Review of Lattice nEDM computations

Tanmoy Bhattacharya

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Lattice Gauge Theory Contributions to New Physics Searches
Instituto de Fisica Teorica UAM/CSIC Madrid

Outline

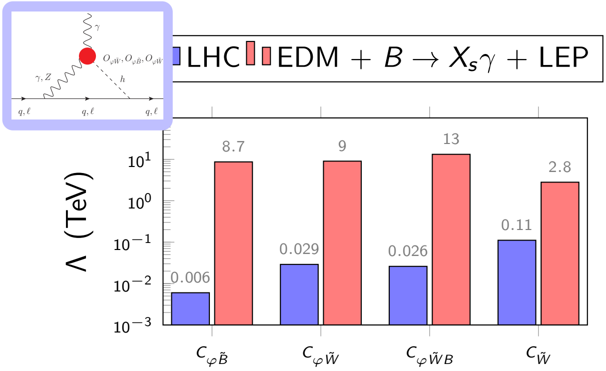
Introduction

Introduction
Standard Model

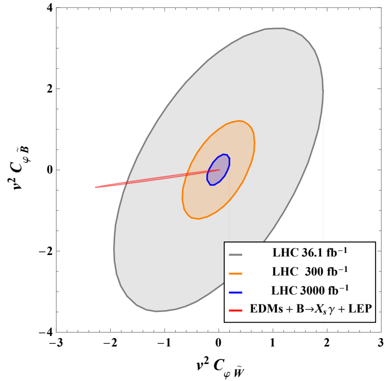
Major issue with standard model of particle physics:

- It is not compatible with standard model of cosmology.
  - The universe is too big and too old: quantum gravity.
  - The universe is too homogeneous and isotropic: dark energy.
  - The universe is too clumpy: dark matter.
  - The universe has too much matter: CP violation.
- Electric Dipole Moments of elementary particles prohibited only by CP symmetry.
  - Standard model CP model very small
    and cannot give electric dipole moments detectable today.
  - EDMs often the best way to detect new physics!!!

Introduction
EDM reach examples I

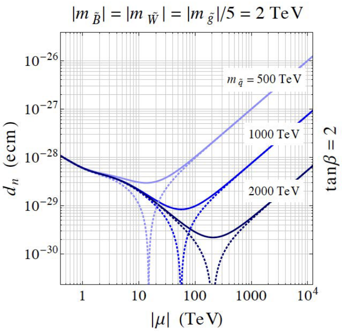


arXiv:1510.00725

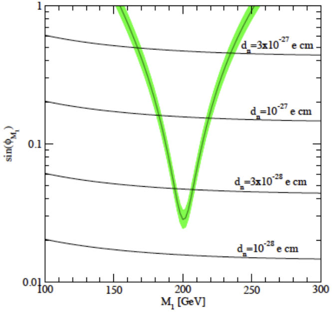


arXiv:1903.03625

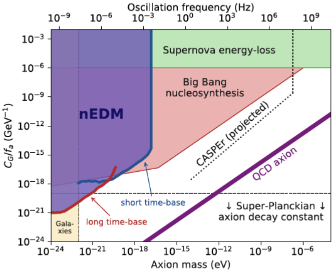
Introduction
EDM reach examples II



arXiv:1308.3653



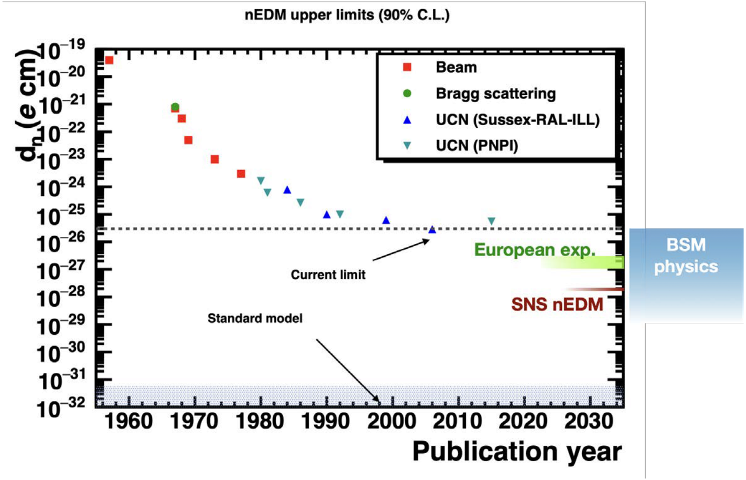
arXiv:0811.1987



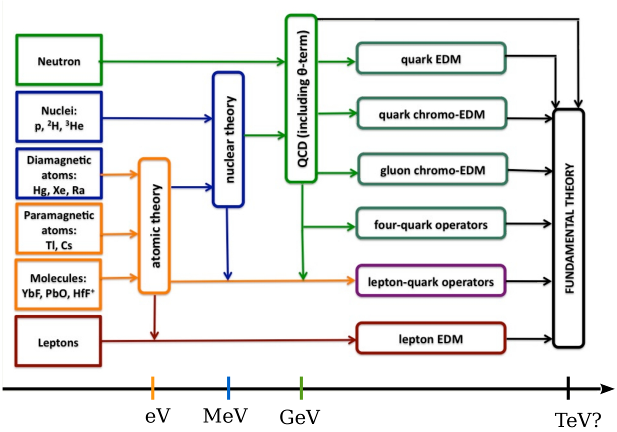
arXiv:1708.06367

Introduction

Neutron EDM: experiments and reach



Introduction
Effective Field Theory



J. de Vries, personal communication

Introduction
BSM Operators

- Dimension 4:
  - CP violating mass $m\bar{\psi}\gamma_5\psi$.
  - Topological charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
    - Equivalent by singlet axial anomaly.
    - Anomalously small.
- Suppressed by $v_{\rm EW}\Lambda_{\rm QCD}/M_{\rm BSM}^2$:
  - Quark Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Quark Chromo-electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $\Lambda_{\rm QCD}^2/M_{\rm BSM}^2$ and $\Lambda_{\rm QCD}^2/v_{\rm EW}^2$:
  - Gluon Chromo-electric Dipole moment: $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators (including standard model).

Introduction
Form Factors

Vector form-factors in standard representation for C, P, and T:
Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$\langle N|V_\mu(q)|N\rangle = \overline{u}_N \left[\gamma_\mu F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2}q_\nu \frac{F_2(q^2)}{2m_N} + (2i\, m_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\frac{F_A(q^2)}{m_N^2} + \frac{[\gamma_\mu,\gamma_\nu]}{2}q_\nu\gamma_5 \frac{F_3(q^2)}{2m_N}\right] u_N$$

Relations:

- $G_E(0) \equiv F_1(0) = 0$.
- $G_M(0)/2M_N \equiv F_2(0)/2M_N$.
- $d_N \equiv F_3(0)/2m_N$
- F_A violates PT; F_3 violates CP.

Nuclear forces also need:

- Axial/pseudoscalar form-factors.
- Direct nucleon-nucleon coupling.

Introduction
Technical Note: States

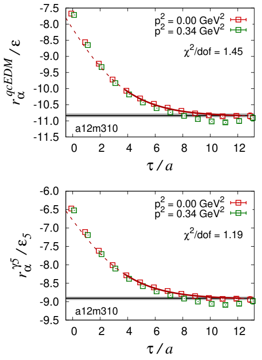
Asymptotic states are 'free' particles: always have a P symmetry.

If interaction does not have these symmetries, the symmetry generator will be different on different asymptotic states.
Change the nucleon interpolating operator

$$\hat{N} \to e^{-i\alpha_N\gamma_5}\hat{N}$$

to get standard parity for nucleon.
α_N can be chosen real if interactions have PT symmetry.
α_N can be calculated in chiral perturbation theory and as

$$\lim_{\tau\to\infty}[r_\alpha(\tau) \equiv \frac{\Im\,\mathrm{Tr}\,\gamma_5(1+\gamma_4)\langle N(\tau)\bar{N}(0)\rangle}{\Re\,\mathrm{Tr}(1+\gamma_4)\langle N(\tau)\bar{N}(0)\rangle}]$$



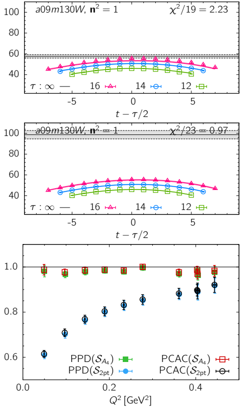
arXiv:2304.09929 [hep-lat]

Introduction
Technical Note: Excited States

- Lattice calculates

$$\lim_{\substack{\beta\to\infty \\ \tau\to\infty \\ t\to\infty}} e^{-\beta H} N(\tau) O(t) \bar{N}(0)$$

- Interpolating operators create many states.
- Weakly interacting multiparticle states are volume suppressed:
  - Often not visible in fits to two-point functions, especially when light excitations.
  - Can contribute more to three-point functions if transition matrix element is enhanced and/or attractive interaction.
  - χ^2 surface often too flat to detect this effect.
- May be importantto correct this to get physics right!
- χPT might provide guidance.



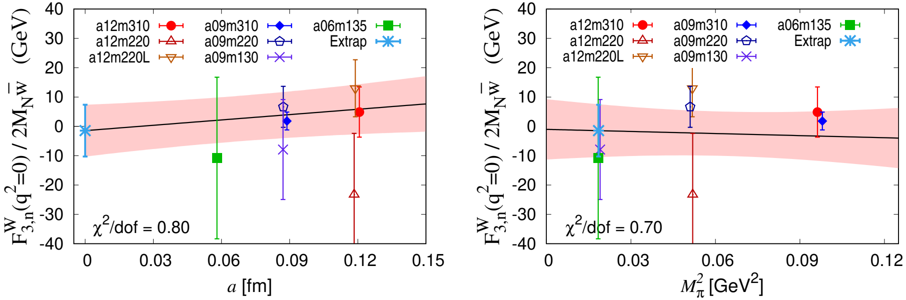
Lattice Calculation

Lattice Calculation
Dimension 6 operators

- CPV semileptonic four-fermion operators
  - Do not give hadronic EDM.
  - Affect atomic and molecular EDMs.
  - Need fermion bilinear charges from lattice QCD or phenomenology.
  - Mostly available with precision.
- No lattice data yet on CPV four-quark operators.
  - One group is starting these calculations.
  - Some Lorentz structures have chirally suppressed mixing with lower-dimensional operators.
- Very preliminary lattice data on CPV Weinberg operator.
  - Gradient-flow scheme to control 'ultraviolet' mixing with lower-dimensional operators.
  - Gives rise to $1/t_{\mathrm{gf}}$ mixing when converted to $\overline{\mathrm{MS}}$ scheme.
  - No result disentangling this mixing yet.

Lattice Calculation
Weinberg term

PRELIMINARY (errors only statistical):



Mixing with the Theta term to be removed.

Data collected with both clover-on-HISQ and clover-on-clover.

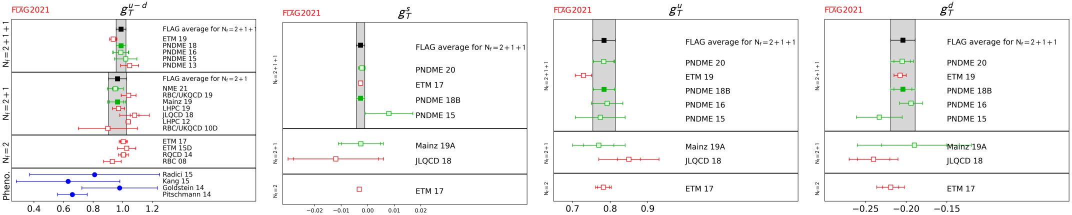
Analysis in progress.

Lattice Calculation
Quark EDM

nEDM due to the quark EDM operator proportional to the tensor charge.

Extraction has small systematics.

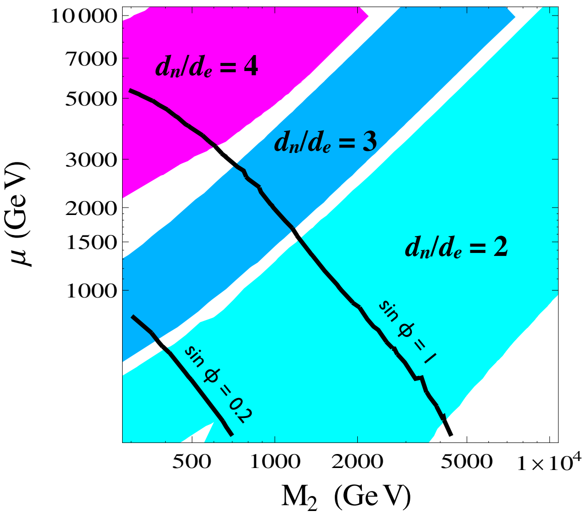
Lattice calculations dominate determination



Our new clover-on-clover analysis in Phys. Rev. D105 (2022) 054505.

Lattice Calculation
Precision targets

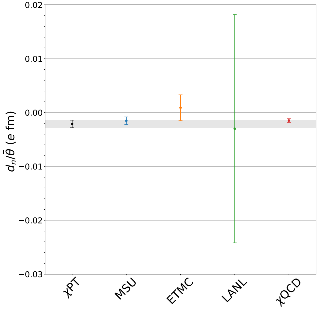
- Tensor charge target precision: 5–10%
- Dimension 5 and 6: 25–50%



QCD Topological term

QCD Topological term
Theta term

- Because of singlet axial ward identity, only $\overline{\Theta} \equiv \Theta + \arg\det m_q$ observable. By nonsinglet chiral symmetry, other fermion mass phases unphysical.
- Unnaturally small: nEDM limits imply $\overline{\Theta} \lesssim 10^{-9}$.
- Other CPV operators often mix to produce $\overline{\Theta}$.
- Peccei-Quinn symmetry: $\overline{\Theta}$ relaxes to a value of the order of the other CP violation in the theory.
- Need to calculate it for understanding higher-dimension operators.
- Different groups claim very different errors: no reliable result yet.



E. Mereghetti, personal communication

QCD Topological term
Lattice Difficulty

Need to reweight vector 3-pt function by the topological charge.

Topological charge noisy: answer depends on topological charge density.

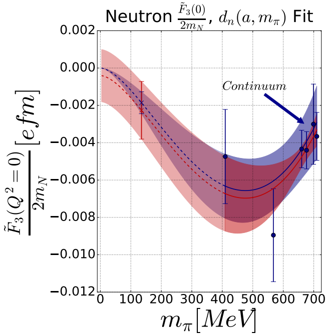
Groups have done different things to tame the problem: typically integrate $G\tilde{G}$ or $\bar{\psi}\gamma_5\psi$ over a finite region

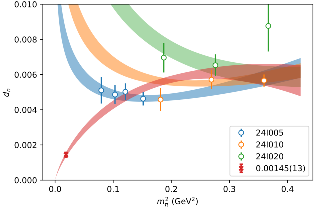
- Around the source
- Around the sink (Vector current momentum nonzero)
- In a tube

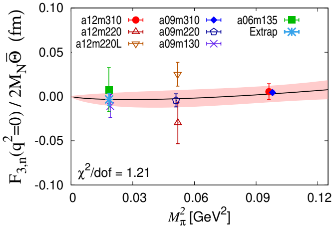
Bias-variance tradeoff is not yet understood.

QCD Topological term
Worrisome chiral behavior

Effects proportional to the harmonic mean, $\bar{m}_q$.



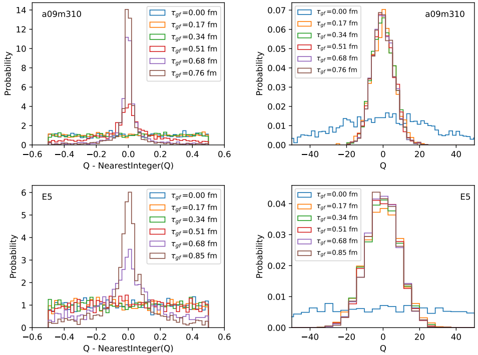




Chiral extrapolation from $M_\pi > 300$ MeV problematic.
Statistical fluctuations not under control for $M_\pi < 300$ MeV.

QCD Topological term
Comparing clover and HISQ sea

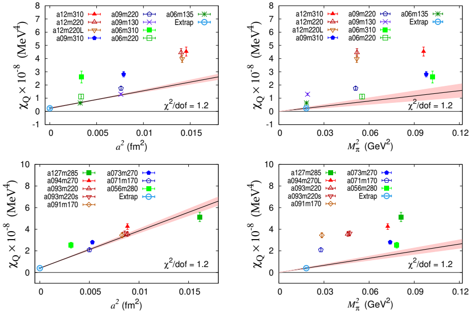
- Previous calculation
  - Fermion: Clover (valence) on HISQ (sea)
  - Lattice spacings: 0.057 fm – 0.151 fm
  - Pion mass: 128 MeV – 320 MeV
  - Number of configurations: 550 – 2200
- New calculation
  - Fermion: Clover (valence) on Clover (sea)
  - Lattice spacings: 0.056 fm – 0.127 fm
  - Pion mass: 167 MeV – 285 MeV
  - Number of configurations: 810 – 2100



QCD Topological term
Topological Susceptibility

$$\chi_Q = [66(9)(4)\ \mathrm{MeV}]^4 \qquad \text{Clover-on-HISQ}$$

$$\chi_Q = [79.5(3.0)\ \mathrm{MeV}]^4 \qquad \text{Clover-on-Clover}$$

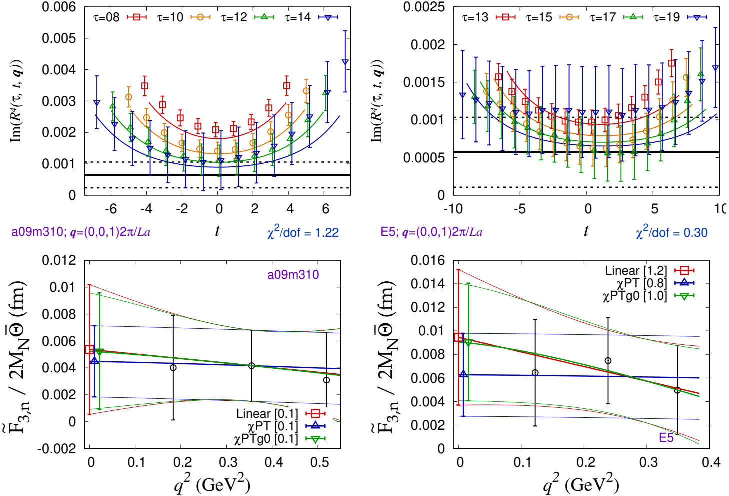


Expectation from χPT

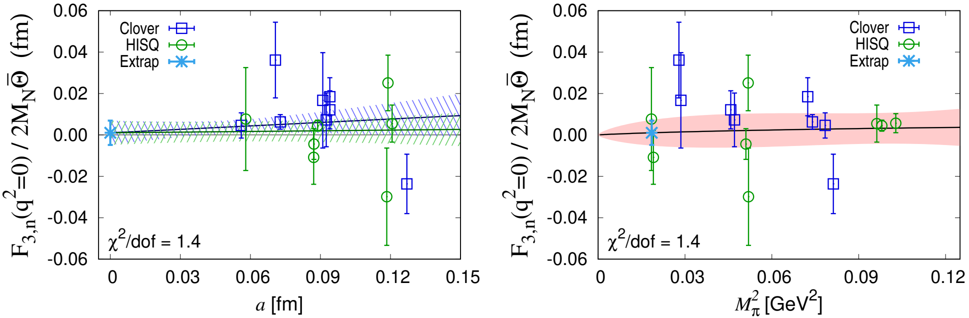
$$\frac{1}{\chi_Q} = \frac{1}{\chi_Q^{\text{quench.}}} + \frac{4}{M_\pi^2 F_\pi^2}\left(1 - \frac{M_\pi^2}{3M_\eta^2}\right)^{-1}$$

$$\chi_Q = [79\ \mathrm{MeV}]^4$$

QCD Topological term
ESC and Q^2 extrapolation



QCD Topological term
Simultaneous fit



- Extrapolation to continuum $a \to 0$ and physical pion mass $M_\pi \to 135\text{MeV}$
- Simultaneous fit to Clover-on-HISQ and Clover-on-Clover results

$$d_N = c_1 M_\pi^2 + c_2 M_\pi^2 \log\left(\frac{M_\pi^2}{M_N^2}\right) + c_3^{\text{HISQ}} a + c_3^{\text{Clover}} a = 0.0010(59)$$

PRELIMINARY; statistical error only.

Quark Chromoelectric Dipole Moment

Quark Chromoelectric Dipole Moment

Technique: quark chromo-EDM

The quark chromo-EDM operator can be handled by the Schwinger source method

$$P \equiv \left[\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu}\right]^{-1} \longrightarrow \left[\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})\right]^{-1}$$

The fermion determinant

$$\exp \mathrm{Tr} \ln \left[1 + i\epsilon\,\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right]$$

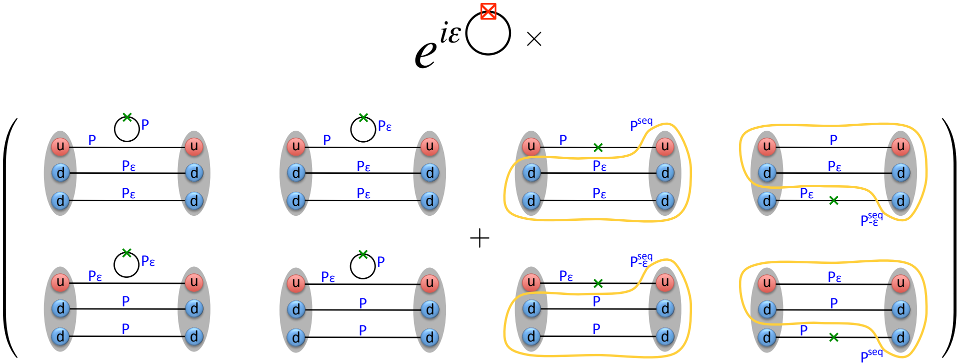
vanishes for the isovector case

Schwinger-source does not work with gradient-flow! Instead, we can use

$$\left[\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})\right]^{-1} \approx P - i\epsilon P\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}P$$

Quark Chromoelectric Dipole Moment

Three-point function



ϵ small to avoid multiple insertions.

Quark Chromoelectric Dipole Moment

quark chromo-EDM status

- HISQ analysis of isovector chromo-EDM available.
- Developed techniques for subtraction of power-divergence.
- Large lattice corrections because of $O(a^2)/am$ corrections.
- Available at arXiv:2304.09929 [hep-lat]
- Isoscalar analysis in progress.

Quark Chromoelectric Dipole Moment

Power divergence

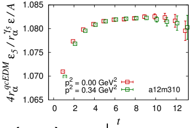
Quark chromo-EDM operator is power-divergent:

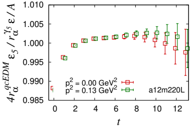
$$\tilde{C} = i\bar{\psi}\Sigma^{\mu\nu}\gamma_5 G_{\mu\nu} T^a \psi - i\frac{A}{a^2}\bar{\psi}\gamma_5 T^a \psi$$

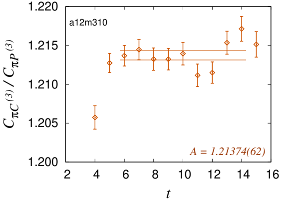
Demanding $\langle \Omega | \tilde{C} | \pi \rangle = 0$ fixes A:

Leading order χPT:

$$\alpha_N(\tilde{C}) = 0 \implies \frac{1}{A}\frac{\alpha_N(C)}{\alpha_N(\gamma_5)} = 1$$







Ensemble	c_{SW}	a (fm)	t-range	A
a12m310	1.05094	0.1207(11)	6–14	1.21374(62)
a12m220L	1.05091	0.1189(09)	7–14	1.21800(33)
a09m310	1.04243	0.0888(08)	8–22	0.99621(30)
a06m310	1.03493	0.0582(04)	14–30	0.77917(24)

Quark Chromoelectric Dipole Moment

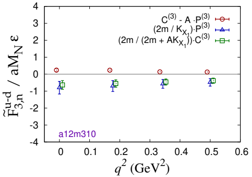
Multiplicative renormalization

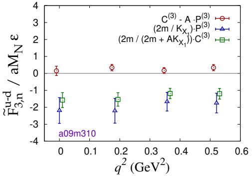
Isovector pseudoscalar can be rotated away up to $O(a)$ effects!
We can determine the $O(a)$ effects non-perturbatively:

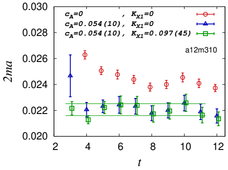
$$\frac{\langle \pi \left[a\partial_\mu A^\mu - \bar{c}_A a^2 \partial^P + \bar{K}(a^2 C - AP) \right] \rangle}{\langle \pi P \rangle} = 2\bar{m}a(1 + O(a^2))$$

So, on-shell zero-momentum

$$\text{M.E. of } P = \text{M.E. of } \frac{x \equiv a^2 \bar{K}}{y \equiv 2\bar{m}a + A\bar{K}} C \,.$$



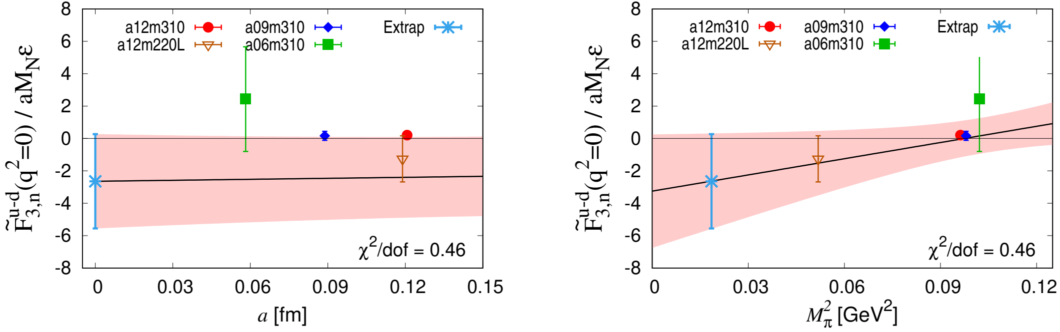




Ensemble	fit-range c_A	fit-range $\bar{K}_{X1}$	$\chi^2/\text{d.o.f}$ c_A	$\chi^2/\text{d.o.f}$ $\bar{K}_{X1}$	c_A	$\bar{K}_{X1}$	$2\bar{m}a$	$\frac{2ma}{K_{X1}}$	$\frac{2ma}{2ma + AK_{X1}}$
a12m310	4–11	3–11	0.66	0.88	0.054(10)	0.097(45)	0.02205(46)	0.23(10)	0.158(58)
a12m220L	4–11	3–11	2.08	3.09	0.0342(77)	0.183(35)	0.01152(21)	0.063(12)	0.0491(86)
a09m310	5–15	4–15	0.99	1.09	0.0277(40)	0.047(15)	0.01684(15)	0.35(11)	0.263(61)
a06m310	6–20	5–20	0.29	1.53	0.0093(17)	0.0272(60)	0.010460(37)	0.385(87)	0.331(50)

Quark Chromoelectric Dipole Moment

Results



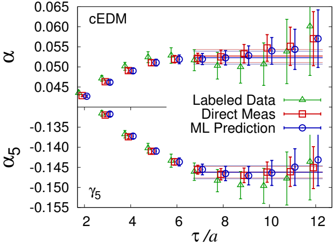
Preliminary data for power-divergent-subtracted tree-level matched operator $\tilde{C}$ in $\overline{\mathrm{MS}}$ scheme 1-loop run to 2 GeV.

Future

Future

Next steps in progress

- Compare with Gradient-flow to control mixing.
  - In Gradient Flow scheme, smearing cEDM allows smooth $a \to 0$ limit at fixed physical smearing t.
  - Continuum calculation mixes matrix elements of cEDM and $t^{-1}\bar{\psi}\gamma_5\psi$ to obtain cEDM in other schemes.
  - Mixing with quark EDM.
- Use machine learning to find correlated observables, and reduce variance.



  - CP violating phase $\alpha_N = 0$ without CP violation.
  - Training on 70 confs, bias correction on 50 confs, prediction for 400 confs.
  - ML algorithm is able to learn the computation of CPV observables from CP conserving one