FINITE TEMPERATURE TOPOLOGICAL SUSCEPTIBILITY

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Calculation of the axion mass based on high-temperature lattice QCD

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Main goal

Compute QCD topological susceptibility

- temperature range $0 < T < 2$ GeV
- physical quark masses m_u, m_d, m_s, m_c
- continuum limit

Using

- $n_f = 2 + 1 + 1$ with isospin breaking corrections
- staggered and overlap quarks
- lattices with $N_t = 8, 10, 12, 16$ and 20

Physics to be captured

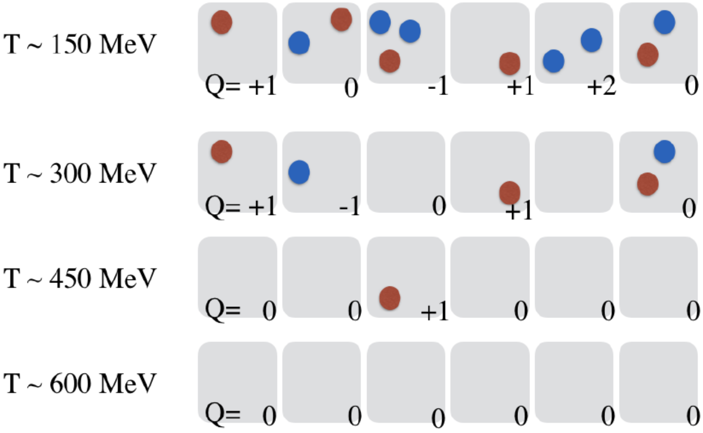
- typical instanton size $1/T$

- instantons suppressed by temperature $\exp[-S_Q/g(T)^2]$ and dilute gas of instantons remain

- zero modes in the quark det suppress them further

$\rightarrow \chi(T)$ falls sharply above T_c

$\chi(T)$ from standard approach

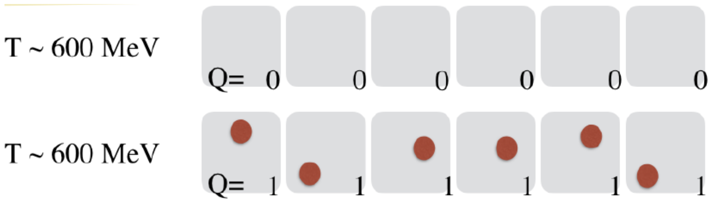


- direct measurement is not feasible, simulate for centuries to get the first $Q > 0$ configuration!

$\chi(T)$ from fixed Q integral

measure temperature derivate and integrate (also [Frison...'16])

$$-\frac{d\log Z_Q/Z_0}{d\log T} = b = 4 + \frac{d\beta}{dT}\langle S_g\rangle_{Q-0} + \sum_f \frac{dm_f}{dT} m_f \langle\overline{\psi}\psi\rangle_{Q-0}$$



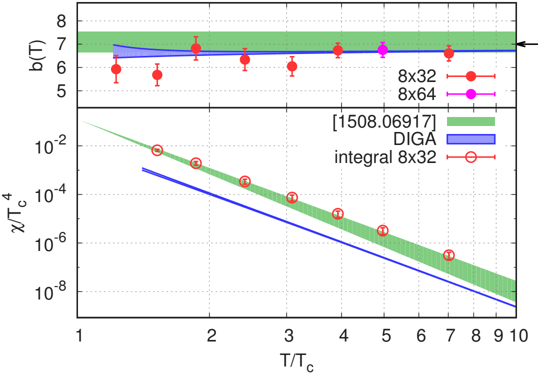
- b is related to the slope $Td\chi/dT$
- better signal using higher Q's
- better to integrate in mass than in β.

Integral method in quenched

- integration gives the ratio:

$$Z_1/Z_0|_T = \exp\left(\int_{T_0}^{T} d\log T'\ b_1(T')\right) Z_1/Z_0|_{T_0}$$

- simulate fixed Q (gradient flow) with acc/rej step

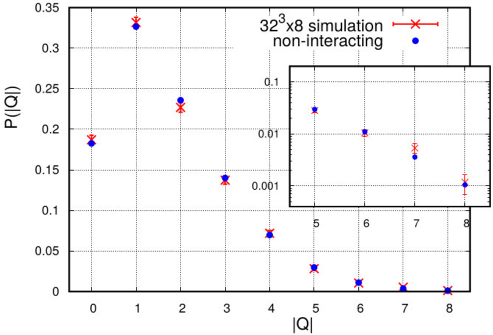


- χ is higher than χ_{DIGA} (slope is same) even at $T \gg T_c$

Dilute instanton gas approx

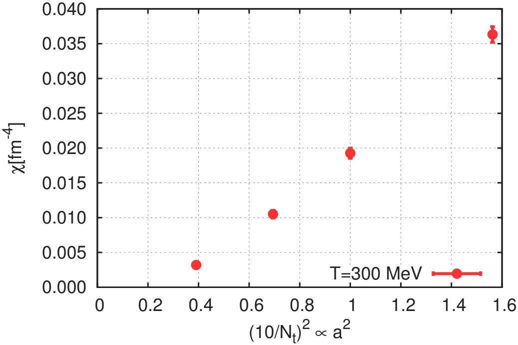
[Gross,Pisarski,Yaffe ’81]

1. instanton interactions can be neglected (ideal gas)
2. compute one instanton probability perturbatively (numerical error corrected [Boccaletti,Nogradi 2001.03383])



- ideal gas already at $T = 1.05\,T_c$ [Kovacs,Vig 2101.01498]
- Z_1/Z_0 is enough to get the susceptibility

Difficult cont. extrapolation



staggered fermions have no exact fermion zero modes

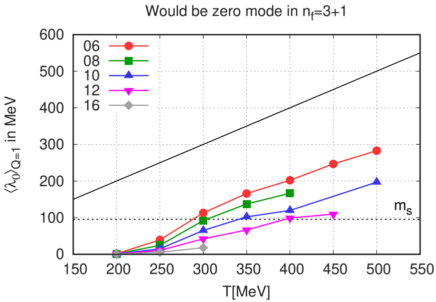
$\det(D+m) \sim (m+\lambda_0)^{|Q|}$ with $\lambda_0 \neq 0$ on the lattice

$\rightarrow$ results in too large $\chi(T)$ and too small slope

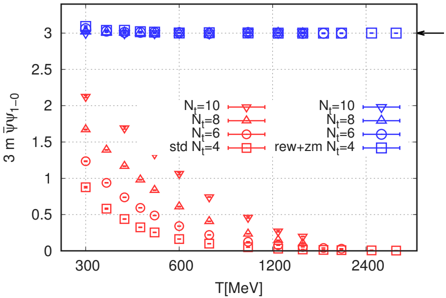
Continuum instanton and zero mode

Lattice instanton and zero mode

Lattice artefacts



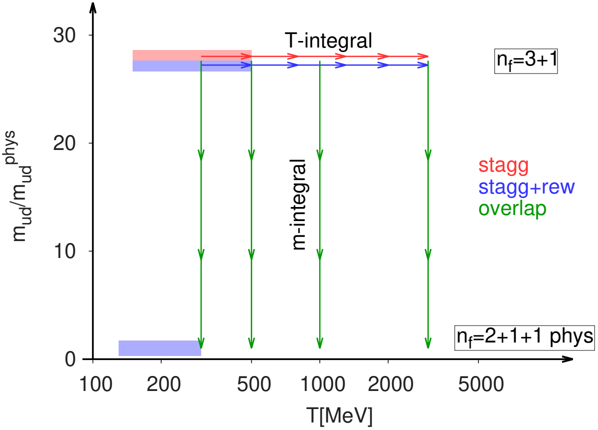
would be zero mode
increases with T



chiral condensate difference
$\to$ slope of $\chi(T)$

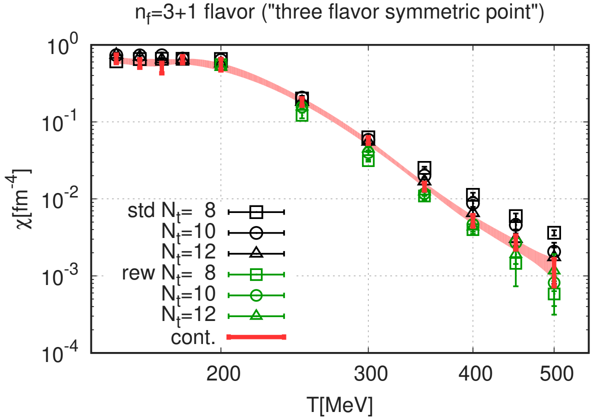
Cont. extrapolation strategy

1. use staggered fermions at $m_{ud} = m_s^{phys}$, continuum extrapolation behaves much better
2. use overlap fermions to calculate difference to m_{ud}^{phys} by integrating in m



$n_f = 3 + 1$ flavor result

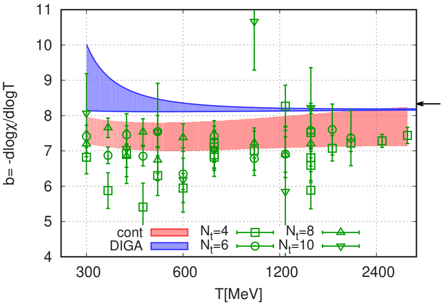
- lesser challenge to work at the strange mass

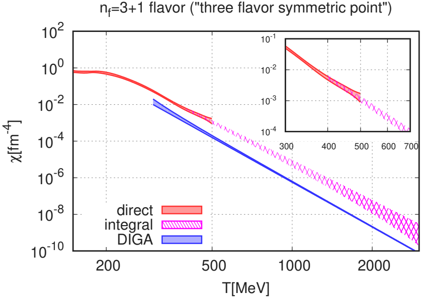


- standard and reweighted approach the continuum from different directions

$n_f = 3 + 1$ flavor result

- combine with the integral technique
- slope is somewhat below DIGA





- value is order of magnitude higher than DIGA

Dynamical overlap lattice setup

- tree-level Symanzik gauge action
- overlap fermions with 2 levels of HEX smearing
- extra Wilson fermion to avoid topology change and extra bosonic field to cancel Wilson-fermion β-shift [Aoki et al 2006]

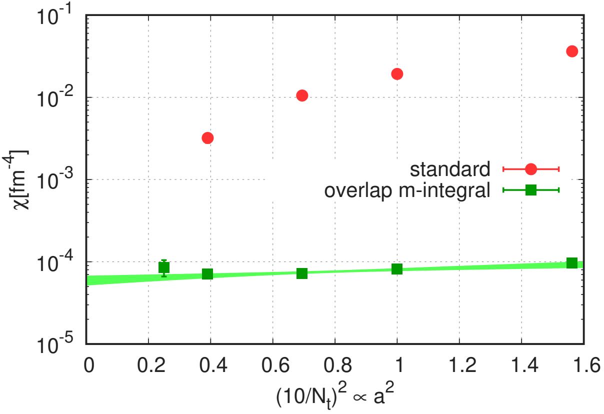
- simulate odd flavor $n_f = 3$ with help of chiral symmetry
- find LCP in the three-flavor symmetric point
- three lattice spacings $N_t = 6, 8, 10$
- multiple volumes $N_s/N_t = 2, 3, 4$

$n_f = 2+1+1$ result

- for $T > 300$ MeV and downto m_{ud}^{phys} we find

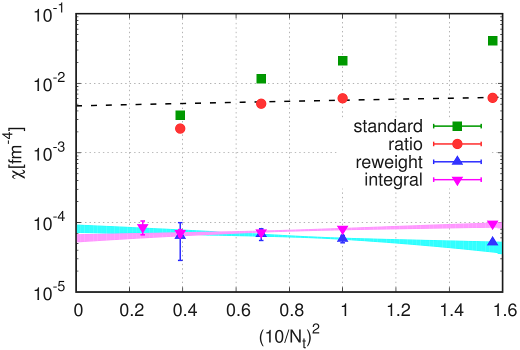
$$m\langle\overline{\psi}\psi\rangle_{1-0} = 1.00(2)$$

- simple rescaling with $(m_{ud}/m_s)^2$ follows



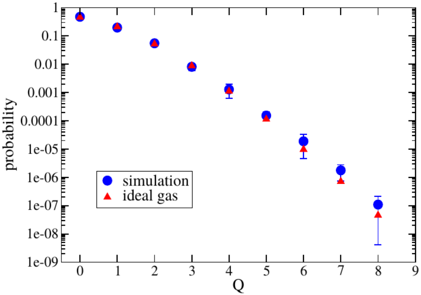
$T = 300$ MeV

Cont. extrapolation with reweighting



1. compute topological charge Q with Wilson-flow
2. identify $|Q|$ would-be zero eigenvalues $\lambda_1, \lambda_2, ..$
3. apply reweighting factor $\prod_i m_q/(\lambda_i + m_q)$

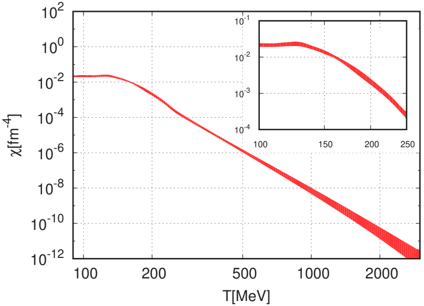
$n_f = 2 + 1 + 1$ flavor result



- ideal gas picture valid already at $T = 180$ MeV

- final result

- include estimates for $m_u \neq m_d$ and m_b

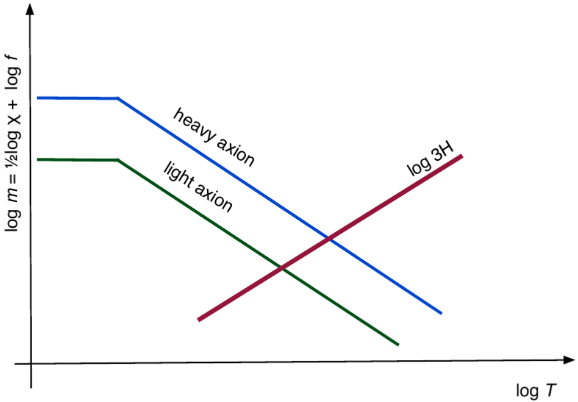


Lighter mass more axions

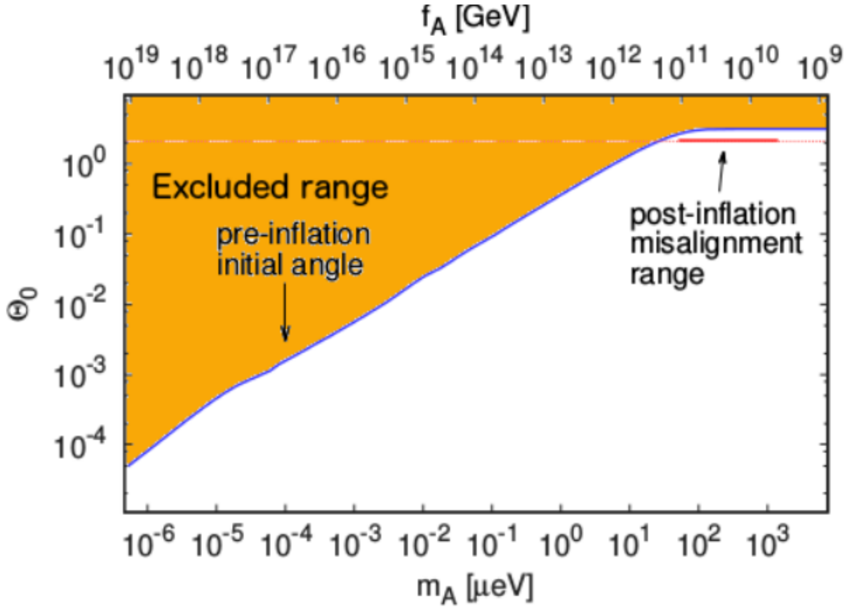
Have to solve

$$\frac{d^2\theta}{dt^2} + 3H(T)\frac{d\theta}{dt} + \frac{\chi(T)}{f_a^2}\sin\theta = 0$$

Rolling starts when $3H(T) \approx \sqrt{\chi(T)}/f_a$

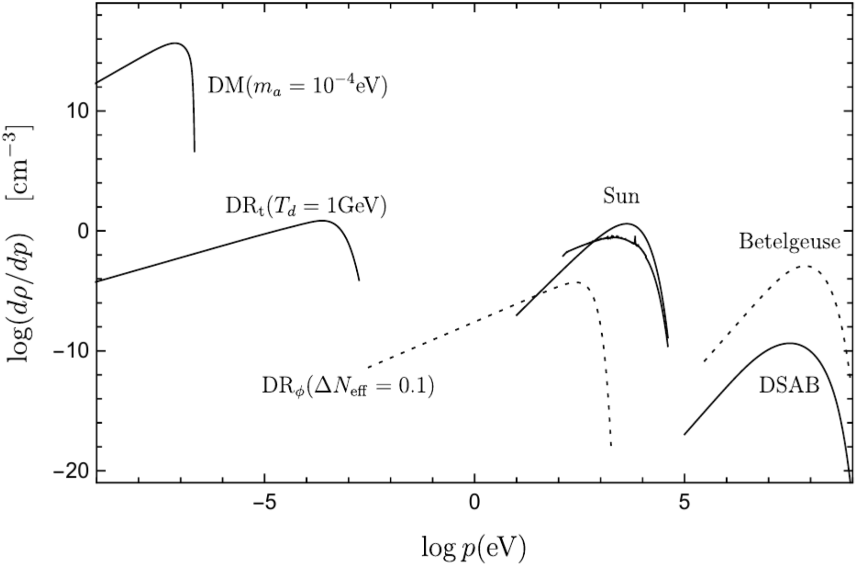


Axion mass and initial angle



Sources of axions

[Irastorza,Redondo ’18]

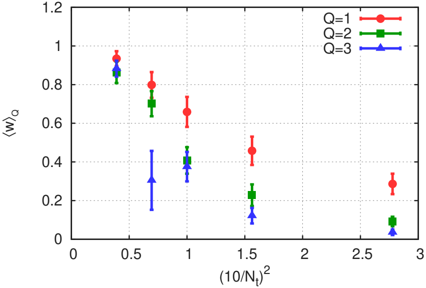


Reweighting

Problem: In continuum weight is m, on the lattice $m + \lambda_0[U]$.

Solution: change weight of configuration by $w[U] \equiv \frac{m}{m+\lambda_0[U]}$

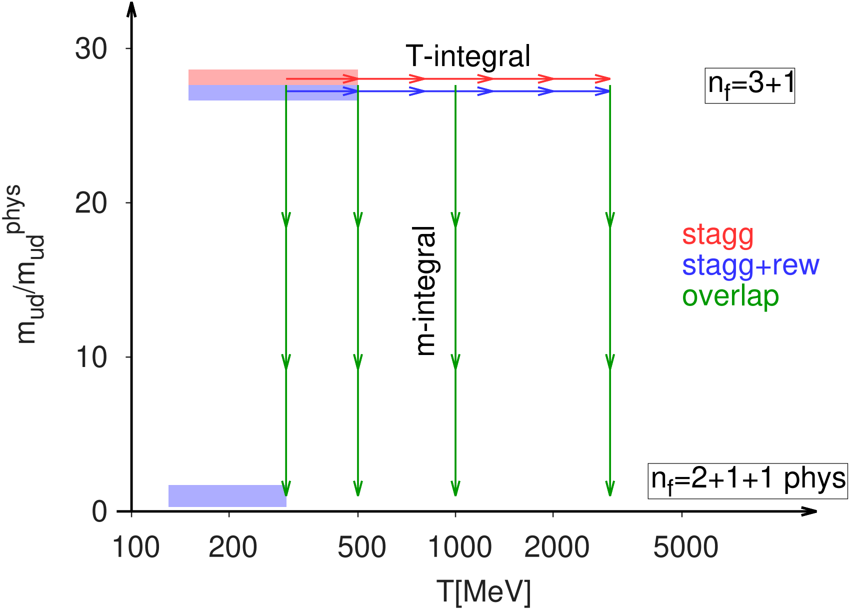
$\langle w \rangle_Q$ must approach 1 in the continuum limit.



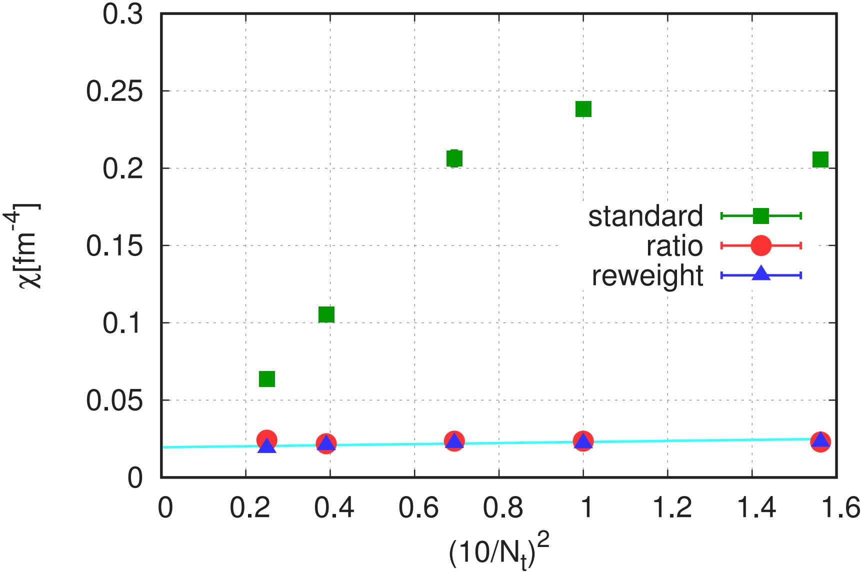
Improves the observable <u>without</u> changing the action.

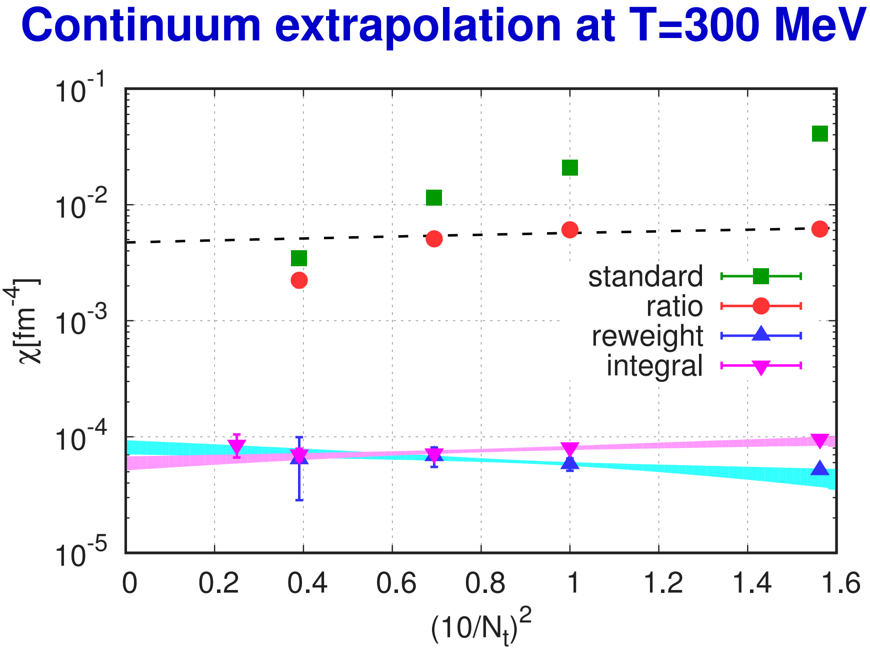
$$\chi = \frac{\sum Q^2 \; Z_Q}{\sum Z_Q} \rightarrow \chi_{\text{rew}} = \frac{\sum Q^2 \; \langle w \rangle_Q \; Z_Q}{\sum \langle w \rangle_Q \; Z_Q}$$

Map of simulations



Continuum extrapolation at T=150 MeV

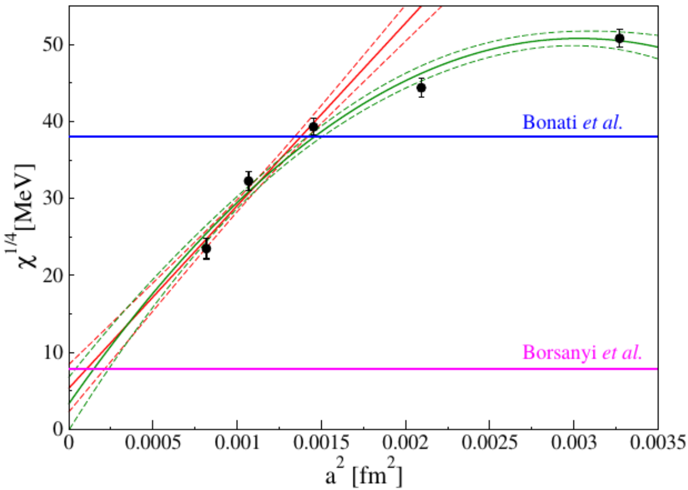




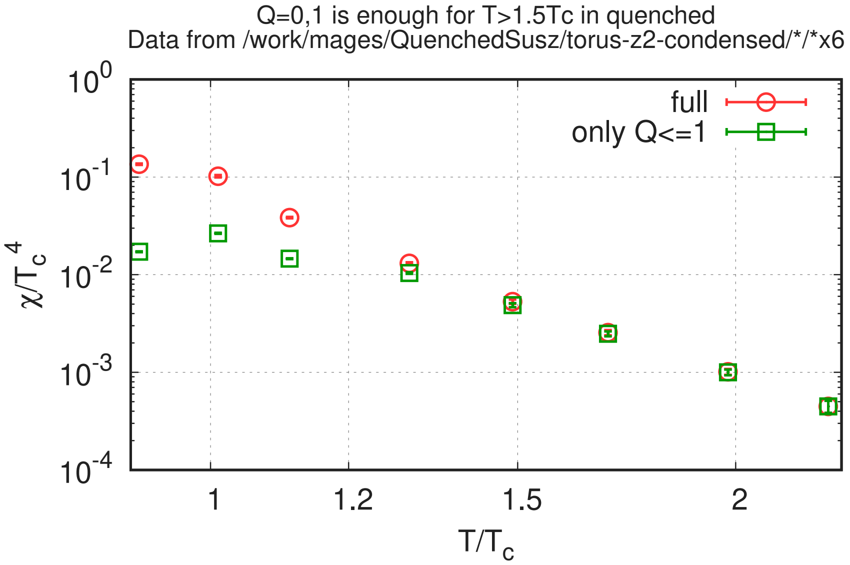
Continuum extrapolation at T=300 MeV
10^-1
10^-2
10^-3
10^-4
10^-5
χ[fm^-4]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
(10/N_t)^2
standard
ratio
reweight
integral

Continuum extrapolation at T=430 MeV

[Bonati et al ’18]

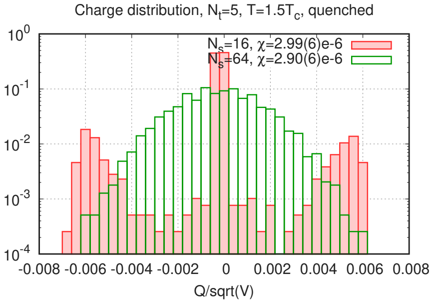


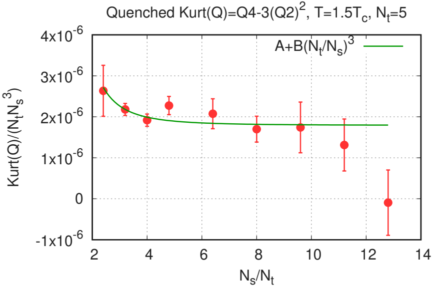
Contribution from $Q = 0, \pm 1$



Volume dependence illustration

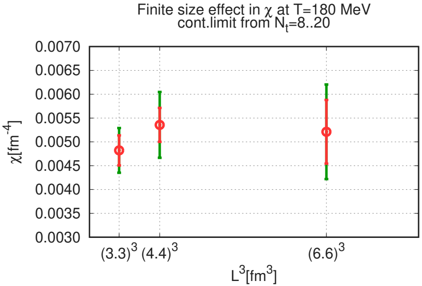
Q distribution depends (extensive quantity)



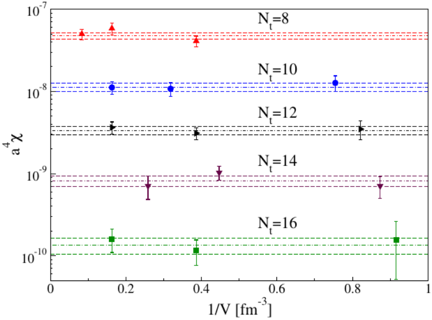


susceptibility, kurtosis (intensive quantitites) not

Volume (in)dependence at the phys point



[WB'16]



[Bonati et al '18]