Muon g-2 – Discussion Session
High-precision lattice calculations of the HVP

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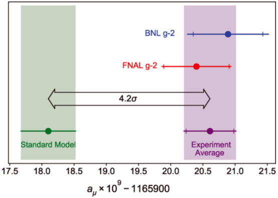
Experiment vs Standard Model prediction

Exp: $\boldsymbol{a_\mu = 0.00116592061(41)}$

SM: $\boldsymbol{a_\mu = 0.00116591810(43)}$

Muon g—2 Coll., Phys. Rev. Lett. 126, 141801



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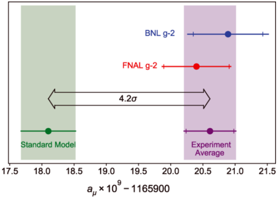
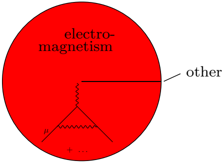
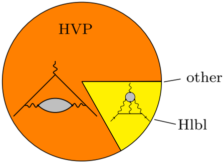


- FNAL reduce error by factor $\sim$ **4**, new upcoming experiment @JPARC
- Breakdown of Standard Model Prediction

contribution to a_μ



contribution to variance $\Delta^2 a_\mu$



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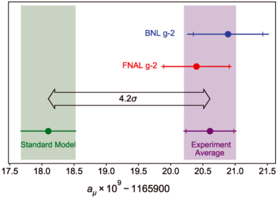

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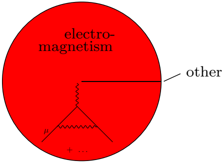

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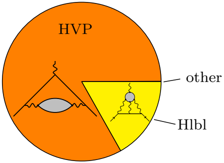
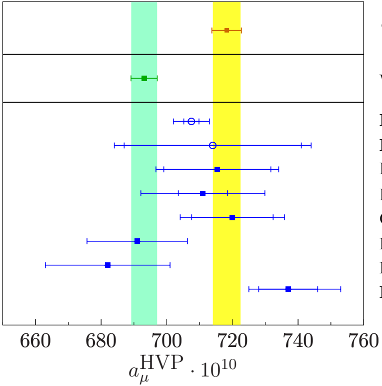


- $a_\mu^{\mathrm{HVP}} = 693.1(4.0) \times 10^{-10}$ from R-ratio
  → need a_μ^{HVP} at ∼ **0.2%** precision to match experimental target

Lattice calculation of HVP

- Comparision of available lattice QCD calculations of HVP



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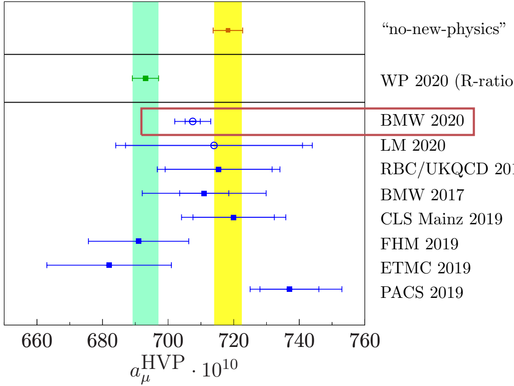
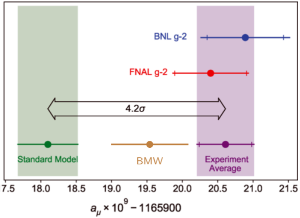


- 2020 lattice result by BMW

$$a_\mu^{\mathrm{HVP}}(\mathrm{BMW}) = \mathbf{707.5(5.5)} \times \mathbf{10^{-10}}$$



Precision targets for lattice calculation of HVP

- $\mathbf{[\qquad]} \sim \mathbf{0.6\%}$ to match current $\boldsymbol{R}$-ratio precision

- $\mathbf{[\qquad]} \sim \mathbf{0.2\%}$ to match experimental target precision

Precision targets for lattice calculation of HVP

- **[(✓)]** $\sim$ **0.6%** to match current ***R***-ratio precision
    - **0.8%** precision by BMW
    - still no second lattice result at similar precision
- **[   ]** $\sim$ **0.2%** to match experimental target precision

Precision targets for lattice calculation of HVP

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- **[ ]** $\sim$ **0.2%** to match experimental target precision
  - still a long way to go

Flavour decomposition of HVP

- a_μ from vector two-point function $C(t)$ [T. Blum (2003); Bernecker and Meyer (2011)]

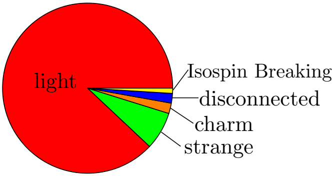
$$a_\mu^{\mathrm{HVP}} = \sum_t w_t C_{ii}(t) \quad \text{with} \quad C_{\mu\nu}(t) = \sum_{\vec{x}} \langle J_\mu(t,\vec{x}) J_\nu(0) \rangle$$

- flavour decomposition

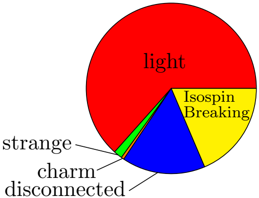
$$C(t) = \frac{5}{9} C^\ell(t) + \frac{1}{9} C^s(t) + \frac{4}{9} C^c(t) + C^{\mathrm{disc}}(t) + C^{\mathrm{IB}}(t)$$

- White Paper lattice average $a_\mu^{\mathrm{HVP}}(\mathrm{lat}) = 711.6(18.4) \times 10^{-10}$

contributions to $a_\mu^{\mathrm{HVP}}(\mathrm{lat})$

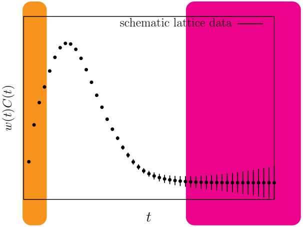


contributions to $\Delta a_\mu^{\mathrm{HVP}}(\mathrm{lat})$



Light-quark contribution

- main challenges:
  - statistical noise at large t
  - finite volume effects (largest at large t)
  - discetisation effects at small t



- statistical noise at large t:
  - noise reduction for "raw" correlator: AMA, LMA
  - Bounding method
  - Reconstruction of long distance tail ($\pi\pi$-spectrum) & improved bounding method
- Finite volume
  - large lattices
  - FV corrections: Meyer-Lellouch-Lüscher-Gounaris-Sakurai, Hansen-Patella, NNLO-χPT
- discretisation effects

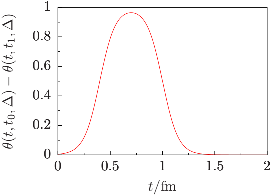
Window method

- a_μ^{HVP} from intermediate window $a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$

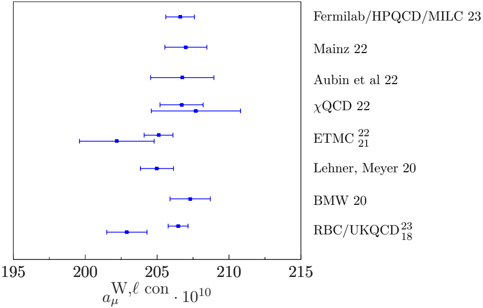
  [T. Blum, P. Boyle, VG *et al* Phys.Rev.Lett. 121 (2018) 022003]

$$a_\mu^{\mathrm{W}} = \sum_t w_t C(t)[\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$

e.g. $t_0 = 0.4$ fm to $t_1 = 1.0$ fm



- comparison light-quark connected window
- absolute errors of $\sim \mathbf{0.7 - 1.5 \times 10^{-10}}$ (propagates to $\sim \mathbf{0.1\% - 0.2\%}$ on total HVP)



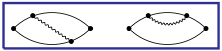
Isospin Breaking Corrections

- lattice calculations usually done in the isospin symmetric limit
- two sources of isospin breaking effects
  - different masses for up- and down quark (of $\mathcal{O}((m_d - m_u)/\Lambda_{\mathrm{QCD}})$)
  - Quarks have electrical charge (of $\mathcal{O}(\alpha)$)
- required for calculation at $\lesssim$ **1%** precision

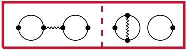
- separation of strong IB and QED effects requires renormalization scheme
- definition of isospin symmetric theory also scheme dependent [presentation by A. Portelli]

- full physical QCD+QED theory is scheme independent

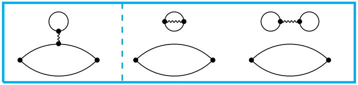
Isospin Breaking Corrections – Status



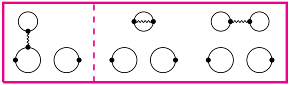
BMW	**−1.27(40)(33)**
RBC/UKQCD	**5.9(5.7)(1.7)**
ETM	**1.1(1.0)**



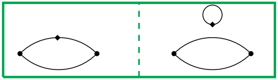
−0.55(15)(11)	BMW
−6.9(2.1)(2.0)	RBC/UKQCD



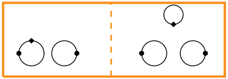
−0.0095(86)(99) **0.42(20)(19)** BMW



0.011(24)(14) **−0.047(33)(23)** BMW

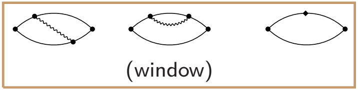


6.59(63)(53)		BMW
10.6(4.3)(6.8)		RBC/UKQCD
6.0(2.3)		ETM
7.7(3.7)	**9.0(2.3)**	FHM
9.0(0.8)(1.2)		LM



−4.63(54)(69) BMW

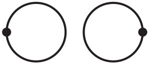
BMW, Nature (2021)
RBC/UKQCD [PRL 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]
Mainz [Phys. Rev. D 106, 114502 (2022)]



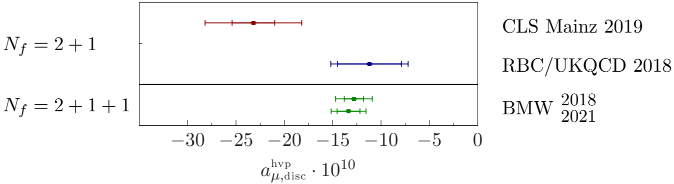
0.70(45) Mainz

Quark-disconnected contribution

- quark-disconnected contribution



- quark-loop $\boldsymbol{D^{-1}(x,x)}$ requires knowledge of full inverse of $\boldsymbol{D}$
  $\rightarrow$ calculate stochastically
- statistical noise reduction, e.g. [V.G. et al, PoS LAT2014 (2014) 128], [T. Blum *et al*, Phys. Rev. Lett. 116, 232002 (2016)], [A. Stathopoulos et al, arXiv:1302.4018], [A. Gérardin *et al*, Phys. Rev. D 100, 014510 (2019)], [L. Giusti et al, Eur.Phys.J.C 79 (2019) 7, 586]
- comparison of lattice results



- errors $\sim \mathbf{2-5\times 10^{-10}}$ $\rightarrow$ propagates to $\mathbf{0.3-0.7\%}$ for total HVP
- status of updates/more results?

Scale Setting

- a_μ^{hvp} depends on the scale through am_μ in the kernel

- scale set by quantity Λ with error $\Delta\Lambda$

$$\Delta a_\mu^{hvp} = \left|\Lambda \, \frac{\mathrm{d}a_\mu^{hvp}}{\mathrm{d}\Lambda}\right| \cdot \frac{\Delta\Lambda}{\Lambda} = \left|M_\mu \frac{\mathrm{d}a_\mu^{hvp}}{\mathrm{d}M_\mu}\right| \cdot \frac{\Delta\Lambda}{\Lambda} \qquad M_\mu = \frac{m_\mu}{\Lambda}$$

→ relative error on Λ amplified by $\approx \mathbf{1.8}$ in relative error for a_μ
[M. Della Morte, VG, *et al*, JHEP 1710 (2017) 020]
→ for **0.2%** error on a_μ^{hvp} need $\lesssim$ **0.1%** on lattice spacing

- precise scale setting needed [presentation by K. Szabo]

Discussion

- Any outstanding questions for this morning's talks
- scale setting [presentation by K. Szabo]
- IB schemes [presentation by A. Portelli]
- Masterfield simulations
- Status/prospects for determinations of long-distance (to $<$ **1%** HVP)
- AOB