Volume complexity of dS bubbles

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arXiv 2302.03584

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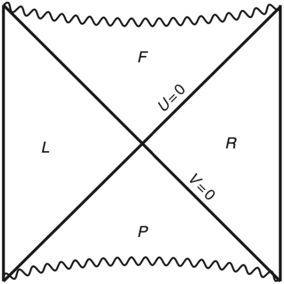
Tha AdS/CFT

AdS/CFT is an interesting theoretical lab for gravity

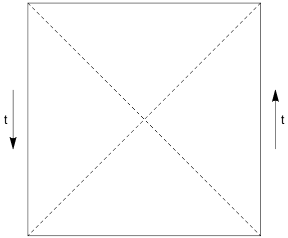


However, we don't live in AdS...
Can we do holography in a cosmological setting?

How are dS symmetries implemented?

How would we recognize a dS dual?

For AdS_d, symmetries: $SO(d-1,2)$
For dS_d we would like to identify $SO(d,1)$



Thermofield double

Symmetry violation in dS

For $d = 3$, we have six generators

- easy generators: H and J preserve static patch
- hard generators: $K_{1,2}$ and $R_{1,2}$ mix the dS thermofield double

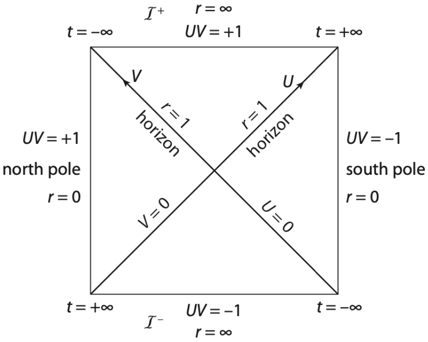
A finite entropy is not compatible with the full symmetry generators and an hermitian H

Goheer, Kleban, Susskind, hep-th/0212209

Symmetries must be violated in eternal dS

Holography for dS ?

No timelike boundary in dS



Where the dual theory is supposed to live ?

Holography for dS ?

- dS/CFT ?
  spacelike infinity. CFT with no time
  (Strominger, 2001)

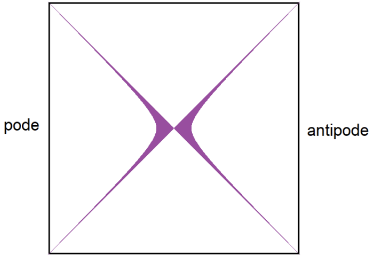
- dual to a system with a finite number of degrees of freedom ?

$$S = \frac{A}{4G}$$

  Finite entropy associated to dS cosmological horizon
  (Gibbons-Hawking, 1977)
  Empty dS maximizes entropy

Stretched horizon holography for dS

dS should be hyperfast scrambler
the static patch should be dual to a quantum system on the stretched horizon

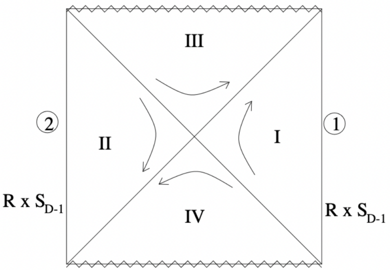


$S = \frac{A}{4G}$ is interpreted as entropy between left and right static patch
Susskind 2109.14104, Shaghoulian 2110.13210

Thermofield double state in AdS

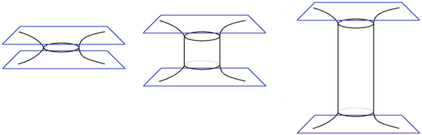
AdS eternal Black Holes are dual to Thermofield double state

$$|\Psi_{TFD}\rangle \propto \sum_n e^{-E_n\beta/2 - iE_n(t_L - t_R)} |E_n\rangle_R |E_n\rangle_L \,.$$

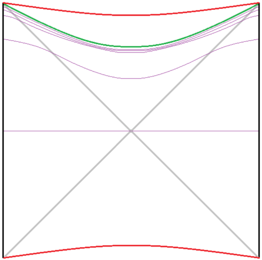


J. M. Maldacena, hep-th/0106112

The growth of Einstein-Rosen bridge



Entanglement is not enough, because is saturates at the thermalization time



L. Susskind, 1411.0690

Complexity

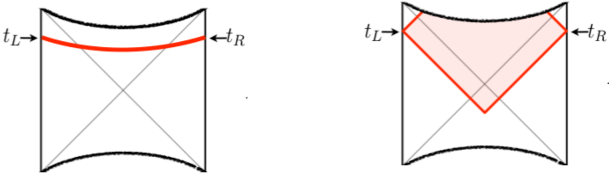
Concept from theoretical computer science: it is heuristically defined as the minimum number of simple unitary operations required to reach a given state from a reference state

Example: a system of n qubits

- Simple state $|0\rangle = |00000\ldots\rangle$
- Generic state $|\psi\rangle = \sum_{i=1}^{2^n} \alpha_i |i\rangle$
- Simple operation: act on 2 qubits

Continuous version of complexity: Nielsen, geodesics in space of unitary evolution

Holographic conjectures, CV & CA



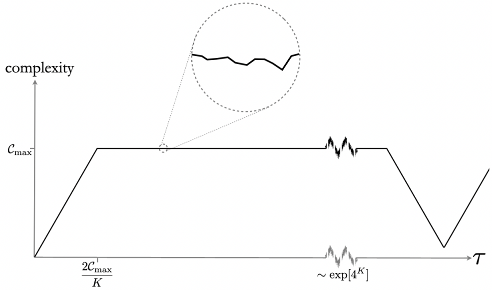
$$\mathcal{C}_V \sim \frac{\mathrm{Max}(V)}{G\,L}\,, \qquad \mathcal{C}_A = \frac{\mathcal{S}}{\pi\hbar}$$

$$\frac{d\mathcal{C}}{dt} \sim T\,S$$

D. Stanford and L. Susskind, 1406.2678
A. R. Brown et al, 1509.07876

Time dependence of complexity in AdS

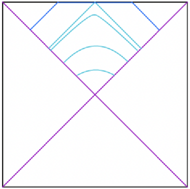
Both from holographic CA and CV, complexity rate approaches a constant at late time



Complexity should reach a plateau in a time which exp in number of degs of freedom [Susskind, 1507.02287]

Volume complexity in dS

Hyperfast complexity growth, Susskind 2109.14104

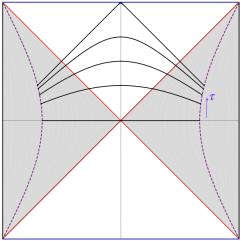


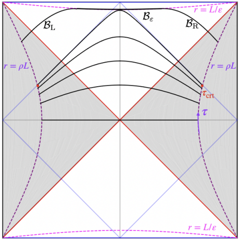
Not k-local Hamiltonian ?

k-local: the Hamiltonian is the sum of terms that simultaneously act at most on k degrees of freedom, where $k = O(1)$ in the limit of a large number of degrees of freedom.

Regularizing volume in dS

Jorstad, Myers, Ruan, 2202.10684





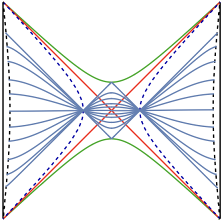
Switchback effect:
Baiguera, Berman, Chapman, Myers, 2304.15008
Anegawa and N. Iizuka, arXiv:2304.14620

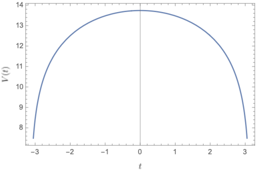
At the crossroads, the Centaurs

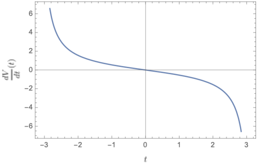
Geometries with a dS_2 region inside AdS_2

Realized in dilaton-gravity theories in 2 dimensions
D. Anninos and D. M. Hofman, 1703.04622

The volume of Centaurs

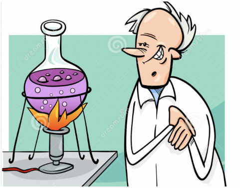






S. Chapman, D. A. Galante and E. D. Kramer, 2110.05522

What about higher dimensions?

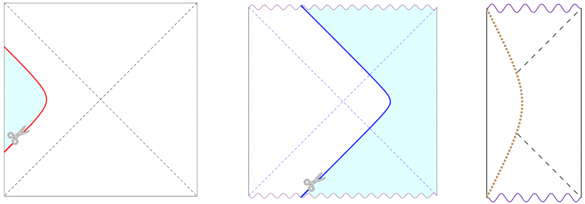


False vacuum bubbles: "Creating a universe in the lab"

For dS bubbles inside a flat region:
Studied in '80ies by Blau, Farhi, Guendelman and Guth

For dS bubbles inside AdS spacetime:
B. Freivogel, V. E. Hubeny, A. Maloney, R. C. Myers,
M. Rangamani and S. Shenker, [arXiv:hep-th/0510046 [hep-th]].

Scissors and glue



Cut dS (red line), AdS (blue line) and glue togheter!

Metric

$$ds_{i,o}^2 = -f_{i,o}(r)\, dt_{i,o}^2 + \frac{dr^2}{f_{i,o}(r)} + r^2 d\theta^2$$

Outside, BTZ black hole: $\qquad f_o(r) = r^2 - \mu$

Inside, dS_3 spacetime: $\qquad f_o(r) = 1 - \lambda\, r^2$

$$r_{i,o}^* = \int \frac{d\tilde{r}}{f_{i,o}(\tilde{r})}\,,$$

Lightcone coordinates:

$$v_{i,o} = t_{i,o} + r_{i,o}^*(r)\,, \qquad u_{i,o} = t_{i,o} - r_{i,o}^*(r)\,.$$

The wall and the juction conditions

Domain wall position as a function of proper time

$$r = R(\tau)\,,$$

Introducing:

$$\beta_{i,o} = (K_\theta^\theta)_{i,o}\, R\,.$$

From Israel's junction conditions

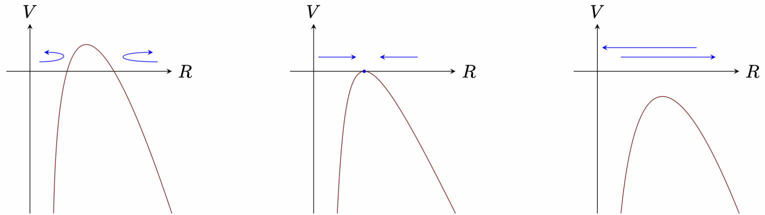
$$\beta_i - \beta_o = \kappa\, R\,, \qquad \kappa = 8\pi G\,\sigma$$

$$\beta_i = \pm\sqrt{\dot{R}^2 + f_i(R)}\,, \qquad \beta_o = \pm\sqrt{\dot{R}^2 + f_o(R)}\,,$$

Blau, Guendelman, Guth, Phys. Rev. D 35 (1987)

Effective potential

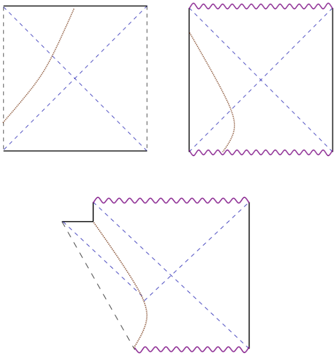
$$\dot{R}^2 + V(R) = 0\,, \qquad V(R) = f_o(R) - \frac{(f_i(R) - f_o(R) - \kappa^2\,R^2)^2}{4\,\kappa^2\,R^2}$$



We restrict to time-reversal symmetric bubbles

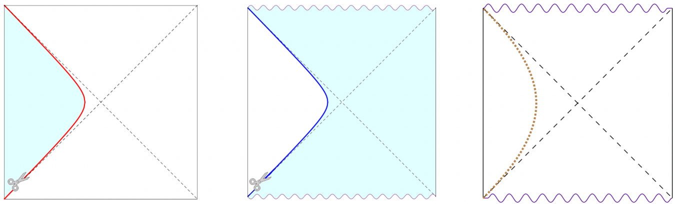
$$R(\tau) = \sqrt{\frac{\beta}{2} \mp \frac{\sqrt{\beta^2 - 4\gamma}}{2}\cosh(2\sqrt{A}\tau)}\,.$$

Without time reversal symmetry



Static bubble

Model specified by λ, κ ("theory parameters") and μ, black hole mass

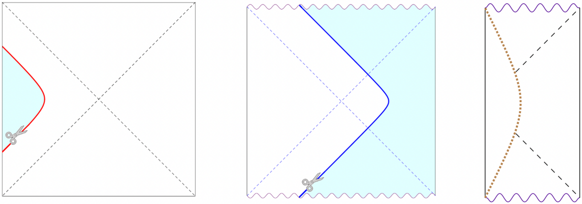


Static bubble for $\mu = \mu_0$

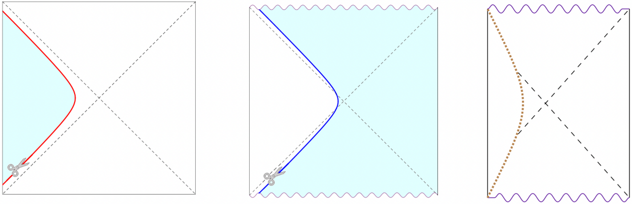
$$\mu_0 = \frac{\sqrt{(\kappa^2 + \lambda - 1)^2 + 4\lambda} - (\kappa^2 + \lambda - 1)}{2\lambda},$$

Small bubbles

Very small bubble, $0 < \mu < \mu_s$

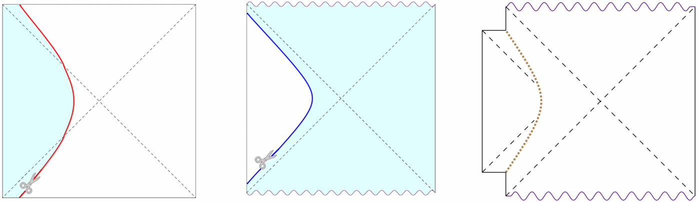


Not so small bubble, $\mu_s < \mu < \mu_0$

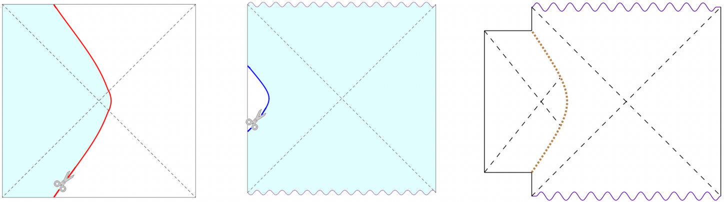


Large bubbles

Not so large bubble, $\mu_h < \mu < \mu_0$



Very large bubble, $0 < \mu < \mu_h$



The entropies of large bubbles

the number of degrees of freedom accessible from the AdS boundary is less than the number of degrees of freedom of the internal dS region

$$S_{\rm BH} = 2\pi\sqrt{\mu}\,, \qquad S_{\rm dS} = \frac{2\pi}{\sqrt{\lambda}}\,.$$

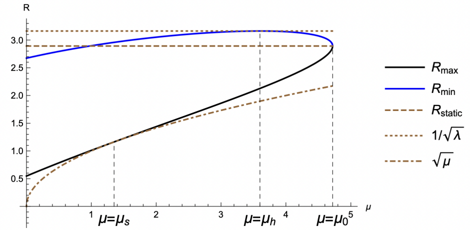
$$\lambda\mu \leq \lambda\mu_0 \leq 1\,,$$

$$S_{\rm BH} \leq S_{\rm dS}\,,$$

Large bubbles are the dual of a density matrix ?
B. Freivogel, V. E. Hubeny, A. Maloney, R. C. Myers, M. Rangamani and S. Shenker, [arXiv:hep-th/0510046 [hep-th]].

As a function of μ

Time-reversal symmetric bubbles



For the same value of $\mu < \mu_0$, we have both a small and a large bubble solution

Volume functional

$$r = r(l)\,, \qquad t = t(l)\,,$$

$$\mathcal{V}_{i,o} = 2\pi \int \mathcal{L}\, dl \qquad \mathcal{L} = r\sqrt{-f_{i,o}(v'_{i,o})^2 + 2r'v'_{i,o}}\,,$$

Conserved quantity

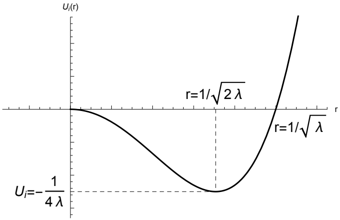
$$P_{i,o} = \frac{\partial \mathcal{L}}{\partial v'_{i,o}} = \frac{r\left(-f_{i,o}v'_{i,o} + r'\right)}{\sqrt{-f_{i,o}(v'_{i,o})^2 + 2r'v'_{i,o}}}\,,$$

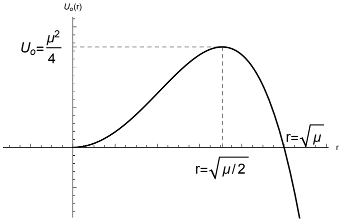
Fixing reparameterization invariance:

$$\sqrt{-f_{i,o}(v'_{i,o})^2 + 2r'v'_{i,o}} = r$$

Effective potential

$$(r')^2 + U_{i,o}(r) = P_{i,o}^2\,, \qquad U_{i,o}(r) = -f_{i,o}(r)\,r^2\,,$$





Left: dS Right: AdS

Joining the interior with the exterior

$$t_o = G(r)\, t_i \, .$$

$$ds^2 = -g(r,t_i)\, dt_i^2 + \frac{dr^2}{f(r,t_i)} + 2h(r,t_i)\, dr\, dt_i + r^2 d\theta^2 \ ,$$

For the interior:

$$g(r,t_i) = f_i \, , \qquad f(r,t_i) = f_i \, , \qquad h(r,t_i) = 0 \, ,$$

For the exterior:

$$g(r,t_i) = G^2 f_o \, , \qquad f(r,t_i) = \frac{f_o}{1 - f_o^2 \left(\frac{dG}{dr}\right)^2 t_i^2} \, ,$$

$$h(r,t_i) = -f_o G \, \frac{dG}{dr} \, t_i \, .$$

On the domain wall

All the derivatives $\partial_{t_i} f$, $\partial_{t_i} h$, $\partial_r f$, $\partial_r h$, will have a Dirac delta contribution localized on the surface of the bubble

$$t_i = T_i(\tau)\,, \qquad r = R(\tau)\,.$$

This delta function contribution is constant on the surface of the bubble and so

$$\partial_r f = \frac{1}{\sqrt{1 + \left(\frac{dR}{dT_i}\right)^2}}\,\delta(r - R(\tau))\,\Delta f\,,$$

$$\partial_{t_i} f = -\frac{dR}{dT_i}\frac{1}{\sqrt{1 + \left(\frac{dR}{dT_i}\right)^2}}\,\delta(r - R(\tau))\,\Delta f\,,$$

A conserved quantity

$$\begin{pmatrix} t_i \\ r \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} s \\ w \end{pmatrix},$$

where

$$\sin\psi = \frac{\dot{R}}{\sqrt{\dot{R}^2 + \dot{T}_i^2}}, \qquad \cos\psi = \frac{\dot{T}_i}{\sqrt{\dot{R}^2 + \dot{T}_i^2}}, \qquad \tan\psi = \frac{dR}{dT_i}.$$

In the approximation in which we consider just the "fast" dependence of the Lagrangian due to discontinuities at the two sides of the domain wall, the Lagrangian density is independent of s.

$$\hat{P} = \frac{\partial \mathcal{L}}{\partial s'} = (hr' - gt_i')\cos\psi + \left(\frac{r'}{f} + ht_i'\right)\sin\psi\,.$$

A refraction law for extremal surfaces

Imposing that $\hat{P}$ is conserved on top of the domain wall, we get a refraction law for the extremal surface, which can be written in a covariant form

$$\frac{dx^{\mu}_{i,o}}{dl} = (t'_{i,o}(l), r'_{i,o}(l)), \qquad \frac{dX^{\mu}_{i,o}}{d\tau} = (\dot{T}_{i,o}(\tau), \dot{R}(\tau)).$$

$$(g_i)_{\mu\nu} \frac{dx^{\mu}_i}{dl} \frac{dX^{\nu}_i}{d\tau} = (g_o)_{\mu\nu} \frac{dx^{\mu}_o}{dl} \frac{dX^{\nu}_o}{d\tau}.$$

$$\rho_{i,o}(R) = r'_{i,o}(l_0) \qquad \text{where} \qquad r_{i,o}(l_0) = R.$$

$$P_i \frac{dT_i}{dR} + \frac{\rho_i(R)}{f_i(R)} = P_o \frac{dT_o}{dR} + \frac{\rho_o(R)}{f_o(R)}.$$

Bubble moving at speed of light

$$\frac{dT_{i,o}}{dR} = \pm \frac{1}{f_{i,o}}\,.$$

depending on the sign

$$V_i' = V_o' \qquad \text{or} \qquad U_i' = U_o'$$

Consistent with:
Balasubramanian, Bernamonti, de Boer, Copland, Craps, et al. 1103.2683
S. Chapman, H. Marrochio and R. C. Myers, 1804.07410,

Complexity from smooth extremal surfaces

A possible way to apply the CV conjecture in asymptotically AdS geometries with an internal dS bubble is to consider extremal surfaces which are anchored at some given time t_b at the AdS boundary and which are smooth in the interior.

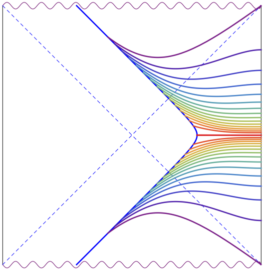
$$P_i = 0$$

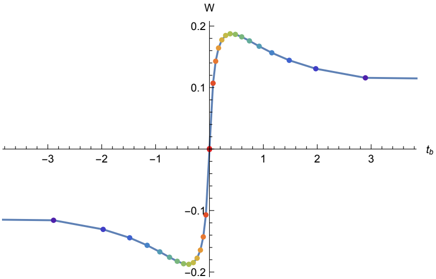
Otherwise there is a curvature singularity at $r = 0$ in the dS interior

In this case we have shown that the complexity rate

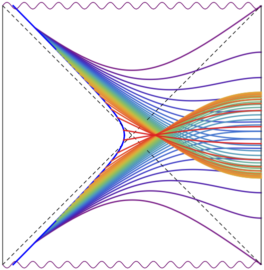
$$W = \frac{1}{2\pi} \frac{d\,\mathcal{V}}{d\,t_b} = P_o$$

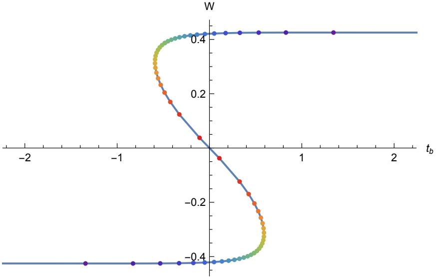
Complexity rate for small bubbles, example 1



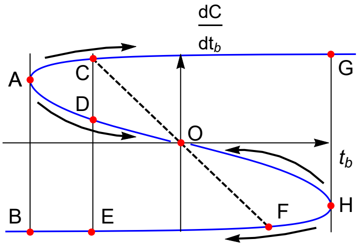


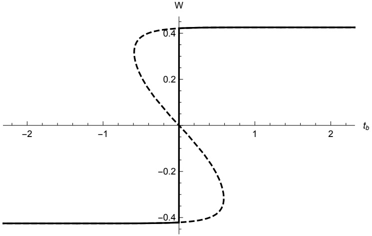
Complexity rate for small bubbles, example 2





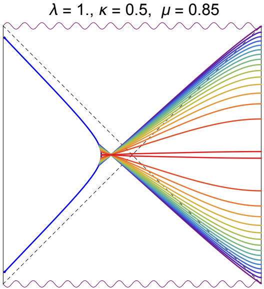
Maximum volume requirement

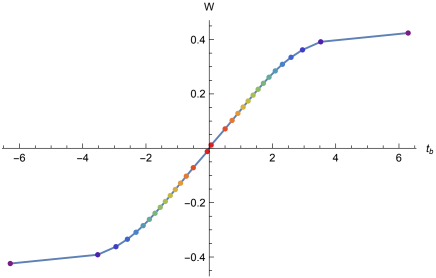




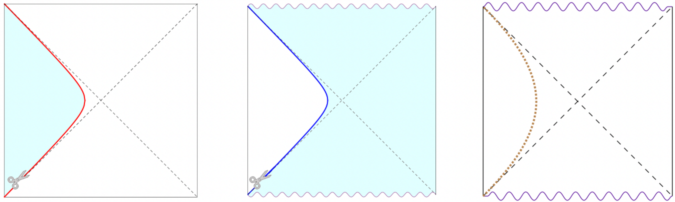
The requirement of maximal volume selects the step-like function rate represented by the solid line

Complexity rate for large bubbles





Static bubble complexity rate



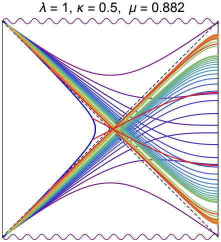
For the strictly static bubble configuration $\mu = \mu_0$,

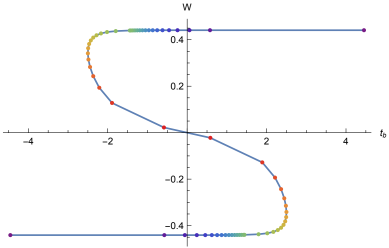
$$P_o = 0$$

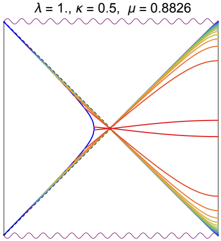
so the complexity rate identically vanishes.

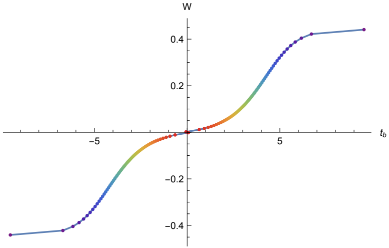
Killing vector $\partial/\partial t$ is not broken by the bubble trajectory

Static bubble limit

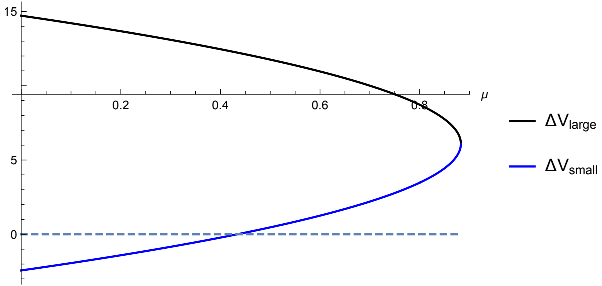








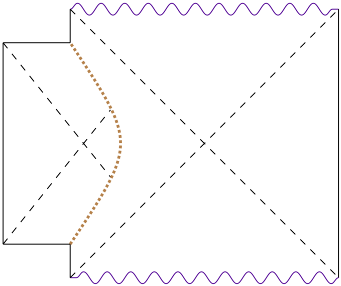
Complexity of formation



$$\Delta\mathcal{V}_{\rm large} = \mathcal{V}_{\rm large} - \mathcal{V}_{\rm BTZ}\,, \qquad \Delta\mathcal{V}_{\rm small} = \mathcal{V}_{\rm small} - \mathcal{V}_{\rm BTZ}\,.$$

Complexity with a dS stretched horizon

For a very large bubble with $0 < \mu < \mu_h$, we can consider a generalization of the thermofield double state with both an AdS and a dS boundary.



Complexity with a dS stretched horizon

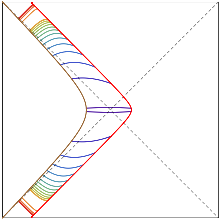
$$r_{\rm sh} = \frac{1}{\sqrt{\lambda}}\,(1-\epsilon)\ ,$$

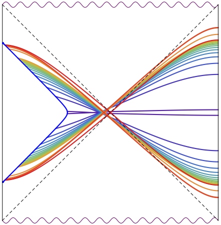
$$t_L = t_i(l=0)\,, \qquad \text{where} \qquad r(l=0) = r_{\rm sh}$$

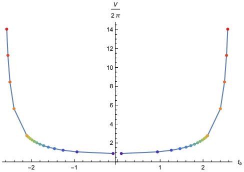
$$t_R = t_o(l=l_\Lambda)\,, \qquad \text{where} \qquad r(l=l_\Lambda) = \Lambda$$

$$t_L = \alpha_t\, t_R\ , \qquad t_b = -t_L = -\alpha_t$$

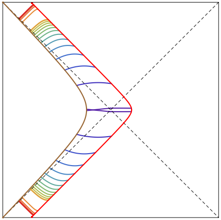
$t_L = t_R$

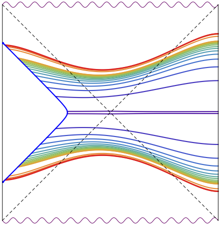


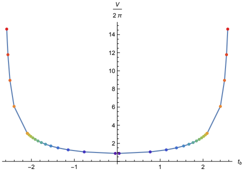




$t_L = -t_R$







Hyperfast growth of complexity

For every α, there is hyperfast behavior of complexity

Complexity diverge at the critical time

$$t_{\mathrm{cr}} = \frac{1}{4\sqrt{\lambda}} \log \frac{4}{\epsilon^2} - Q \,,$$

where Q is an integration constant

Conclusions

- We first focused on extremal surfaces attached just at the AdS boundary and smooth everywhere into the interior spacetime. With the exception of the static bubble configuration, we found that complexity asymptotically grows linearly as a function of time, with the same rate as for the BTZ black hole.

- The static bubble configuration gives rise to a time-independent complexity, so it does not match the expectation, generically satisfied by AdS black holes, that complexity rate at late time is of the same order of magnitude as TS, with T the temperature and S the entropy of the system

Conclusions

- If the limit $\mu \to \mu_0$ in the parameter space is approached from the large bubble configuration, the complexity rate remains frozen to zero for an initial amount of time which tends to infinity for $\mu \to \mu_0$.
- If the limit $\mu \to \mu_0$ is approached from the small bubble region of the parameter space, the static behavior of complexity emerges from a class of extremal surfaces with non-maximal volume. The discarded solutions would give rise to a negative complexity rate, as for the two-dimensional centaur geometries.

Conclusions

- Hyperfast growth is recovered in the very large bubble case if we consider extremal surfaces anchored both at the AdS boundary and at the dS stretched horizon. This choice should correspond to a thermofield double state which involves both an AdS and a dS boundary.

- This suggest that: The volume of smooth extremal surfaces anchored just at the boundary of AdS is proportional to the complexity of a mixed CFT state, obtained by tracing over the dS degrees of freedom in the thermofield double state. Instead, the volume of the extremal surface anchored both at the AdS boundary and at the dS static patch horizon is proportional to the complexity of the pure product thermofield double state.

Thank you!