Scale precision for HVP

- first moment of vacuum polarization $\Pi_1 = d\Pi/dQ^2$

$$\delta a/a = 1\% \to \delta\Pi_1/\Pi_1 = 2\%$$

(there might be corrections due to change in physical point)

- magnetic moment a_μ [Mainz'17]

$$\delta a/a = 1\% \to \delta a_\mu/a_\mu = 1.8\%$$

sub-percent needs scale determination with few per-mill

- window observable

$$\delta a/a = 1\% \to \delta a_{\mu,\mathrm{win}}/a_{\mu,\mathrm{win}} = 0.5\%$$

Pseudoscalar decay constant

leptonic decay rates of pions/kaons:

$$\Gamma(\pi \to \ell\bar{\nu}_\ell) = |V_{ud}|^2 \times f_\pi^2(1+\delta_\pi) \times \ldots$$

$$\Gamma(K \to \ell\bar{\nu}_\ell) = |V_{us}|^2 \times f_K^2(1+\delta_K) \times \ldots$$

- $f_\pi = 130.56(2)_{\mathrm{exp}}(13)_\delta(2)_{\mathrm{V_{ud}}}$ MeV [FLAG]

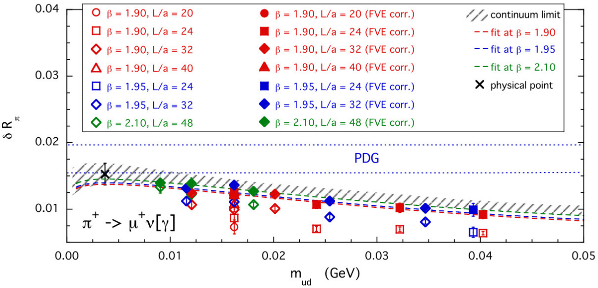
  $\to$ 0.10% accuracy

- electromagnetic corrections are complicated

- CKM matrix elements

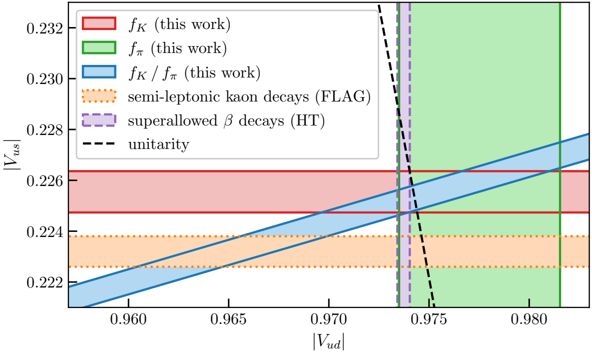
Radiative corrections

- theory worked out Rome/Southampton [1502.00257]



- keep in mind their isospin scheme
- $\delta_\pi = 0.0153(19)$ and $\delta_K = 0.0024(10)$ [1904.08731] latter is in disagreement with pheno $\delta_K = 0.0107(21)$
- recently RBC/UKQCD also joined [2211.12865]

Cabibbo-anomaly



- V_{ud} from superallowed beta decays 0.03% error, but values might change 0.97417(21) $\to$ 0.97373(31)
- V_{us} from semileptonic kaon decays plus lattice
- $2 - 3\sigma$ disrepancies

Omega baryon mass

- $M_\Omega = 1672.45(29)$ MeV or $M_\Xi = 1316.9(3)$ MeV

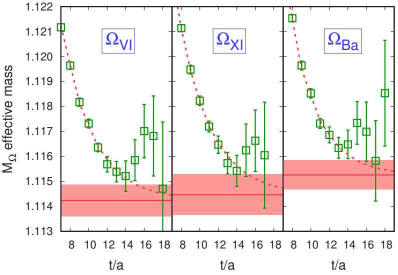
  $\rightarrow$ 0.02% accuracy

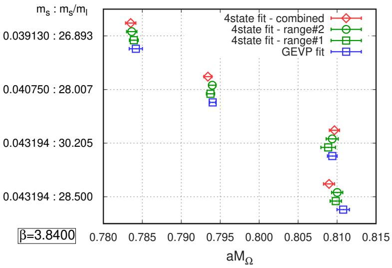
- electromagnetic corrections straightforward

- fast inversions (strange), finite volume effects small

- plateau fits difficult $\rightarrow$ use more operators and GEVP, large no of sources, multi state fits, . . .

Plateau fits - BMW collaboration

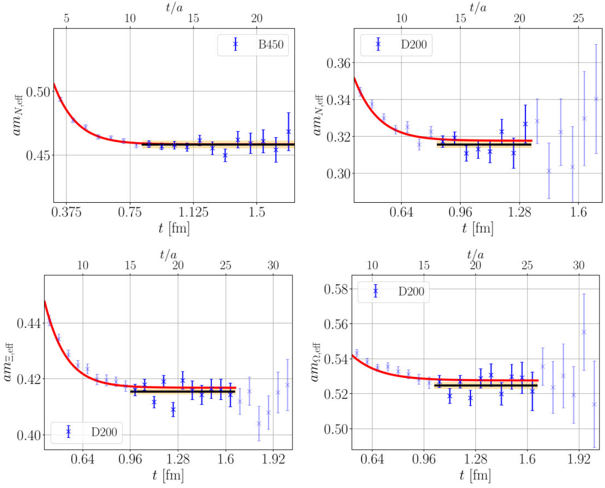




- multiple operators (differing in staggered taste)
- two-state fits with different fit ranges
- GEVP based approach [Aubin,Orginos 1010.0202]

$w_0 = 0.17236(70)$ fm (0.4% accuracy)

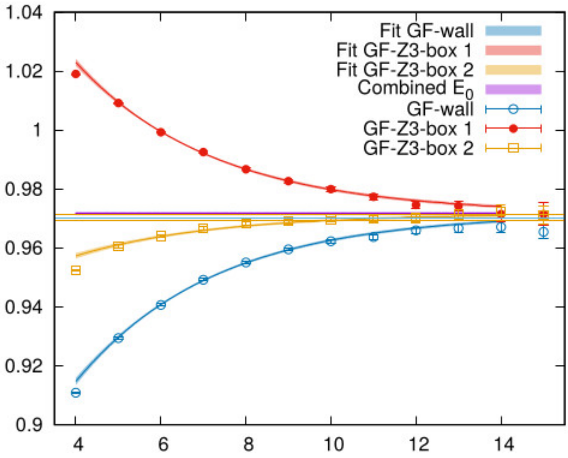
Plateau fits - RQCD collaboration



- single-state fit, fitrange from two-state fit

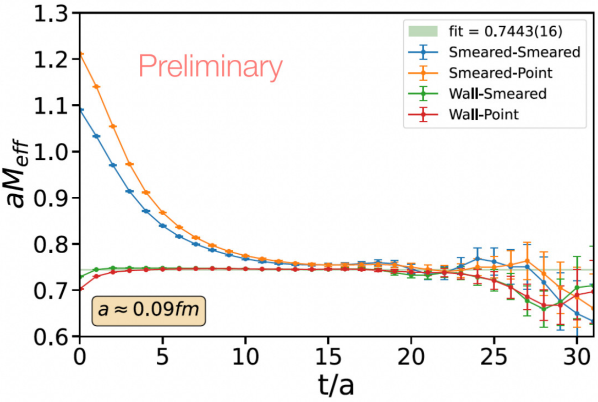
$\sqrt{t_0} = 0.1449(8)$ fm (0.6% accuracy)

Plateau fits - RBC/UKQCD collaboration



- multiple operators with different excited state contribution

Plateau fits - FNAL/MILC collaboration

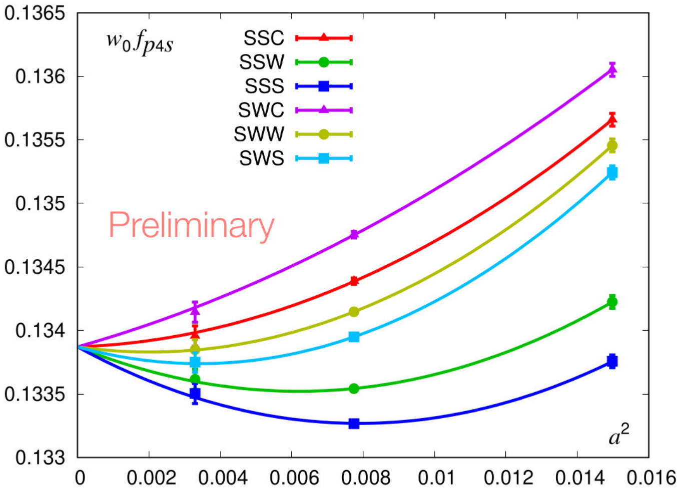


- two different operators
- 0.2% accuracy

Gradient flow observables

- purely gluonic, dont bother about em effects
- high accuracy, but also note high autocorrelation
- intermediate distance $\sqrt{t_0} \sim 0.4$ fm, can be sensitive to lattice artefacts (eg. high level of smearing)

Gradient flow cont. extrap



FNAL/MILC collaboration [Gottlieb ’23]

FLAG

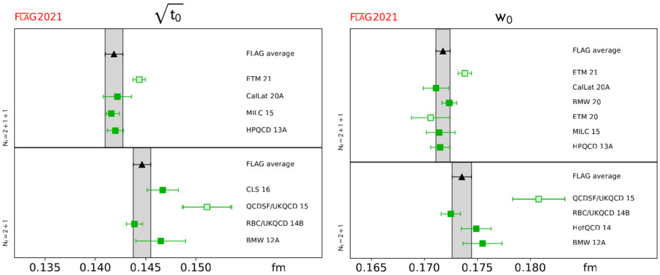


Figure 51: Results for gradient flow scales.

- $w_0 = 0.17177(67)$ fm [FLAG]

  $\rightarrow$ 0.35% accuracy

- note, recent ETM result isnt included and too high

- Having multiple ways of setting the scale (f_π, M_Ω, ...) is as important as having multiple fermion actions.