The Euclidean Adler Function

Interplay with $\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}$, α_s and $(g-2)_\mu$

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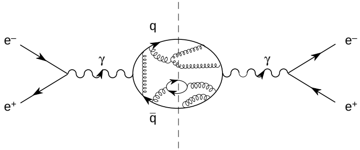
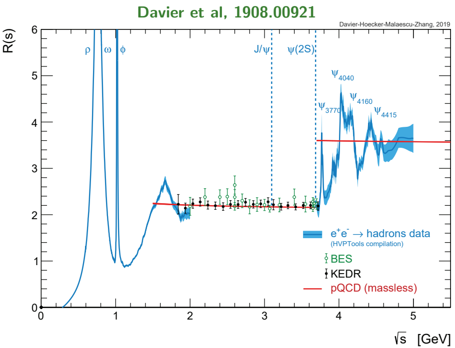


In collaboration with Michel Davier, David Díaz-Calderón, Bogdan Malaescu, Antonio Rodríguez-Sánchez and Zhiqing Zhang, arXiv:2302.01359

Lattice Gauge Theory Contributions to New Physics Searches

IFT (UAM–CSIC), 12-16 June 2023

Major tensions in hadronic e^+e^- data



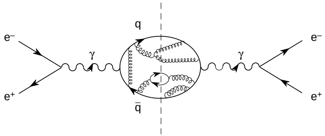
Experimental Discrepancies

- BaBar/KLOE disagreement at $\rho(\pi\pi)$
- Differences in $\phi(K^+K^-)$ largely exceed the quoted uncertainties
- Inclusive results larger than exclusive ones around 2 GeV

- Compilation of different data sets with quite different systematics
- Energy scanning versus Initial State Radiation method (BaBar, KLOE)
- Modelling of Final State Radiation needed (usually with scalar QED)
- The most relevant discrepancy for $a_\mu^{\mathrm{HVP,LO}}$ is Babar vs KLOE at $\rho(\pi\pi)$

$$R(s) \equiv \frac{3s}{4\pi\alpha}\,\sigma^0(e^+e^- \to \text{hadrons}\,(\gamma)) = 12\pi\,\text{Im}\Pi(s)$$

σ^0 = bare cross section (vacuum polarization and ISR subtracted)



$$\mathcal{J}^{\mu}_{\text{em}} = \sum_i \mathcal{Q}_i\, \bar{q}_i \gamma^\mu q_i$$

$$\Pi^{\mu\nu}(q) \equiv i\int d^4x\, e^{-iqx}\,\langle 0|\,T\left(\mathcal{J}^\mu_{\text{em}}(x)\,\mathcal{J}^\nu_{\text{em}}(0)\right)|0\rangle = \left(q^\mu q^\nu - g^{\mu\nu}q^2\right)\Pi(q^2)$$

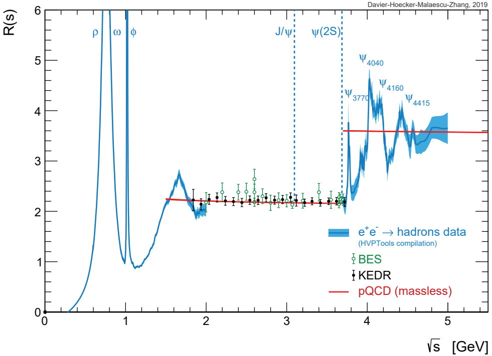
DHMZ Data Compilation

Davier et al, 1908.00921

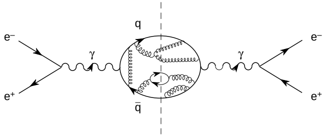
Perturbative QCD in

$1.8\ \text{GeV} < \sqrt{s} < 3.7\ \text{GeV}$

and $\sqrt{s} > 5\ \text{GeV}$



The Euclidean Adler Function: $Q^2 \equiv -q^2$

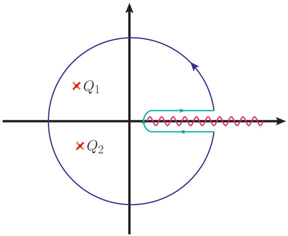


$$\mathcal{J}^\mu_{\mathrm{em}} = \sum_i \mathcal{Q}_i \, \bar{q}_i \gamma^\mu q_i$$

$$D(Q^2) \equiv -12\pi^2 \, Q^2 \, \frac{d\Pi(Q^2)}{dQ^2} = \frac{3\pi}{\alpha} \, Q^2 \, \frac{d\Delta\alpha_{\mathrm{had}}(Q^2)}{dQ^2}$$

Dispersion relations

(analyticity)



$$D(Q^2) = Q^2 \int_{s_{th}}^\infty ds \, \frac{R(s)}{(s+Q^2)^2}$$

$$\Delta\alpha_{\mathrm{had}}(Q^2) = \frac{\alpha Q^2}{3\pi} \int_{s_{th}}^\infty ds \, \frac{R(s)}{s\,(s+Q^2)}$$

Lattice Adler Function:

Cè et al (Mainz), 2203.08676

$$D(Q^2) \equiv -12\pi^2\, Q^2\, \frac{d\Pi(Q^2)}{dQ^2} = \frac{3\pi}{\alpha}\, Q^2\, \frac{d\Delta\alpha_{\rm had}(Q^2)}{dQ^2}$$

$$\Pi(0) - \Pi(Q^2) \approx \frac{\sum_{n=1}^{3} a_n x^n}{1+\sum_{n=1}^{3} b_n x^n} = \frac{0.1094\,(23)\,x + 0.093\,(15)\,x^2 + 0.0039\,(6)\,x^3}{1 + 2.85\,(22)\,x + 1.03\,(19)\,x^2 + 0.0166\,(12)\,x^3}$$

$$0.1 \le x \equiv Q^2/\mathrm{GeV}^2 \le 7$$

$$\mathrm{corr}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 & & & & & \\ 0.455 & 1 & & & & \\ 0.17 & 0.823 & 1 & & & \\ 0.641 & 0.946 & 0.642 & 1 & & \\ 0.351 & 0.977 & 0.915 & 0.869 & 1 & \\ 0.0489 & -0.0934 & 0.0667 & -0.044 & -0.115 & 1 \end{pmatrix}$$

$\Longrightarrow$ $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73 \pm 0.000\,15$

QCD Adler Function at Euclidean Q^2

Away from physical cut → **Reliable short-distance methods**

$\overline{\mathrm{MS}}$ scheme (running & matching): $m_{u,d} \ll m_s \ll \Lambda_{\mathrm{QCD}} \ll m_c \ll m_b \ll m_t$

Previous analyses in (decoupling) MOM scheme limited to $\mathcal{O}(\alpha_s^2)$ (Eidelman et al, hep-ph/9812521)

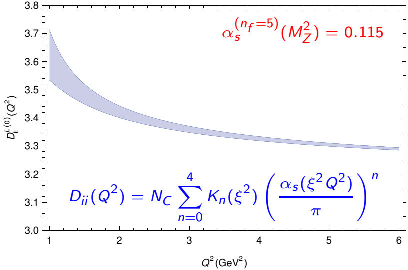
Perturbative $D(Q^2)$ at low Q^2 (3 flavours):

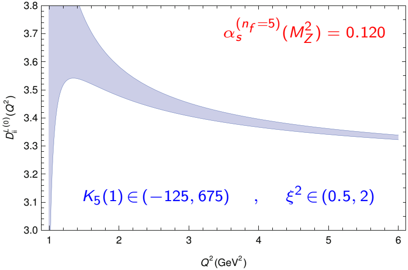
- Massless correlator and matching conditions at $\mathcal{O}(\alpha_s^4)$
- m_s^2/Q^2 corrections at $\mathcal{O}(\alpha_s^3)$
- $\mathcal{O}(\alpha_s^2)$ charm corrections to light-quark correlators (suppressed by $(\frac{Q^2}{4m_c^2})^n$)
- Contributions from heavy-quark correlators at $\mathcal{O}[\alpha_s^2\,(\frac{Q^2}{4m_c^2})^{30}\,,\;\alpha_s^3\,(\frac{Q^2}{4m_c^2})^{10}]$
- Leading QED corrections

Non-perturbative corrections (OPE): $\mathcal{O}[(\Lambda_{\mathrm{QCD}}^2/Q^2)^{D/2}]$ with $D=4,6$

$\mathcal{J}^{\mu}_{\rm em} = \sum_i \mathcal{Q}_i\, \bar{q}_i \gamma^\mu q_i$ ➡ $D(Q^2) = \sum_{i,j} \mathcal{Q}_i \mathcal{Q}_j\, D_{ij}(Q^2)$

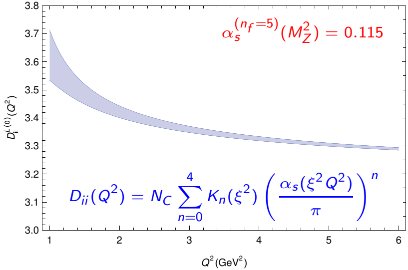
Perturbative light-quark contribution $(m_q = 0)$

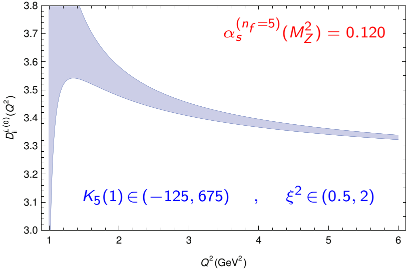




$$\mathcal{J}^{\mu}_{\rm em} = \sum_i \mathcal{Q}_i\, \bar{q}_i \gamma^\mu q_i \quad \longrightarrow \quad D(Q^2) = \sum_{i,j} \mathcal{Q}_i \mathcal{Q}_j\, D_{ij}(Q^2)$$

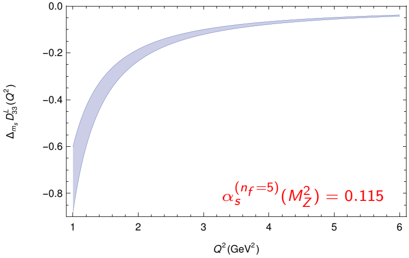
Perturbative light-quark contribution $(m_q = 0)$

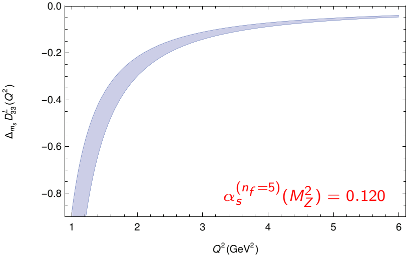




Strange-mass correction

(large relative error but small contribution: $\mathcal{Q}_s^2 = \frac{1}{9}$)

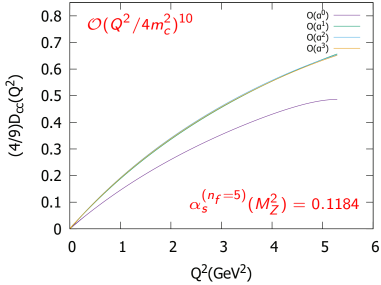


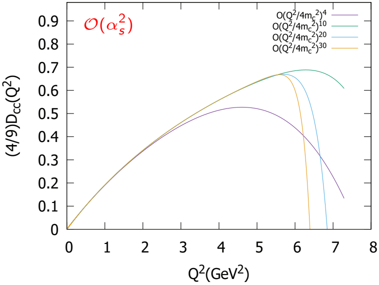


Charm contributions:

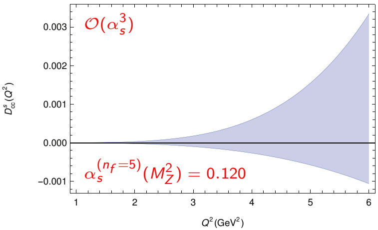
$D(Q^2) = \sum_{i,j} \mathcal{Q}_i \mathcal{Q}_j \, D_{ij}(Q^2)$

Non-singlet charm correlator

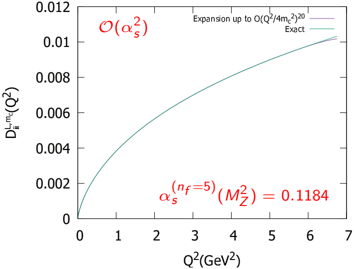




Singlet charm correlator



Light-quark correlators



Non-perturbative Power Corrections (OPE)

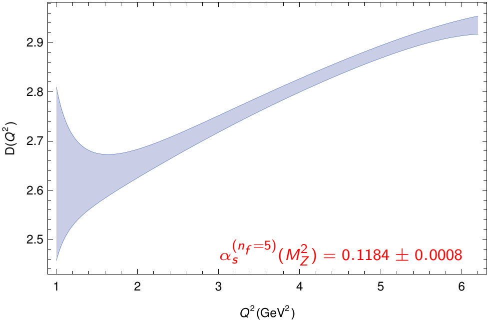
$$\delta D_{\rm em}^{L,D=4} = \frac{4\pi^2}{3Q^4}\left\{\left(1+\frac{7}{6}\frac{\alpha_s}{\pi}\right)\langle\frac{\alpha_s}{\pi}GG\rangle + 4\left[1+\frac{1}{3}\frac{\alpha_s}{\pi}+\left(\frac{27}{8}+4\,\zeta_3\right)\left(\frac{\alpha_s}{\pi}\right)^2\right] m_s\langle\bar{s}s\rangle]\right\}$$

$$\left.\begin{aligned}\langle\tfrac{\alpha_s}{\pi}GG\rangle &= (0.012\pm 0.012)\,\mathrm{GeV}^4\\ m_s\langle\bar{s}s\rangle &= -F_K^2 M_K^2\left[1-\delta_{\mathcal{O}(p^4,m_{u,d})}\right]\\ &\approx -(1.3\pm 0.7)\cdot 10^{-3}\;\mathrm{GeV}^4\end{aligned}\right\} \quad\longrightarrow\quad \delta D_{\rm em}^{L,D=4}\approx\frac{(0.10\pm 0.18)\,\mathrm{GeV}^4}{Q^4}$$

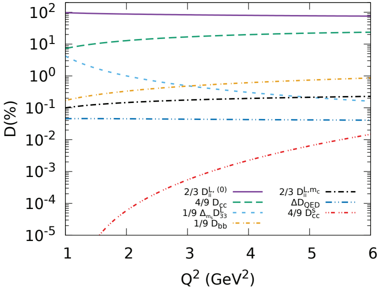
$$\left.\begin{aligned}\mathcal{O}^{\tau}_{6,V-A} &= (-0.0035\pm 0.0009)\,\mathrm{GeV}^6\\ \mathcal{O}^{N_C\to\infty}_{6,V+A} &= -\tfrac{2}{9}\,\mathcal{O}^{N_C\to\infty}_{6,V-A} < |\mathcal{O}_{6,V-A}|\end{aligned}\right\} \quad\longrightarrow\quad \mathcal{O}_{6,V} = (-0.0015\pm 0.0015)\,\mathrm{GeV}^6$$

$$\longrightarrow\quad D_{\rm em}^{L,D=6}\approx 24\pi^2\,\frac{\mathcal{O}_{6,V}}{Q^6} = \frac{-(0.36\pm 0.36)\,\mathrm{GeV}^6}{Q^6}$$

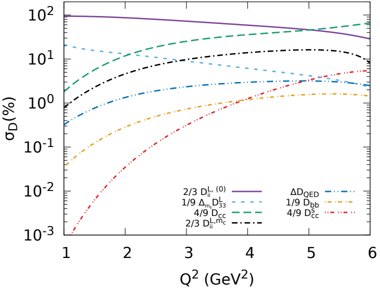
Perturbative Adler Function



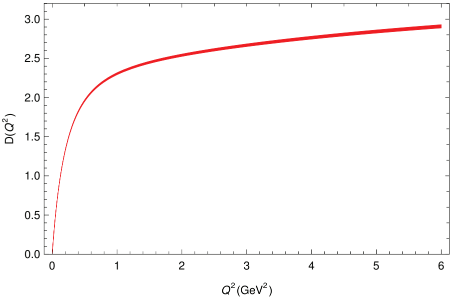
Separate contributions (%)



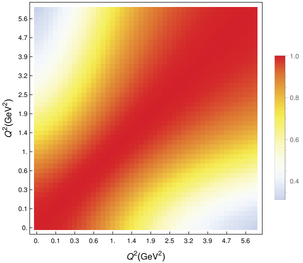
Error contributions (%)



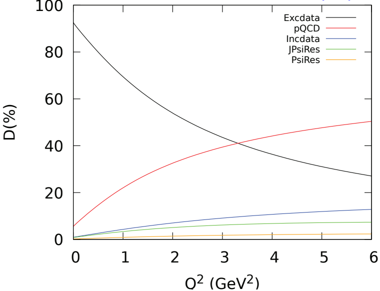
Dispersive Adler Function



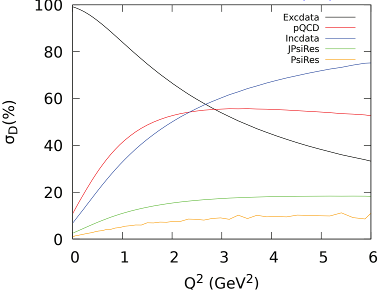
Correlations



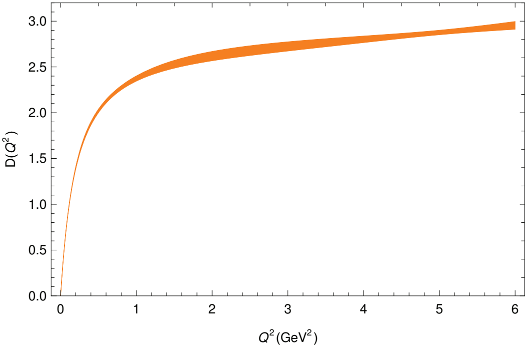
Separate contributions (%)



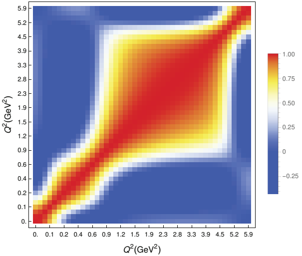
Error contributions (%)



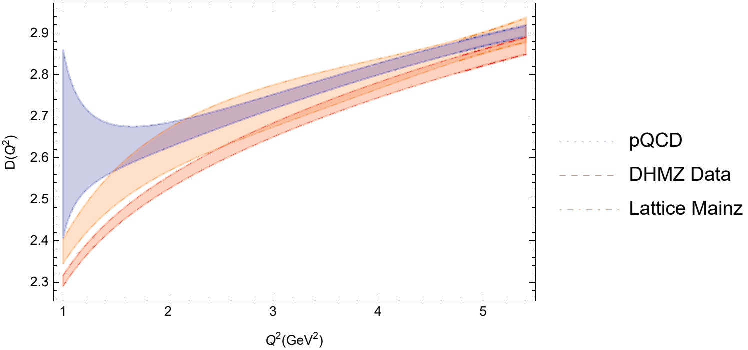
Lattice Adler Function



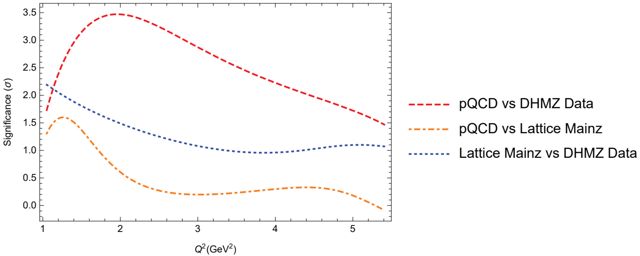
Correlations



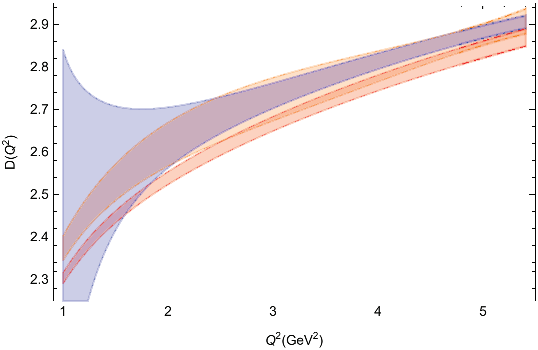
Comparison of the three Adler functions



Statistical significance of differences



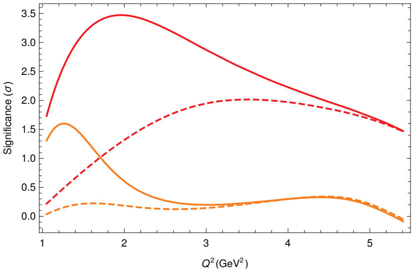
Comparison of the three Adler functions



Power corrections included

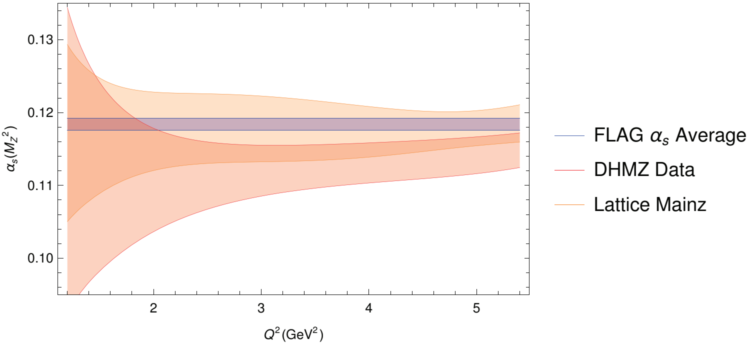
OPE
DHMZ Data
Lattice Mainz

Statistical significance of differences



Determination of $\alpha_s(M_Z^2)$

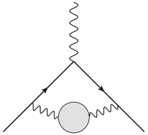
$$D^{\rm OPE}(Q^2) - D^{\rm data}(Q^2) = 0 \quad \longrightarrow \quad \alpha_s^{n_f=3)}(Q^2) \quad \longrightarrow \quad \alpha_s^{n_f=5)}(M_Z^2)$$



$$\longrightarrow \quad \alpha_s^{(n_f=5)}(M_Z^2) = \begin{cases} 0.1136 \pm 0.0025 & (e^+e^- \text{ data}) \\ 0.1179 \pm 0.0025 & (\text{Lattice}) \end{cases}$$

FLAG average: $\alpha_s^{(n_f=5)}(M_Z^2) = 0.1184 \pm 0.0008$

LO Hadronic Vacuum Polarization



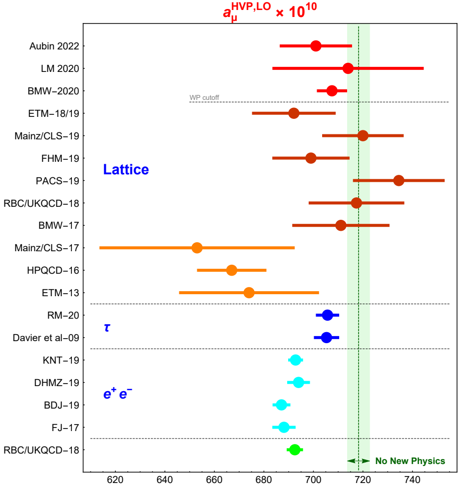
$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2 m_\mu^2}{9\pi^2} \int_{s_{\mathrm{th}}}^{\infty} \frac{ds}{s^2}\, \hat{K}(s)\, R(s)$$

↑

Dominated (75%) by 2π contribution

$$R(s) = \frac{\sigma^0[e^+e^- \to \text{hadrons}\,(\gamma)]}{4\pi\alpha^2/(3s)}$$

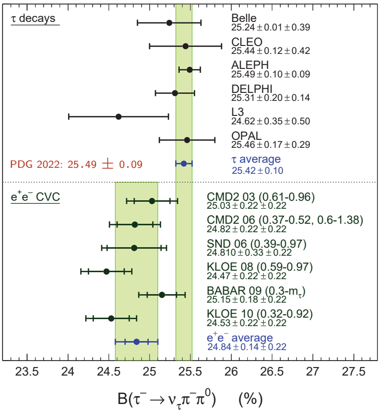
$$\frac{d\Gamma(\tau^- \to \nu_\tau V^-)}{ds} \propto \sigma^{I=1}(e^+e^- \to V^0)$$



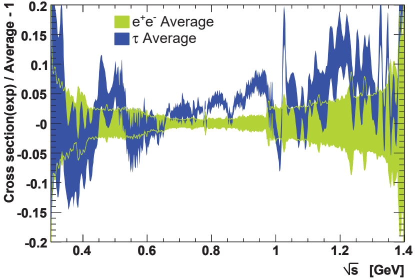
The BMW-2020 and τ results were not included in the WP 2020 value

e^+e^- versus τ data

Davier et al, 0906.5443



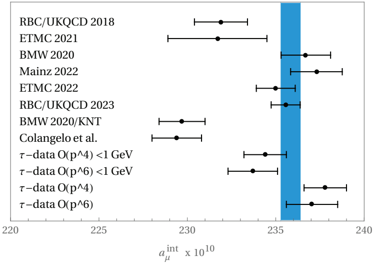
Davier et al, 0908.4300



Isospin-breaking corrections: Cirigliano et al, hep-ph/0104267, hep-ph/0207310
Flores-Baez et al, hep-ph/0608084

Updated in Miranda-Roig, 2007.11019

Euclidean Windows with $\tau \to 2\pi\nu_\tau$ Data

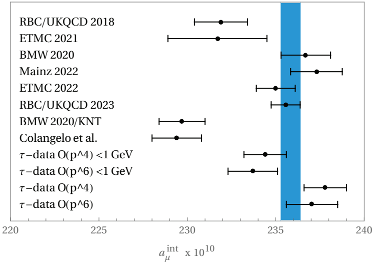


Masjuan-Miranda-Roig 2305.20005

e^+e^- data from Colangelo et al

e^+e^- data from DHMZ / KNT

Euclidean Windows with $\tau \to 2\pi\nu_\tau$ Data



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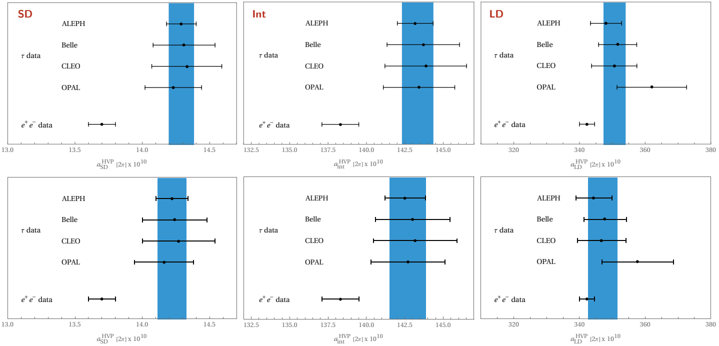


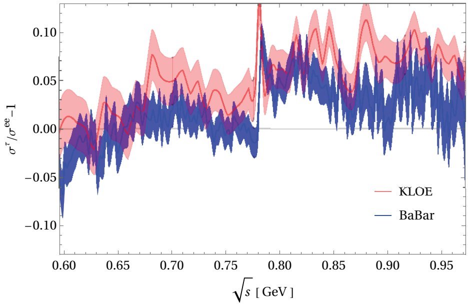
2π data only

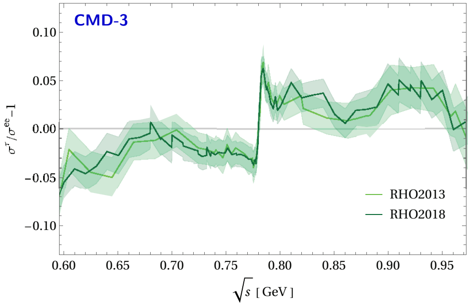
$\mathcal{O}(p^4)$ IB corrections

$\mathcal{O}(p^6)$ IB corrections

$\tau \to 2\pi\nu_\tau$ & $e^+e^- \to 2\pi$ Spectral Functions

Masjuan-Miranda-Roig 2305.20005





Summary

- The QCD Euclidean Adler function is $\sim 2\sigma$ higher than its dispersive evaluation from e^+e^- data
- The QCD Euclidean Adler function agrees with Lattice data
- Consistent hints from Lattice, τ data and CMD-3
- Unaccounted systematics in the e^+e^- data?
- Better data samples needed

  Belle-II, Beijing, Novosibirsk

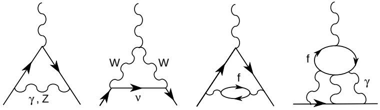
- Forthcoming MUonE experiment at CERN: $\sigma(\mu e \to \mu e)$

  Measure $\Pi_{\mathrm{em}}(Q^2)$ with space-like data

The μ anomaly does not necessarily imply New Physics

Backup

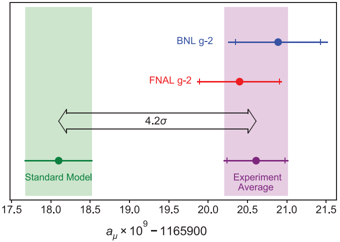
μ Anomalous Magnetic Moment



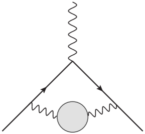
White Paper (2020) — G. Colangelo, Moriond EW 2021

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment (E821)	116 592 089(63)
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	279(76)

$$a_\mu = \frac{1}{2}\,(g-2)_\mu$$



$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (251 \pm 59)\cdot 10^{-11} \qquad (4.2\sigma)$$



LO Hadronic Vacuum Polarization

Aoyama et al, 2006.04822

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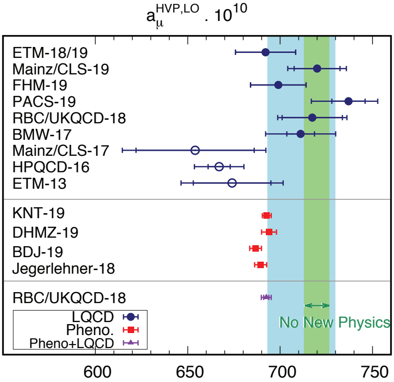
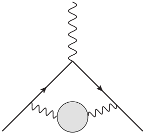
↑

Dominated (75%) by 2π contribution



$$R(s) = 12\pi\, {\rm Im}\Pi_{\rm em}(s)$$
$$= \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

2020 ➡ $\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (279 \pm 76) \cdot 10^{-11}$ (3.7σ)



LO Hadronic Vacuum Polarization

Aoyama et al, 2006.04822

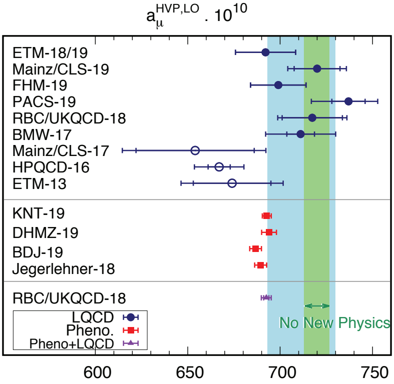
$$a_\mu^{\rm HVP,LO} = \frac{\alpha^2 m_\mu^2}{9\pi^2} \int_{s_{\rm th}}^{\infty} \frac{ds}{s^2}\, \hat{K}(s)\, R(s)$$

$\uparrow$

Dominated (75%) by 2π contribution

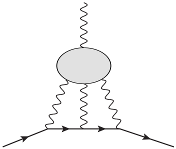
$$R(s) = \frac{\sigma^0[e^+e^- \to \text{hadrons}\,(\gamma)]}{4\pi\alpha^2/(3s)}$$

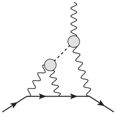
Data need to be undressed $\longrightarrow$ MC



2020 $\Rightarrow$ $\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (279 \pm 76)\cdot 10^{-11}$ $\quad (3.7\sigma)$

Light-by-Light Contributions





Colangelo, Moriond EW 2021

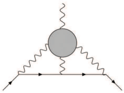
Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

A lot of work since Glasgow consensus (Prades, de Rafael, Vainshtein, 2009):

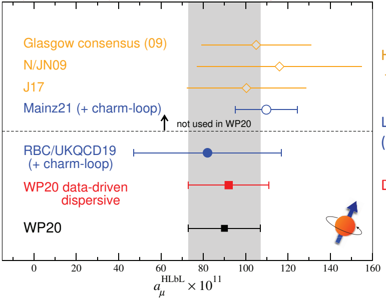
Masjuan, Sánchez-Puertas (17); Colangelo, Hagelstein, Hoferichter, Laub, Procura, Stoffer (17-20); Hoferichter, Hoid, Kubis, Leupold, Schneider (18); Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20, 21); . . .

Errors reduced, size unchanged ➡ **Cannot account for the anomaly**

Hadronic light-by-light scattering



[Aoyama et al., Phys. Rep. 887 (2020) 1; Colangelo et al., arXiv:2203.15810]



- Hadronic models, data-driven method and Lattice QCD produce consistent results
- White paper recommended value:

$$a_\mu^{\rm hlbl} = (92 \pm 18) \cdot 10^{-11}$$

- Recent lattice calculations (Mainz):

$$a_\mu^{\rm hlbl} = (109.6 \pm 14.7) \cdot 10^{-11}$$

[Chao et al., EPJC 81 (2021) 651; EPJC 82 (2022) 664]

$a_\mu^{\rm hlbl}$: **Uncontroversial** — contributes 0.15 ppm to the total SM uncertainty of 0.37 ppm

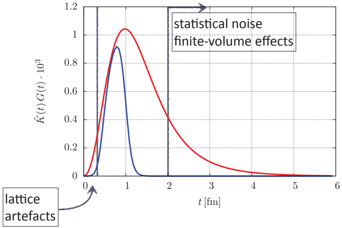
→ Focus on refinements and further reduction of uncertainty

Lattice: Intermediate-Distance Window

Time-momentum representation: $a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\; \tilde{K}(t)\, G(t)$

$G(t) = -a^3 \sum_{\vec{x}} \langle J_k^{em}(\vec{x}, t)\, J_k^{em}(0)\rangle$, $\tilde{K}(t)$ known kernel

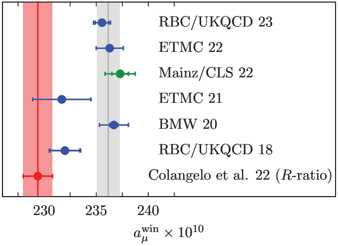
H. Wittig, Moriond 2023



$$a_\mu^{\rm hvp,win} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\; \tilde{K}(t)\, G(t)\, W(t; t_0, t_1)$$

Integration restricted to unproblematic regions

Uncertainty dominated by statistics



$$a_\mu^{\rm win}\Big|_{Latt} - a_\mu^{\rm win}\Big|_{e^+e^-} = (6.8 \pm 1.8) \times 10^{-10}$$

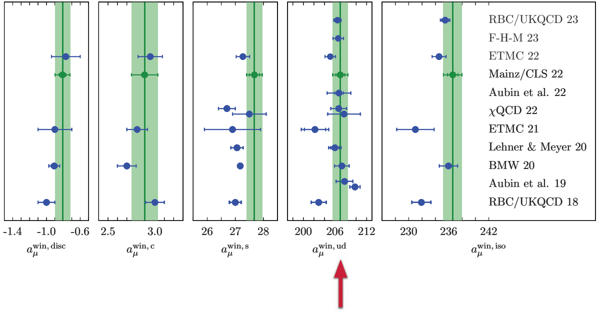
Intermediate window accounts for 50% of the discrepancy between 2020 BMW and WP results

Lattice: Intermediate-Distance Window

H. Wittig, Moriond 2023

Intermediate window observable in Lattice QCD

Results for individual quark flavours / quark-disconnected contribution in isospin limit

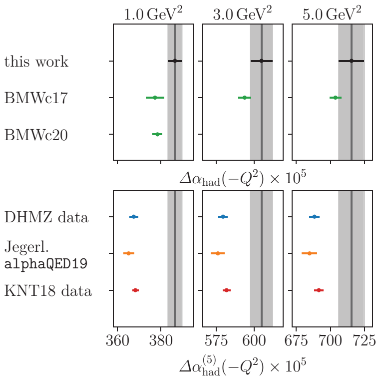


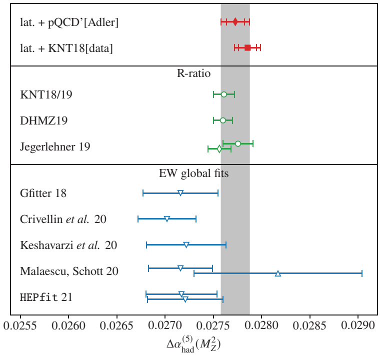
[Blum et al., arXiv:2301.08696]
[Bazavov et al., arXiv:2301.08274]
[Alexandrou et al., arXiv:2206.15084]
[Cè et al., Phys Rev D106 (2022) 114502]

Result for the dominant, light-quark connected contribution confirmed for wide range of different discretisations with sub-percent precision

Lattice Evaluation of $\Delta\alpha_{\text{had}}^{\text{QED}}$

Cè et al, 2203.08676





$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \Delta\alpha_{\text{had}}^{(5)}(-Q_0^2) \quad \leftarrow \text{ lattice QCD}$$

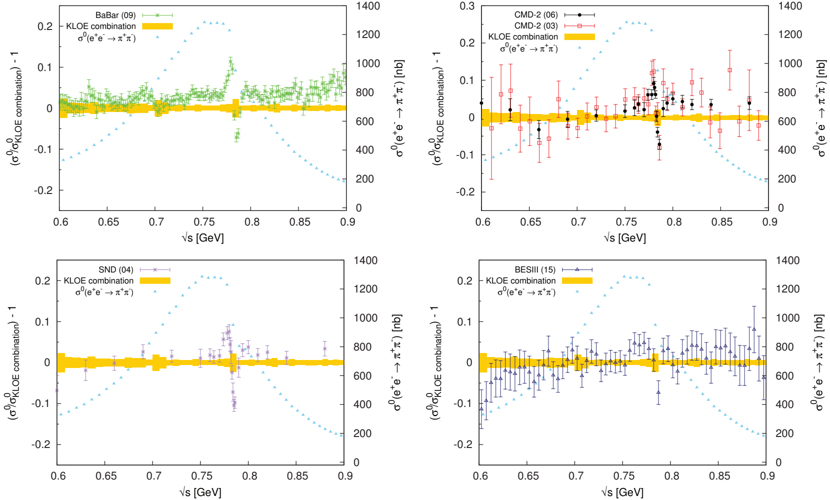
$$+[\Delta\alpha_{\text{had}}^{(5)}(-M_Z^2) - \Delta\alpha_{\text{had}}^{(5)}(-Q_0^2)] \quad \leftarrow \text{ perturbative Adler function}$$

$$+[\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) - \Delta\alpha_{\text{had}}^{(5)}(-M_Z^2)] \quad \leftarrow \text{ pQCD}$$

$$\Rightarrow \quad \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.027\,73\,(9)_{\text{latt}}\,(2)_b\,(12)_{\text{pQCD}}$$

KLOE data vs other experiments

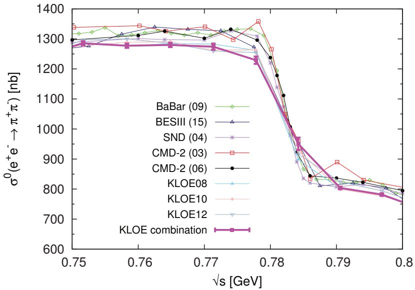
KLOE-2, 1711.03085



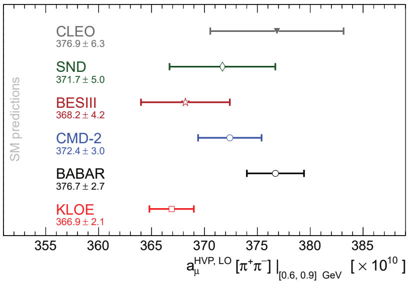
Internal tensions also among the three KLOE datasets: 2008, 2010, 2012

KLOE data vs other experiments

KLOE-2, 1711.03085



Aoyama et al, 2006.04822

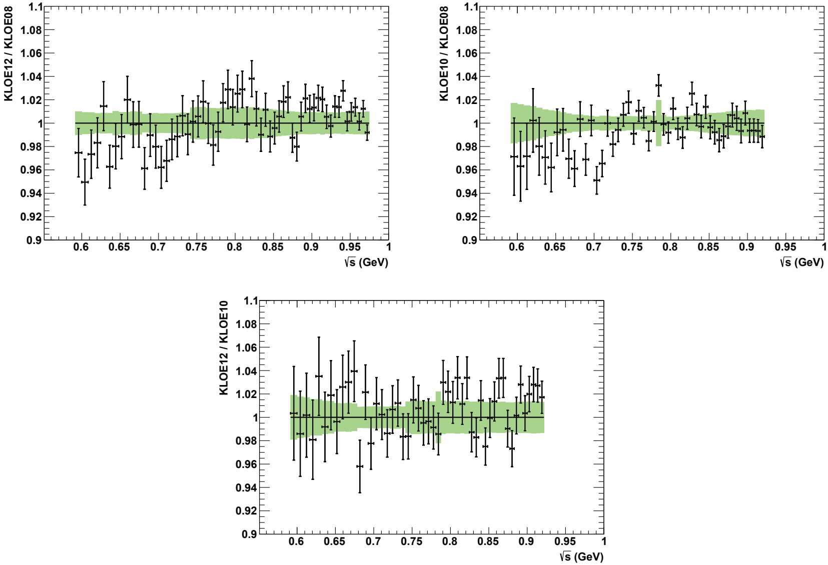


Discrepancies in the differential distribution are much larger than what gets reflected in the integral over the mass spectrum

Compensating effects ➡ **Underestimated systematics**

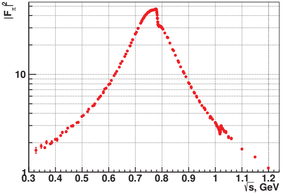
Comparison among KLOE datasets

KLOE-2, 1711.03085



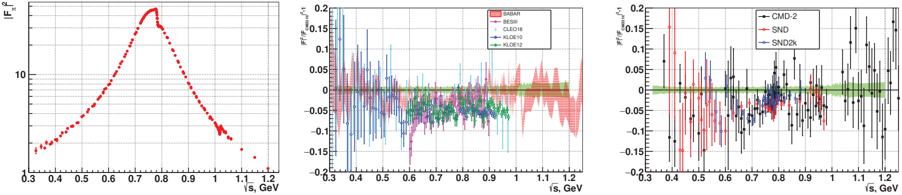
2023 CMD-3 $e^+e^- \to \pi^+\pi^-$ Data

2302.08834



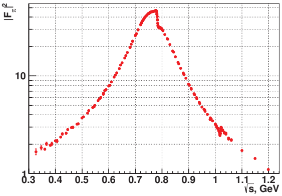
2023 CMD-3 $e^+e^- \to \pi^+\pi^-$ Data

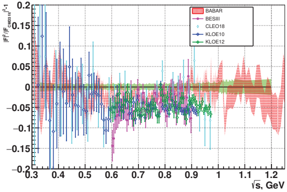
2302.08834

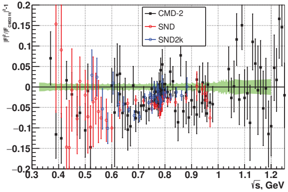


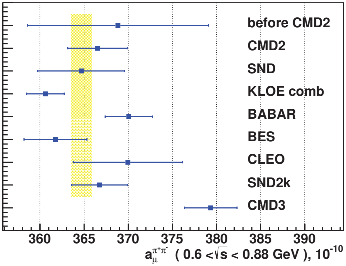
2023 CMD-3 $e^+e^- \to \pi^+\pi^-$ Data

2302.08834







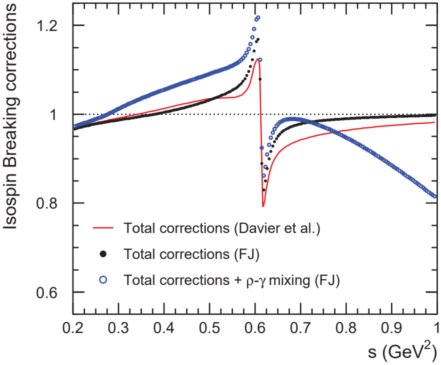


Experiment	$a_\mu^{\pi^+\pi^-,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

$$\Delta a_\mu \approx 14.6 \times 10^{-10}$$

Isospin-breaking corrections applied to τ data

Z. Zhang, 1511.05405



In order to achieve compatibility with e^+e^- data, FJ introduces huge IB corrections (blue line), which are not supported by the explicit calculations available

The growing with energy of the ρ-γ mixing correction does not make sense