

University  
of Glasgow

Lattice Gauge Theory Contributions to New Physics Searches

# Review of Heavy-Quark Flavour Physics

Judd Harrison  
University of Glasgow

# Outline of the Talk

I will mostly focus on  $b$  decays:

- Lattice methods for simulating  $b$  quarks
- $b \rightarrow c \ell \bar{\nu} : B_q \rightarrow D_q^{(*)} \ell \nu, R(D_{(s)}^{(*)}), A_{FB}$
- $b \rightarrow s \ell^+ \ell^- : B \rightarrow K^{(*)}, R(K^{(*)}), P'_5, B_S \rightarrow \phi \ell^+ \ell^-$
- $b \rightarrow u/d : B \rightarrow \pi, B_S \rightarrow K$

# Outline of the Talk

Disclaimer: This talk will  
contain my own opinions!

I will mostly focus on  $b$  decays:

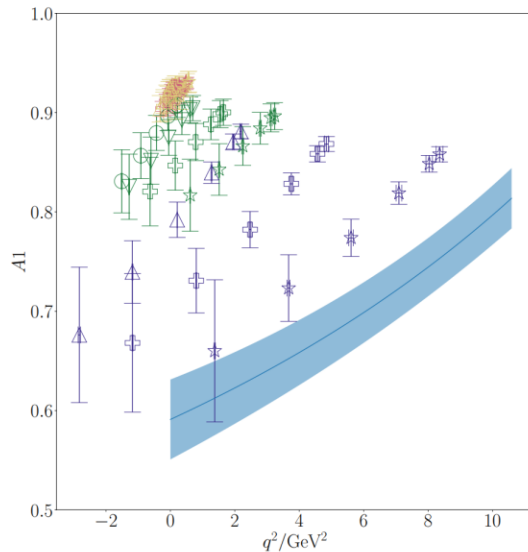
- Lattice methods for simulating  $b$  quarks
- $b \rightarrow c \ell \bar{\nu} : B_q \rightarrow D_q^{(*)} \ell \nu, R \left( D_{(s)}^{(*)} \right), A_{FB}$
- $b \rightarrow s \ell^+ \ell^- : B \rightarrow K^{(*)}, R(K^{(*)}), P_5', B_S \rightarrow \phi \ell^+ \ell^-$
- $b \rightarrow u/d : B \rightarrow \pi, B_S \rightarrow K$

# Relativistic $b$ decays on the lattice

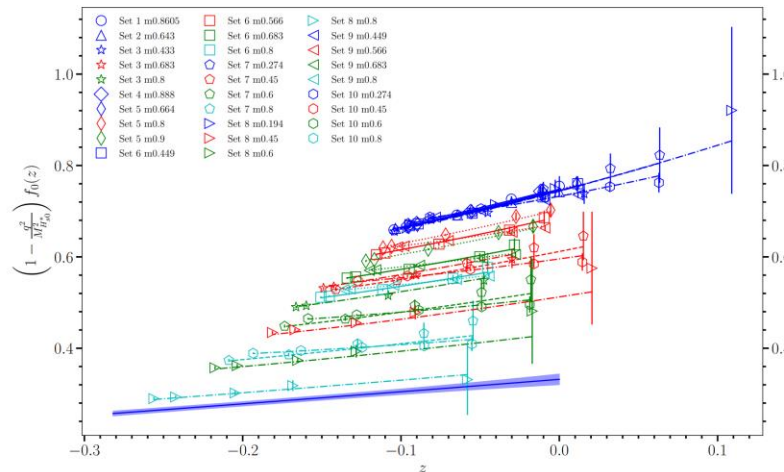
Several recent and currently ongoing lattice calculations of hadronic matrix elements using relativistic  $b$  quarks, e.g.  $B_c$ -meson decays [2003.00914, 2007.06957, 2108.11242],  $B_s$ -meson decays [1906.00701, 2105.11433],  $B$ -meson decays [2203.04938, 2207.12468, 2301.09229, 2304.03137]

## Common approach:

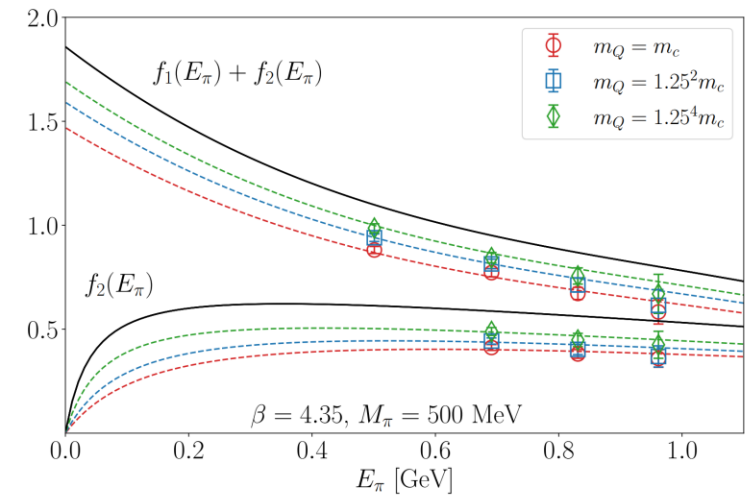
- Perform lattice calculation at multiple  $b$  quark masses at and below  $m_b$ , using the same action for all quarks
- fit results using HQET-like form to disentangle physical mass dependence and control discretisation effects



$B_S \rightarrow D_S^*$  [2105.11433]



$B \rightarrow K$  [2207.12468]



$B \rightarrow \pi$  [2203.04938]

# Relativistic $b$ decays on the lattice

## Advantages:

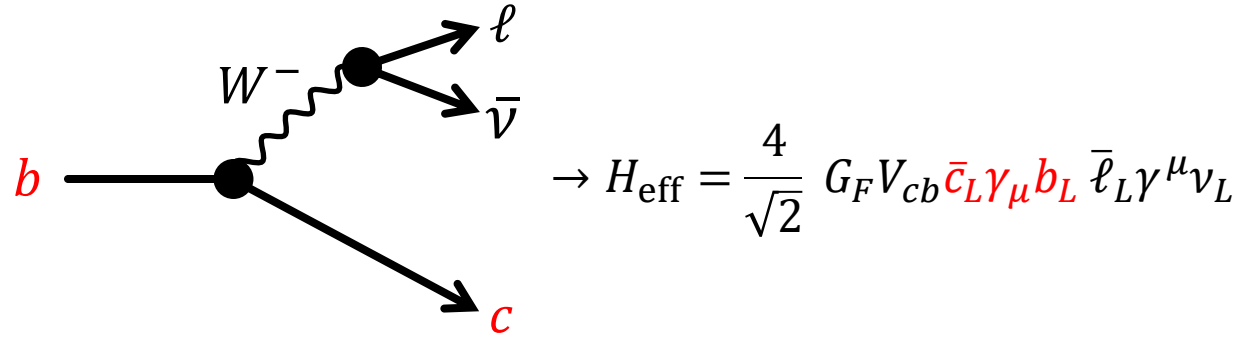
- Allows for nonperturbative renormalisation of currents using, e.g. RI-SMOM, partially conserved current relations.
- Connects  $b$ - and  $c$ -decays and gives heavy-mass dependence - test of HQET.
- Statistics limited

## Challenges:

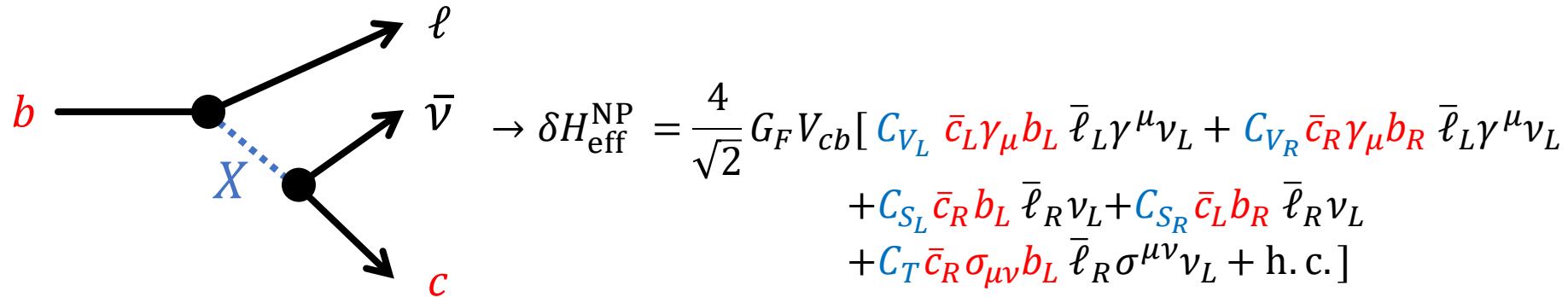
- Must compute and analyse many more correlation functions
- Fitting correlation functions simultaneously is difficult
- Some subtlety in choice of fit function, e.g. which basis to use for form factors

# $b \rightarrow c \ell \bar{\nu}$

- In Standard Model (SM) mediated at tree level by the weak interaction



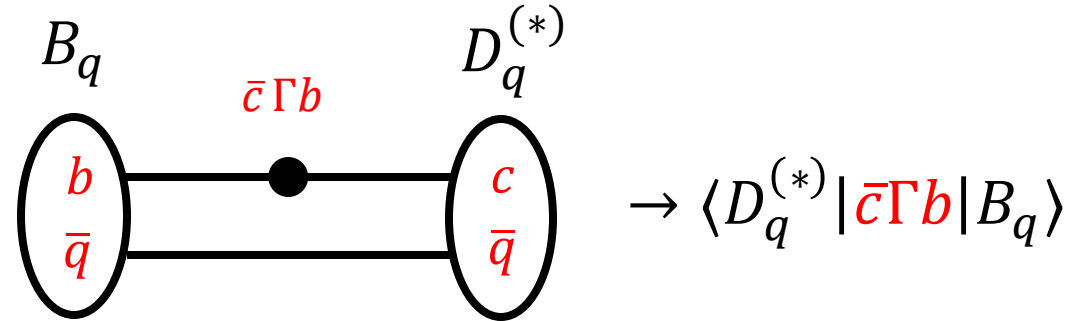
- New Physics could modify this coupling, e.g. leptoquarks (LQ)



# Exclusive $b \rightarrow c \ell \bar{\nu}$ Decays

In nature,  $b$  and  $c$  quarks appear confined within hadrons

- Theory predictions require nonperturbative matrix elements of operators in  $H_{\text{eff}}$  between QCD bound states



These are typically parameterised in terms of **form factors (FFs)** according to the Lorentz and helicity structure of the decay

- 3 independent FFs for P to P, 7 for P to V

$$\langle D_q | \bar{c} b | B_q \rangle = \sqrt{M_{B_q} M_{D_q}} (w + 1) h_S(w),$$

$$\langle D_q | \bar{c} \gamma^\mu b | B_q \rangle = \sqrt{M_{B_q} M_{D_q}} [h_+(w)(v + v')^\mu + h_-(w)(v - v')^\mu],$$

$$\langle D_q | \bar{c} \sigma^{\mu\nu} b | B_q \rangle = i \sqrt{M_{B_q} M_{D_q}} [h_T(w)(v'^\mu v^\nu - v'^\nu v^\mu)].$$

$$\langle D_q^* | \bar{c} \gamma^5 b | B_q \rangle = -\sqrt{M_{B_q} M_{D_q}} h_P(w) (\epsilon^* \cdot v),$$

$$\langle D_q^* | \bar{c} \gamma^\mu b | B_q \rangle = i \sqrt{M_{B_q} M_{D_q}} h_V(w) \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* v'_\alpha v_\beta,$$

$$\langle D_q^* | \bar{c} \gamma^\mu \gamma^5 b | B_q \rangle = \sqrt{M_{B_q} M_{D_q}} [h_{A_1}(w)(w + 1) \epsilon^{*\mu} -$$

$$h_{A_2}(w) (\epsilon^{*\mu} \cdot v) v^\mu - h_{A_3}(w) (\epsilon^{*\mu} \cdot v) v'^\mu],$$

$$\langle D_q^* | \bar{c} \sigma^{\mu\nu} b | B_q \rangle = -\sqrt{M_{B_q} M_{D_q}} \epsilon^{\mu\nu\alpha\beta} [h_{T_1}(w) \epsilon_\alpha^* (v + v')_\beta +$$

$$h_{T_2}(w) \epsilon_\alpha^* (v - v')_\beta + h_{T_3}(w) (\epsilon^* \cdot v) v_\alpha v'_\beta].$$

$$v = p_{B_q} / M_{B_q}, \quad v' = p_{D_q^{(*)}} / M_{D_q^{(*)}}, \quad w = v \cdot v'$$

# Exclusive $b \rightarrow c\ell\bar{\nu}$ Decays

Lattice calculations of the FFs are progressing rapidly, with many new results in the last few years:

Fermilab-MILC: 2+1 asqtad, Wilson-clover  $b$  and  $c$  quarks

HPQCD: 2+1+1 HISQ, heavy-HISQ  $b$

JLQCD: 2+1 Möbius domain-wall, Möbius domain-wall  $b$

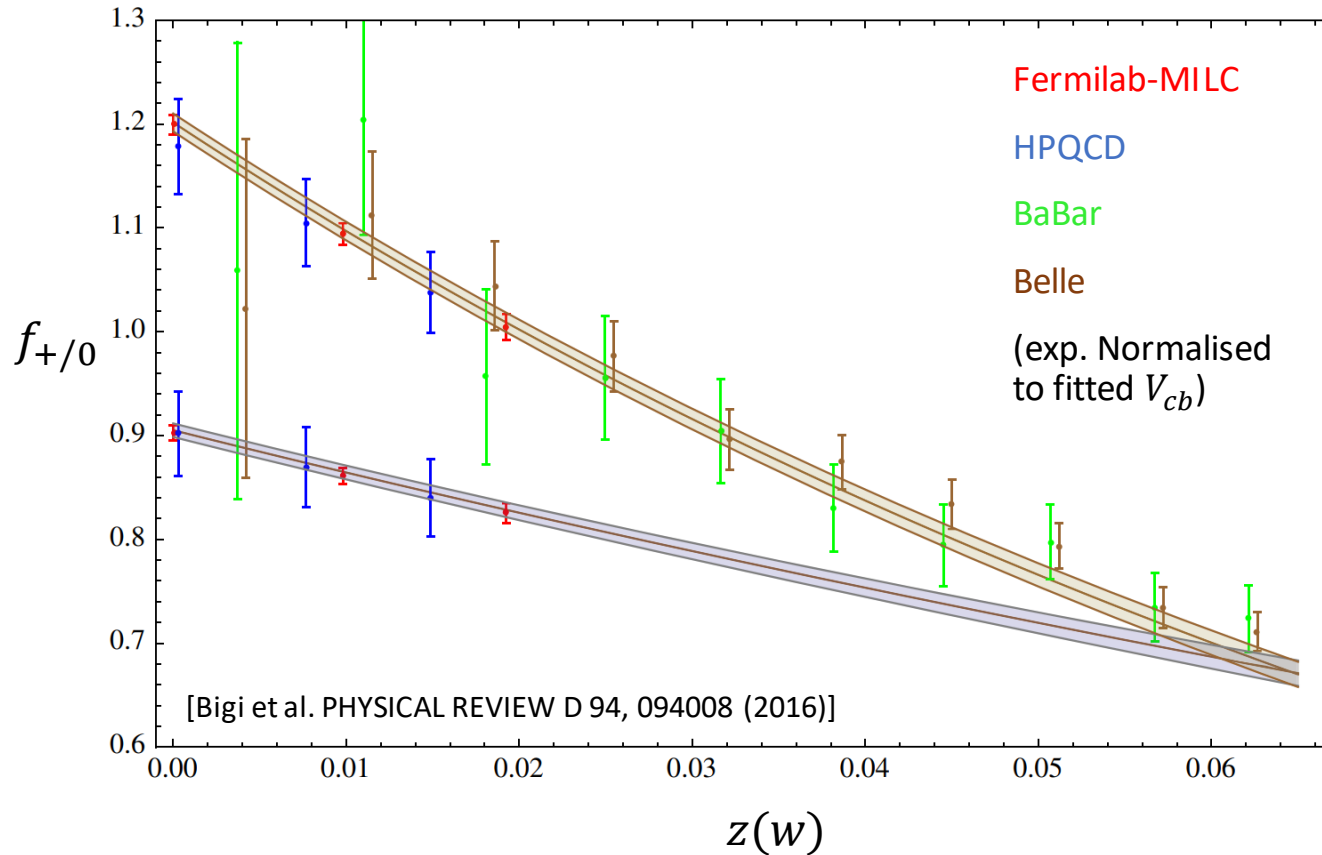
|                                  | $h_{\pm}(w)$                             | $h_T(w)$ | $h_{A_{1,2,3},V}(w)$                                   | $h_{T_{1,2,3}}(w)$ |
|----------------------------------|--|----------|--|--------------------|
| $B \rightarrow D^{(*)}$          | ✓ [1503.07237]<br>✓ [1505.03925*]<br>(✓) | (✓)      | ✓ [2105.14019]<br>(✓) [2304.03137]<br>(✓) [2306.05657] | (✓) [2304.03137]   |
| $B_S \rightarrow D_S^{(*)}$      | ✓ [1906.00701]                           |          | ✓ [2105.11433]<br>(→[2304.03137])                      | (✓) [2304.03137]   |
| $B_c \rightarrow J/\psi(\eta_c)$ |  |          | ✓ [2007.06957]   |                    |

\* 2+1 asqtad, NRQCD  $b$  quarks, HISQ  $c$  quarks



# $B \rightarrow D \ell \bar{\nu}$

Good agreement between lattice calculations for SM FFs and also with experiment!



$$\frac{d\Gamma}{dw} = |V_{cb}|^2 G(w)^2 R(w),$$

where for  $\ell = \mu$  or  $e$

$$G(w) = \frac{2\sqrt{M_B M_D}}{M_D + M_B} \times f_+(w).$$

This gives the most recent averages

$$V_{cb}^{\text{HFLAV}} = 39.14 \pm 0.92_{\text{exp}} \pm 0.36_{\text{th}} \times 10^{-3}$$

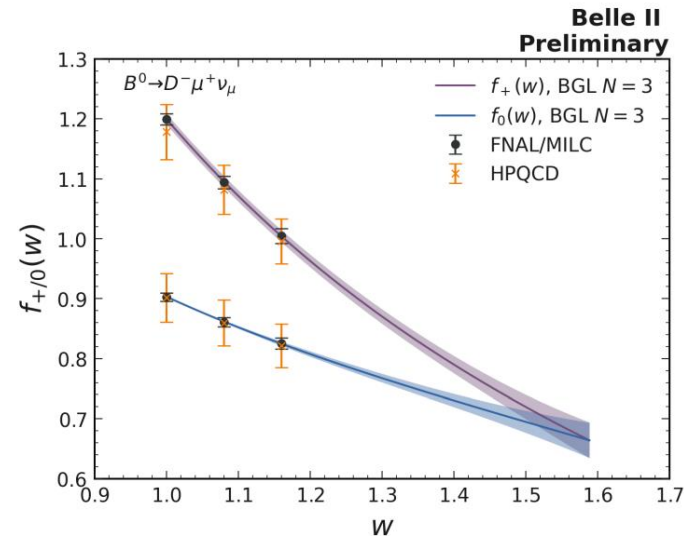
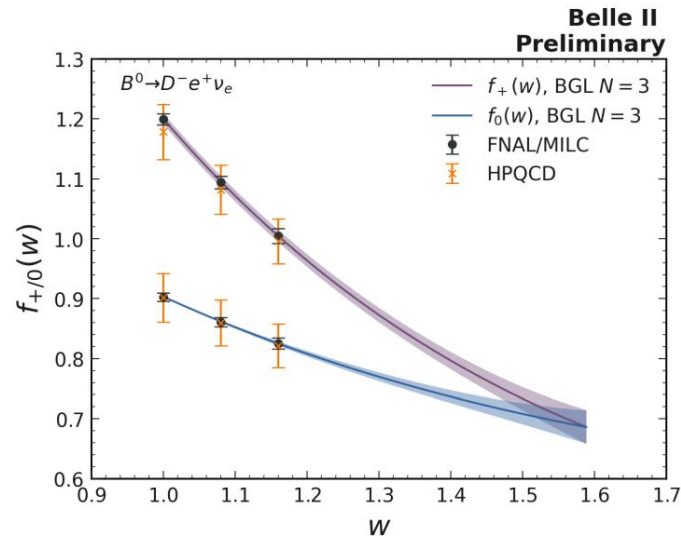
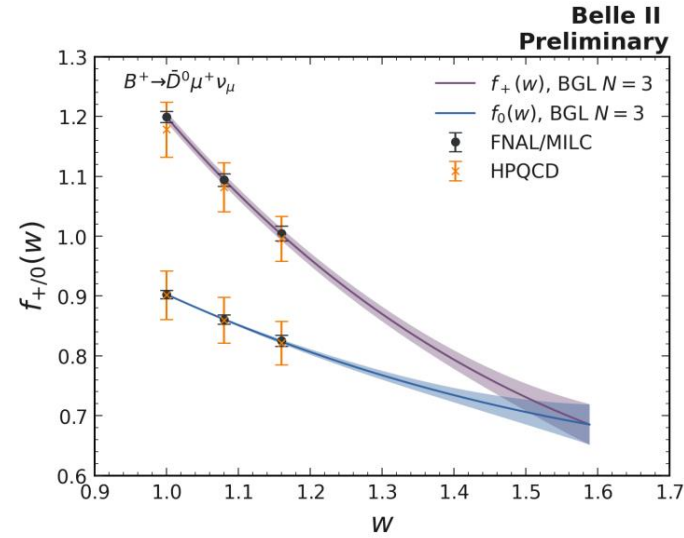
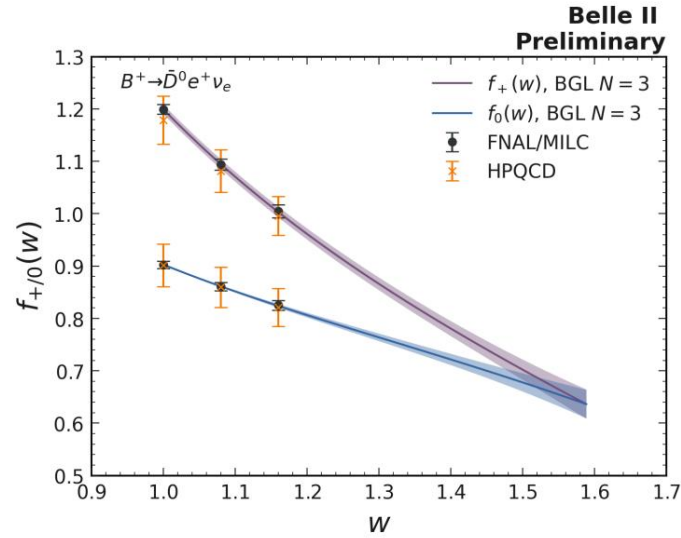
$$R_{\text{th}}^{\text{HFLAV}}(D) = \frac{\Gamma(B \rightarrow D\tau\bar{\nu}_\tau)}{\Gamma(B \rightarrow D\mu\bar{\nu}_\mu)} = 0.298 \pm 0.004$$

$$R_{\text{exp}}^{\text{HFLAV}}(D) = 0.339 \pm 0.030$$

# $B \rightarrow D \ell \bar{\nu}$ Belle II [2210.13143]

New results from Belle II using  $189\text{fb}^{-1}$  integrated luminosity also agree well with theory

- Note the limited  $w$  range of old lattice calculations
- Calculations underway at Fermilab-MILC collaboration to update with all-HISQ calculation



Clockwise from top left:  
 $\frac{\chi^2}{\text{dof}} = \frac{18.2}{14}, \frac{12.3}{14}, \frac{11.0}{14}, \frac{15.1}{14}$

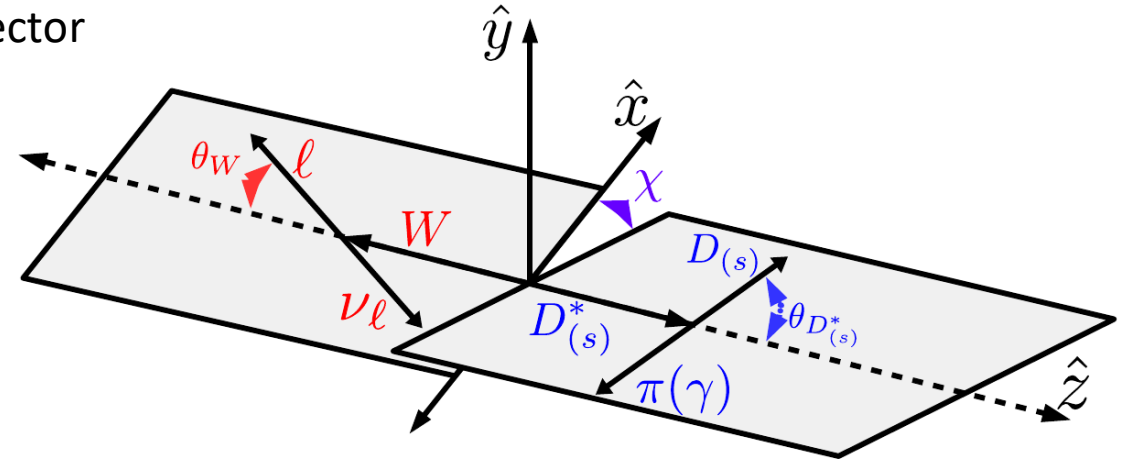
# $B \rightarrow D^* \ell \bar{\nu}$

- Lattice calculation harder than for  $B \rightarrow D$  due to noisier vector and larger number of FFs.
- Rich angular structure due to vector  $D^*$  final state
- Angular asymmetry observables, e.g.

$$A_{FB} = \frac{1}{\Gamma} \left[ \int_0^1 - \int_{-1}^0 \right] \frac{d\Gamma}{d\cos(\theta_W)} d\cos(\theta_W)$$

$$A_{\lambda_\ell} = \frac{\Gamma^{\lambda_\ell = -\frac{1}{2}} - \Gamma^{\lambda_\ell = +\frac{1}{2}}}{\Gamma}$$

$$F_L = \frac{\Gamma^{\lambda_{D^*} = 0}}{\Gamma}$$



3 LQCD results for 4 SM FFs away from zero recoil

Published:

**Fermilab-MILC: 2+1 asqtad, Wilson-clover  $b$  and  $c$  quarks**

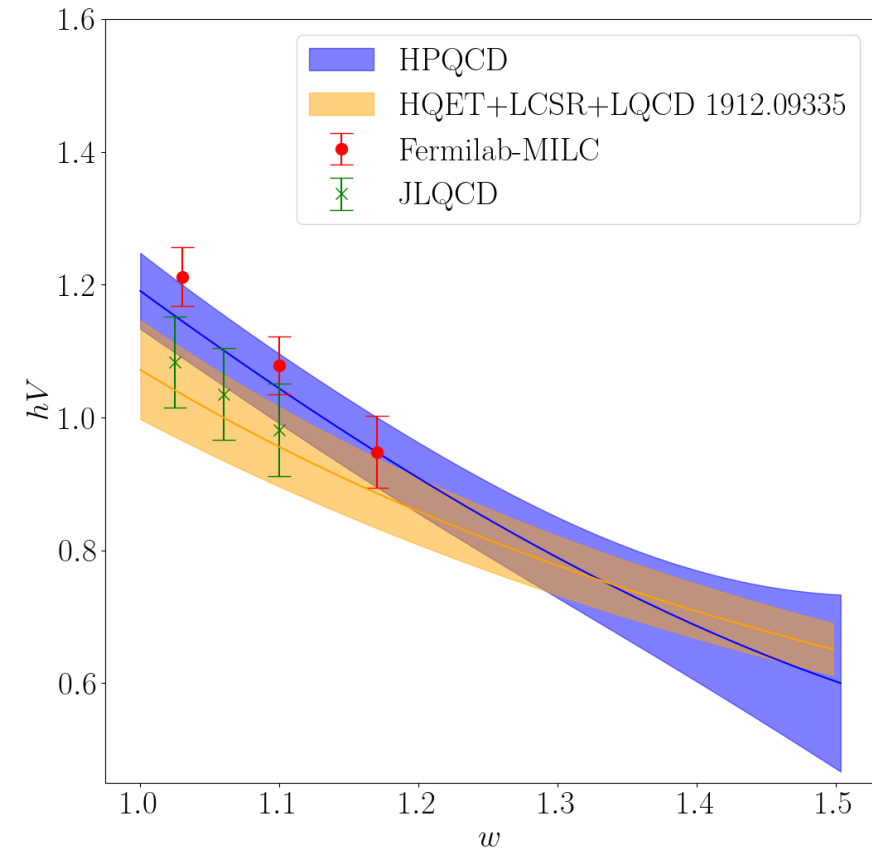
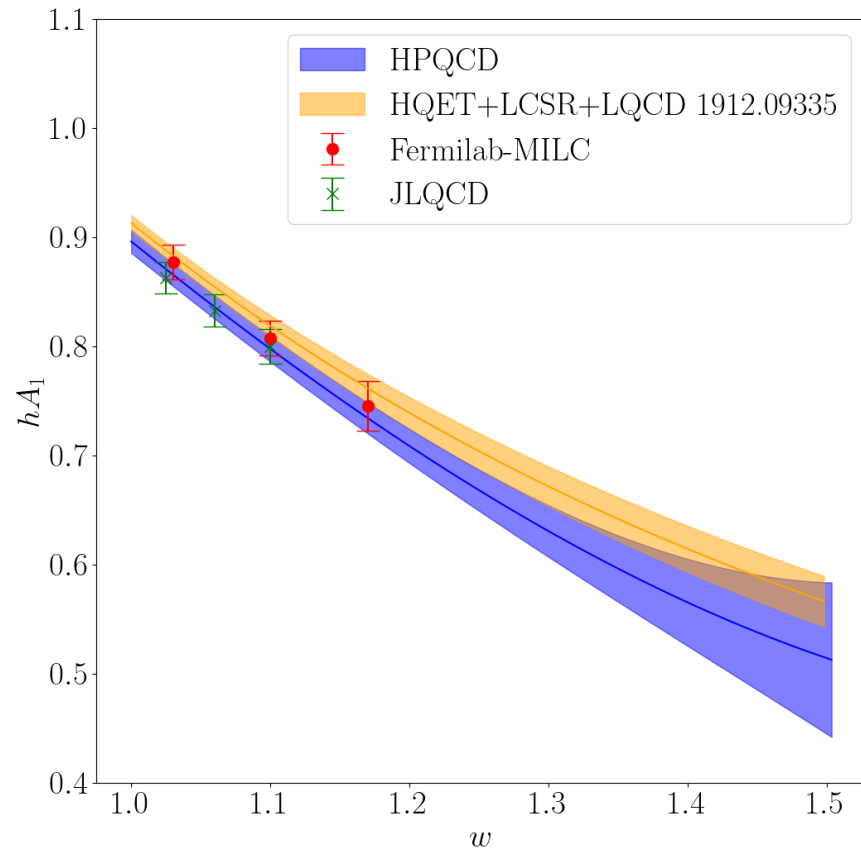
arxiv:

**HPQCD: 2+1+1 HISQ, heavy-HISQ  $b$  (+ Tensor FFs)**

**JLQCD: 2+1 Möbius domain-wall**

# $B \rightarrow D^* \ell \bar{\nu}$

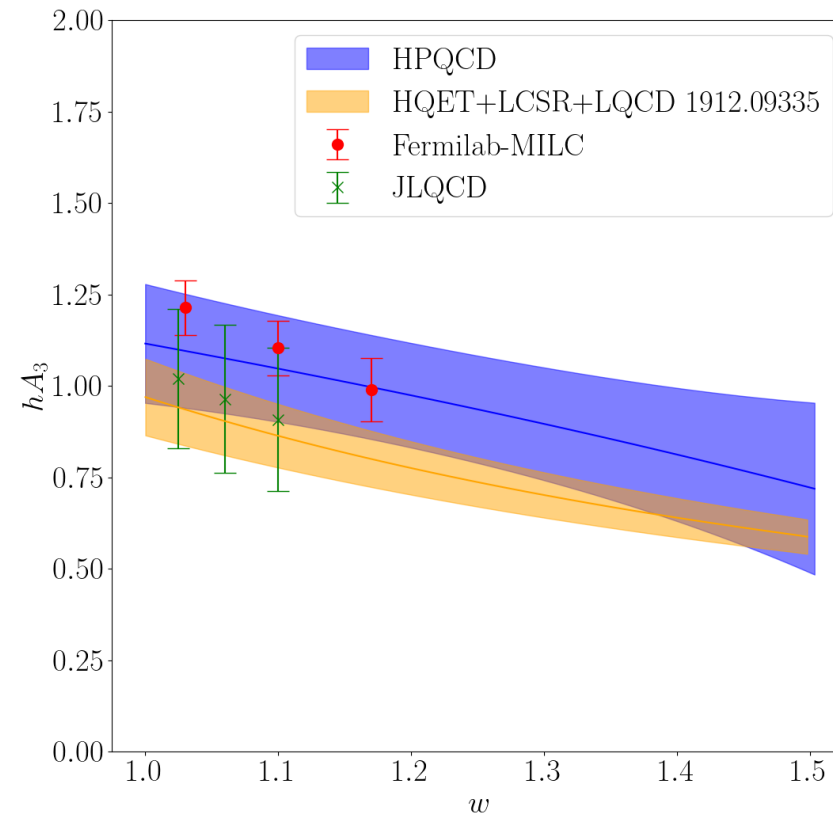
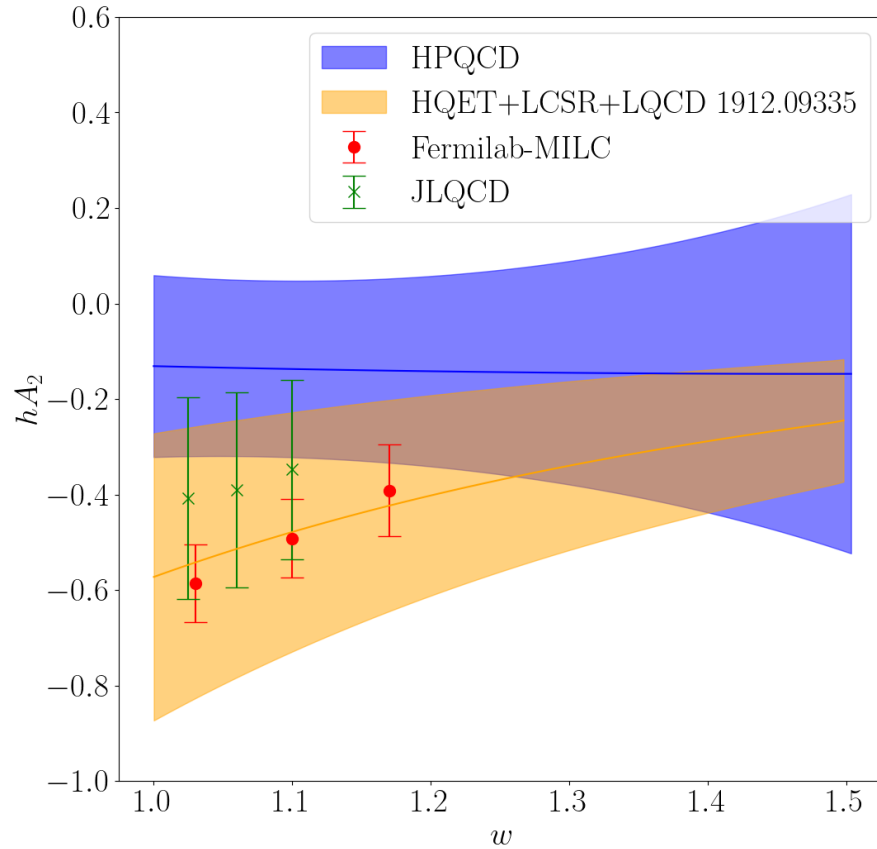
Good agreement between lattice calculations of the SM FFs  $h_{A_1}$  and  $h_V$



[2105.14019, 2304.03137, 2306.05657]

# $B \rightarrow D^* \ell \bar{\nu}$

agreement between lattice calculations of SM FFs  $h_{A_2}$  and  $h_{A_3}$  less good



[2105.14019, 2304.03137, 2306.05657]

# $B \rightarrow D^* \ell \bar{\nu}$

Some tension with experimental shape  $\approx 2\sigma$ .

Exclusive, model-independent  $V_{cb}$  using full range of Exp. data and lattice FFs

$$|V_{cb}^{\text{FNAL}}| = 38.40 \pm 0.78 \times 10^{-3}$$

$$|V_{cb}^{\text{HPQCD}}| = 39.31 \pm 0.74 \times 10^{-3}$$

Both in good agreement with 2021 HFLAV exclusive average, using  $B \rightarrow D^{(*)} \ell \bar{\nu}$  and  $B_s \rightarrow D_s^{(*)} \ell \bar{\nu}$ :

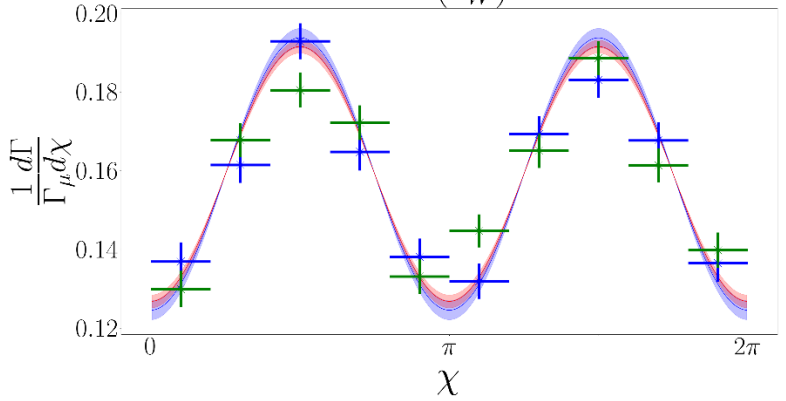
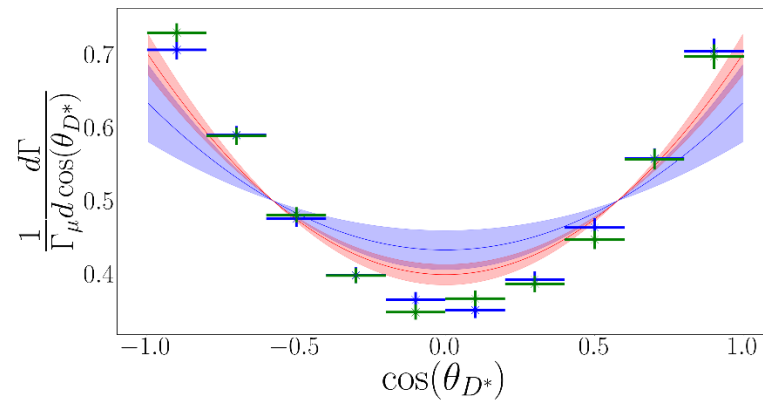
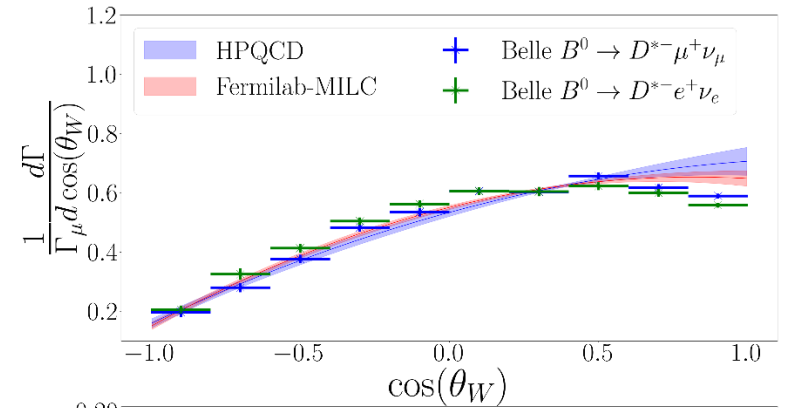
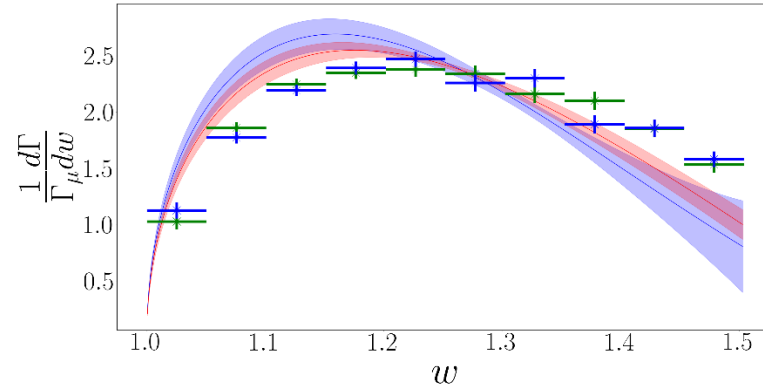
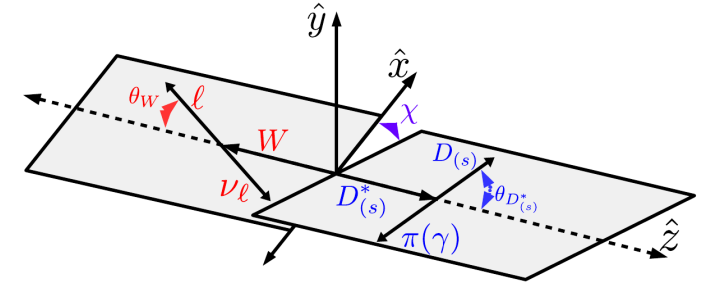
$$|V_{cb}^{\text{HFLAV}}| = 38.90 \pm 0.53 \times 10^{-3}$$

Compare the inclusive result:

$$|V_{cb}^{\text{HFLAV}}| = 42.19 \pm 0.78 \times 10^{-3}$$

In tension at the level of  $\approx 3.5\sigma$

$$V_{cb}^{\text{JLQCD}} = 39.19(90) \times 10^{-3}$$



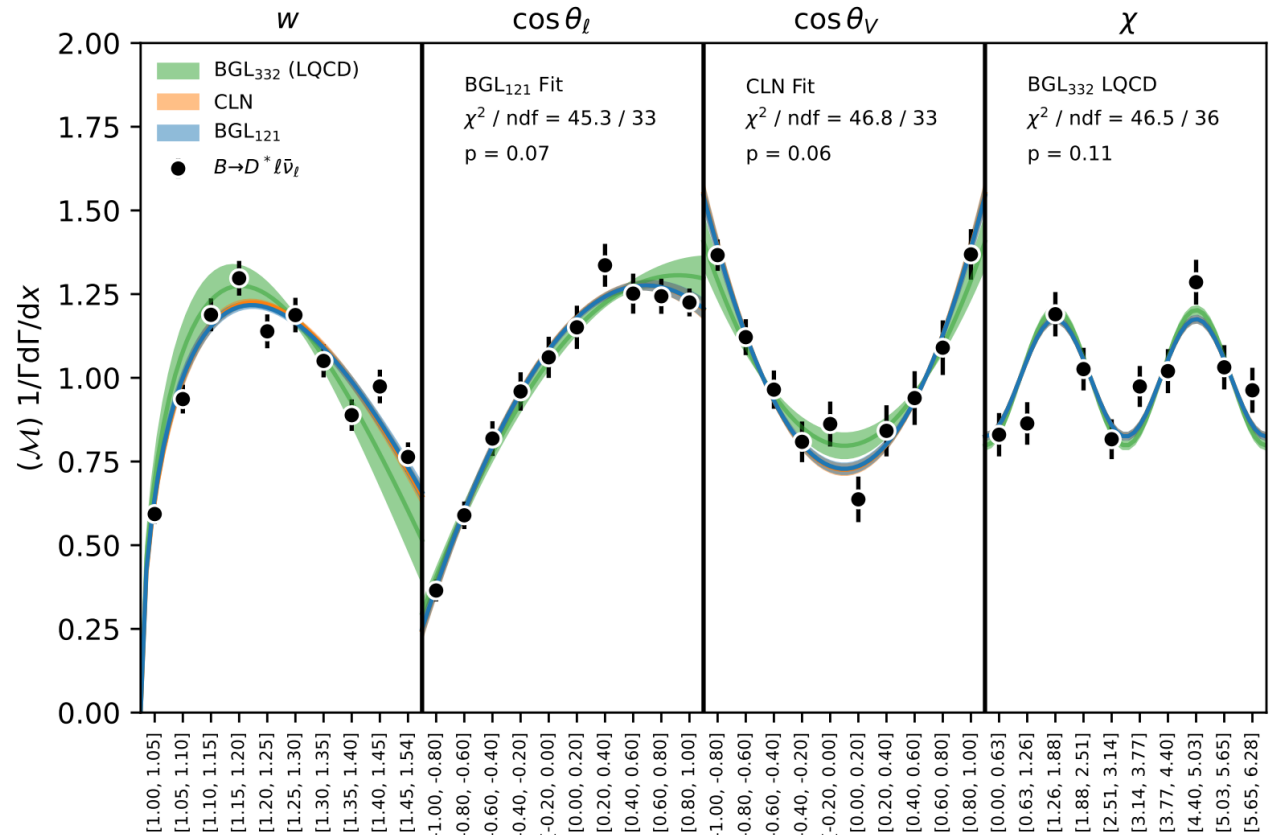
# $B \rightarrow D^* \ell \bar{\nu}$ Belle [2301.07529]

New results from Belle for  $\bar{B}^0$  and  $B^-$  mode shapes (Green BGL332 band is Fermilab-MILC 2105.14019) using full  $711\text{fb}^{-1}$ . Using zero recoil lattice results together with HFLAV branching fractions they find:

$$|V_{cb}|_{\text{BGL}} = 40.6 \pm 0.9 \times 10^{-3}.$$

Also find angular asymmetry variables for light modes consistent with lattice-only SM results

|   | Belle<br>2301.07529 | HPQCD<br>2304.03137  |
|---|---------------------|----------------------|
| $A_{FB}^{\ell=e}$                         | $0.230 \pm 0.019$   | $0.274 \pm 0.023$    |
| $A_{FB}^{\ell=\mu}$                       | $0.252 \pm 0.020$   | $0.270 \pm 0.024$    |
| $A_{FB}^{\ell=\mu} - A_{FB}^{\ell=e}$     | $0.022 \pm 0.027$   | $-0.0035 \pm 0.0009$ |
| $\frac{F_L^{\ell=e} + F_L^{\ell=\mu}}{2}$ | $0.501 \pm 0.012$   | $0.430 \pm 0.036$    |



# $R(D^*)$

$$R^{JLQCD}(D^*) = 0.252(22)$$

With lattice FFs across the full kinematic range, can determine a theory-only  $R(D^*)$  without including experimental information on FFs. Find:

$$R(D^*) = \frac{\Gamma(B \rightarrow D^* \tau \bar{\nu}_\tau)}{\Gamma(B \rightarrow D^* \mu \bar{\nu}_\mu)}$$

Using lattice results only,

$$R^{\text{FNAL}}(D^*) = 0.265 \pm 0.013,$$

$$R^{\text{HPQCD}}(D^*) = 0.279 \pm 0.013.$$

However, fitting FFs to lattice+experiment gives

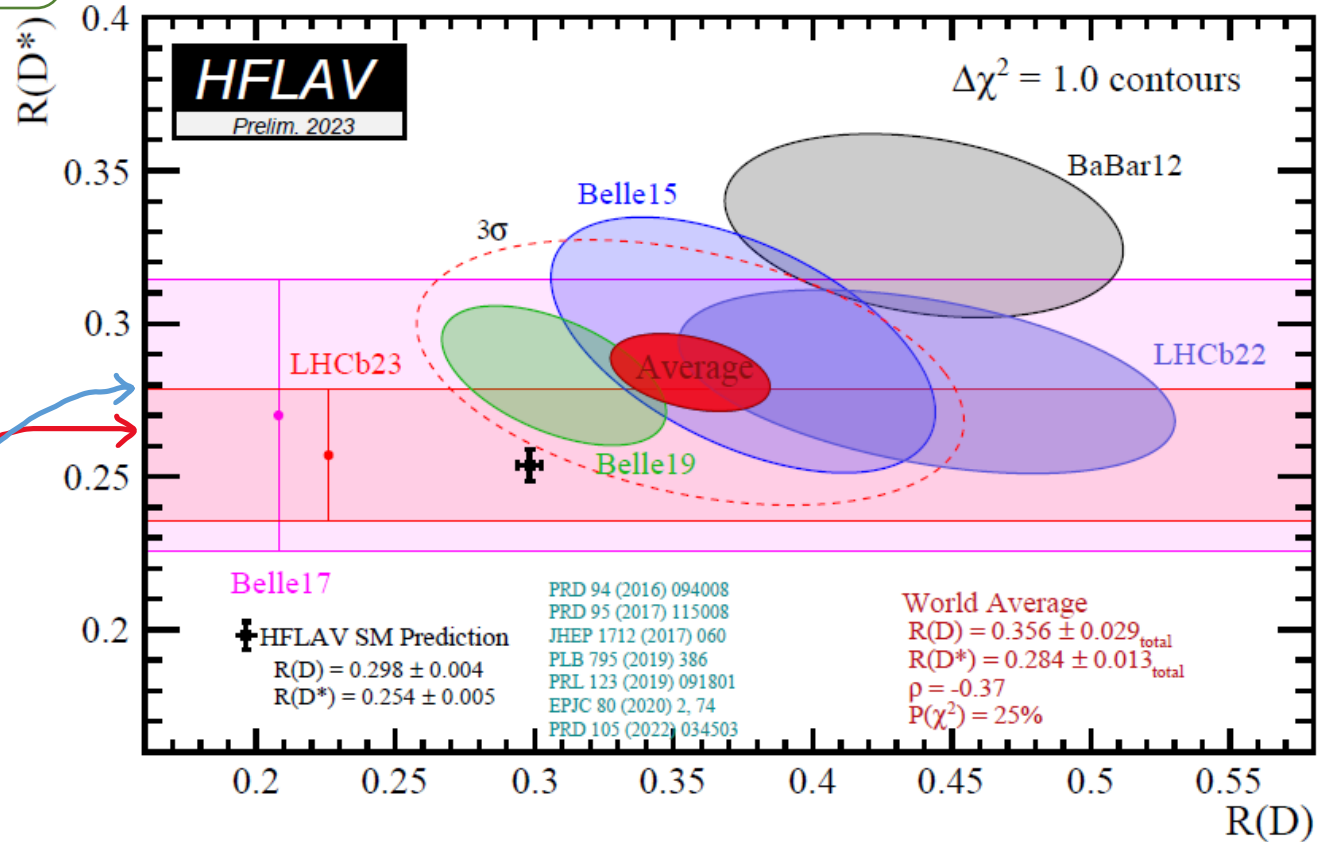
$$R_{\text{latt+exp}}^{\text{FNAL}}(D^*) = 0.2484 \pm 0.0013,$$

$$R_{\text{latt+exp}}^{\text{HPQCD}}(D^*) = 0.2471 \pm 0.0019.$$

Most recent measurements (including new 2023 LHCb measurement!) in good agreement with SM, but  $\approx 3\sigma$  tension remains with average.

Need to improve understanding of the shape of the lattice FFs for  $B \rightarrow D^* \ell \bar{\nu}$

- HQET fits to lattice  $B \rightarrow D$  + zero recoil lattice  $B \rightarrow D^*$  + QCDSR + LCSR agree with determinations using exp. data as input





# Constraining NP in $b \rightarrow c\ell\bar{\nu}$ using $B \rightarrow D^{(*)}\ell\bar{\nu}$

These measurements can be used to constrain NP appearing in the light leptonic mode.

- The patterns of Wilson coefficients generated by different tree-level models are [1801.01112]:

$$\delta H_{\text{eff}}^{\text{NP}} = \frac{4}{\sqrt{2}} G_F V_{cb} [$$

$$C_{V_L} \bar{c}_L \gamma_\mu b_L \bar{\ell}_L \gamma^\mu \nu_L$$

$$+ C_{V_R} \bar{c}_R \gamma_\mu b_R \bar{\ell}_L \gamma^\mu \nu_L$$

$$+ C_{S_L} \bar{c}_R b_L \bar{\ell}_R \nu_L$$

$$+ C_{S_R} \bar{c}_L b_R \bar{\ell}_R \nu_L$$

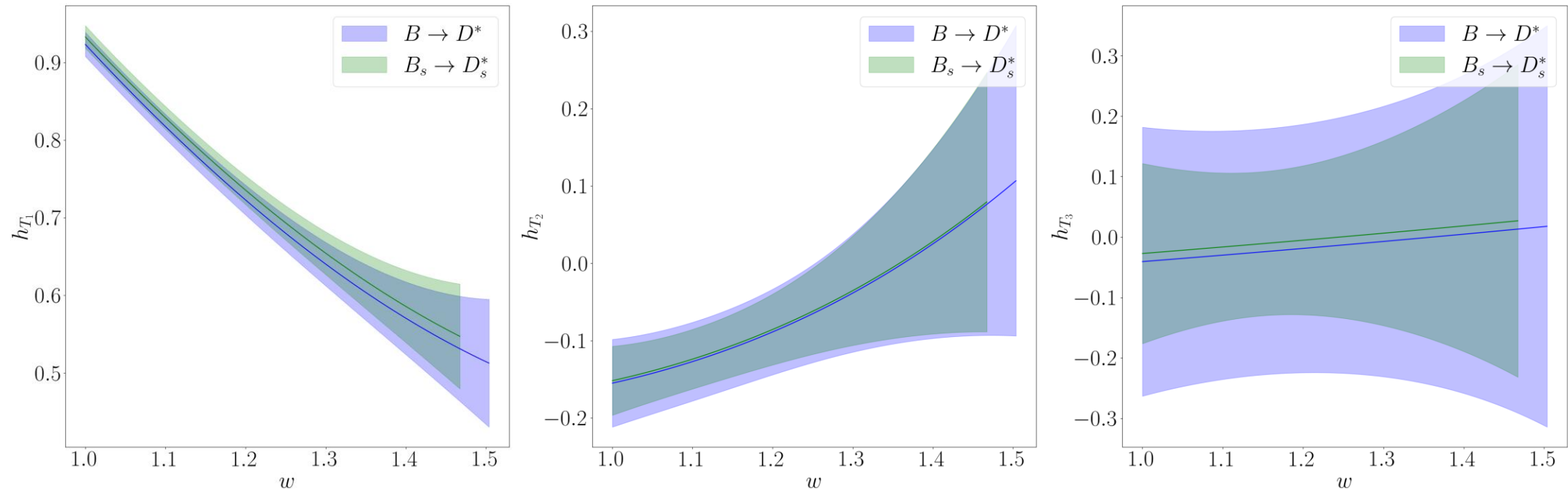
$$+ C_T \bar{c}_R \sigma_{\mu\nu} b_L \bar{\ell}_R \sigma^{\mu\nu} \nu_L + \text{h. c.}]$$

Note that  $C_{V_L}$  may be absorbed into  $V_{cb}$

| Tree-Level Models   | $C_{V_L}$ | $C_{V_R}$ | $C_{S_L}$ | $C_{S_R}$ | $C_T = C_{S_L}/4$ | $C_T = -C_{S_L}/4$ |
|---------------------|-----------|-----------|-----------|-----------|-------------------|--------------------|
| Vector-like singlet | ✓         |           |           |           |                   |                    |
| Vector-like doublet |           | ✓         |           |           |                   |                    |
| $W'$                | ✓         |           |           |           |                   |                    |
| $H^\pm$             |           |           | ✓         | ✓         |                   |                    |
| $S_1$               | ✓         |           |           |           |                   | ✓                  |
| $R_2$               |           |           |           |           | ✓                 |                    |
| $S_3$               | ✓         |           |           |           |                   |                    |
| $U_1$               | ✓         | ✓         |           |           |                   |                    |
| $V_2$               |           |           |           | ✓         |                   |                    |
| $U_3$               | ✓         |           |           |           |                   |                    |

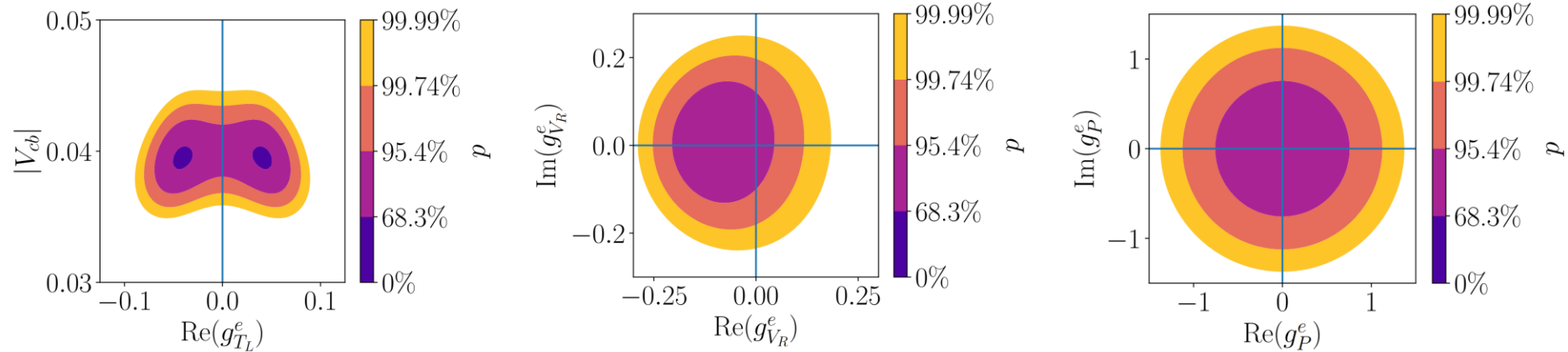
# Constraining NP in $b \rightarrow c\ell\bar{\nu}$ using $B \rightarrow D^{(*)}\ell\bar{\nu}$

Lattice tensor FFs are now available for  $B \rightarrow D^*\ell\bar{\nu}$  and  $B_s \rightarrow D_s^*\ell\bar{\nu}$  from HPQCD [2304.03137]:



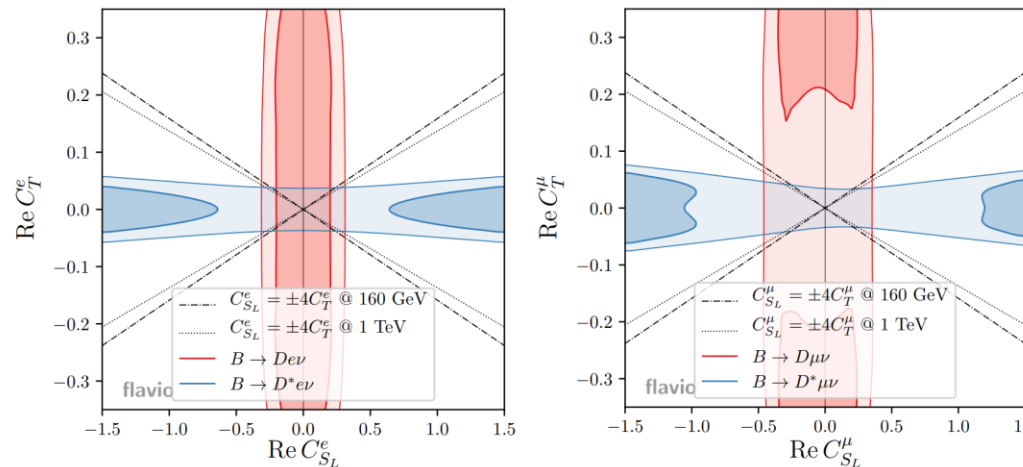
# Constraining NP in $b \rightarrow c\ell\bar{\nu}$ using $B \rightarrow D^{(*)}\ell\bar{\nu}$

Constraints for Wilson coefficients using just  $B \rightarrow D^{(*)}\ell\bar{\nu}$  with e.g.  $\ell = e$  are all consistent with the SM [2304.03137]



Note that  $B \rightarrow D^*\ell\bar{\nu}$  is much more sensitive to a tensor coupling than  $B \rightarrow D\ell\bar{\nu}$ , and vice-versa for the scalar coupling

→ Ideally want fully correlated lattice calculation of all  $B \rightarrow D^{(*)}\ell\bar{\nu}$  FFs

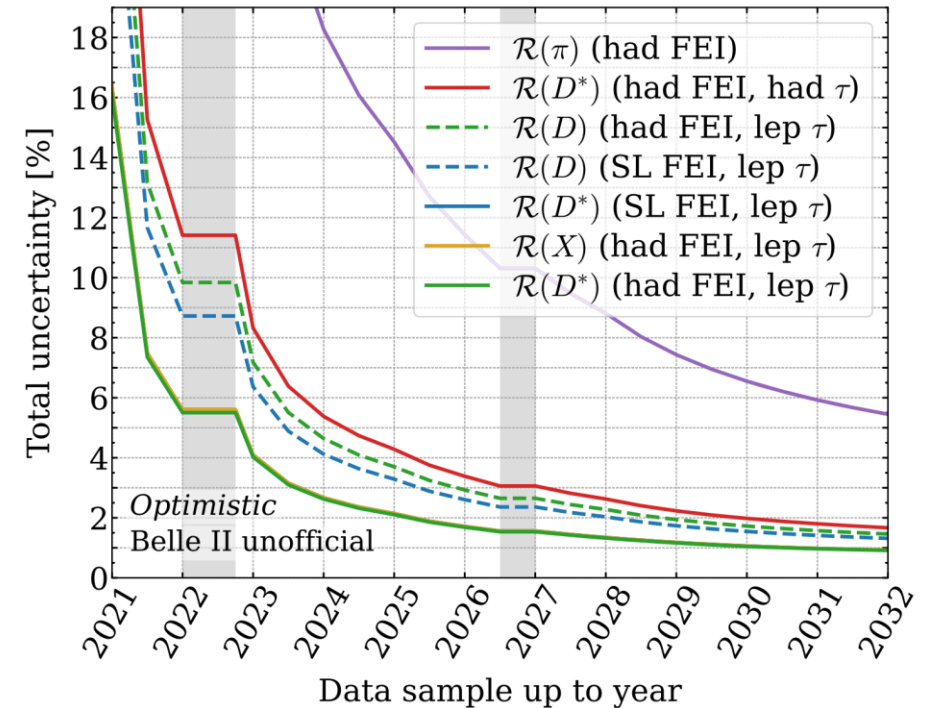
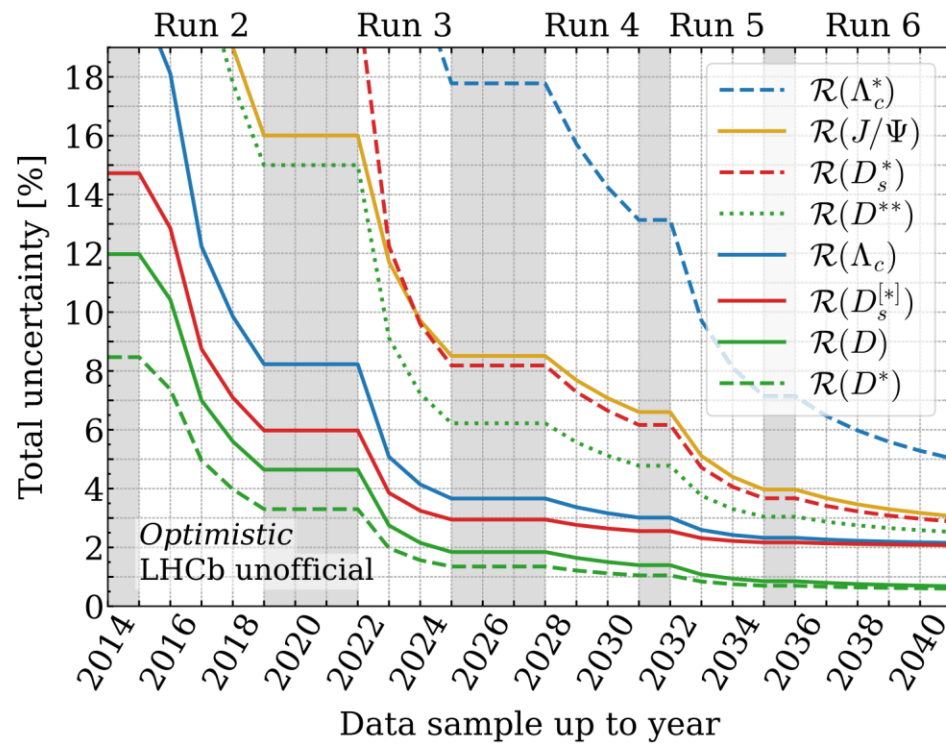


[Jung et al. JHEP01(2019)009]

# Experimental Outlook for $b \rightarrow c\ell\bar{\nu}$

Optimistically, uncertainties of  $R$ -ratio measurements may reach percent level

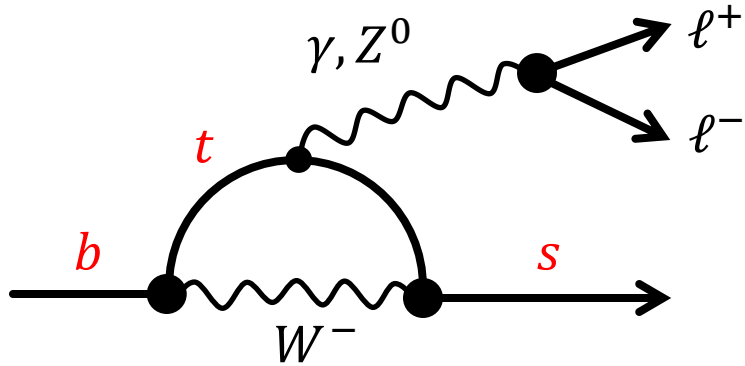
- Commensurate uncertainties on the theory side would require treatment of QED effects
- Most recent lattice only results give  $R(J/\psi) = 0.2582(38)$ ,  $R(D_s^*) = 0.265(9)$ , much more precise than experiments are likely to obtain soon
- Differential decay rate data from Belle II will allow for further tests of angular asymmetries



[[Bernlochner et al. Rev. Mod. Phys. 94, 015003](#)]

# $b \rightarrow s \ell^+ \ell^-$

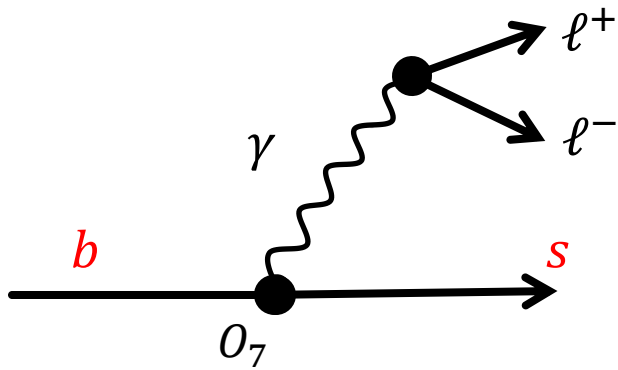
1-loop in the Standard Model



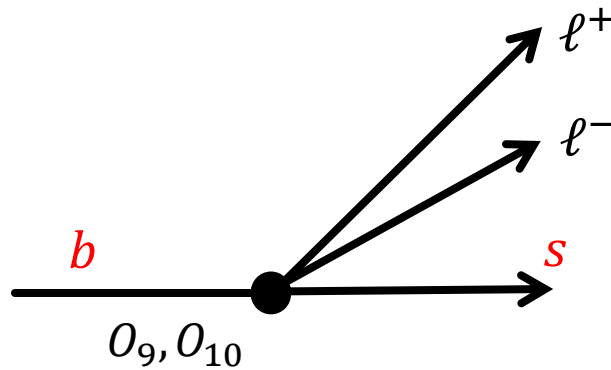
$$\rightarrow H_{\text{eff}} = -\frac{4}{\sqrt{2}} G_F \left[ V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + V_{ub} V_{us}^* \times \dots \right]$$

$$\mu = m_b$$

Main contributions from  $H_{\text{eff}}$  are from local hadronic operators



$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$$



$$O_9 = \frac{e^2}{16\pi^2} m_b \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \ell,$$

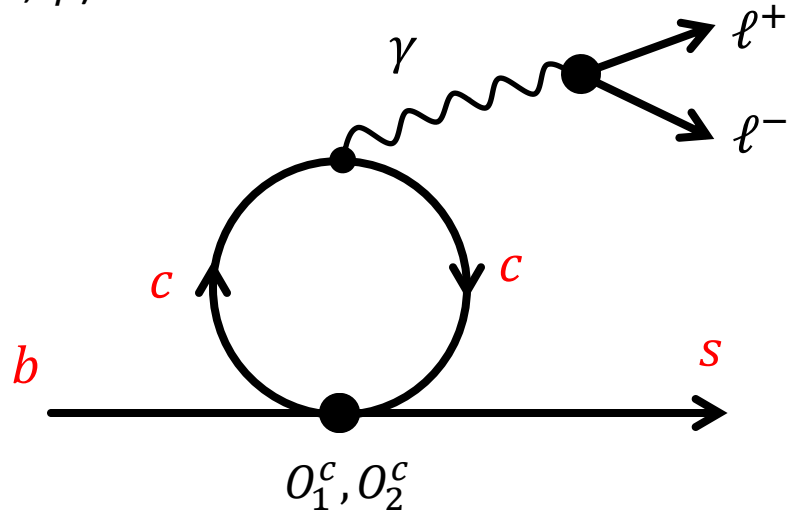
$$O_{10} = \frac{e^2}{16\pi^2} m_b \bar{s}_L \gamma^\mu b_L \bar{\ell} \gamma_\mu \gamma_5 \ell$$

→ 'local' FFs:

$$\langle M(p') | O_i^{\text{had}} | B(p) \rangle \equiv F_\lambda^{B \rightarrow M}(q^2) S_\lambda(p', p)$$

$$b \rightarrow s \ell^+ \ell^-$$

Non-local contributions from  $O_1^c, O_2^c$  coupling to  $J/\psi, \psi(2S)$ , as well as on-shell states coupling to  $\gamma$  from  $O_7$  (e.g.  $\rho, \omega, \phi$ )



$$O_1^c = \bar{s}_L \gamma^\mu c_L \bar{c}_L \gamma_\mu b_L, \quad O_2^c = \bar{s}_L^j \gamma^\mu c_L^i \bar{c}_L^i \gamma_\mu b_L^j$$

→ ‘non-local’ FFs:

$$H_\lambda^{B \rightarrow M}(q^2) S_\lambda(p', p)$$

$$\equiv \langle M(p') | T \{ j_\mu^{\text{em}}, \sum_{i=1}^2 C_i O_i^c + \sum_{i=3}^6 C_i O_i + C_8 O_8 \} | B(p) \rangle$$

These non-local contributions are not well understood close to resonances

- Dispersive bound for non-local FFs give model independent constraints, control truncation error, include data for e.g.  $B \rightarrow K J/\psi \ell^+ \ell^-$  [2011.09813, 2206.03797]
- Usual solution: exclude veto regions with  $q^2$  around  $M_{\text{res}}^2$
- Local FFs still dominate uncertainties

$$b \rightarrow s \ell^+ \ell^-$$

Lattice calculations for  $b \rightarrow s$  FFs are less advanced than for  $b \rightarrow c$

Fermilab-MILC: 2+1 asqtad, Wilson-clover  $b$

HPQCD: 2+1+1 HISQ, heavy-HISQ  $b$

|                             | $h_{\pm}(w)$                     | $h_T(w)$                         | $h_{A_{1,2,3},V}(w)$ | $h_{T_{1,2,3}}(w)$ |
|-----------------------------|----------------------------------|----------------------------------|----------------------|--------------------|
| $B \rightarrow K^{(*)}$     | ✓ [2207.12468]<br>✓ [1509.06235] | ✓ [2207.12468]<br>✓ [1509.06235] | ✓ [1310.3722*]       | ✓ [1310.3722*]     |
| $B_s \rightarrow \phi$      |                                  |                                  | ✓ [1310.3722*]       | ✓ [1310.3722*]     |
| $B_c \rightarrow D_s^{(*)}$ | ✓ [2108.11242]                   | ✓ [2108.11242]                   |                      |                    |

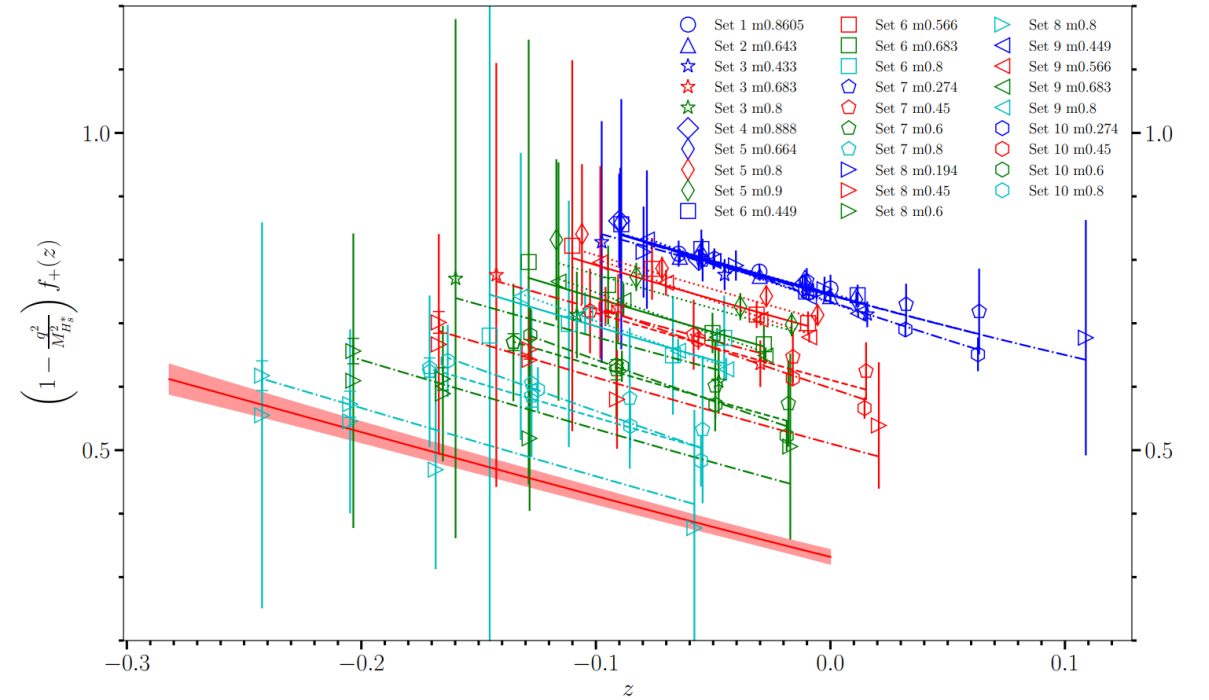
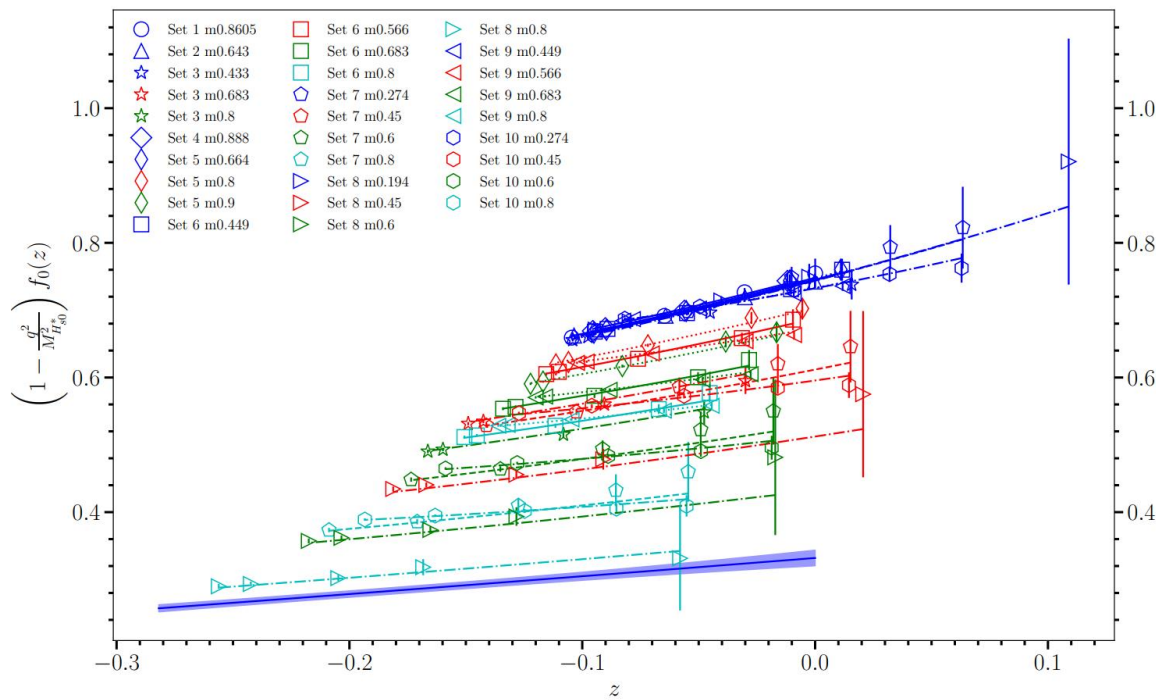
\* 2+1 asqtad, NRQCD  $b$  quarks

Note that the HPQCD calculation of the  $B \rightarrow K$  SM+Tensor FFs [2207.12468] also included SM+Tensor  $D \rightarrow K$  FFs. The Fermilab-MILC collaboration has also computed the  $D \rightarrow \pi$ ,  $D \rightarrow K$  and  $D_s \rightarrow K$  SM FFs using 2+1+1 HISQ for all quarks [2212.12648], with work in progress to extend this calculation to the  $B$ .

# $B \rightarrow K \ell^+ \ell^-$ [2207.12468]

W. G. Parrott, C. Bouchard, and C. T. H. Davies: 2+1+1 HISQ, heavy-HISQ  $b$

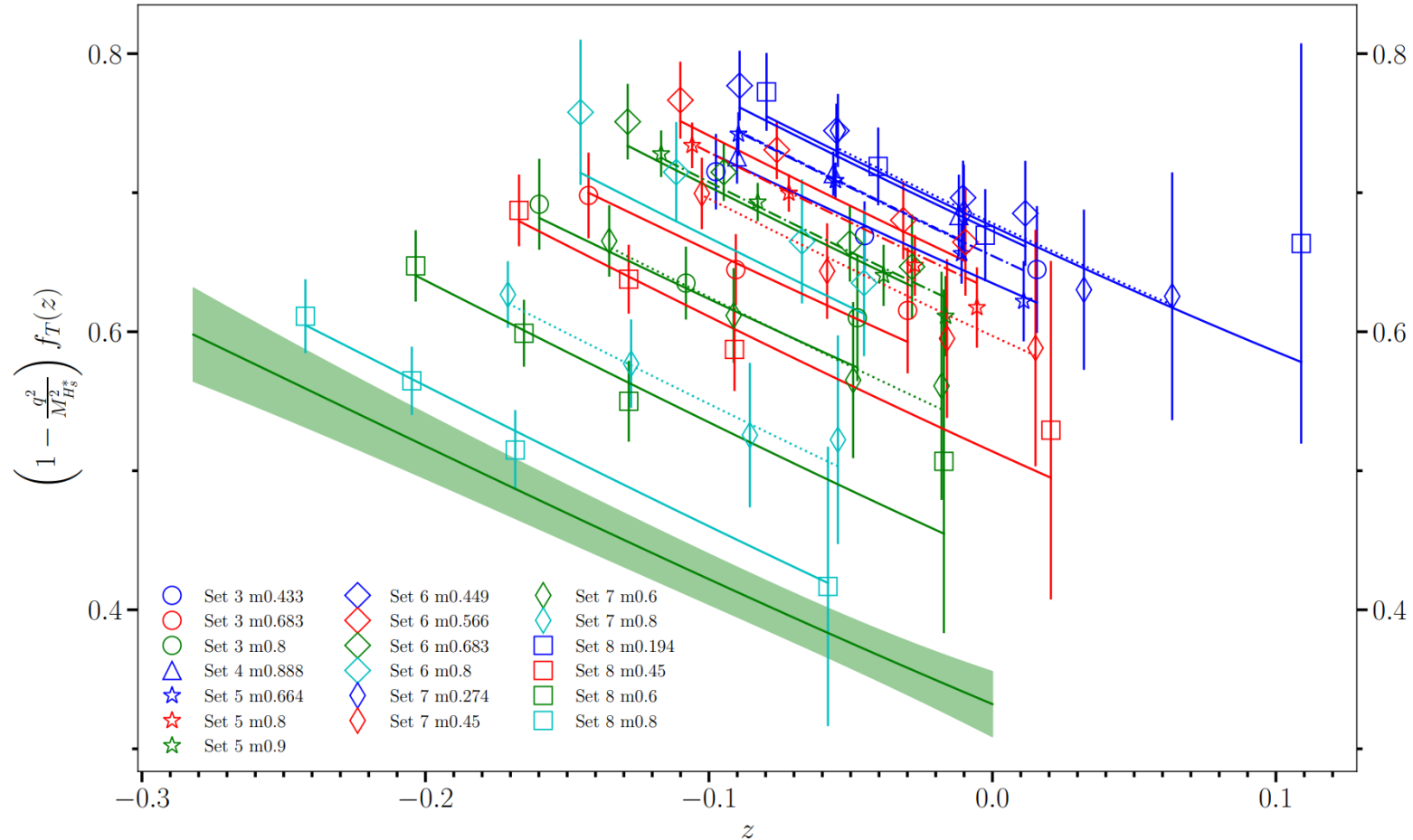
BCL parameterisation:  $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$ ,  $f^Y(q^2) \sim L_\chi \times P(q^2, M_{b\bar{s}}^{\text{res}_Y^2}) \times \sum a_n^Y \left( \frac{\Lambda_{\text{QCD}}}{M_H}, am_h, a\Lambda_{\text{QCD}} \right) z^n$





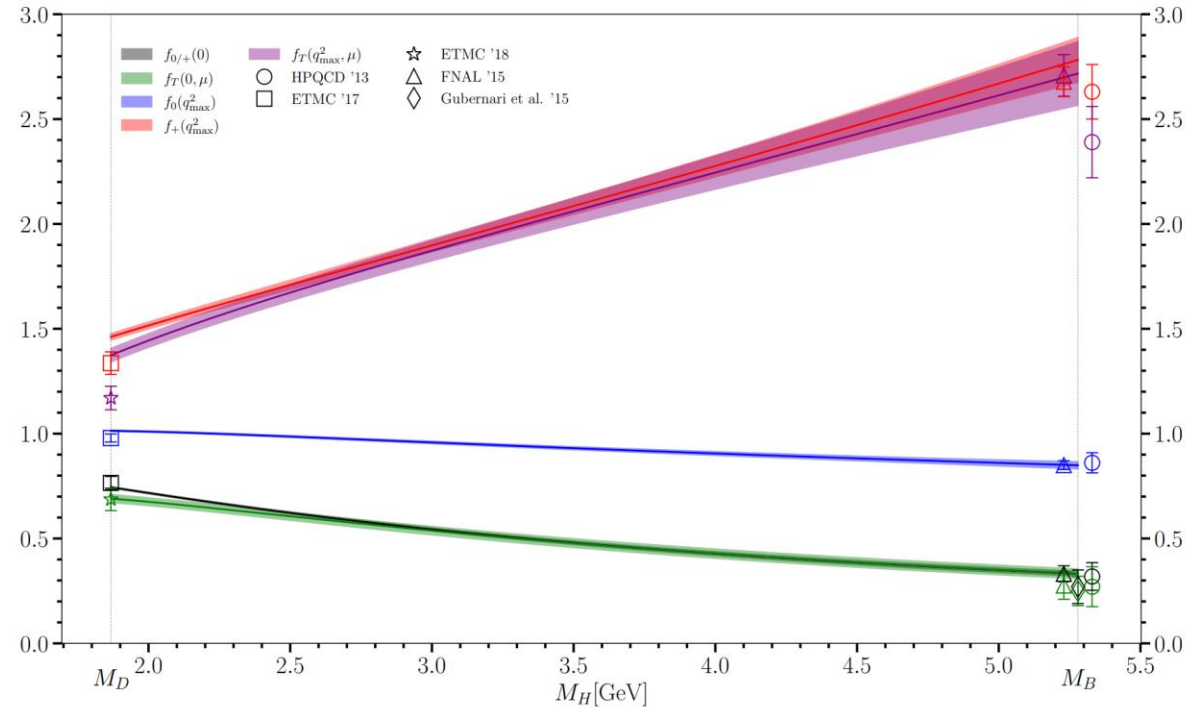
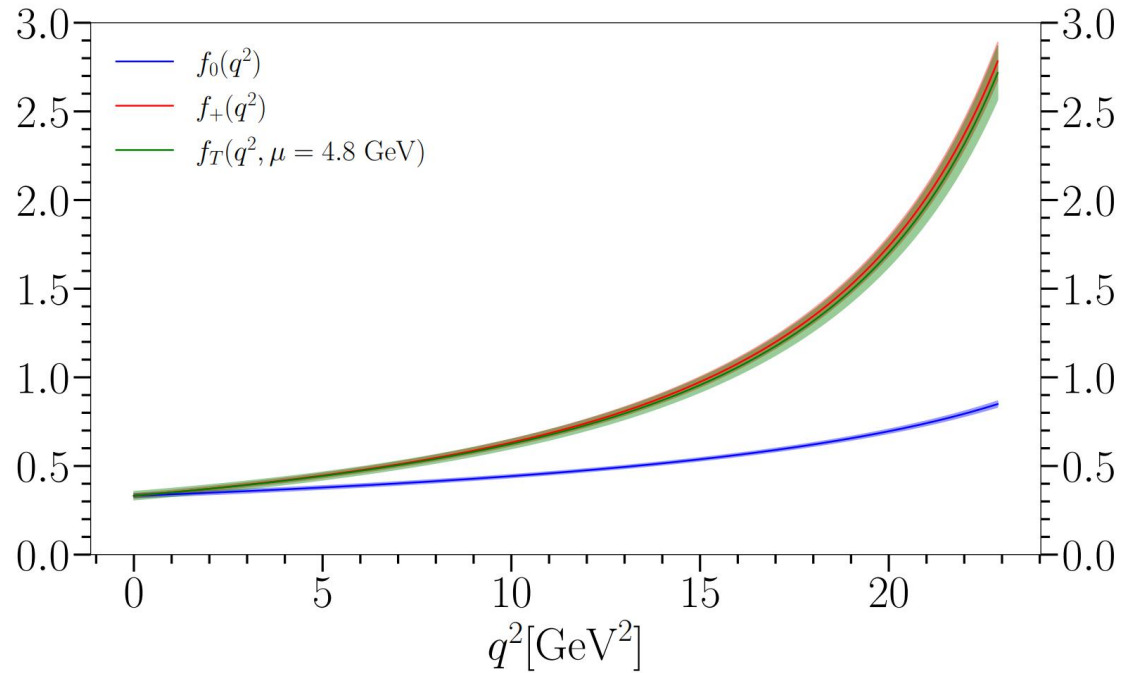
# $B \rightarrow K \ell^+ \ell^-$ [2207.12468]

W. G. Parrott, C. Bouchard, and C. T. H. Davies: 2+1+1 HISQ, heavy-HISQ  $b$



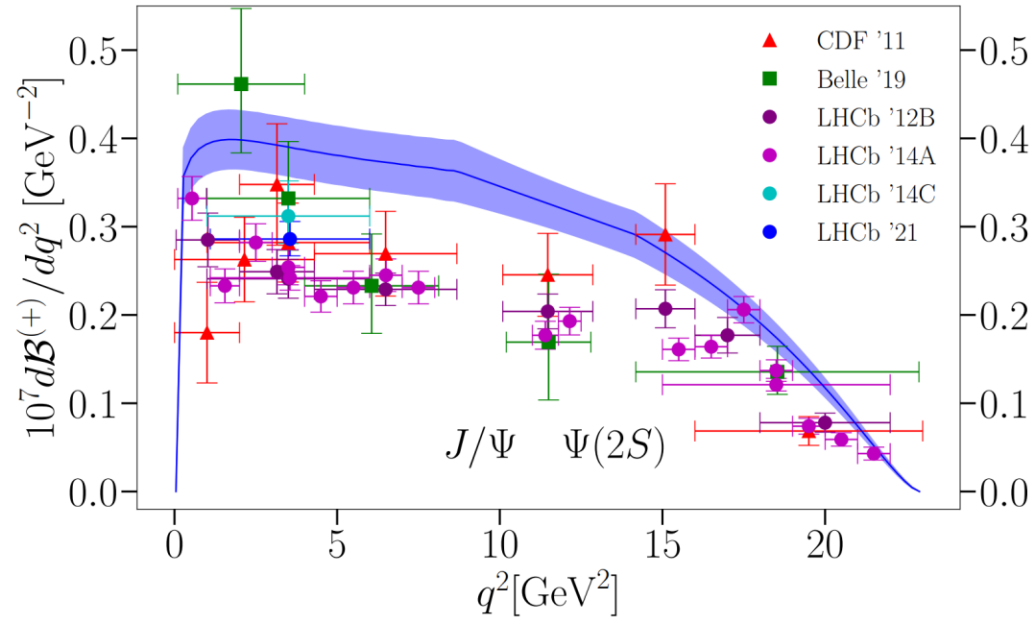
# $B \rightarrow K \ell^+ \ell^-$ [2207.12468]

W. G. Parrott, C. Bouchard, and C. T. H. Davies: 2+1+1 HISQ, heavy-HISQ  $b$



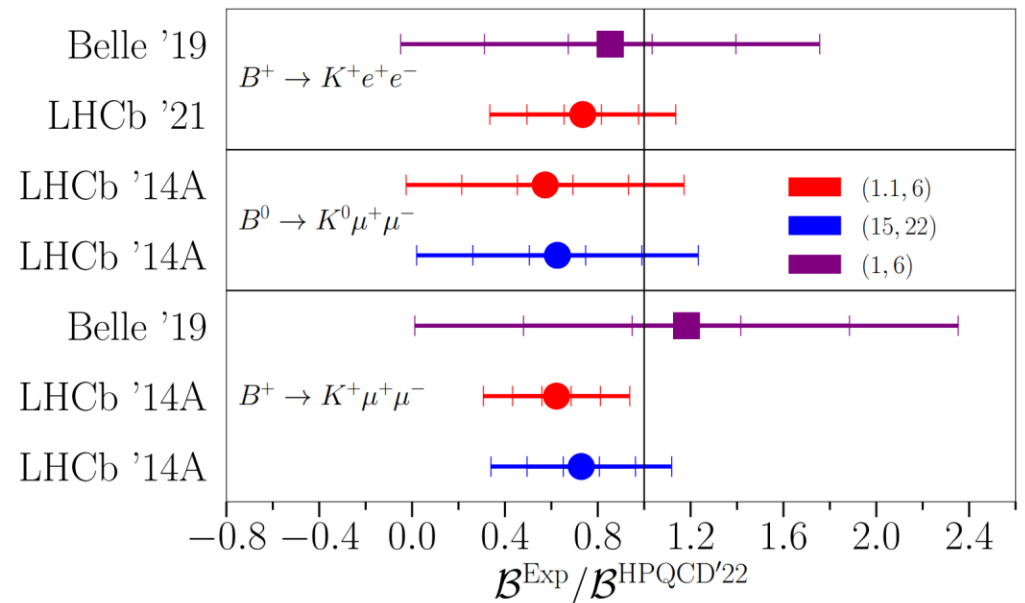
# $B \rightarrow K \ell^+ \ell^-$ [2207.13371]

W. G. Parrott, C. Bouchard, and C. T. H. Davies: 2+1+1 HISQ, heavy-HISQ  $b$



- Integrating over allowed regions gives tension at the level of  $\approx 3 - 5\sigma$  with recent experimental measurements.

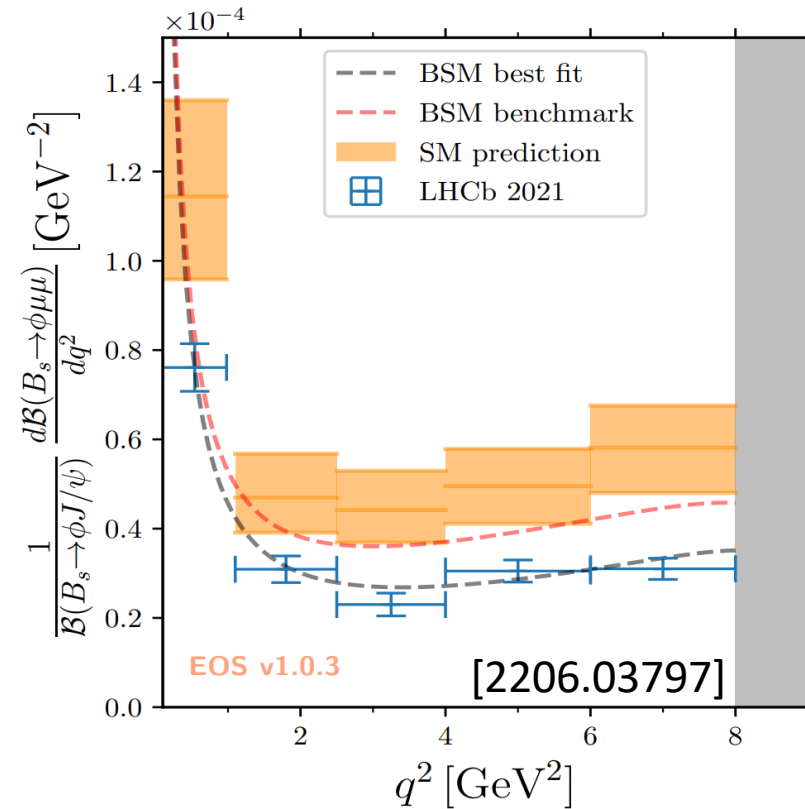
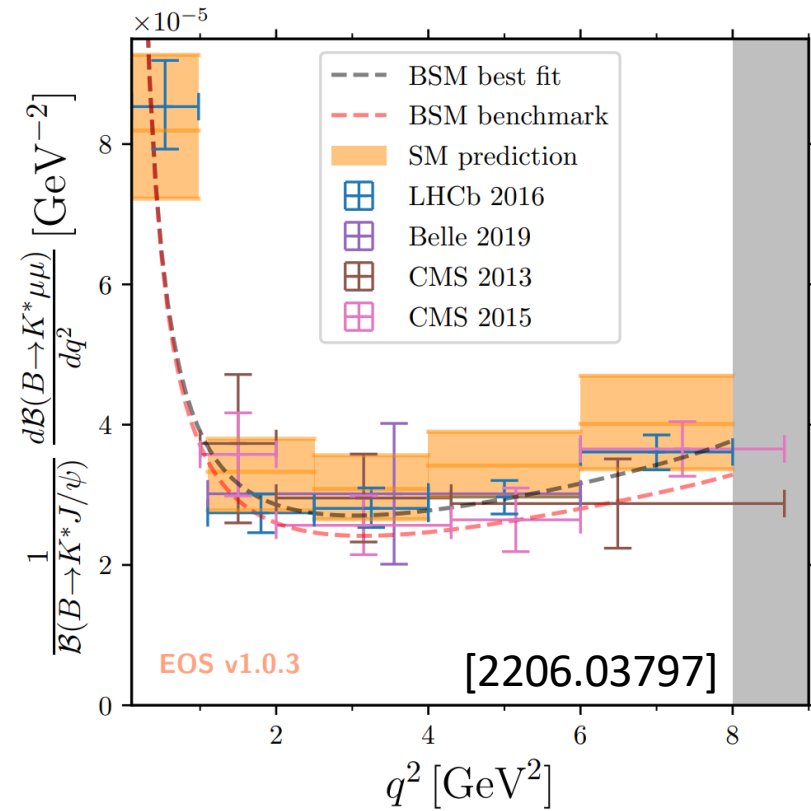
- $B^+ \rightarrow K^+ \ell^+ \ell^-$  differential branching fraction shows clear discrepancy with experimental measurements [2207.13371]
- Similar situation for  $B^0 \rightarrow K^0 \ell^+ \ell^-$  and  $B^- \rightarrow K^- \ell^+ \ell^-$  modes.



$$B \rightarrow K^* \ell^+ \ell^-, B_s \rightarrow \phi \ell^+ \ell^-$$

Dispersive bound for local and non-local FFs combined with older lattice results and LCSR [2206.03797]

- Similar discrepancy to  $B \rightarrow K$  in both cases.



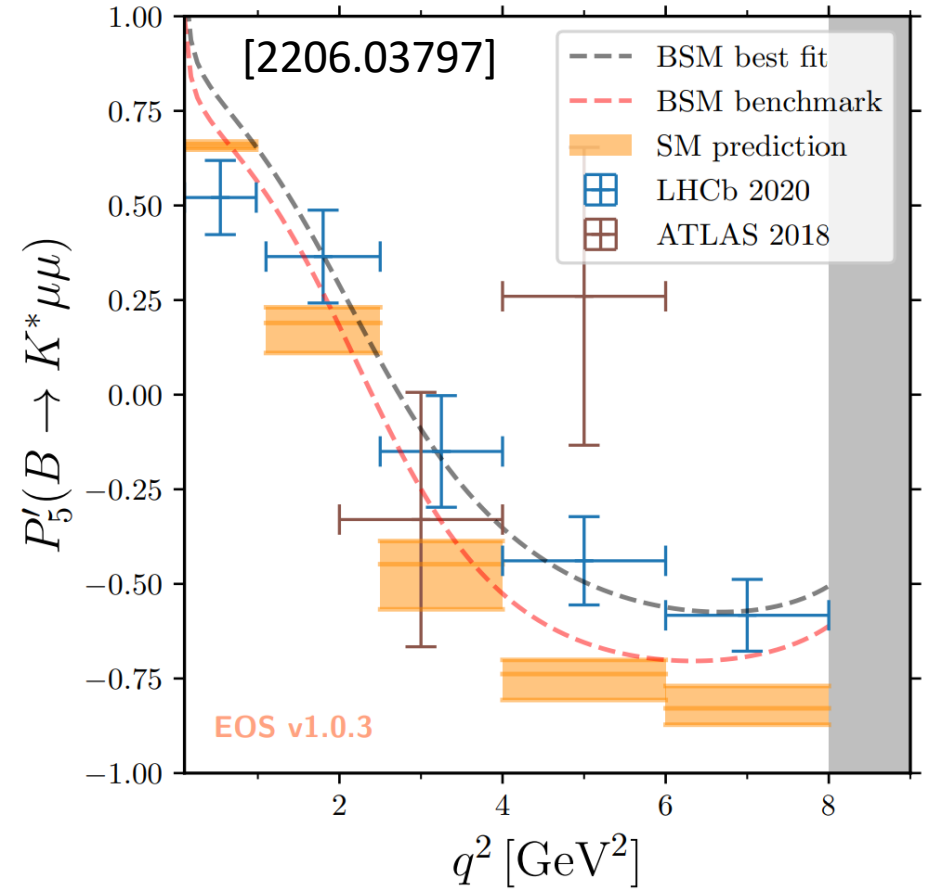
$$B \rightarrow K^* \ell^+ \ell^-, B_s \rightarrow \phi \ell^+ \ell^-$$

Similar level of discrepancy for  $P'_5$

$$P'_5(q^2) = \frac{S_5(q^2)}{\sqrt{F_L(q^2)(1-F_L(q^2))}}$$

$$S_5(q^2) = -\frac{4}{3} \left[ \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} - \int_0^{\frac{\pi}{2}} - \int_{\frac{3\pi}{2}}^{2\pi} \right] d\phi \left[ \int_0^1 - \int_{-1}^0 \right] d\cos(\theta_K) \\ \times \frac{d^3(\Gamma - \bar{\Gamma})}{dq^2 d\cos(\theta_K) d\phi} / \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$$

$$F_L(q^2) = \frac{d\Gamma^{\lambda_{K^*}=0}}{dq^2} / \frac{d\Gamma}{dq^2}$$



# $b \rightarrow s \ell^+ \ell^-$ : BSM analysis

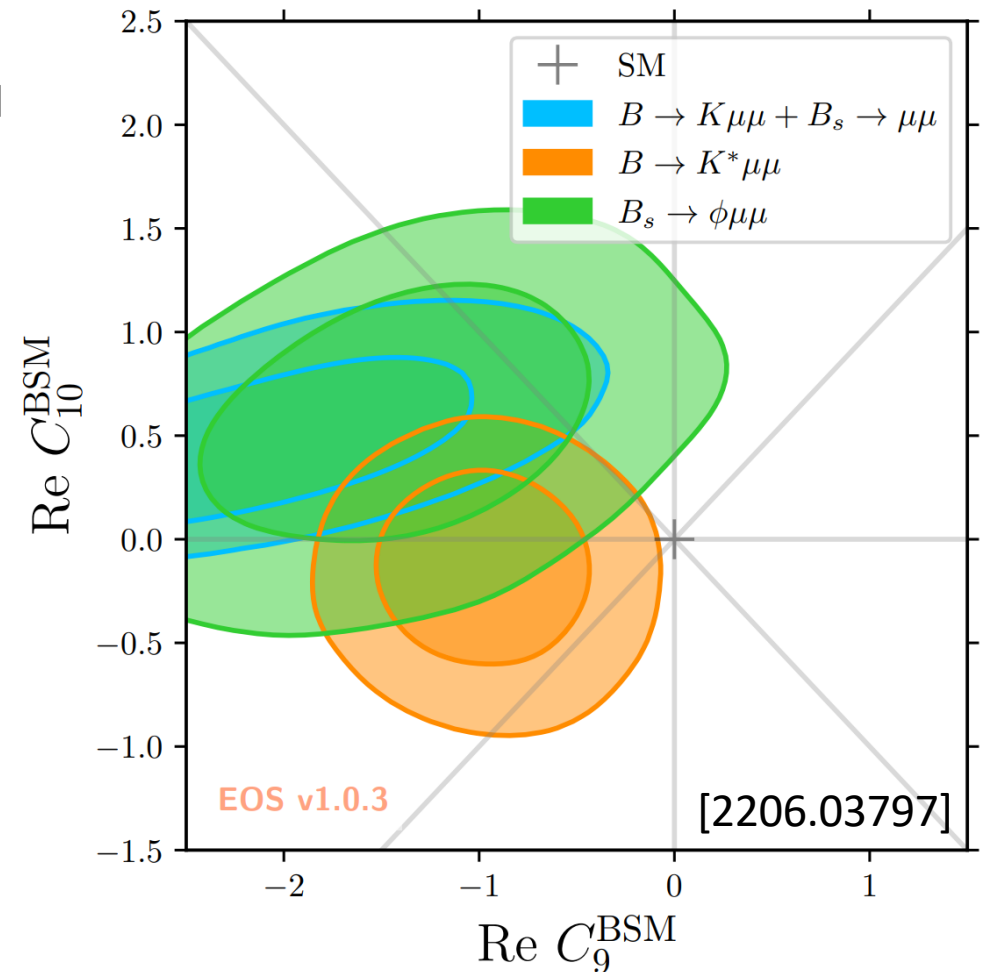
Combine experimental results with LQCD FFs with LCSR, improved dispersive bounds to constrain  $C_9$  and  $C_{10}$  [2206.03797]

$$B \rightarrow K \mu^+ \mu^-, B_s \rightarrow \mu^+ \mu^-$$

$$B \rightarrow K^* \mu^+ \mu^-$$

$$B \rightarrow \phi \mu^+ \mu^-$$

→ Look forward to simultaneous BSM analysis using new LQCD (e.g. W. G. Parrott et al.) and new experimental results in these channels

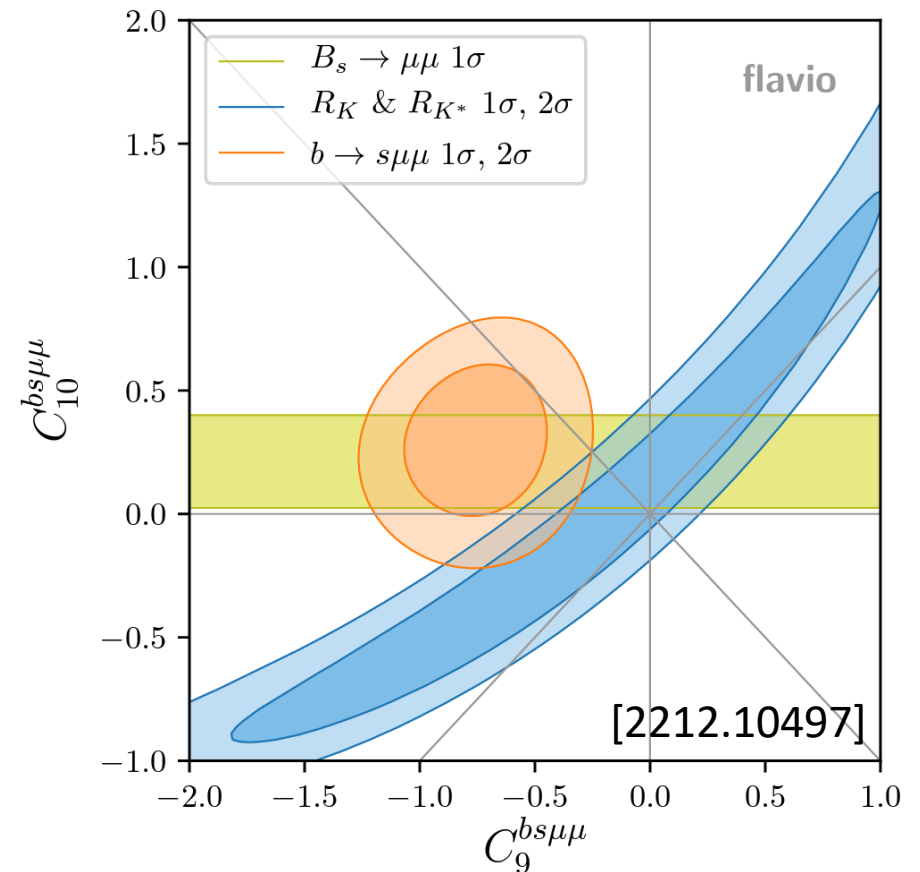
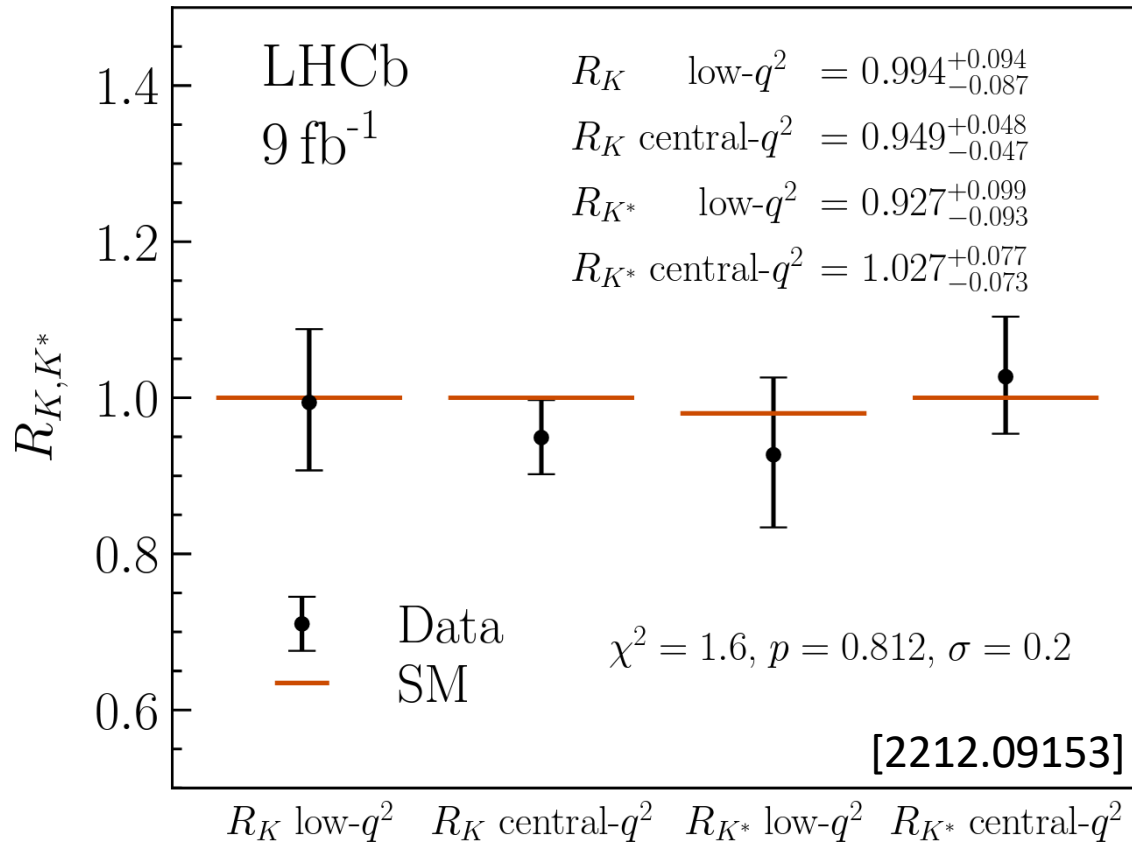


(this figure does not include HPQCD '22  $B \rightarrow K$  or CMS  $B_s \rightarrow \mu^+ \mu^-$  [2212.10311], bounds for each channel computed separately)

# $b \rightarrow s \ell^+ \ell^-$ : BSM analysis - LFU

New measurement of  $R_{K^{(*)}}$  by LHCb asks if deviations from SM seen in  $b \rightarrow s \mu^+ \mu^-$  can be explained consistently.

- Best performing 1D LFU NP case,  $C_9^{\text{univ}}$  [2212.10497]
- QCD effects could contribute  $\rightarrow$  understanding non-local contributions very important



# $b \rightarrow u \ell \bar{\nu}$

Form factors much more expensive computationally due to light quarks, especially for physical pions

Fermilab-MILC: 2+1 asqtad, Wilson-clover  $b$  and  $c$  quarks

HPQCD: 2+1+1 HISQ, heavy-HISQ  $b$

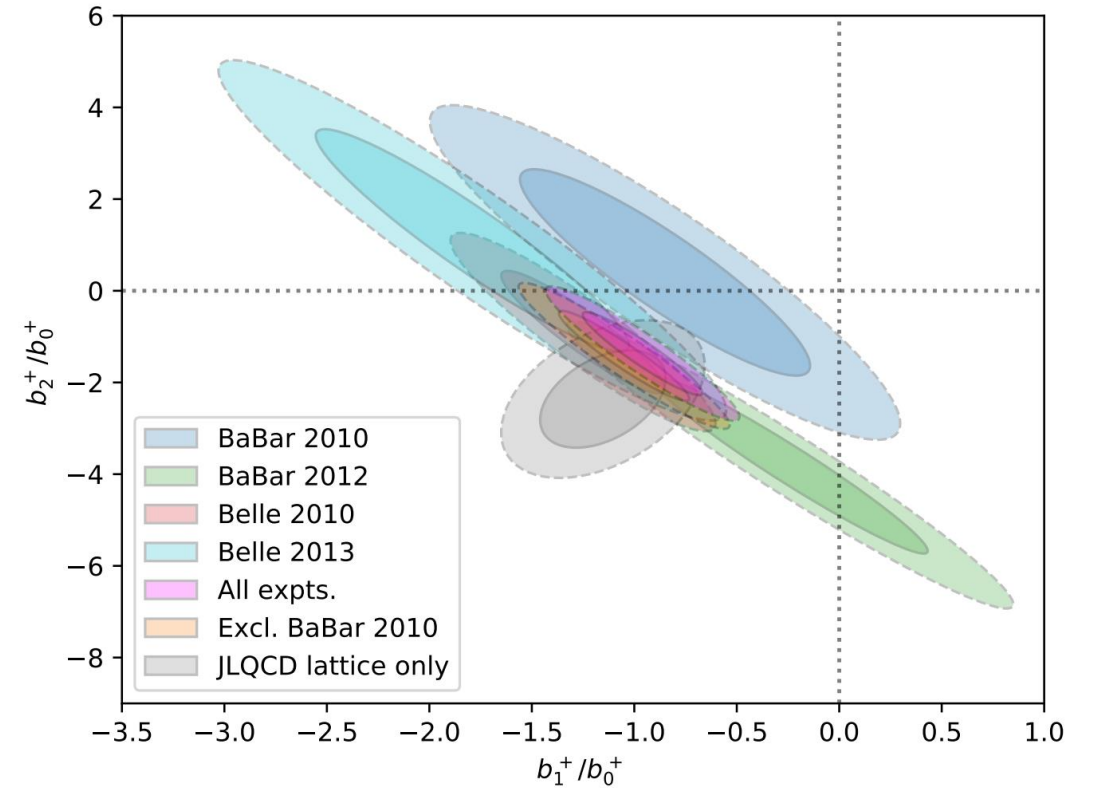
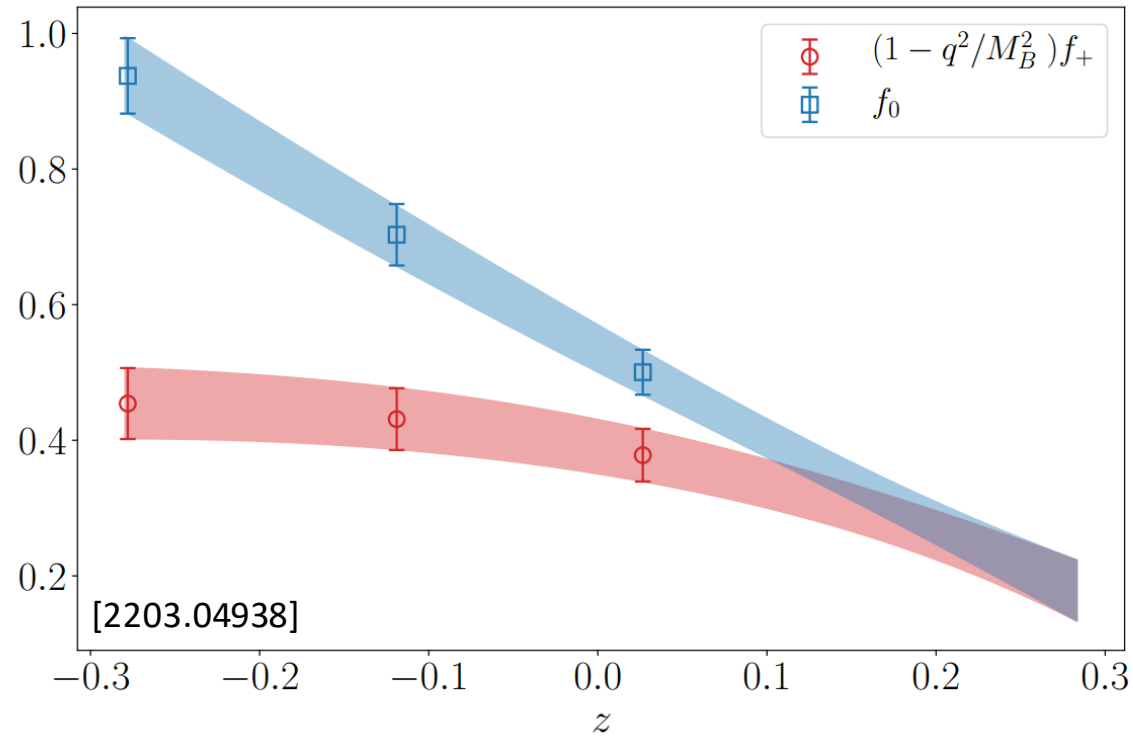
JLQCD: 2+1 Möbius domain-wall, Möbius domain-wall  $b$

|  | $h_{\pm}(w)$                   | $h_T(w)$       | $h_{A_{1,2,3},V}(w)$ | $h_{T_{1,2,3}}(w)$ |
|--|--------------------------------|----------------|----------------------|--------------------|
| $B \rightarrow \pi / B \rightarrow \rho$ | ✓ [2203.04938]<br>[1503.07839] |                |                      |                    |
| $B_S \rightarrow K^{(*)}$                | ✓ [1901.02561]                 |                |                      |                    |
| $B_C \rightarrow D^{(*)}$                | ✓ [2108.11242]                 | ✓ [2108.11242] |                      |                    |



# $B \rightarrow \pi \ell \bar{\nu}$ [2203.04938]

Recent JLQCD calculation of  $B \rightarrow \pi$  form factors

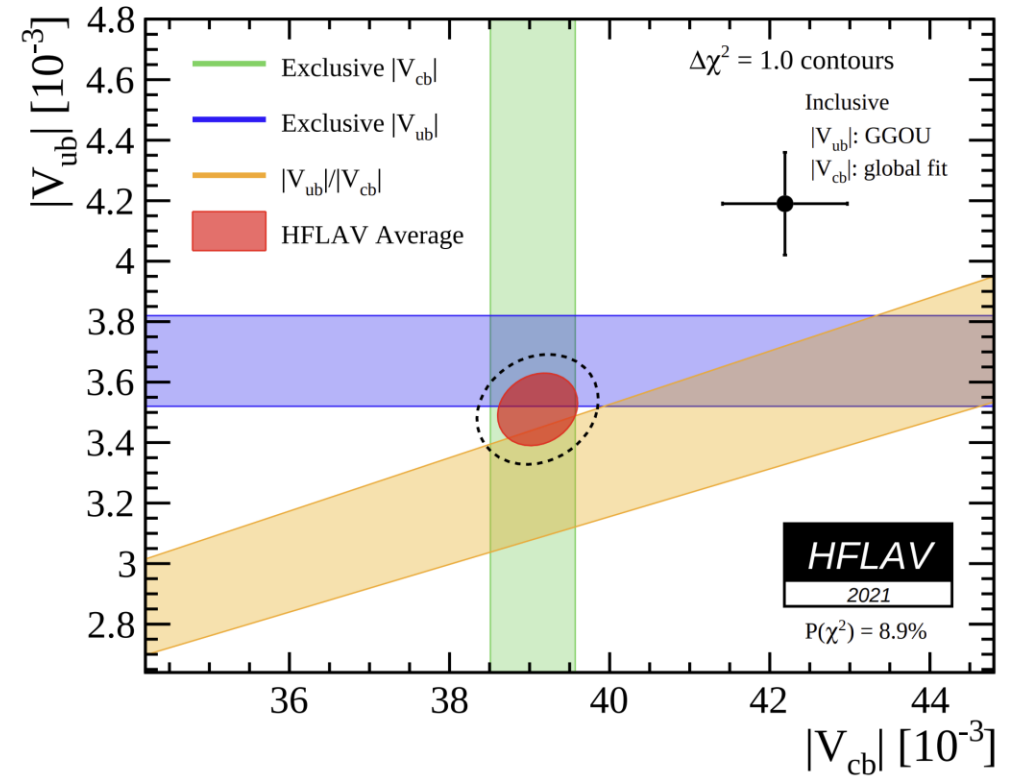
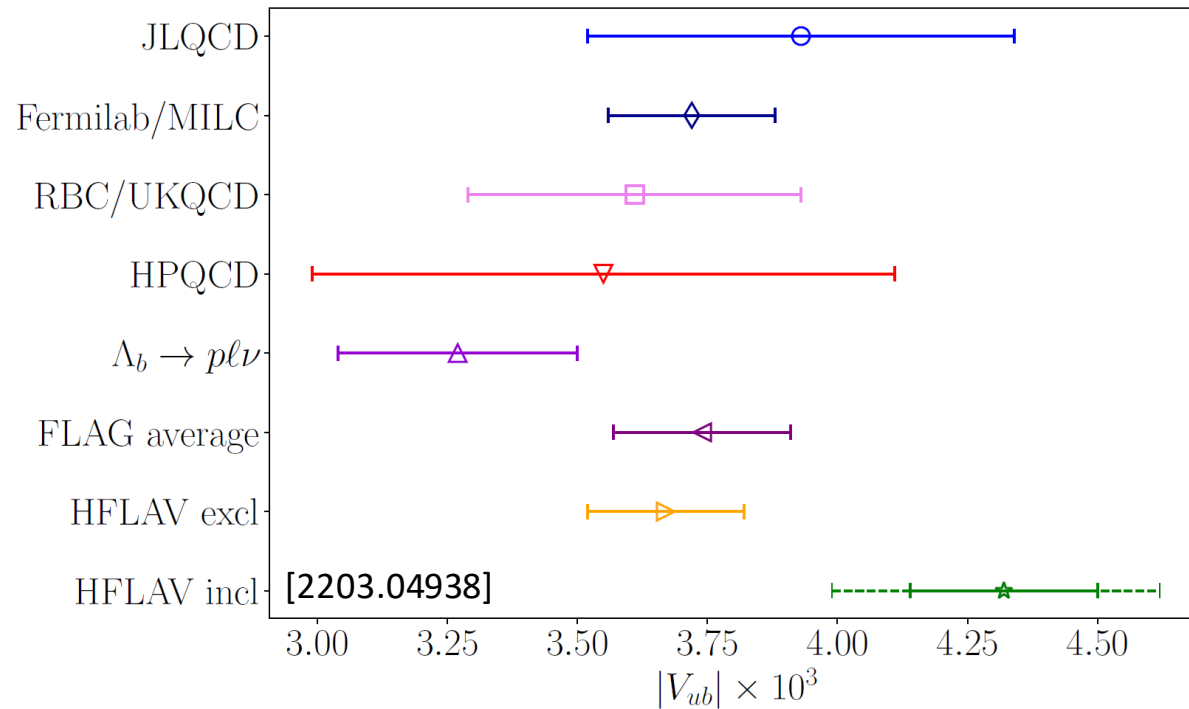


Good agreement between lattice shape parameters and experimental measurements

# $B \rightarrow \pi \ell \bar{\nu}$ [2203.04938]

$B \rightarrow \pi \ell \bar{\nu}$  provides a means to compute the CKM matrix element  $|V_{ub}|$

- JLQCD find  $V_{cb} = 3.93 \pm 0.41 \times 10^{-3}$
- Work in progress by both HPQCD and Fermilab-MILC collaborations

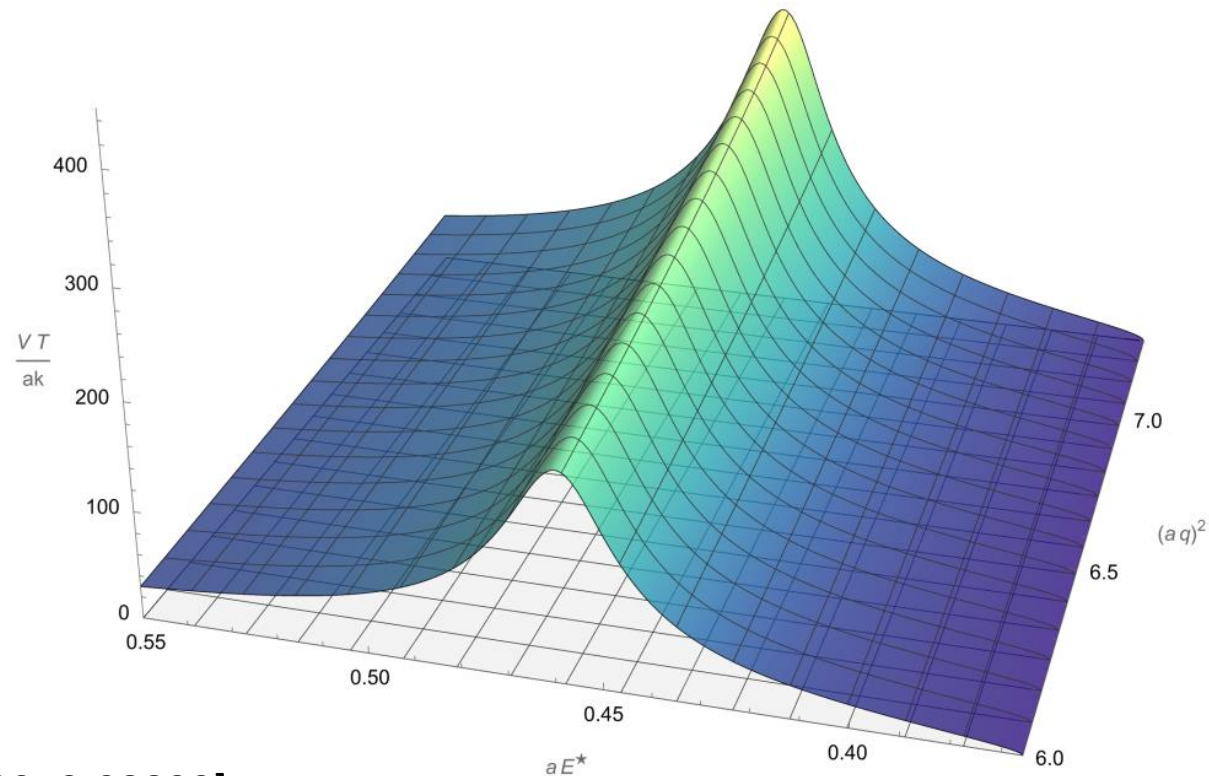


Also offers a test of LFU through the ratio  $R(\pi) = \Gamma(B \rightarrow \pi \tau \bar{\nu}_\tau) / \Gamma(B \rightarrow \pi \ell \bar{\nu}_\ell)$ , expected to be measured by Belle II with precision of  $\approx 14\%$

# $B \rightarrow \rho(\rightarrow \pi\pi)\ell\bar{\nu}$ [2212.08833]

$B \rightarrow \rho(\rightarrow \pi\pi)\ell\bar{\nu}$  provides a complementary determination of the CKM matrix element  $|V_{ub}|$

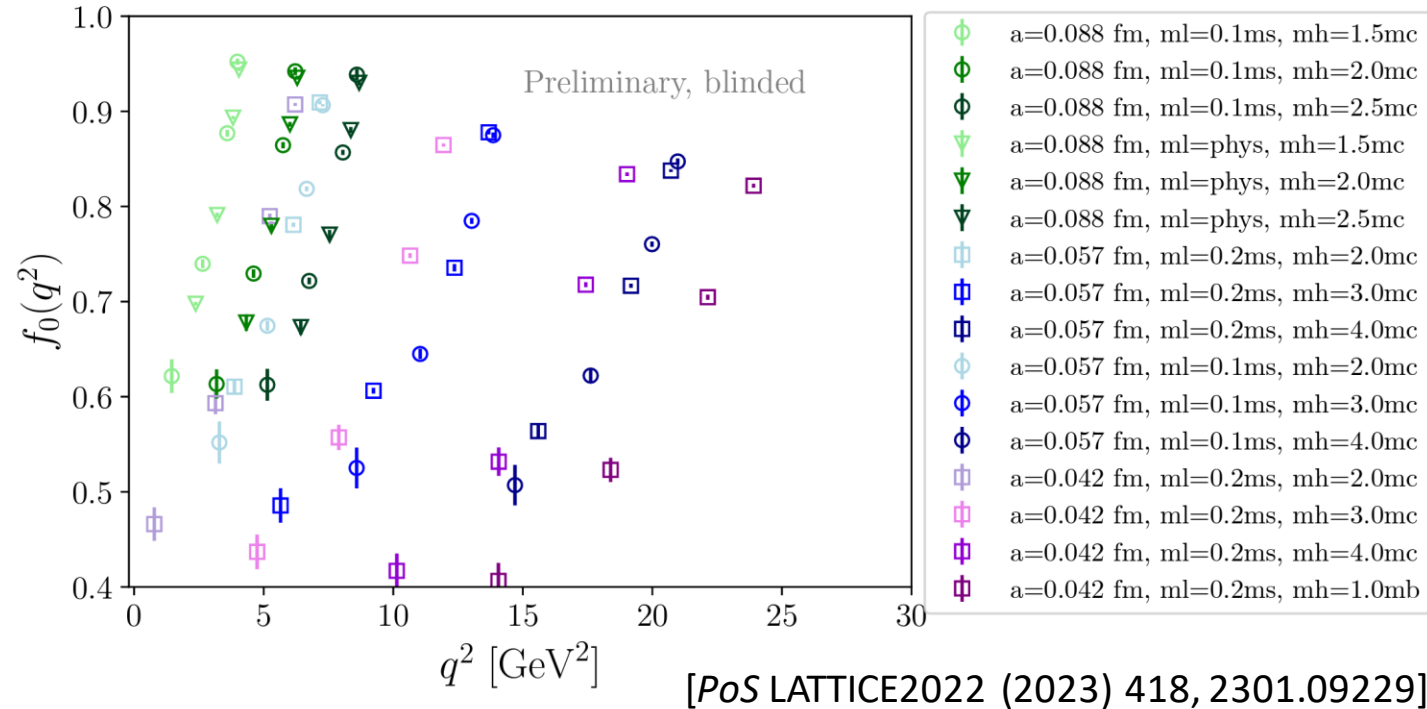
- Challenging on the lattice, due to  $\rho$  resonance.
- 2212.08833 follows the approach of Briceño, Hansen, and Walker-Loud (e.g. [1406.5965]) to compute the transition amplitude,  $\frac{VT}{ak}$ .
- Currently, only preliminary results for the vector current at a single lattice spacing.
- Nevertheless, demonstrates feasibility of such calculations
- Experimental data available from BaBar, Belle and recently Belle II [2211.15270] but hadronic matrix elements not yet well known.



[2212.08833]

# $B_S \rightarrow K \ell \bar{\nu}$ [2203.04938]

Work in progress by Fermilab-MILC on  $B_S \rightarrow K$  using 2+1+1 HISQ gauge configurations and HISQ heavy quarks, e.g



# Summary

## $b \rightarrow c\ell\bar{\nu}$

- New  $B_{(s)} \rightarrow D_{(s)}^* \ell\bar{\nu}$  SM+Tensor FFs from HPQCD,  $B \rightarrow D^* \ell\bar{\nu}$  SM FFs from JLQCD, WIP by Fermilab-MILC on  $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell\bar{\nu}$
- Lattice  $R(D^*)$  seems to disagree with HQET predictions, some discrepancy with semimuonic shape and asymmetry measurements from Belle and Belle II
- Need to look carefully at ingredients of lattice calculations
- New experimental results expected in  $B \rightarrow D^* \ell\bar{\nu}$  and other channels soon

## $b \rightarrow s\ell^+\ell^-$

- Recent SM+Tensor FFs from HPQCD confirm tension seen between theory and experiment in branching ratios in  $B \rightarrow K$  [2207.12468, 2207.13371], WIP also at Fermilab-MILC on  $B \rightarrow K$
- LHCb  $R_{K^{(*)}}$  [2212.09153] highlights importance of understanding non-local contributions -> look to new dispersive bound calculations [2011.09813, 2206.03797]
- WIP on  $B_S \rightarrow \phi$  and  $B \rightarrow K^*$  FFs at HPQCD will clarify situation in these channels where current discrepancy is based on older nonrelativistic calculations

## $b \rightarrow u\ell\bar{\nu}$

- $B \rightarrow \pi\ell\bar{\nu}$  SM FFs from JLQCD in agreement with experimentally measured shape, give exclusive  $V_{cb}$  compatible with both inclusive and exclusive averages [2203.04938]
- WIP by Leskovec et al. on  $B \rightarrow \rho(\rightarrow \pi\pi)\ell\bar{\nu}$  [2212.08833], treating the  $\rho$  resonance using the Lellouch-Lüscher method
- WIP by HPQCD on  $B \rightarrow \pi\ell\bar{\nu}$
- WIP by Fermilab-MILC on  $B_S \rightarrow K\ell\bar{\nu}$  and  $B \rightarrow \pi\ell\bar{\nu}$

# Thanks for listening!

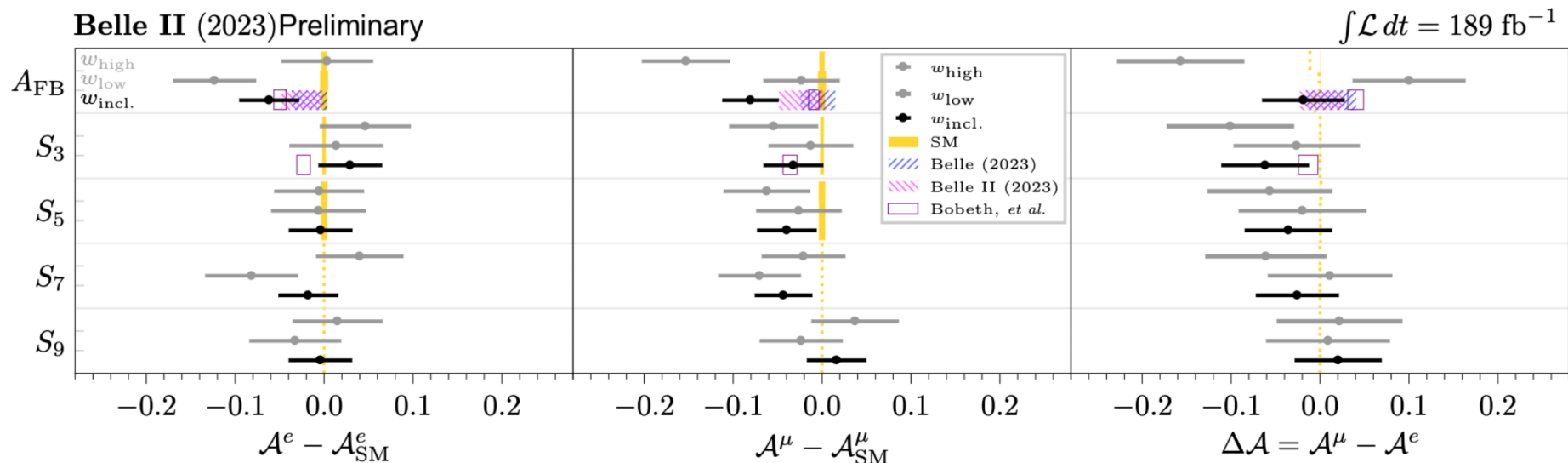
# Backup Slides

# $B^0 \rightarrow D^{*-} \ell^+ \nu$ Belle II Preliminary [2305.10746]

Recently, Belle II reported preliminary results for a measurement of  $|V_{cb}|$  using  $189\text{fb}^{-1}$  of  $e^+e^-$  collisions at the  $\Upsilon(4S)$  resonance and  $18\text{fb}^{-1}$  of collisions  $60\text{MeV}$  below the  $\Upsilon(4S)$  resonance.

Using LQCD for the normalisation at zero recoil:

$$|V_{cb}|_{\text{BGL}} = 40.6 \pm 0.3^{\text{stat}} \pm 1.0^{\text{syst}} \pm 0.6^{\text{theo}} \times 10^{-3}.$$



$w \in [1.0, 1.275]$  ( $w_{\text{low}}$ ),

$w \in [1.275, \approx 1.5]$  ( $w_{\text{high}}$ )