Entanglement and complexity in various braneworld models

Complexity: Between Field Theory and Gravity

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Based on work with Rob Myers, Shan-Ming Ruan and work in progress with Sergio Aguilar-Gutierrez, Ben Craps, Mikhail Khramtzov, Maria Knysh and Ashish Shukla

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Outline

Geometry from entanglement

Entanglement entropy

Connection between entanglement and geometry

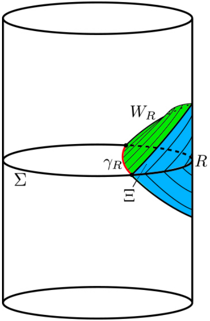
- HRT formula

$$S(R) = \min_{\partial\gamma_R = \partial R} \frac{A(\gamma_R)}{4G}$$

- EE from 1st law

$$\delta S = \delta \langle H \rangle \to E^{g}_{ab} = 0$$

- Entanglement wedge reconstruction

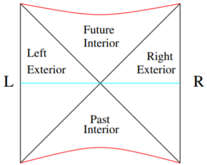


Geometry from entanglement

Entanglement entropy is not enough [Susskind]

Bulk regions inaccessible to entanglement entropy

- Entanglement entropy thermalizes fast
- HRT surfaces don't reach deep interior regions
- Deep interior regions not probed by entanglement entropy



Geometry from entanglement

Entanglement entropy is not enough

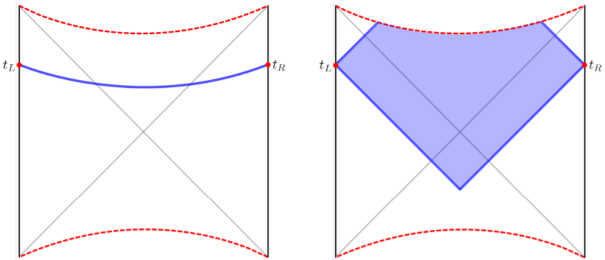
Solution: Study other geometric quantities which reach deeper into the bulk

Complexity

Holographic complexity

Two proposals: complexity=volume (C_V) and complexity=action (C_A)

$$C_V(R) = \max_{\partial\Sigma_R = R} \frac{V(\Sigma_R)}{G\ell}\,, \quad C_A(R) = \frac{I_{WdW}(R)}{\pi\hbar}$$



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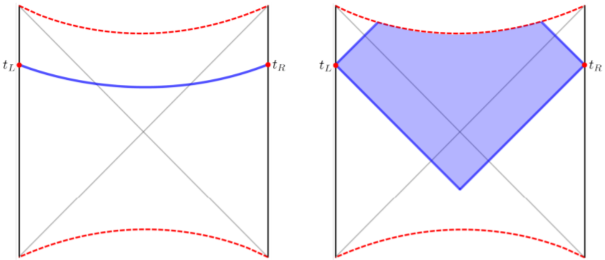


More proposals: CV2.0 and complexity = (almost) anything

Complexity

Circuit complexity

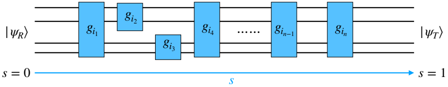
What is holographic complexity on the field theory side?

- Linear increase in time
- Slow to thermalize
- Reach very large values
- Switchback effect

State/circuit complexity

$$C_{\Psi_R}(\Psi_T) = \min_U D(U)$$

$$s.t. \quad U|\Psi_R\rangle = |\Psi_T\rangle$$

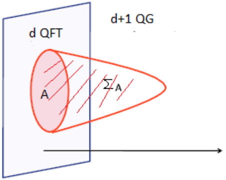


Complexity

Subregion complexity

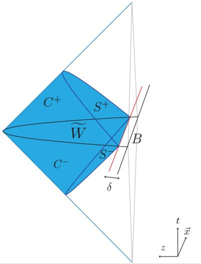
Considering subregion duality

- Holographic dual to reduced density matrix is its entanglement wedge
- Can we define the complexity of a subregion state?



Holographic subregion complexity

- Restriction of holographic proposals to entanglement wedge

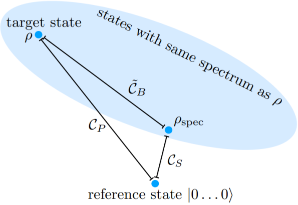


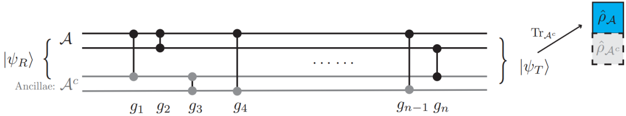
Complexity

Mixed state complexity [Agón, Headrick, Swingle]

What is holographic subregion complexity on the field theory side?

- Purification complexity
- Spectrum complexity
- Basis complexity
- ...





Outline

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Information paradox

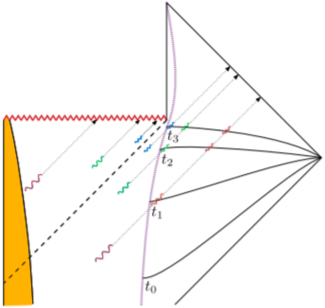
Entropy curve

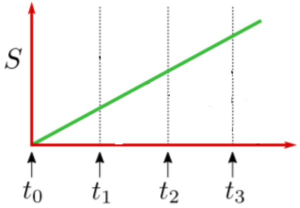
Hawking radiation

- Particle pair creation
- Near horizon
- Fall/escape of partners

Entanglement of black hole

- Pairs are entangled
- Constant Hawking radiation
- Linear increase in entropy





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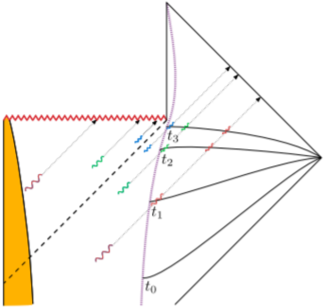
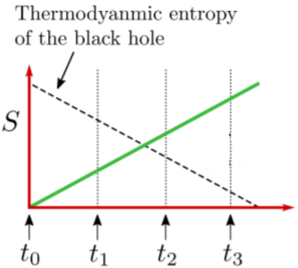




Generalized entropy and island rule

For a theory with gravity, generalized entropy

$$S_{gen}(R) = S_{EE}(R) + \frac{A(\partial R)}{4G}$$

In addition, the island rule

$$S_{gen}(R) = \min_{I} \left(S_{EE}(R \cup I) + \frac{A(\partial R)}{4G} + \frac{A(\partial I)}{4G} \right)$$

$\frac{A(\partial I)}{4G}$ is very big. Islands only relevant when there is a lot of entanglement between R and I

The island rule

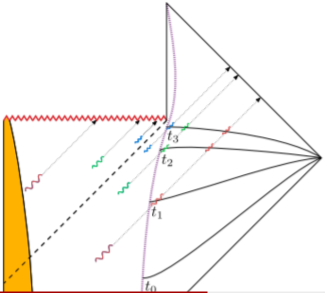
[Almheiri, Mahajan, Maldacena, Zhao]

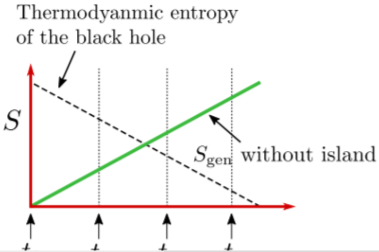
Island rule

- Generalized entropy
- When gravity is included
- Allow for "entanglement islands"

Page curve

- Early: no islands
- Entanglement increases
- Late: island configuration
- Entropy decreases





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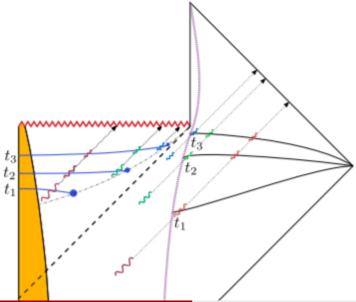

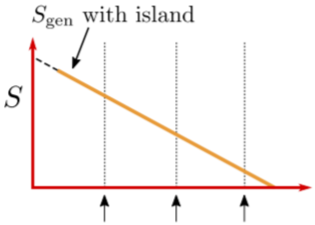

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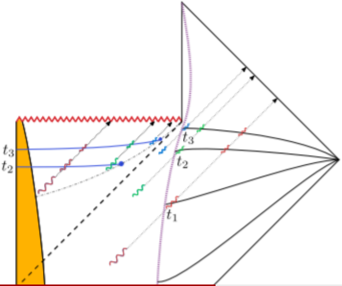

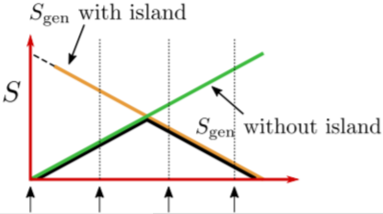

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   - Islands
   - Easy islands model
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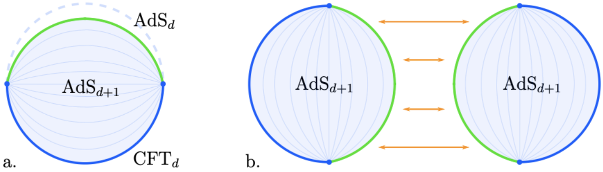
3. Bounds from entanglement and complexity
   - Entanglement velocity and Lloyd bounds
   - Black holes + ETW branes
   - Bounds on intrinsic gravity on the brane
4. Conclusion

QEIs made easy

[Chen, Myers, Neuenfeld, Reyes, Sandor]

We use a Randall-Sundrum braneworld model



With action

$$I_{\text{bulk}} + I_{\text{brane}} = \frac{1}{16\pi G_N} \int d^{d+1}x \sqrt{-g} \left[R + \frac{d(d-1)}{L^2} \right] - T_0 \int d^d x \sqrt{-h} \,.$$

The location of the brane is determined by the Israel junction conditions

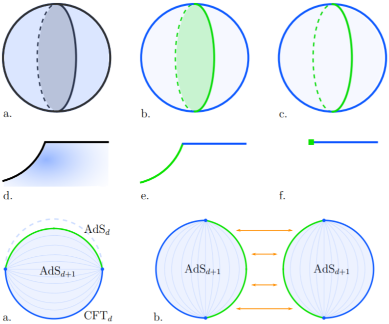
$$\Delta K_{ij} - h_{ij}\, \Delta K^k{}_k = -8\pi G_N\, T_0\, h_{ij} \,.$$

QEIs made easy

[Chen, Myers, Neuenfeld, Reyes, Sandor]

Two layers of holography, so three perspectives

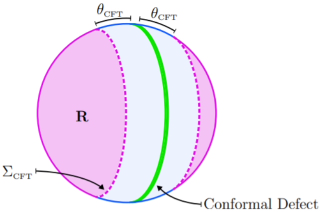
- Bulk: Asymptotically AdS_{d+1} + co-dimension 1 brane
- Brane: CFT in S_d coupled to brane with RS gravity + CFT
- Boundary: CFT in S_d with conformal defect

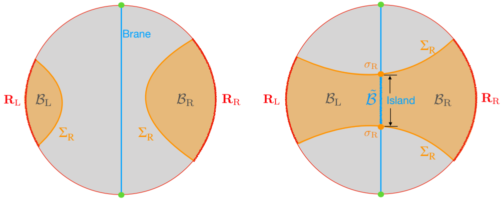


QEIs made easy

Island phase transition

The island phase transition is due to the conventional transition found in holographic EE





Fefferman-Graham expansion

Ambient metric construction

$$ds^2 = g_{\mu\nu}\, dy^\mu\, dy^\nu = \frac{L^2}{z^2}\left(dz^2 + g_{ij}(z, x^i)dx^i dx^j\right)$$

Einstein equations fix g_{ij} in terms of boundary $\overset{(0)}{g}_{ij}$ (and $\overset{(d/2)}{g}_{ij}$)

$$g_{ij}(z, x^i) = \overset{(0)}{g}_{ij}(x^i) + \frac{z^2}{L^2}\overset{(1)}{g}_{ij}(x^i) + \cdots + \frac{z^d}{L^d}\left(\overset{(d/2)}{g}_{ij}(x^i) + f_{ij}(x^i)\log\left(\frac{z}{L}\right)\right) + \cdots$$

For example

$$\overset{(1)}{g}_{ij}(x^i) = -L^2 P_{ij}[\overset{(0)}{g}] = -\frac{L^2}{d-2}\left(R_{ij}[\overset{(0)}{g}] - \frac{\overset{(0)}{g}_{ij}}{2(d-1)} R[\overset{(0)}{g}]\right)$$

Induced gravity action

FG expansion into the action, and integrating out the z direction

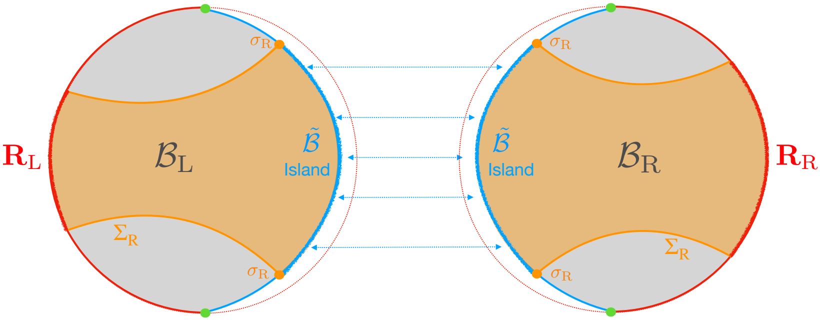
$$
\begin{aligned}
I_{\mathrm{eff}} &= \frac{1}{16\pi G_{\mathrm{eff}}} \int d^d x \sqrt{-\tilde{g}} \left[\frac{(d-1)(d-2)}{\ell_{\mathrm{eff}}^2} + \tilde{R}(\tilde{g}) \right] \\
&+ \frac{1}{16\pi G_{\mathrm{RS}}} \int d^d x \sqrt{-\tilde{g}} \left[\frac{L^2}{(d-4)(d-2)} \left(\tilde{R}^{ij} \tilde{R}_{ij} - \frac{d}{4(d-1)} \tilde{R}^2 \right) + \cdots \right]
\end{aligned}
$$

where

$$
\frac{1}{G_{\mathrm{eff}}} = \frac{1}{G_{\mathrm{RS}}} = \frac{2L}{(d-2)\,G_{\mathrm{bulk}}}\,, \qquad \frac{L^2}{\ell_{\mathrm{eff}}^2} \approx \frac{z_B^2}{L^2} \approx 2 - \frac{8\pi G_{\mathrm{bulk}} L T_o}{d-1}\,.
$$

Locality and small curvature limit

Induced action is a series in $L^2 \times \tilde{R} \sim L^2/\ell_{\text{eff}}^2 \sim z_B^2/L^2$
Brane close to asymptotic boundary $z_B/L \ll 1$



Natural expansion parameter z_B/L

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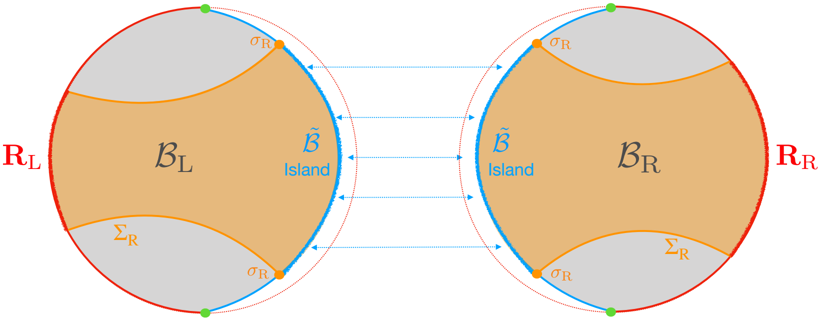

Natural expansion parameter z_B/L
Note: lengths on the brane scale as L/z_B

Intrinsic gravitational action

Add a gravitational action to the brane

$$I_{\text{brane}} = -(T_0 - \Delta T)\int d^d x\sqrt{-\tilde{g}} + \frac{1}{16\pi G_{\text{brane}}}\int d^d x\sqrt{-\tilde{g}}\tilde{R},$$

were ΔT ensures the location of the brane doesn't change.
FG expansion + z integration lead to same induced gravity action, but now

$$\frac{1}{G_{\text{eff}}} = \frac{1}{G_{\text{RS}}} + \frac{1}{G_{\text{brane}}} = \frac{2L}{(d-2)G_{\text{bulk}}} + \frac{1}{G_{\text{brane}}}$$

For d=2, I_{eff} is non-local $\sim \tilde{R}\log\tilde{R} \sim$ on-shell Polyakov action
For d=2, I_{EH} is topological, so once can instead add a I_{JT} on the brane

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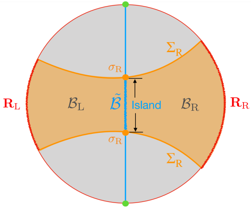
Entanglement entropy of the island

Holographic entanglement entropy

$$S(R) = \underset{\partial\Sigma_R = \partial R}{\mathrm{ext}} \frac{A(\Sigma_R)}{4G_{\mathrm{bulk}}}$$



Applying a Fefferman-Graham expansion

$$x^i(z,\sigma^a) = \overset{(0)}{x^i}(\sigma^a) + \frac{z^2}{L^2}\overset{(1)}{x^i}(\sigma^a) + \cdots, \quad \overset{(1)}{x^i} = \frac{L}{2(d-2)} K \overset{(0)}{n^i}$$

about the brane on holographic entanglement entropy

$$S(R) = \frac{A(\Sigma_R)}{4G_{\mathrm{bulk}}} = UV(\partial R) + \frac{\widetilde{A}(\sigma_R)}{4G_{\mathrm{eff}}} + \cdots$$

Entanglement entropy of the island

[Chen, Myers, Neuenfeld, Reyes, Sandor] and [JH, Myers, Ruan]

Including subleading terms, the contribution to enanglement entropy is the Wald-Dong entropy for the induced action on the brane

$$S(R) = UV(\partial R) + S_{WD}(\sigma_R) + \cdots$$

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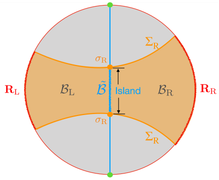
$$S(R) = UV(\partial R) + S_{WD}(\sigma_R) + \cdots$$

- Since RT surface Σ_R extremizes S, it follows that QES σ_R extremizes S_{WD}
- $UV(\partial R) \sim S_{WD}(\partial R) \sim$ UV entanglement across ∂R
- $S_{WD}(\sigma_R) \sim \mathcal{O}(z_B/L)$ entanglement across QES σ_R
- Recall z_B/L is locality scale of the brane

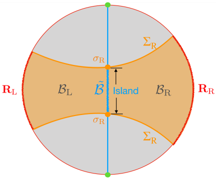


Complexity of the island

[JH, Myers, Ruan]

Subregion complexity=volume

$$C_V(R) = \max_{\partial\mathcal{B}=R\cup\Sigma_R} \frac{V(\mathcal{B})}{G_{\mathrm{bulk}}\ell}$$



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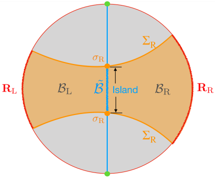

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Applying a Fefferman-Graham expansion about the brane on holographic complexity=volume

$$C_V(R) = \frac{V(\mathcal{B})}{G_{\text{bulk}}\ell} = UV(R) + \frac{\widetilde{V}(\tilde{\mathcal{B}})}{G_{\text{eff}}\ell} + \cdots$$

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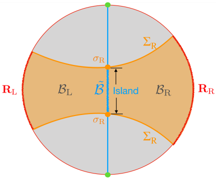

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UV divergences cancel when considering "mutual complexity"

$$C_V(R_L) + C_V(R_R) - C_V(R) = \frac{\widetilde{V}(\tilde{\mathcal{B}})}{G_{\text{eff}}\ell} + \cdots$$

Complexity of the island

[JH, Myers, Ruan]

Including subleading terms lead to a "generalized volume"

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- $\widetilde{W}$ derived from the action in parallel to S_{WD}

Complexity of the island

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- $UV(R) = \frac{\widetilde{W}(R)}{G_{\text{eff}}\,\ell} \sim$ complexity of UV entanglement structure in R

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- Unlike EE, subregion complexity is discontinuous at the island transition
- The complexity gained in the island phase is that of the most complex island anchored at the QES

Gravity on the brane

[JH, Myers, Ruan]

We can add an intrinsic gravitational term to the brane action

$$I_{\rm brane} = -(T_o - \Delta T) \int d^d x \sqrt{-\tilde{g}} + \frac{1}{16\pi G_{\rm brane}} \int d^d x \sqrt{-\tilde{g}}\, \tilde{R}$$

Because of the intrinsic gravity on the brane, C_V gets an extra brane contribution

$$C_V(R) = \max_{\partial\mathcal{B}=R\cup\Sigma_R} \frac{V(\mathcal{B})}{G_{\rm bulk}\ell} + \frac{\widetilde{V}(\tilde{\mathcal{B}})}{G_{\rm brane}\ell'}$$

The same result

$$C_V(R) = UV(R) + \frac{\widetilde{W}(\tilde{\mathcal{B}})}{G_{\rm eff}\ell} + \cdots$$

where

$$\frac{1}{G_{\rm eff}} = \frac{2L}{(d-2)G_{\rm bulk}} + \frac{1}{G_{\rm brane}}$$

Same for entropy

Generalized CV

And higher curvature bulk gravity

Propose a generalized C_V

$$C_V(R) = \max_{\partial\mathcal{B}=R\cup\Sigma_R} \frac{W(\mathcal{B})}{G_{\text{bulk}}\ell} + \frac{\widetilde{W}(\tilde{\mathcal{B}})}{G_{\text{brane}}\ell'}$$

and verify

$$C_V(R) = UV(R) + \frac{\widetilde{W}(\tilde{\mathcal{B}})}{G_{\text{eff}}\ell} + \cdots$$

for Gauss-Bonnet and $F(R)$ gravity

$$\lambda_{GB}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\,, \quad F(R)$$

Closing comments

Entanglement entropy and complexity of island

- (Sub)leading contribution in the island phase: Wald entropy/complexity of island
- Scales with locality of induced gravity in the brane
- Double holography, what about more generally?

Discontinuity of subregion complexity

- General in holography
- Discontinuity of EW
- Large N effect

Outline

Bounds from entanglement and complexity

In braneworld models, the entanglement/complexity surfaces can sometimes intersect the branes

For branes with intrinsic gravity, there is an additional contact term that should be included in the entanglement entropy/complexity proposals

The gravitational couplings on the brane can influence the behaviour of entanglement/complexity

Bounds on behaviour for entanglement/complexity can limit the allowed values of gravitational couplings on the brane

Outline

Entanglement velocity bound

[Afkhami-Jeddi, Hartman]

Normalized rate of entanglement growth

$$v_E \equiv \frac{\partial_t S(A)}{s_{th}(\beta)|\partial A|}\,,$$

For 2d CFTs, states with uniform energy density, the entanglement velocity is upper bounded

$$|v_E| \leq \coth\frac{\pi|A|}{\beta}\,.$$

In the thermodynamic limit $|A| \gg \beta$,

$$|v_E| \leq 1\,.$$

Lloyd bound

"Energy limits speed of computation" [Lloyd]

Consider the "ultimate laptop"

- Average temperature E
- Bound on rate of logical operations
- Operations/second $\leq \frac{2E}{\pi\hbar}$

Bound on complexity growth

- $\frac{dC}{dt} \leq \frac{2E}{\pi\hbar}$
- Saturated for holographic complexity of black holes
- Black holes as "ultimate computers"

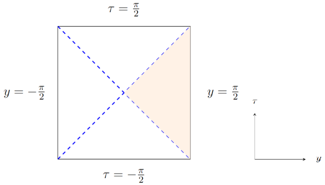
Outline

Black hole + ETW brane

[Hartman, Maldacena], [Takayanagi] and [Fujita, Takayanagi, Tonni]

Single sided BH microstate geometry

- Double sided planar black hole
- Capped by ETW brane on one side
- BH microstate with semiclassical geometry



Total action

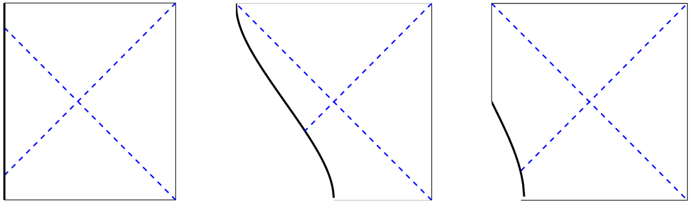
$$I = \frac{1}{16\pi G_N}\left[\int_{\text{bulk}} d^{d+1}x\sqrt{-g}\left(R + \frac{d(d-1)}{L^2}\right) + 2\int_{\partial AdS} d^d x\sqrt{-h}\,K \right.$$

$$\left. +2\int_{\text{brane}} d^d x\sqrt{-h}\left(K - \frac{T_0}{L^{d-1}}\right)\right].$$

Black hole + ETW brane

[Lee, Shukla, Neuenfeld]

Israel junction conditions determine location of the brane



(a) Subcritical: $0 \leq T_0 < 1$ (b) Critical: $T_0 = 1$ (c) Supercritical: $T_0 > 1$

In addition, we can add gravity action on the brane

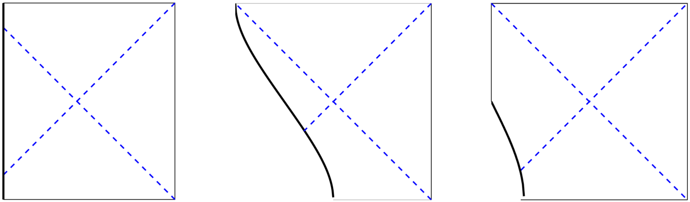
$$I_{\text{brane}} = \frac{1}{9\pi G_N} \int_{\text{brane}} d^d x \sqrt{-h} \left(K - \frac{T}{L^{d-1}} \right)$$

$$+ \frac{1}{16\pi G_N^{\text{brane}}} \int_{\text{brane}} d^d x \sqrt{-h}\, R^{\text{brane}}$$

Black hole + ETW brane

[Lee, Shukla, Neuenfeld]

Israel junction conditions determine location of the brane



(a) Subcritical: $0 \leq T_0 < 1$ (b) Critical: $T_0 = 1$ (c) Supercritical: $T_0 > 1$

In addition, we can add intrinsic JT gravity on the brane

$$I_{\mathrm{JT}} = \frac{1}{16\pi G_N^{\mathrm{brane}}} \int d^2x \sqrt{-h}\, \varphi \left(R^{\mathrm{brane}} - 2\Lambda^{\mathrm{brane}} \right)$$

$$+ \frac{1}{16\pi G_N^{\mathrm{brane}}} \int d^2x \sqrt{-h}\, \Phi_0 R^{\mathrm{brane}} \,.$$

Brane location and dilaton profile

[Lee, Shukla, Neuenfeld]

The location of the brane is determined by the cosmological constant

$$-\frac{1}{L^2}\cos^2 y_{\rm brane} = \Lambda^{\rm brane}\,,$$

and the dilaton profile is found by varying the metric on the brane

$$\varphi(\tau_{\rm brane}) = \varphi_0 + \varphi_1 \sin\tau_{\rm brane},$$

where $\varphi_0 = G_N^{\rm brane} K/2G_N \Lambda^{\rm brane}$ and φ_1 are constants.

Brane location and dilaton profile

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where $\varphi_0 = G_N^{\mathrm{brane}} K/2G_N \Lambda^{\mathrm{brane}}$ and φ_1 are constants.

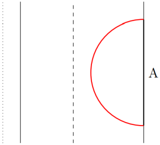
There are bounds on the JT coupling φ_1 due to entanglement and complexity growth bounds.

Outline

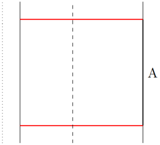
Entanglement bounds

[Lee, Shukla, Neuenfeld]

Consider an interval A large enough that the connected HRT surface still dominates.



(a) Thermal surface



(b) Connected surface

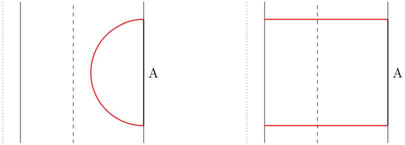
Include contact term due to instrinsic gravity on the brane

$$S(A) = \frac{A(\Sigma)}{4G_N} + \frac{\phi(\Sigma \cap \mathrm{brane})}{4G_N^{\mathrm{brane}}}\,.$$

Entanglement bounds

[Lee, Shukla, Neuenfeld]

Consider an interval A large enough that the connected HRT surface still dominates.



(a) Thermal surface (b) Connected surface

Include contact term due to instrinsic gravity on the brane

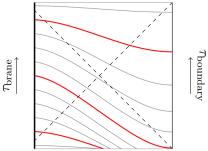
$$S(A) = \frac{A(\Sigma)}{4G_N} + \frac{\phi(\Sigma \cap \mathrm{brane})}{4G_N^{\mathrm{brane}}}.$$

Impose the upper bound on entanglement velocity $|v_E| \leq 1$.

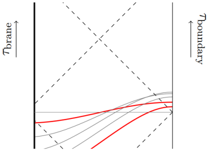
Entanglement bounds

Two bounds on JT coupling $\alpha \equiv \frac{G_N \varphi_1}{G_N^{\mathrm{brane}} L}$

- Rapid growth of entanglement
- Discontinuous jump in entanglement



(c) $\alpha = 1.0$



(d) $\alpha = 1.0$

Entanglement bounds

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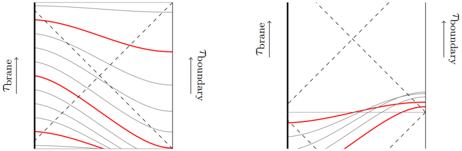

(c) $\alpha = 1.0$ (d) $\alpha = 1.0$

First bound:

$$|\alpha| \leq \frac{1}{1 + \sin y_{\text{brane}}} .$$

Second bound:

$$|\alpha| \leq \frac{1}{1 - \sin y_{\text{brane}}} .$$

Entanglement bounds

Two bounds on JT coupling $\alpha \equiv \frac{G_N \varphi_1}{G_N^{\mathrm{brane}} L}$

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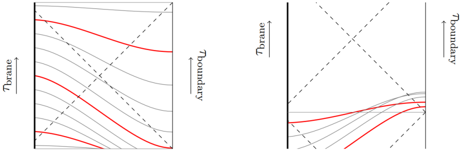


(c) $\alpha = 1.0$ (d) $\alpha = 1.0$

First bound:

$$|\alpha| \leq \frac{1}{1 + \sin y_{\mathrm{brane}}} .$$

Second bound:

$$|\alpha| \leq \frac{1}{1 - \sin y_{\mathrm{brane}}} .$$

Entanglement velocity asymptotes to its upper bound

$$v_E = 1 - 2\mathcal{K}_1 e^{\frac{-4\pi t}{\beta}} + \cdots ,$$

where $\mathcal{K}_1 = \frac{1-\alpha-\alpha \sin y_{\mathrm{brane}}}{1+\alpha-\alpha \sin y_{\mathrm{brane}}}$.

Complexity bounds

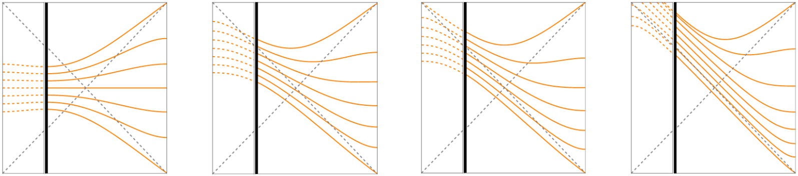
[Aguilar-Gutierrez, Craps, JH, Shulka, Khramtzov, Knysh] (w.i.p)

Recall the Lloyd bound on complexity

$$\frac{dC}{dt} \leq \frac{2M}{\pi \hbar}$$

Compute the growth rate of complexity=volume. Include contact term due to intrinsic gravity on the brane

$$C_V(R) = \frac{V(\mathcal{B})}{G_N L} + \frac{1}{G_N^{\mathrm{brane}} L} \int_{\mathcal{B}\cap \mathrm{brane}} \phi \,.$$



(a) $\alpha = 0$ (b) $\alpha = 1$ (c) $\alpha = 1.42$ (d) $\alpha = 5$

Complexity bounds

Tuning the Lloyd bound

- Definitions of hologaphic complexity are ambiguous
- Universal late time growth rate
- Proportional to M
- Intuition from entanglement velocity, and BHs as "ultimate computers"

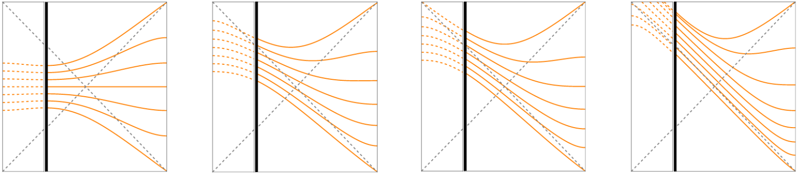
Use this universal late time value as the Lloyd bound.

Complexity bounds

Tuning the Lloyd bound

- Definitions of holographic complexity are ambiguous
- Universal late time growth rate
- Proportional to M
- Intuition from entanglement velocity, and BHs as "ultimate computers"

Use this universal late time value as the Lloyd bound. With this tuning, complexity surface projected to fall into singularity = violation of Lloyd bound



(a) $\alpha = 0$ (b) $\alpha = 1$ (c) $\alpha = 1.42$ (d) $\alpha = 5$

Complexity bounds

The bound on the JT coupling $\alpha \equiv \frac{G_N \varphi_1}{G_N^{\rm brane} L}$ is

$$|\alpha| \leq -\frac{\tan y_{\rm brane}}{\cos y_{\rm brane}} \,,$$

where, recall

$$-\frac{1}{L^2}\cos^2 y_{\rm brane} = \Lambda^{\rm brane} \,.$$

Entanglement and complexity bounds

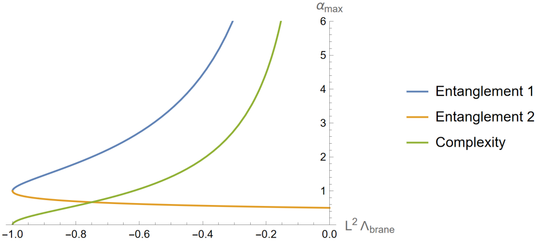
The bounds on the JT coupling α from entanglement velocity are

$$|\alpha| \leq \frac{1}{1 - \sin y_{\mathrm{brane}}}, \quad |\alpha| \leq \frac{1}{1 + \sin y_{\mathrm{brane}}}.$$

The bounds on α from Lloyd bound is

$$|\alpha| \leq -\frac{\tan y_{\mathrm{brane}}}{\cos y_{\mathrm{brane}}},$$

where $-\frac{1}{L^2}\cos^2 y_{\mathrm{brane}} = \Lambda^{\mathrm{brane}}$.



Outline

Closing comments

Entanglement entropy and complexity of island

- Double holography, what about more generally?

Discontinuity of subregion complexity

- General in holography
- Discontinuity of EW

Entanglement and complexity bounds

- Currently no strict bound on entanglement velocity in higher dimensions
- There are late time bounds
- Could apply to other braneworld models

Open questions

Entanglement and complexity of islands

- C_A, $CV2.0$ of the island
- C=anything of the island
- Higher order corrections to S_{WD}, W
- W from first law of causal diamonds (Bueno et al.)

Entanglement and complexity bounds

- Bounds from complexity in higher dimension
- Bounds on entanglement velocity in higher dimension
- Bounds from other holographic complexities
- Entanglement and complexity bounds in other braneworld models