Status of the HPQCD-Fermilab-MILC muon g-2 programme

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Introduction

In Phys.Rev.D 100 (2019) 3, 034506 we published

$$10^{10} a_{\mu}^{\mathrm{HVP,LO}} = 699(15)_{u,d}(1)_{s,c,b}$$

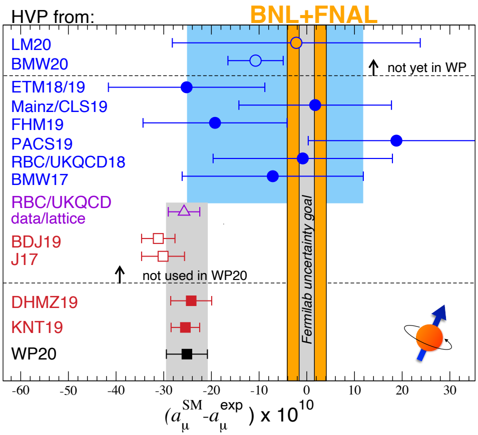
- We have calculated the LO-HVP contribution with an error of 2.2%.
- Our current target is for an error of 0.5%
- As usual in physics the errors are a mixture of systematic and statistical errors.
- This talk will describe the various calculations in progress to reduce the overall error.

There has been increased interest in windowed quantities, so I will discuss two recent papers by our collaboration.

Why a target error of 0.5 % ?

- An error of 0.5% will give an accuracy similar to the phenomenological estimates.

Plot from 2203.15810, Contribution to Snowmass 2021



Overview of the measurements

Essentially measure correlator of two vector currents.

$$G_{ff'}(t) = Q_f Q_{f'} \sum_{\vec{x}} Z_V^2 \langle j_f^i(\vec{x}, t) j_{f'}^i(0) \rangle \, .$$

where f is a flavor index, Q_f is the charge, and Z_V is the vector renormalization factor.

The contribution to a_μ from $G_{ff'}(t)$ is then given by an integral over time:

$$a_{\mu,ff'}^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G_{ff'}(t) K_G(t) \, , \qquad (1)$$

with a known kernel $K_G(t)$.

- We use Peter's g2tools `https://github.com/gplepage/g2tools` to compute the integral.

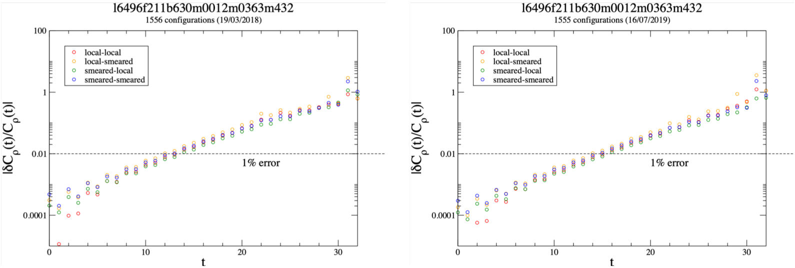
Lattice Ensembles

- We use Nf=2+1+1 HISQ ensembles from the MILC collaboration with physical light quark masses
- Results from above ensembles appeared in arXiv:1902.04223
- These use physical light quark masses.

$\approx a$ (fm)	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	M_{π_5} (MeV)	$(L/a)^3 \times (T/a)$	$N_{\rm conf.}$
0.15	0.00235/0.0647/0.831	1.13670(50)	133.04(70)	$32^3 \times 48$	997
0.15	0.002426/0.0673/0.8447	1.13215(35)	134.73(71)	$32^3 \times 48$	9362
0.12	0.00184/0.0507/0.628	1.41490(60)	132.73(70)	$48^3 \times 64$	998
0.09	0.00120/0.0363/0.432	1.95180(70)	128.34(68)	$64^3 \times 96$	1557
0.06	0.0008/0.022/0.260	3.0170(23)	134.95(72)	$96^3 \times 192$	1230

- One ensemble with 1+1+1+1 sea quarks at 0.15 fm with 5000 configurations (1710.11212).

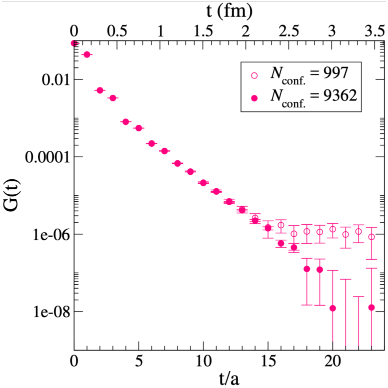
Reducing the errors on the connected correlator



- Connected vector correlator on the 0.09 fm ensemble.
- On the left we have only 16 sloppy solves per configuration, and on the right we have 48 sloppy solves
- Note that 1% error goes from t=12 to t=14.
- Multigrid algorithms exist in QUDA but they only start working at lattice spacing less than 0.09 fm (2212.12559)

Vector correlator at $a \approx 0.15$ fm

- Beyond about 2 fm the vector correlator gets very noisy. Higher statistics help, but improved measurement techniques are probably better.
- We fit the propagator at shorter distance and use fit to extend to $t > t^{\star}$.



Introduction to window analysis

$$a_{\mu,ff'}^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G_{ff'}(t) K_G(t)\,,$$

where $G_{ff'}(t)$ is the correlator of two lattice vector currents, and f and f' are flavour indices.
Note

- Noise on $G_{ff'}(t)$ increases with time t.
- At small t there may be larger lattice spacing corrections.
- Effective field theory works better at large time.

Introducing a clever window function in time allows windowed $a_{\mu,ff'}^{\mathrm{HVP}}$ to be constructed to compare lattice calculations and the results from e^+e^- data.
Bernecker and Meyer (1107.4388), Lehner (1710.06874)

Computing windowed quantities is an intermediate step to computing full $a_{\mu,ff'}^{\mathrm{HVP}}$

One sided Window analysis

Windows on the hadronic vacuum polarization contribution to the muon anomalous magnetic moment Fermilab Lattice and MILC and HPQCD, Collaborations C.T.H. Davies et al. Phys.Rev.D 106 (2022) 7, 074509, 2207.04765

One-sided window then extends from $t = 0$ to t_1 with a rounded edge of width Δt. Window function multiplies the integrand

$$\Theta(t, t_1, \Delta t) = \frac{1}{2}\left[1 - \tanh\left(\frac{t - t_1}{\Delta t}\right)\right] .$$

The contribution to a_μ from the window

$$a_\mu^w(t_1, \Delta t) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, G_{ff'}(t) K_G^w(t)\,,$$

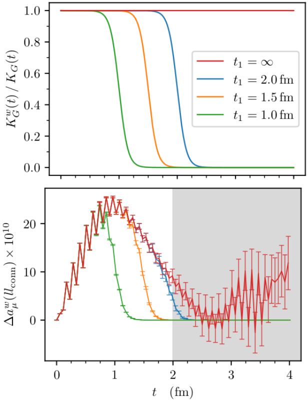
with a modified kernel,

$$K_G^w(t) \equiv K_G(t)\Theta(t, t_1, \Delta t).$$

One sided kernel

Top Plot of the weight function $\Theta(t, t_1, \Delta t)$ as a function of t_1.

Bottom Weighted integrand



$a_\mu^{\rm HVP}$ from $R_{e^+e^-}$

Definition of $a_\mu^{\rm HVP}$ from $R_{e^+e^-}$

$$a_\mu^{\rm HVP} = \left(\frac{\alpha^2}{3\pi^2}\right) \int_{m_\pi^2}^{\infty} ds \frac{R_{e^+e^-}}{s} K_R(s) \, .$$

- The results for $R_{e^+e^-}$ as a function of $\sqrt{s}$ must be collected. This includes summing over exclusive channels, reconciling different experimental results, allowing for correlations and applying QED radiative corrections.
- This is a specialized task.
- The results for $R_{e^+e^-}$ from KNT19 upto 11 GeV were used.
- $\mathcal{O}(\alpha_s^2)$ perturbation theory used for for higher energies.
- The data did not include the recent CMD result arXiv:2302.08834.

Convenient to transform the results for $R_{e^+e^-}$ into a 'lattice correlation function' as a function of Euclidean time, $G_R(t)$, using (Bernecker and Meyer, 1107.4388)

$$G_R(t) \equiv \frac{1}{12\pi^2} \int_0^\infty dE\, E^2\, R_{e^+e^-}\, \mathrm{e}^{-E|t|}\,,$$

where $E = \sqrt{s}$ is the centre of mass energy.

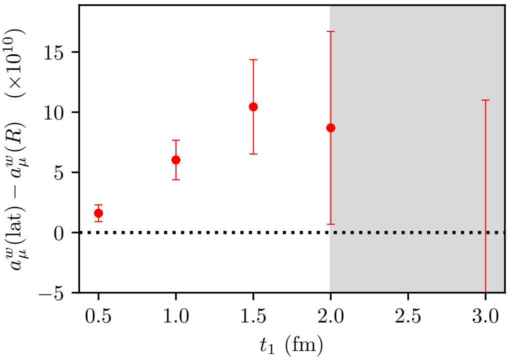
- Evaluate the integral by fitting the integrand to a monotonic (Steffen) spline.
- Evaluate $G_R(t)$ for a discrete set of t, corresponding to a very fine lattice.

Overview of the analysis

Overview analysis (2207.04765) using one sided window

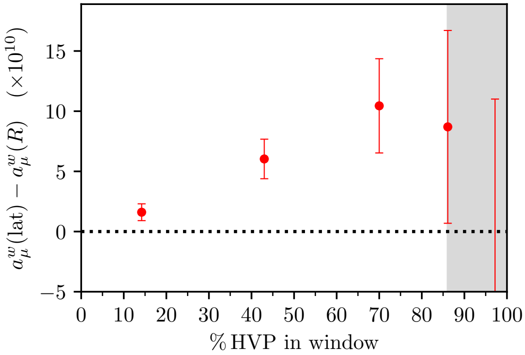
- Use light connected correlators from first analysis (Phys.Rev.D 100 (2019) 3, 034506)
- Use disconnected correlators from 2112.11339 at a single lattice spacing.
- Errors added for missing QED + isospin breaking
- Finite volume and taste corrections from the chiral model.

Compare one sided window from lattice calculation and $R_{e^+e^-}$ data.



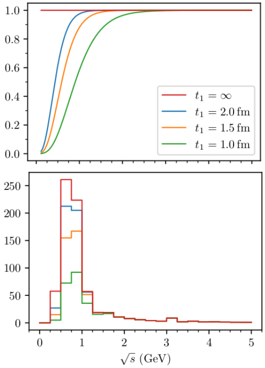
Percentage of window included

Show the amount of $a_\mu^{\mathrm{HVP,LO}}$ included in the one sided window.



Window as a function of s.

Visualize window in energy.



Two sided Window analysis

Light-quark connected intermediate-window contributions to the muon g-2 hadronic vacuum polarization from lattice QCD, Bazavov et al., 2301.08274 to appear in Phys. Rev. D (PhD thesis of Shaun Lahert)

Summary of the update of the analysis

- Calculation of the results for two "standard windows."
- Update in statistics over 2019 paper.
- Analysis was blinded, by adding a random number to the results.
- Use of Bayesean model averaging to incorporate different effective field theory treatment of taste, volume and mass shift.
- Only light quark contribution calculated, so compare to other lattice QCD calculations.

Definition of two sided Window

$$a_\mu^{\mathrm{win}(t_0,t_1,\Delta)} = 4\alpha^2 \int_0^\infty \mathrm{d}t\, C(t)\tilde{K}(t)\mathcal{W}(t, t_0, t_1, \Delta),$$

$$\mathcal{W}(t, t_0, t_1, \Delta) = \frac{1}{2}\left[\tanh\left(\frac{t-t_0}{\Delta}\right) - \tanh\left(\frac{t-t_1}{\Delta}\right)\right] + (t \to -t).$$

- t_1 and t_2 control the location of the window.
- Δ controls the sharpness of the edges of the windows

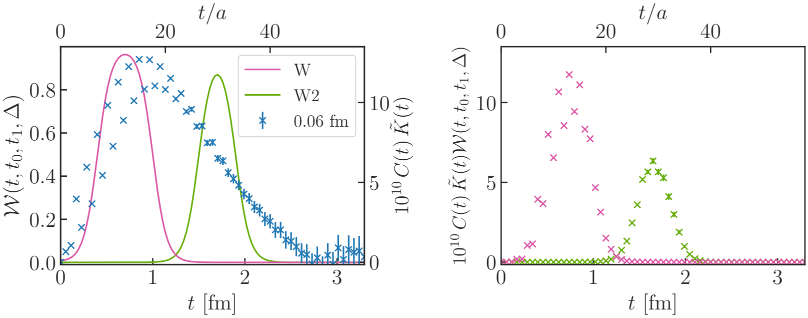
$$a_\mu^{\mathrm{W}} \equiv a_\mu^{\mathrm{win}(0.4,\ 1,\ 0.15)}$$
$$a_\mu^{\mathrm{W2}} \equiv a_\mu^{\mathrm{win}(1.5,\ 1.9,\ 0.15)}$$

- a_μ^{W} a standard window used as a benchmark for calculations
- a_μ^{W2} window introduced by Aubin et al. 2204.12256 where EFT caculations are more reliable.

Plot of the windows function

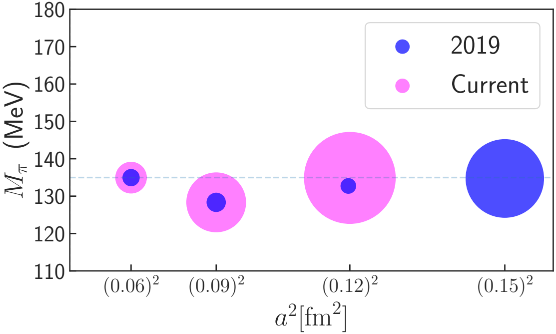
Left The W (magenta) and W2 (green) window functions overlaid with raw lattice data for the integrand from the ensemble with a=0.06 fm.

Right The windowed integrand, product of lattice data with window.



Additional statistics

Statistics of some ensembles increased



Disk areas are proportional to the size of each data set ($N_{\text{conf}} \times N_{\text{loose sources}}$.

Effective field theory

In the 2019 calculation we used a Chiral Model (CM), including a vector meson with chiral perturbation theory, to correct for taste, finite volume and mass corrections.

We included additional effective theory calculations.

- Chiral Perturbation Theory (χPT) at next-to-leading order (NLO) and next-to-next-to-leading order (NNLO).
- The Meyer-Lellouch-Lüscher-Gounaris-Sakurai (MLLGS) approach combines the pion form-factor parameterization of Gounaris-Sakurai with the mapping (due to Meyer-Lellouch-Lüscher
- The relativistic-pion effective-field-theory approach by Hansen and Patella (HP) for finite-volume effects

Continnum extrapolation

To perform our continuum extrapolation we consider fit functions of the form:

$$a_\mu^{ll}(a, \{m_f\}) = a_\mu^{ll}\left(1 + F^{\text{disc.}}(a) + F^m(\{\delta m_f\})\right), \qquad (2)$$

where

$$F^{\text{disc.}}(a) = C_{a^2,n}\left[(a\Lambda)^2\alpha_s^n\right] + C_{a^4}(a\Lambda)^4 + C_{a^6}(a\Lambda)^6 \qquad (3)$$

$$F^m(\{\delta m_f\}) = C_{\text{sea}} \sum_{f=l,l,s} \delta m_f/\Lambda. \qquad (4)$$

- $F^{\text{disc.}}(a)$ describes discretization effects
- $F^m(\{\delta m_f\})$ accounts for quark mass differences in the sea

Bayesian model averaging

See: 2008.01069 Jay and Neil, 2208.14983 Neil and Sitison.

- A "model" M is defined as the set of analysis choices that yield a given result for the desired continuum, infinite-volume, physical observable from a single data set D
- D consists of the unmodified correlation function data

$$\mathrm{pr}(M \mid D) \equiv \mathrm{pr}(M) \exp\left[-\frac{1}{2}\left(\chi^2_{\mathrm{data}}(\mathbf{a}^\star) + 2k + 2N_{\mathrm{cut}}\right)\right]$$

k is the number of parameters.

$$\langle a_\mu \rangle = \sum_i \langle a_\mu \rangle_i \, \mathrm{pr}\left(M_i \mid D\right)$$

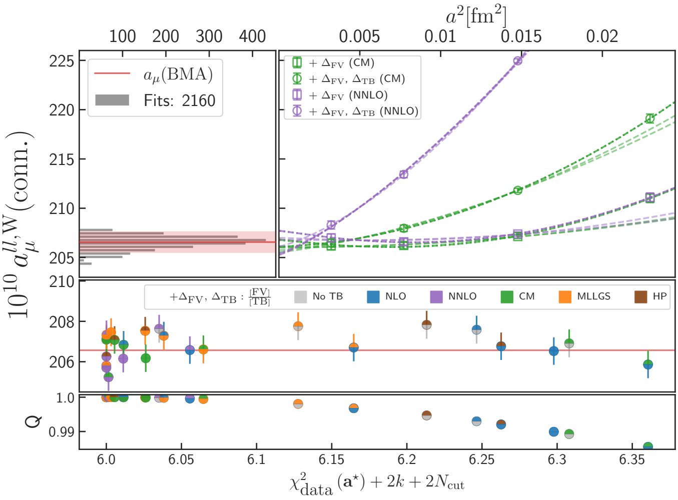
- "Bayesian Akaike information criterion" (BAIC)
- The factor $\mathrm{pr}(M)$ is the prior probability of a given M; we adopt a flat prior.

Comment on Model Averaging

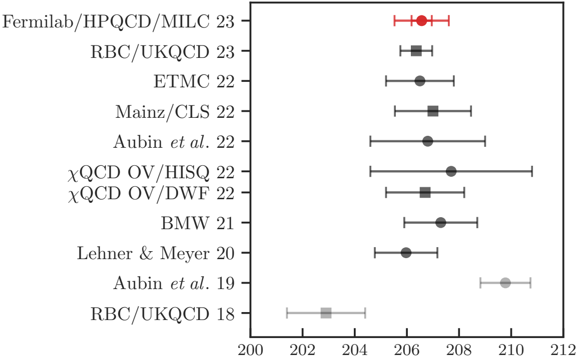
- The US nuclear physics communty working nuclear dynamics, such as properties of nuclei are heavy users of Bayesian model averaging and other similar techniques.
- Get on the BAND Wagon: A Bayesian Framework for Quantifying Model Uncertainties in Nuclear Dynamics, Phillips et al. arXiv:2012.07704
  `https://bandframework.github.io/`

Eventually our calculatiom of $a_\mu^{\mathrm{HVP,LO}}$ will be more data driven, by using dedicated finite volume studies.

Results of Bayesian Model Averaging

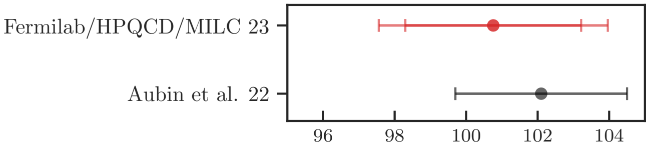


Summary of results for a_μ^{W} (0.4 to 1 fm)



- RBC/UKQCD has been updated from a blinded analysis in 2301.08696 Blum et al.

Summary of results for a_μ^{W2} (1.5 to 1.9 fm)



Aubin et al 2204.12256 use a subset of the HISQ configurations with a slightly different valence action.

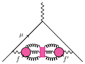
Ensembles used in the disconnected analysis

- The main update from lattice 2021 (2112.11339) in the data generation is that the statistics on the fine ensemble has increased.
- The disconnected correlator analysis is blinded in a simple manner.
- There are currently no plans to run on a ensemble with a smaller lattice spacing.

Ensemble	a fm	m_π MeV	L fm	Eigenmodes	N_{meas}
Very coarse	0.15	134.7	4.8	300	1692
Coarse	0.12	134.9	5.8	-	787
Fine	0.09	128.3	5.8	1000	488

Table: HISQ ensembles used in the disconnected analysis

Estimating loops



The measurement of the loops requires the computation of

$$L_{ls}(t) = \mathrm{Tr}\left(\gamma_\mu \frac{1}{\not{D} + m_l} - \gamma_\mu \frac{1}{\not{D} + m_s}\right)$$

where $\not{D}$ is the massless HISQ Dirac operator and γ_μ is the taste singlet vector operator.
The difference in the above equation can be trivially written down as:

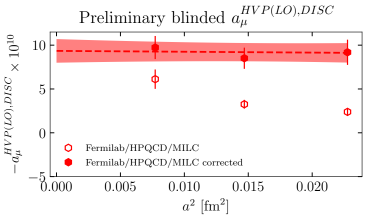
$$L_{ls}(t) = \mathrm{Tr}\left(\gamma_\mu \frac{(m_s - m_l)}{(\not{D} + m_l)(\not{D} + m_s)}\right)$$

Used by the ETM collaboration 0803.0224, and Giusti et al. 1903.10447.

Isospin symmetric disconnected

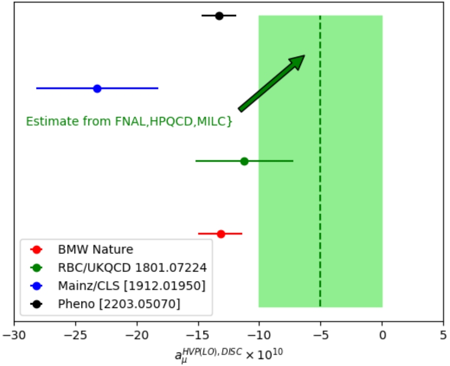
Results from lattice 2021 proceedings (Continuum extrapolation with $O(a^2)$.)

- The taste/volume corrections come from the Chiral model with a 20 % error estimate.
- We are implementing Bayesian model averaging for this analysis. This will allow us to include fits without taste corrections.



Summary of isospin symmetric disconnected

- Green band is the estimate of disconnected contributions from FNAL/HPQCD/MILC Phys.Rev.D 100 (2019) 3, 034506 (1902.04223).



QED corrections to a_μ^s and a_μ^l

Calculating the QED correction to the hadronic vacuum polarisation on the lattice, Gaurav Ray et al., 2212.12031, Lattice 2022 presentation.

Including quenched QED with QCD

- Non-compact $A_\mu(k)$ generated in Feynman gauge for each QCD gluon field configuration.
- Electroquenched.
- Use the QED_L formulation to deal with zero modes.

$$U_\mu^{QCD+QED} = \exp(ieQA_\mu) U_\mu^{QCD}$$

- We are currently computing **connected** QED+QCD correlators.
- The value of Z_V has been computed including quenched QED using the RI-SMOM scheme. (Hatton et al., HPQCD, PhysRevD.100.114513)

Follow HPQCD (2005.01845) study QED contributions to charmonium mesons.

Details of the quenched QED+QCD calculation

- Random sources wall sources.
- The analysis is blinded
- Truncated solver method with 16 sloppy and 1 fine inversion on each lattice.
- Use charge averaging over $+Q, -Q$

We measure neutral vector and pseudo-scalar correlators.

Ensemble	$L^3 \times T$	a[fm]	no. meas	masses
very coarse	$32^3 \times 48$	0.15	1844	m_u m_d $3/5/7m_l$ m_s
coarse	$48^3 \times 64$	0.12	967	$3/5/7m_l$ m_s
fine	$64^3 \times 96$	0.09	596	$3/5/7m_l$ m_s

Ensembles have physical pion masses, but because of noise increassing we use $3/5/7m_l$ valence quark masses (following BMW) and extrapolate to m_l.

Definitions of δa_μ^s

QED correction to the $a_\mu^{\mathrm{HVP,LO}}$, δa_μ^f,

$$\delta a_\mu^{(f)} \equiv a_\mu^f(m_f, Q_f) - a_\mu^f(m_f, 0),$$

where f labels the quark flavour and the difference is evaluated at **equal renormalised quark mass.**
The QED correction to the connected strange $a_\mu^{\mathrm{HVP,LO}}$is then

$$\delta a_\mu^{(s)} = a_\mu^s(m_s, -1/3e) - a_\mu^s(m_s, 0),$$

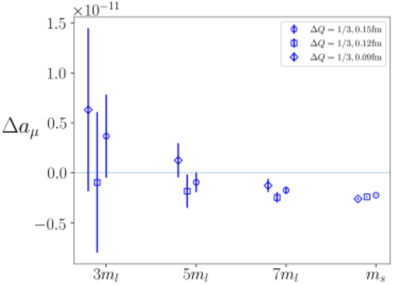
We originally extract QED corrections to a_μ at fixed bare quark mass (Δa_μ) and then convert to δa_μ using

$$\delta a_\mu = \Delta a_\mu - \delta m_q \frac{\partial a_\mu}{\partial m_q}$$

- We are not currently including QED in the lattice spacing determination.

Noise in Δa_μ^q

- The error increases in Δa_μ^q as the light quark mass is reduced
- It is not clear that the noise follows a Lepage like argument for the signal to noise.
- The noise on the up quark contribution is much larger than for the down quark contribution.



Dashen-like scheme

- We have initially used a Dashen-like scheme developed by MILC (1807.05556) and BMW
- We did explore using the GRS scheme (QCD+QED and QCD results compared at fixed renormalized mass) used by the ETM collaboration.
- We followed the prescription used by MILC/FNAL lattice (1807.05556)

$$M^2_{uu'} = M^2_{dd'} = M^2_{nn'} \equiv M^2_{\pi^0}$$

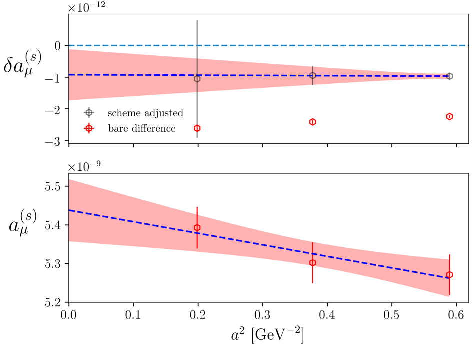
The parameters δ_u and δ_d are defined from the above.

$$m_u = m_l(1 - \delta_u)$$

$$m_d = m_l(1 - \delta_d) \quad , m_S = m_s(1 - \delta_d)$$

δa_μ^s

From lattice 2022, 2212.12031



- Choice of scheme has increased the errors.
- We have the same issue with δa_μ^u, so we are investigating other schemes.

Plans

Improvements to existing calculations. All the improvements below are in progress.

- Develop better schemes for comparing $a_\mu^{\mathrm{HVP,LO}}$ in QCD and QCD+QED.
- Estimate of QED on setting the lattice spacing from Omega baryon.
- Including two pion states in the connected vector correlator (arXiv:2112.11647)
- Using eigenmodes in the connected vector correlator to reduce the errors.
- Inclusion of physical $a \approx 0.042$ fm configurations from MILC in the analysis to improve continuum extrapolations.
- New ensembles to do data driven finite volume studies.

Future work

New Calculations of missing quantities required to compute all contributions to $a_\mu^{\mathrm{HVP,LO}}$

- Including QED in the disconnected diagrams. (In progress)
- Estimating QED contributions to the sea.

Low mode averaging

- The measurement of lattice QCD correlators is based on matrix operations on the quark fermion matrix and gauge fields.
- The eigenmodes of the quark fermion matrix can be used to reduce the errors on the correlators.

The computation of the disconnected loops at 0.09 fm uses the low eigenmodes with the difference trick.

- It is computationally expensive to compute 1000s of eigenvalues of large matrices with orders over million. See Jeong et al. 2201.03755

FNAL/HPQCD/MILC are using low mode averaging for connected correlators starting at 0.09 fm and finer ensembles.
On the physical point ensemble at 0.09 fm Aubin et al (2204.12256) use 78 lattices with 8000 eigenmodes, compared to FNAL/HPQCD/MILC (1902.04223) with 1557 lattices with 0 eigenmodes.

Conclusions

- The results of a two sided window analysis is a good way to compare different lattice QCD calculations.
- One side window analysis is a useful way to see the difference between dispersion analysis and lattice calculations for $a_\mu^{\mathrm{HVP,LO}}$.
- Given the CMD result arXiv:2302.08834 in the mass energy range from 0.32 to 1.2 GeV, is it possible to create windows in energy, but perhaps this is an inverse problem.
- I have listed a number of project in progress to reduce the errors.