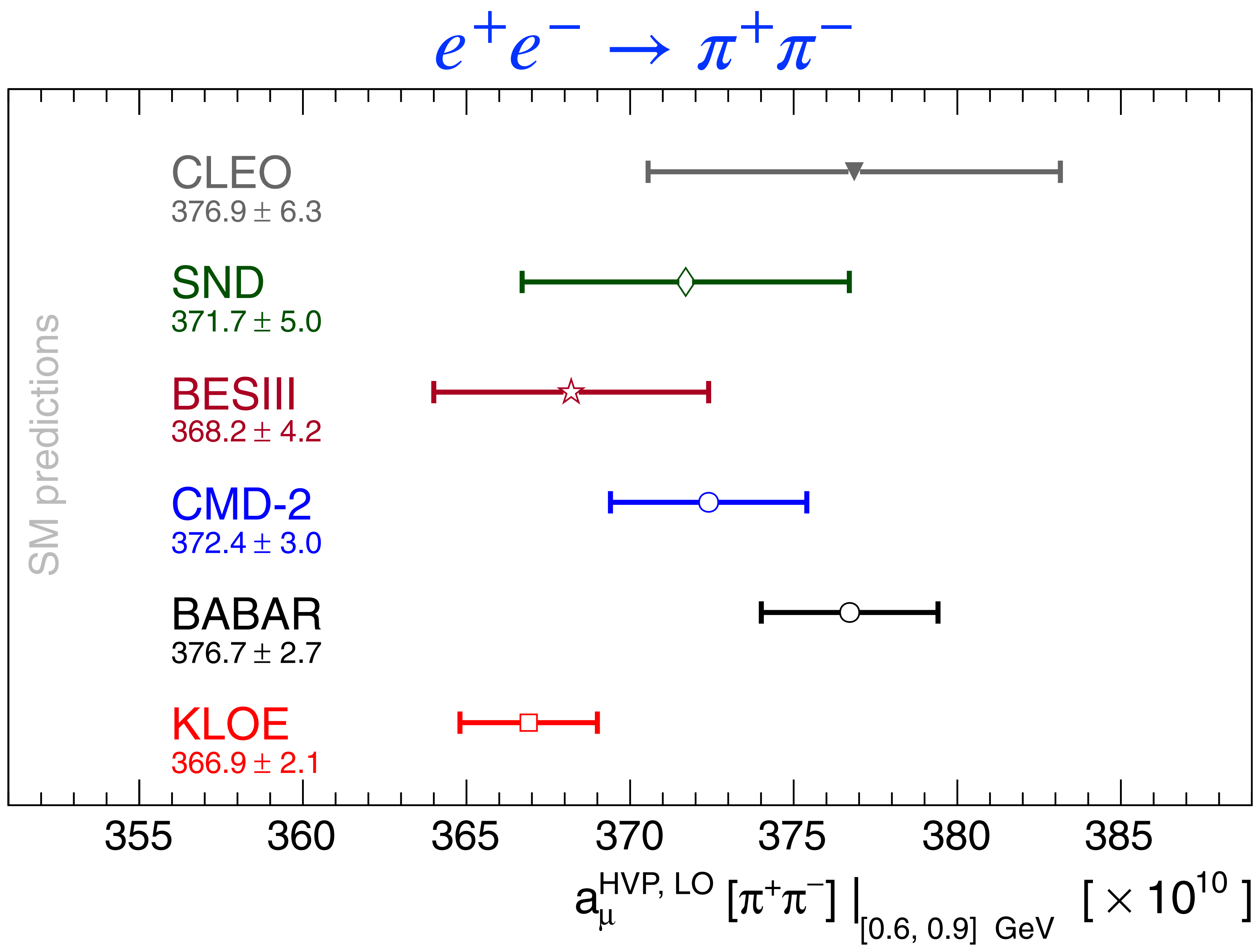
[Davier, Díaz-Calderón, Malaescu, Pich, Rodríguez-Sánchez, Zhang, JHEP 04 (2023) 067, arXiv:2302.01359]

And finally….



- There is a tension of 2.7σ in the dominant $\pi^+\pi^-$ channel between BaBar and KLOE

And finally….



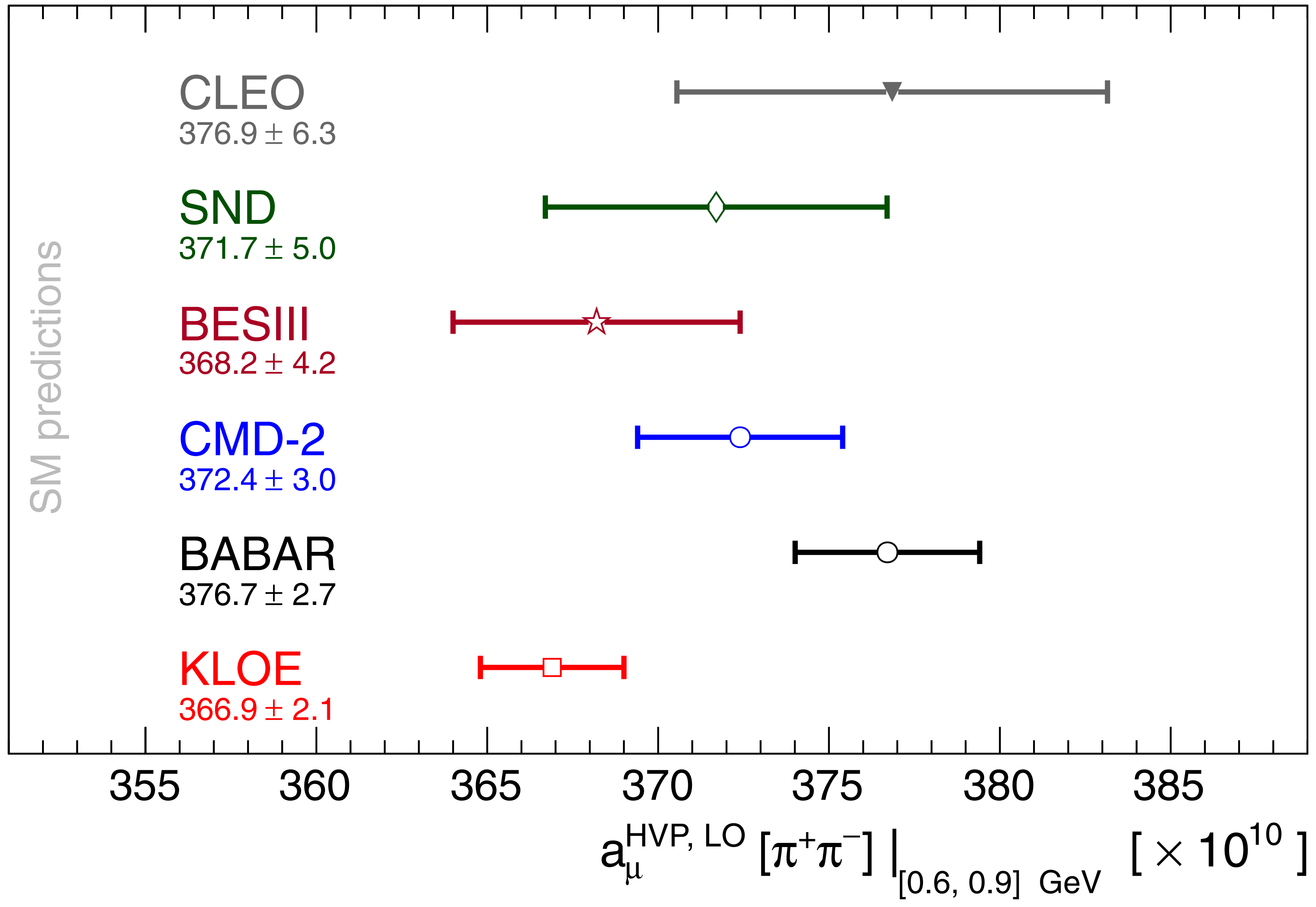
- There is a tension of 2.7σ in the dominant $\pi^+\pi^-$ channel between BaBar and KLOE

$$a_\mu^{\text{hvp}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{syst}}(0.7)_{\text{DV+QCD}} \cdot 10^{-10}$$

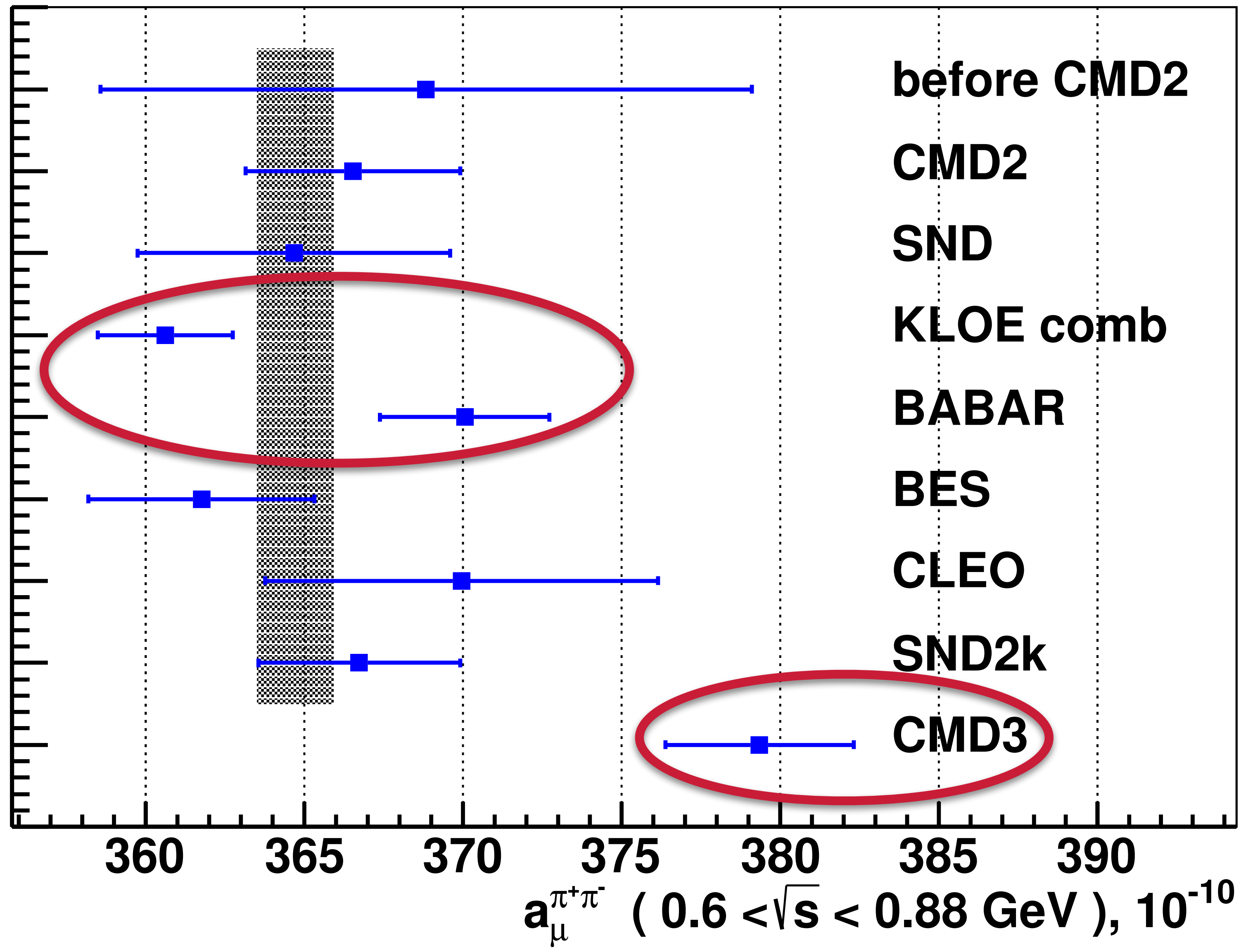
(accounts for tension in the data and among analyses)

And finally….

$e^+e^- \to \pi^+\pi^-$



[Ignatov et al. (CMD-3 Collab.), arXiv:2302.08834]



- There is a tension of 2.7σ in the dominant $\pi^+\pi^-$ channel between BaBar and KLOE

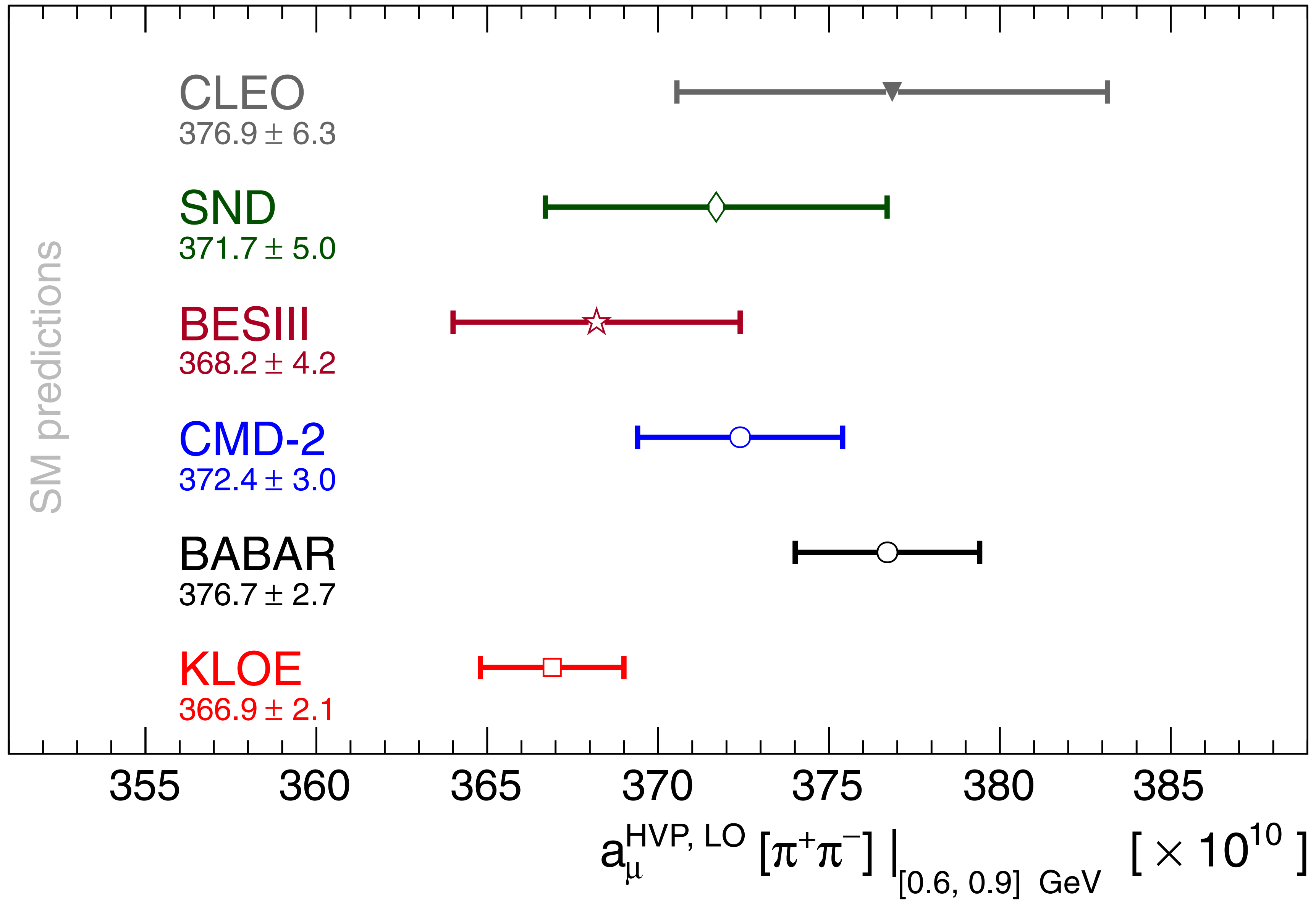
$$a_\mu^{\mathrm{hvp}} = 693.1(2.8)_{\mathrm{exp}}(2.8)_{\mathrm{syst}}(0.7)_{\mathrm{DV+QCD}} \cdot 10^{-10}$$

(accounts for tension in the data and among analyses)

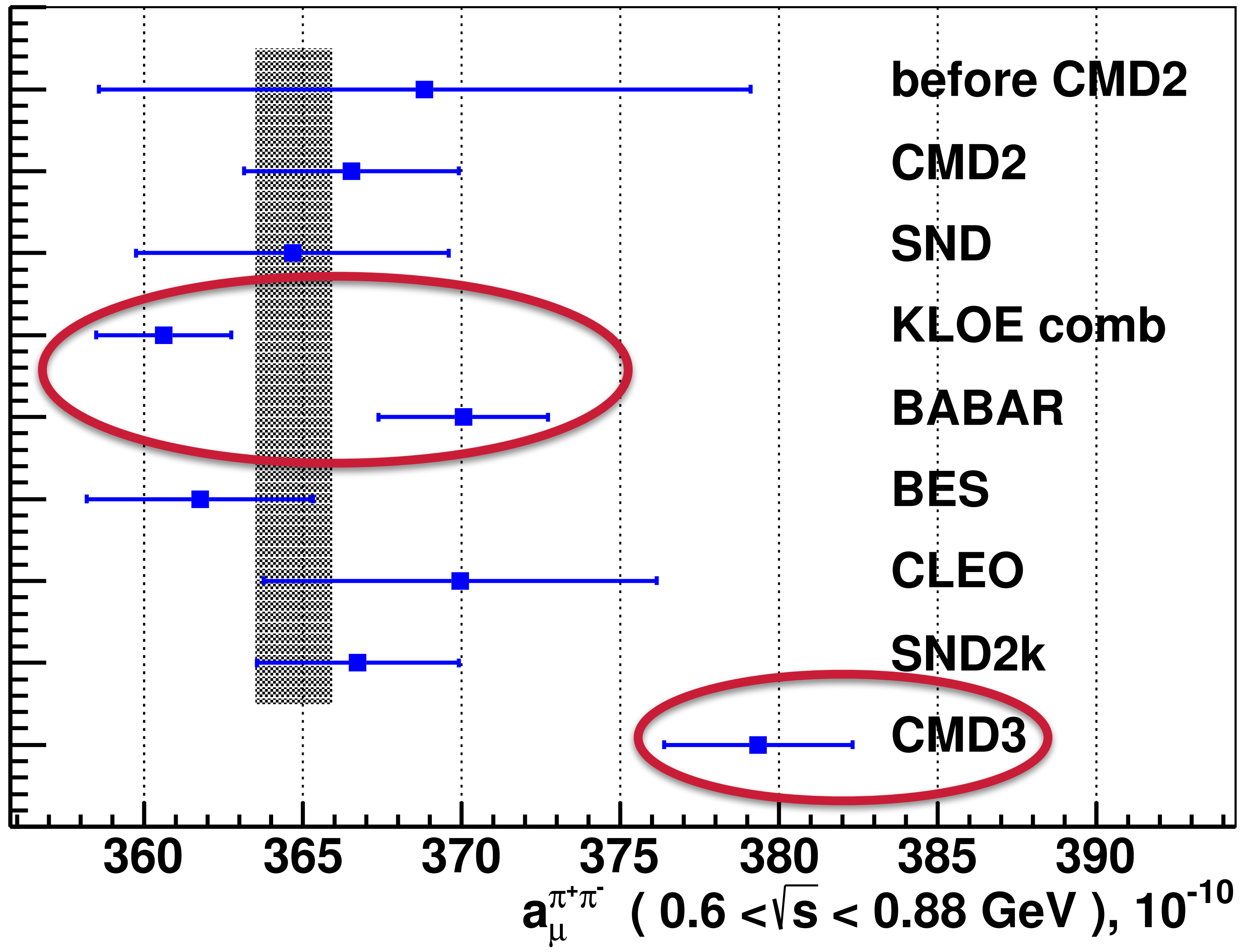
- There is a tension of 4σ in the dominant $\pi^+\pi^-$ channel between CMD-3 and all previous measurements

And finally….

$e^+e^- \to \pi^+\pi^-$



[Ignatov et al. (CMD-3 Collab.), arXiv:2302.08834]



- There is a tension of 2.7σ in the dominant $\pi^+\pi^-$ channel between BaBar and KLOE

$$a_\mu^{\rm hvp} = 693.1(2.8)_{\rm exp}(2.8)_{\rm syst}(0.7)_{\rm DV+QCD} \cdot 10^{-10}$$

(accounts for tension in the data and among analyses)

- There is a tension of 4σ in the dominant $\pi^+\pi^-$ channel between CMD-3 and all previous measurements

$$a_\mu^{\rm hvp} = 707.6(3.4)_{\rm exp}(0.7)_{\rm DV+QCD} \cdot 10^{-10}$$

(my own estimate)

Issues

Badly needed next steps

- Independent check of the HVP result by BMWc with comparable precision

  → in preparation

- Sort out the tension among e^+e^- cross section data

  → new analysis of BaBar data in progress; CMD-3 result is being scrutinised

- Clarify the role of τ-decay data as alternative to e^+e^-

Spares

But what about electroweak precision data?

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$
$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$
$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]

- Agreement between lattice QCD and R-ratio estimate
- Contradiction with tension observed at low energies?
  Not in the correlated difference!