Axions and the lattice

Introduction and statement of the problem

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Adapted from G. Aarts et al, Phase Transitions in Particle Physics - Results and Perspectives from Lattice Quantum Chromo-Dynamics, 2301.04382 [hep-lat]

Lattice Gauge Theories Contributions to New Physics Searches, IFT, Madrid, 12-16 June 2023

Axion mass and topological susceptibility

QCD Action

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_0 + \mathcal{L}_\theta + \mathcal{L}_a$$

Emerging CP-violating term

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\text{Tr}\left(G^{\mu\nu}G^{\rho\sigma}\right)$$

Axion action

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{a}{f_a}q(x)$$

CP-violating term cancelled from the condensation of the axion

Dynamically generated mass

$$m_a^2 f_a^2 = \lim_{V\to\infty}\frac{\langle Q^2\rangle}{V} = \chi$$

Determination of χ

In the chiral limit, χ can be calculated using ChPT: $\chi \propto \prod_{i=1}^{N_f} m_i$

At (very) high temperature, one can use the Diluted Instanton Gas Approximation (DIGA):

$$\chi(T) \propto T^{-d}$$

Lattice calculations enable first principle determinations of χ in the region of temperatures above T_c

Cosmological evolution of the axion field

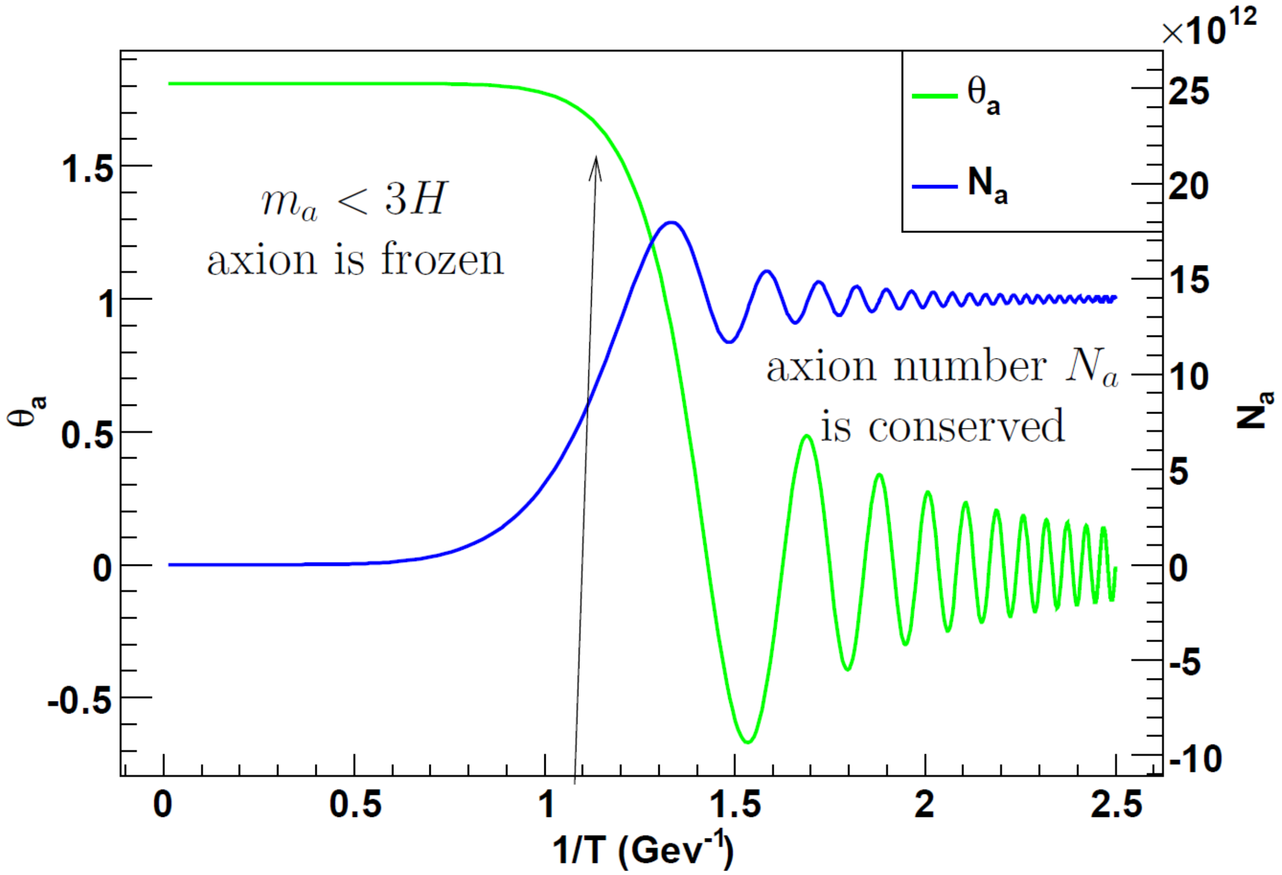

Figure adapted from "Axion Cosmology Revisited", O. Wantz and E.P.S. Shellard, Phys. Rev. D 82 (2010) 123508 [arXiv:0910.1066]

A non-perturbative determination of χ at finite temperature is needed

Topology on the lattice

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Compute χ from Q using the definition $\chi = \lim_{V \to \infty} \frac{\langle Q^2 \rangle}{V}$

Naïve discretization of q replaces F -> Im(U) in the continuum formula

Better control on lattice artefacts using the clover leaf definition

$$\mathcal{C}_{\mu\nu}(x) \equiv \frac{1}{8}\Big\{ U_\mu(x)U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x) + U_\nu(x)U^\dagger_\mu(x+\hat{\nu}-\hat{\mu})U^\dagger_\nu(x-\hat{\mu})U_\mu(x-\hat{\mu}) + $$
$$+ U^\dagger_\mu(x-\hat{\mu})U^\dagger_\nu(x-\hat{\nu}-\hat{\mu})U_\mu(x-\hat{\nu}-\hat{\mu})U_\nu(x-\hat{\nu}) + U^\dagger_\nu(x-\hat{\nu})U_\mu(x-\hat{\nu})U_\mu(x-\hat{\nu}+\hat{\mu})U^\dagger_\mu(x) - \text{h.c.} \Big\}$$

Topology on the lattice - Continued

Lattice topological charge density defined as

$$q_L(x) \equiv \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\, \mathcal{C}_{\mu\nu}(x) \mathcal{C}_{\rho\sigma}(x)$$

Technical points:

- Ultraviolet fluctuations need to be tamed (e.g., using the Gradient Flow)
- A sensible prescription to project Q onto integers need to be used

Computational challenges

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- X proportional to quark masses → large statistics needed for accurate determination in the chiral limit (and sensitive to lattice artefacts due to fermion discretization)

- Q highly suppressed at high temperature → the computational cost gets significantly worse as they increase

- Finite size effects at high T (constant number of sites as β increases)

- Topological freezing towards the continuum limit