Status of the RBC/UKQCD muon $g-2$ programme

Mattia Bruno

Lattice Gauge Theory Contributions to New Physics searches
IFT, Madrid, Spain, June 12th

The RBC & UKQCD collaborations

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BNL and BNL/RBRC
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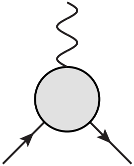
Anomalous magnetic moment



scattering of particle mass m off external photon (μ, q)

$$-ie\Big[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m} F_2(q^2)\Big]\,, g = 2(F_1(0) + F_2(0))$$

$$F_1(0) = 1 \rightarrow F_2(0) = a = (g-2)/2$$

A rich history

electron a_e measured in experiment [Kusch, Foley '48]

confirms radiative corrections [Schwinger '48] $\rightarrow$ success of QFT

muon a_μ measured in experiement [Columbia exp. '59]

"muon is heavy electron" $\rightarrow$ families of leptons

Back to the future

new physics contribution to a: $(a - a^{\mathrm{SM}}) \propto m^2/\Lambda^2_{\mathrm{NP}}$

a_τ experimentally inaccessible, a_μ most promising

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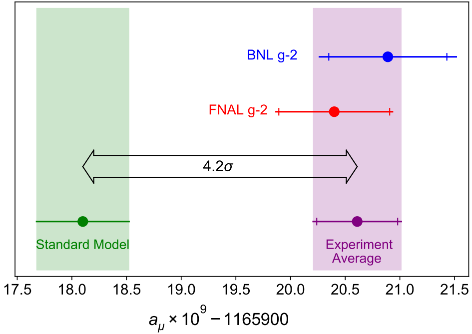
Muon $(g-2)$
Status

$(g-2)$ theory initiative

[White Paper '20]

SM contributions to $a_\mu [\times 10^{10}]$

5-loop QED	11 658 471.8853(36)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.9)



Theory error dominated by hadronic physics HVP and HLbL

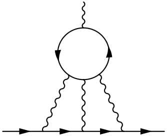
Hadronic Vacuum-Polarization and Light-by-Light

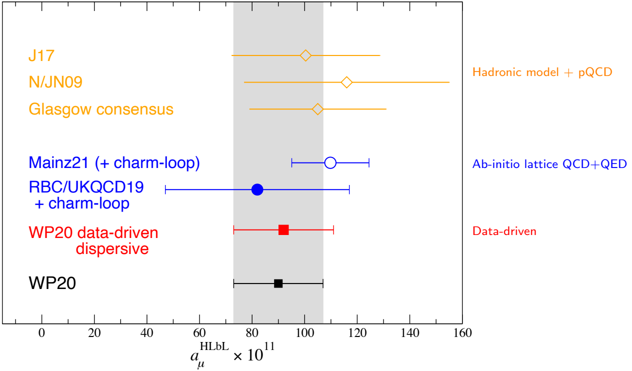
Precision goal for Fermilab $\times 4$ better

implies knowing HVP at 0.2-0.3 % accuracy

Hadronic Light-by-Light

Status





Consistency between lattice QCD+QED and dispersive

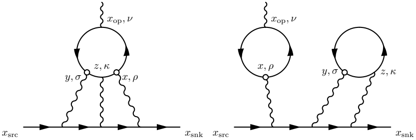
novel update $124.7(11.5)(9.9) \cdot 10^{11}$ [RBC/UKQCD '23]

Hadronic Light-by-Light

Lattice

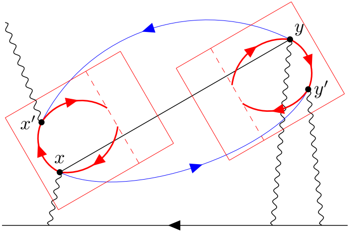
Stochastic point-source sampling

[RBC/UKQCD '17]

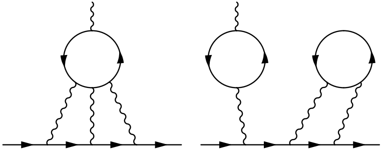


Isolation of $\pi^0(\to\gamma\gamma)$ piece

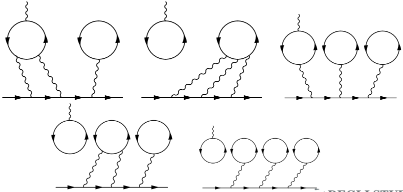
[RBC/UKQCD '23]



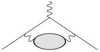
leading



subleading

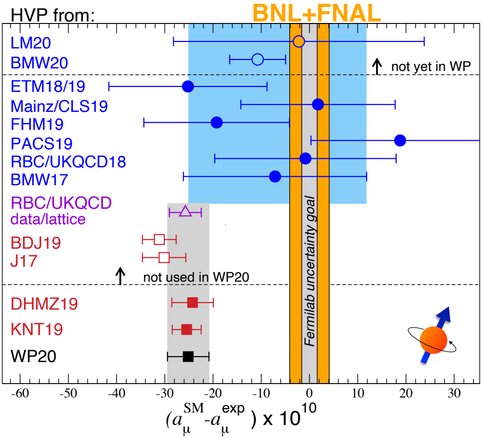


Hadronic Vacuum Polarization



Overview

[Snowmass '21]



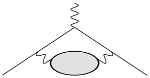
BMW20 first complete Lattice QCD+QED calculation below 1%

Lattice QCD+QED

data-driven/dispersive

WP20: $g-2$ theory initiative community White Paper

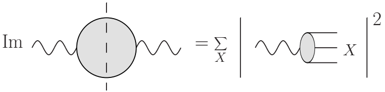
→ only data-driven/dispersive used in current best estimate



$$a_\mu = \frac{\alpha}{\pi} \int \frac{ds}{s} \, K(s, m_\mu) \, \frac{\mathrm{Im}\Pi(s)}{\pi}$$ [Brodsky, de Rafael '68]

analyticity $$\hat{\Pi}(s) = \Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} dx \frac{\mathrm{Im}\Pi(x)}{x(x - s - i\varepsilon)}$$

unitarity



$$\frac{4\pi^2\alpha}{s} \frac{\mathrm{Im}\Pi(s)}{\pi} = \sigma_{e^+e^- \to \gamma^\star \to \mathrm{had}}$$

At present $O(30)$ channels: $\pi^0\gamma, \pi^+\pi^-, 3\pi, 4\pi, K^+K^-, \cdots$

- $K(s, m_\mu) \to \pi^+\pi^-$ dominates due to ρ resonance
- $\pi\pi$ channel is $\sim 70\%$ of signal and $\sim 70\%$ of error

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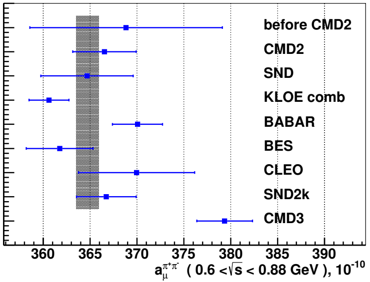
Large tensions among experiments: BaBar, KLOE, now CMD3

[CMD3 2302.08834]



very difficult to combine different experiments

what is the error of $\pi\pi$ contribution to a_μ?

motivates even more first-principles Lattice QCD calculations

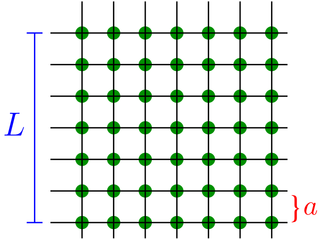
Lattice field theories

Non-perturbative predictions

lattice spacing $a \to$ regulate UV divergences

finite size $L \to$ infrared regulator

Continuum theory $a \to 0$, $L \to \infty$



Euclidean metric $\to$ Boltzman interpretation of path integral

$$\langle O \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S[U]} O(U) \approx \frac{1}{N} \sum_{i=1}^{N} O[U_i]$$

Very high dimensional integral $\to$ Monte-Carlo methods

Markov Chain of gauge field configs $U_0 \to U_1 \to \cdots \to U_N$

Electro-magnetic current $j_\mu(x) = i \sum_f Q_f \overline{\psi}(x)\gamma_\mu \psi(x)$

$$a_\mu = 4\alpha^2 \int dQ^2 K(Q^2)[\Pi(Q^2) - \Pi(0)] \quad (Q^2 \text{ euclidean})$$

[Blum '03]

$\Pi_{\mu\nu}(Q^2) = \int d^4x e^{iQ\cdot x} \langle j_\mu(x) j_\nu(0)\rangle$ on the lattice

Time-momentum representation [Bernecker, Meyer, '11]

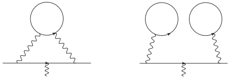
$$C(t) = \frac{1}{3}\sum_k \int d\vec{x} \, \langle j_k(x) j_k(0)\rangle \qquad \langle\cdot\rangle = \text{QCD+QED exp. value}$$

$$a_\mu = 4\alpha^2 \int_0^\infty dt \, w(t) \, C(t) \,, \quad w(t) \text{ muon kernel (weights)}$$

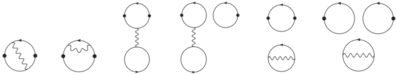
HVP from Lattice

Diagrams

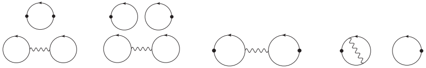
Isospin limit



QED corrections

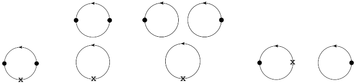


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

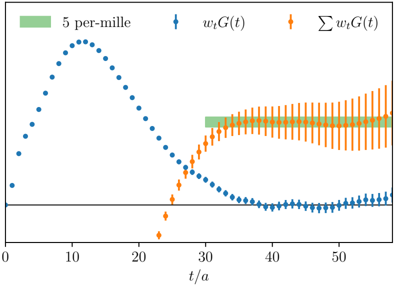
Strong isospin breaking

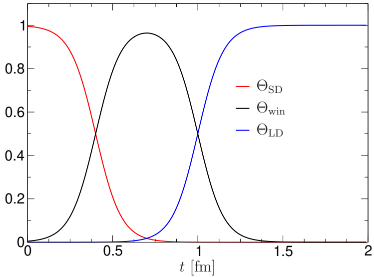


(a) M (b) R (c) R_d (d) O

Euclidean windows

A novel paradigm





Smoothly divide integral in several parts

$$a_\mu = 4\alpha^2 \sum_t w_t \Big[\Theta_{\rm SD}(t) + \Theta_{\rm W}(t) + \Theta_{\rm LD}(t)\Big] G(t)$$

[RBC/UKQCD '18]

- short-distance $\rightarrow$ cutoff effects
- long-distance $\rightarrow$ Monte-Carlo noise
- intermediate window: accessible today w/ current resources
  - most collaborations precision of 0.4 - 0.6 %

HVP from Lattice

Theoretical advances

Formulation isospin-breaking schemes, isosymmetric points

[RM123][RBC/UKQCD 18][BMW 20][WP20][Portelli Lat22][Tantalo Lat22][...]

Analytic control of finite-volume effects

[Hansen, Patella '19 '20][Lehner, Meyer '20][Bijnens et al '19]

Improved understanding of scaling violations

[Mainz 20][Husung, Marquard, Sommer '22][Husung '23][Sommer Lat22]

Numerical setup

Domain-wall fermions

ID	a^{-1}/GeV	$L^3 \times T \times L_s/a^4$	m_π/MeV	m_K/MeV	m_{D_s}/GeV	$m_\pi L$
48I	1.7312(28)	$48^3 \times 96 \times 24$	139.32(30)	499.44(88)	–	3.9
64I	2.3549(49)	$64^3 \times 128 \times 12$	138.98(43)	507.5(1.5)	–	3.8
96I	2.6920(67)	$96^3 \times 192 \times 12$	131.29(66)	484.5(2.3)	–	4.7
1	1.7310(35)	$32^3 \times 64 \times 24$	208.1(1.1)	514.0(1.8)	–	3.8
2	1.7257(74)	$24^3 \times 48 \times 32$	285.4(2.9)	537.8(4.6)	–	4.0
3	1.7306(46)	$32^3 \times 64 \times 24$	211.3(2.3)	603.8(6.1)	–	3.9
4	1.7400(73)	$24^3 \times 48 \times 24$	274.8(2.5)	530.1(3.1)	–	3.8
5	1.7498(73)	$24^3 \times 48 \times 24$	279.8(3.5)	539.1(5.3)	1.9902(69)	3.8
7	1.7566(81)	$24^3 \times 48 \times 24$	272.5(5.9)	523(10)	1.3882(57)	3.7
A	1.7556(83)	$24^3 \times 48 \times 8$	307.4(3.5)	557.3(5.7)	–	4.2
24ID	1.0230(20)	$24^3 \times 64 \times 24$	142.96(30)	515.7(1.0)	–	3.4
32ID	1.0230(20)	$32^3 \times 64 \times 24$	142.96(30)	515.7(1.0)	–	4.5

Iwasaki gauge acton + Möbius DWF [Brower at al '12][RBC '14]

- 3-level stout smearing for charm quarks
- GRID+gpt open-source software libraries

Isosymmetric $N_f = 2 + 1$ world(s)

Tuning

1. at fixed a, hadronic masses $m_X(a, m_l, m_s)$ with $X = \pi^+, K^+, \Omega^-$
   derivatives w.r.t. bare parameters m_l, m_s

2. solve system of linear equations for $\Delta m_l, \Delta m_s$

$$m_X(m_l, m_s) + \sum_{f=l,s} (\partial_{m_f} m_K)\, \Delta m_f = m_X^{\text{target}} \quad X = \pi, K$$

3. shift m_Ω for lattice spacing and all other quantities

[RBC/UKQCD 18]

1. $m_\pi = 0.135\ \text{GeV}\,, m_K = 0.4957\ \text{GeV}\,, m_\Omega = 1.67225\ \text{GeV}$

[BMW 20]

2. $m_\pi = 0.13497\ \text{GeV}\,, m_{ss*} = 0.6898\ \text{GeV}\,, w_0 = 0.17236\ \text{fm}$

3. $m_{D_s} = 1.96847\ \text{GeV}$ [RBC/UKQCD 23]

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Observable $\mathcal{O}(s,x)$ at Monte Carlo time s and position x_μ [Lüscher '17]

true expectation value $\langle\mathcal{O}\rangle$

$\langle\langle\overline{\mathcal{O}}\rangle\rangle$ our best estimator for $\langle\mathcal{O}\rangle$

Stochastic locality implies $\lim_{V\to\infty}\langle\langle\overline{\mathcal{O}}\rangle\rangle = \langle\mathcal{O}\rangle$ (even at fixed N)

$\langle\big[\langle\langle\overline{\mathcal{O}}\rangle\rangle - \langle\mathcal{O}\rangle\big]^2\rangle = \frac{1}{VN}\sum_{s,x}\Gamma(s,x)$

$\Gamma(s,x) = \langle\big[\mathcal{O}(s,x)-\langle\mathcal{O}\rangle\big]\big[\mathcal{O}(0)-\langle\mathcal{O}\rangle\big]\rangle$

In practice our best estimator for Γ is given by

$\overline{\Gamma}(s,x) = \frac{1}{V}\frac{1}{N-s}\sum_{s',x'}\delta\mathcal{O}(s+s',x+x')\delta\mathcal{O}(s',x')$

w/ $\delta\mathcal{O}(s,x) = \mathcal{O}(s,x) - \langle\langle\overline{\mathcal{O}}\rangle\rangle$

Estimator of the error from truncated sum

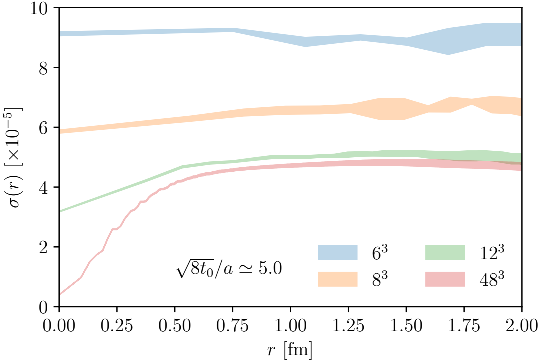
$$\sigma^2 = \frac{1}{V}\left[\sum_{|x|\le r}\overline{\Gamma}(x) + O(e^{-mr}) + O(V^{-1/2})\right]$$

Master-field errors

Gauge-noise limit

$\mathcal{O}$ known on subset of points (common for fermionic observables)

e.g. on a grid of points w/ equal distance



[MB et al, in prep]

error of E_{t_0}

$m_\pi L \simeq 6.7$

less points smaller accuracy

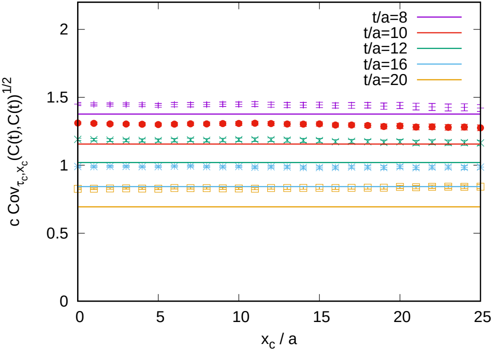
master-field analysis in space L^3

checking saturation point useful to stop fermionic measurements

similar to check scaling with number random sources

Master-field errors

RBC/UKQCD 23



96I ensemble, phys. pion

660 random point sources
33 configs

vector correlator

plateau in MC time not shown

solid lines = Jackknife errs

Among authors [RBC/UKQCD 23] 5 non-overlapping analysis groups b

$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t)$$

b_0, b_1, b_2 randomly chosen for each analysis group b

$|b_1 a^2| < 0.05$ and $|b_2 a^4| < 0.0025$

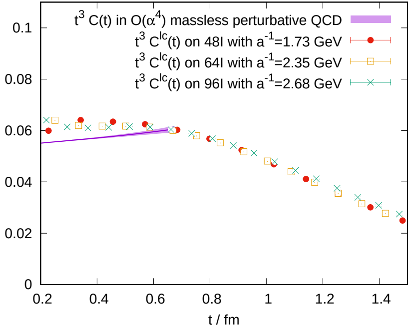
---

1. every group complete analysis of window quantities
   tuning + cont. limit + inf.vol limit
2. once analysis code fixed, re-run on unblinded data

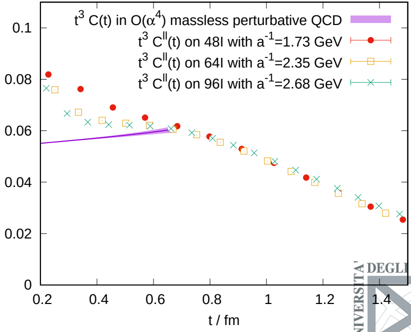
Local current $Z_V j^l_\mu(x)$ vs conserved current $j^c_\mu(x)$

$$C(t) = \frac{c_0(\alpha_s)}{t^3}\left[1 + O\left(a^2/t^2\right) + O\left(m^2/t^2\right)\right]$$

local-conserved



local-local



Continuum limit

Intermediate window

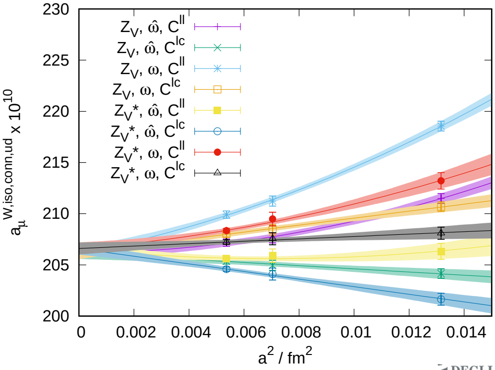
$$a_\mu^W = \sum_t \Theta_{\rm W}(t)\, w_t\, C(t)$$

x2: w_t cont vs discretized

x2: Z_V 3pt pion charge
ratio local/conserved

x2: $C(t)$ from ll or lc

$=$ 8 trajectories $a \to 0$
(correlated though)



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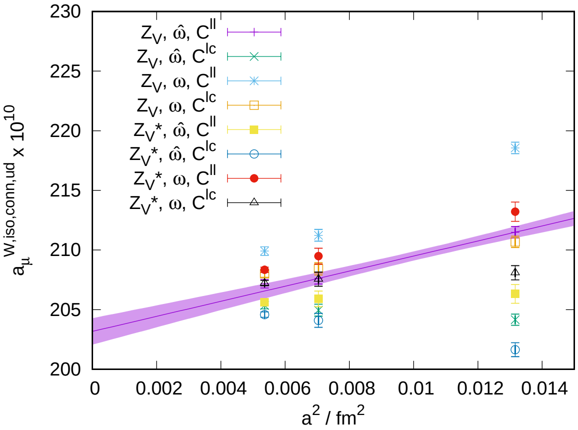
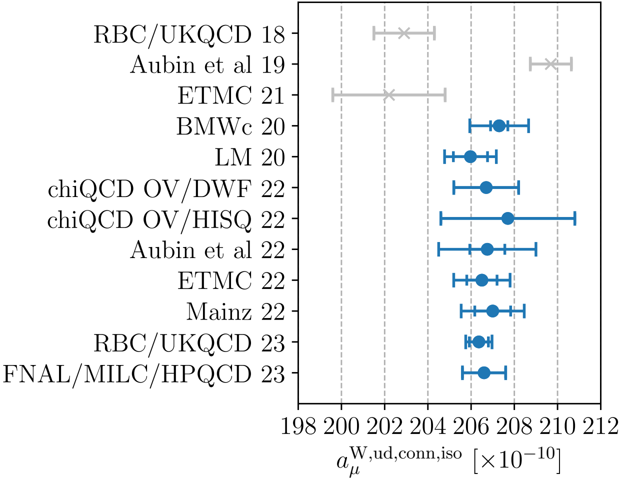
Intermediate window

RBC/UKQCD 18

Older result reproduced and understood as under-estimated syst. errs from cont limit



isosymmetric intermediate window: internal lattice cross-checks



New puzzles forming

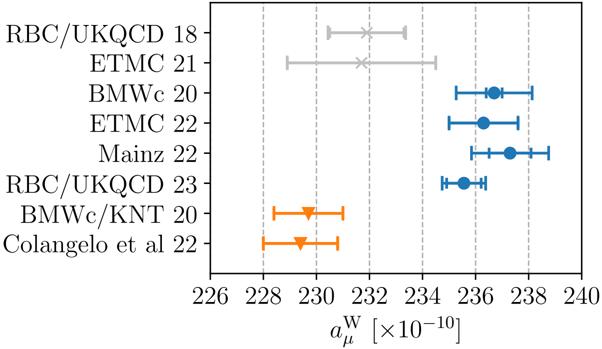
Comparison with data

Windows calculable starting from $R(s)$: compare w/ Lattice QCD+QED

isosymmetric updated value [RBC/UKQCD 23]

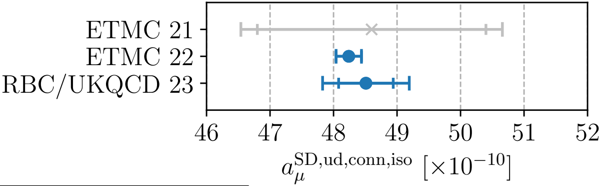
isospin-breaking + strange + charm + disc [RBC/UKQCD 18]

Situation before CMD3 (see also [Aubin et al/CL/KNT 19])

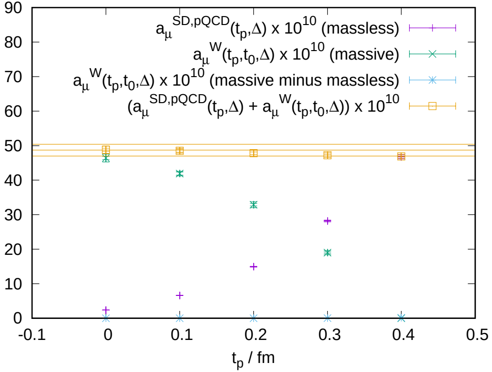


Short-distance window

Results



[RBC/UKQCD '23]



isospin limit

dominated by pert. theory

[ETMC 22]

[Sommer Lat22][Lehner Lat22]

Outlooks

1. **Long-distance window**

2. **Isospin-breaking**

3. Update strange, charm, disconnected as well

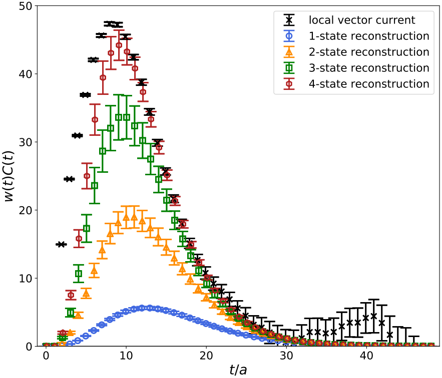
4. τ-decay data

Numerical strategy

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \sum_n e^{-E_n t}|\langle n|\hat{\mathcal{O}}|0\rangle|^2 \overset{t\gg 0}{\approx} \sum_n^N e^{-E_n t}|\langle n|\hat{\mathcal{O}}|0\rangle|^2$$

dedicated calculation to resolve lowest N states [H. Meyer '12]

$\rightarrow$ partially cured signal-to-noise growth



[MB, Meyer, Lehner, Izubuchi PoS '19]

naive full sum

$$\delta a_\mu = 38 \times 10^{-10}$$

truncated sum (bounding method)

$$\delta a_\mu = 16 \times 10^{-10}$$

3-state reconstruction

$$\delta a_\mu = 5 \times 10^{-10}$$

area = HVP contribution to a_μ

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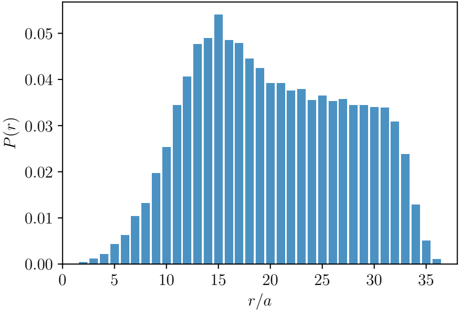
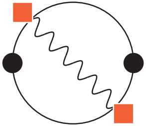
Propagators on disk from HLbL project [Phys.Rev.Lett. 118 (2017)]

$$\tilde{V}_\Gamma(x_0, z_0, r) = \sum_{\vec{x},\vec{z}} \mathrm{tr}\Big[\Gamma D^{-1}(x,0)\gamma_\nu D^{-1}(0,z)\Gamma D^{-1}(z,r)\gamma^\nu D^{-1}(r,x)\Big]$$

$$V_\Gamma(|x_0 - z_0|) = \sum_r \Delta(r)\tilde{V}_\Gamma(x_0, z_0, r)$$

$O(10^3)$ points $\rightarrow$ $O(10^6)$ pairs





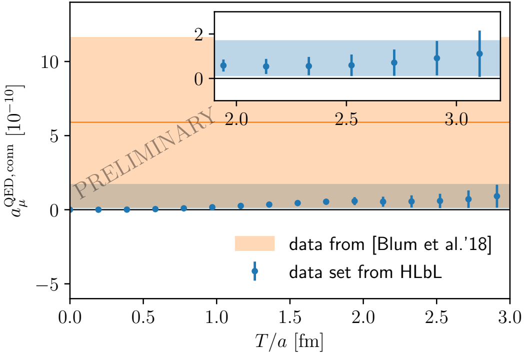
contract photon offline
$\rightarrow$ study QED_L vs QED_∞

QED valence connected

Preliminary

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagrams V, S to a_μ



Coarse ensemble 32ID

$\sim 3 \cdot 10^3$ point pairs

$O(10)$ configurations

preliminary (rough) analysis

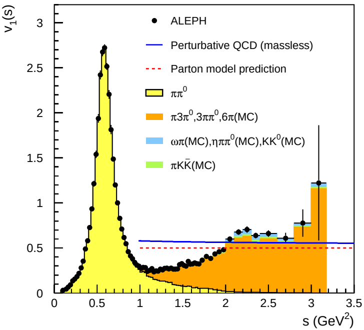
plain sum up to 3 fm

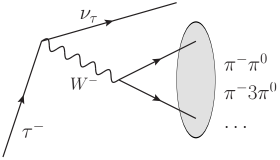
$\times 4$ reduction in stat. error

only stat. error showed

expected QED conn. error $\leq 3 \times 10^{-10} \rightarrow$ matches target

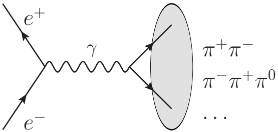
τ DECAYS





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\rightarrow$ 72% of total Hadronic LO

$\rightarrow$ competitive precision on a_μ^W

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Conclusions

Intermediate window in isospin limit
- reached target accuracy

Short-distance window isospin limit
- dominated by perturbation theory
- sufficient control a^2 effects for precision target

Intermediate window in Standard Model
- significant tensions among experiments
- remarkable agreement among lattice results
  - significant tension lattice vs experiment

Inclusion long-distance window and remaining effects
- 1 lattice complete (impressive) result [BMW 20]
- RBC/UKQCD aiming at complete prediction soon

Thanks for the attention

Charm tuning strategy

Stoch. loc./master-field for improved estimators = truncated vol. sums

$$\partial_\beta \mathcal{O} = \langle \mathcal{O}\, W \rangle - \langle \mathcal{O} \rangle \langle W \rangle$$

$$\partial_m \langle \mathcal{O} \rangle_{N_f=1} = \langle \mathcal{O}\, \mathrm{tr} \tilde{D}^{-1}(m) \rangle_{N_f=1} - \langle \mathcal{O} \rangle \langle \mathrm{tr} \tilde{D}^{-1}(m) \rangle_{N_f=1}$$

$$\tilde{D}^{-1} = \tfrac{1}{1-m}(D^{-1}(m) - 1)$$

flowed energy density $E(t = t_0/2)$