Lattice QCD inputs for CKM matrix elements determinations

Elvira Gámiz

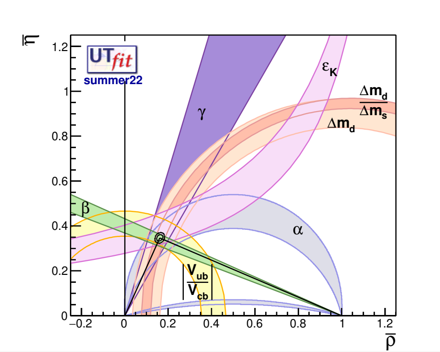
UNIVERSIDAD
DE GRANADA

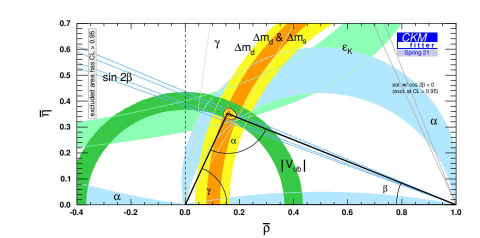
· Lattice Gauge Theory Contributions to New Physics Searches ·
· 12-16 June 2023 ·

Introduction

Precise tests of CKM unitarity and internal consistency within SM description.

Indirect probe of NP.





Good consistency but some tensions: Cabbibo anomaly, $V_{cb(ub)}$ exc. vs inc., ...

Experimental program: **LHCb, Belle II, BESIII,** ATLAS/CMS, NA62, KLOE-2, PIONEER

Contents

$$
V_{CKM} = \begin{pmatrix}
\begin{matrix} |V_{ud}| \\ \pi \to \ell\nu \\ \text{nucleon charges} \\ \text{RC} \end{matrix} & \begin{matrix} |V_{us}| \\ K \to \ell\nu \\ K \to \pi\ell\nu \\ \text{Hyperon decays} \end{matrix} & \begin{matrix} |V_{ub}| \\ B \to \pi\ell\nu \\ B_s \to K\ell\nu \\ B \to \ell\nu \\ \Lambda_b \to p\ell\nu \end{matrix} \\
\begin{matrix} |V_{cd}| \\ D \to \ell\nu \\ D \to \pi\ell\nu \\ \\ B_c \to B^0\ell\nu \ldots \end{matrix} & \begin{matrix} |V_{cs}| \\ D_s \to \ell\nu \\ D \to K\ell\nu \\ \Lambda_c \to \Lambda\ell\nu \\ B_c \to B_s^0\ell\nu \ldots \end{matrix} & \begin{matrix} |V_{cb}| \\ B \to D^{(*)}\ell\nu \\ B_s \to D_s^{(*)}\ell\nu \\ \Lambda_b \to \Lambda_c\ell\nu \\ B \to X_c\ell\nu \end{matrix} \\
\begin{matrix} |V_{td}| \\ \Delta M_d \\ B \to \pi\ell\ell \end{matrix} & \begin{matrix} |V_{ts}| \\ \Delta M_s \\ B \to K\ell\ell \end{matrix} & \begin{matrix} |V_{tb}| \\ \\ \\ \end{matrix}
\end{pmatrix}
$$

Radiative Corrections:
Talk by Matteo Di Carlo

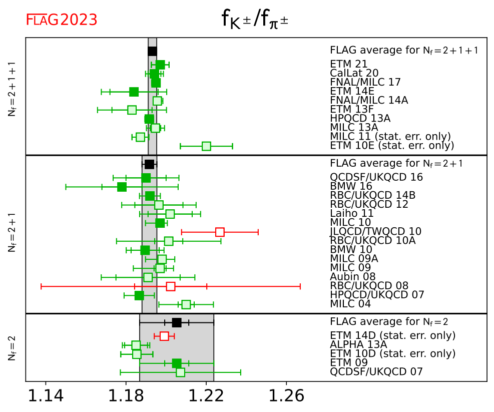
$|V_{ub,cb}|$**:**
Talk by Judd Harrison

HFLAV21, Phys. Rev. D 107 (2023) 5

FLAG21, Eur. Phys.J. C 82 (2022) 10, 869 [2023 web update]

Tests of first-row CKM unitarity

Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$



Pure QCD including $SU(2)$ IB corrections.

$N_f = 2+1+1$ **FLAG21**

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.1934(19)$$

0.16% error

Reduction of errors in the last years thanks to physical light quark masses, improved actions, NPR or no renormalization.

Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$

Following **W. Marciano** proposal in PRL93, 231803 (2004), hep-ph/0402299

$$\frac{\Gamma(K^+ \to l^+ \nu_l(\gamma))}{\Gamma(\pi^+ \to l^+ \nu_l(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_{K^\pm}^2}{f_{\pi^\pm}^2} \frac{\left(1+\delta^l_{EM,K}\right)}{\left(1+\delta^l_{EM,\pi}\right)}$$

δ^l_{EM} includes structure dependent EM corrections

Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$

Following **W. Marciano** proposal in PRL93, 231803 (2004), hep-ph/0402299

$$\frac{\Gamma(K^+ \to l^+\nu_l(\gamma))}{\Gamma(\pi^+ \to l^+\nu_l(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f^2_{K^\pm}}{f^2_{\pi^\pm}}\frac{\left(1+\delta^l_{EM,K}\right)}{\left(1+\delta^l_{EM,\pi}\right)} \to \frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(29)(24)$$

δ^l_{EM} includes structure dependent EM corrections, traditionally estimated phenomenologically within ChPT, **Cirigliano & Neufeld** PLB700 (2011) 7, **Knecht et al** EPJC12 (2000) 469

- Experimental data PDG 21

Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$

Following **W. Marciano** proposal in PRL93, 231803 (2004), hep-ph/0402299

$$\frac{\Gamma(K^+\to l^+\nu_l(\gamma))}{\Gamma(\pi^+\to l^+\nu_l(\gamma))}\propto\frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_{K^\pm}^2}{f_{\pi^\pm}^2}\frac{\left(1+\delta^l_{EM,K}\right)}{\left(1+\delta^l_{EM,\pi}\right)}\to\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^\pm}}{f_{\pi^\pm}}=0.27599(29)(24)$$

δ^l_{EM} includes structure dependent EM corrections, traditionally estimated phenomenologically within ChPT, **Cirigliano & Neufeld** PLB700 (2011) 7, **Knecht et al** EPJC12 (2000) 469

* Experimental data PDG 21
* $N_f=2+1+1$ FLAG21 average for $\frac{f_{K^\pm}}{f_{\pi^\pm}}$:

$$\frac{|V_{us}|}{|V_{ud}|}=0.23126(24)_{exp}(20)_{RC}(37)_{latt}=0.23126(49)$$

Leptonic decays of light mesons: $|V_{us}|/|V_{ud}|$

Calculate leptonic decay rates including QCD and QED on the lattice **See M. Di Carlo talk**

- For $K_{\mu 2}/\pi_{\mu 2}$, EM and SIB effects $\delta_{SU(2)} + \delta_{EM}$ found to be very compatible with $ChPT$ estimates: -1.26(14)% vs -1.12(21)%. **But smaller errors.**

  (Compatible result from **RBC/UKQCD, P.Boyle et al.** 2211.12865: -0.86$\binom{+41}{-40}$ %)

  Carrasco et al, 1502.00257, **Giusti et al** 1711.06537, **Di Carlo et al** 1904.08731

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\Big|_{Di\,Carlo\,et\,al}^{Moulson\,CKM21} = 0.27679(28)(20)$$

- Together with the $N_f = 2+1+1$ isospin-symmetric average $\frac{f_K}{f_\pi} = 1.1978(22)$

  (**Moulson** CKM21 from LQCD $N_f = 2+1+1$ results except FNAL/MILC 14A):

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23108(23)_{exp}(16)(42)_{latt} = 0.23108(51)$$

How to define isospin-symmetric quantities? $X = X^{iso} + \delta_{QED} + \delta_{SU(2)}$

Leptonic decays of light mesons: $|V_{us}|$

Extraction of $|V_{us}|$ need external input for $|V_{ud}|$:

- Most precise determination from **superallowed β decays.**
  - * Recent updates of universal single-nucleon radiative corrections (focused on the γW-box diagrams): **Seng et al** 1812.03352, 1807.10197, 2003.11264, **Czarnecki, Marciano & Sirlin** 1907.06737, **Hayen** 2010.07262, **Shiells, Blunden, Melnitchouk** 2012.01580
    - ** **Seng et al** 2003.11264: Use LQCD result for π axial γW-box
    - ** Also relevant for neutron decay

$\Delta_R^V = 0.02467(27)$ **Cirigliano, Crivellin, Hoferichter, Moulson** 2208.11707 $\longrightarrow$ Shift central value

- * New nuclear structure-dependent corrections **Gorchtein** 1812.04229, **Seng et al** 1812.03352 $\longrightarrow$ Increase the errors.

$$|V_{ud}|^{0^+\to 0^+} = 0.97367(11)_{exp}(13)_{\Delta_V^R}(27)_{NS} = 0.97367(32)$$

Hardy&Towner 21+**Cirigliano, Crivellin, Hoferichter, Moulson** 2208.11707

Leptonic decays of light mesons: $|V_{us}|$

Extraction of $|V_{us}|$ need external input for $|V_{ud}|$:

- Most precise determination from **superallowed β decays.**
  - * Recent updates of universal single-nucleon radiative corrections (focused on the γW-box diagrams): **Seng et al** 1812.03352, 1807.10197, 2003.11264, **Czarnecki, Marciano & Sirlin** 1907.06737, **Hayen** 2010.07262, **Shiells, Blunden, Melnitchouk** 2012.01580
    - ** **Seng et al** 2003.11264: Use LQCD result for π axial γW-box
    - ** Also relevant for neutron decay

$\Delta_R^V = 0.02467(27)$ **Cirigliano, Crivellin, Hoferichter, Moulson** 2208.11707 $\longrightarrow$ Shift central value

  - * New nuclear structure-dependent corrections **Gorchtein** 1812.04229, **Seng et al** 1812.03352 $\longrightarrow$ Increase the errors.

$$|V_{ud}|^{0^+\to 0^+} = 0.97367(11)_{exp}(13)_{\Delta_V^R}(27)_{NS} = 0.97367(32)$$

Hardy&Towner 21+**Cirigliano, Crivellin, Hoferichter, Moulson** 2208.11707

- **Neutron decays:** Clean system, large exp. errors. From **PDG21**

$$|V_{ud}|_{\rm n}^{\rm PDG} = 0.97441(31)_f(13)_{\Delta R}(82)_\lambda(28)_{\tau_n} = 0.97441(88)$$

where f =phase-space factor, $\lambda = g_A/g_V$, τ_n=neutron lifetime.

Leptonic decays of light mesons: $|V_{us}|$

Extraction of $|V_{us}|$ need external input for $|V_{ud}|$:

- Most precise determination from **superallowed β decays.**
  - * Recent updates of universal single-nucleon radiative corrections (focused on the γW-box diagrams): **Seng et al** 1812.03352, 1807.10197, 2003.11264, **Czarnecki, Marciano & Sirlin** 1907.06737, **Hayen** 2010.07262, **Shiells, Blunden, Melnitchouk** 2012.01580
    - ** **Seng et al** 2003.11264: Use LQCD result for π axial γW-box
    - ** Also relevant for neutron decay

$\Delta_R^V = 0.02467(27)$ **Cirigliano, Crivellin, Hoferichter, Moulson** 2208.11707 $\longrightarrow$ Shift central value

  - * New nuclear structure-dependent corrections **Gorchtein** 1812.04229, **Seng et al** 1812.03352 $\longrightarrow$ Increase the errors.

$$|V_{ud}|^{0^+\to 0^+} 0.97367(11)_{exp}(13)_{\Delta_V^R}(27)_{NS} = 0.97367(32)$$

Hardy&Towner 21+**Cirigliano, Crivellin, Hoferichter, Moulson** 2208.11707

- **Neutron decays:** New EFT evaluation of RC **Cirigliano et al** 2306.03138

$$|V_{ud}|^{\text{Cirigliano et al}}_{\text{n,PDG}} = 0.97430(2)_{\tilde{\Delta}_f}(13)_{\tilde{\Delta}_R}(82)_{\lambda}(28)_{\tau_n} = 0.97430(88)$$

where f =phase-space factor, $\lambda = g_A/g_V$, τ_n=neutron lifetime.

Leptonic decays of light mesons: $|V_{us}|$

Extraction of $|V_{us}|$ need external input for $|V_{ud}|$:

- **Pion β decay $\pi^{\pm} \to \pi^0 e^{\pm} \nu_e$:** Experimental errors still large (PIONEER can improve)

$$|V_{ud}|^{\pi} = 0.9739(29)$$

Leptonic decays of light mesons: $|V_{us}|$

Experimental average from PDG21 and $N_f = 2+1+1$ FLAG21 average for $\frac{f_{K^\pm}}{f_{\pi^\pm}}$:

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23126(24)_{exp}(20)_{RC}(37)_{latt} = 0.23126(49)$$

plus V_{ud} from superallowed β decays

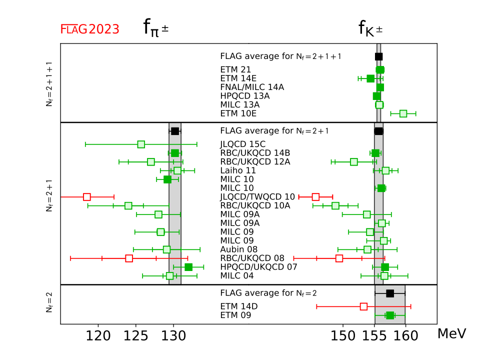
$$|V_{ud}|^{0^+\to 0^+} = 0.97367(11)_{exp}(13)_{\Delta_V^R}(27)_{NS} = 0.97367(32)$$

give

$$\boxed{|V_{us}|^{K_{\ell 2}/\pi_{\ell 2}} = 0.22517(24)_{exp}(36)_{latt}(24)_{RC}(6)_{NS} = 0.22517(48)}$$

* Error dominated by uncertainty in the ratio V_{us}/V_{ud}.

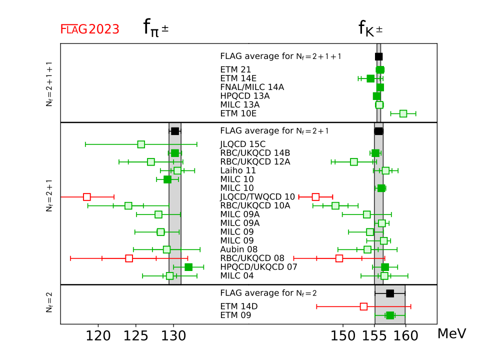
Leptonic decays of light mesons: $f_{\pi^\pm}$ and $f_{K^\pm}$



$f_{K^\pm}^{N_f=2+1+1} = 155.7(3)$ 0.19% error

But many existing LQCD calculations use f_π to set the lattice scale (implicitly rely on $|V_{ud}|$ and the SM).

$\rightarrow$ use a different external input?



$$f_{K^\pm}^{N_f=2+1+1} = 155.7(3)$$ 0.19% error

But many existing LQCD calculations use f_π to set the lattice scale (implicitly rely on $|V_{ud}|$ and the SM).

→ use a different external input?

Without relying on f_π to set the scale:

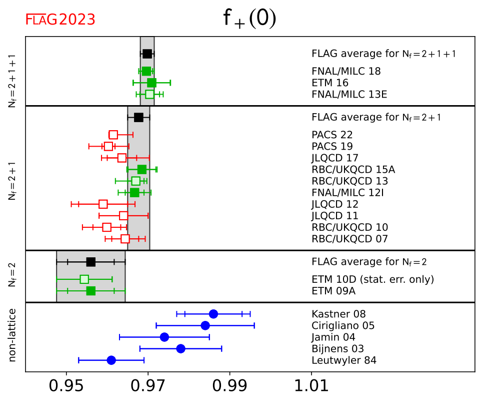
- With $f_{K^\pm}^{FLAG21,2+1} = 155.7(7)$ MeV and $|V_{us} f_{K^\pm}|^{PDG21} = 35.09(4)_{exp}(4)_{RC}$ MeV

$$|V_{us}| = 0.2254(10)_{latt}(3)_{exp}(3)_{RC}$$

- With $|V_{us} f_K| = 35.23(4)_{exp}(2)_{RC}$ MeV **Di Carlo et al** 1904.08731, $f_K^{2+1} = 156.0(7)$ MeV

$$|V_{us}| = 0.2258(10)_{latt}(3)_{exp}(1)_{RC}$$

K semileptonic decays: Direct $|V_{us}|$



$$f_+(0)^{FLAG21}_{N_f=2+1+1} = 0.9698(17)$$

0.18% error

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \eta_{EW}^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

$\eta_{EW} = 1.0232(3)$ Universal SD EW correction. δ^{Kl}_{EM} includes structure-dependent EM corrections and the SIB $\delta^{K\pi}_{SU(2)}$ is defined as a correction with respect to the K^0 decay. Traditionally estimated phenomenologically within ChPT, Cirigliano et al 1107.6001

K semileptonic decays: Direct $|V_{us}|$

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \eta_{EW}^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

$\delta_{SU(2)}$ looks solid at current level of precision, ChPT estimate for δ_{EM} into question (plus relevant source of error)

K semileptonic decays: Direct $|V_{us}|$

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \eta_{EW}^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

$\delta_{SU(2)}$ looks solid at current level of precision, ChPT estimate for δ_{EM} into question (plus relevant source of error)

* Calculation of $\delta_{EM}^{Ke(\mu)}$ based on Sirlin's representation of RC **Seng et al** 1910.13208, 2009.00459, 2103.00975, 2103.04843, 2203.05217, ChPT and new lattice QCD inputs for $\gamma W - box$ diagrams **Ma et al** 2102.12048
  * Consistent with previous ChPT estimates but with smaller errors.
  * New results for π and K box diagrams in **Yoo et al** 2305.03198

K semileptonic decays: Direct $|V_{us}|$

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \eta_{EW}^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

$\delta_{SU(2)}$ looks solid at current level of precision, ChPT estimate for δ_{EM} into question (plus relevant source of error)

* Calculation of $\delta_{EM}^{Ke(\mu)}$ based on Sirlin's representation of RC **Seng et al** 1910.13208, 2009.00459, 2103.00975, 2103.04843, 2203.05217, ChPT and new lattice QCD inputs for $\gamma W - box$ diagrams **Ma et al** 2102.12048
  ** Consistent with previous ChPT estimates but with smaller errors.
  ** New results for π and K box diagrams in **Yoo et al** 2305.03198

Using experimental average in **Cirigliano et al**, 2208.11707 (includes new RC above)

$$|f_+(0)V_{us}| = 0.21656(35)_{\rm exp+IB}$$

* New KLOE-2 result **D. Babusci et al**, 2208.04872 $(K_S \to \pi e \nu)$

And $N_f = 2+1+1$ FLAG21 average for $f_+(0) = 0.9698(17)$

$$|V_{us}|^{\rm semil} = 0.22330(35)_{exp}(39)_{latt}(8)_{IB} = 0.22330(53)$$

Other determinations of $|V_{us}/V_{ud}|$ and $|V_{us}|$

* $\frac{\Gamma(K\to\pi\ell\nu(\gamma))}{\Gamma(\pi^+\to\pi^0 e^+\nu(\gamma))}\propto\left|\frac{V_{us}}{V_{ud}}\right|^2\left|\frac{f_+^{K\pi}(0)}{f_+^{\pi^+\pi^0}(0)}\right|^2$ proposed by **Czarnecki, Marciano, Sirlin**, 1911.04685

With $f_+^{K\pi}(0)=0.9698(17)$, $f_+^{\pi^+\pi^0}(0)\approx 1$ and including new RC for K_{e3} and π_{e3} **Feng et al** 2003.09798, **Ma et al** 2102.12048, **Seng et al** 2103.00975, 2107.14708

$$\left|\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 3}/\pi_{\ell 3}}=0.22908(66)_{exp,\pi}(41)_K(40)_{f_+^K(0)}(2)_{\tau_\pi^+}(1)_{RC\pi}=0.22908(88)$$

Other determinations of $|V_{us}/V_{ud}|$ and $|V_{us}|$

* $\frac{\Gamma(K \to \pi\ell\nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e^+\nu(\gamma))} \propto \left|\frac{V_{us}}{V_{ud}}\right|^2 \left|\frac{f_+^{K\pi}(0)}{f_+^{\pi^+\pi^0}(0)}\right|^2$ proposed by **Czarnecki, Marciano, Sirlin**, 1911.04685

  With $f_+^{K\pi}(0) = 0.9698(17)$, $f_+^{\pi^+\pi^0}(0) \approx 1$ and including new RC for K_{e3} and π_{e3} **Feng et al** 2003.09798, **Ma et al** 2102.12048, **Seng et al** 2103.00975, 2107.14708

$$\left|\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 3}/\pi_{\ell 3}} = 0.22908(66)_{exp,\pi}(41)_K(40)_{f_+^K(0)}(2)_{\tau_\pi^+}(1)_{RC\pi} = 0.22908(88)$$

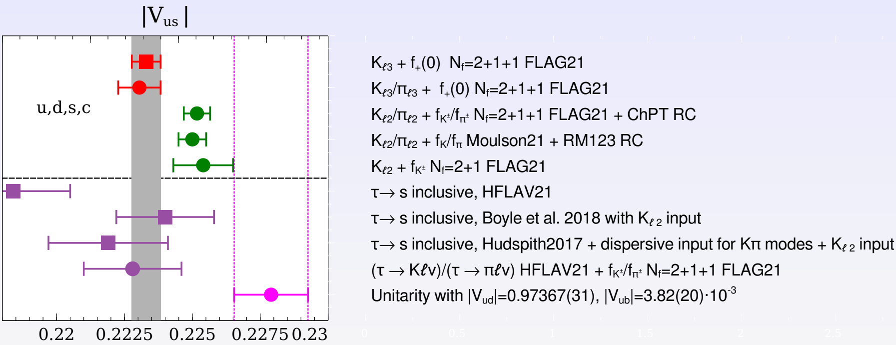
* **Inclusive hadronic τ decays:** $D > 4$ condensates from the lattice **Hudspith et al** 1702.01767, replacing OPE expansion by lattice HVP functions and optimizing weight functions **RBC/UKQCD Boyle et al** 1803.07228. Updates **T. Izubuchi** talk at CKM18 and **K. Maltman** talk at TAU18

  ** **ETMC**: New method based on reconstruction of smeared spectral densities from Euclidean lattice correlators: talk by **A. Evangelista** at Lattice22, 2301.00796

* **Hyperon decays**: $|V_{us}|^{\rm hyp}_{PDG\,22} = 0.2250(27)$

  ** Preliminary work by **RBC/UKQCD** on the calculation of form factors for $\Sigma^- \to n\ell^-\bar{\nu}$ presented at Lattice21, talk by **R. Hodgson.**

First-row: Consistency CKM description

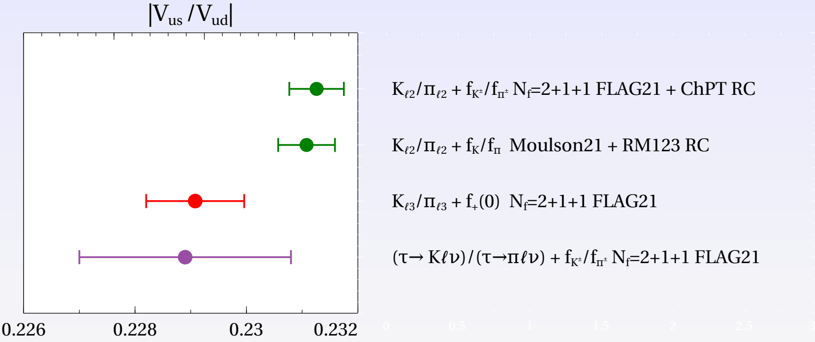


Circles: Use $|V_{ud}|^{0^+\to 0^+} = 0.97367(32)$ to get $|V_{us}|$.

$(\tau \to K\ell\nu)/(\tau \to \pi\ell\nu)$ HFLAV21 includes new RC calculation by **Arroyo-Ureña et al** 2107.04603

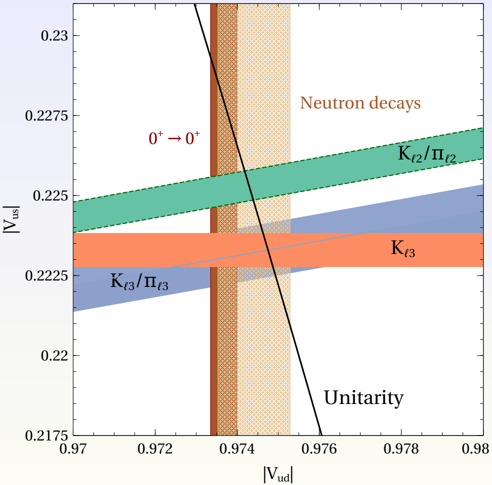
- Internal tensions between leptonic and semileptonic determinations of $|V_{us}|$ (with $|V_{ud}|$ as external input): $\sim 3\sigma$

First-row: Consistency CKM description



Tests of first-row CKM unitarity

Tensions with first-row unitarity at $\sim 2-3\sigma$ level



$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$

- V_{us}^{semil} and $|V_{ud}^{0^+\to 0^+}|$

  $\Delta_u = -0.0021(2)_{V_{us}}(6)_{V_{ud}}$ $\sim \mathbf{3\sigma}$

- V_{us}^{semil} and $|V_{us}|/|V_{ud}|^{K_{\ell 2}\,/\,\pi_{\ell 2}}$

  $\Delta_u = -0.018(5)_{V_{us}}(4)_{V_{us}/V_{ud}}$ $\sim \mathbf{3\sigma}^\dagger$

  Correlated analysis: Theory and experiment ($|K_{\mu 3}/K_{\mu 2}|_{exp}$ could have large impact Cirigliano, Crivellin, Hoferichter, Moulson, 2208.11707)

- $|V_{ud}^{0^+\to 0^+}|$ and $|V_{us}|/|V_{ud}|^{K_{\ell 2}\,/\,\pi_{\ell 2}}$

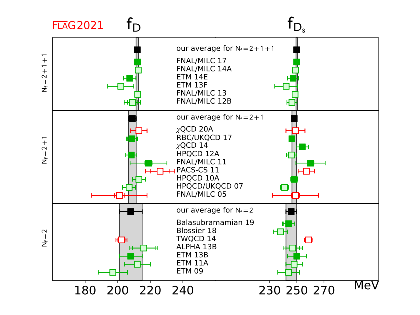
  $\Delta_u = -0.0013(7)_{V_{ud}}(2)_{V_{us}/V_{ud}}$ $\sim \mathbf{2\sigma}$

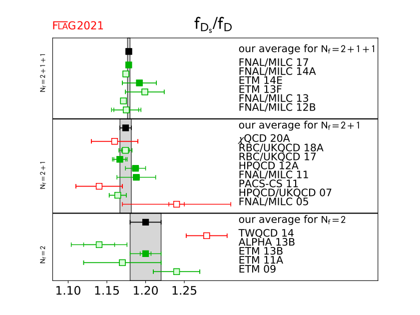
† $\sim 2.5\sigma$ with $|V_{us}|/|V_{ud}|^{K_{\ell_2}/\pi_{\ell_2}}|^{Moulson\,CKM21}_{Di\,Carlo\,et\,al} = 0.23108(51)$

Tests of second-row CKM unitarity

Leptonic decays of D and D_s mesons

Errors at 0.33-0.20%, 0.14% for the ratio: physical light quark masses, improved actions, NPR or no renormalization + small lattice spacings + same action for all flavors

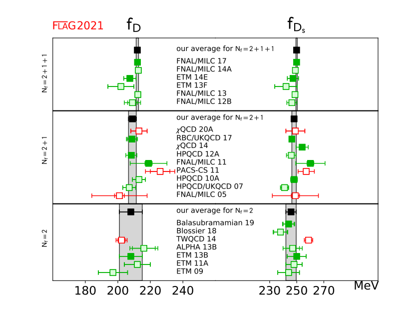


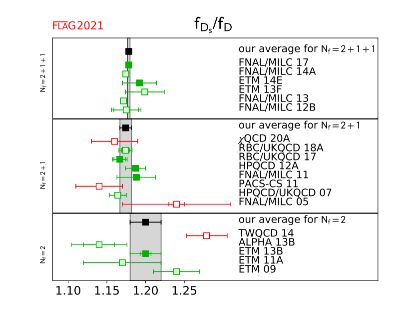


* On-going work on $N_f = 2+1$ CLS ensembles, see **A.Conigli** talk at Lattice 22

Leptonic decays of D and D_s mesons

Errors at 0.33-0.20%, 0.14% for the ratio: physical light quark masses, improved actions, NPR or no renormalization + small lattice spacings + same action for all flavors





FLAG21 $N_f = 2+1+1$ averages:

$$f_D = 212.0(0.7)\ \text{MeV} \quad f_{D_s} = 249.9(0.5)\ \text{MeV} \quad f_{D_s}/f_D = 1.1783(0.0016)$$

With dominant SIB from **FNAL/MILC** 1712.09262:

$$f_{D+} = 212.6(0.7)\ \text{MeV}\ f_{D_s} = 249.9(0.5)\ \text{MeV}\ f_{D_s}/f_{D+} = 1.1751(16)$$

Leptonic D decays: Extraction of $|V_{cd(cs)}|$

$$\Gamma(D^+_{(s)} \to \ell^+\nu(\gamma)) \propto \eta^2_{EW}\,(1+\delta_{\rm EM})\,|V_{cd(cs)}|^2 f^2_{D^+_{(s)}}$$

- **Experimental data:** CLEO-c, Belle, Babar, and BESIII. **HFLAV21** averages

$$\eta_{EW}|V_{cd}|f_{D^+} = 46.2(1.1)\ \text{MeV} \qquad \eta_{EW}|V_{cs}|f_{D_s^+} = 245.4(2.4)\ \text{MeV}$$

(averages do not include recent **BESIII** results for $D_s \to \tau\nu$, 2303.22600, 2303.12468)

Leptonic D decays: Extraction of $|V_{cd(cs)}|$

$$\Gamma(D^+_{(s)} \to \ell^+\nu(\gamma)) \propto \eta^2_{EW}\,(1+\delta_{\rm EM})\,|V_{cd(cs)}|^2 f^2_{D^+_{(s)}}$$

- **Experimental data:** CLEO-c, Belle, Babar, and BESIII. **HFLAV21** averages

  $|V_{cd}|f_{D+} = 45.8(1.1)(0.3)$ MeV $\qquad |V_{cs}|f_{D_s^+} = 243.2(2.4)(1.7)$ MeV

  (averages do not include recent **BESIII** results for $D_s \to \tau\nu$, 2303.22600, 2303.12468)

- **EW and EM corrections:** Accounted for in the exp. rates. **PDG21** adds 1.4% uncertainty to purely leptonic decay rate. Adding that to **HFLAV21** averages:
  - * Short-distance EW corrections $\sim$1.8% **Sirlin** NP82
  - * Structure-dependent EM: Reduce $\sim$1% μ channels **Dobrescu, Kronfeld** 0803.0512
  - * Long-distance EM: removed with PHOTOS.

Together with $N_f = 2+1+1$ **FLAG21** averages (with dominant SIB corrections):

$$|V_{cd}| = 0.2154(52)_{exp}(14)(7)_{latt} \qquad |V_{cs}| = 0.973(10)_{exp}(7)(2)_{latt}$$

* EW+EM corrections important source of error. First results for radiative decay rates on the lattice: **Desiderio et al** 2006.05358, **Giusti, Kane, Lehner, Meinel, Soni** 2302.01298

Semileptonic D decays: Extraction of $|V_{cd(cs)}|$

$$\frac{d\Gamma(D \to P\ell\nu)}{dq^2} \propto \eta_{EW}^2 \left(1 + \delta_{\rm EM}\right) |V_{cd(cs)}|^2 \times \left[|f_+^{DP}(q^2)|^2 + h(q^2, M_P^2, M_D^2, m_\ell) |f_0^{DP}(q^2)|^2 \right]$$

(with $h((q^2, M_P^2, M_D^2, 0) = 0)$

* Neglecting contribution from $|f_0^{DP}(q^2)|^2$ in $D \to P\mu\nu$ could shift $V_{cd,cs}$ by a few percent **Bazavov et al** 2212.12648.

Experimental data: CLEO-c, Belle, Babar, and BESIII. 2-3% errors for total branching fractions (μ and e)

Semileptonic D decays: Extraction of $|V_{cd(cs)}|$

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} \propto \eta_{EW}^2\,(1+\delta_{\rm EM})\,|V_{cd(cs)}|^2 \times \left[\,|f_+^{DP}(q^2)|^2 + h(q^2,M_P^2,M_D^2,m_\ell)|f_0^{DP}(q^2)|^2\,\right]$$

(with $h((q^2,M_P^2,M_D^2,0)=0)$

* Neglecting contribution from $|f_0^{DP}(q^2)|^2$ in $D\to P\mu\nu$ could shift $V_{cd,cs}$ by a few percent **Bazavov et al** 2212.12648.

Experimental data: CLEO-c, Belle, Babar, and BESIII. 2-3% errors for total branching fractions (μ and e)

EW and EM corrections: Should be accounted for in exp. rates.

* Short-distance EW corrections $\sim$1.8% **Sirlin** NP82
* Structure-dependent EM: use RC $K_{\ell 3}$ calculations to estimate uncertainty $\sim 1\%$ ($V_{cd,cs}$), $\sim 0.5\%$ (V_{cd}/V_{cs}, LFU ratios).
* Long-distance EM: removed with PHOTOS.

Semileptonic D decays: Extraction of $|V_{cd(cs)}|$

$$\frac{d\Gamma(D \to P\ell\nu)}{dq^2} \propto \eta^2_{EW}\,(1+\delta_{\rm EM})\,|V_{cd(cs)}|^2 \times \left[\,|f_+^{DP}(q^2)|^2 + h(q^2, M_P^2, M_D^2, m_\ell)|f_0^{DP}(q^2)|^2\,\right]$$

(with $h((q^2, M_P^2, M_D^2, 0) = 0)$

* Neglecting contribution from $|f_0^{DP}(q^2)|^2$ in $D \to P\mu\nu$ could shift $V_{cd,cs}$ by a few percent **Bazavov et al** 2212.12648.

Theoretically: New $N_f = 2+1+1$ results in the last couple of years $\to$ form factors errors reduced to $\sim 0.5 - 1\%$ (at $q^2 = 0$)

* $D \to K\ell\nu$: f_0, f_+ **HPQCD** 2104.09883
* $D \to K$: f_0, f_+, f_T **HPQCD** 2207.12468
* $D_{(s)} \to K\ell\nu$, $D \to \pi\ell\nu$: f_0, f_+ **FNAL/MILC** 2212.12648

Semileptonic D decays: HPQCD

HPQCD 2104.09883 $D \to K\ell\nu$ form factors on $N_f = 2+1+1$ MILC HISQ ensembles.

* Relativistic (HISQ) description of all flavors: very small discretization errors
* 5 lattice spacings $a \approx 0.15 - 0.042$ fm.

  m_s and m_c close to physical, 3 ensembles with physical m_l.
* NPR imposing Ward identities at q^2_{max}

  $(M_D - M_K) Z_V \langle K|V^0|D\rangle_{q^2_{max}} = (m_c - m_s)\langle K|S|D\rangle_{q^2_{max}}$
* Modified z-expansion: chiral interpolation, mass mistunings, continuum extrapol. and q^2 dependence.
* tbc: Four momenta for each ensemble

Semileptonic D decays: HPQCD

HPQCD 2104.09883 $D \to K\ell\nu$ form factors on $N_f = 2+1+1$ MILC HISQ ensembles.

* Relativistic (HISQ) description of all flavors: very small discretization errors
* 5 lattice spacings $a \approx 0.15 - 0.042$ fm.
  m_s and m_c close to physical, 3 ensembles with physical m_l.
* NPR imposing Ward identities at q^2_{max}
  $(M_D - M_K)Z_V\langle K|V^0|D\rangle_{q^2_{max}} = (m_c - m_s)\langle K|S|D\rangle_{q^2_{max}}$
* Modified z-expansion: chiral interpolation, mass mistunings, continuum extrapol. and q^2 dependence.
* tbc: Four momenta for each ensemble

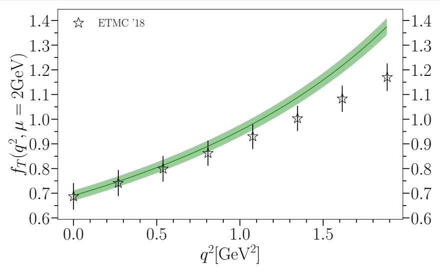
Three methods to extract V_{cs}: differ in experimental data included

* Total branching fraction.
* $f_+^{DK}(0)$
* q^2 binned differential decay rates.

Semileptonic D decays: HPQCD

HPQCD 2207.12468 $D(B) \to K$ on $N_f = 2+1+1$ MILC HISQ ensembles.

* Same data as HPQCD 2021.
* Includes $f_T(q^2)$
* Reanalysis include: Tensor-current correlators (combined correlator fits), heavier-than-charm data and $D_s \to \eta_s$ data.
* Results for $f_{0,+}(q^2)$ within 1σ of 2021 results.



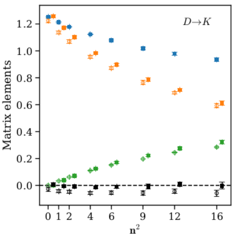
Plot from **HPQCD** 2207.12468

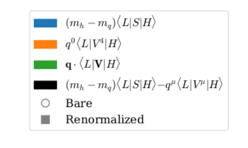
ETMC'18: $N_f = 2+1+1$ twisted mass, **Lubicz et al** 1803.04807

Semileptonic decays of D mesons: FNAL/MILC

FNAL/MILC 2212.12648 $D \to K(\pi)\ell\nu$, $D_s \to K\ell\nu$ **blinded** analysis on $N_f = 2+1+1$ MILC HISQ ensembles.

* Relativistic (HISQ) description of all flavors: small discretization errors
* 4 lattice spacings $a \approx 0.12 - 0.042$ fm.
  m_s close to physical, several $m_h \sim 0.9m_c - 2.2m_c$, 3 ensembles with physical m_l.
* NPR imposing Ward identities at all simulated q^2

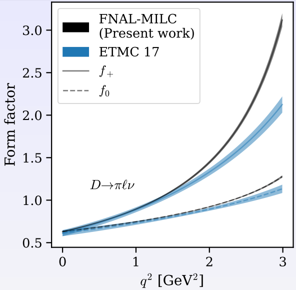




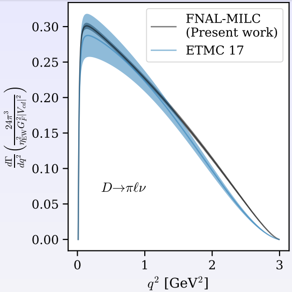
physical quark masses 0.12 fm ensemble

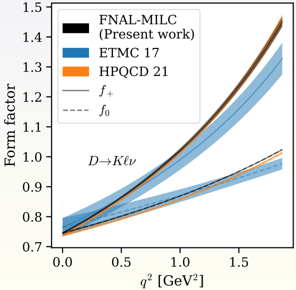
* Chiral interpolation + continuum extrapolation: hard pion/kaon (staggered) SU(2) ChPT for f_0 ($\langle S\rangle$) and f_+ ($\langle S\rangle$ and $\langle V^i\rangle$).
* (BCL) z-expansion
* Momenta (pbc): 8 for each ensemble $\boldsymbol{p} = 2\pi\boldsymbol{n}/N_s a$

Semileptonic D decays: $D_{(s)} \to K(\pi)\ell\nu$

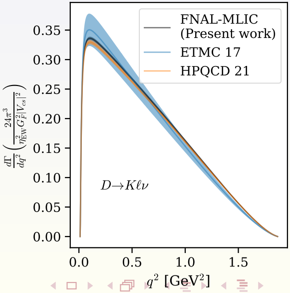


$D \to \pi$



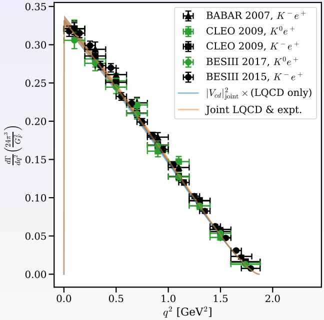


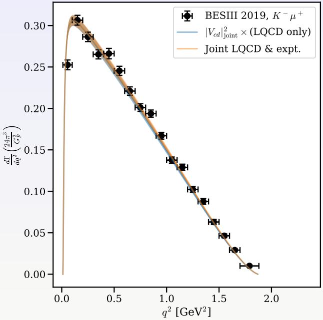
$D \to K$

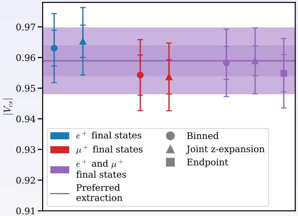


Semileptonic decays of D mesons: FNAL/MILC

Three methods to extract $|V_{cd,cs}|$: **Joint experimental+LQCD z-expansion fit**, $f_+^{DK}(0)$, q^2 binned differential decay rates.







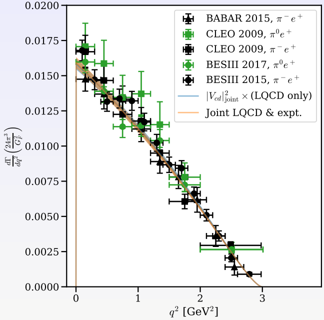
inner bands/bars: Without QED errors.

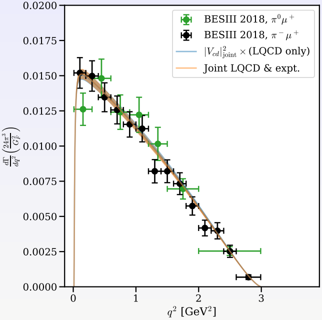
$$|V_{cs}|^{D\to K} = 0.9589(23)_{exp}(40)_{latt}(15)_{EW}(05)_{SIB}[95]_{QED}$$

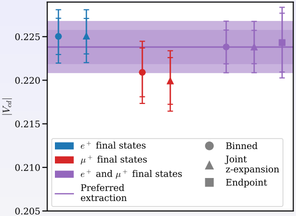
Statistics is the dominant error in the LQCD FF calculation for all three channels.

Semileptonic decays of D mesons: FNAL/MILC

Three methods to extract $|V_{cd,cs}|$: **Joint experimental+LQCD z-expansion fit**, $f_+^{DK}(0)$, q^2 binned differential decay rates.





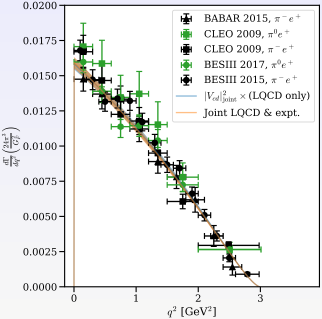


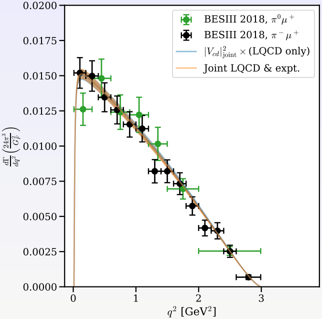
inner bands/bars: Without QED errors.

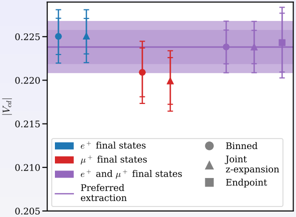
$$|V_{cd}|^{D\to\pi} = 0.2238(11)_{exp}(15)_{latt}(04)_{EW}(02)_{SIB}[22]_{QED}$$

Semileptonic decays of D mesons: FNAL/MILC

Three methods to extract $|V_{cd,cs}|$: **Joint experimental+LQCD z-expansion fit**, $f_+^{DK}(0)$, q^2 binned differential decay rates.







inner bands/bars: Without QED errors.

$$|V_{cd}|^{D\to\pi} = 0.2238(11)_{exp}(15)_{latt}(04)_{EW}(02)_{SIB}[22]_{QED}$$

$$|V_{cd}|^{D_s\to K} = 0.258(15)_{exp}(01)_{latt}[03]_{QED}$$ First determination, based on : **BESIII** 1811.02911

* $D \to \pi$ and $D_s \to K$ FF agree at $\lesssim 2\%$ level throughout the kinematic range.

Alternative ways of extracting $|V_{cd(cs)}|$

- In progress $D_{(s)} \to \pi(K)$: **RBC/UKQCD** M. Marshall at Lattice2022 (2201.02680), **ALPHA/CLS** J. Frison at Lattice2022 (PoS (LATTICE2022) 408)
- $\Lambda_c \to \Lambda \ell^+ \nu_\ell$: $N_f = 2+1$ form factors calculated by **Meinel** 1611.09696. Combined with **new BESIII** 2306.02624:

$$|V_{cs}| = 0.937(24)_{latt}(7)_{\tau_{\Lambda_c}}(14)_{\mathcal{B}}$$

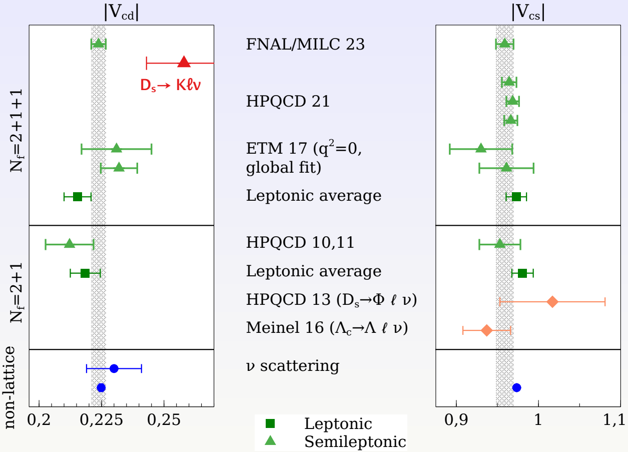
    * Need to reduce LQCD error.

- $B_c \to B^0_{(s)}$ **HPQCD** 2003.00914 $N_f = 2+1+1$ calculation of form factors over complete phys. q^2 range. Measurable at LHCb.
- $\Xi_c \to \Xi \ell^+ \nu_\ell$: First calculation of form factors with LQCD **Q.A. Zhang et at** 2103.07064 gives

  $|V_{cs}| = 0.834(74)_{latt}(127)_{exp}$ together with **Belle** 2103.06496 data

  $|V_{cs}| = 0.883(88)_{latt}(167)_{exp}$ together with **ALICE, J. Zhu,** PoS ICHEP2020 (2021) 524

Second row unitarity

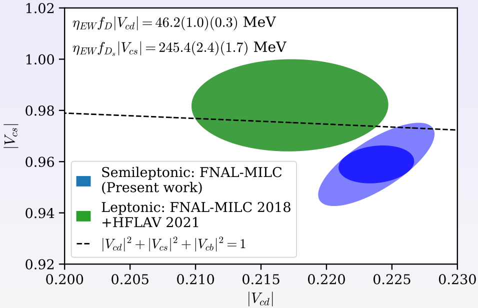


* Exper. inputs for leptonic and semil. $D_{(s)}$:
  ** HFLAV21, except in semileptonic FNAL/MILC23, HPQCD21 and ETM17 (global fit)
  ** $\eta_{EW} = 1.009(2)$
* **Grey bands**: FNAL/MILC 2212.12648 (for visual guidance)

* Large improvement in semileptonic $|V_{cs,cd}|$, but large QED uncertainties.
* Overall good consistency.

Second row unitarity

Second row unitarity is fulfilled within $\sim 1\sigma$.



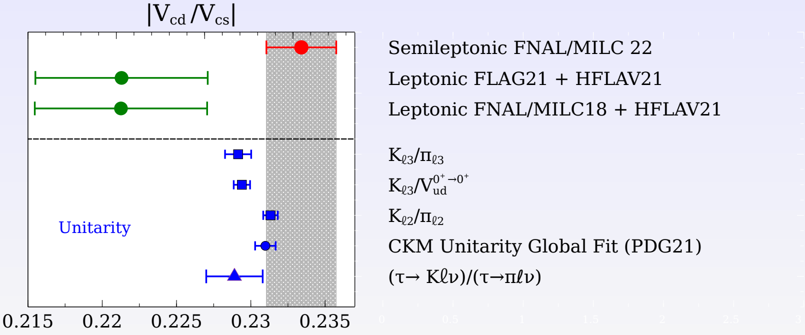
* dotted line: Unitarity with $|V_{cb}|^{inc.+exc.}_{PDG21} = (40.8 \pm 1.4)10^{-3}$
* Inner blue ellipse: No QED errors.

$$\Delta_c = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1$$

For example, using correlated semileptonic results in **FNAL/MILC** 2212.12648:

$$\Delta_c = -0.0286(44)_{exp}(78)_{latt}(28)_{EW}[194]_{QED} = -0.029(22)$$

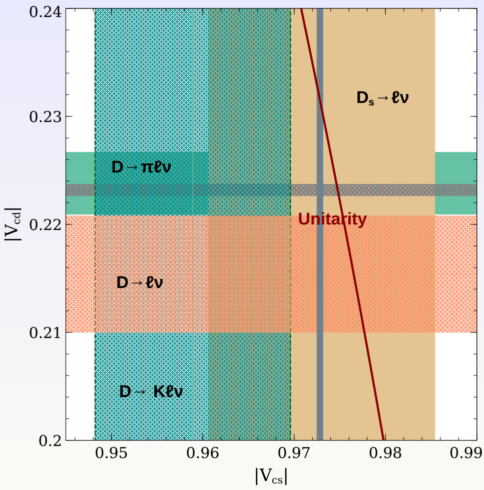
First and second row: consistency checks



- Agreement-tension between leptonic and semileptonic determination at the 2σ level.

(Results for $|V_{us}/V_{ud}|$ are translated to $|V_{cd}/V_{cs}|$ assuming unitarity and correcting up to $\mathcal{O}(\lambda^4)$ using parameters from a global unitarity fit (**PDG21**).)

First and second row: First-column unitarity



Grey lines: $|V_{cd}|_{K\ell 3}$, $|V_{cs}|_{V_{ud}^{0^+\to 0^+}}$
(assumming unitarity and including corrections at $\mathcal{O}(\lambda^4)$)

$$\Delta_d \equiv |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 - 1$$

* Using Hardy&Towner 21 and Cirigliano et al 2208.11707

$V_{ud}^{0^+\to 0^+} = 0.97367(32)$, and
$V_{td} = 8.67(23)\cdot 10^{-3}$ from a global unitarity fit PDG21

and $|V_{cd}|$ from leptonic decays

$$\Delta_d = -0.0054(6)_{V_{ud}}(23)_{V_{cd}}(0)_{V_{td}} = 0.0054(24)$$

or $|V_{cd}|$ from semileptonic decays FNAL/MILC22

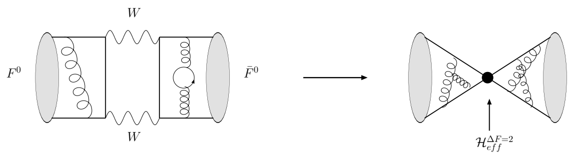
$$\Delta_d = -0.00018(6)_{V_{ud}}(13)_{V_{cd}}(0)_{V_{cb}} = -0.0018(14)$$

Dominant error: $V_{cd}^{\rm semil.}$ QED, $V_{cd}^{\rm lep.}$ experimental (to be improved by Belle II, BESIII to $\sim 1.1\%$ BESIII 2204.08943).

Third row: Consistency tests

B-meson mixing: $|V_{td(ts)}|$

In the Standard Model and beyond, short-distance contributions to the mixing can be described via a $\mathcal{H}_{eff}^{\Delta F=2}$.



In general: $\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i \mathcal{O}_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{\mathcal{O}}_i$

SM:

$$\mathcal{O}_1 = \left(\bar{f}^\alpha \gamma_\mu L\, q^\alpha\right) \left(\bar{f}^\beta \gamma^\mu L\, q^\beta\right)$$
$$\mathcal{O}_2 = \left(\bar{f}^\alpha L\, q^\alpha\right) \left(\bar{f}^\beta L\, q^\beta\right)$$
$$\mathcal{O}_3 = \left(\bar{f}^\alpha L\, q^\beta\right) \left(\bar{f}^\beta L\, q^\alpha\right)$$

BSM:

$$\mathcal{O}_4 = \left(\bar{f}^\alpha L\, q^\alpha\right) \left(\bar{f}^\beta R\, q^\beta\right)$$
$$\mathcal{O}_5 = \left(\bar{f}^\alpha L\, q^\beta\right) \left(\bar{f}^\beta R\, q^\alpha\right)$$

$\widetilde{\mathcal{O}}_{1,2,3} = \mathcal{O}_{1,2,3}$ with the replacement $L(R) \to R(L)$

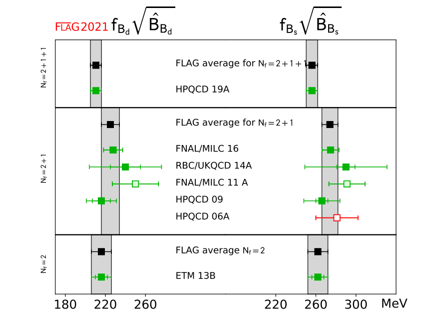
- Recent and on-going lattice calculations of K, D, and B mixing matrix elements for all five operators $\to$ constraints on BSM physics

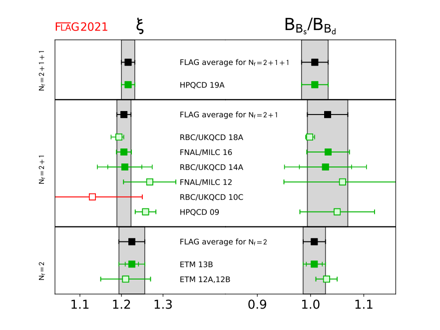
$B-\bar{B}$ mixing

For the mass differences, in the **Standard Model**

$$\Delta M_q \propto \left|V_{tq}^{*}V_{tb}\right|^2 f_{B_q}^2 \hat{B}_{B_q}^{(1)}, \text{ where } \frac{8}{3} f_{B_q}^2 B_{B_q}^{(1)}(\mu) M_{B_q}^2 = \langle \bar{B}^0|\mathcal{O}_1^q|B^0\rangle(\mu)$$

and the $SU(3)$-breaking ratio $\xi = \sqrt{\dfrac{f_{B_s}^2 \hat{B}_{B_s}^{(1)}}{f_{B_d}^2 \hat{B}_{B_d}^{(1)}}} \propto \dfrac{\Delta M_s}{\Delta M_d}\dfrac{|V_{td}|^2}{|V_{ts}|^2}$





* Some tensions for $f_{B_{d(s)}}^2 \hat{B}_{B_{d(s)}}^{(1)}$ need to be understood.

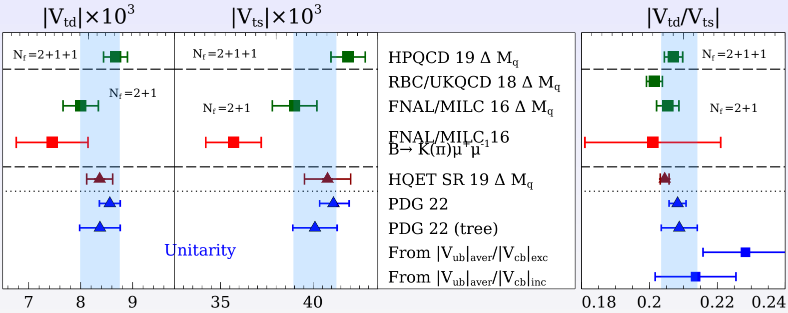
$B - \bar{B}$ mixing

* Work in progress: **RBC/UKQCD + JLQCD**, F. Erben talk at Flavour@TH 2023, CERN, 8-12 May 2023 Both matrix elements and ratios with DWF and NPR.

Room for improvement in lattice calculations: statistics, matching, heavy quarks (relativistic and with a contolled extrapolation to m_b), ...

* 1% error for the bag parameters achievable in the next few years.

$B-$ mixing: $V_{td,ts}$



Full/tree CKM unitarity from PDG22 using all inputs/only tree-level observables.

- B-mixing results dominated by LQCD errors: **HPQCD** 1907.01025, **RBC/UKQCD** 1812.08791, **FNAL/MILC**, 1602.03560
- $B \to K(\pi)\mu^+\mu^-$ results from **FNAL/MILC**, D. Du et al, 1509.06235,1510.02349
  - New experimental data available from **BaBar, Belle,** and **LHCb**
  - New **HPQCD** 2207.13371: $B \to K$ form factors with fully relativistic description. Imply very low values of $|V_{ts}|$.

J. Harrison talk

Conclusions and outlook

Quantity	error(%)	Latt. improvement	Experiments
$\|V_{ud}\|$	-	RC, f_π, nucleon charges	
$\|V_{us}\|_{sem}$ $\|V_{us}\|_{lep}$ $\|V_{us}/V_{ud}\|$	0.18±0.16±0.04 0.4±0.13±0.04 $\sim$ 0.2±0.1±0.1	correlated analyses, f_K (without f_π), QCD+QED	NA62, KLOE-2
$\|V_{us}\|_\tau$		inclus. hadronic decays	Belle II
$\|V_{cd}\|_{lep}$ $\|V_{cd}\|_{sem}$ $\|V_{cs}\|_{lep}$ $\|V_{cs}\|_{sem}$	$0.3 \pm 2.4 \pm 0.7$ $0.7 \pm 0.5 \pm 1.0$ $0.2 \pm 1.0 \pm 0.7$ $0.4 \pm 0.25 \pm 1.0$	FF calculations with other formulations, QCD+QED	BESIII, Belle II
$\|V_{cs}\|_{\Lambda_c}$	2.6 ± 1.7	improve FF calculations	BESIII
$\|V_{td}\|_{\mathrm{B-mix}}$ $\|V_{ts}\|_{\mathrm{B-mix}}$	$2.5 \pm 0.25 \pm 1?$ $2.1 \pm 0.0 \pm 1?$	Reduce error in bag parameters	LHCb, Belle II

Error: LQCD± Exper.± IB/RC/EW