Gravitation from optimized computation

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Complexity: between field theory and gravity
IFT, Madrid workshop

Black hole thermodynamics (Bekenstein '73), (Hawking '75)

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}(\mathcal{H})}{4G}$$

- A world with gravity is *holographic*
- Classical gravity is *emergent*

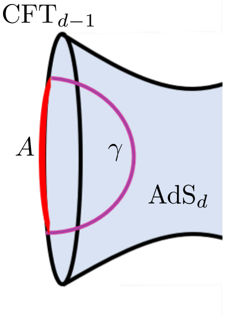
'Spacetime thermodynamics' (Jacobson '95)

$$\delta Q = T\delta S_{\mathrm{BH}} \Longrightarrow G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Local regions of spacetime assumed to obey holographic 'first law'
- Extends to *any* theory of gravity (Parikh, Svesko '17)
- Underlying microscopics?

Holographic entanglement entropy
(Ryu, Takayanagi '06)

$$S_{\mathrm{vN}}^{\mathrm{CFT}}(A) = \min_{\gamma \sim A} \frac{\mathrm{Area}(\gamma)}{4G}$$



- Gravity has information theoretic character
- Entanglement probes spacetime

'Spacetime entanglement' (Lashkari,... '14), (Faulkner,... '14)

$$\delta_\rho S_{\mathrm{vN}}(A) = \delta_\rho \langle H_A \rangle \Longrightarrow G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Entanglement = Geometry (Raamsdonk '10), (Swingle,...;Bianchi... '14)
- Extends to *any* theory; non-linear (Faulkner,... '17), (Haehl,... '17)
- 'Entanglement equilibrium' (Jacobson,... '15)

- 'Entanglement not enough' to describe late-time physics of BH interior (Maldacena,...'13), (Liu,... '13), (Susskind, '14)
- Computational complexity? Min number of gates to prepare a state

$$|\psi_f\rangle = g_n \dots g_2\, g_1 |\psi_i\rangle = U_{fi} |\psi_i\rangle$$

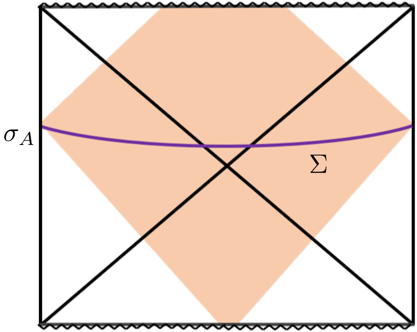
Holographic complexity

- complexity=volume (CV) (Susskind, '14)

$$\mathcal{C}(\sigma_A) = \frac{1}{G\ell} \max_{\Sigma \sim A} V(\Sigma)$$

- complexity=action (CA) (Brown,... '15)

$$\mathcal{C} = \frac{1}{\pi \hbar} I_{\text{WdW}}$$

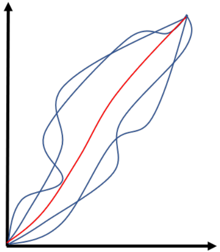


- Beyond: CV2.0 (Couch,... '16); complexity=anything (Belin,... '21)

Optimized computation is fundamental

- Principle of least action:

$$\delta_q I \Rightarrow E_q = 0$$



- Optimal path is solves EOM

EOM reduce cost of computing system dynamics

- 'Nature is thrifty in its actions.'

Complexity quantifies optimal computation

- Operator growth $\Rightarrow$ gravitational attraction (Susskind '19), (Barbon,... '20)
- 2D gravity governs complexity of (Virasoro) circuits (Caputa...'19)
- Quantum gravity as quantum computation (Lloyd '06)

'Spacetime complexity' (PRSWD, '21,'22)

Gravitational equations of motion result of spacetime minimizing cost of computing its own dynamics

Gravity from first law of holographic complexity

$$\delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta V \Longrightarrow \delta E_{\mu\nu} = 0$$

- $\mathcal{C} \sim$ min. number of sources $\{\lambda_f\}$ to prepare state $|\psi_f\rangle$
- Bulk $\leftrightarrow$ bdry symplectic form (Belin,... '18)

Outline:

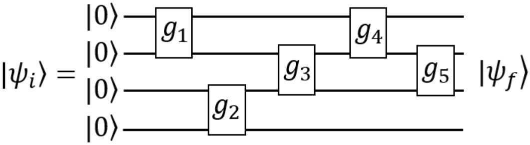
- Complexity in terms of holographic state prep. (Belin,... '18)
- Linearized gravity EOM from 1st law (PRSWD, '21), (CPSWD, '23)
- Adding bulk quantum corrections (CPSWD, '23)

$$\delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta V + \int_M \delta c_{\text{bulk}}\,, \quad \delta c_{\text{bulk}} = \frac{1}{2}\delta_Y g_{\mu\nu}\langle\delta T^{\mu\nu}\rangle$$

- Visualize $\mathcal{C}$ in terms of Lorentzian 'threads' (PRSWD, '21)

Computational complexity as state prep.

Computational (circuit) complexity



$$|\psi_f\rangle = g_n \dots g_2\, g_1 |\psi_i\rangle = U_{fi}|\psi_i\rangle$$

- Given $|\psi_i\rangle$ and $\{g_k\}$, complexity $\mathcal{C}(|\psi_f\rangle)$ = min. number of gates
- $\mathcal{C}$ – optimal cost required to prepare target state given reference state; 'shortest circuit'

Geometrization of circuit complexity (Nielsen '05)

quantum circuits $\leftrightarrow$ geodesics in space of operations

$\mathcal{C}$ = length of minimal geodesic

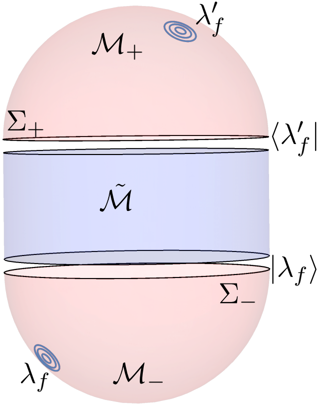
Holographic state preparation

AdS/CFT: bulk Lorentzian spacetime = time evolution of CFT state

(Skenderis,...'08), (Botta-Cantcheff,...'16), (Marolf,...'17)

$$|\lambda_f\rangle = T e^{-\int_{\tau<0} d\tau d\vec{x} \sum_\alpha \lambda_\alpha(\tau,\vec{x}) \mathcal{O}_\alpha(\tau,\vec{x})} |\lambda_i\rangle$$

- Prep. $|\lambda_f\rangle$ on Σ_- from sources $\{\lambda_f\}$
- $|\lambda_i\rangle = |0\rangle \equiv \int_{\tau<0} [D\Phi] e^{-I_E^{\text{CFT}}}$
- Bdry values of bulk fields in $\mathcal{M}_-$ specify $|\lambda_i\rangle$ and $\{\lambda_f\}$
- Time evolution in $\tilde{\mathcal{M}}$ follows from solving bulk EOMs given initial data Σ_-
- Close contour $\langle\lambda'_f|$



Complexity in space of sources

Geometrizing space of sources

- Space of states $|\lambda\rangle$ = manifold coordinatzed by $\{\lambda_\alpha\}$
- Distances given by metric η_{ab}
- Minimal path = minimizing 'cost function', e.g., $F \equiv \eta_{ab}\dot{\lambda}^a\dot{\lambda}^b$

Complexity in space of sources

- $\mathcal{C}(|\lambda_f\rangle)$ = trajectory minimizing F (Belin,...'18)

$$\mathcal{C}(s_i, s_f) = \int_{s_i}^{s_f} ds \eta_{ab}\dot{\lambda}_a\dot{\lambda}_b$$

- $\{\lambda_f\}$ act as gates $\{g_k\}$
- Clarifies role of reference state

A first law of complexity

$$\delta_{\lambda_f}\mathcal{C} = (\dot{\lambda}^a|_{\lambda_f})\eta_{ab}\delta\lambda_f^b$$

- Vary $\{\lambda_f\}$; look for variations minimizing computational cost

Geometrizing space of sources

- Space of states $|\lambda\rangle$ = manifold coordinatzed by $\{\lambda_\alpha\}$
- Distances given by (Kahler) metric η_{ab}
- Symplectic manifold with coordinates $\tilde{\lambda} = (\lambda_\alpha, \lambda_\alpha^*)$; 2-form Ω_{bdry}

$$\Omega_{\text{bdry}}(\delta_1\tilde{\lambda}, \delta_2\tilde{\lambda}) = i(\delta_1^*\delta_2 - \delta_2^*\delta_1)\log\langle\lambda|\lambda\rangle$$

- For CFTs, $Z_{\text{CFT}}[\lambda] = \langle\lambda|\lambda\rangle$

A first law of complexity

- Special deformations of sources $\delta_Y\lambda$

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda})$$

- $\dot{\lambda}^a|_{\lambda_f} = J[\delta_Y\lambda]$, with complex structure J
- Purely a boundary first law; no gravity

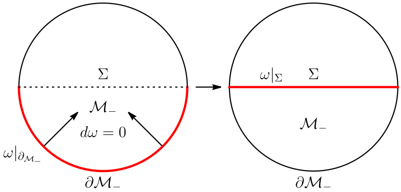
- Standard AdS/CFT dictionary

$$Z_{\mathrm{CFT}}[\tilde{\lambda}] = \langle \lambda | \lambda \rangle = e^{-I_{E,\mathrm{grav}}^{\text{on-shell}}[\tilde{\lambda}]}$$

- $\tilde{\lambda}$ set bcs for bulk fields ϕ via 'extrapolate' dictionary

$$\Omega_{\mathrm{bdry}}(\delta_1\tilde{\lambda}, \delta_2\tilde{\lambda}) = i \int_{\partial\mathcal{M}_-} \omega^E_{\mathrm{bulk}}(\phi, \delta_1\phi, \delta_2\phi)$$

- Symplectic current form $\omega \equiv \delta_1\theta(\phi, \delta_2\phi) - \delta_2\theta(\phi, \delta_1\phi)$
- Assumes on-shell field configurations, $E_\phi = 0$
- Linearized EOM $\delta_{1,2}E_\phi = 0$, $d\omega_{\mathrm{bulk}} = 0$



$$\Omega_{\mathrm{bdry}}(\delta_1\tilde{\lambda}, \delta_2\tilde{\lambda}) = \int_{\Sigma} \omega^L_{\mathrm{bulk}}(\phi, \delta_1\phi, \delta_2\phi) = \Omega_{\mathrm{bulk}}(\delta_1\phi, \delta_2\phi)$$

Holographic first law of complexity

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) = \Omega_{\text{bulk}}(\delta_Y\phi, \delta\phi)$$

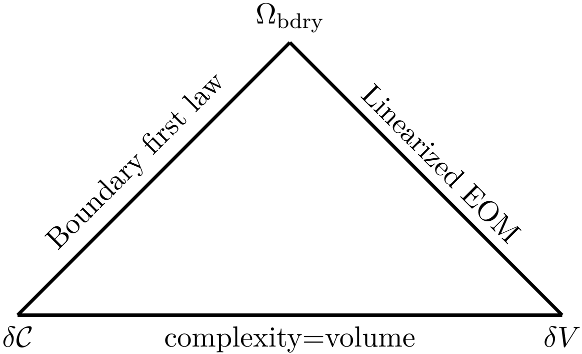
- Assuming $\delta_Y E_\phi = \delta E_\phi = 0$
- δ_Y – ‘new York’ deformation (Belin,...’18)

$$\delta_Y h_{ab} = 0 \ , \quad \delta_Y K_{ab} = -\alpha h_{ab}$$

- ADM variables (h_{ab}, π^{ab}). CMC slices

$$\Omega_{\text{bulk}}(g, \delta_Y g, \delta g) = \int_{\Sigma_t} \delta_Y(\pi^{ab}\delta h_{ab}) = \frac{(d-2)\alpha}{8\pi G}\delta V$$

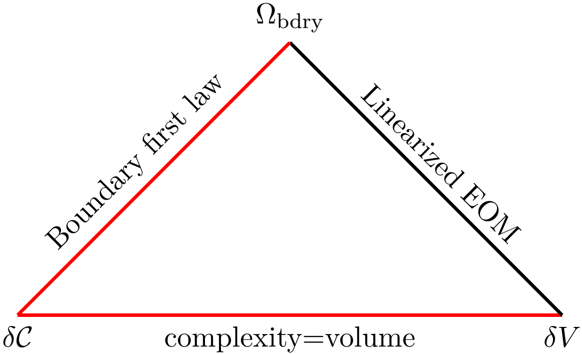
- $V = \int_\Sigma \sqrt{h}$
- On-shell only on *maximal* slices, $K = 0$.
- δ_Y translation in ‘York time’ (York ’72); diffeomorphism about empty AdS



$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) = \Omega_{\text{bulk}}(\delta_Y\phi, \delta\phi) = \frac{1}{G\ell}\delta V$$

- Varying complexity $\leftrightarrow$ linearized gravity

Linearized Einstein from varying complexity



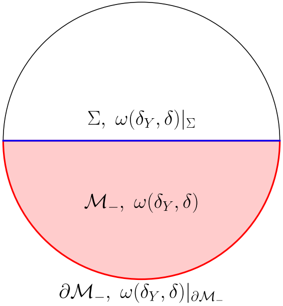
$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) \text{ and } \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta V \implies \delta E_{\mu\nu} = 0$$

- Varying complexity $\implies$ linearized gravity

Stokes' Theorem

$$i\int_{\mathcal{M}_-} d\omega^E_{\text{bulk}}(g,\delta_Y g,\delta g) = i\left(\int_{\partial\mathcal{M}_-}\omega^E_{\text{bulk}}(g,\delta_Y g,\delta g) - \int_\Sigma \omega^E_{\text{bulk}}(g,\delta_Y g,\delta g)\right)$$
$$= \Omega_{\text{bdry}}(\delta_Y\tilde\lambda,\delta\tilde\lambda) - \frac{1}{G\ell}\delta V$$

- Holds for all variations that yield real Lorentzian initial data on Σ
- Using extrapolate dictionary



First law of complexity and CV

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda},\delta\tilde{\lambda}) \;\text{ and }\; \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta V$$

Then

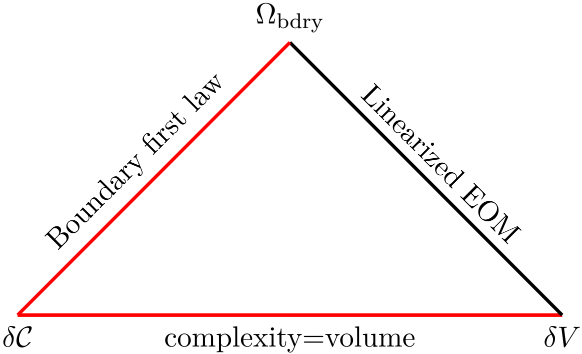
$$\int_{\mathcal{M}_-} d\omega^E_{\text{bulk}} = 0 \Rightarrow 0 = \delta_Y E^{\mu\nu}\delta g_{\mu\nu} - \delta E^{\mu\nu}\delta_Y g_{\mu\nu}$$

- Deformations about empty AdS; $\delta_Y E^{\mu\nu} = 0$ (diffeo)
- $\delta_Y g_{\mu\nu} \neq 0$

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda},\delta\tilde{\lambda}) \;\text{ and }\; \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta V \Longrightarrow \delta E_{\mu\nu} = 0$$

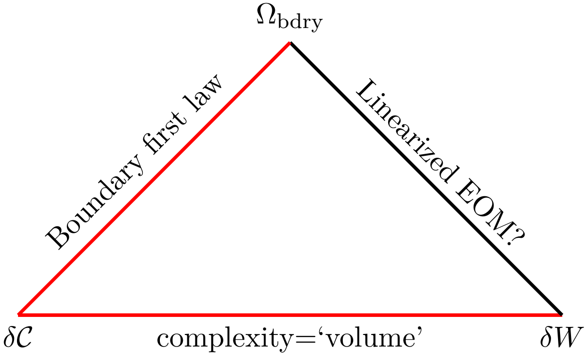
- Holds for all Lorentzian maximal slices which provide initial data

‘Nature is thrifty in its *computations*’



$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) \;\text{ and }\; \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta V \implies \delta E_{\mu\nu} = 0$$

- Varying complexity $\implies$ linearized Einstein’s equations
- Spacetime dynamics from optimized computation
- Holds for perturbations over general states, not just empty AdS



Modify legs to account for other types of gravity theories

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) \;\text{ and }\; \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta W$$

CFTs dual to higher-order theories expect general geometric functionals

Generalized volume (Bueno,...'16)

$$\mathcal{C} = \frac{1}{G\ell} W_{\text{gen}} \,, \quad W_{\text{gen}} = \frac{1}{(d-2)P_0} \int_\Sigma \epsilon_\Sigma \left(P^{\mu\nu\rho\sigma} n_\mu n_\sigma h_{\nu\rho} - P_0 \right)$$

- Analog of Wald entropy for black holes; $P^{\mu\nu\rho\sigma} = \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} + \ldots$

Generalized volume with corrections (Hernandez,...'20)

$$\mathcal{C}(\sigma_A) = \frac{1}{G\ell} \max_{\Sigma \sim A} \left[W_{\text{gen}}(\Sigma) + W_K(\Sigma) \right]$$

- Analog of Camps-Dong HEE; $W_K(\Sigma)$ ext. curvature corrections

Deriving linearized EOM

Assume

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) \;\text{ and }\; \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta W_{\text{gen}}$$

- Amounts to showing

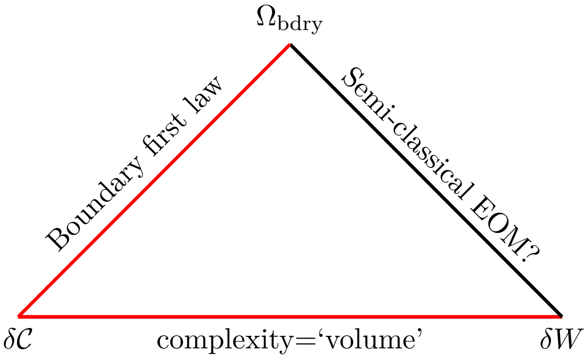
$$\Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) = \Omega_{\text{bulk}}(g, \delta_Y g, \delta g) = \frac{1}{G\ell}\delta W_{\text{gen}}$$

- Valid for perturbations about empty AdS
- GR: $\Omega_{\text{bdry}}(\delta_Y, \delta) = \Omega_{\text{bulk}}(\delta_Y, \delta)$ on $K = 0$, δ_Y on-shell
- Higher-order: δ_Y on-shell for more complicated constraint
- Match extremization of W (Hernandez,...'20) (future work)

Higher-order gravity from 1st law (CPSWD,...'23)

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\text{bdry}}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) \;\text{ and }\; \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta W_{\text{gen}} \implies \delta E_{\mu\nu} = 0$$

Adding quantum corrections?



- Modify legs to account for semi-classical quantum corrections
- $1/N$ corrections CFT $\leftrightarrow$ bulk quantum corrections

$$\delta_{\lambda_f}\mathcal{C} = \Omega_{\rm bdry}(\delta_Y\tilde{\lambda}, \delta\tilde{\lambda}) \quad \text{and} \quad \delta_{\lambda_f}\mathcal{C} = \frac{1}{G\ell}\delta W$$

Task: Determine W when bulk quantum corrections included

- Find δ_Y accounting presence of bulk quantum fields
- Possible in 2D dilaton gravity

Semi-classical JT gravity

$$I_{\mathrm{SJT}} = I_{\mathrm{JT}} + I_{\mathrm{Poly}} + I_{\mathrm{GHY}}$$

$$I_{\mathrm{JT}} = \frac{1}{16\pi G_2} \int_{\tilde{\mathcal{M}}} d^2x \sqrt{-g} \left(\left(\Phi_0 + \Phi\right) R + \frac{2\Phi}{L^2} \right)$$

$$I_{\mathrm{Poly}} = -\frac{c}{24\pi} \int_{\tilde{\mathcal{M}}} d^2x \sqrt{-g} \left[\chi R + (\nabla\chi)^2 \right]$$

- Effective theory with 2D CFT χ in AdS_2
- Problem of backreaction solvable

CV complexity in 2D (Brown,...'18), (Schneiderbauer,...'19), (Anegwa,...'23), (Patra,...'23)

$$\mathcal{C} \sim V_{\mathrm{JT}} \ , \quad V_{\mathrm{JT}} = \int_\Sigma dy \sqrt{h}(\Phi_0 + \Phi)$$

- Motivation: $\mathcal{C}$ grows with # d.o.f. $\sim S \sim (\Phi_0 + \Phi)$

A new 'new York' deformation (CPSWD, '23)

- Look for deformations δ_Y such that $\Omega_{\mathrm{bulk}}(\delta_Y, \delta) \sim \delta V_{\mathrm{JT}}$

$$\delta_Y \pi^{ab} = \frac{\alpha}{2}\sqrt{h} h^{ab}(\Phi + \Phi_0) \ , \quad \delta_Y \pi_\Phi = \alpha\sqrt{h} \ , \quad \delta_Y \chi = 0$$

Then,

$$\Omega_{\mathrm{bulk}}(\delta_Y, \delta) = \frac{1}{G\ell}\delta V_{\mathrm{JT}} + \int_{\mathcal{M}_-} \delta_Y g_{\mu\nu} \langle \delta T^{\mu\nu}_\chi \rangle$$

Quantum CV and semi-classical EOM

CV with quantum corrections

Suggestive to introduce 'bulk complexity' c_{bulk}

$$\Omega_{\text{bulk}}(\delta_Y, \delta) = \frac{1}{G\ell}\delta V_{\text{JT}} + \int_{\mathcal{M}_-} \delta c_{\text{bulk}} \,, \qquad \delta c_{\text{bulk}} \equiv \delta_Y g_{\mu\nu}\langle \delta T^{\mu\nu}_{\chi}\rangle$$

Consistent with generalized CV (Hernandez,...'20)

$$\mathcal{C} = \max_{\Sigma\sim A}\left[\frac{W_{\text{gen}}(\Sigma) + W_K(\Sigma)}{G\ell} + \mathcal{C}_{\text{bulk}}\right]$$

- Analog of QES formula for entanglement entropy

Semi-classical gravity from first law

- Assuming first law and generalized CV

$$\text{“}\delta G_{\mu\nu} = 8\pi G\langle \delta T_{\mu\nu}\rangle\text{”}$$

- Holds in 2D; valid for perturbations about vacuum
- Semi-classical Einstein? Formally consistent

Lorentzian threads and CV

Reformulate CV (PRSWD, '21)

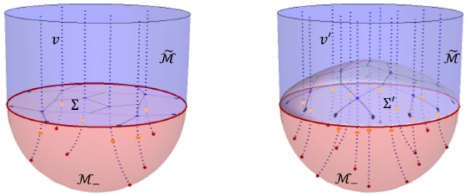
$$\mathcal{C}(\sigma_A) = \max_{\Sigma\sim A} \frac{V(\Sigma)}{G\ell} = \min_{v\in\mathcal{F}} \int_A v\,, \quad \mathcal{F} = \left\{ v \,|\, v^0 > 0,\ \nabla\cdot v = 0,\ |v| \geq \frac{1}{G\ell} \right\}$$

- Lorentzian MinFlow-MaxCut theorem (Headrick & Hubeny, '17)

Gateline interpretation

- Threads or 'gatelines' cut through physical tensors

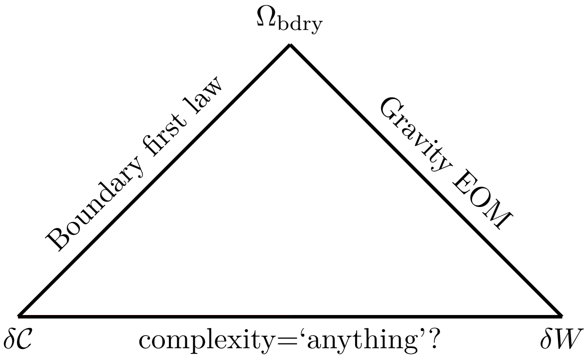
$$\mathcal{C} \sim \#\ \text{tensors in the network} \sim \#\ \text{gatelines}$$



- Unitaries (gates) attached to each thread
- Einstein's equations encoded in threads

$$\boxed{v \leftrightarrow \omega_{\text{bulk}}(\delta_Y\phi, \delta\phi)}$$

Summary and future work



- *Spacetime dynamics* $\leftrightarrow$ *optimized computation*
- Modify legs to account for other theories
- Non-linear corrections? Extrinsic curvature?
- Other holographic duals? (Belin,... '21)
- Other measures of bdry complexity?
- Thread description?