The QCD-axion Sum Rule

Lattice Gauge Theory Contributions to New Physics Searches
June 12-16 2023

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H2020

Why axions or ALPs ?

The spin 0 window

The SM Higgs is a ~ doublet of $SU(2)_L$

Is the Higgs the only (fundamental?) scalar in nature?

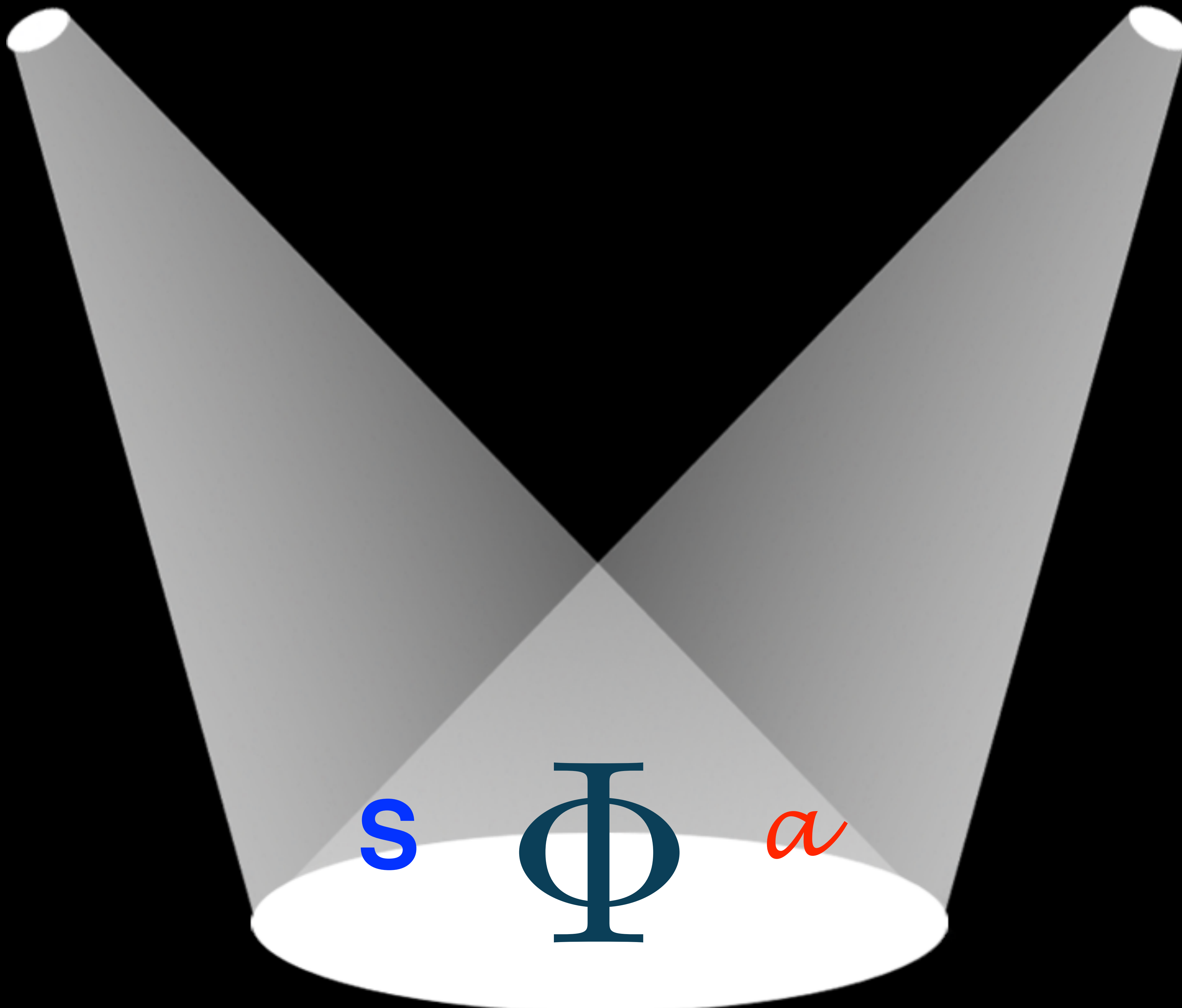
Or simply the first one discovered?

The spin 0 window

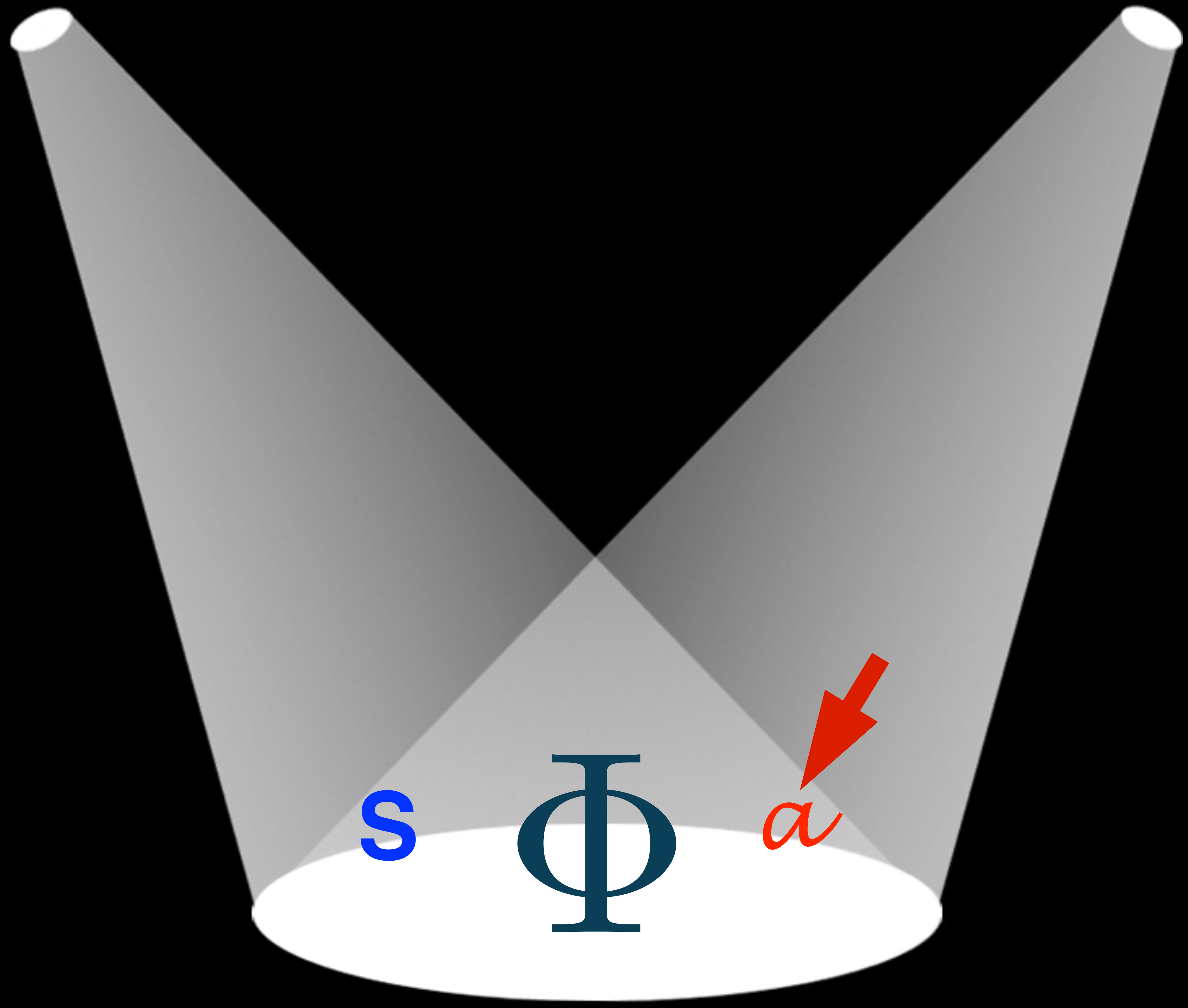
The SM Higgs is a ~ doublet of $SU(2)_L$

What about a singlet (pseudo) scalar?

Strong motivation from fundamental problems of the SM



Rocio del Rey



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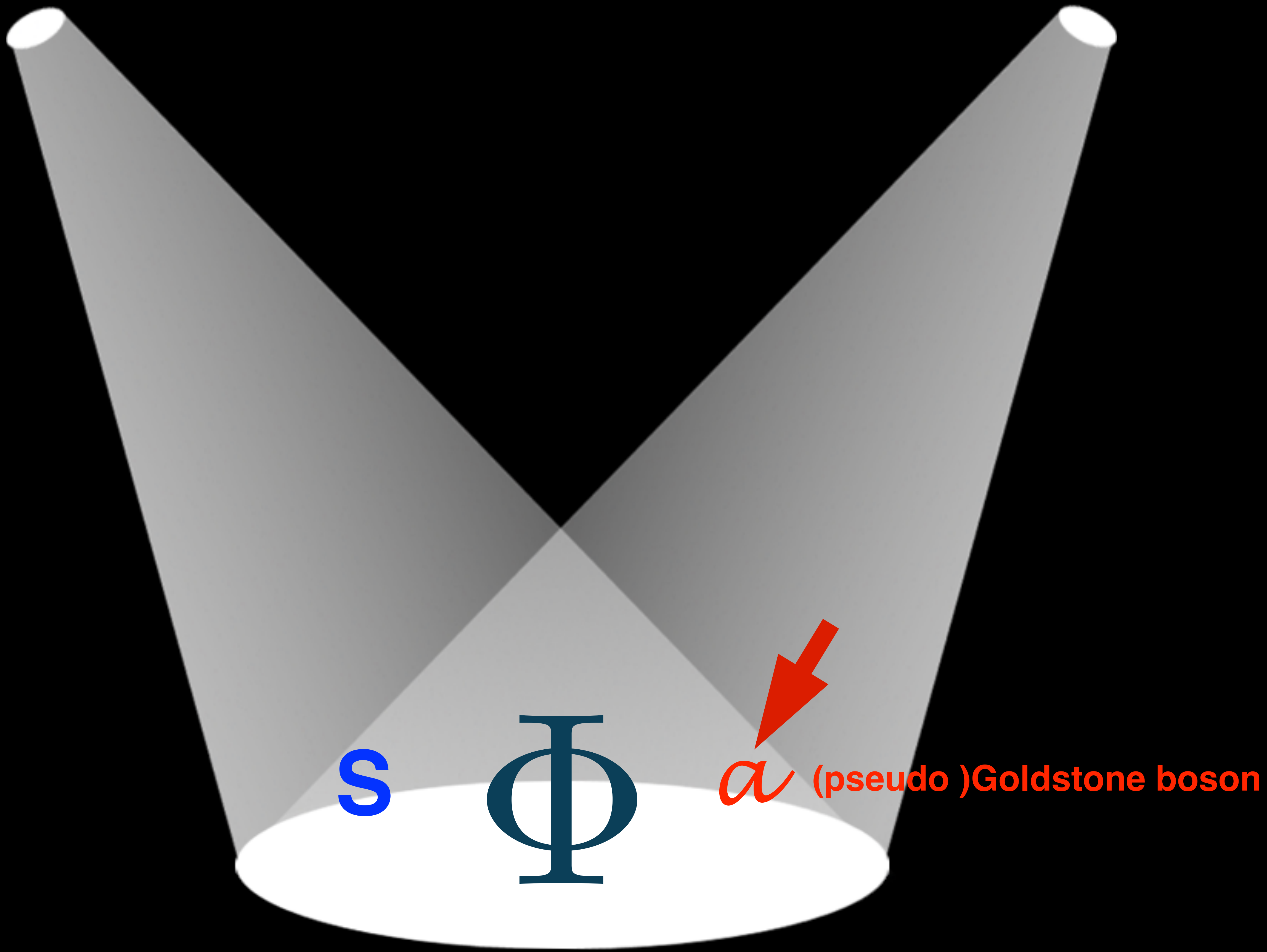


Rocio del Rey

Search for (pseudo)Goldstone bosons

=

search for hidden symmetries of Nature

They are spin 0 particles, with interactions proportional to its momentum and a tiny mass

Axions and ALPs a

are the tell-tale of hidden symmetries awaiting discovery

Many small unexplained SM parameters

Hidden symmetries
can explain small parameters

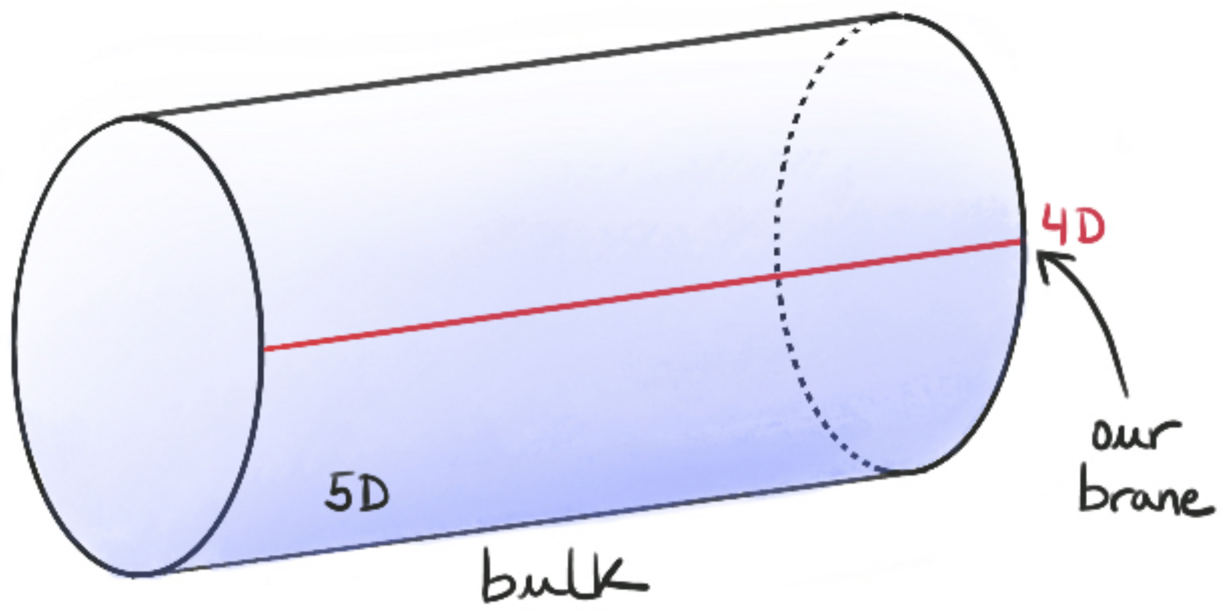
↓

If spontaneously broken:
Goldstone bosons a

—> derivative couplings to SM particles

(Pseudo)Goldstone Bosons appear in many BSM theories

* e.g. Extra-dim Kaluza-Klein: 5d gauge field compactified to 4d
The Wilson line around the circle is a GB, which behaves as an axion in 4d



* Majorons, for dynamical neutrino masses

* From string models

* The Higgs itself may be a pGB ! ("composite Higgs" models)

* Axions a that solve the strong CP problem, and ALPs (axion-like particles)

……

Strong motivation for singlet (pseudo)scalars from fundamental SM problems

The strong CP problem: Why is the QCD **θ** parameter so small?

$$\mathcal{L}_{QCD} = G_{\mu\nu}\, G^{\mu\nu}$$

Strong motivation for singlet (pseudo)scalars from fundamental SM problems

The strong CP problem: Why is the QCD θ parameter so small?

$$\mathcal{L}_{QCD} = G_{\mu\nu} G^{\mu\nu} + \theta \, G_{\mu\nu} \tilde{G}^{\mu\nu}$$

where $\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$

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$$\vec{E}^2 - \vec{B}^2 \qquad\qquad \theta\, \vec{E} \cdot \vec{B}$$

(CP even) (CP odd)

experimentally (neutron EDM): $\bar{\theta} \leq 10^{-10}$ **?**

Strong motivation for singlet (pseudo)scalars from fundamental SM problems

The strong CP problem: Why is the QCD **θ** parameter so small?

$\bar{\theta} \leq 10^{-10}$

$\mathcal{L}_{QCD} \supset \theta \, G_{\mu\nu} \tilde{G}^{\mu\nu}$

$\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$

↓

A dynamical $U(1)_A$ solution ?

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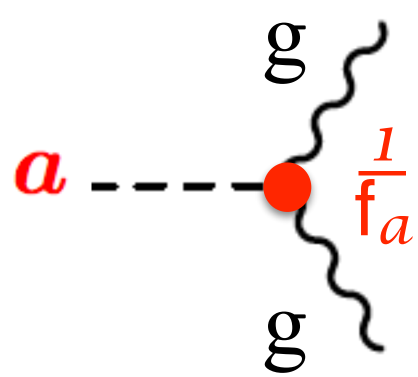
A dynamical $U(1)_A$ solution ?

It substitutes θ by a spin 0 particle a, i.e. a field $a(x)$, which has a small potential with minimum at zero

Strong motivation for singlet (pseudo)scalars from fundamental SM problems

The strong CP problem: Why is the QCD θ parameter so small?



$$\mathcal{L}_{QCD} \supset \left(\frac{a}{f_a} - \theta\right) G_{\mu\nu}\tilde{G}^{\mu\nu}$$

scale f_a

A dynamical $U(1)_A$ solution

[Peccei+Quinn 77]
[Weinberg, 78]
[Wilczek, 78]

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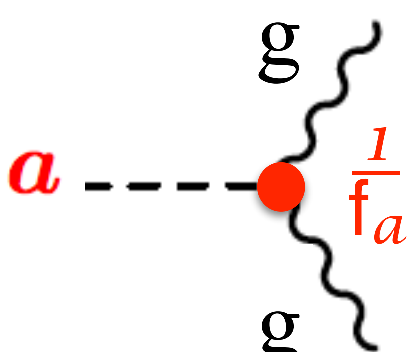

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with minimum at θf_a: $a = \theta f_a + a'$

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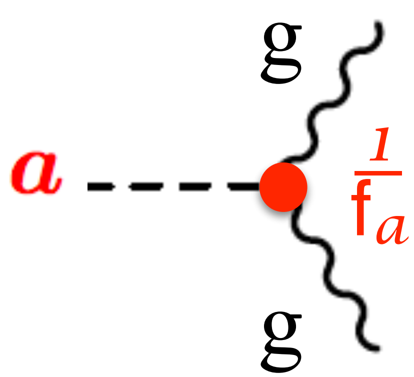

$$\mathcal{L}_{QCD} \supset \frac{a'}{f_a} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

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$$a = \theta f_a + a'$$

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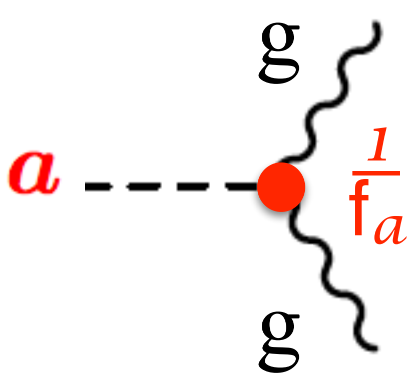

$$\mathcal{L}_{QCD} \supset \frac{a}{f_a} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

A dynamical U(1)$_A$ solution

→ the axion a

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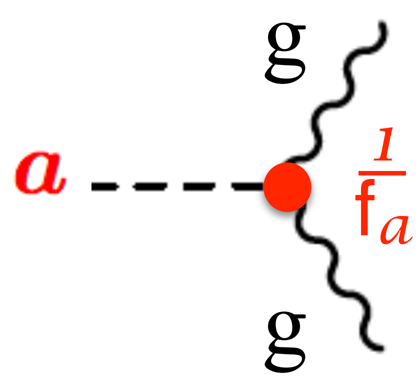

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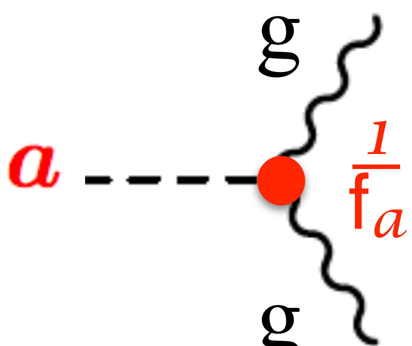
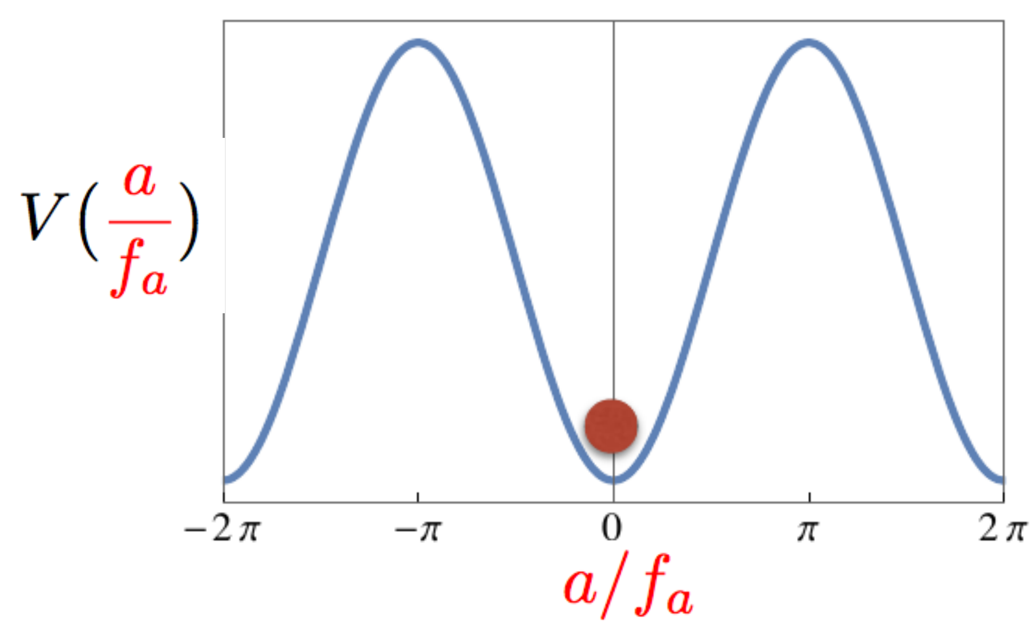


$\mathcal{L}_{QCD} \supset \frac{a}{f_a} G_{\mu\nu}\tilde{G}^{\mu\nu}$

A dynamical $U(1)_A$ solution

→ the axion a

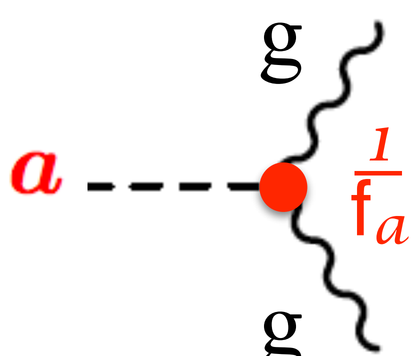
It is a **pGB**:



$\mathbf{m}_a \neq 0$

Strong motivation for singlet (pseudo)scalars from fundamental SM problems

The strong CP problem: Why is the QCD **θ** parameter so small?



$$\mathcal{L}_{QCD} \supset \frac{a}{f_a} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

A dynamical $U(1)_A$ solution

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[Peccei+Quinn 77]
[Weinberg, 78]
[Wilczek, 78]

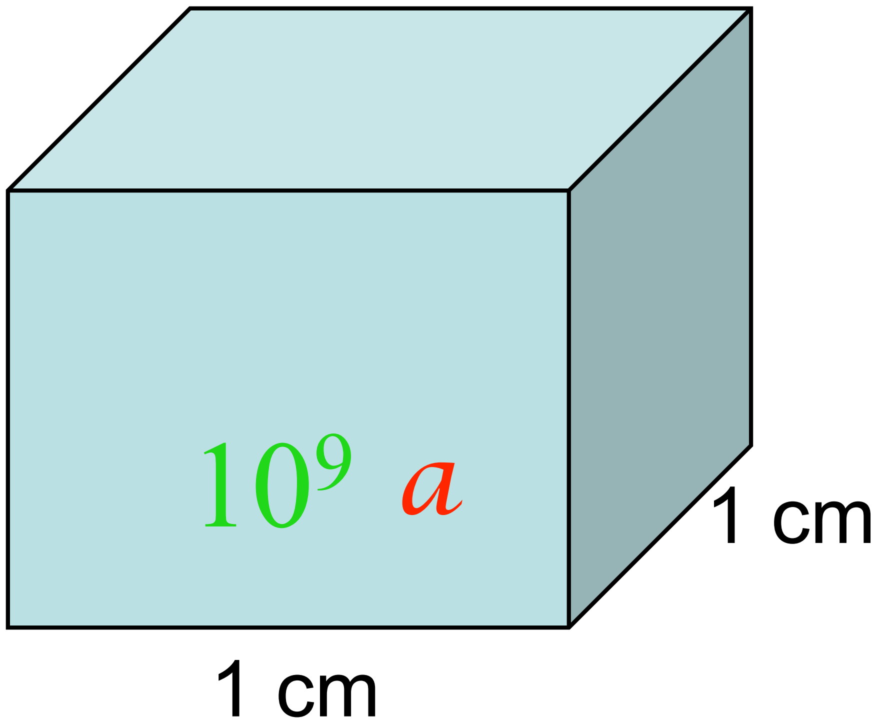
It is a pGB: ~mainly derivative couplings $\partial_\mu a$

Also excellent DM candidate $m_a \neq 0$

[Abbot+Sikivie, 83]
[Dine and W. Fischler, 83]
[Preskil et al, 91]

If axions are the dark matter of the universe

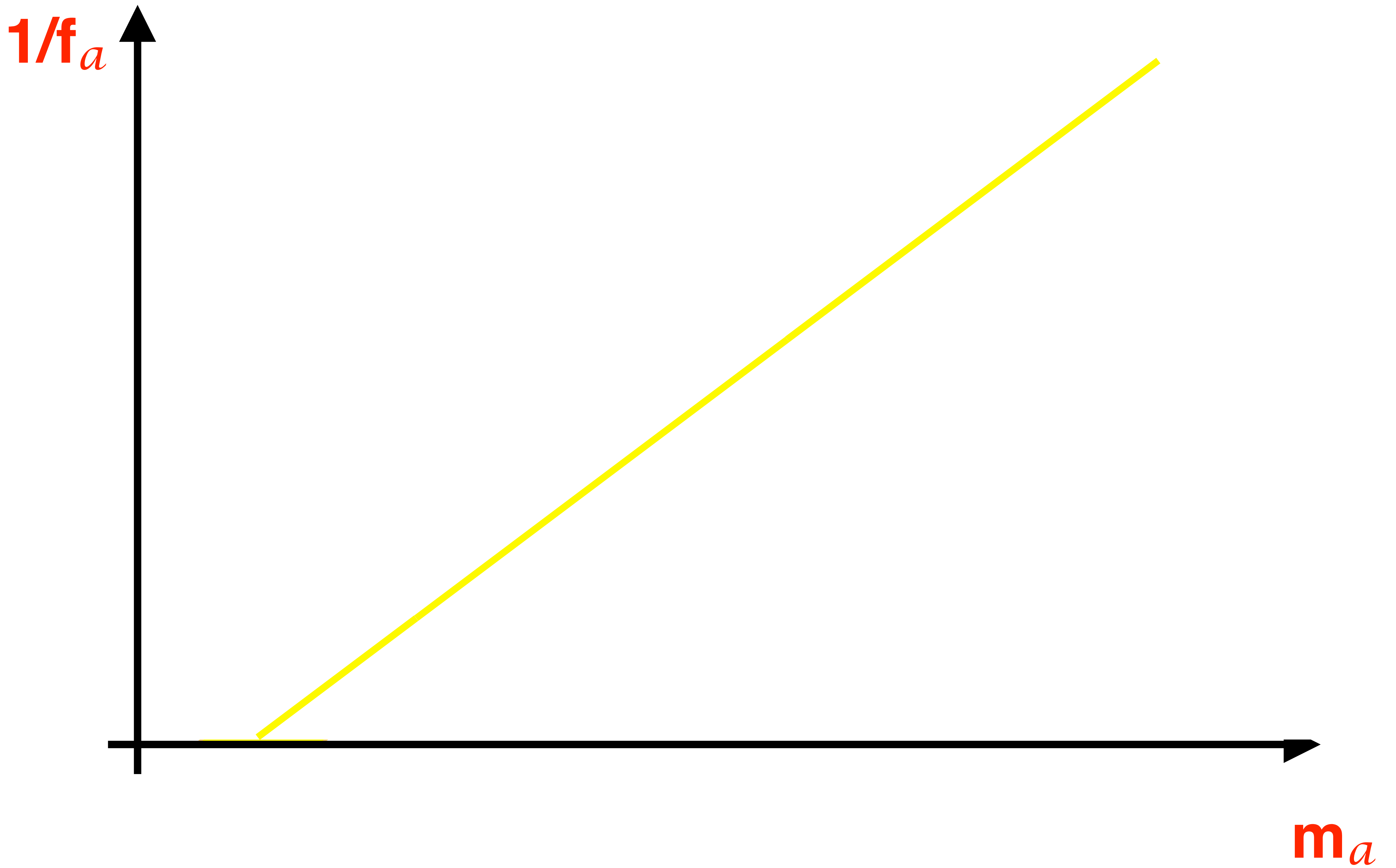
e.g. for $m_a = 10^{-6}$ eV, inside each cm^{-3} there must be



about one thousand million axions per cm^{-3} !

In "true axion" models (= which solve the strong CP problem):

$m_a f_a = \text{cte.}$



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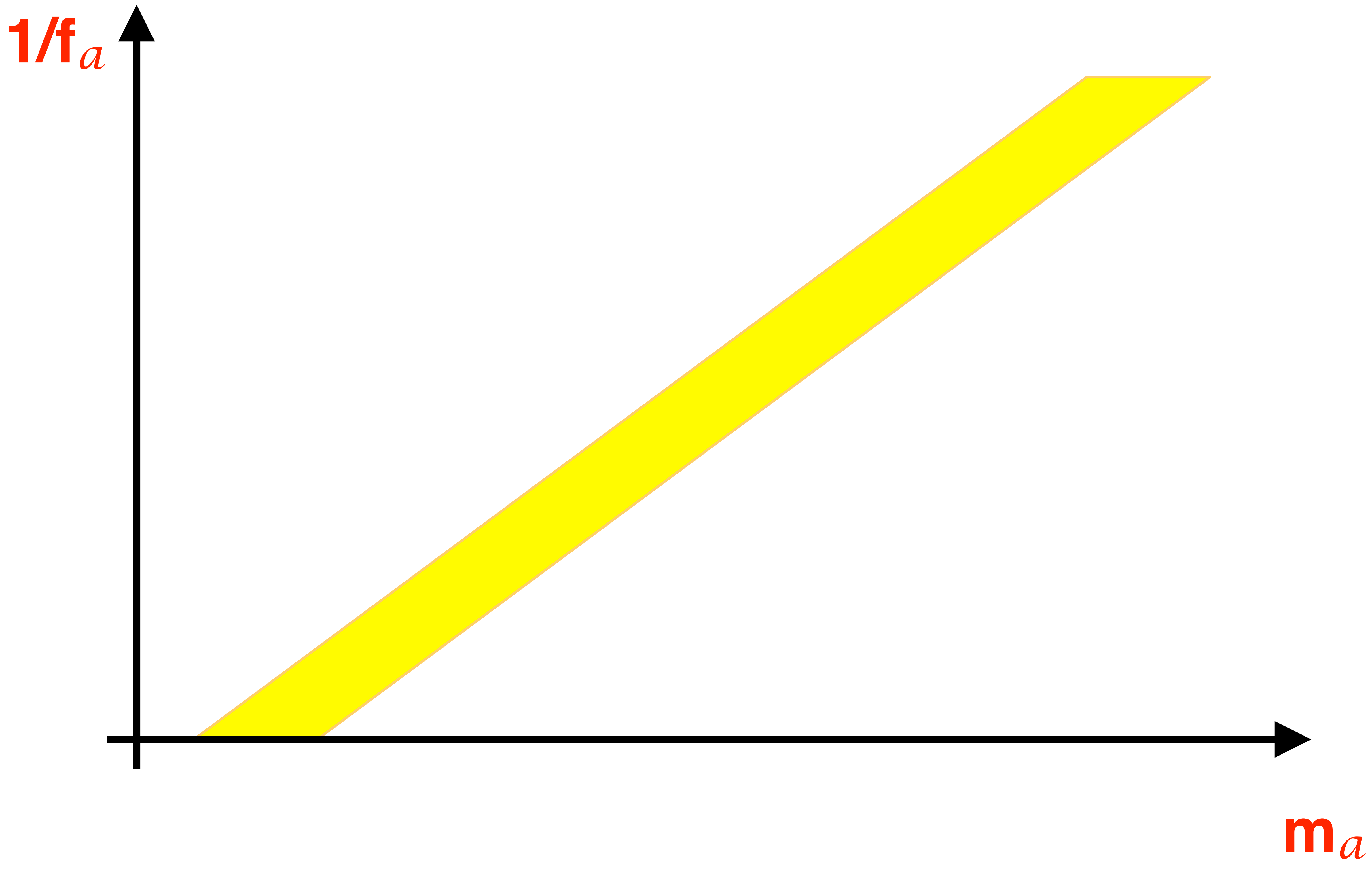

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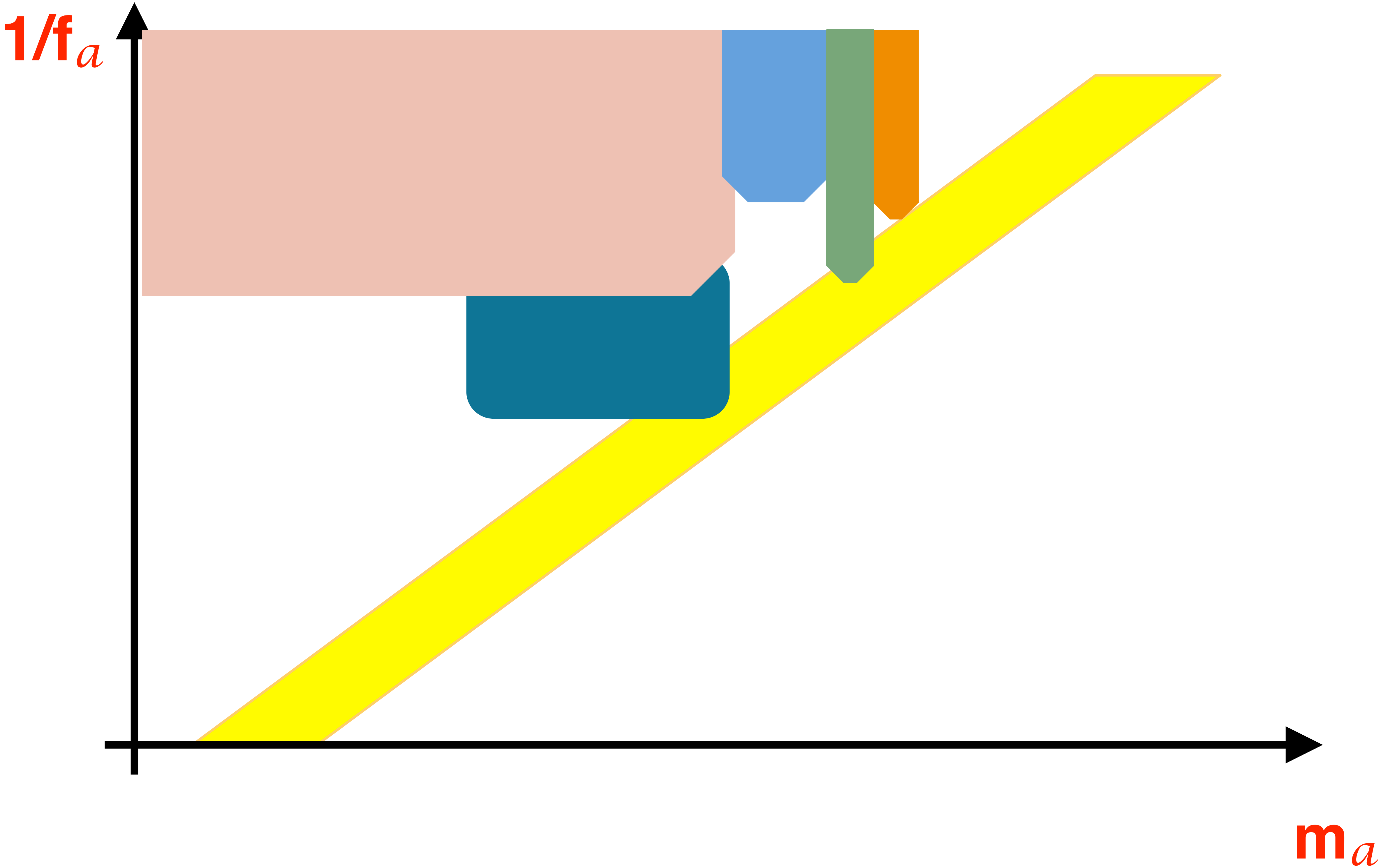

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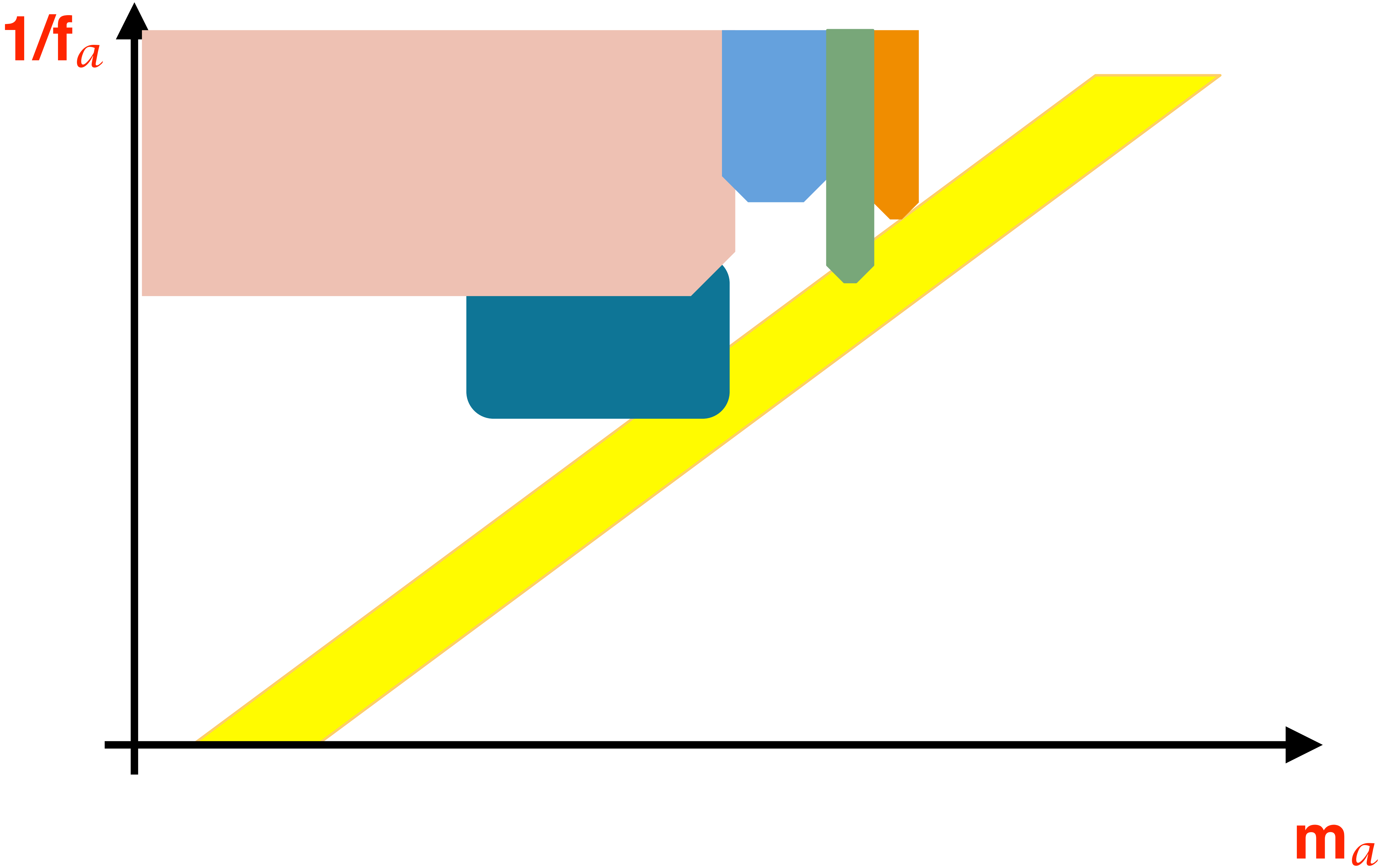
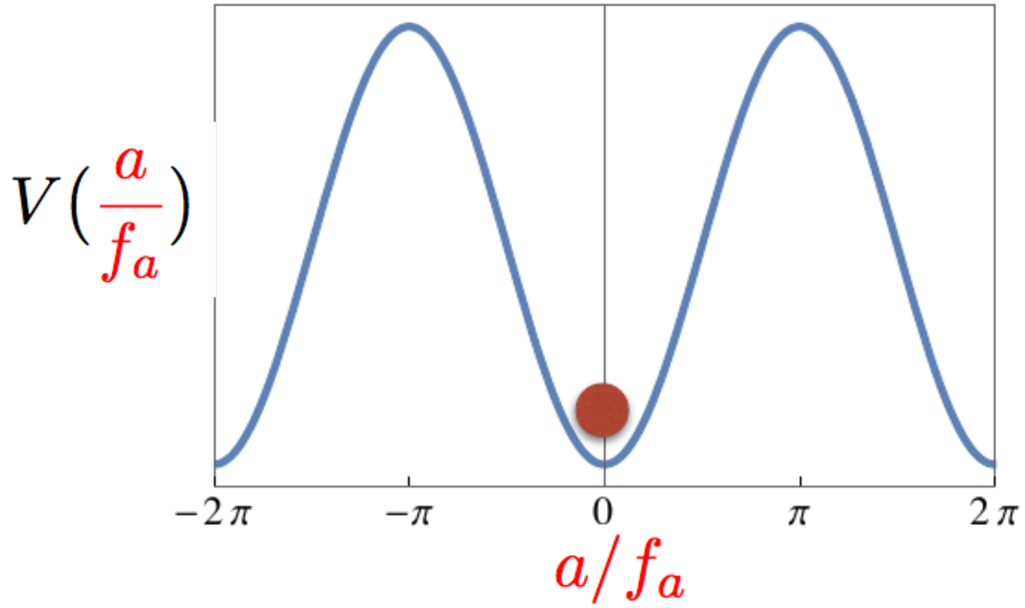


The value of the constant is determined by the strong gauge group

In "true axion" models (= which solve the strong CP problem)

$$m_a f_a = \text{cte.}$$

* If the confining group is QCD:



$\longleftarrow \frac{a}{f_a} G_{\mu\nu}\tilde{G}^{\mu\nu}$

$$V_{SM}\left(\frac{a}{f_a}\right) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2 f_a}\right)}$$

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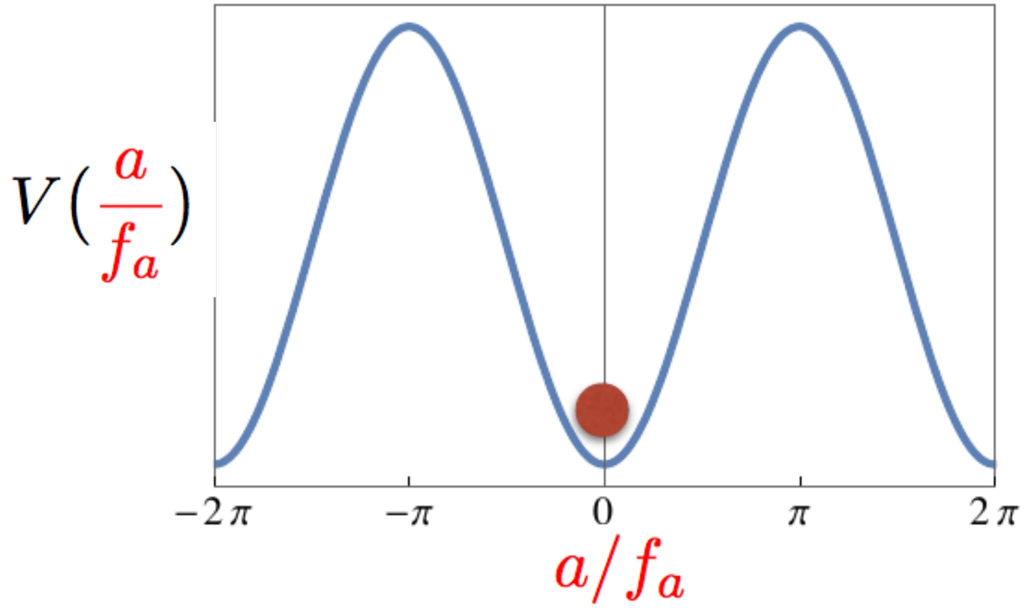

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$$m_a^2 f_a^2 = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

canonical QCD axion

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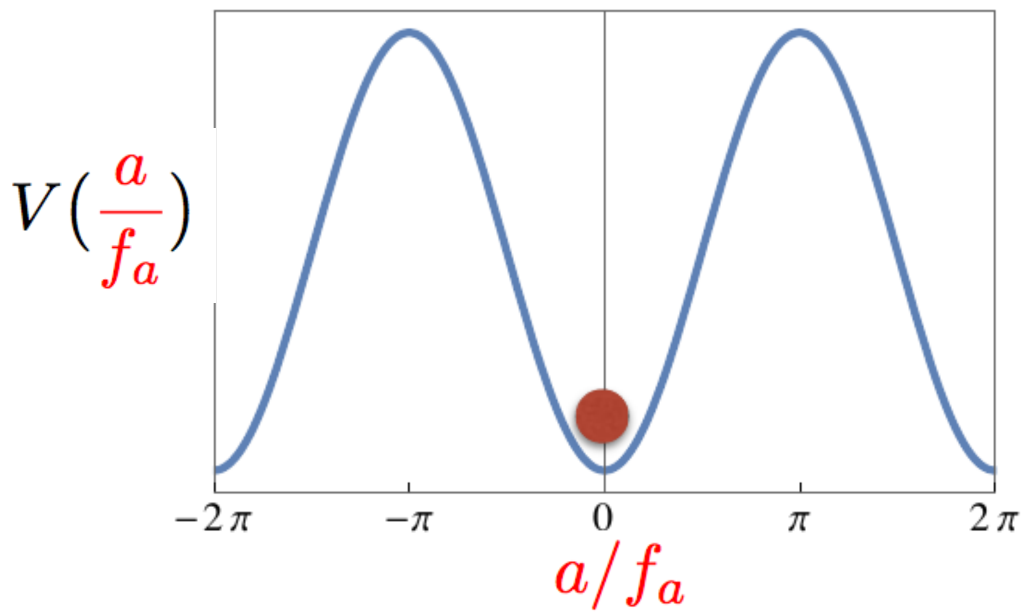

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QCD topological susceptibility = $\chi_{\rm QCD}$

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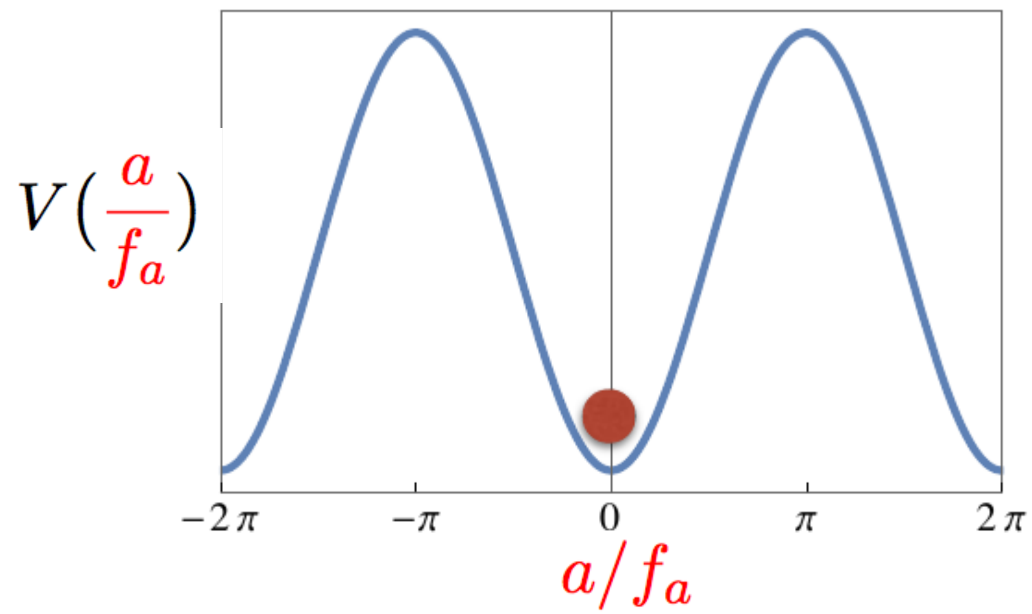

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standard QCD axion

QCD topological susceptibility = χ_{QCD}

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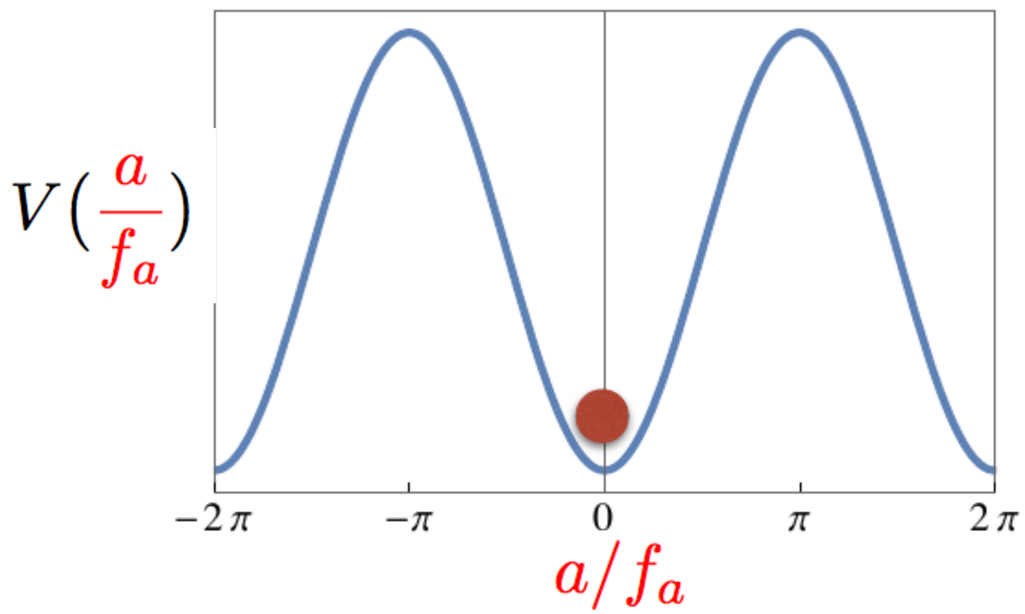

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invisible axion

QCD topological susceptibility = χ_{QCD}

How come the QCD axion mass is NOT $\sim\Lambda_{QCD}$

Because two pseudo scalars couple to the QCD anomalous current :

N_{ps} : η'_{QCD} $\quad a_1$

With only QCD:

one combination must be (almost) **massless**

–> "Invisible axion"

N_{inst} : $G\tilde{G}$

QCD

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N_{ps} : η'_{QCD} a_1

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QCD

With only QCD:

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–> "Invisible axion"

The tiny axion mass is due to mixing with η' and pion:

$$m_a^2 f_a^2 \sim m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

relation independent of the UV axion model

In "true axion" models (= which solve the strong CP problem):

$$\mathbf{m}_a\,\mathbf{f}_a = \text{cte.}$$

*** If the confining group is QCD:** $m_a^2 f_a^2 = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$

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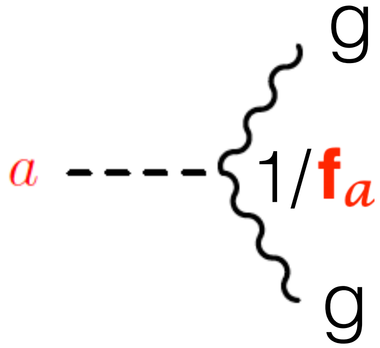
$10^{-5} < m_a < 10^{-2}$ eV , $10^{9} < f_a < 10^{12}$ GeV

Because of SN and hadronic data,
if axions light enough to be emitted

"Invisible axion"

Intensely looked for experimentally…

$\{m_a, 1/f_a\}$: direct a - **gluon coupling**



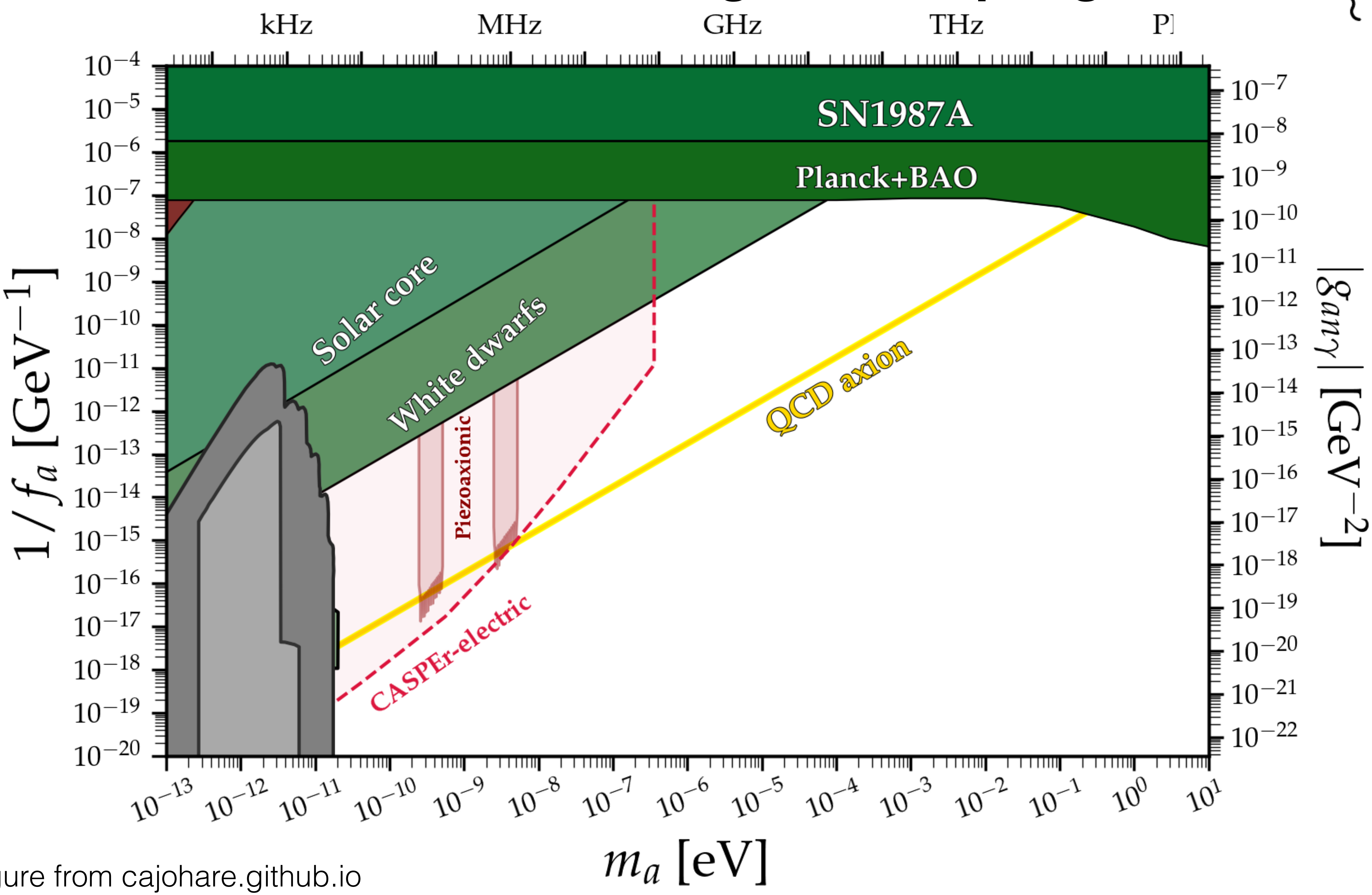
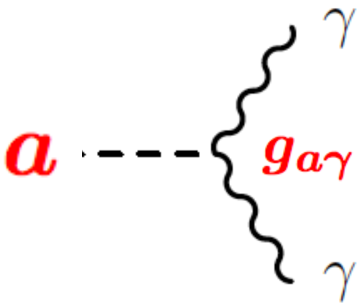


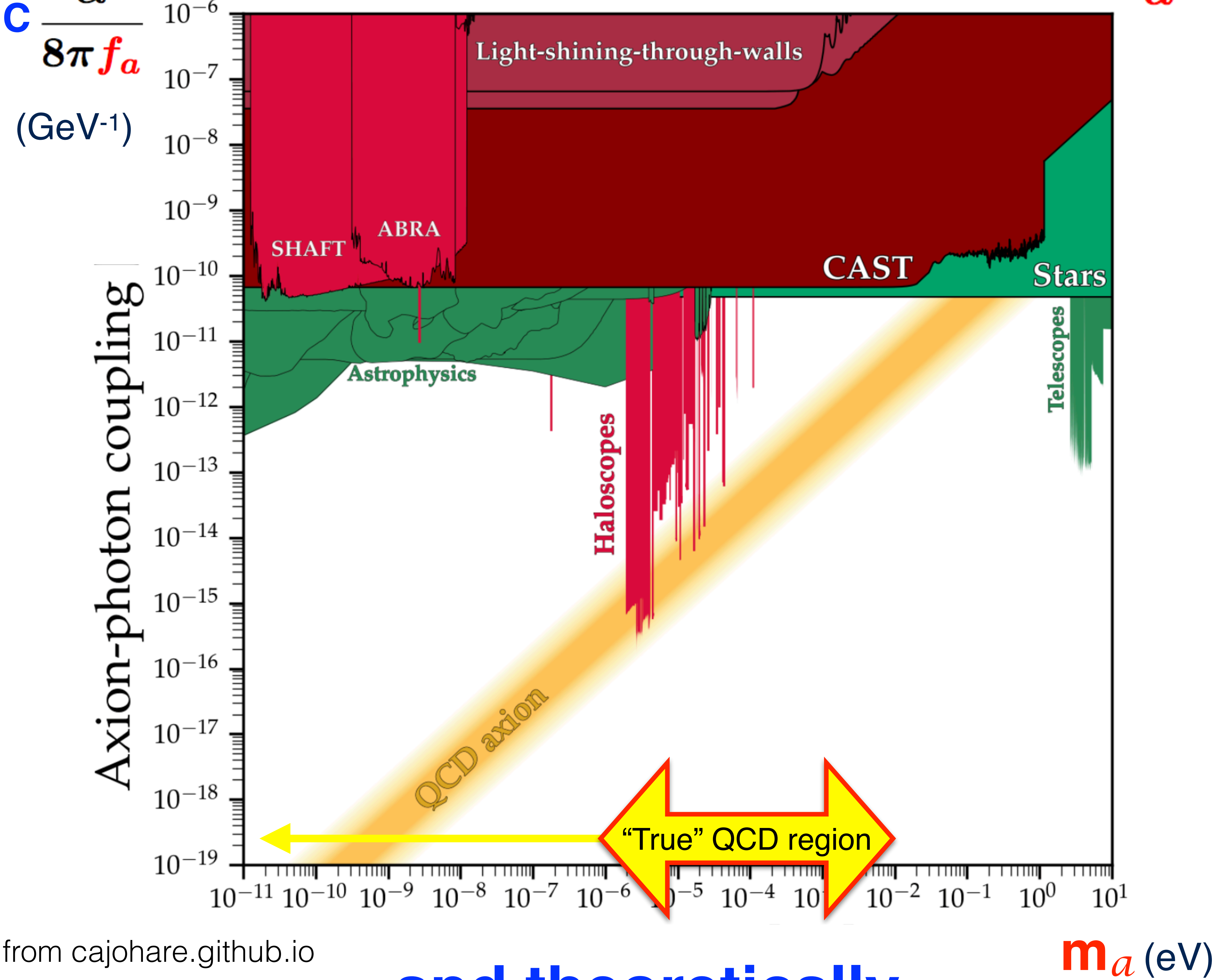
figure from cajohare.github.io

… and theoretically

Intensely looked for experimentally…

$$g_{a\gamma} \sim C\,\frac{\alpha}{8\pi f_a}$$





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… and theoretically

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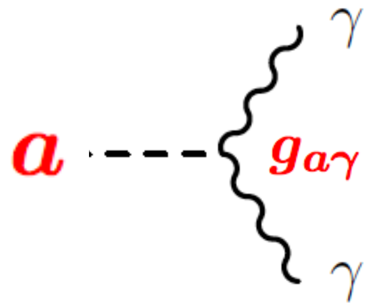
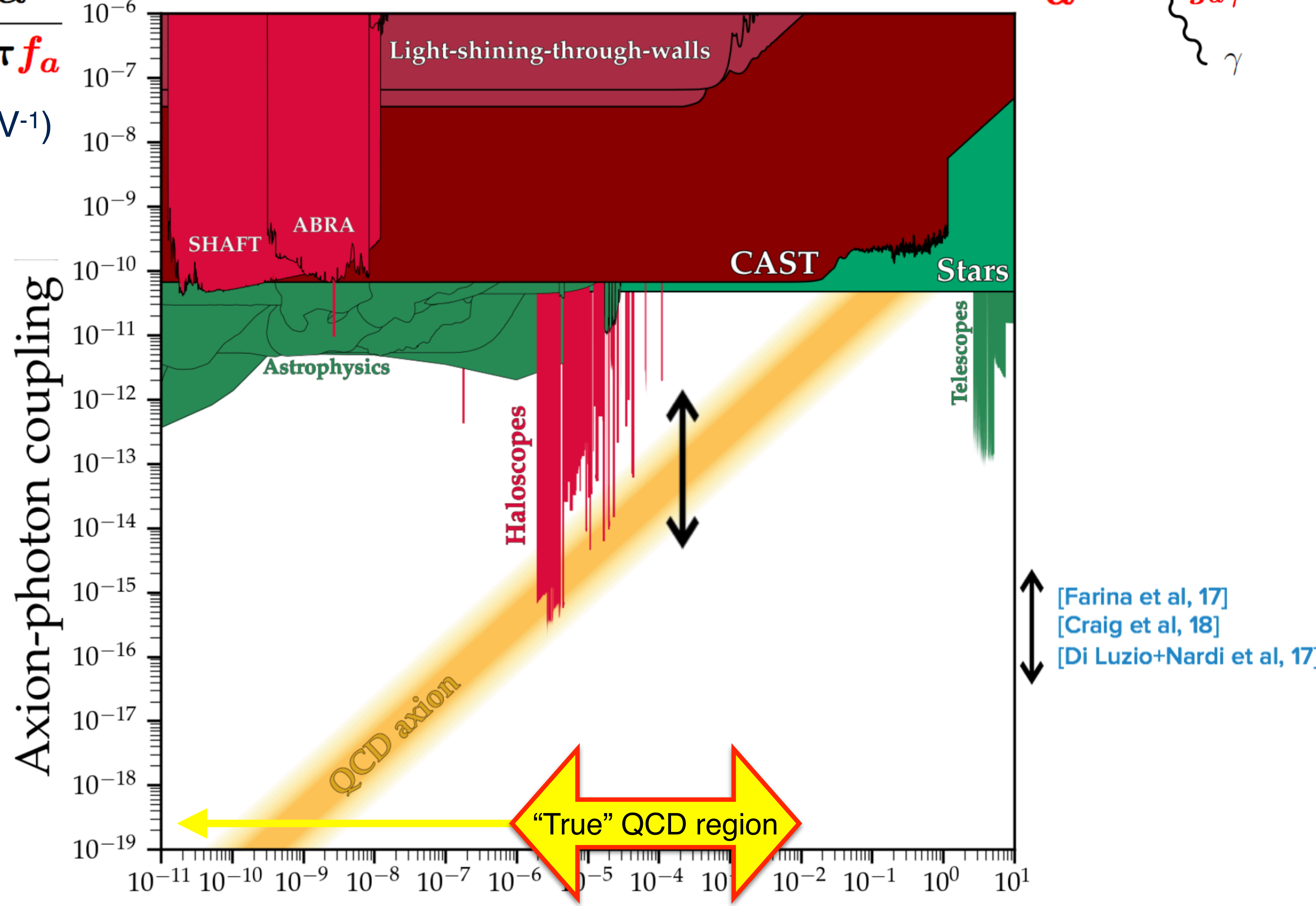
$g_{a\gamma} \sim C \frac{\alpha}{8\pi f_a}$ (GeV^{-1})



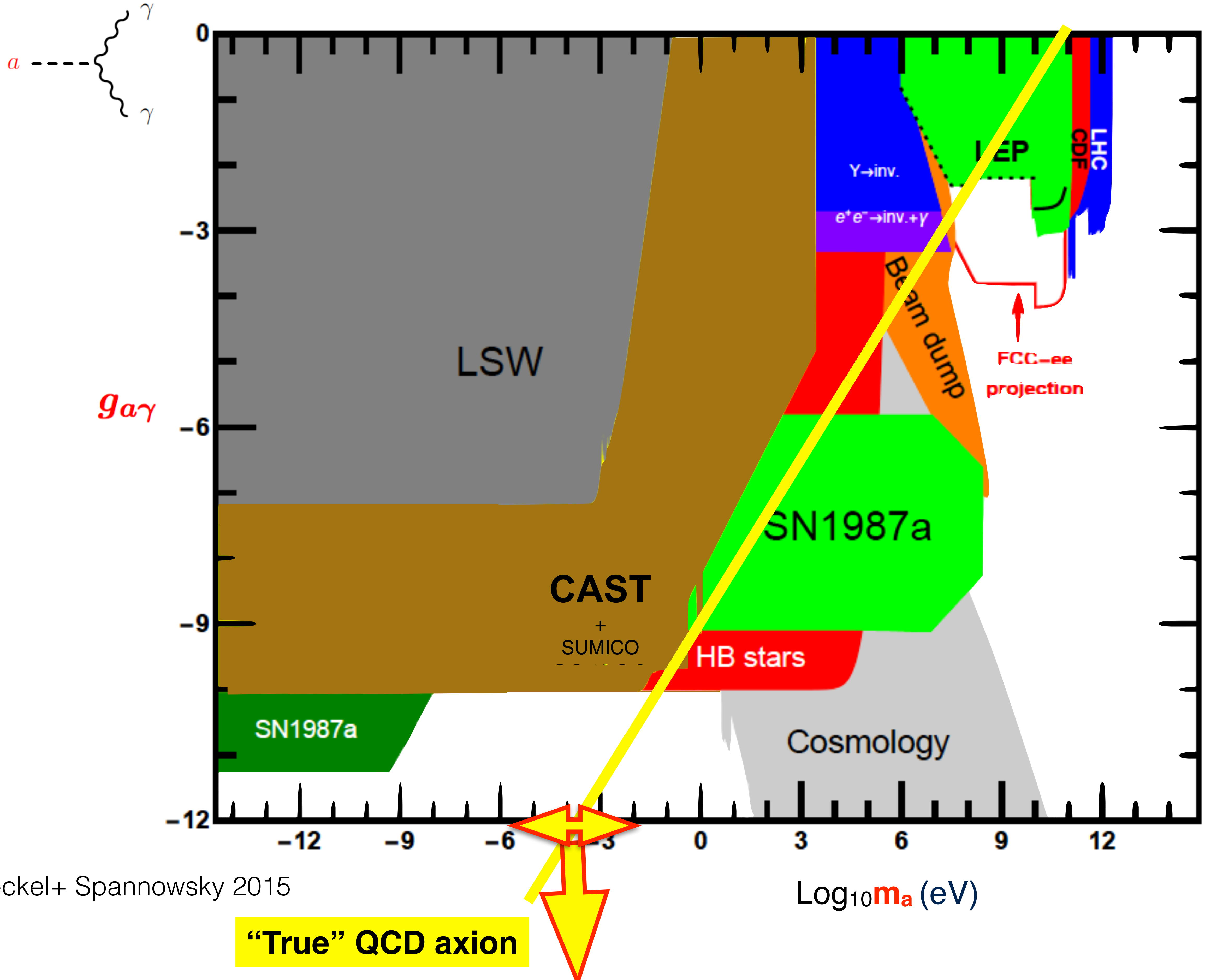


m_a (eV)

figure from cajohare.github.io

… and theoretically

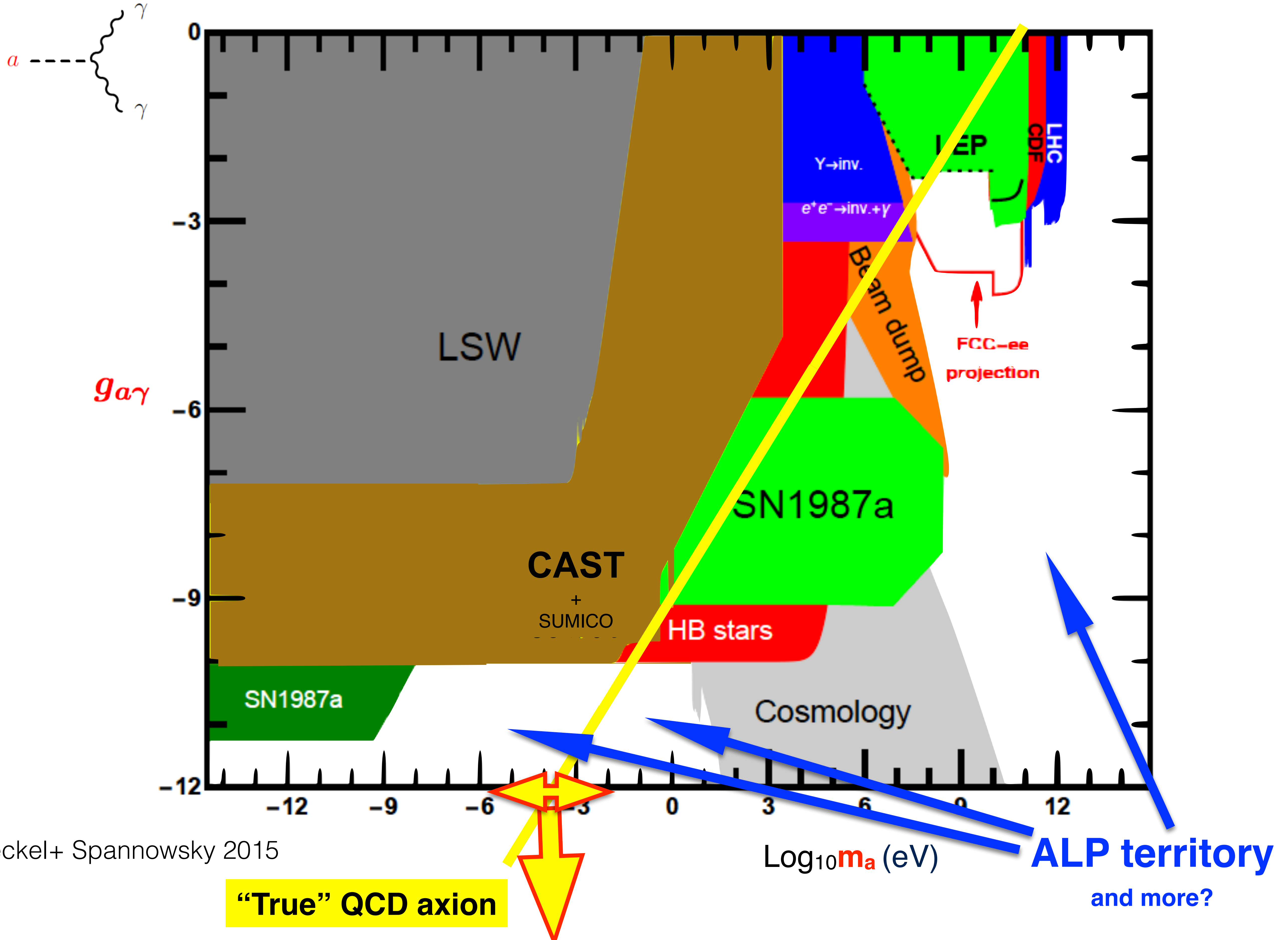
ALPs (axion-like particles) territory



Jaeckel+ Spannowsky 2015

“True” QCD axion

ALPs (axion-like particles) territory



Jaeckel+ Spannowsky 2015

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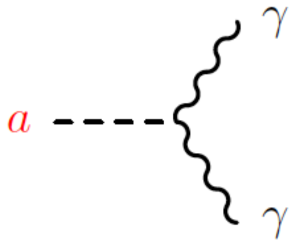
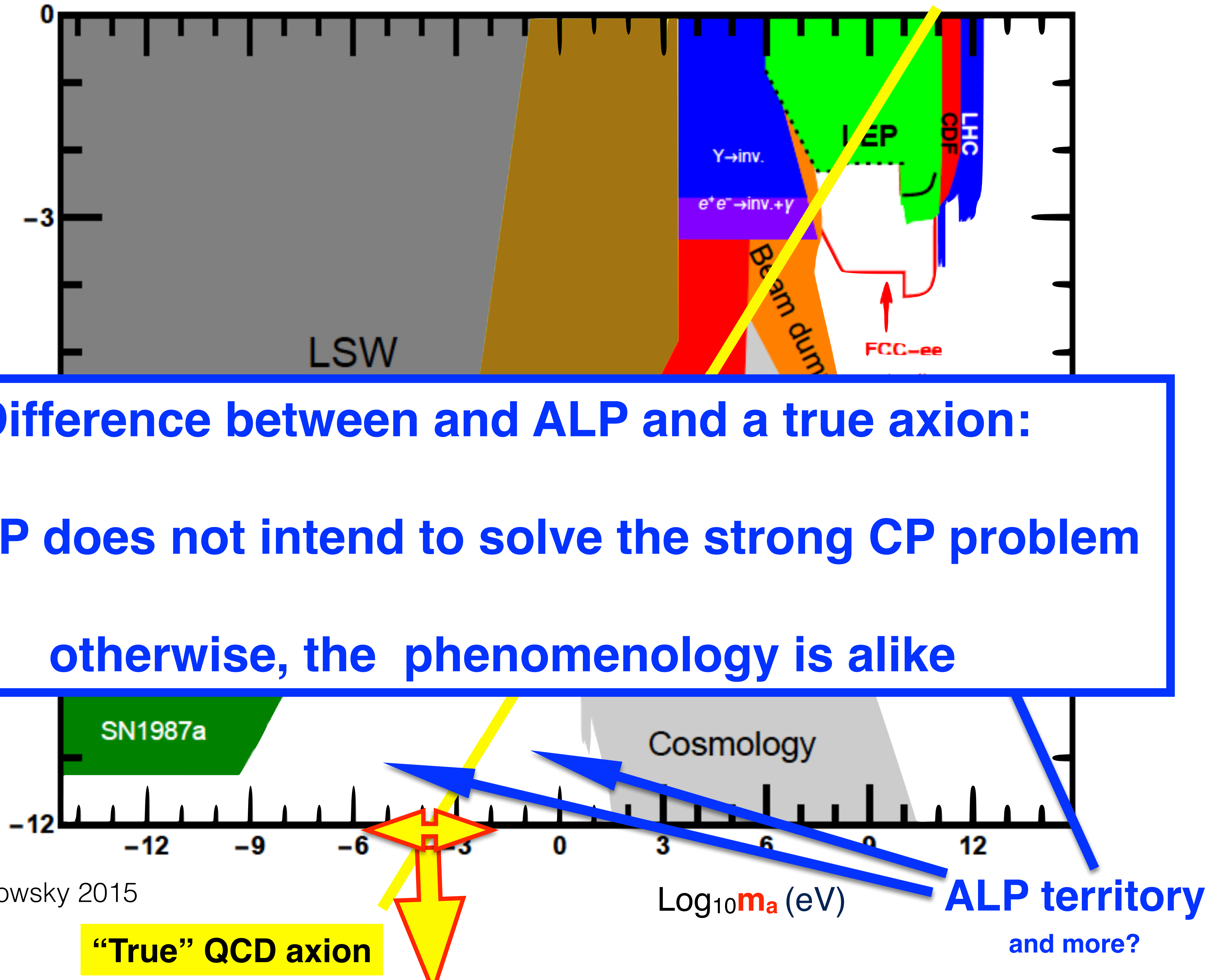




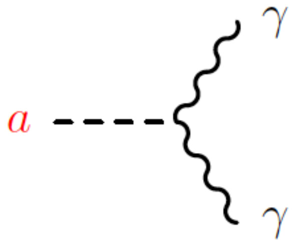
Difference between and ALP and a true axion:

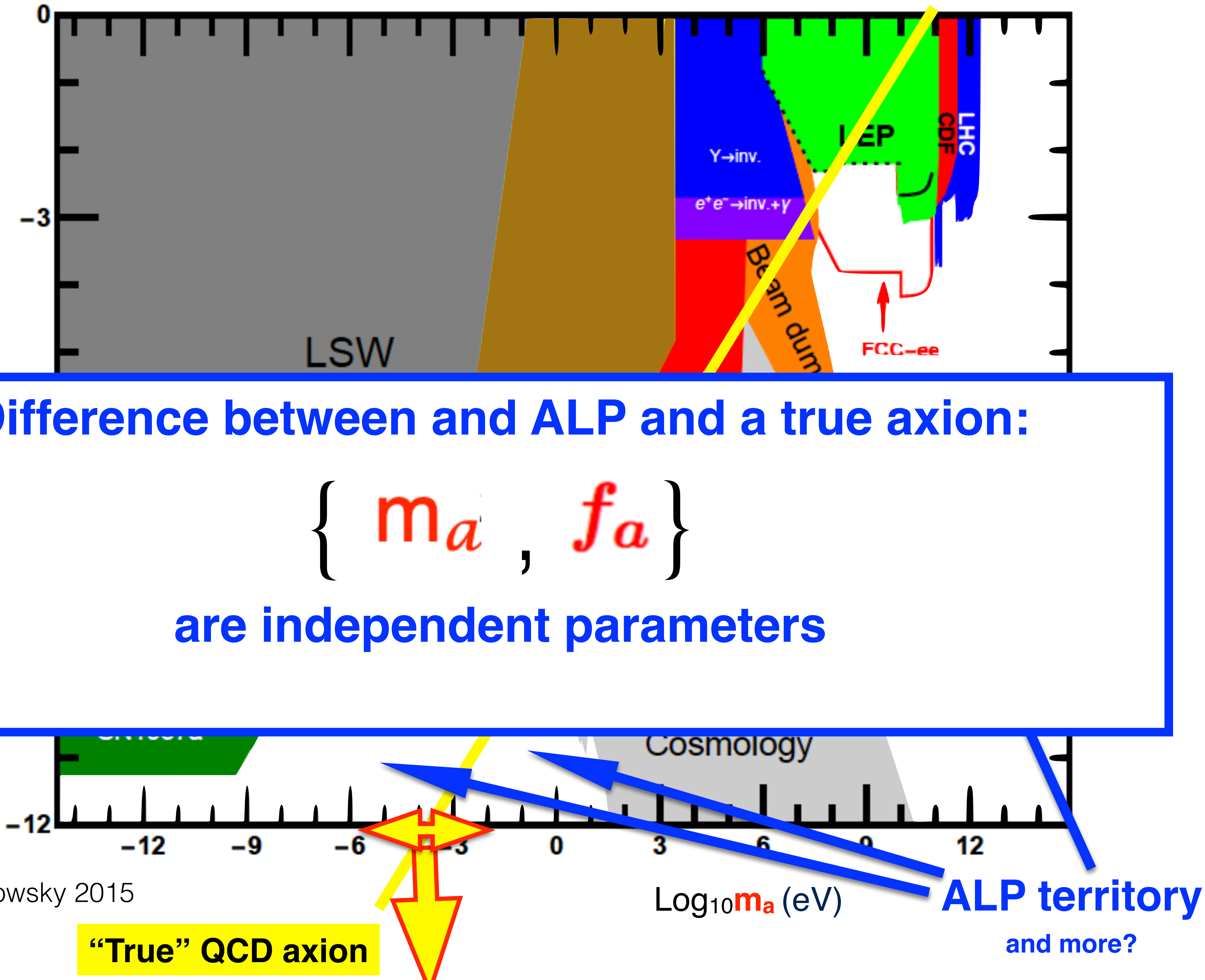
an ALP does not intend to solve the strong CP problem

otherwise, the phenomenology is alike

Jaeckel+ Spannowsky 2015

ALPs (axion-like particles) territory





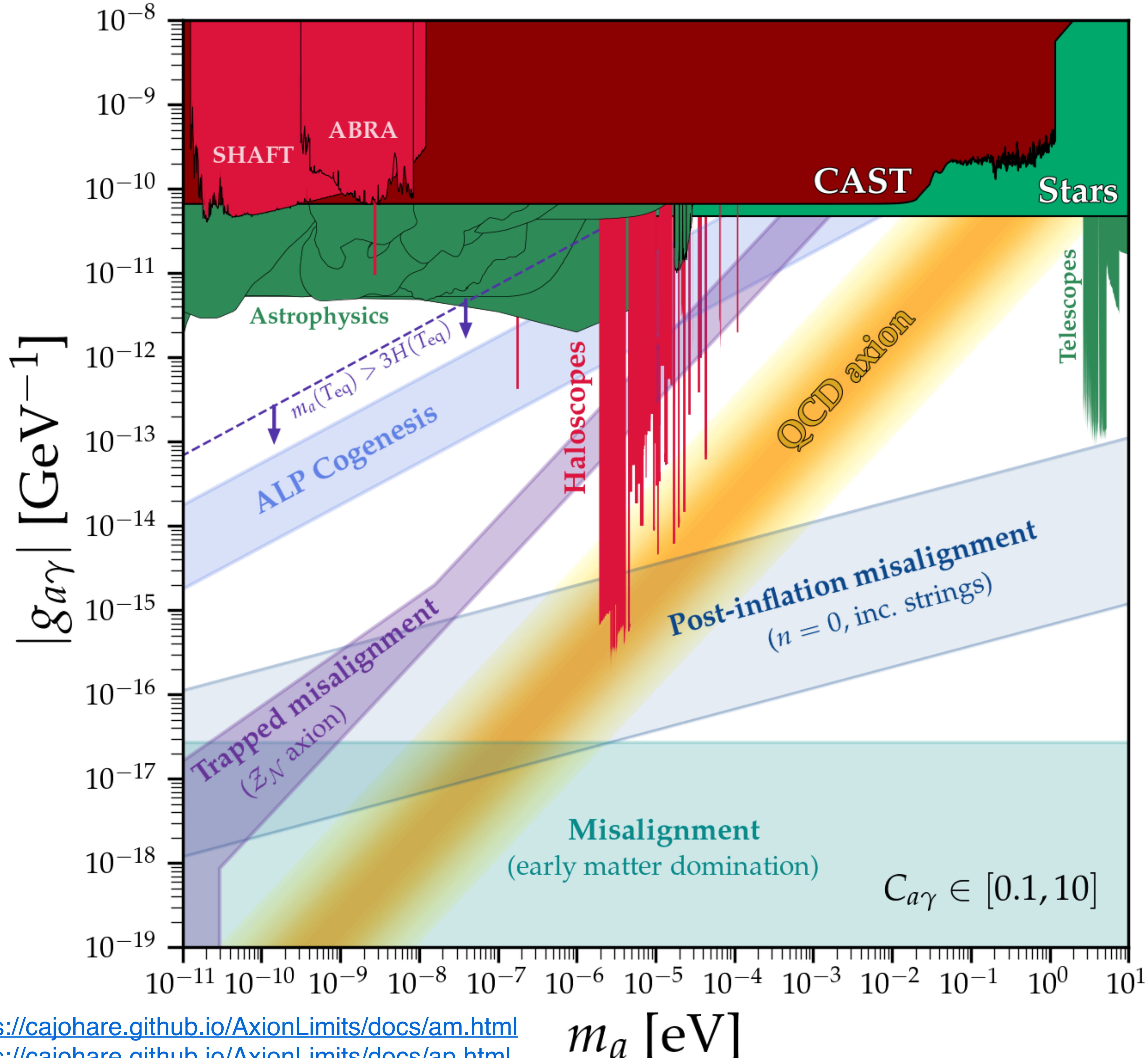
Difference between and ALP and a true axion:

$$\{\ m_a\ ,\ f_a\ \}$$

are independent parameters

Jaeckel+ Spannowsky 2015

Axions and ALPs can explain Dark Matter



within the blueish bands axions/ALPs would account for all the DM

https://cajohare.github.io/AxionLimits/docs/am.html
https://cajohare.github.io/AxionLimits/docs/ap.html

The field is BLOOMING

in Experiment … and Theory

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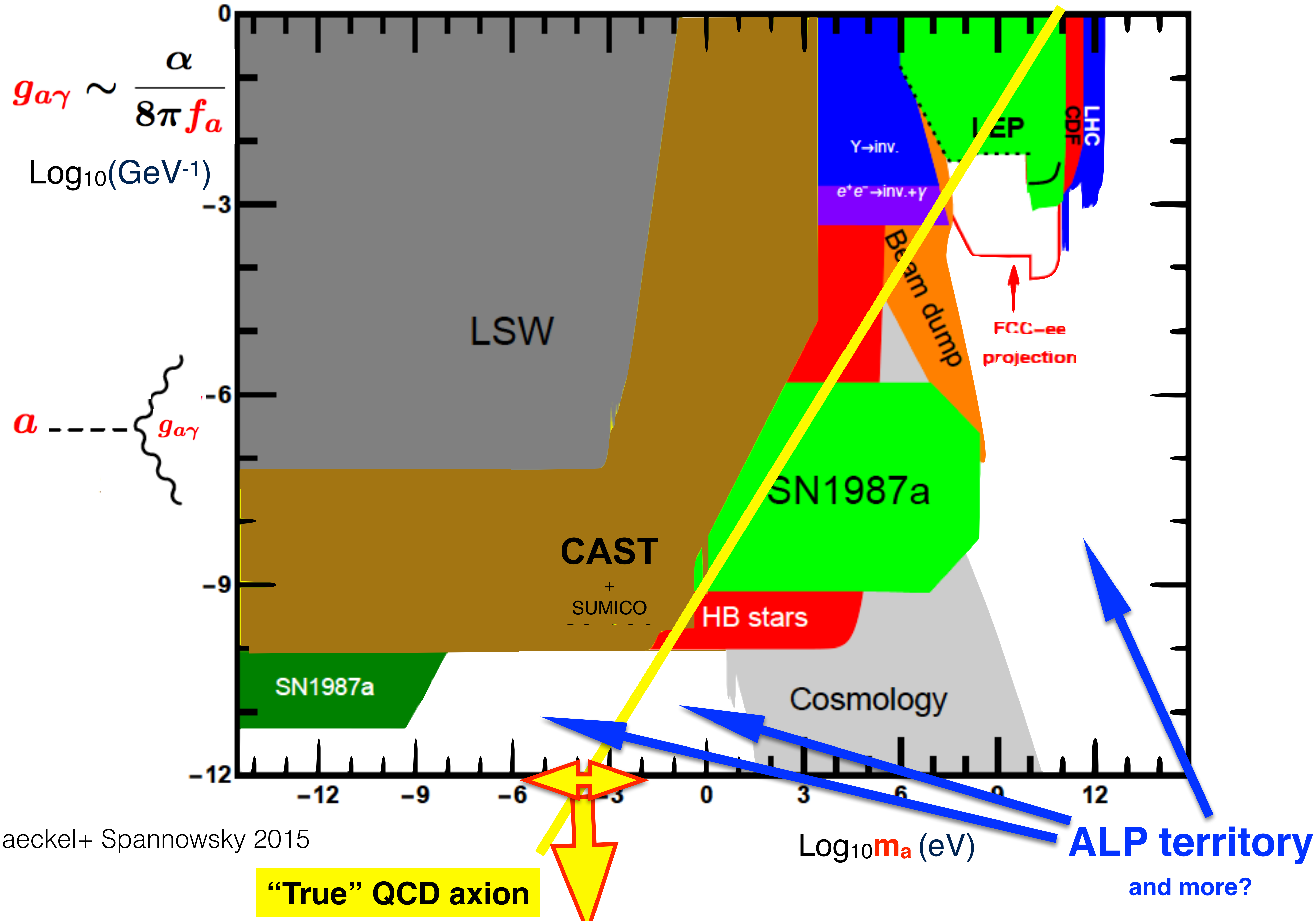
in Experiment … and Theory

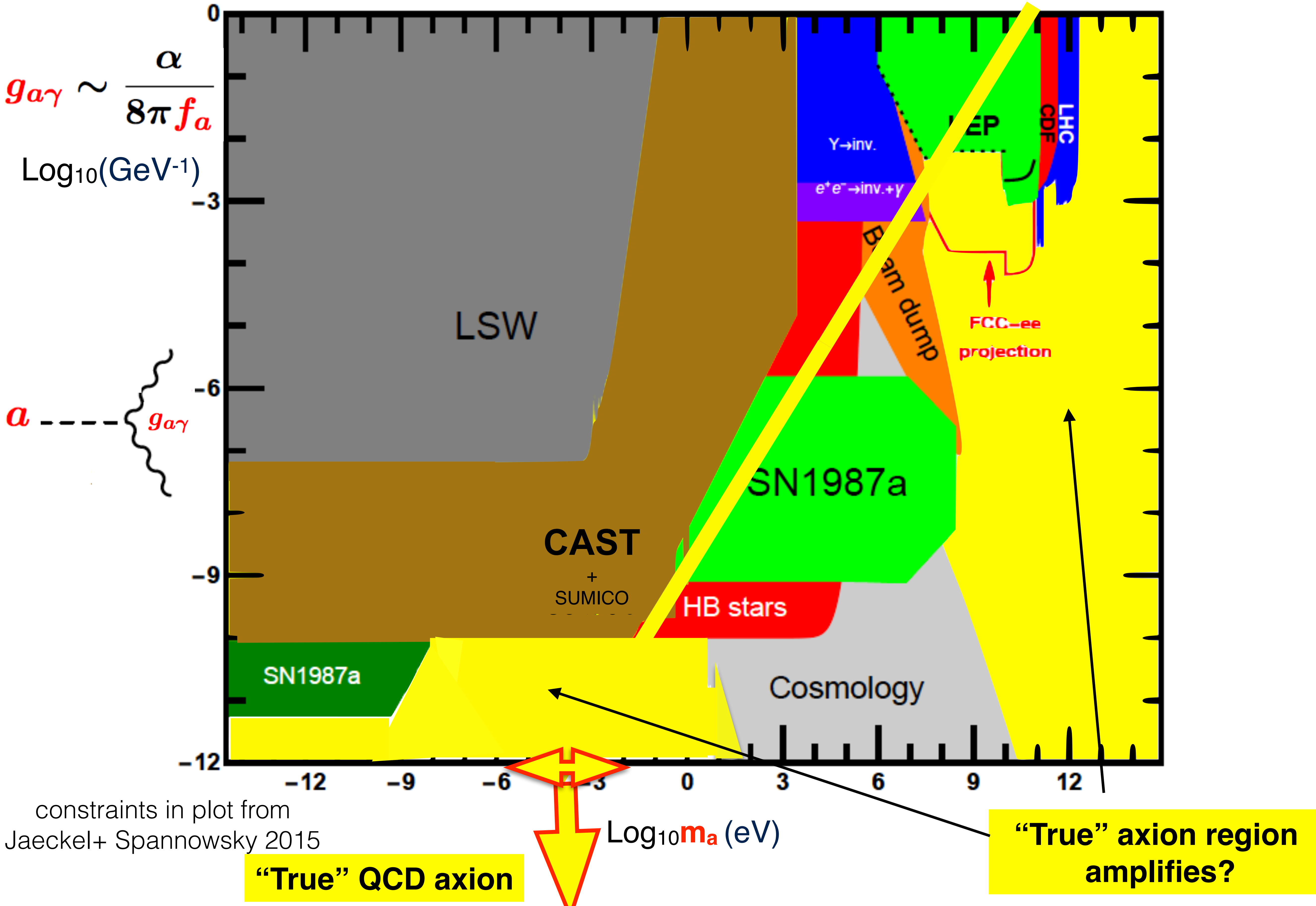
My task today: **can ALPs be true axions ?(i.e. solve strong CP)**

$$g_{a\gamma} \sim \frac{\alpha}{8\pi f_a}$$



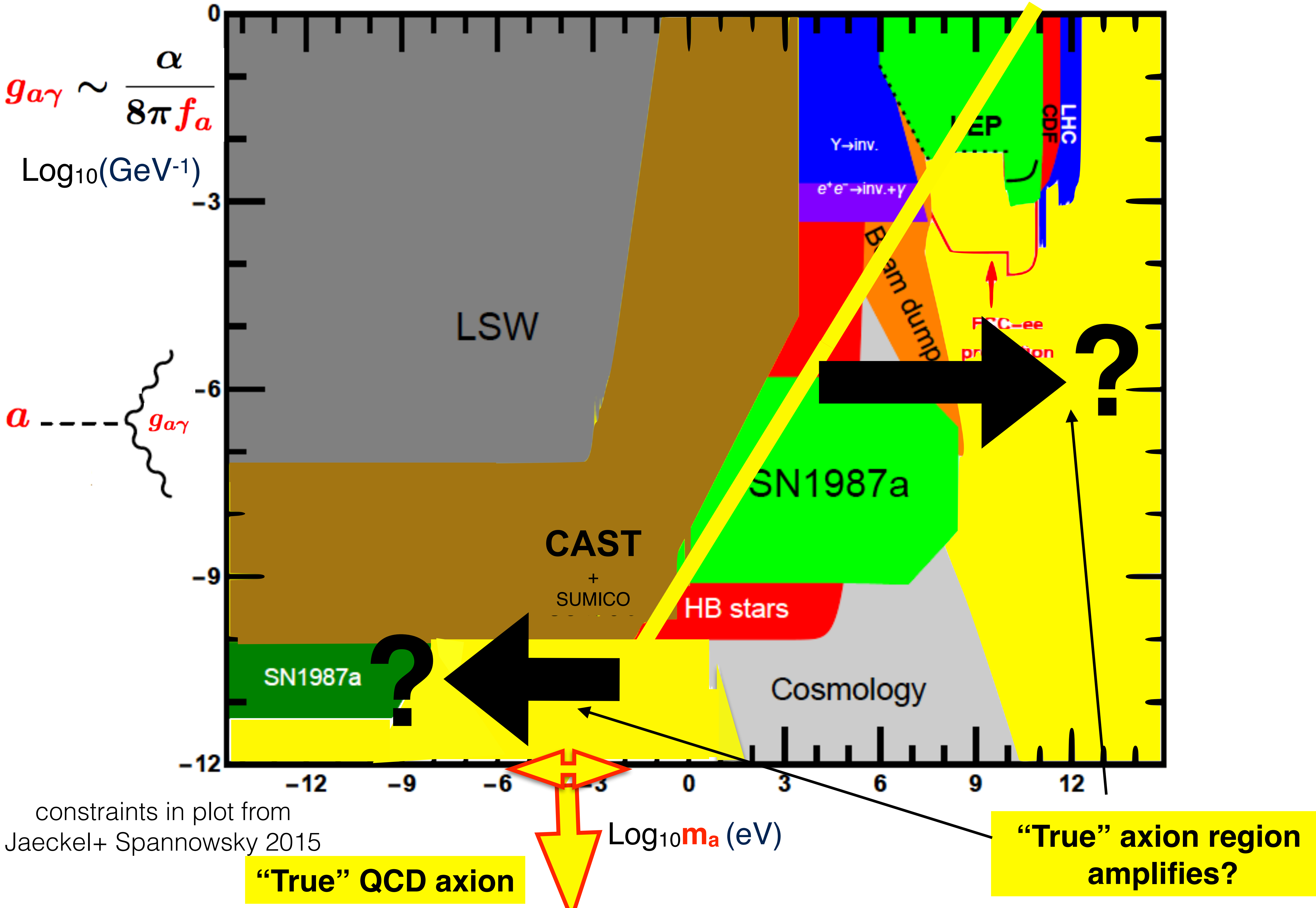
Jaeckel+ Spannowsky 2015

ALPs territory: can they be true axions ?

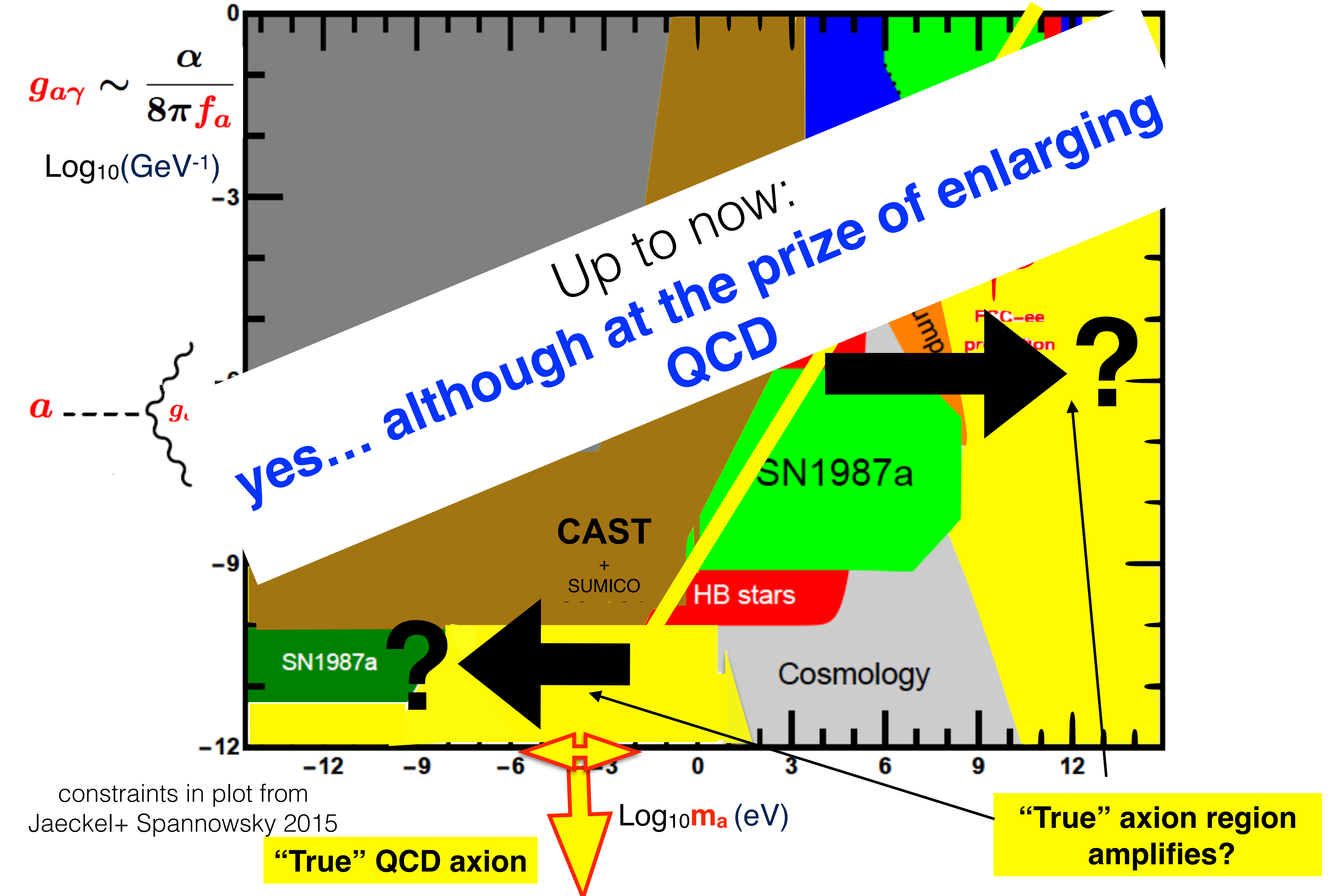


constraints in plot from
Jaeckel+ Spannowsky 2015

ALPs territory: can they be true axions ?



ALPs territory: can they be true axions ?



constraints in plot from Jaeckel+ Spannowsky 2015

"True" QCD axion

"True" axion region amplifies?

but…..

Let me revisit, and challenge,

the standard QCD wisdom

In "true axion" models (= which solve the strong CP problem):

$m_a f_a$ = cte.

* If the confining group is QCD:

$$m_a^2 f_a^2 = \underbrace{m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}}$$

I am going to challenge this !

QCD topological susceptibility = χ_{QCD}

``Multiple QCD axion”

with Pablo Quilez and Maria Ramos, arXiv2305.15465

``The QCD axion sum rule”

with Pablo Quilez and Maria Ramos, arXiv2305.15465

The Peccei-Quinn symmetry

PQ symmetry = a global $U(1)_A$ symmetry,

exact at classical level

but explicitly broken only by QCD instantons

→ **QCD axion** a

The Peccei-Quinn symmetry

PQ symmetry = a global $U(1)_A$ symmetry,

exact at classical level

but explicitly broken only by QCD instantons

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This connection assumes that a is a mass eigenstate

ungranted !

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to be more precise:
that it only mixes
with the η', i.e.:

QCD eigenstate
= mass eigenstate

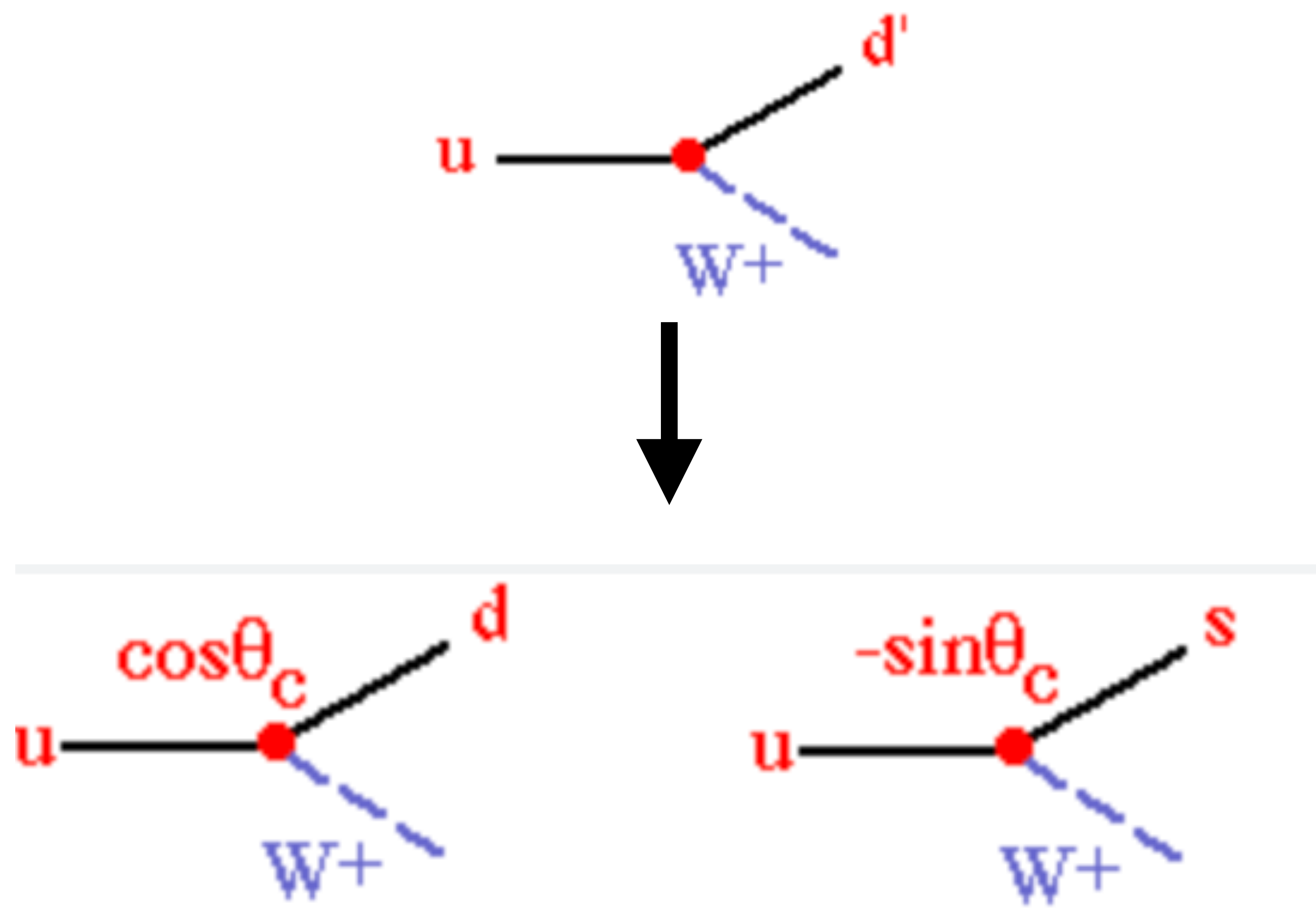
ungranted !

Remember:

In SM electroweak interactions families mix because

The weak interaction basis $\neq$ the mass basis

(they are not simultaneously diagonal, unlike for QCD or QED)



In QCD-axion interactions, axions may mix because

The gluon interaction basis $\neq$ the mass basis

(they are not necessarily simultaneously diagonal)

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The axion field may not be the only singlet scalar in Nature. Let us allow it to mix with other singlet scalars

As long as the total scalar potential has a PQ symmetry, the strong CP problem is solved

coupling to gluons

Standard QCD axion:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}\right) G\widetilde{G}$$

$$\Rightarrow m_a^2 f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

Instead, we can have:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V'(\hat{a}_{G\widetilde{G}}, \ldots, \hat{a}_N)$$

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$$\Rightarrow m_i^2 f_i^2 = g_i \chi_{\rm QCD}$$

within QCD

Axion-exotic scalars mixing has appeared before in other constructions (clockwork, GUT, multiHiggs…)

Kim, Niles, Peloso 2005
Choi, Kim, Yun 2014
Kaplan, Ratazzi 2016
Giudice, McCullough 2017
Di Luzio et al. 2018
Fraser, Reece 2020
Darme et al. 2021
Chen at al. 2022
Agrawal Nee, Reig 2022

but, either by choice or by construction,
they took the limit where all but one decouple

Plan

1) A toy model with N=2 scalars

2) N fields and the most general PQ-invariant potential

N=2 toy example

$$\mathcal{L}_{N=2} = \left(\frac{a_{G\widetilde{G}}}{F} + \theta\right) G\widetilde{G} - V(a_{G\tilde{G}}, a_{\perp})$$

$$\frac{a_{\widetilde{G}}}{F} \equiv \frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}$$

or equivalently:

$$\mathcal{L}_{N=2} = \left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \theta\right) G\widetilde{G} - V(\hat{a}_1,\, \hat{a}_2)$$

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After confinement (and for $\hat{f}_1 = \hat{f}_2 = \hat{f}$ and $r \equiv \frac{\hat{m}_2^2 \hat{f}^2}{\chi_{\rm QCD}}$):

$$\mathbf{M^2} = \frac{\chi_{\rm QCD}}{\hat{f}^2} \begin{pmatrix} 1 & 1 \\ 1 & 1+r \end{pmatrix} \rightarrow \begin{matrix} \{m_1, f_1\} \\ \{m_2, f_2\} \end{matrix}$$

N=2 toy example

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Both eigenstates couple to gluons: $\mathcal{L} \supset \frac{\alpha_s}{8\pi}\left[\frac{a_1}{f_1} + \frac{a_2}{f_2}\right] G\widetilde{G}$

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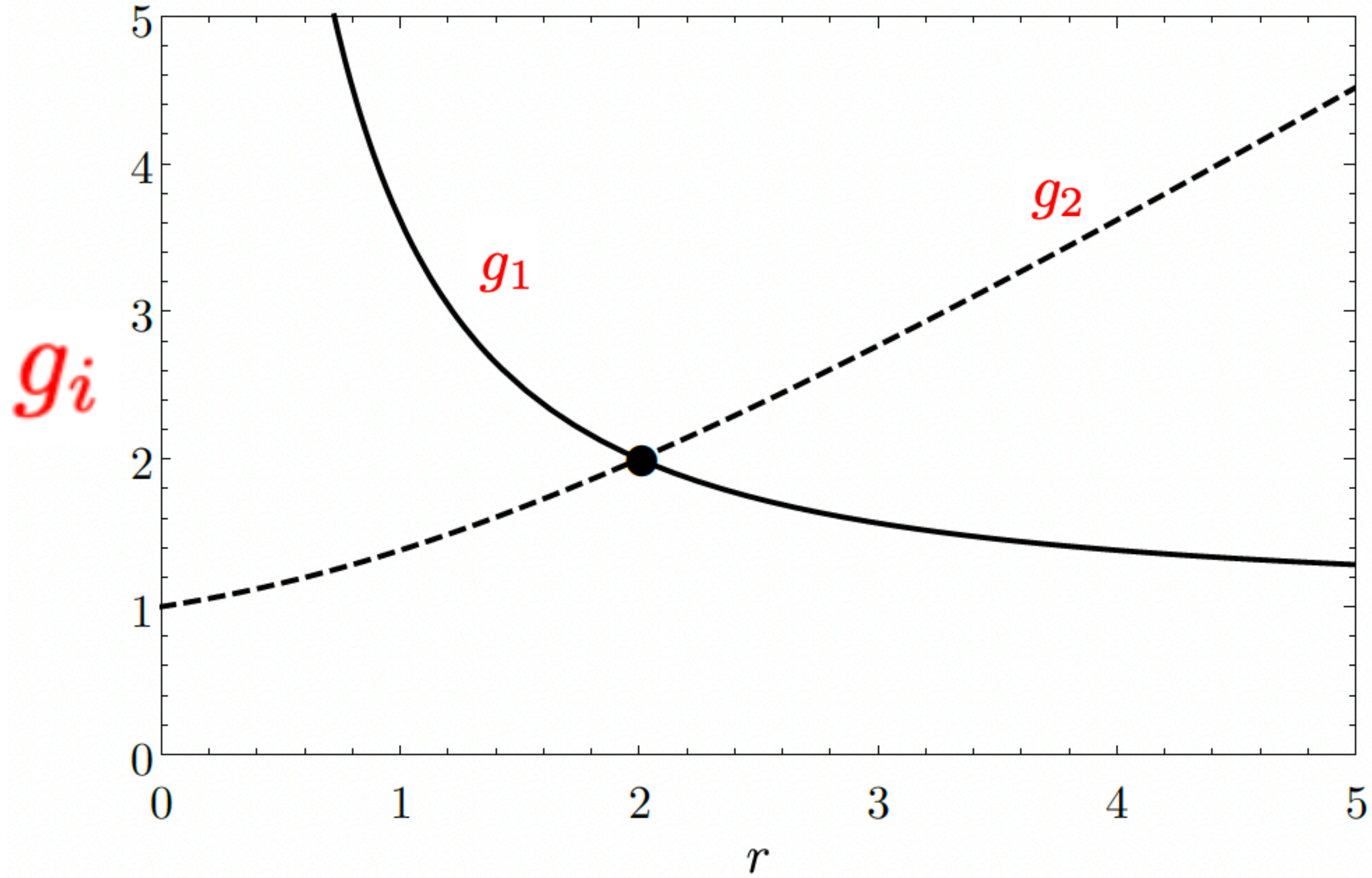
Distance to standard QCD band $\{m_a, 1/f_a\}$

$$g_i \equiv \frac{m_i^2 f_i^2}{m_a^2 f_a^2}\Big|_{\text{single QCD axion}} \qquad \beta_i \equiv \frac{1}{g_i}$$

in the N=2 toy model: $$g_{1(2)} = \frac{2\sqrt{4+r^2}}{\sqrt{4+r^2} \pm (r-2)}$$



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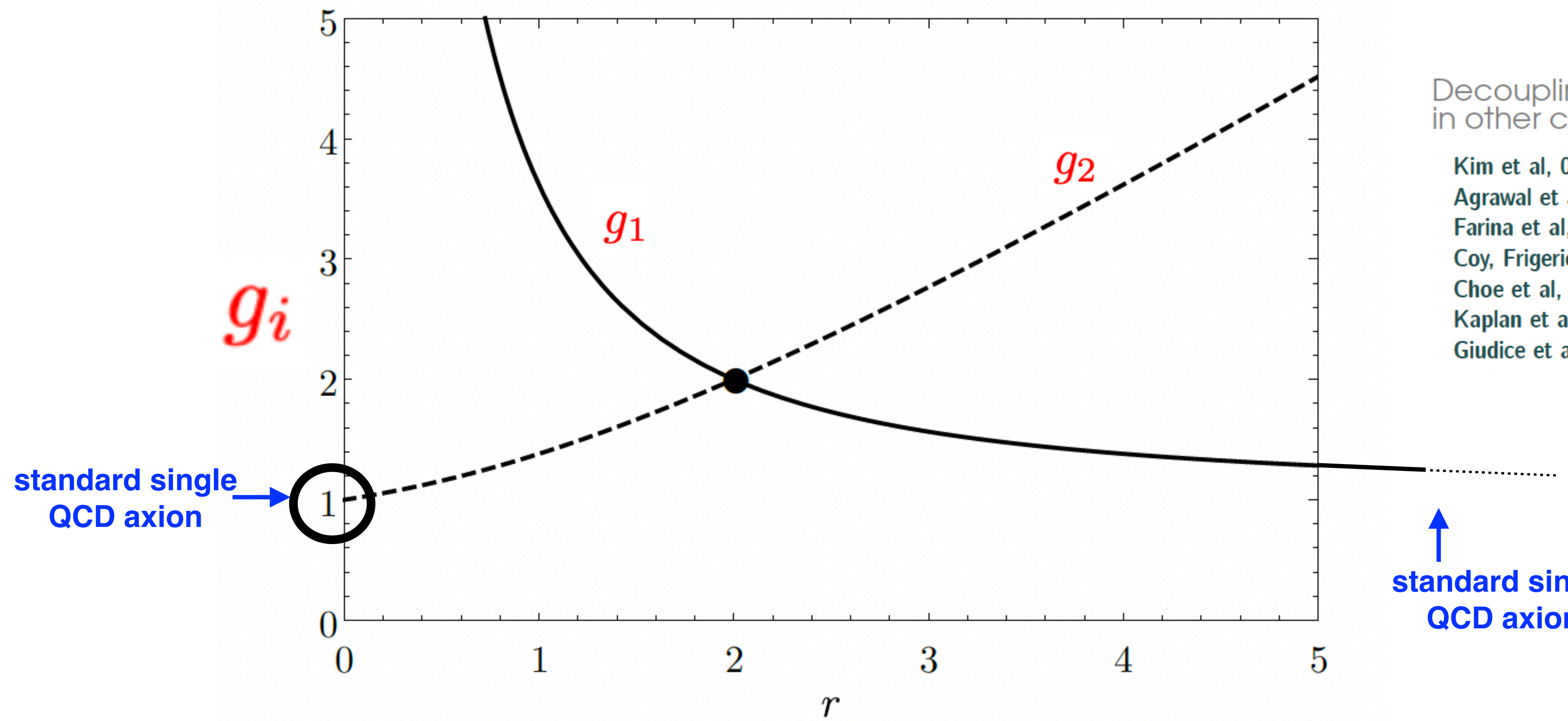

Decoupling limit assumed in other constructions

Kim et al, 04
Agrawal et al, 17 and 22
Farina et al, 17
Coy, Frigerio, 17
Choe et al, 14 and 16
Kaplan et al, 16
Giudice et al, 16

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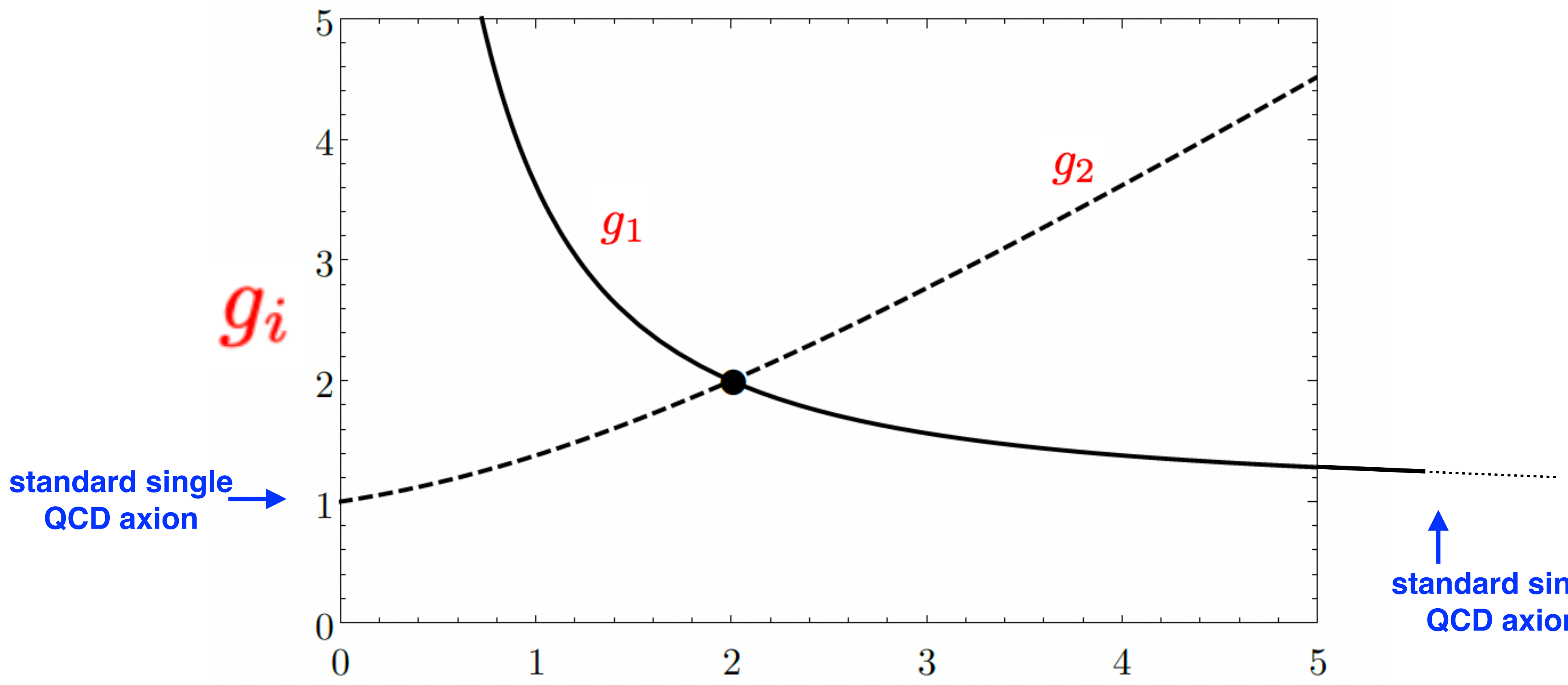

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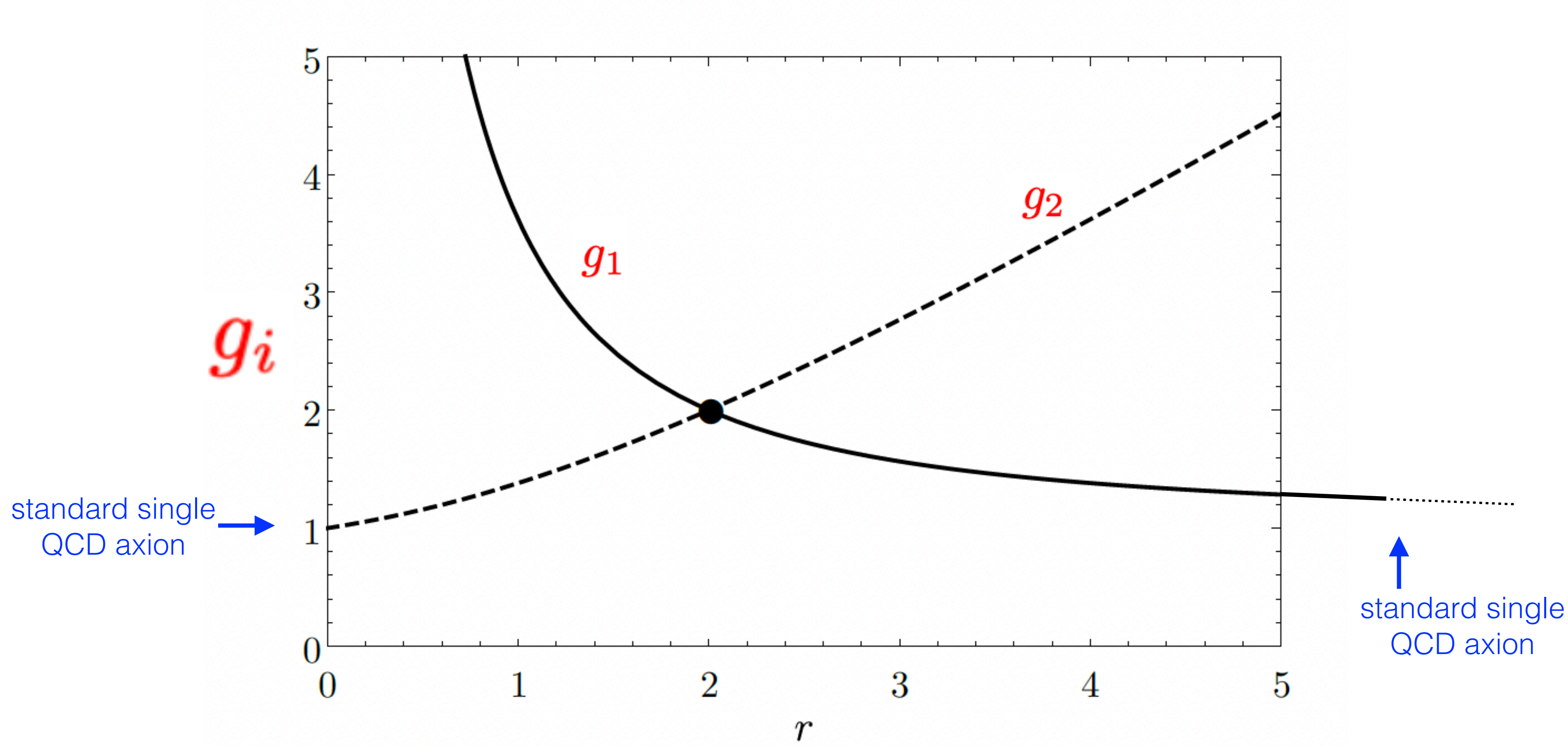

$$\beta_1 + \beta_2 = 1$$

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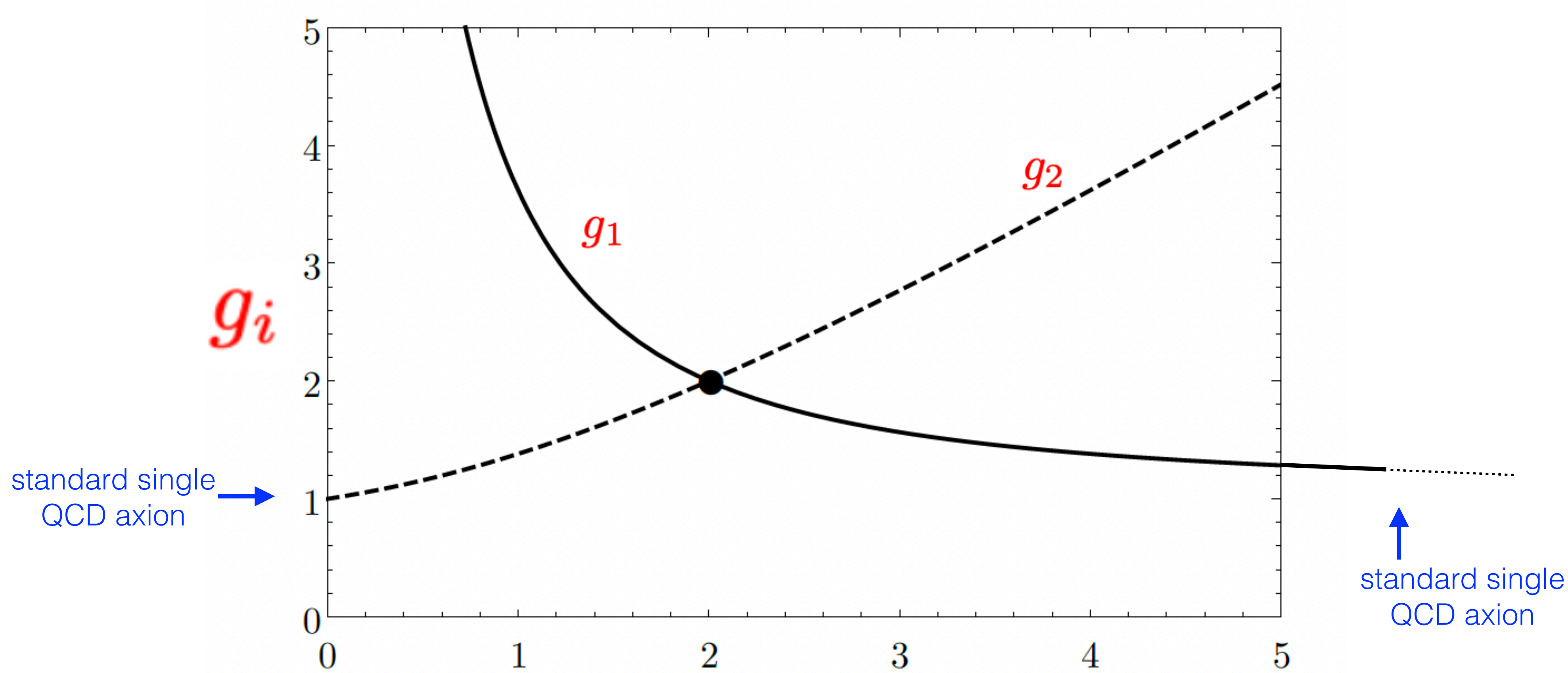

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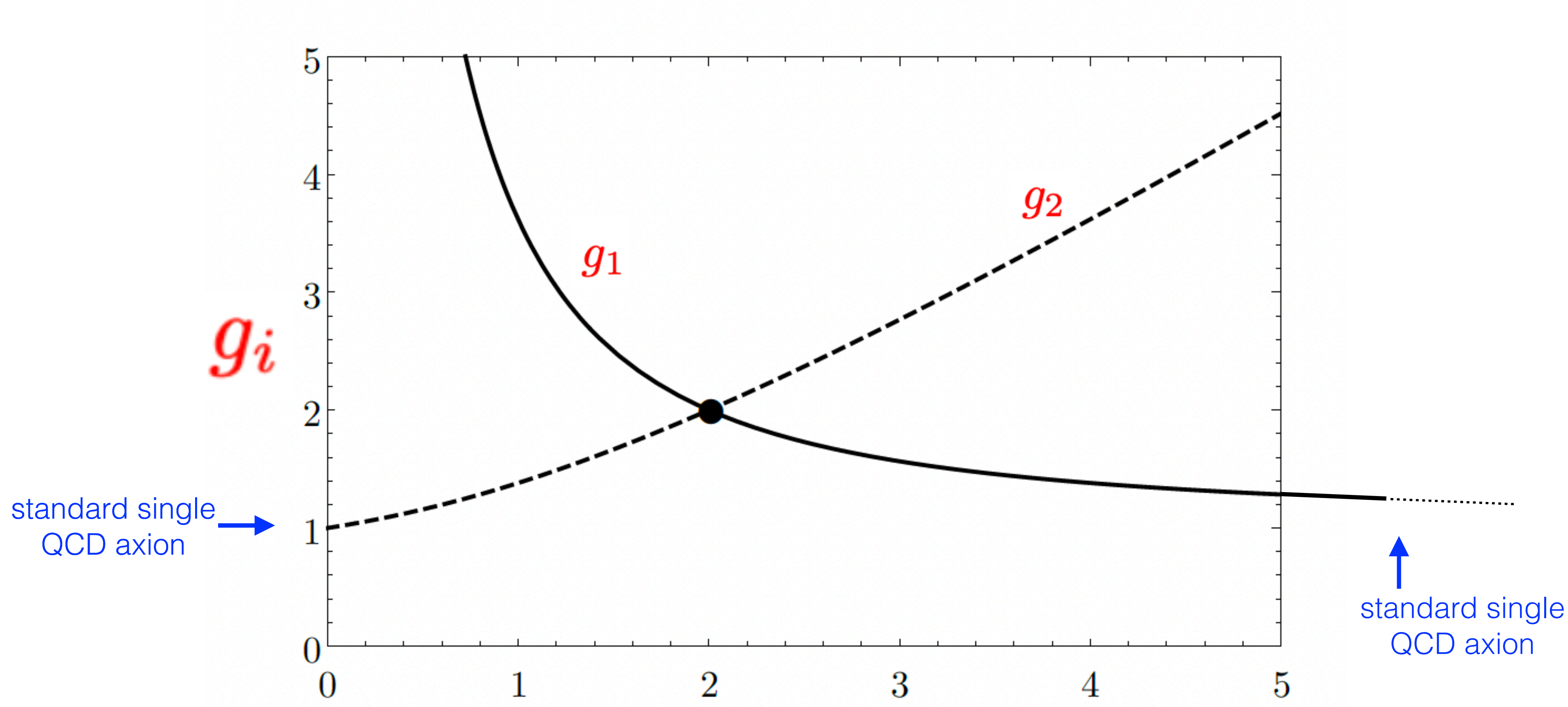

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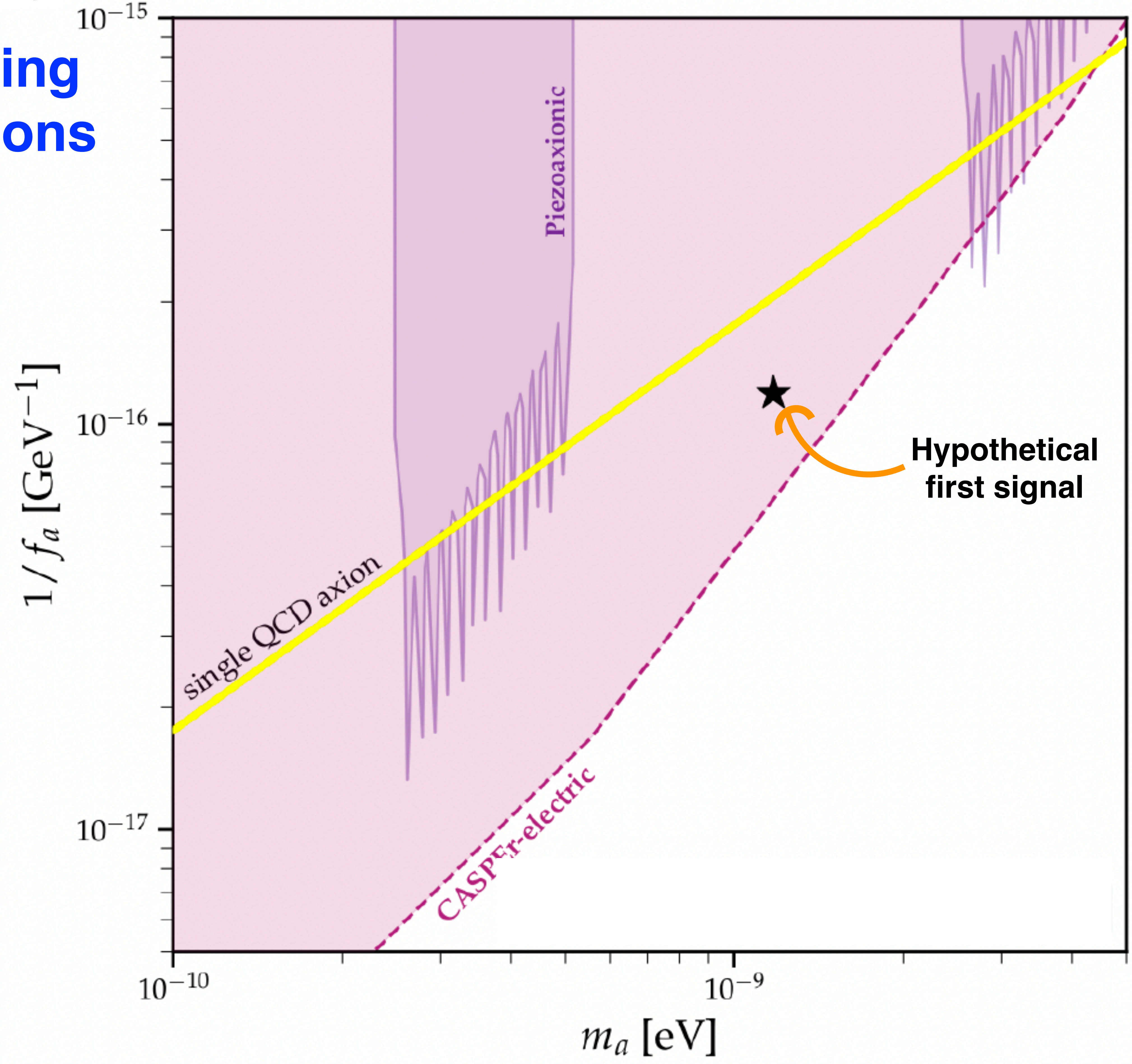
$\{m_a, 1/f_a\}$

coupling to gluons



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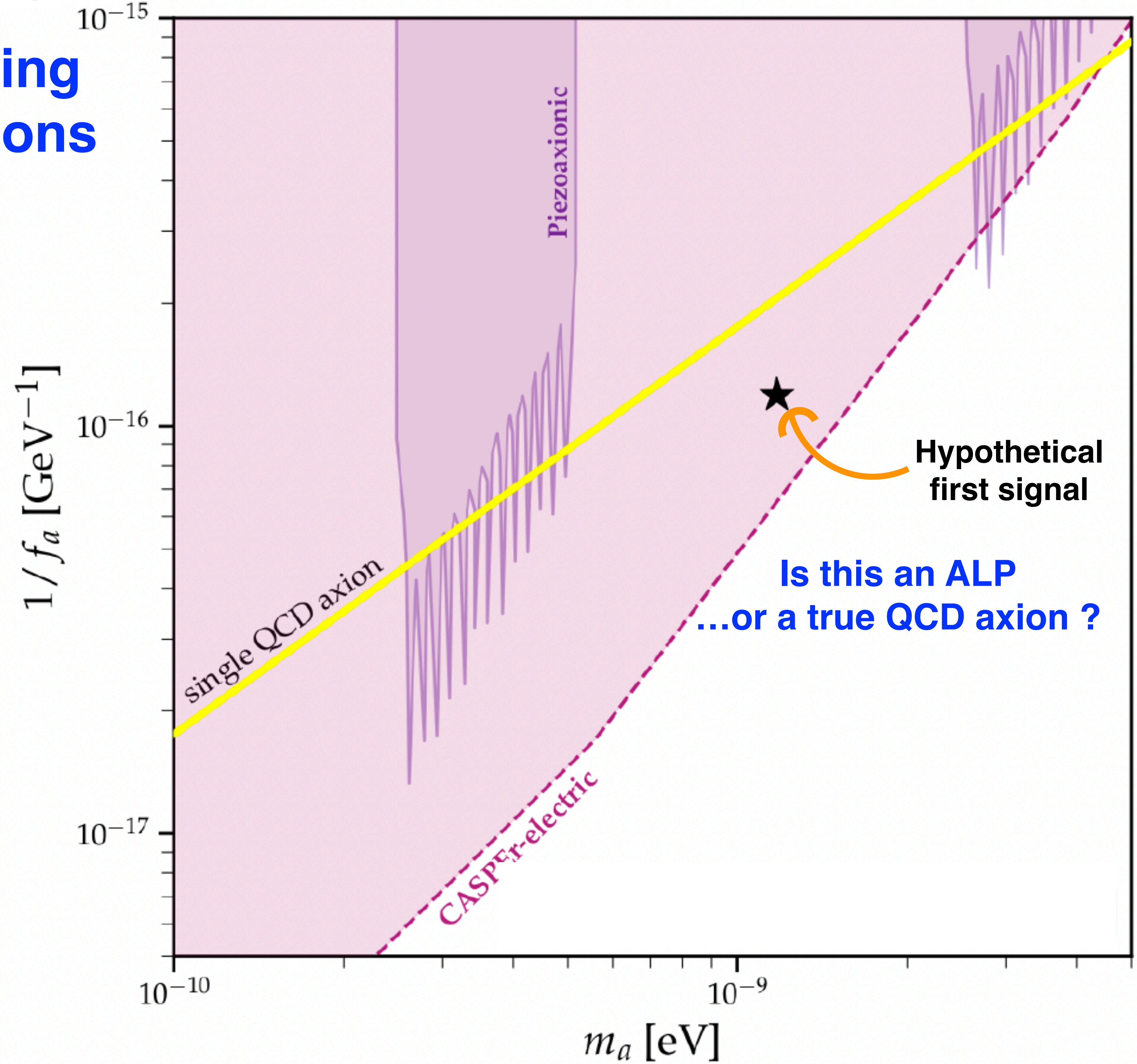

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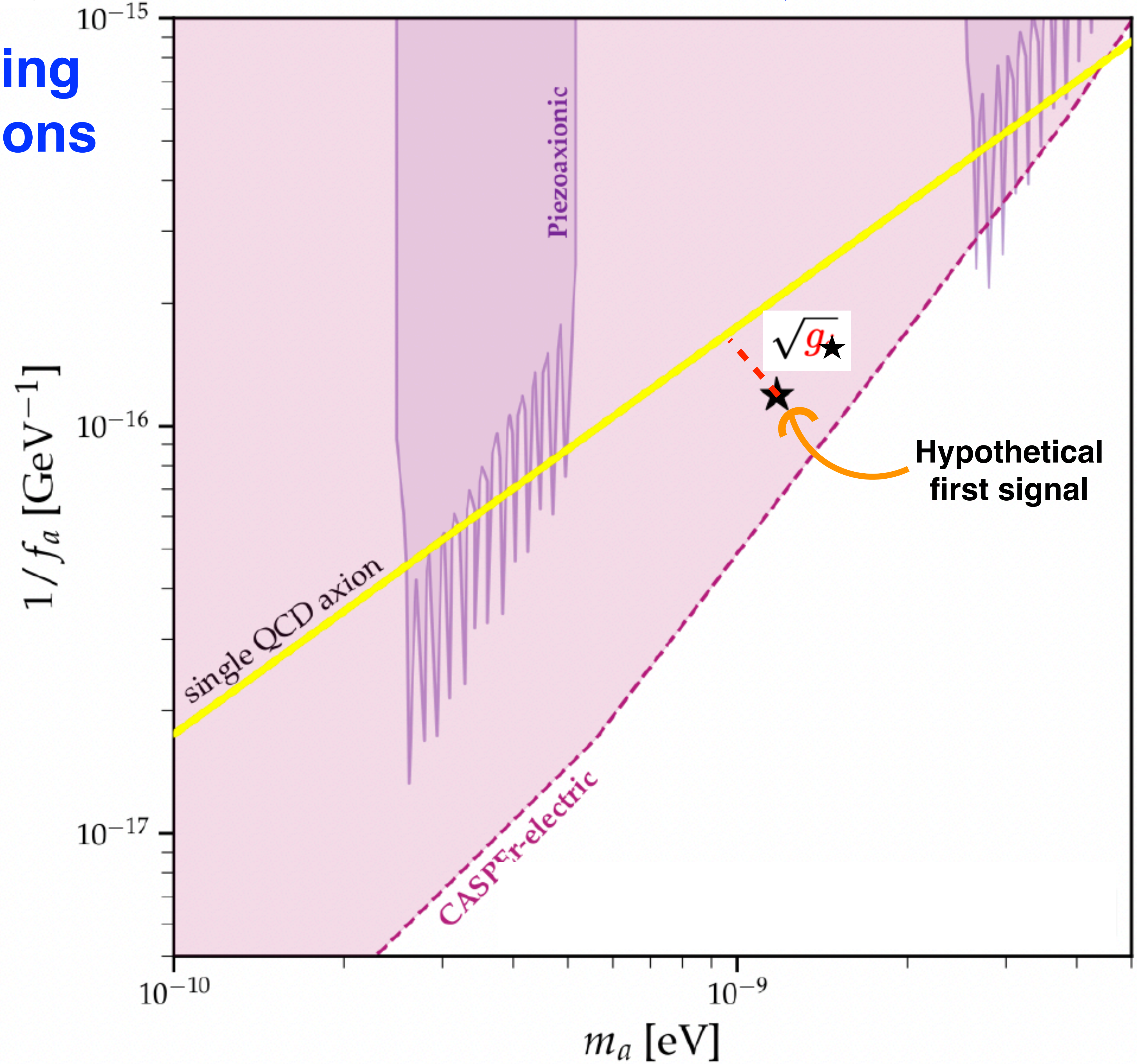
Distance to standard QCD band

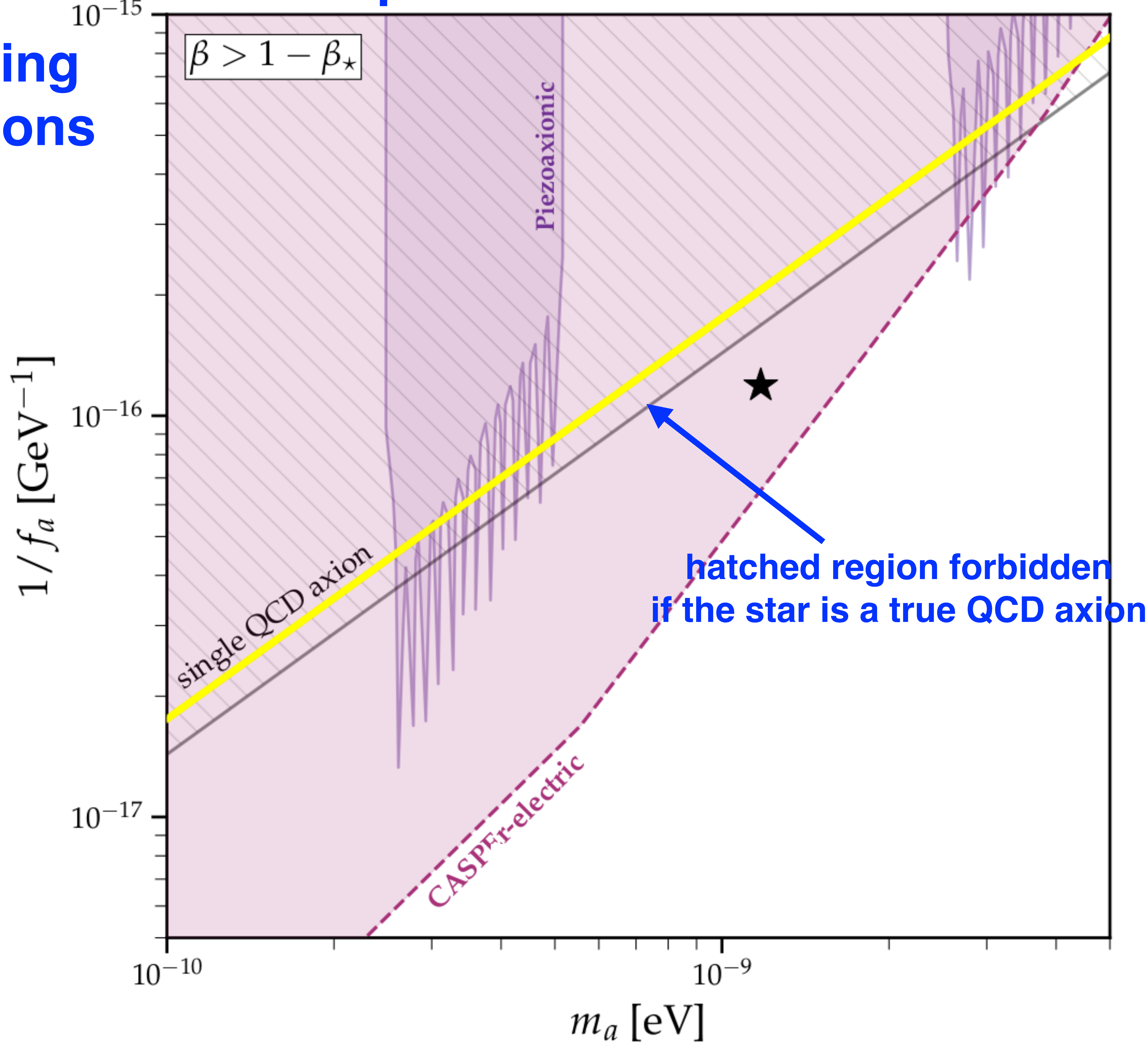
coupling to gluons



$\{m_a, 1/f_a\}$

Multiple QCD axion for N=2

coupling to gluons



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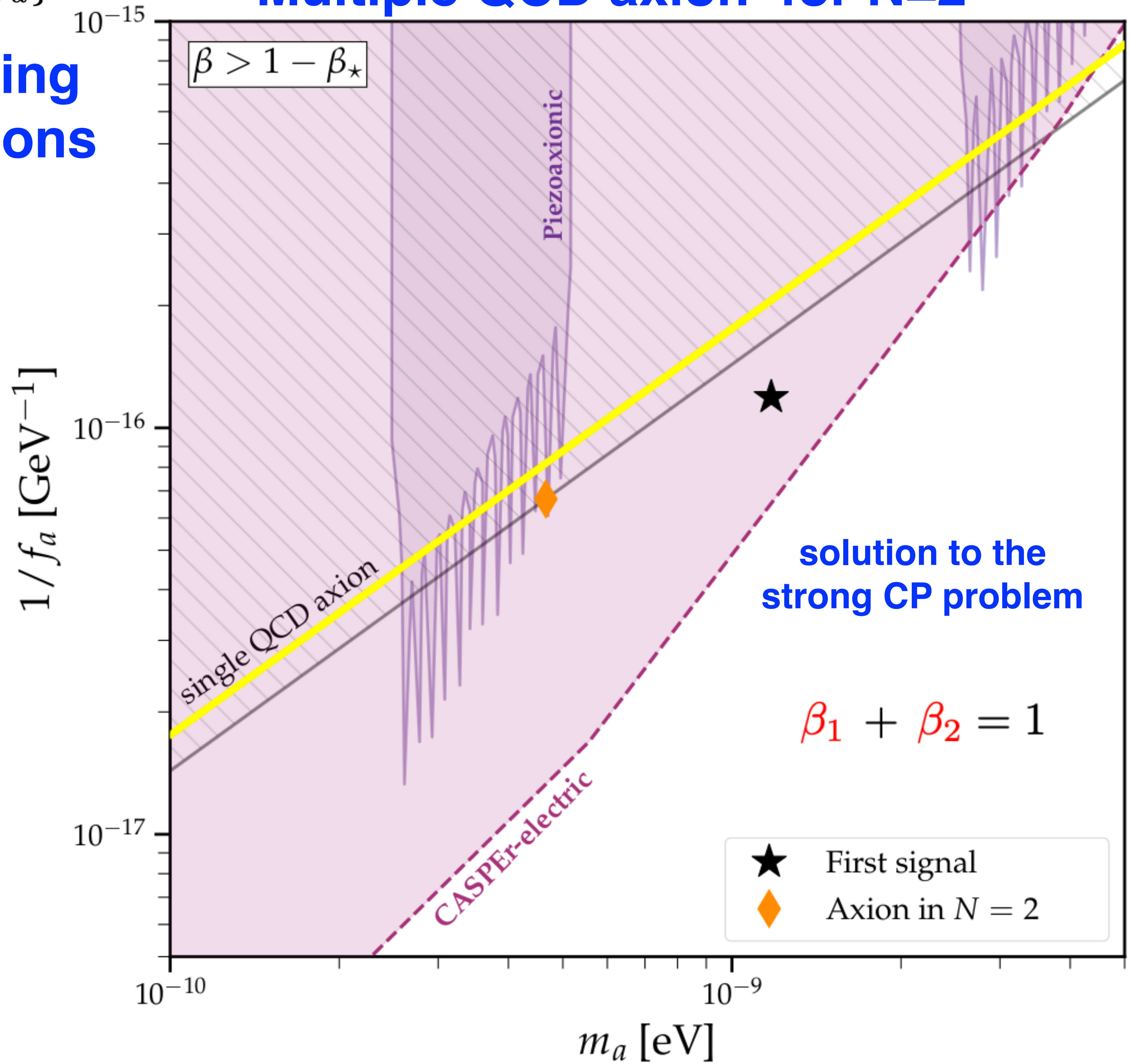

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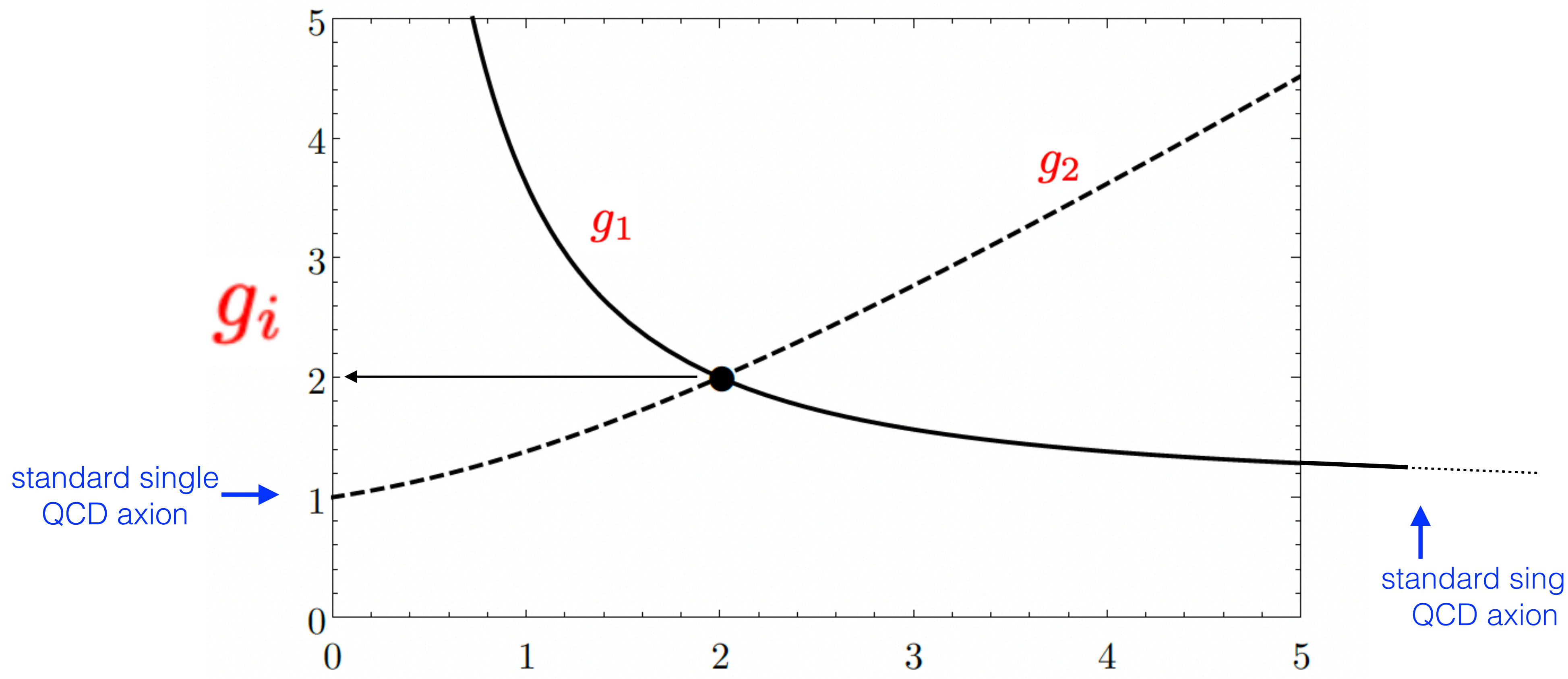

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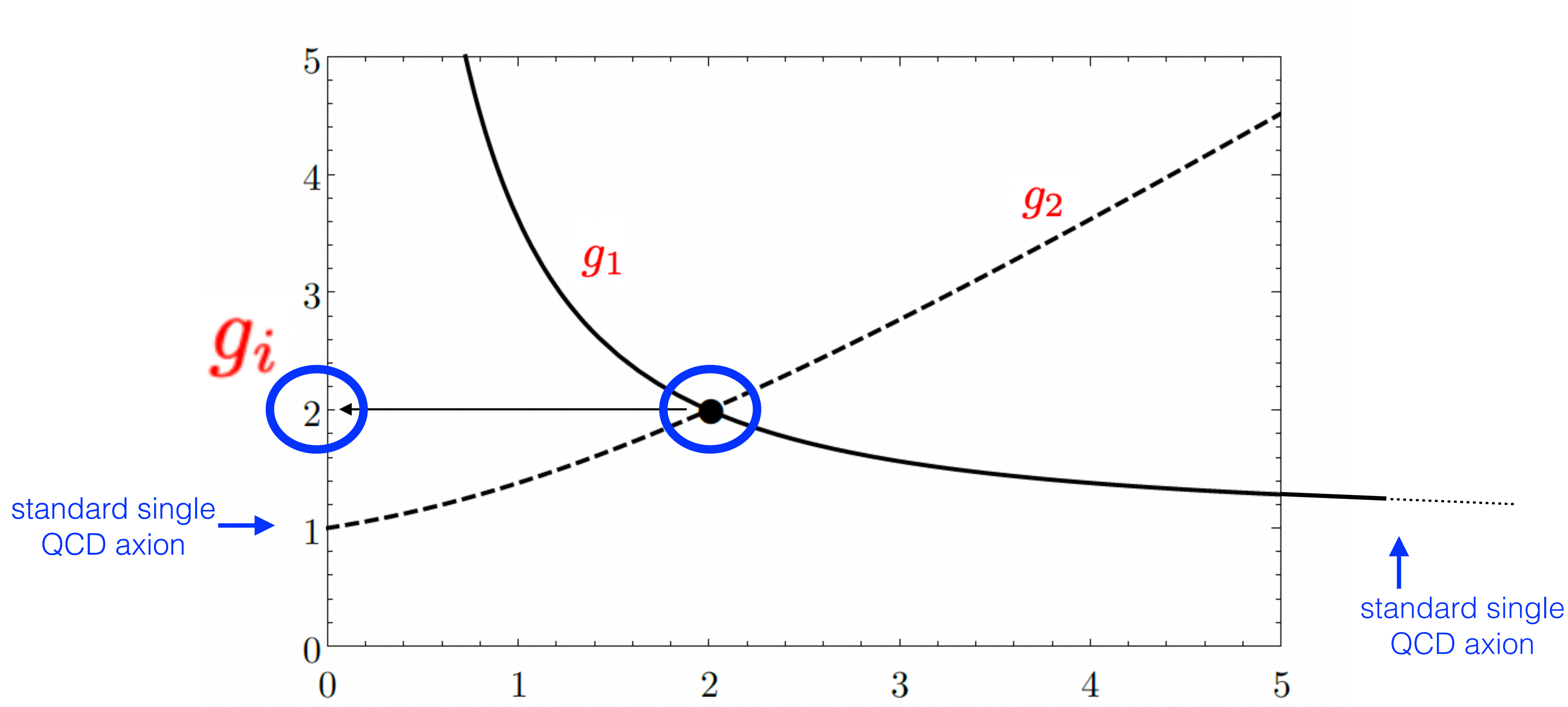

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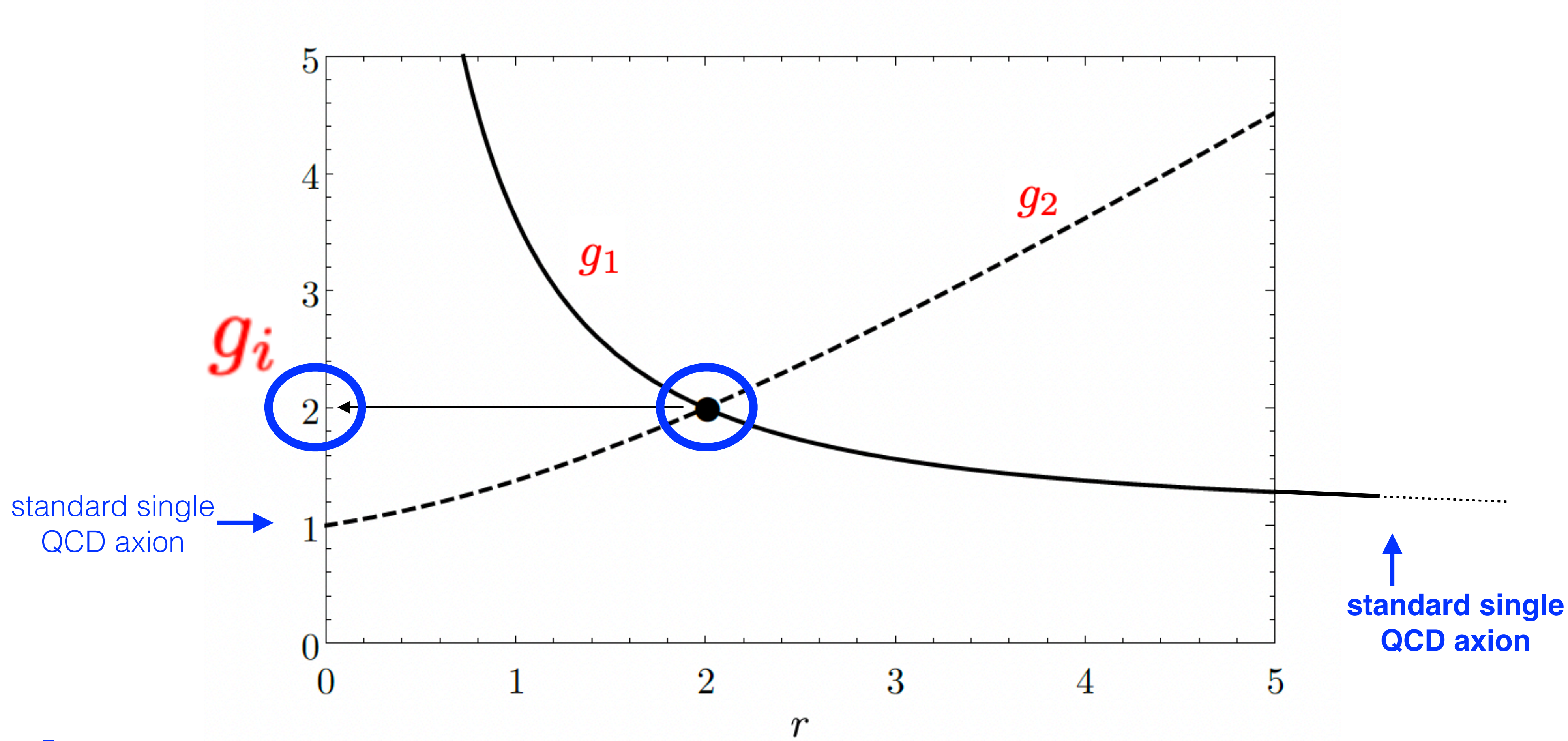
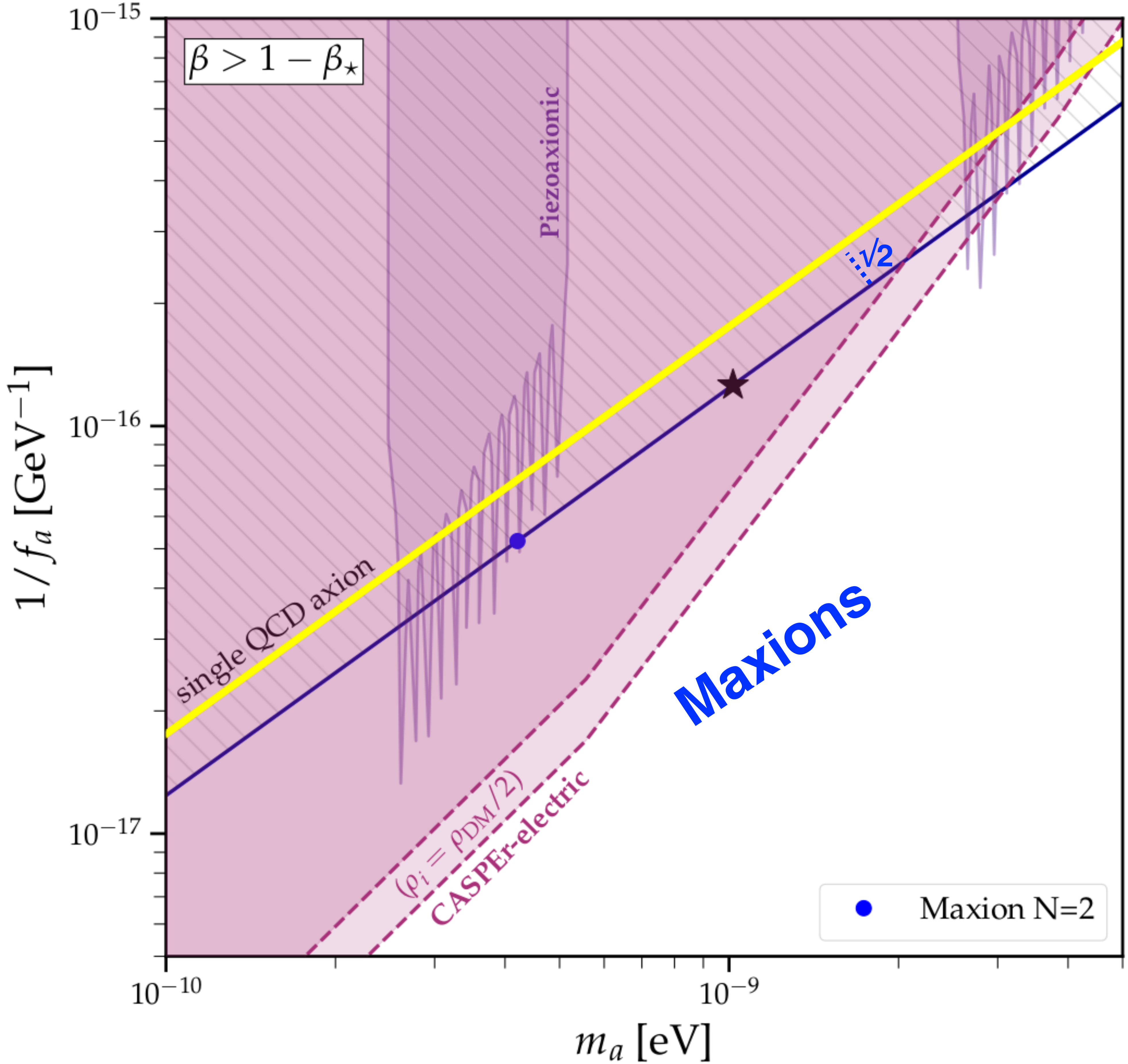


Maxions (maximally deviated QCD axions): the maximal distance possible for the closest axion eigenstate is… **2**, and $g_1 = g_2 = 2$

N=2 QCD MAXION

$\{m_a, 1/f_a\}$



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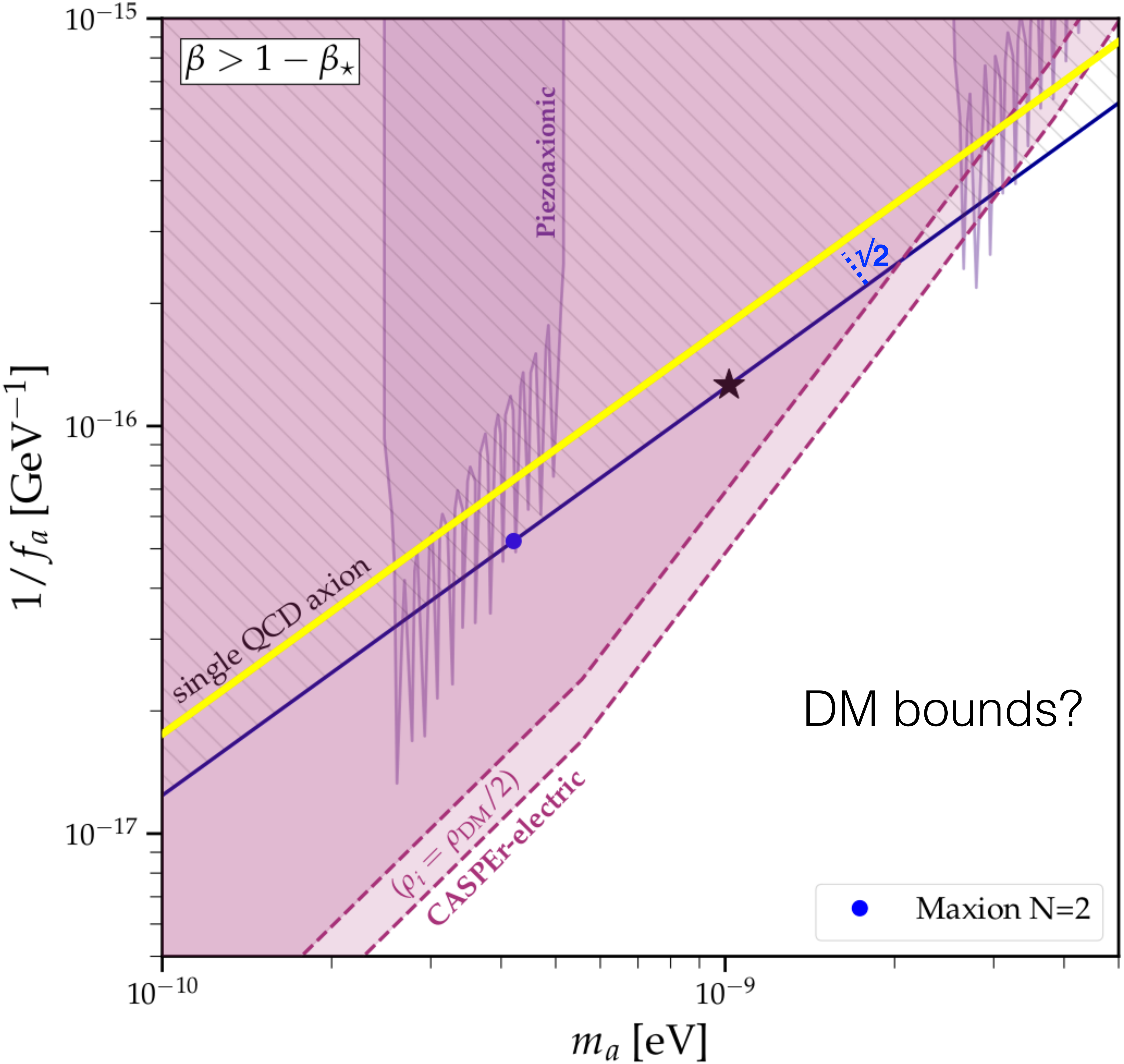
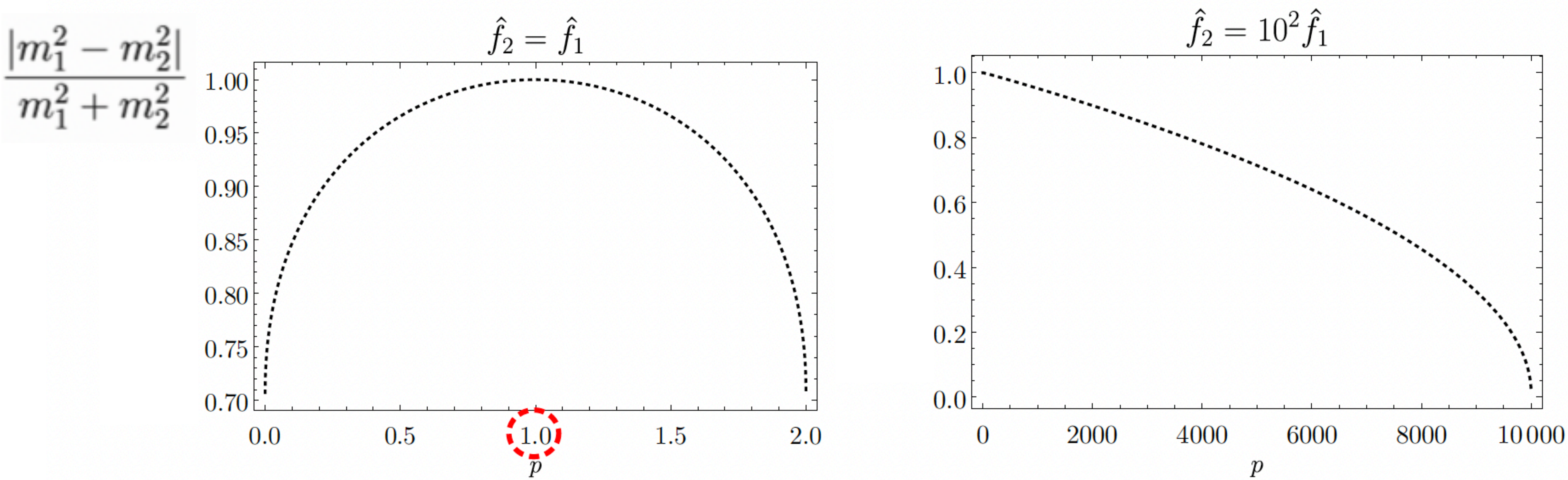


General MAXION condition for N=2

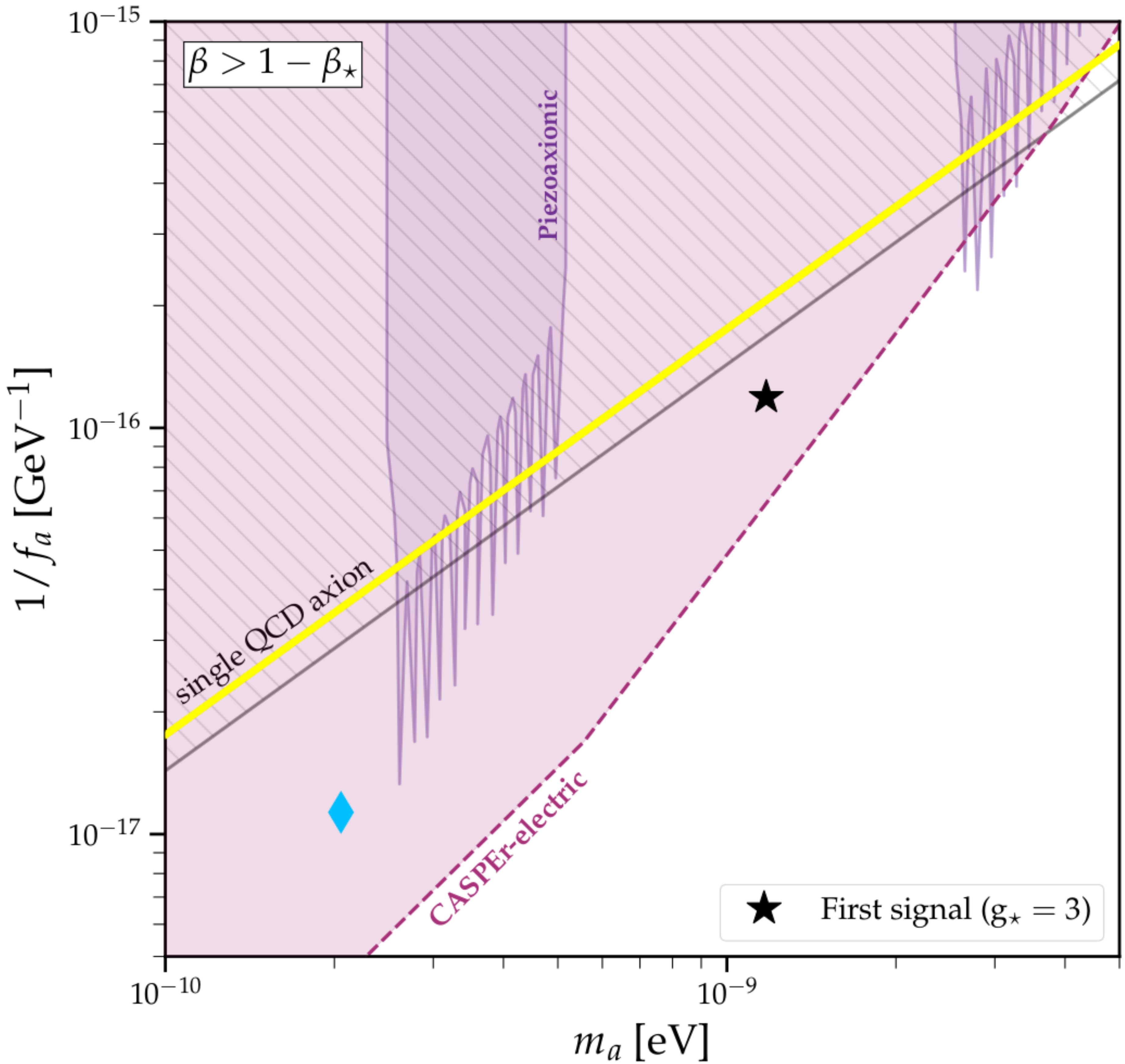
In general, N(N+1)/2 maxion families

$$\mathbf{M}^2_{N=2} = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 2-p & 1+\sqrt{p(2-p)} \\ 1+\sqrt{p(2-p)} & 1+p \end{pmatrix}$$

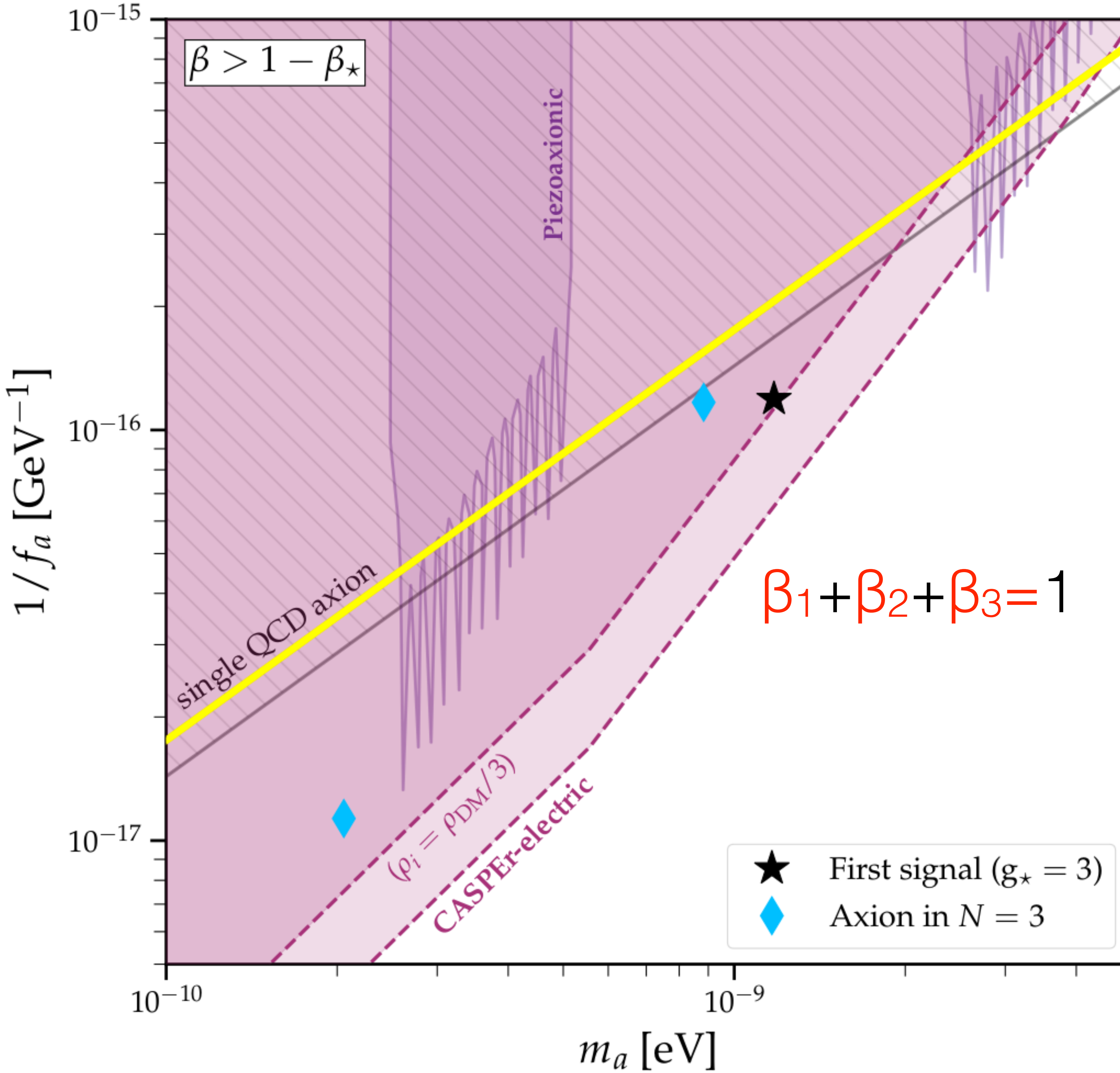


Limiting case: Massless state has no mixing with gluons, the heavy one with mass $\sim 4\frac{\chi_{\rm QCD}}{\hat{f}^2}$

and what if $\beta_1+\beta_2<1$



a multiple QCD axion for N=3



General potential for arbitrary N scalars

Exact results and sum rules

Multiple QCD axion for any N

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N)$$

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V_B^{\mathrm{R}}(\underbrace{a_{G\widetilde{G}}, \ldots}_{N})$$

$$\frac{1}{F^2} = \sum_{k=1}^{N}\frac{1}{\hat{f}_k^2}$$

Multiple QCD axion for any N

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Multiple QCD axion for any N and arbitrary potential

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\frac{a_{G\widetilde{G}}}{F} G\widetilde{G} - V_B^{\rm R}(\underbrace{a_{G\widetilde{G}}, \ldots}_{N})$$

$$\mathbf{M}^2 = \mathbf{M}^2_{\rm QCD} + \mathbf{M}^2_{\rm ext} = \frac{\chi_{\rm QCD}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\rm QCD}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix}$$

eigenvalues: $$m_i^2 = \beta_i\, m_i^2 + \langle a_i | \mathbf{M}^2_{\rm ext} | a_i \rangle$$

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β_i is the fraction of the total m_i due to QCD: the *QCD-axionness*

$$\beta_i \equiv \frac{1}{g_i}$$

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N eigenvectors a_i *coupled to* $G\widetilde{G}$

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i}G\widetilde{G} \qquad \frac{1}{F^2} = \sum_{i=1}^{N}\frac{1}{f_i^2} \qquad g_i \equiv \frac{m_i f_i}{m_a f_a\big|_{\text{single QCD axion}}} \geq 1$$

1 PQ field —> multiple QCD axions, displaced to the right

Multiple QCD axion for any N and arbitrary potential

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\frac{a_{G\widetilde{G}}}{F}G\widetilde{G} - V_B^{\mathrm{R}}(\underbrace{a_{G\widetilde{G}}, \dots}_{N})$$

$$\mathbf{M}^2 = \mathbf{M}^2_{\mathrm{QCD}} + \mathbf{M}^2_{\mathrm{ext}} = \frac{\chi_{\mathrm{QCD}}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix}$$

eigenvalues: $m_i^2 = \beta_i\, m_i^2 + \langle a_i|\mathbf{M}^2_{\mathrm{ext}}|a_i\rangle$

N eigenvectors a_i *coupled to* $G\widetilde{G}$

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi}\frac{a_i}{f_i}G\widetilde{G} \qquad \frac{1}{F^2} = \sum_{i=1}^{N}\frac{1}{f_i^2} \qquad g_i \equiv \frac{m_i f_i}{m_a f_a|_{\text{single QCD axion}}} \geq 1$$

1 PQ field —> multiple QCD axions, displaced to the right

Several exact results follow from the eigenvalue-eigenvector theorem

Jacobi…..

PETER B. DENTON, STEPHEN J. PARKE, TERENCE TAO, AND XINING ZHANG

ABSTRACT. If A is an $n \times n$ Hermitian matrix with eigenvalues $\lambda_1(A), \ldots, \lambda_n(A)$ and $i, j = 1, \ldots, n$, then the j^{th} component $v_{i,j}$ of a unit eigenvector v_i associated to the eigenvalue $\lambda_i(A)$ is related to the eigenvalues $\lambda_1(M_j), \ldots, \lambda_{n-1}(M_j)$ of the minor M_j of A formed by removing the j^{th} row and column by the formula

$$|v_{i,j}|^2 \prod_{k=1; k \neq i}^{n} (\lambda_i(A) - \lambda_k(A)) = \prod_{k=1}^{n-1} (\lambda_i(A) - \lambda_k(M_j)) \, .$$

We refer to this identity as the *eigenvector-eigenvalue identity*

https://arxiv.org/pdf/1908.03795.pdf

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{a_i}{f_i} G\widetilde{G} \qquad \text{with} \qquad \frac{1}{f_i} = \frac{\langle a_{G\widetilde{G}} | a_i \rangle}{F} \equiv \frac{v_{i1}}{F} \implies \sum_{i=1}^{N} \frac{1}{f_i^2} = \frac{1}{F^2}$$

Peccei-Quinn condition for arbitrary M

$$\lim_{\chi_{\rm QCD}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_{\rm ext} = 0 .$$

$$\frac{1}{F^2} = \sum_{i=1}^{N} \frac{1}{f_i^2}$$

$$\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{f_\pi^2 m_\pi^2}{F^2} \frac{m_u m_d}{(m_u + m_d)^2}$$

PQ-invariance condition for arbitrary potential

Peccei-Quinn condition for arbitrary M

$$\lim_{\chi_{\rm QCD}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_{\rm ext} = 0 \,.$$

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$$\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{f_\pi^2 m_\pi^2}{F^2} \frac{m_u m_d}{(m_u + m_d)^2}$$

PQ-invariance condition for arbitrary potential

$$\exists\, U(1)_{PQ} \implies \sum_{i=1}^{N} \frac{1}{g_i} = 1$$

PQ sum-rule

or equivalently

$$\exists\, U(1)_{PQ} \implies \sum_{i=1}^{N} \beta_i = 1\,; \quad \beta_i = \frac{1}{g_i}$$

QCD-axionness is shared

An intuitive view of the *QCD-axionness*

$$\beta_i \equiv \frac{1}{g_i}$$

eigenstate

field (or combination of) that has shift symmetry

field (or combination of) that couples to $G\widetilde{G}$

$$\beta_i = \frac{1}{g_i} = \frac{\langle a_{\rm PQ} \mid a_i \rangle \langle a_i \mid a_{G\widetilde{G}} \rangle}{\langle a_{\rm PQ} \mid \hat{a}_{G\widetilde{G}} \rangle}$$

An intuitive view of the *QCD-axionness*

$$\beta_i \equiv \frac{1}{g_i}$$

eigenstate

field (or combination of) that has shift symmetry

field (or combination of) that couples to $G\widetilde{G}$

$$\beta_i = \frac{1}{g_i} = \frac{\langle a_{\mathrm{PQ}} \mid a_i \rangle \langle a_i \mid a_{G\widetilde{G}} \rangle}{\langle a_{\mathrm{PQ}} \mid \hat{a}_{G\widetilde{G}} \rangle}$$

a_{PQ}

* e.g. in the N=2 toy model: $\mathcal{L}_{N=2} = \left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \theta \right) G\widetilde{G} - \frac{1}{2}\hat{m}_2^2\, \hat{a}_2^2$

$a_{G\widetilde{G}}$

An intuitive view of the *QCD-axionness* $\beta_i \equiv \frac{1}{g_i}$

$|a_i\rangle$: eigenstates

$|a_{G\widetilde{G}}\rangle$: field(s) that couple to $G\widetilde{G}$

$|a_{PQ}\rangle$: field(s) that maintain shift invariance

then $$\beta_i = \frac{1}{g_i} = \frac{\langle a_{\rm PQ} \mid a_i\rangle \langle a_i \mid a_{G\widetilde{G}}\rangle}{\langle a_{\rm PQ} \mid a_{G\widetilde{G}}\rangle}$$

and it can be proven that:

$$1 = \frac{\langle a_{\rm PQ} \mid a_{G\widetilde{G}}\rangle}{\langle a_{\rm PQ} \mid a_{G\widetilde{G}}\rangle} = \sum_i^N \frac{\langle a_{\rm PQ} \mid a_i\rangle \langle a_i \mid a_{G\widetilde{G}}\rangle}{\langle a_{\rm PQ} \mid a_{G\widetilde{G}}\rangle} = \sum_i^N \frac{\chi_{\rm QCD}}{m_i^2 f_i^2} = \sum_i^N \frac{1}{g_i}$$

Maxions (maximally deviated QCD axions): N relations

$$p_{\mathbf{M}^2}(\lambda) \equiv \sum_{k=0}^{N} c_k^{\mathbf{M}} \lambda^k$$

$$\boxed{c_k^{\mathbf{M}} = -N \, \frac{\chi_{\rm QCD}}{F^2(N-k)} \, c_k^{\mathbf{M}_1}}$$

Maxion conditions

Maxions (maximally deviated QCD axions):

$$\max\left\{\min_i\{g_i\}\right\} = N \qquad \Longrightarrow \boxed{g_i = N,}\ \forall\, i$$

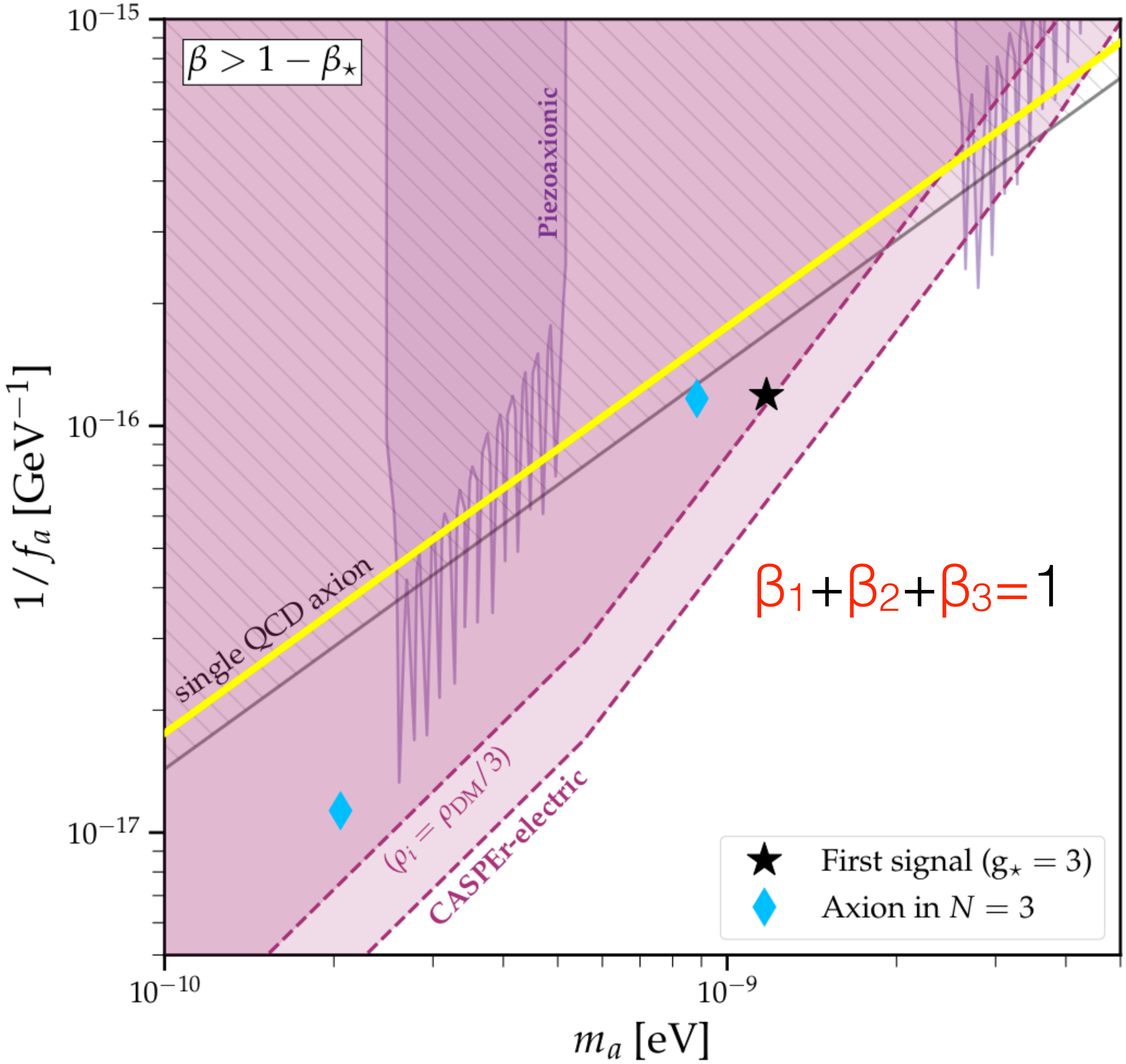
$$\boxed{\mathrm{Tr}[\mathbf{M}^2] = \sum_i m_i^2 = N\frac{\chi_{\mathrm{QCD}}}{F^2}}$$

Examples of N>2 axions

and Maxions

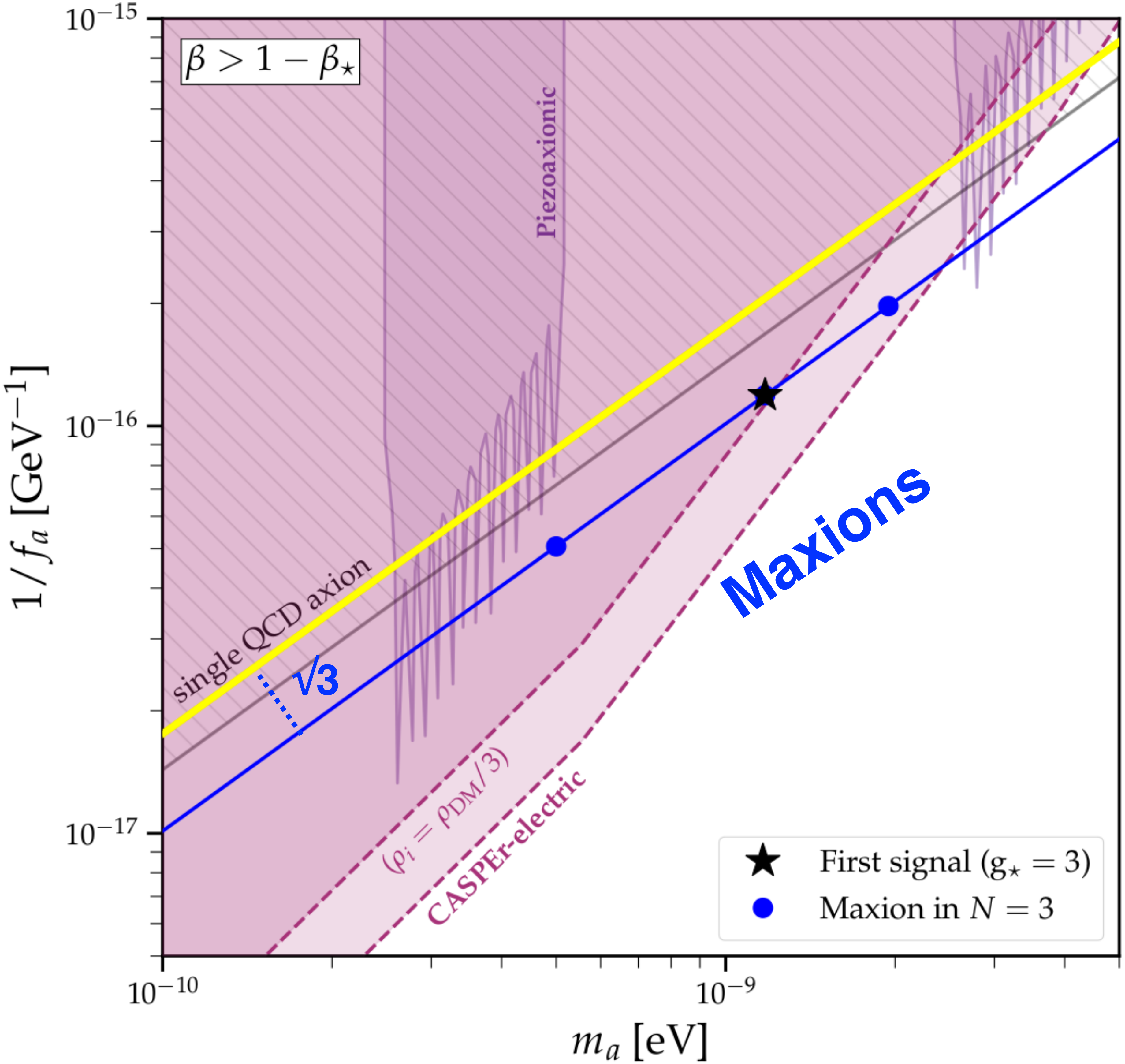
a multiple QCD axion for N=3

$\{m_a, 1/f_a\}$



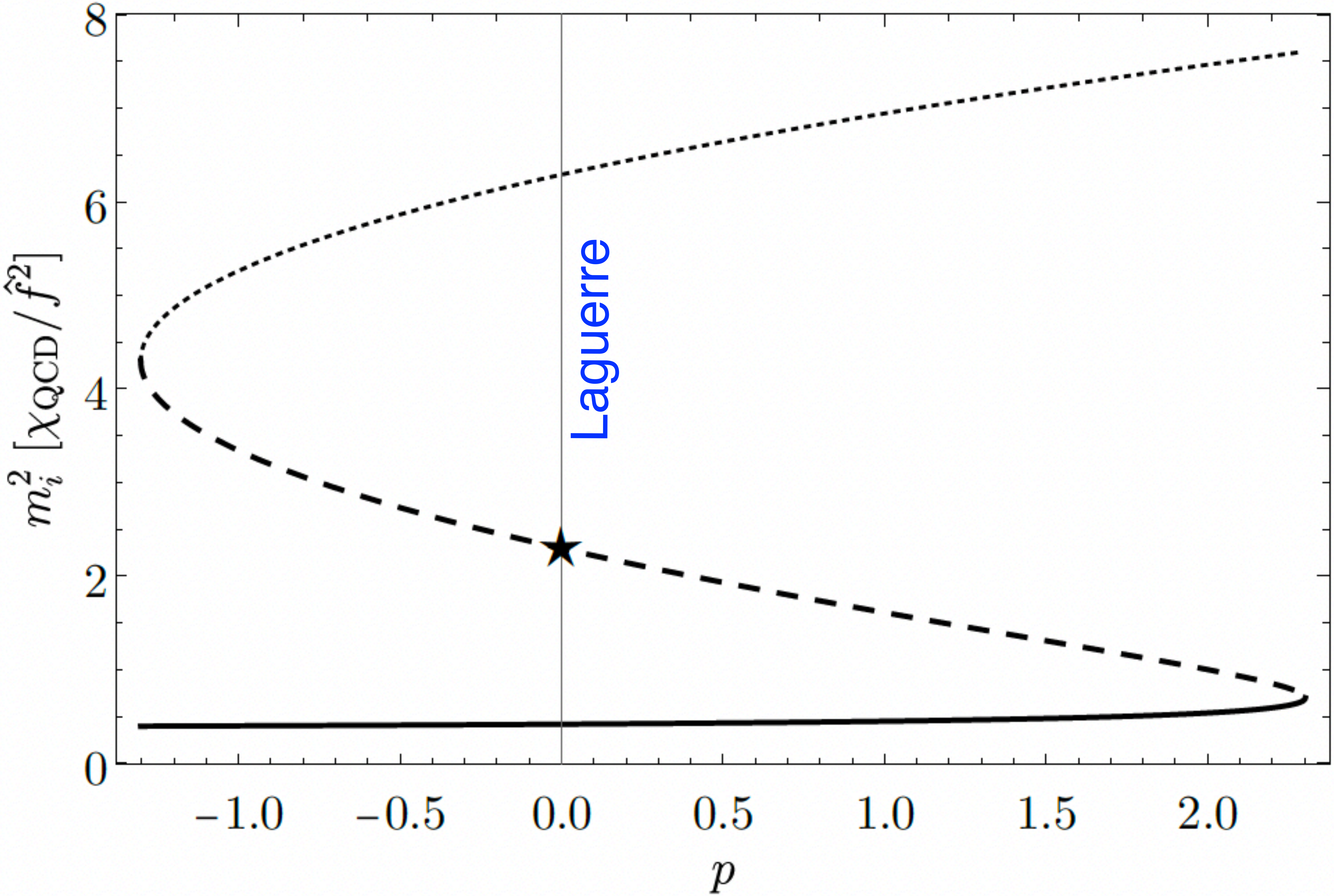
a N=3 Maxion

$\{m_a, 1/f_a\}$



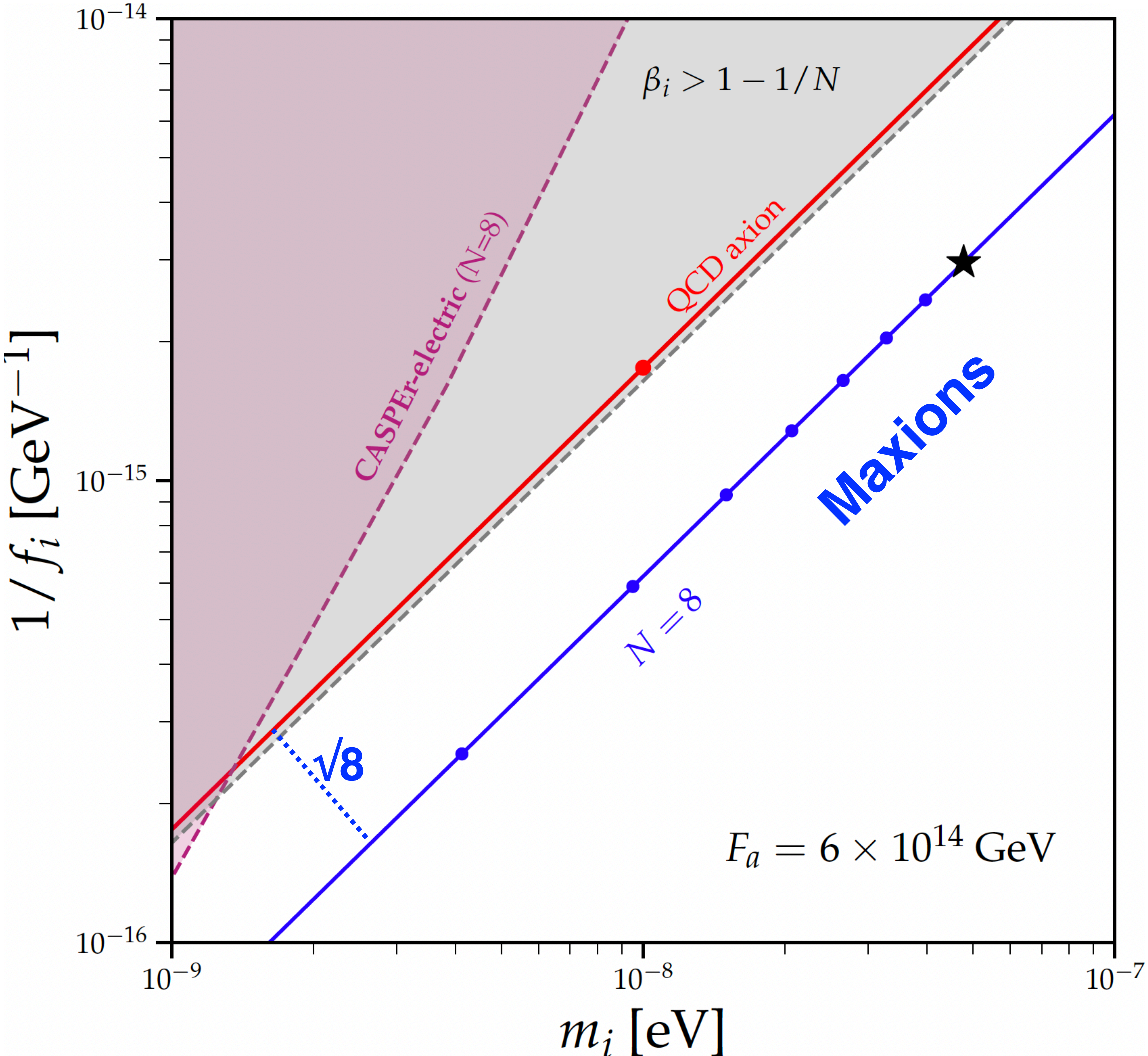
Potential=
Laguerre
matrices

Maxions

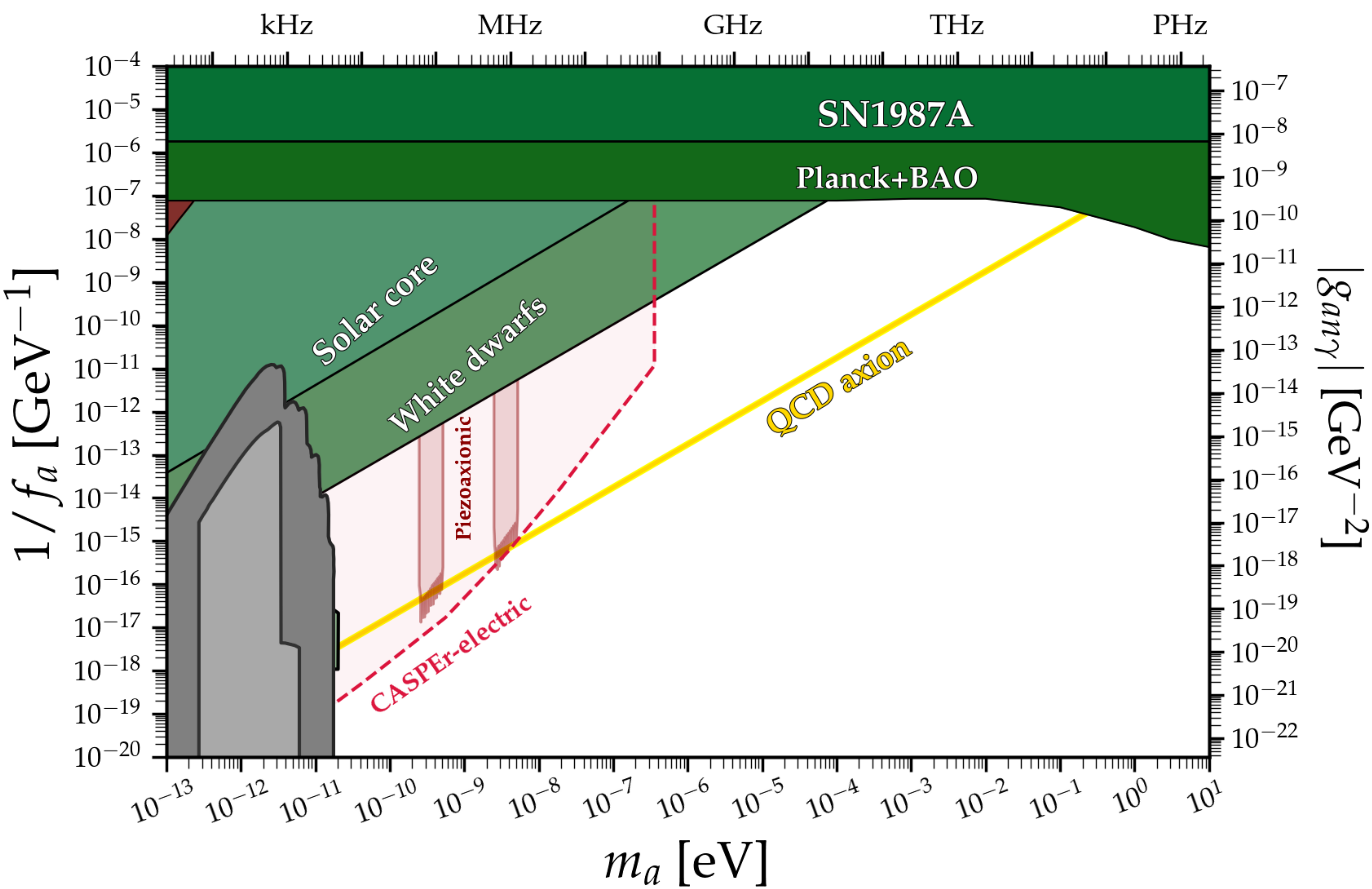


a N=8 Maxion

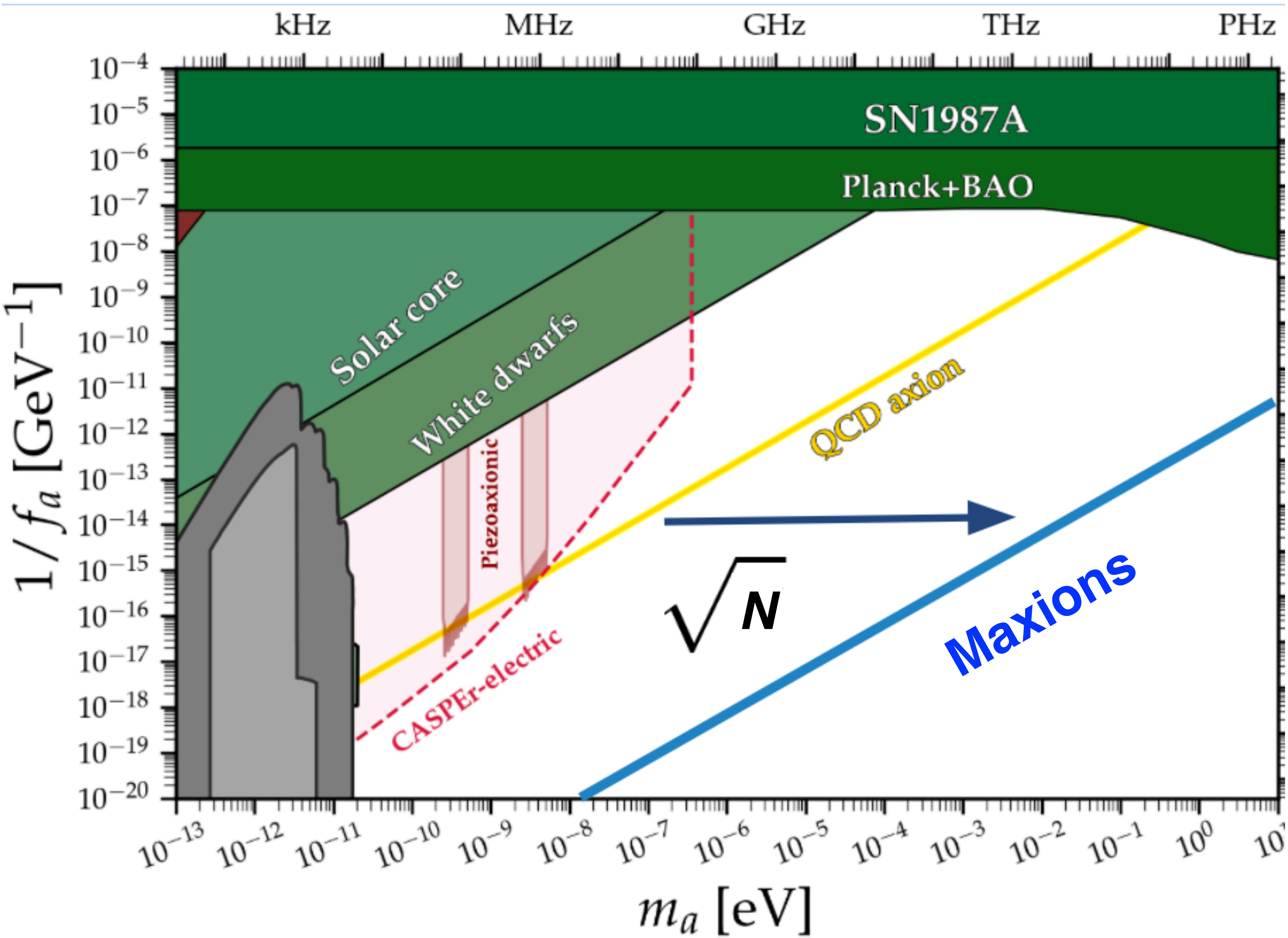
$\{m_a, 1/f_a\}$



$\{m_a, 1/f_a\}$: coupling to gluons



$\{m_a, 1/f_a\}$: **coupling to gluons**



Coupling to photons

Coupling to photons for the multiple QCD axion

Standard single QCD axion: $$\mathcal{L} \supset \frac{\alpha_{em}}{2\pi}\left[\frac{E}{\mathcal{N}} - 1.92\right]\frac{a}{f_a}F\widetilde{F}$$

↑

model-dependent

Coupling to photons for the multiple QCD axion

Standard single QCD axion:

$$\mathcal{L} \supset \frac{\alpha_{em}}{2\pi}\left[\frac{E}{\mathcal{N}} - 1.92\right]\frac{a}{f_a}F\widetilde{F}$$

Multiple QCD axion:

$$\mathcal{L} \supset \frac{\alpha_{em}}{2\pi}\sum_i\left[\frac{E_i}{\mathcal{N}_i} - 1.92\right]\frac{a_i}{f_i}F\widetilde{F}$$

↑ model-dependent

Coupling to photons for the multiple QCD axion

Standard single QCD axion: $\mathcal{L} \supset \frac{\alpha_{em}}{2\pi}\left[\frac{E}{\mathcal{N}} - 1.92\right]\frac{a}{f_a}F\widetilde{F}$

Multiple QCD axion: $\mathcal{L} \supset \frac{\alpha_{em}}{2\pi}\sum_i\left[\frac{E_i}{\mathcal{N}_i} - 1.92\right]\frac{a_i}{f_i}F\widetilde{F}$

↑ model-dependent

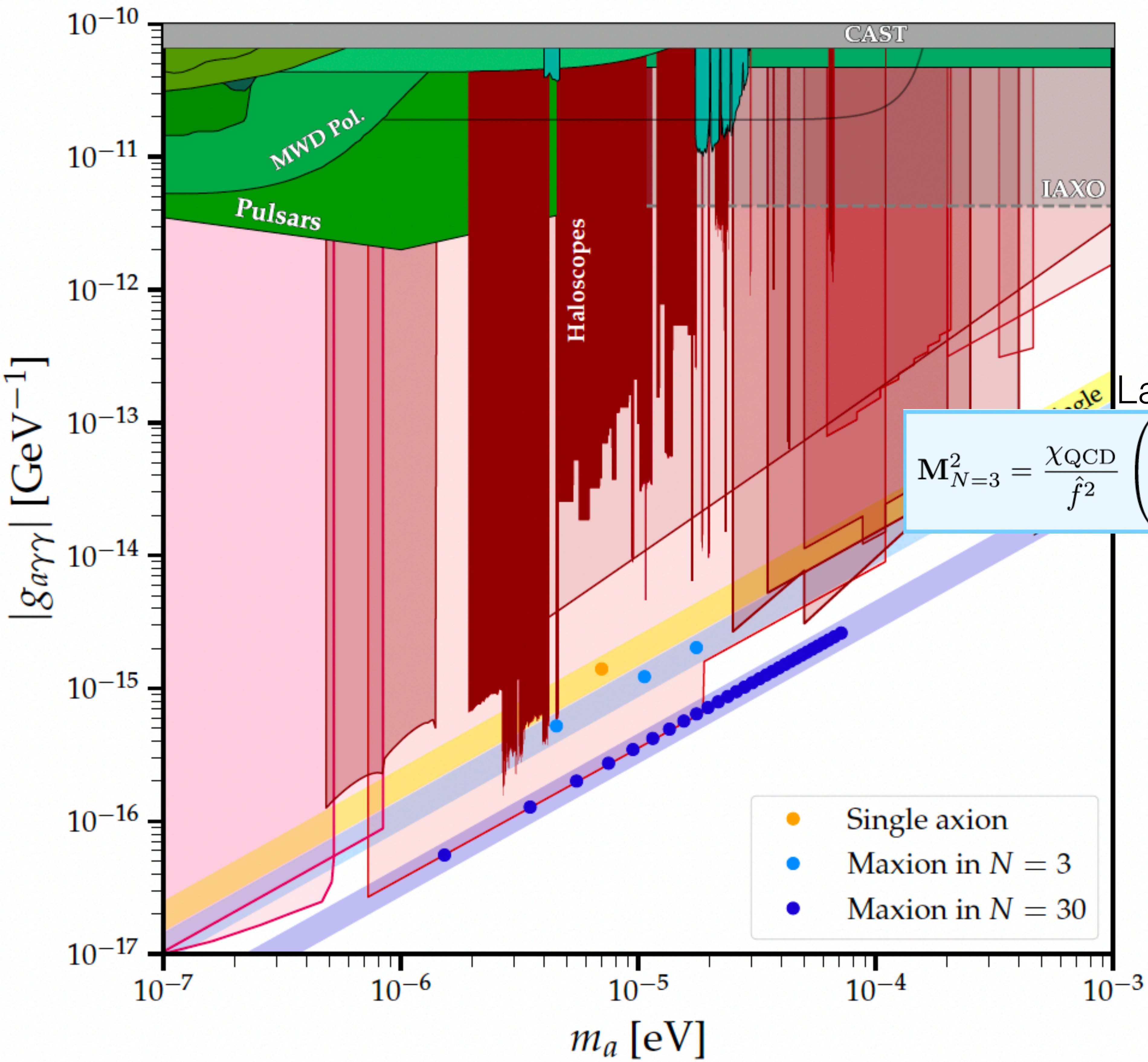
if $E_i\,/\,\mathcal{N}_i$ universal: $\mathcal{L} \supset \frac{\alpha_{em}}{2\pi}\left[\frac{E}{\mathcal{N}} - 1.92\right]\sum_i\frac{a_i}{f_i}F\widetilde{F}$

$$\frac{{m_i}^2}{{g_{a_i\gamma\gamma}}^2} = \frac{m_a^2}{g_{a\gamma\gamma}^2}\Bigg|_{\text{single QCD axion}} \times g_i$$

$$\frac{(2\pi)^2\chi_{\text{QCD}}}{\alpha_{em}^2}\left[\frac{E}{\mathcal{N}} - 1.92\right]^{-2}\sum_{i=1}^{N}\frac{g_{a_i\gamma\gamma}}{{m_i}^2} = 1$$

sum-rule

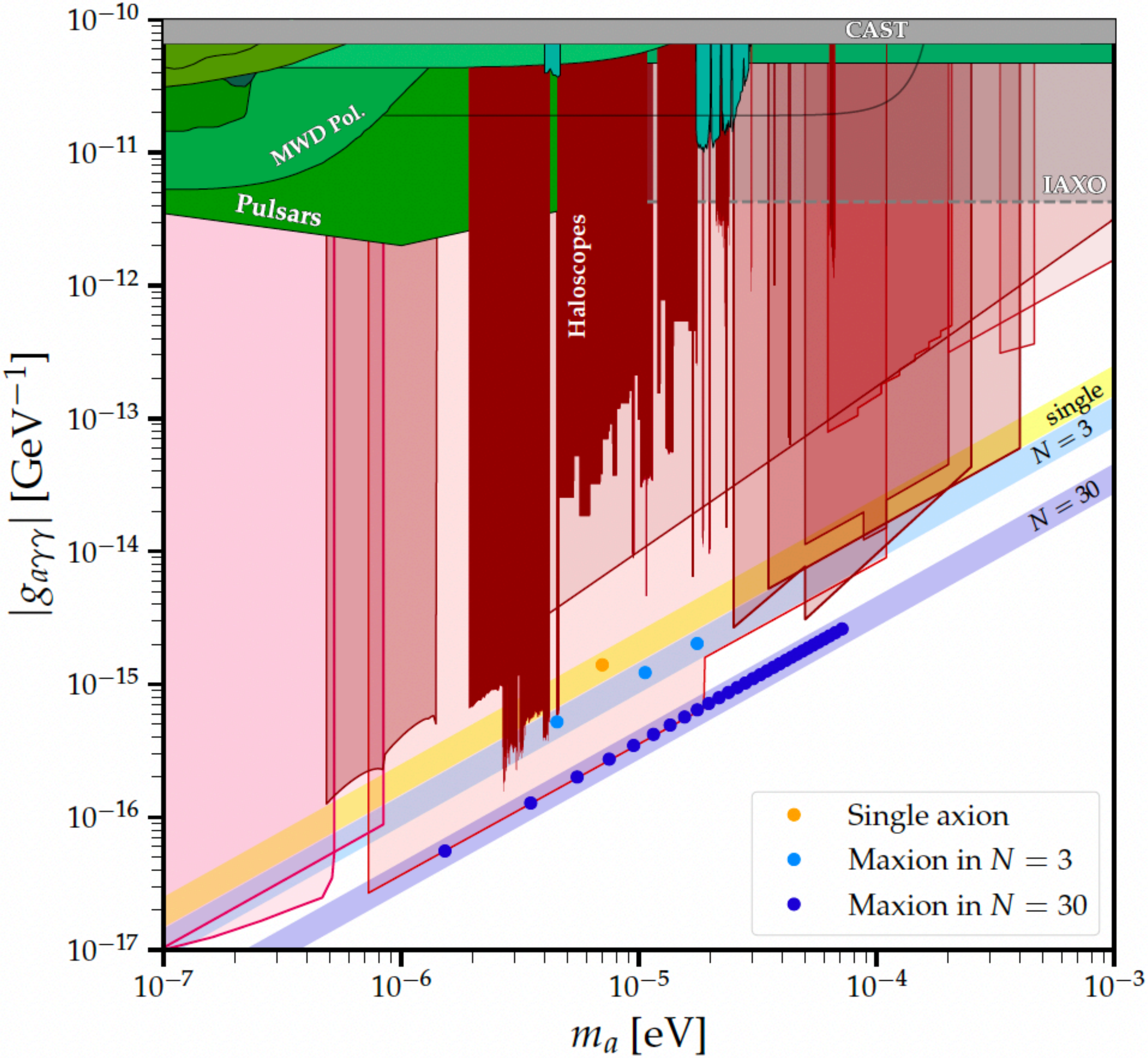
Coupling to photons for Maxions



Laguerre maxions

$$\mathbf{M}^2_{N=3} = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4-\sqrt{3} & 1 \\ 1 & 1 & 4+\sqrt{3} \end{pmatrix}$$

Coupling to photons for Maxions



UV completions: one example

a simple KSVZ with 2 true QCD axions

$$\Psi_{1,2} \sim (3,1,0) \qquad S_{1,2} \sim (1,1,0)$$

$$\mathcal{L}_{\rm UV} = |\partial_\mu S_1|^2 + |\partial_\mu S_2|^2 + \overline{\Psi}_1 \mathrm{i} \not{D} \Psi_1 + \overline{\Psi}_2 \mathrm{i} \not{D} \Psi_2 - \left[y_1 \overline{\Psi}_1 \Psi_1 S_1 + y_2 \overline{\Psi}_2 \Psi_2 S_2 + \text{h.c.} \right] - V(S_{1,2})$$

$$S_i = \frac{1}{\sqrt{2}} \left(\hat{f}_i + \rho_i \right) e^{\mathrm{i} \hat{a}_i / \hat{f}_i}$$

for instance $V(S_{1,2}) \sim S_2^4$ reduces the system to just one PQ

and gives precisely the first N=2 mass matrix I showed you !

a simple KSVZ with 2 true QCD axions

$$\Psi_{1,2} \sim (3,1,0) \qquad S_{1,2} \sim (1,1,0)$$

$$\mathcal{L}_{\rm UV} = |\partial_\mu S_1|^2 + |\partial_\mu S_2|^2 + \overline{\Psi}_1 i \not{D} \Psi_1 + \overline{\Psi}_2 i \not{D} \Psi_2 - \left[y_1 \overline{\Psi}_1 \Psi_1 S_1 + y_2 \overline{\Psi}_2 \Psi_2 S_2 + \text{h.c.} \right] - V(S_{1,2})$$

$$S_i = \frac{1}{\sqrt{2}} \left(\hat{f}_i + \rho_i \right) e^{i \hat{a}_i / \hat{f}_i}$$

for instance $V(S_{1,2}) = \lambda S_1^3 S_2 + \text{h.c.}$ reduces the system to just one PQ

$$V_{\rm eff} = \frac{1}{2} \chi_{\rm QCD} \left(\frac{\hat{a}_1 + \hat{a}_2}{\hat{f}} - \bar{\theta} \right)^2 + \frac{\lambda}{4} \hat{f}^4 \left(\frac{3\hat{a}_1 + \hat{a}_2}{\hat{f}} \right)^2$$

$(\hat{f}_1 = \hat{f}_2 = \hat{f})$

$$\mathbf{M}^2 = \frac{\chi_{\rm QCD}}{F^2} \begin{pmatrix} 2 + 8r & -4r \\ -4r & 2r \end{pmatrix} \qquad 1/F^2 = 2/\hat{f}^2$$

Maxion solution for r=1/5

a simple KSVZ with 2 true QCD axions

$$\Psi_{1,2} \sim (3,1,0) \qquad S_{1,2} \sim (1,1,0)$$

$$\mathcal{L}_{\rm UV} = |\partial_\mu S_1|^2 + |\partial_\mu S_2|^2 + \overline{\Psi}_1 i \not{D} \Psi_1 + \overline{\Psi}_2 i \not{D} \Psi_2 - \left[y_1 \overline{\Psi}_1 \Psi_1 S_1 + y_2 \overline{\Psi}_2 \Psi_2 S_2 + \text{h.c.} \right] - V(S_{1,2})$$

$$S_i = \frac{1}{\sqrt{2}} \left(\hat{f}_i + \rho_i \right) e^{i \hat{a}_i / \hat{f}_i}$$

for instance $V(S_{1,2}) = \lambda S_1^3 S_2 + \text{h.c.}$ reduces the system to just one PQ

$$V_{\rm eff} = \frac{1}{2} \chi_{\rm QCD} \left(\frac{\hat{a}_1 + \hat{a}_2}{\hat{f}} - \bar{\theta} \right)^2 + \frac{\lambda}{4} \hat{f}^4 \left(\frac{3\hat{a}_1 + \hat{a}_2}{\hat{f}} \right)^2$$

$(\hat{f}_1 = \hat{f}_2 = \hat{f})$

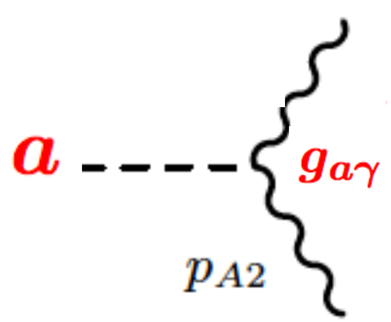
$$\mathbf{M}^2 = \frac{\chi_{\rm QCD}}{F^2} \begin{pmatrix} 2 + 8r & -4r \\ -4r & 2r \end{pmatrix} \qquad 1/F^2 = 2/\hat{f}^2$$

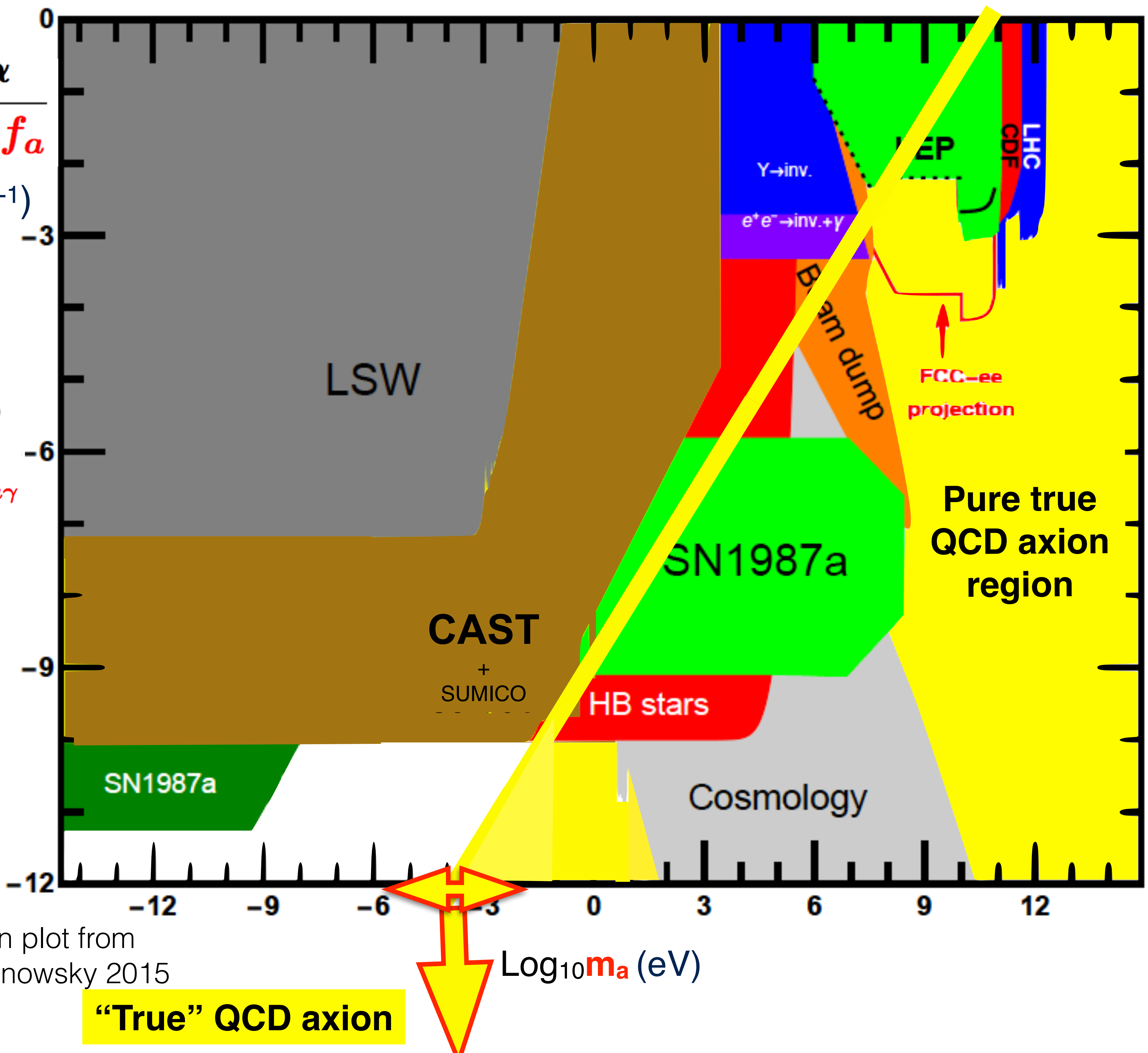
Maxion solution for r=1/5 ← $\text{Tr}[\mathbf{M}^2] = \sum_i m_i^2 = N \frac{\chi_{\rm QCD}}{F^2}$

right **ALP territory: they can be pure QCD axions**

$$g_{a\gamma} \sim \frac{\alpha}{8\pi f_a}$$

Log10(GeV-1)





constraints in plot from
Jaeckel+ Spannowsky 2015

Conclusions

*** The PQ solution to the strong CP problem leads in all generality to multiple QCD axion signals**

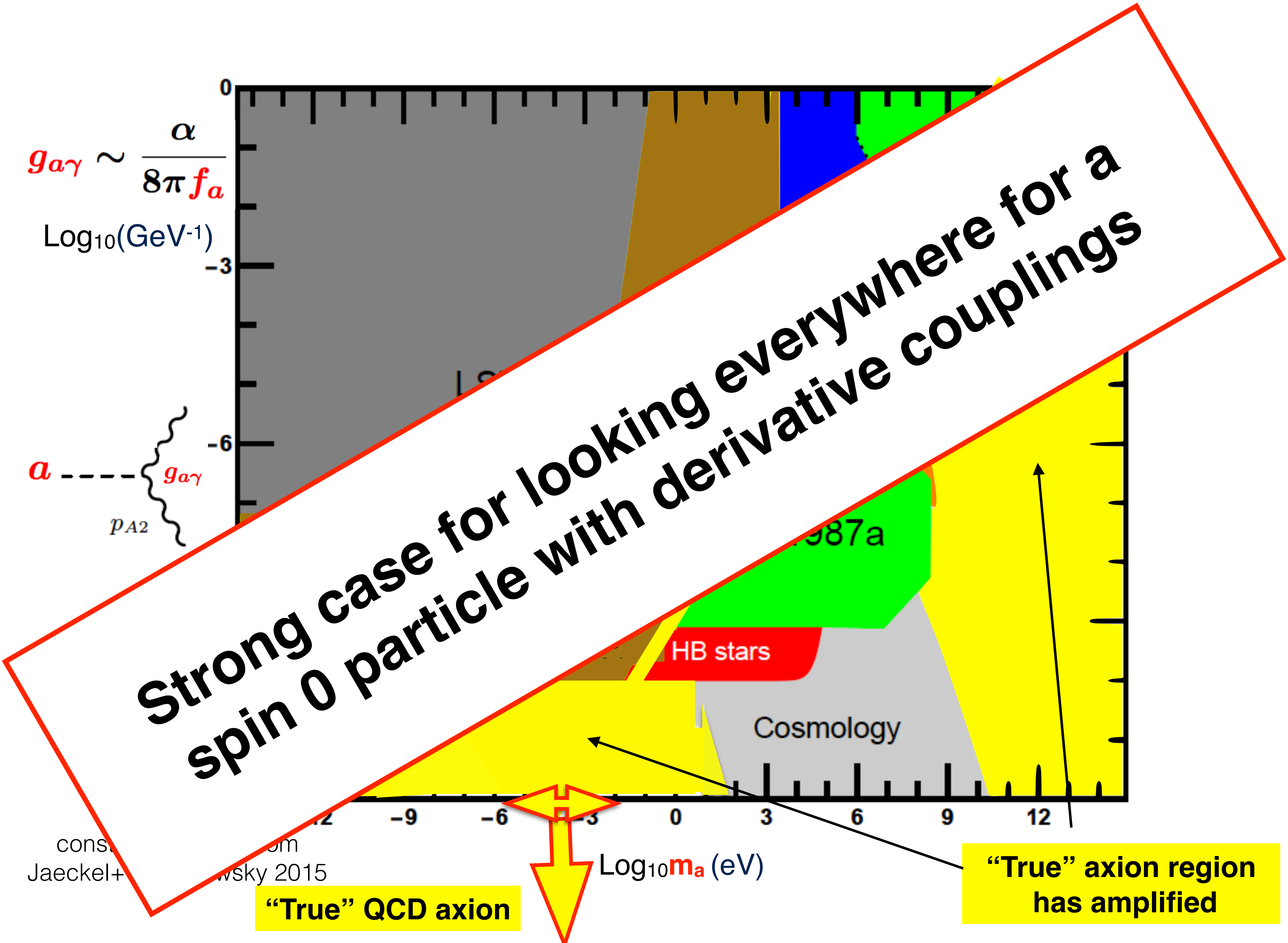
* They are displaced to the right of the canonical QCD band. The usual single QCD axion is just one limit of the solutions

*** The smoking gun is the multiplicity of signals.**

* **Exact PQ invariance condition and exact PQ sum rule**.

* The main experimental impact is from scales not far from the QCD contribution

* We encourage experiments to hunt for several signals. Beautiful synergy between different experiments.



$g_{a\gamma} \sim \frac{\alpha}{8\pi f_a}$
Log10(GeV-1)
0
-3
-6
a
$g_{a\gamma}$
p_{A2}
1987a
HB stars
Cosmology
-9
-6
-3
0
3
6
9
12
Log10ma (eV)
Jaeckel+
wsky 2015
"True" QCD axion
"True" axion region has amplified
Strong case for looking everywhere for a spin 0 particle with derivative couplings

Conclusions / Outlook

It is a deep pleasure to be here today

Thank you very very much for the invitation!

Backup

Many UV complete QCD axion models:

* KSVZ: new exotic fermions with QCD color… + scalar S

* DFSZ: SM fermions plus 2 Higgs doublets… + scalar S

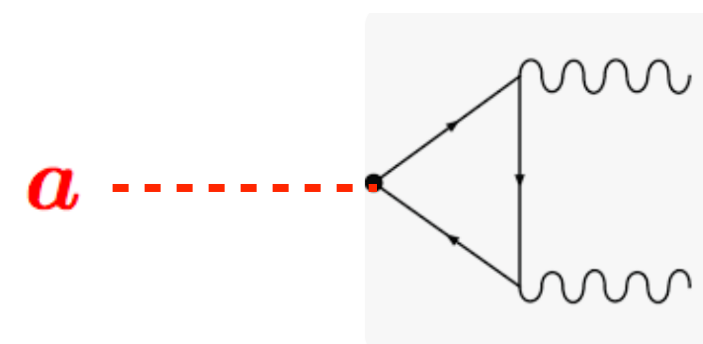
$$\text{e.g. } S = (f_a + \rho) \exp (i\, a / f_a)$$

* Composite axion models: new exotic massless fermions confined by a new force

etc.

All of them have in common:

1) some QCD coloured fermions



$G_{\mu\nu}\tilde{G}^{\mu\nu}$

2) QCD –>

$$m_a^2 f_a^2 = m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$$

Appendix D: Comparison with clockwork scenario

In general, clockwork matrices do not generate maxions (one exception being the model comprising only 2 scalars). The reason being that the next neighbor interactions of clockwork scenarios are engineered to generate exponentially small mixings whereas our maxions require sizable mixings. To see a concrete example, we focus on a scenario with three scalars, where the typical clockwork mass matrix reads, including the QCD contribution therefore reads:

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix} + r\,\frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix}1 & -q & 0\\ -q & 1+q^2 & -q\\ 0 & -q & q^2\end{pmatrix}\,. \tag{D1}$$

with $q = 3$ and where it was assumed that only one field in the 3rd-site develops couplings to gluons.

One can easily check that the PQ condition $\det \mathbf{M}^2/\det \mathbf{M}_1^2 = \chi_{\rm QCD}/F^2$ is indeed satisfied, as $F^2 = \hat{f}^2$. To prove that this model does not generate maxions, we have to show that the remaining two maxions conditions spanned by Eq. 44 cannot be satisfied for the same r. Indeed,

$$\mathrm{Tr}\,\mathbf{M}^2 = N\,\frac{\chi_{\rm QCD}}{F^2} \Leftrightarrow r = \frac{1}{10}\,, \tag{D2}$$

$$\mathrm{Tr}^2\,\mathbf{M}^2 - \mathrm{Tr}\,\mathbf{M}^2\cdot\mathbf{M}^2 = N\,\frac{\chi_{\rm QCD}}{F^2}\mathrm{Tr}\,\mathbf{M}_1^2 \Leftrightarrow r = 0 \vee r = \frac{11}{182}\,. \tag{D3}$$

$\{m_a, 1/f_a\}$: **coupling to gluons**

The single QCD axion line

$$\mathcal{L} \supset \frac{a}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

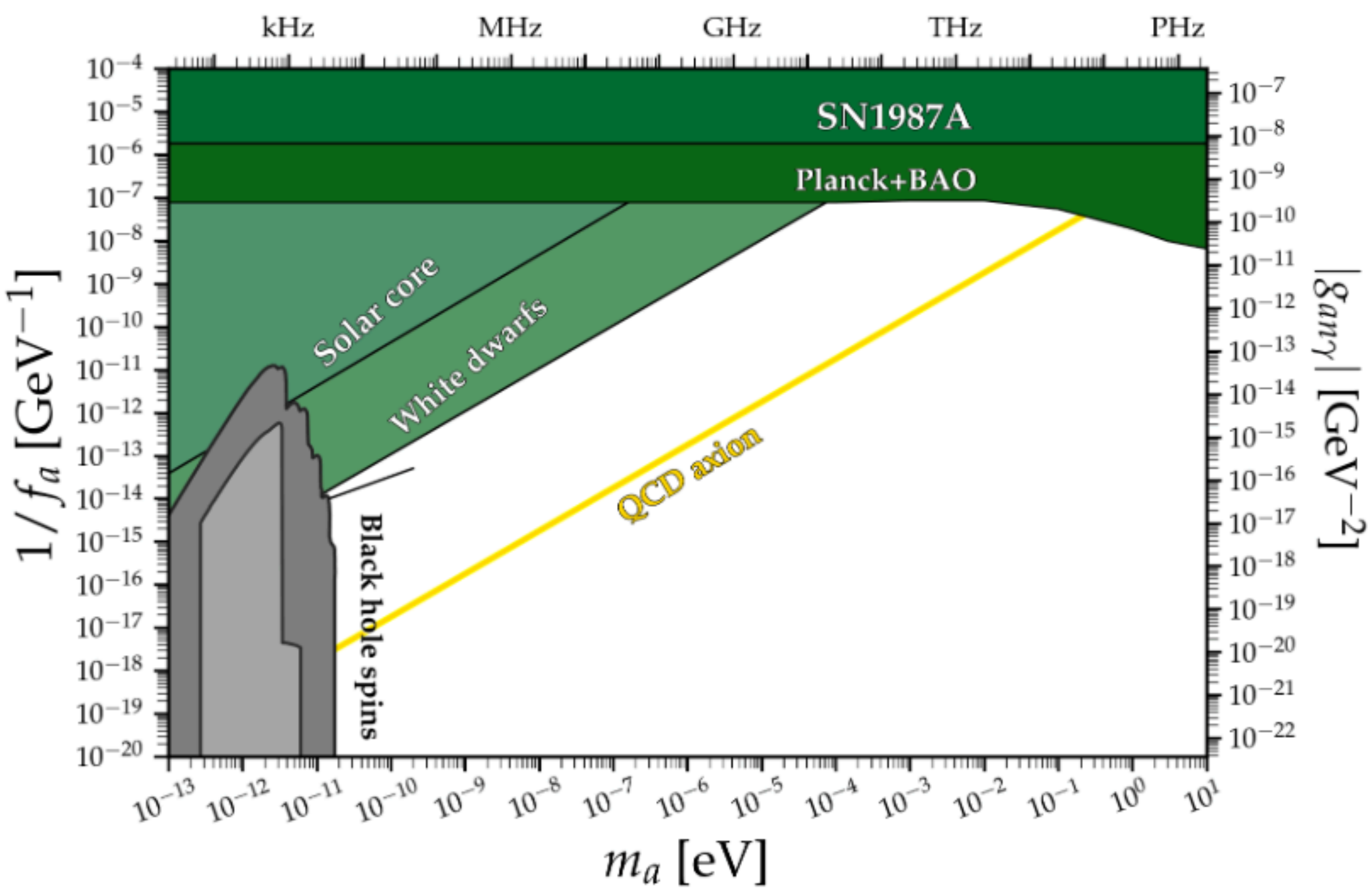
$$\delta\mathcal{L} \equiv -\frac{i}{2}\frac{0.011\, e}{m_n}\frac{a}{f_a}\bar{n}\sigma_{\mu\nu}\gamma_5 n F^{\mu\nu}$$

$$\equiv g_{a\gamma n}$$

$$m_a^2 f_a^2 \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2}$$

Coupling to the nEDM

Axion mass



Adapted from AxionLimits [Ciaran O'hare, 20]

Ringwald+Sokolov 2022: assume magnetic monopoles or dyons in UV theory

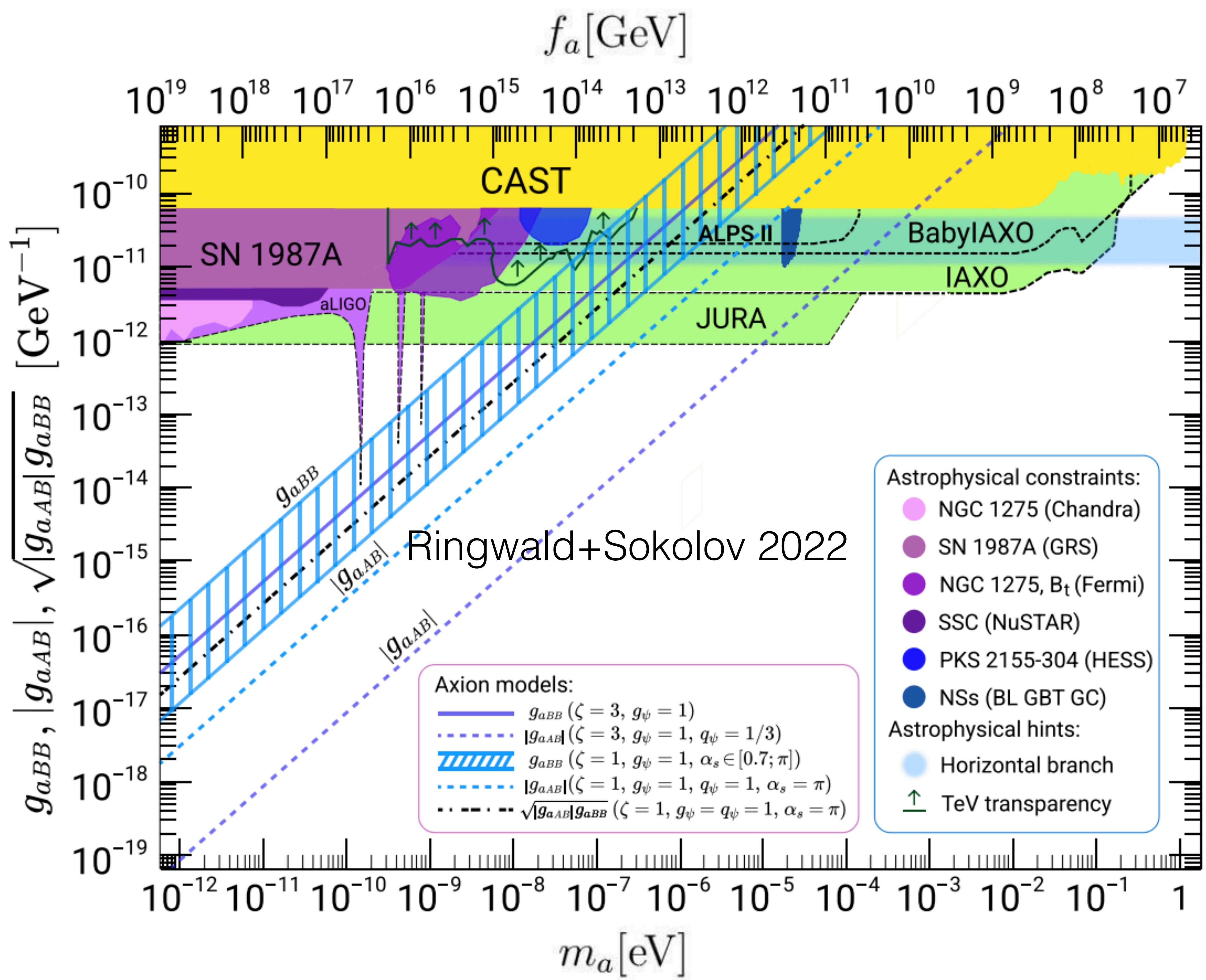


Figure 1. Existing and projected (dashed lines) constraints on the parameter space of ALP-photon g_{aBB} and g_{aAB} couplings versus ALP mass and decay constant together with the lines corresponding to g_{aBB} (solid), $|g_{aAB}|$ (dashed) and $\sqrt{|g_{aAB}|\, g_{aBB}}$ (dash-dotted) in different hadronic axion models with one heavy PQ-charged fermion ψ with the parameters given in a box and N_{DW} =