Strategies to investigate tensions between R-ratio and lattice HVP computations

Peter Stoffer
Physik-Institut, University of Zurich
and Paul Scherrer Institut

June 13, 2023

Lattice Gauge Theory Contributions to New Physics Searches
IFT UAM/CSIC Madrid

University of Zurich[UZH]

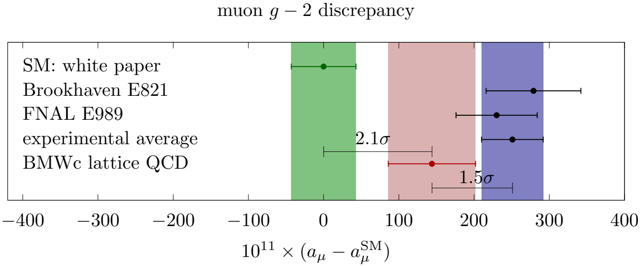
PAUL SCHERRER INSTITUT PSI

funded by Swiss National Science Foundation

Motivation

- 4.2σ discrepancy between $g-2$ experiments and White Paper SM prediction
- **2.1σ tension** between R-ratio and BMWc lattice-QCD for HVP
- increases to **3.7σ for intermediate window**
- recent results from ETMC, Mainz, RBC/UKQCD confirm BMWc intermediate window
- motivates **ongoing scrutiny** of R-ratio results
- new $e^+e^- \to \pi^+\pi^-$ data from CMD-3 agree with lattice, **incompatible with previous experiments**

Tension between R-ratio and lattice



Two-pion contribution to HVP

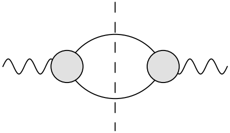
- $\pi\pi$ contribution amounts to **more than 70%** of HVP contribution
- responsible for a similar fraction of HVP uncertainty
- can be expressed in terms of **pion vector form factor** $\Rightarrow$ constraints from analyticity and unitarity

  $\rightarrow$ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006

Unitarity and analyticity

implications of unitarity (two-pion intermediate states):

1. $\pi\pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi\pi$ scattering
3. $\pi\pi$ scattering—$\pi\pi$ scattering



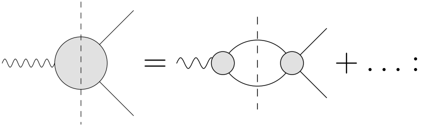
: $\sigma(e^+e^- \to \pi^+\pi^-) \propto |F_\pi^V(s)|^2$

analyticity $\Rightarrow$ dispersion relation for HVP contribution

Unitarity and analyticity

implications of unitarity (two-pion intermediate states):

1. $\pi\pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi\pi$ scattering
3. $\pi\pi$ scattering—$\pi\pi$ scattering



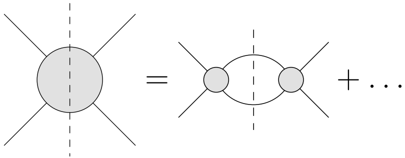
$$F_\pi^V(s) = |F_\pi^V(s)|e^{i\delta_1^1(s)+\ldots}$$

analyticity $\Rightarrow$ dispersion relation for pion VFF

Unitarity and analyticity

implications of unitarity (two-pion intermediate states):

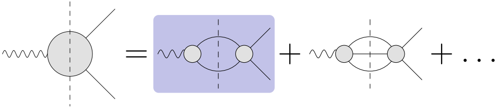
1. $\pi\pi$ contribution to HVP—pion vector form factor (VFF)
2. pion VFF—$\pi\pi$ scattering
3. $\pi\pi$ scattering—$\pi\pi$ scattering



analyticity, crossing, PW expansion $\Rightarrow$ Roy equations

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



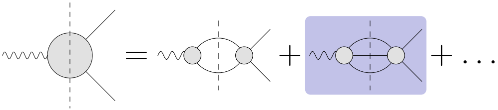
$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

- **Omnès function** with elastic $\pi\pi$-scattering P-wave phase shift $\delta_1^1(s)$ as input:

$$\Omega_1^1(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^\infty ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}$$

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\text{in}}^N(s)$$

- isospin-breaking 3π intermediate state: negligible apart from ω **resonance** (ρ–ω interference effect)

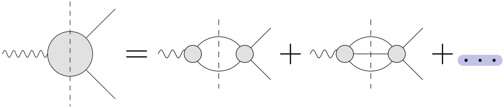
$$G_\omega(s) = 1 + \frac{s}{\pi}\int_{9M_\pi^2}^\infty ds' \frac{\text{Im} g_\omega(s')}{s'(s'-s)} \left(\frac{1-\frac{9M_\pi^2}{s'}}{1-\frac{9M_\pi^2}{M_\omega^2}}\right)^4,$$

$$g_\omega(s) = 1 + \epsilon_\omega \frac{s}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s}$$

ϵ_ω: a priori a free **real** parameter

Dispersive representation of pion VFF

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006



$$F_\pi^V(s) = \Omega_1^1(s) \times G_\omega(s) \times G_{\rm in}^N(s)$$

- heavier intermediate states: 4π (mainly $\pi^0\omega$), $\bar{K}K, \ldots$
- described in terms of a **conformal polynomial** with cut starting at $\pi^0\omega$ threshold

$$G_{\rm in}^N(s) = 1 + \sum_{k=1}^{N} c_k(z^k(s) - z^k(0))$$

- correct P-wave threshold behavior imposed

Input and systematic uncertainties

- elastic $\pi\pi$-scattering P-wave phase shift $\delta_1^1(s)$ from Roy-equation analysis, including uncertainties

  $\rightarrow$ Ananthanarayan et al., 2001; Caprini et al., 2012

- high-energy continuation of phase shift above validity of Roy equations
- ω width
- systematics in conformal polynomial: order N, one mapping parameter

Free fit parameters

- value of the elastic $\pi\pi$-scattering P-wave **phase shift** δ_1^1 at two points (0.8 GeV and 1.15 GeV): number of free parameters dictated by Roy equations
- **ρ–ω mixing parameter** ϵ_ω
- **ω mass**
- **energy rescaling** for the experimental input, which allows for a calibration uncertainty
- $N - 1$ coefficients in the **conformal polynomial**

VFF fit to the following data

- time-like e^+e^- cross-section data
- space-like VFF data from NA7
- Eidelman–Łukaszuk bound on inelastic phase:

  $\rightarrow$ Eidelman, Łukaszuk, 2004
- iterative fit routine including full experimental covariance matrices and avoiding D'Agostini bias

  $\rightarrow$ D'Agostini, 1994; Ball et al. (NNPDF) 2010

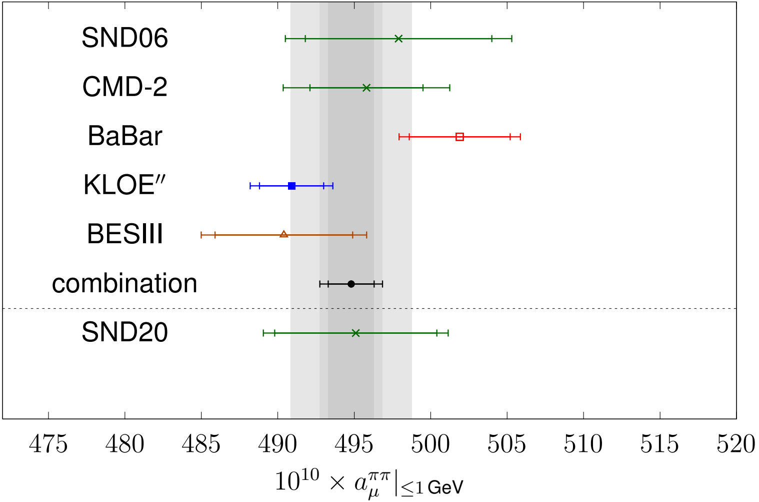
Updated results for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV

→ Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \mathrm{Re}(\epsilon_\omega)$
SND06	1.40	5.3%	$781.49(32)(2)$	$2.03(5)(2)$
CMD-2	1.18	14%	$781.98(29)(1)$	$1.88(6)(2)$
BaBar	1.14	5.7%	$781.86(14)(1)$	$2.04(3)(2)$
KLOE	1.36	7.4×10^{-4}	$781.82(17)(4)$	$1.97(4)(2)$
KLOE″	1.20	3.1%	$781.81(16)(3)$	$1.98(4)(1)$
BESIII	1.12	25%	$782.18(51)(7)$	$2.01(19)(9)$
SND20	2.93	3.3×10^{-7}	$781.79(30)(6)$	$2.04(6)(3)$
all w/o SND20	1.23	3.0×10^{-5}	$781.69(9)(3)$	$2.02(2)(3)$

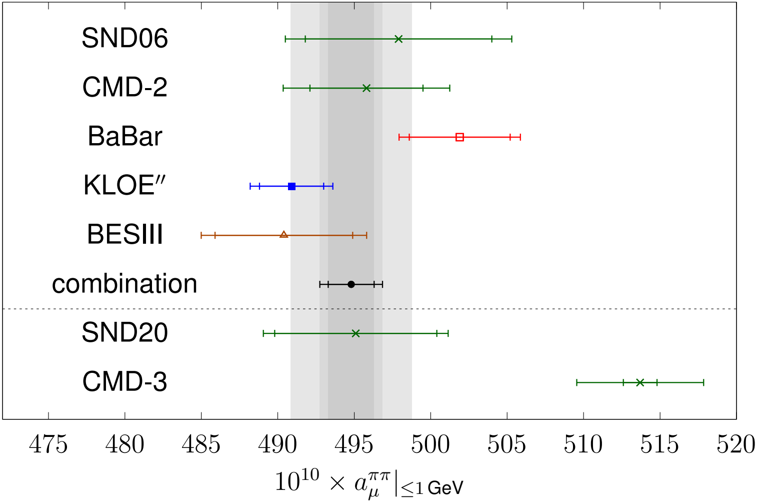
Results for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV

→ Colangelo, Hoferichter, Stoffer, JHEP **02** (2019) 006
Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032



More tensions: CMD-3

→ F. Ignatov et al. (CMD-3), 2302.08834 [hep-ex]



Tension with lattice QCD

$\rightarrow$ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- implications of changing HVP?
- modifications at high energies affect **hadronic running of $\alpha_{\text{QED}}^{\text{eff}}$** $\Rightarrow$ clash with global EW fits

  $\rightarrow$ Passera, Marciano, Sirlin (2008), Crivellin, Hoferichter, Manzari, Montull (2020), Keshavarzi, Marciano, Passera, Sirlin (2020), Malaescu, Schott (2020)
- lattice studies point at region < 2 GeV
- $\pi\pi$ **channel** dominates
- relative changes in other channels would need to be huge

Tension with lattice QCD

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- force a different HVP contribution in VFF fits by including "lattice" datum with tiny uncertainty
- three different scenarios:
  - "low-energy" physics: $\pi\pi$ phase shifts
  - "high-energy" physics: inelastic effects, c_k
  - all parameters free
- study effects on pion charge radius, hadronic running of $\alpha_{\mathrm{QED}}^{\mathrm{eff}}$, phase shifts, cross sections

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

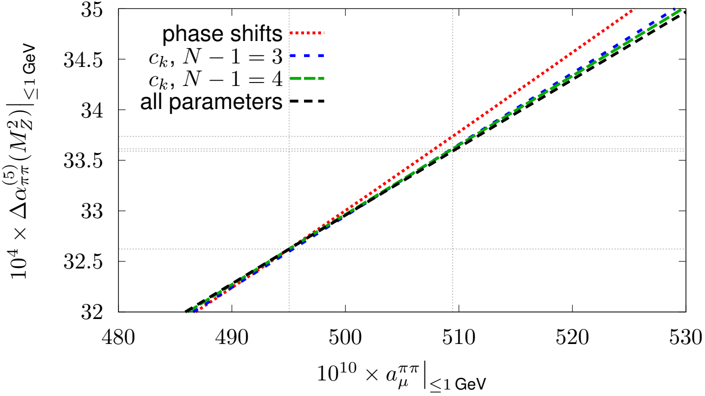
→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

- "low-energy" scenario requires large local changes in the cross section in the ρ region
- "high-energy" scenario has an impact on **pion charge radius** and the space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

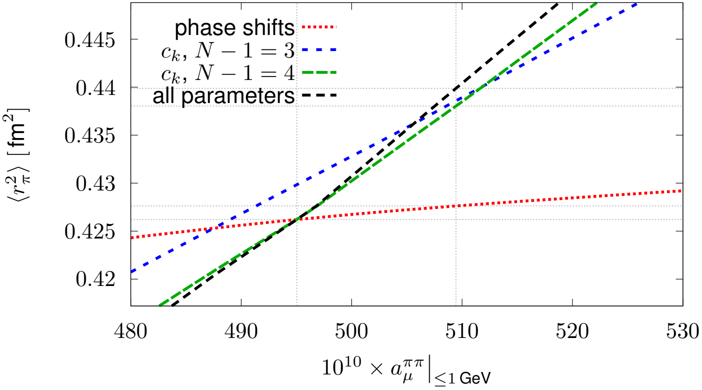
correlations between $a_\mu^{\pi\pi}$ and $\Delta\alpha_{\pi\pi}^{(5)}(M_Z^2)$



Modifying $a_\mu^{\pi\pi}|_{\leq 1\text{ GeV}}$

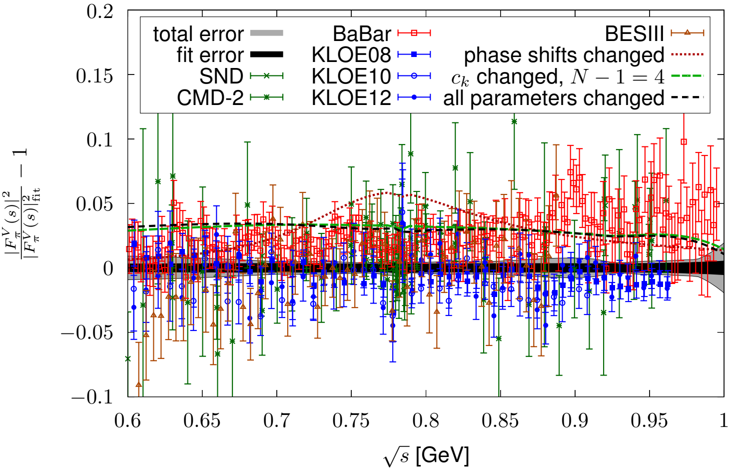
→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

correlations between $a_\mu^{\pi\pi}$ and $\langle r_\pi^2 \rangle$



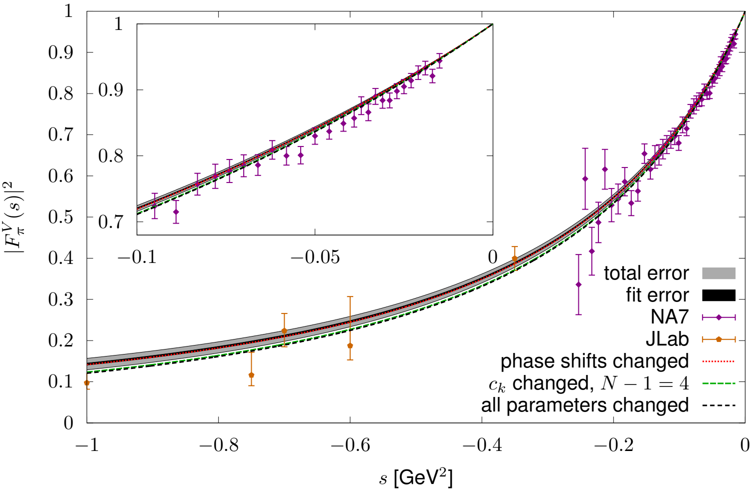
Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073

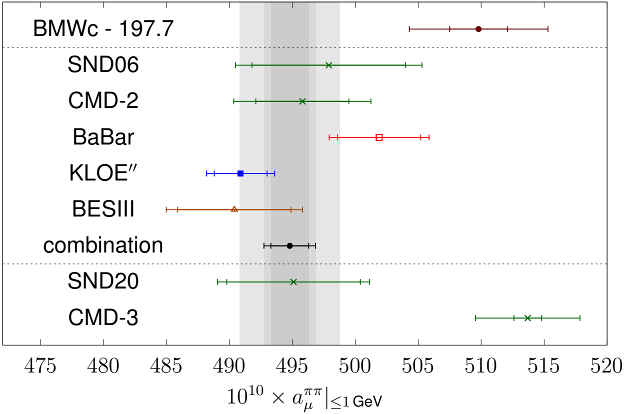


Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

→ Colangelo, Hoferichter, Stoffer, PLB **814** (2021) 136073



Results for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV



Assumption: suppose all changes occur in $\pi\pi$ channel < 1 GeV

$$\Rightarrow a_\mu^{\mathrm{total}}[\mathrm{WP20}] - a_\mu^{2\pi,<1\,\mathrm{GeV}}[\mathrm{WP20}] = 197.7 \times 10^{-10}$$

CMD-3 vs. all the rest

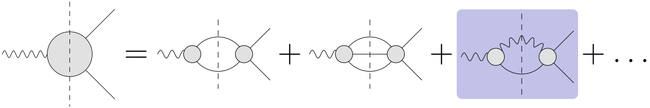
discrepancy	$a_\mu^{\pi\pi}\big\vert_{[0.60,0.88]\text{ GeV}}$	$a_\mu^{\pi\pi}\big\vert_{\leq 1\text{ GeV}}$	int window
SND06	1.8σ	1.7σ	1.7σ
CMD-2	2.3σ	2.0σ	2.1σ
BaBar	3.3σ	2.9σ	3.1σ
KLOE″	5.6σ	4.8σ	5.4σ
BESIII	3.0σ	2.8σ	3.1σ
SND20	2.2σ	2.1σ	2.2σ
Combination	4.2σ (6.1σ)	3.7σ (5.0σ)	3.8σ (5.7σ)

(discrepancies in brackets exclude systematic effect due to BaBar–KLOE tension)

- p-value of fit to CMD-3: 20%
- $\pi\pi$ phase shifts reasonable, main effect in conformal polynomial
- effect on charge radius as expected for rather uniform cross-section shift

Resonantly enhanced isospin-breaking effects

- with the given approximations, ϵ_ω is real by construction
- however, **additional radiative corrections** can be effectively mapped onto a phase in ϵ_ω
- additional channels in unitarity relation:



Resonantly enhanced isospin-breaking effects

- with the given approximations, ϵ_ω is real by construction
- however, **additional radiative corrections** can be effectively mapped onto a phase in ϵ_ω
- e.g., dominant $\pi^0\gamma$ channel can be implemented as

$$G_\omega(s) = 1 + \frac{s}{\pi}\int_{9M_\pi^2}^\infty ds' \frac{\text{Re}\epsilon_\omega}{s'(s'-s)} \text{Im}\left[\frac{s'}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s'}\right] \left(\frac{1-\frac{9M_\pi^2}{s'}}{1-\frac{9M_\pi^2}{M_\omega^2}}\right)^4 + \frac{s}{\pi}\int_{M_{\pi^0}^2}^\infty ds' \frac{\text{Im}\epsilon_\omega}{s'(s'-s)} \text{Re}\left[\frac{s'}{(M_\omega - \frac{i}{2}\Gamma_\omega)^2 - s'}\right] \left(\frac{1-\frac{M_{\pi^0}^2}{s'}}{1-\frac{M_{\pi^0}^2}{M_\omega^2}}\right)^3$$

- resonance enhancement: **details of implementation irrelevant** (similar results with complex ϵ_ω in $g_\omega(s)$)

Effective phase in ρ–ω mixing parameter

- narrow-resonance estimate:

$$\mathrm{Im}\epsilon_\omega \simeq \frac{\sqrt{\Gamma[\omega \to \pi^0\gamma]\Gamma[\rho \to \pi^0\gamma]}}{3M_V}$$

- analogous relation for other intermediate states
- estimate leads to phases of $2.8^\circ(\pi^0\gamma)$, $0.4^\circ(\pi^+\pi^-\gamma)$, $0.2^\circ(\eta\gamma)$, $0.02^\circ(\pi^0\pi^0\gamma)$
  $\Rightarrow$ expect a phase $\arg(\epsilon_\omega) \approx 3.5(1.0)^\circ$

Updated results for $a_\mu^{\mathrm{HVP},\pi\pi}$ below 1 GeV

→ Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \mathrm{Re}(\epsilon_\omega)$
SND06	1.40	5.3%	$781.49(32)(2)$	$2.03(5)(2)$
CMD-2	1.18	14%	$781.98(29)(1)$	$1.88(6)(2)$
BaBar	1.14	5.7%	$781.86(14)(1)$	$2.04(3)(2)$
KLOE	1.36	7.4×10^{-4}	$781.82(17)(4)$	$1.97(4)(2)$
KLOE″	1.20	3.1%	$781.81(16)(3)$	$1.98(4)(1)$
BESIII	1.12	25%	$782.18(51)(7)$	$2.01(19)(9)$
SND20	2.93	3.3×10^{-7}	$781.79(30)(6)$	$2.04(6)(3)$
all w/o SND20	1.23	3.0×10^{-5}	$781.69(9)(3)$	$2.02(2)(3)$

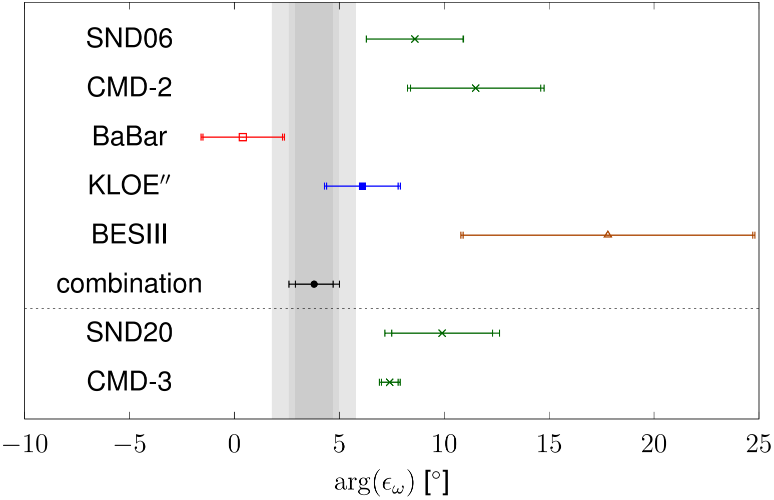
Including phase in ϵ_ω

→ Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \mathrm{Re}(\epsilon_\omega)$	$\arg(\epsilon_\omega)$ [°]
SND06	1.08	35%	$782.11(32)(2)$	$1.98(4)(2)$	$8.5(2.3)(0.3)$
CMD-2	1.01	45%	$782.64(33)(4)$	$1.85(6)(4)$	$11.4(3.1)(1.0)$
BaBar	1.14	5.5%	$781.93(18)(4)$	$2.03(4)(1)$	$1.3(1.9)(0.7)$
KLOE	1.27	6.7×10^{-3}	$782.50(25)(6)$	$1.94(5)(2)$	$6.8(1.8)(0.5)$
KLOE″	1.13	10%	$782.42(23)(5)$	$1.95(4)(2)$	$6.1(1.7)(0.6)$
BESIII	1.02	44%	$783.05(60)(2)$	$1.99(19)(7)$	$17.6(6.9)(1.2)$
SND20	1.87	4.1×10^{-3}	$782.37(28)(6)$	$2.02(5)(2)$	$10.1(2.4)(1.4)$
all w/o SND20	1.19	4.8×10^{-4}	$782.09(12)(4)$	$1.97(2)(2)$	$4.5(9)(8)$

Results for $\arg(\epsilon_\omega)$

→ Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032



Extraction of IB contribution due to ρ–ω mixing

$\rightarrow$ Colangelo, Hoferichter, Kubis, Stoffer, JHEP **10** (2022) 032

- extracted from full result vs. HVP integral with $\epsilon_\omega = 0$
- **similar size as FSR** contribution (sQED):

$\arg(\epsilon_\omega)$	$0°$	$4.5(1.2)°$
$10^{10} \times a_\mu^{\rho\text{–}\omega}$	$4.37(4)(7)$	$3.68(14)(10)$
$10^{10} \times a_\mu^{\pi\pi,\text{FSR}}$	$4.23(1)(2)$	$4.24(1)(2)$

- since we are considering 1-photon-irreducible HVP, entire effect to be assigned to $\mathcal{O}(m_u - m_d)$

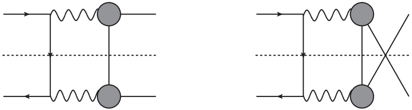
  $\rightarrow$ thanks to Pablo Sanchez-Puertas for pointing this out

Re-examination of RCs to $e^+e^- \to$ hadrons

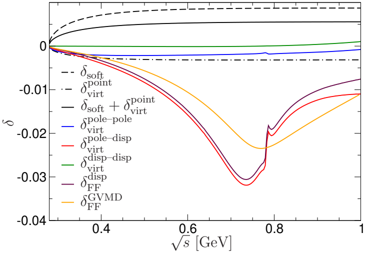
- central discussion item at “5th Workstop / Thinkstart: RC and MC tools for Strong 2020” (Zurich University)
- aiming at NNLO for leptonic part and improvement of **structure-dependent** NLO effects
- employing **dispersion relations** for radiative corrections to F_π^V

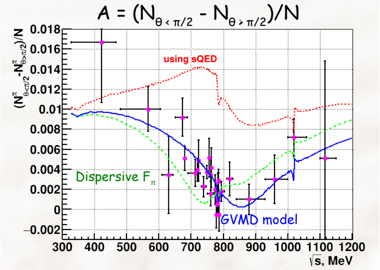
  $\to$ G. Colangelo, M. Cottini, J. Monnard, J. Ruiz de Elvira, work in progress
- scan experiments rely on MCGPJ, ISR experiments on Phokhara: only **one MC generator** in each case
- Phokhara: FSR modeled by sQED $\times$ pion VFF **outside loop integrals** + resonance models

Forward-backward asymmetry



→ talk by G. Colangelo at UZH "WorkStop"

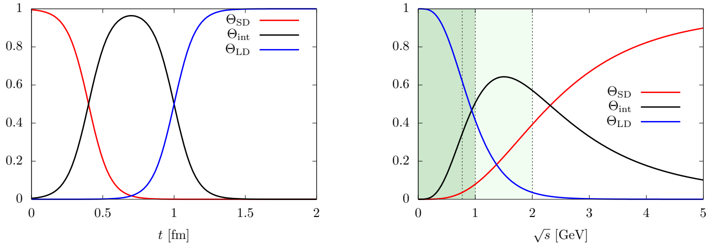




→ Colangelo, Hoferichter, Monnard, Ruiz de Elvira (2022)

→ Ignatov, Lee (2022), talk by F. Ignatov at UZH

Some insights from the window quantities



- smooth window weight functions in Euclidean time

  → Blum et al. [RBC/UKQCD], PRL **121** (2018) 022003

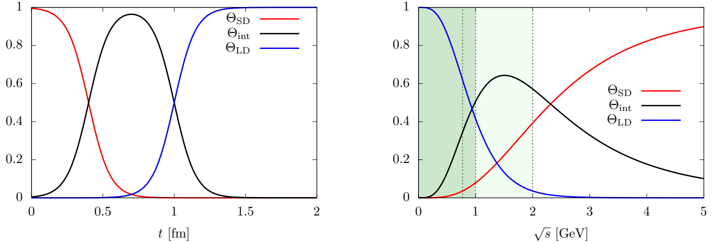
- total discrepancy:

  $$a_\mu[\text{BMWc}] - a_\mu[\text{WP20}] = 14.4(6.8) \times 10^{-10}$$

- intermediate window: → Colangelo et al., PLB **833** (2022) 137313

  $$a_\mu^{\text{int}}[\text{BMWc}] - a_\mu^{\text{int}}[e^+e^-] = 7.3(2.0) \times 10^{-10}$$

Some insights from the window quantities



- using form of weight functions:
  at least $\sim 40\%$ from **above 1 GeV**

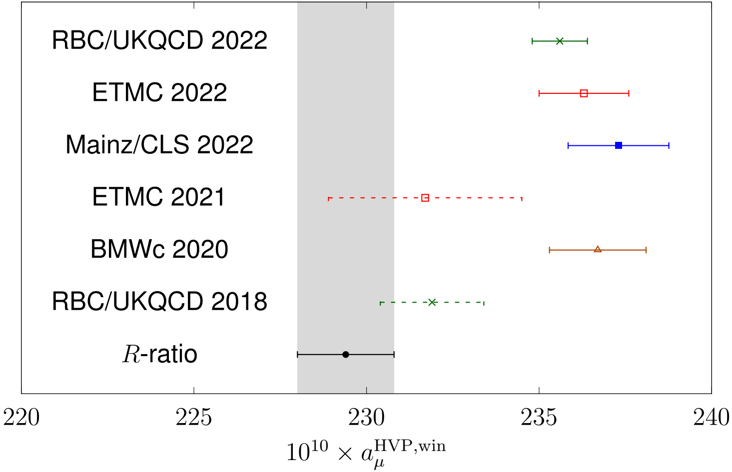
- assumptions:
  - rather uniform shifts in low-energy $\pi\pi$ region
  - no significant negative shifts

Data-driven evaluation of window quantities

→ Colangelo et al., PLB **833** (2022) 137313

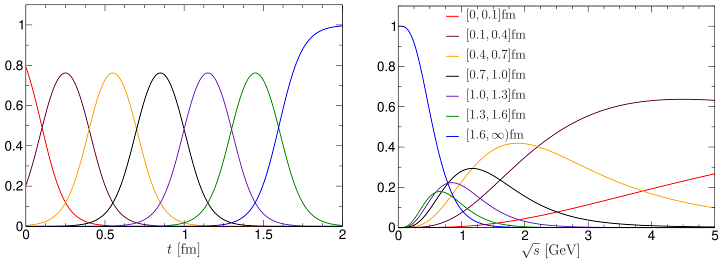
- standard windows: $[0, 0.4]$ fm, $[0.4, 1.0]$ fm, $[1.0, \infty)$ fm with $\Delta = 0.15$ fm
- **additional windows**: cuts at $\{0.1, 0.4, 0.7, 1.0, 1.3, 1.6\}$ fm
- **data-driven evaluation** based on merging of KNT and CHHKS
- systematic effect due to BaBar vs. KLOE tension close to the WP estimate
- full covariance matrices for windows provided

Results for intermediate window



R-ratio result: → Colangelo et al., PLB **833** (2022) 137313

Additional Euclidean-time windows



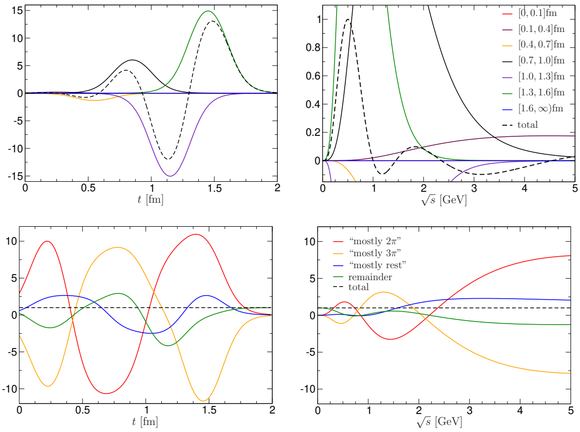
→ Colangelo et al., PLB **833** (2022) 137313

Localization in time-like region possible?

→ see also talk by D. Boito

- better **localization in time-like region** could be achieved by taking linear combinations of Euclidean-time windows
- typically leads to **large cancellations** in Euclidean-time integral
- reflecting ill-posed inverse Laplace transform
- assessing usefulness requires knowledge of **full covariances**
- combinations dominated by **exclusive hadronic channels** suffer from similar problems

Localization in time-like region possible?



→ Colangelo et al., PLB **833** (2022) 137313

Conclusions

- unitarity/analyticity enable **independent checks** via pion VFF and $\langle r_\pi^2 \rangle$
- analysis of resonantly enhanced IB effects point at **systematic differences** between data sets
  - phase of mixing parameter
  - ω mass
- no good fit to SND20 data set possible
- CMD-3: compatible with constraints from unitarity/analyticity

Conclusions

- BMWc result: **window quantities** and **analyticity constraints** point at an effect $\lesssim 8 \times 10^{-10}$ below 1 GeV, $\gtrsim 6 \times 10^{-10}$ above 1 GeV
- more detailed analysis might be possible with additional windows and knowledge of **correlations**