



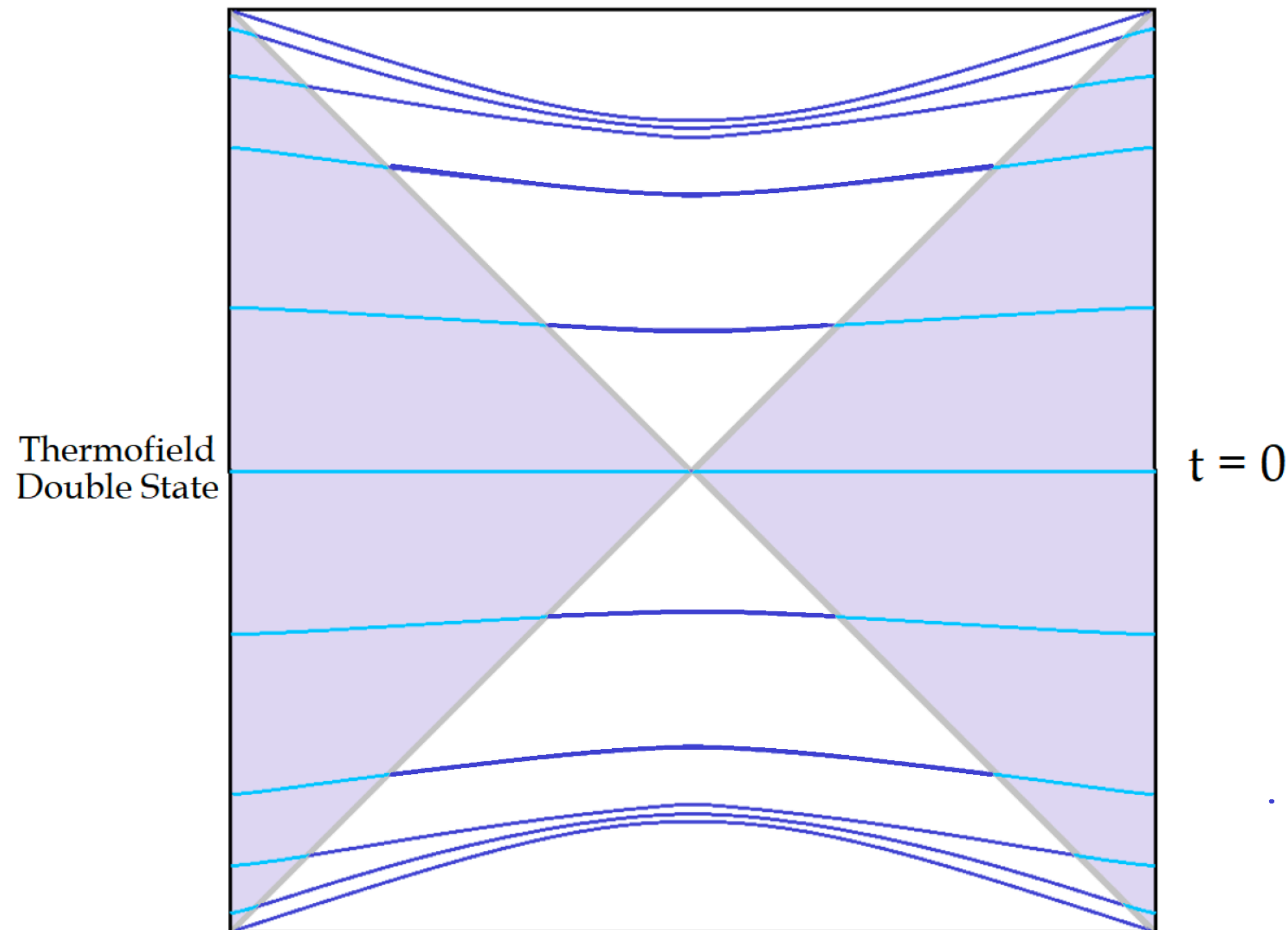
# A BULK MANIFESTATION OF KRYLOV COMPLEXITY

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*Complexity in Field Theory and Gravity. IFT, 23rd May 2023*

# INTRODUCTION

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(Susskind, 2018).

- Goal: Boundary description of the growth of the Einstein-Rosen Bridge (ERB).
- Proposal: (Susskind, 2016): Captured by complexity of the boundary state evolving in Lorentzian time.
- Controversies: Ambiguities in complexity definition (tolerance parameter, gates...), absence of explicit matching (Belin et al., 2021).
- Our work:
  - Low-dimensional instance of holography.
  - Krylov complexity.

# KRYLOV SPACE (OPERATORS)

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- Initially defined for operators. *(Parker et al., 2018)*
- A notion of complexity adapted to time evolution of an initial operator  $\mathcal{O} \equiv \mathcal{O}(0)$ .
- Take a Hilbert space of states  $\mathcal{H}$  with  $\dim \mathcal{H} = D$ . Operator space is  $\widehat{\mathcal{H}}$ .
- Time evolution generator in  $\widehat{\mathcal{H}}$  is the Liouvillian  $\mathcal{L} := [H, \cdot]$ , as:

$$\mathcal{O}(t) = e^{iHt} \mathcal{O} e^{-iHt} = e^{it\mathcal{L}} \mathcal{O} = \mathcal{O} + it[H, \mathcal{O}] - \frac{t^2}{2} [H, [H, \mathcal{O}]] + \dots$$

- Define **Krylov space** as  $\mathcal{H}_{\mathcal{O}} := \text{span} \left\{ \mathcal{L}^n \mathcal{O} \right\}_{n=0}^{+\infty} \leq \widehat{\mathcal{H}}$

$\implies$  Always contains  $\mathcal{O}(t)$ . Dimension:  $K \leq D^2 - D + 1$

*(E. Rabinovici, ASG, R. Shir, J. Sonner, 2020.)*

*(V. S. Viswanath & G. Muller, 1994; Parker et al., 2018.)*

# KRYLOV COMPLEXITY (STATES)

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- State evolving in Schrödinger picture:  $|\phi(t)\rangle = e^{-iHt} |0\rangle$ .
- Note that  $|\phi(t)\rangle \in \text{span} \{ |0\rangle, H|0\rangle, H^2|0\rangle, \dots \} =: \mathcal{H}_\phi$  for all  $t$ .
- The **Lanczos algorithm** provides an orthonormal basis for this **Krylov space**:

$$|A_{n+1}\rangle = (H - a_n) |n\rangle - b_n |n-1\rangle, \quad |n+1\rangle = \frac{1}{b_{n+1}} |A_{n+1}\rangle$$

With **Lanczos coefficients**:  $a_n = \langle n | H | n \rangle$ ,  $b_{n+1} = \sqrt{\langle A_{n+1} | A_{n+1} \rangle}$ .

State K-complexity: Given  $|\phi(t)\rangle = \sum_n \phi_n(t) |n\rangle$ ,

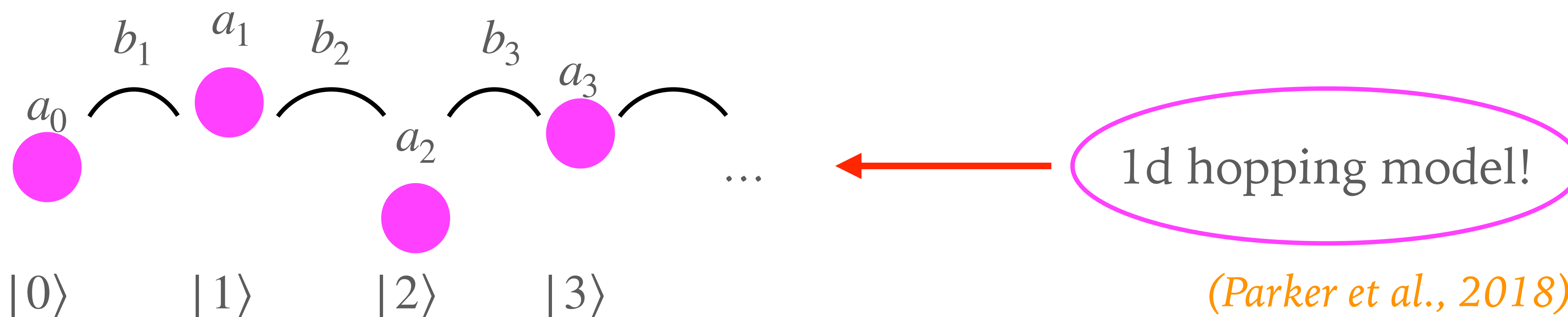
*(Balasubramanian et al., 2022)*

$$C_K(t) = \langle \phi(t) | \hat{n} | \phi(t) \rangle = \sum_n n |\phi_n(t)|^2$$

# KRYLOV CHAIN AS A ONE-DIMENSIONAL HOPPING MODEL

- The Hamiltonian takes a tridiagonal form in the Krylov basis:

$$H = \sum_{n=0}^{K-2} b_{n+1} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) + \sum_{n=0}^{K-1} a_n |n\rangle\langle n|$$



- **Krylov chain**, with localized states  $|n\rangle$ , potential energies  $a_n$  and hopping amplitudes  $b_n$ .

Initial condition  $\phi_n(t=0) = \delta_{n0}$  spreads along the chain as it evolves,  $\phi_n(t)$ .

# SURVIVAL AMPLITUDE

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- Defined as **fidelity** of the evolving state:

$$\phi_0(t) = \langle 0 | \phi(t) \rangle = \sum_n \frac{(-it)^n}{n!} M_n$$

With **moments**:

$$M_n = \langle 0 | H^n | 0 \rangle$$

- There is a **bijjective** correspondence:  $\{a_n, b_n\} \iff \{M_n\}$
- *Note:* If  $\phi_0(t)$  **even**  $\implies M_{2n+1} = 0 \implies a_n = 0$  (as in operator case)

For TFD:  $|0\rangle = |TFD\rangle = \sum_E e^{-\beta E/2} |E\rangle \otimes |E\rangle$

$$\implies \phi_0(t) = Z(\beta + it) \quad (\text{Balasubramanian et al., 2022})$$

# DOUBLE-SCALED SYK (DSSYK)

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► Analytically solvable version of SYK. Allows to approach Schwarzian sector.

- Hamiltonian:

*(M. Berkooz et al., 2019; H. Lin, 2022)*

$$H = i^{p/2} \sum_{1 \leq i_1 < \dots < i_p \leq N} J_{i_1 \dots i_p} \psi_{i_1} \dots \psi_{i_p}$$

- **Double-scaling** limit:

$$\lambda := \frac{2p^2}{N}$$

- Disordered model:

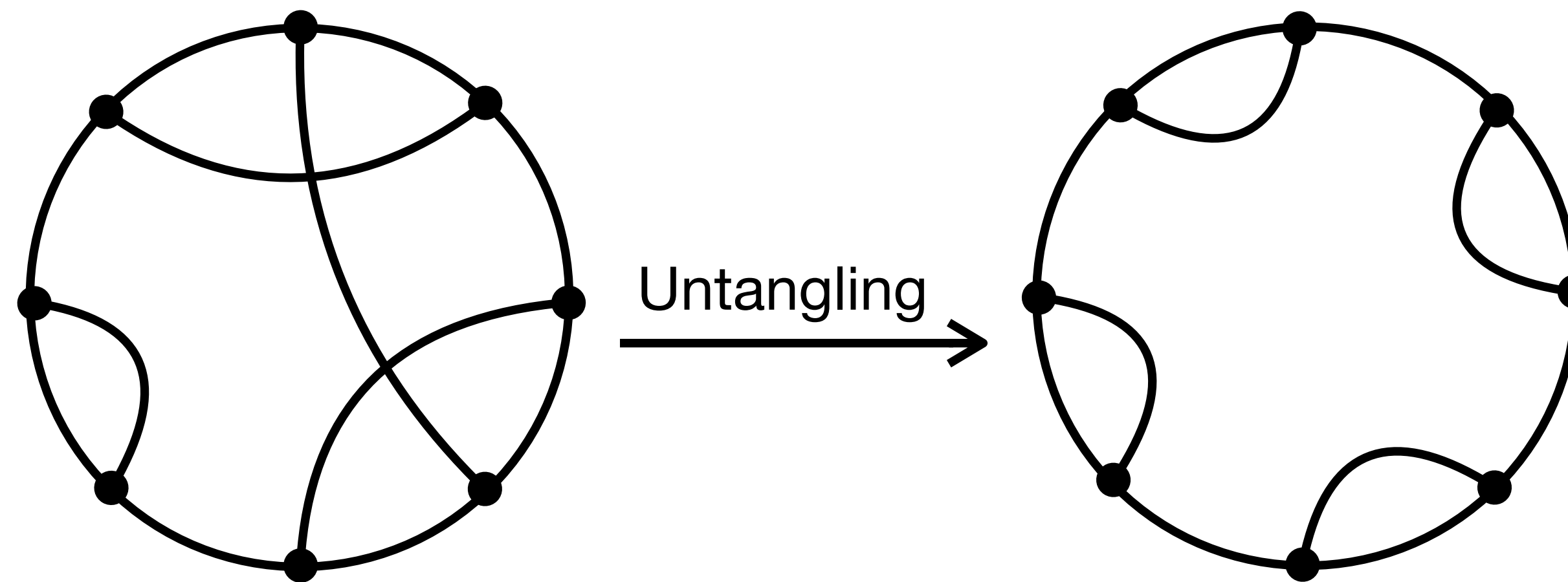
$$\langle J_{i_1 \dots i_p} \rangle = 0, \quad \langle J_{i_1 \dots i_p}^2 \rangle = \frac{1}{\lambda} \binom{N}{p}^{-1} J^2.$$

- Note:  $\lambda \rightarrow 0$  recovers model with  $1 \ll p \ll N$  *(Maldacena & Stanford. 2016)*

# CHORD DIAGRAMS

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- Moments of the partition function:  $M_{2k} = \langle \text{Tr} (H^{2k}) \rangle$
- Allows for a diagrammatic representation:



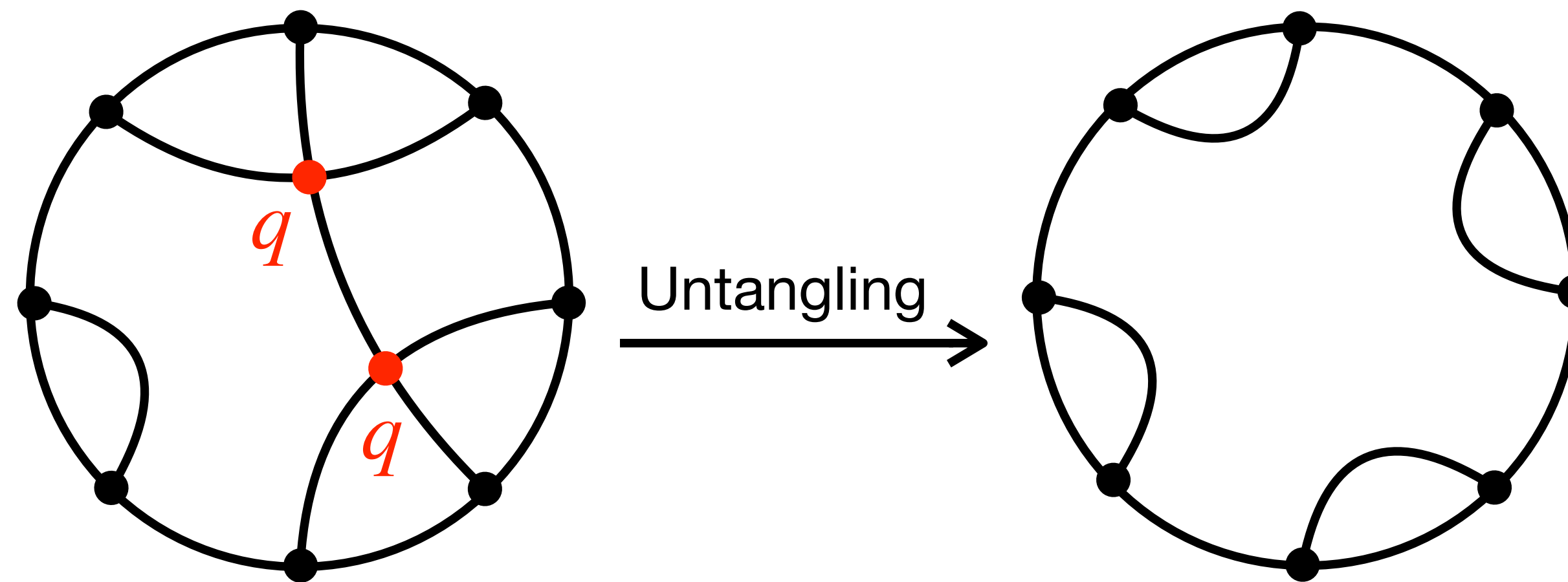
$$M_{2k} = \frac{J^{2k}}{\lambda^k} \sum_{\text{diagrams with } k \text{ chords}} q^{\text{number of intersections}}, \quad q = e^{-\lambda}.$$

(M. Berkooz et al., 2019)



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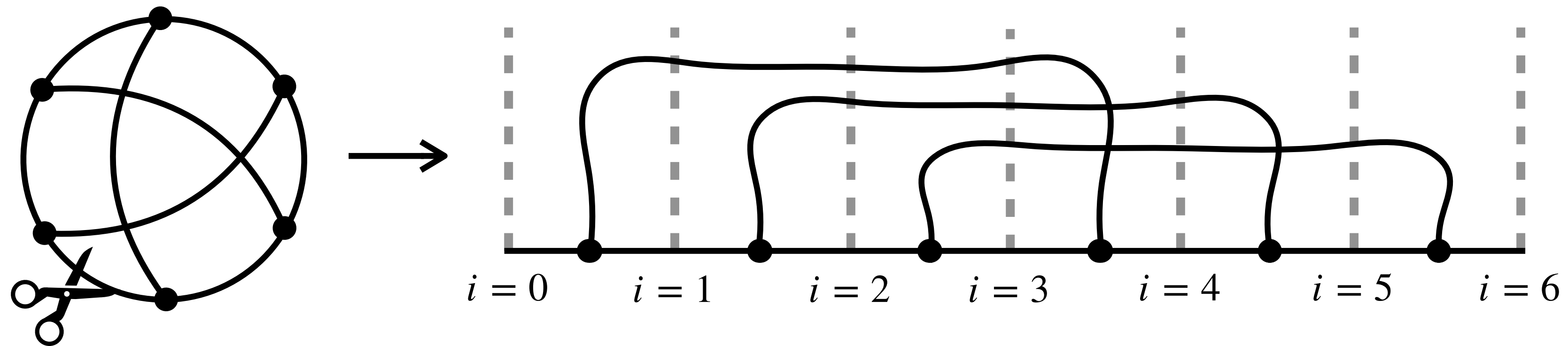


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# CHORD HILBERT SPACE

- Idea: Interpret a chord diagram as a transition from 0 chords back to 0.



- Introduce Hilbert space:  $\text{span} \{ |n\rangle \}_{n \geq 0}$  where  $n =$  number of *open chords*.

Diagrams of  $i$  steps:  $|\psi^{(i)}\rangle = \sum_{n \geq 0} \psi_n^{(i)} |n\rangle$

Recursion: 
$$\psi_n^{(i+1)} = \frac{J}{\sqrt{\lambda}} \psi_{n-1}^{(i)} + \frac{J}{\sqrt{\lambda}} (1 + q + \dots + q^n) \psi_{n+1}^{(i)}$$

(M. Berkooz et al., 2019)

# TRANSFER MATRIX (NON-SYMMETRIC VERSION)

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► Can write:  $|\psi^{(i)}\rangle = T^i |0\rangle$ , with:

$$T \stackrel{*}{=} \frac{J}{\sqrt{\lambda}} \begin{pmatrix} 0 & \frac{1-q}{1-q} & 0 & 0 & \dots \\ 1 & 0 & \frac{1-q^2}{1-q} & 0 & \dots \\ 0 & 1 & 0 & \frac{1-q^3}{1-q} & \dots \\ 0 & 0 & 1 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$T$ : Transfer matrix (*effective Hamiltonian in the averaged theory*)

$$M_{2k} = \langle 0 | T^{2k} | 0 \rangle = \langle 0 | \psi^{(2k)} \rangle = \psi_0^{(2k)} .$$

(M. Berkooz et al., 2019)

# TRANSFER MATRIX (SYMMETRIC VERSION)

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- There exists a *diagonal similarity transformation* such that: (Berkooz et al., 2019)

$$T \stackrel{*}{=} \frac{J}{\sqrt{\lambda}} \begin{pmatrix} 0 & \sqrt{\frac{1-q}{1-q}} & 0 & 0 & \dots \\ \sqrt{\frac{1-q}{1-q}} & 0 & \sqrt{\frac{1-q^2}{1-q}} & 0 & \dots \\ 0 & \sqrt{\frac{1-q^2}{1-q}} & 0 & \sqrt{\frac{1-q^3}{1-q}} & \dots \\ 0 & 0 & \sqrt{\frac{1-q^3}{1-q}} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Interpreted as *renormalization* of states such that  $\langle n | n' \rangle = \delta_{n,n'}$ . (H. Lin, 2022)

# THE EFFECTIVE HAMILTONIAN

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► Can write in operator language:

$$T = \frac{J}{\sqrt{\lambda}}(\alpha + \alpha^\dagger)$$

With  $\alpha = \sum_{n \geq 0} \sqrt{[n+1]_q} |n\rangle\langle n+1|$ , where  $[n]_q \equiv \frac{1-q^n}{1-q} = \sum_{k=0}^{n-1} q^k$

→ *q-deformed oscillator* (M. Berkooz et al., 2019)

Can write as:  $\alpha^\dagger = \sqrt{\frac{1-q^{\hat{n}}}{1-q}} D^\dagger$ , with  $D^\dagger = \sum_{n \geq 0} |n+1\rangle\langle n| = e^{-ip}$

→ *p* is conjugate momentum of  $\hat{n}$ .

# EFFECTIVE HAMILTONIAN AND TRIPLE-SCALING LIMIT

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- Defining  $\lambda n =: \frac{l}{L}$ ,

$$T = \frac{J}{\sqrt{\lambda(1-q)}} \left( e^{i\lambda Lk} \sqrt{1 - e^{-\frac{l}{L}}} + \sqrt{1 - e^{-\frac{l}{L}}} e^{-i\lambda Lk} \right)$$

- Triple-scaling limit:  $\lambda \rightarrow 0$ ,  $l \rightarrow \infty$ ,  $\frac{e^{-\frac{l}{L}}}{(2\lambda)^2} =: e^{-\frac{\tilde{l}}{L}}$  fixed.

- The Hamiltonian takes the form: *(Liouville!!  $\implies$  JT)* *(H. Lin, 2022)*

$$\tilde{T} = E_0 + 2\lambda J \left( \frac{L^2 k^2}{2} + 2e^{-\frac{\tilde{l}}{L}} \right) + O(\lambda^2), \quad E_0 \sim -\frac{2J}{\lambda}$$

# LANCZOS COEFFICIENTS IN DSSYK

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► The chord basis **automatically performs** the Lanczos algorithm:

1. Each state  $|n\rangle$  is a linear combination of  $\{|\psi^{(k)}\rangle = T^k |0\rangle\}_{k=0}^n$ .

2.  $\{|n\rangle\}_{n \geq 0}$  forms an orthonormal basis.

3. In this basis  $T$  takes tridiagonal form with positive items.

$\implies \{|n\rangle\}_{n \geq 0}$  is the Krylov basis for  $|0\rangle$  and  $T$ .

**Lanczos coefficients:**

$$a_n = 0, \quad b_n = J \sqrt{\frac{1 - q^n}{\lambda(1 - q)}} = \frac{J}{\sqrt{\lambda}} \sqrt{[n]_q}$$

# STATE-DEPENDENCE OF THE LANCZOS COEFFICIENTS

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► We have performed the Krylov construction for:  $|\phi(t)\rangle = e^{-itT} |0\rangle$

Survival probability:  $\langle 0 | e^{-itT} | 0 \rangle = \sum_{k=0}^{+\infty} \frac{(-it)^{2k}}{(2k)!} M_{2k}$

$M_{2k} = \langle 0 | T^{2k} | 0 \rangle = \langle \text{Tr} (H^{2k}) \rangle$

It is the **partition function**:

$$\langle 0 | e^{-itT} | 0 \rangle = \langle \text{Tr} [e^{-itH}] \rangle = \langle Z(\beta + it) \rangle \Big|_{\beta=0}$$

► Thus,  $|0\rangle$  plays the role of the  $\beta = 0$  **thermofield “double”** in the **effective** (averaged)

theory:  $|\Omega\rangle := \frac{1}{\sqrt{\mathcal{N}}} \sum_E |E\rangle$ , *Cf Harlow & Jafferis, 2018*

$\text{Tr} [e^{-itH}] = \langle \Omega | e^{-itH} | \Omega \rangle$  *Effective theory*  $\longrightarrow \langle \text{Tr}[e^{-itH}] \rangle = \langle \langle \Omega | e^{-itH} | \Omega \rangle \rangle = \langle 0 | e^{-itT} | 0 \rangle$



# ILLUSTRATION: FIRST COEFFICIENTS FROM RECURSION RELATION

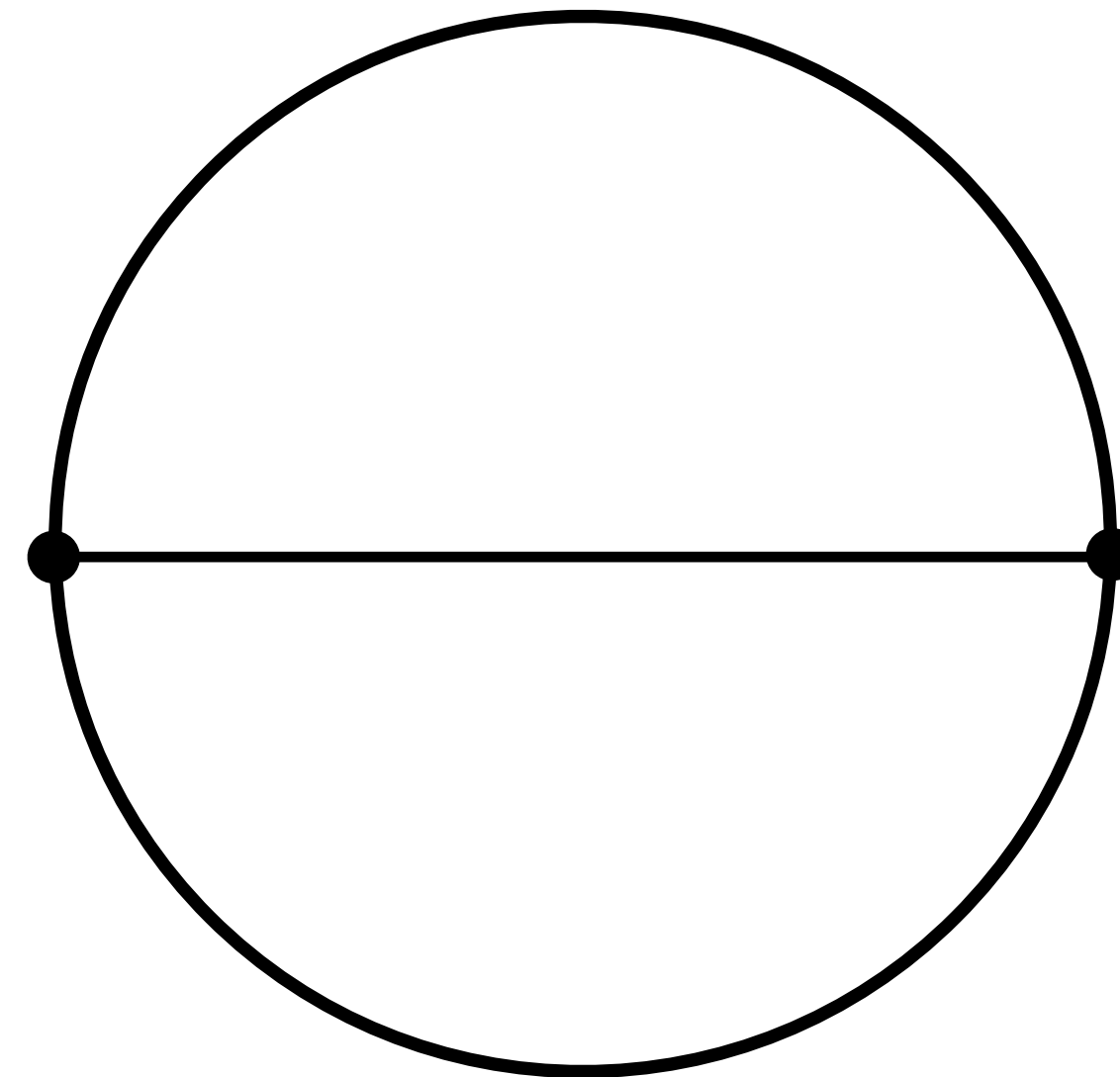
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- Can obtain the Lanczos coefficients from  $M_{2k}$  computed with chord diagrams using the recurrence relation.

$$M_2 = \frac{J^2}{\lambda}$$

$$M_4 = \frac{J^4}{\lambda^2}(2 + q)$$

$$M_6 = \frac{J^6}{\lambda^3}(5 + 6q + 3q^2 + q^3)$$



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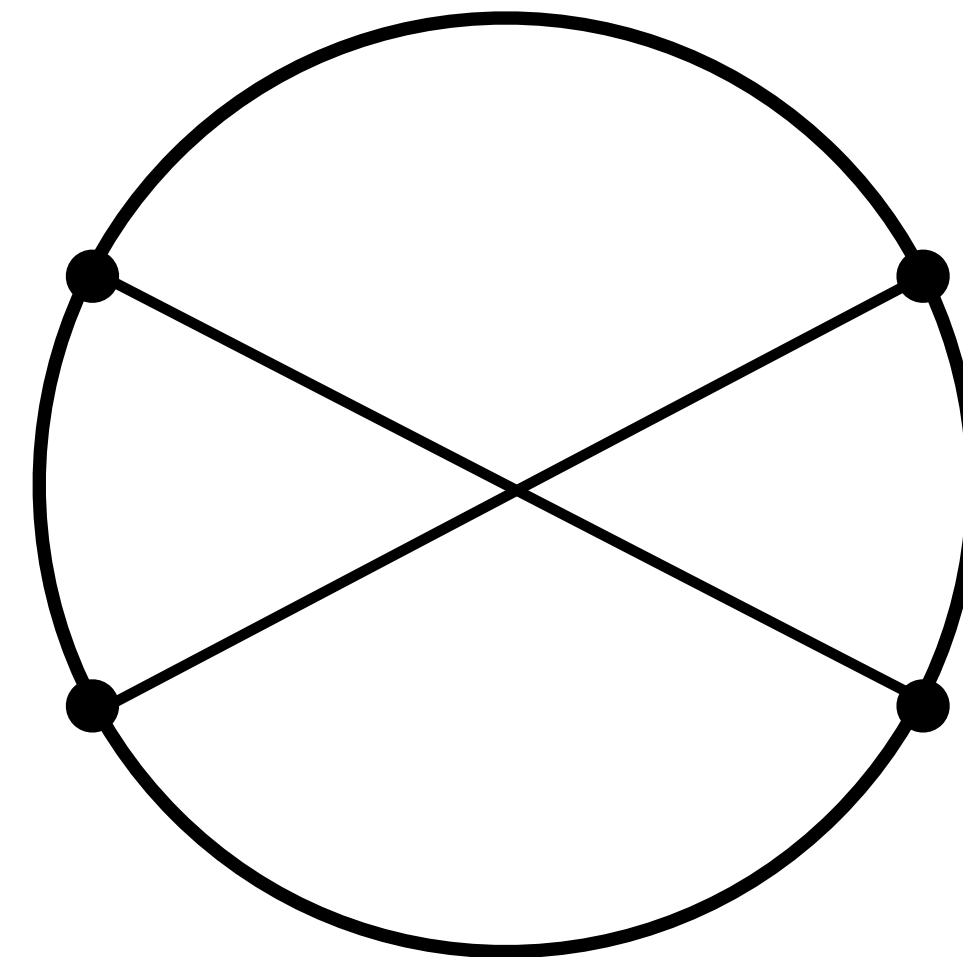
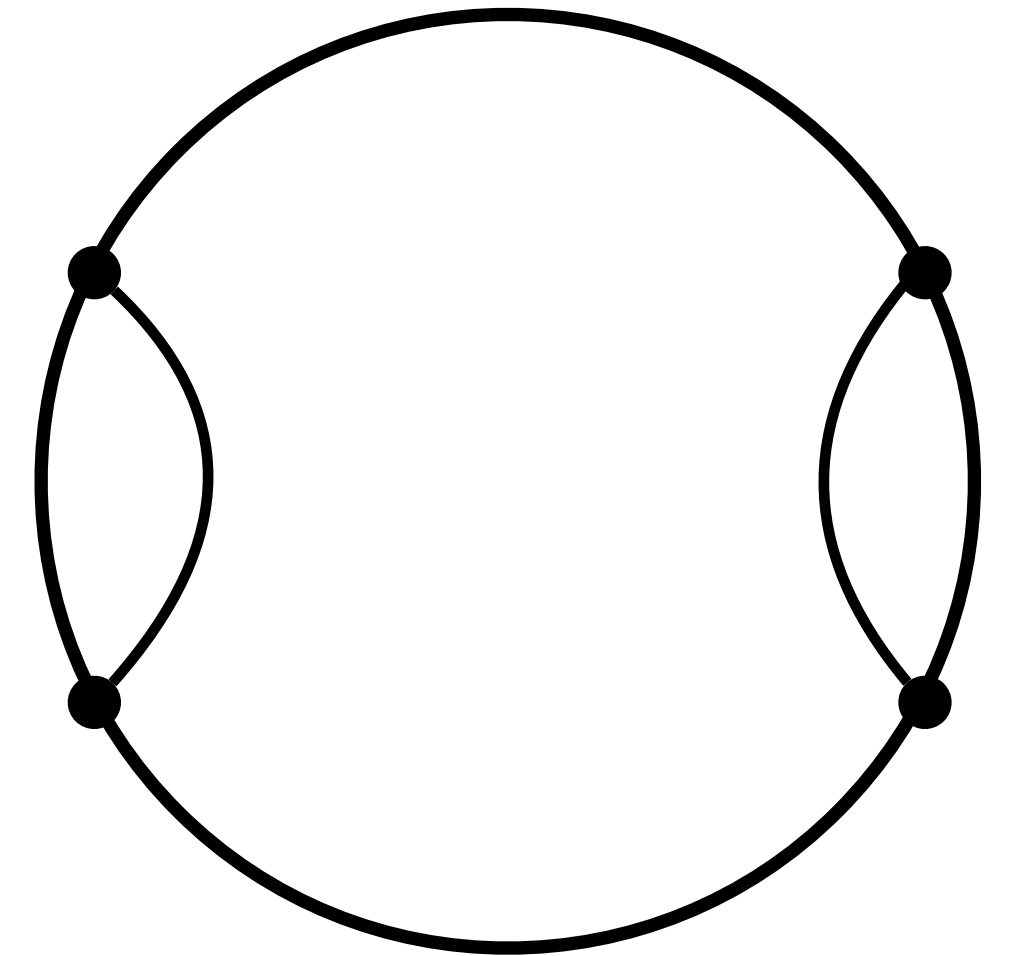
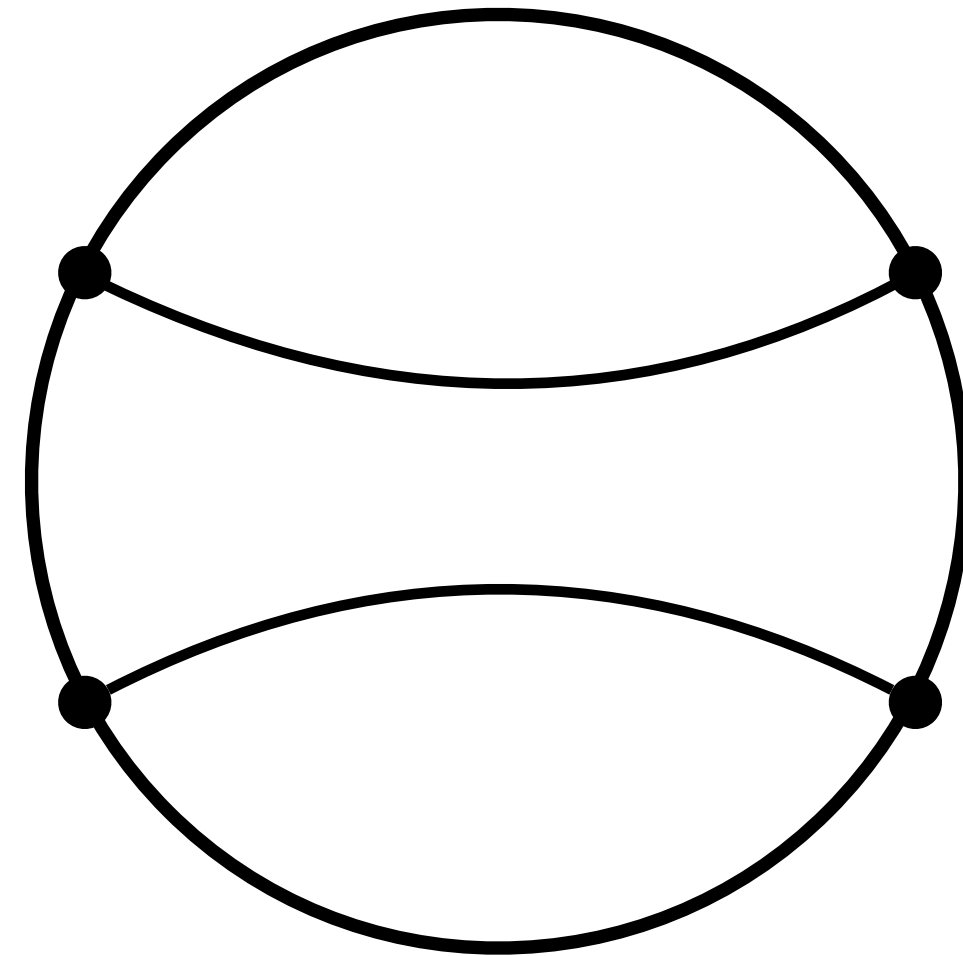
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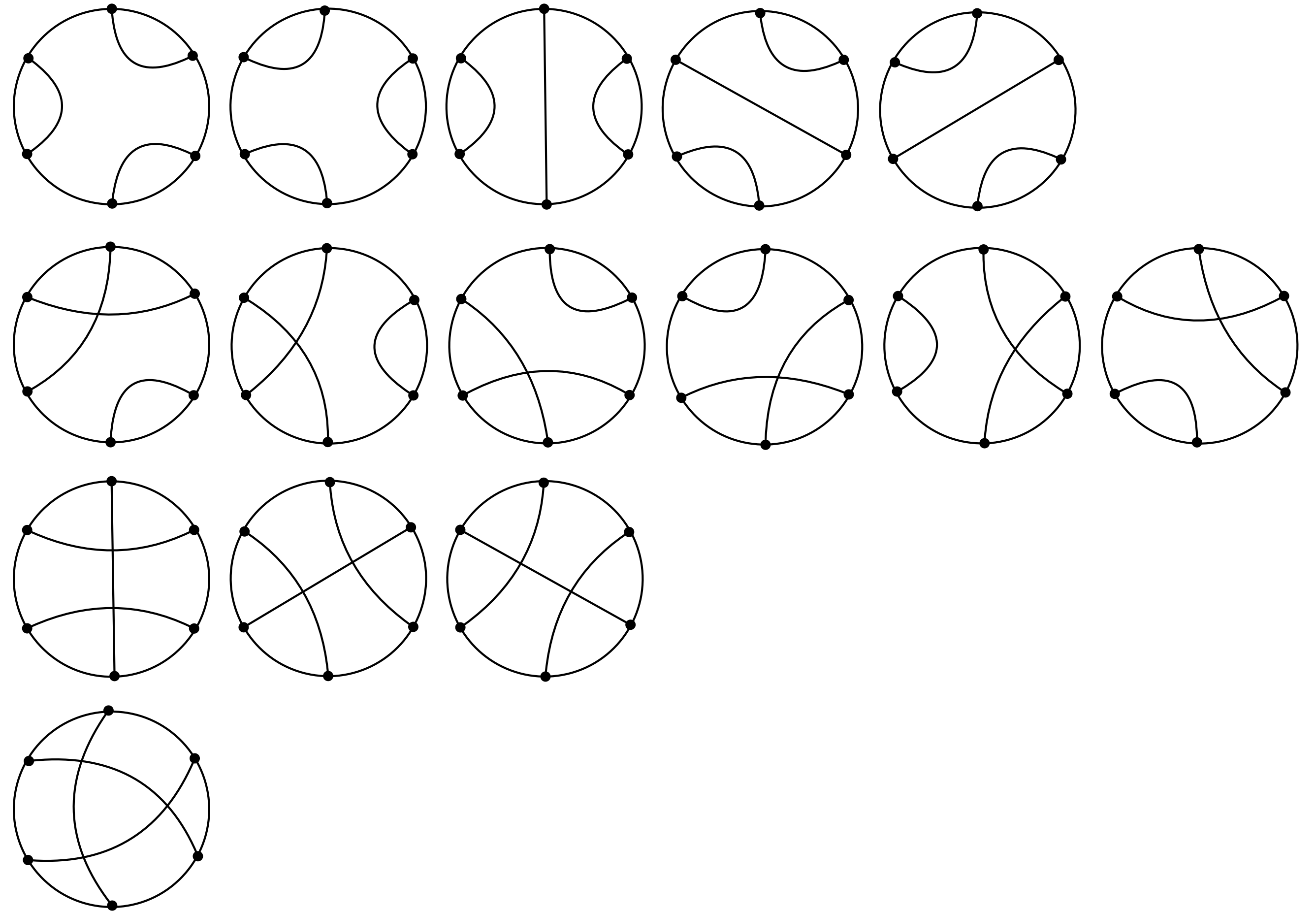
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$$b_1^2 = M_2 = \frac{J^2}{\lambda} = \frac{J^2}{\lambda}[1]_q$$

$$b_2^2 = \frac{M_4}{M_2} - M_2 = \frac{\frac{J^4}{\lambda^2}(2 + q)}{\frac{J^2}{\lambda}} - \frac{J^2}{\lambda} = \frac{J^2}{\lambda}(1 + q) = \frac{J^2}{\lambda}[2]_q$$

$$b_3^2 = \frac{\frac{M_6}{M_2} - M_4}{\frac{M_4}{M_2} - M_2} - \frac{M_4}{M_2} = (\dots) = \frac{J^2}{\lambda}(1 + q + q^2) = \frac{J^2}{\lambda}[3]_q .$$

# K-COMPLEXITY REGIMES: EARLY TIMES

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For  $n \ll \frac{1}{\lambda}$ :

$$b_n = J \sqrt{\frac{1 - e^{-\lambda n}}{\lambda(1 - q)}} \approx J \sqrt{\frac{n}{1 - q}}$$

$\implies T \approx \gamma (a + a^\dagger)$ , where  $a, a^\dagger$  are SHO ladder operators.

$$\implies \phi_n(t) = e^{-\frac{\gamma^2 t^2}{2}} \frac{(-i\gamma t)^n}{\sqrt{n!}} \implies C_K(t) = \sum_{n=0}^{+\infty} n |\phi_n(t)|^2 = \gamma^2 t^2$$

*(P. Caputa et al., 2022)*

i.e. **coherent states** propagating on the Krylov chain.

$$\text{Transition time: } C(t_*) \approx \frac{1}{\lambda} \implies t_* = \frac{1}{J} \sqrt{\frac{1 - e^{-\lambda}}{\lambda}} \xrightarrow{\lambda \rightarrow 0} J^{-1}$$

# K-COMPLEXITY REGIMES: LATE TIMES

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For  $n \gg \frac{1}{\lambda}$ :

$$b_n = J \sqrt{\frac{1 - e^{-\lambda n}}{\lambda(1 - q)}} \approx \frac{J}{\sqrt{\lambda(1 - q)}} \left( 1 - \frac{e^{-\lambda n}}{2} \right) \xrightarrow{n \rightarrow \infty} \frac{J}{\sqrt{\lambda(1 - q)}} \equiv b_\infty$$

► Given  $b_n = b_\infty$ ,  $\exists$  exact solution for  $\phi_n(t)$

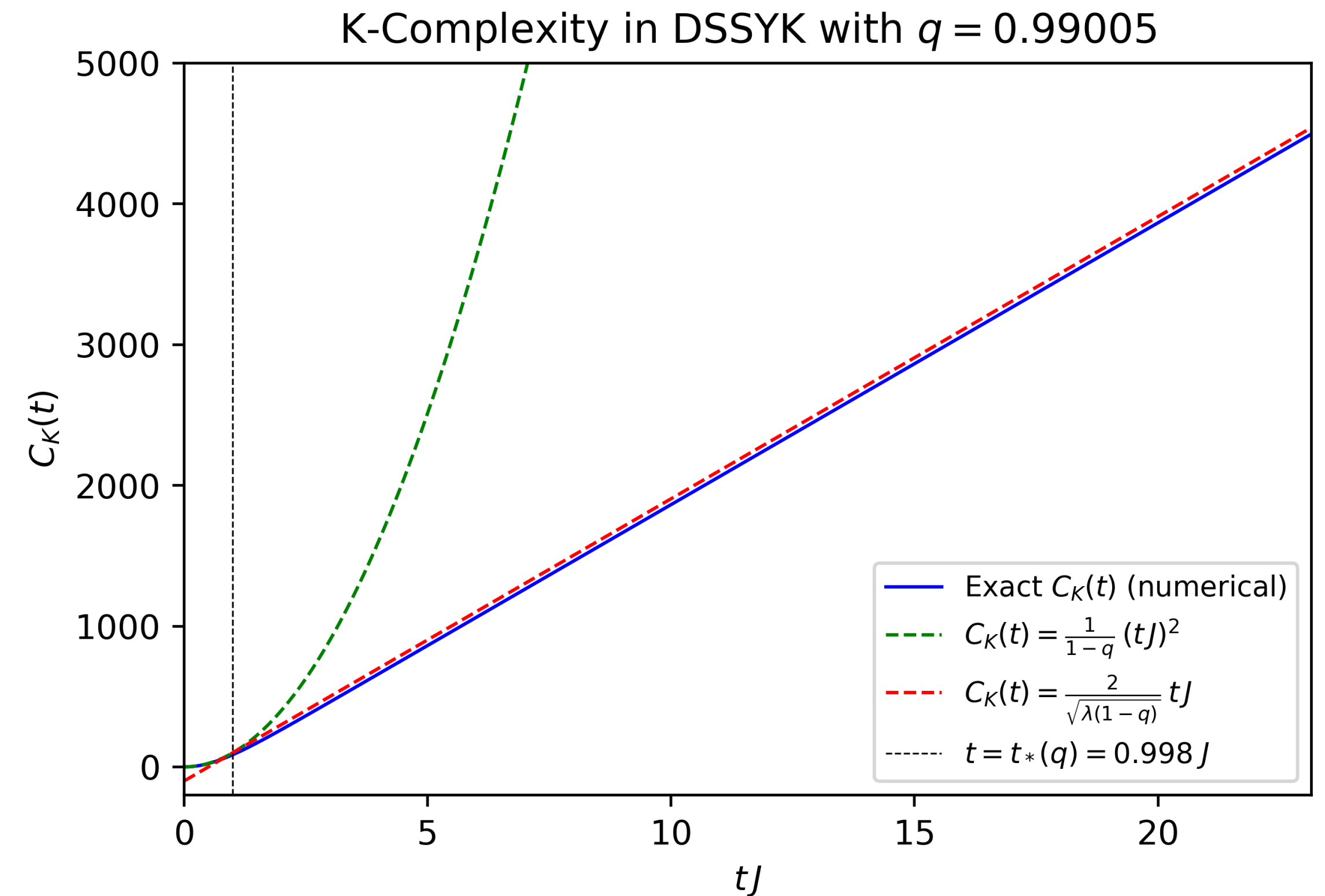
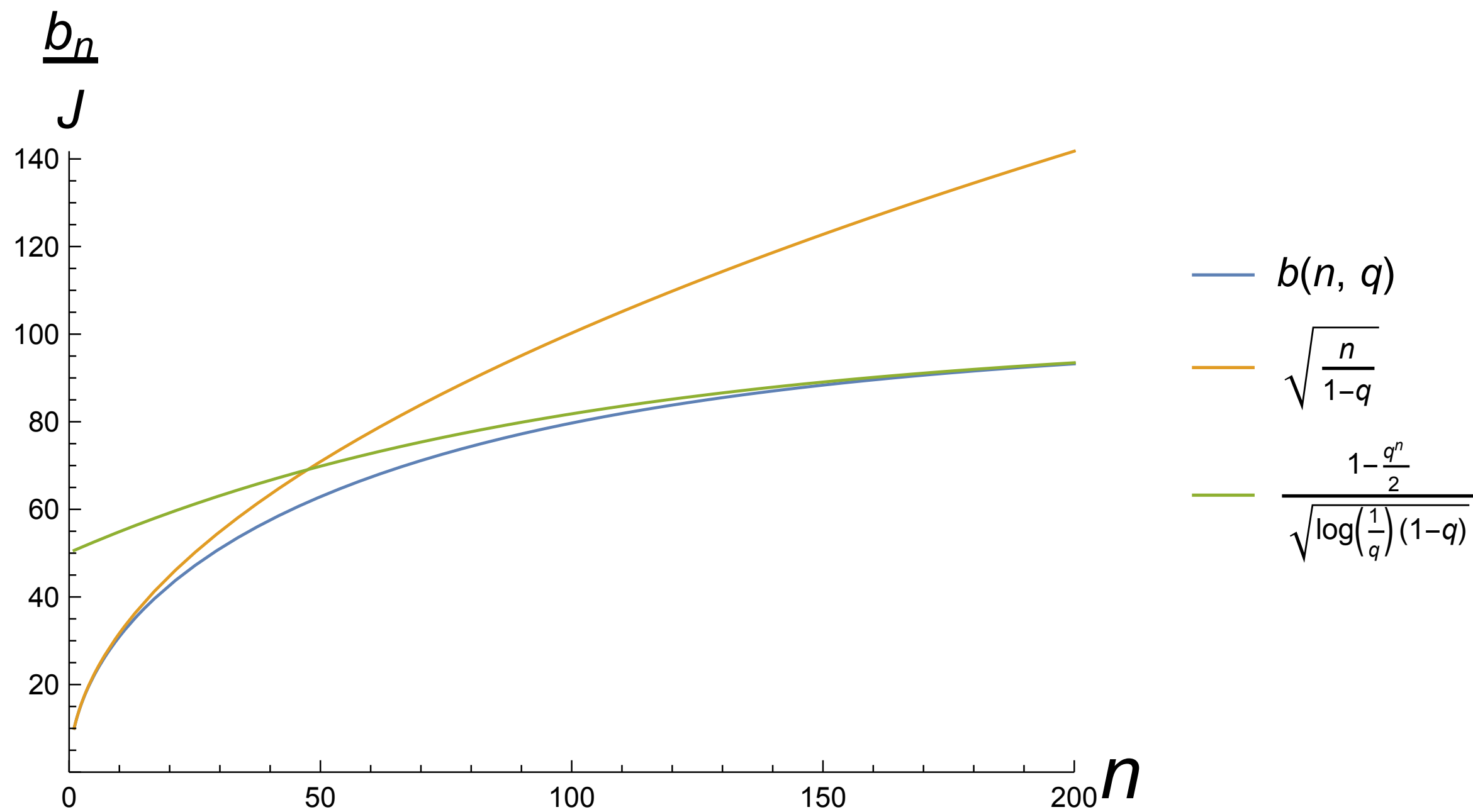
*(J. Barbón et al., 2019)*

*(E. Rabinovici, ASG, R. Shir, J. Sonner, 2023)*

Using front-most peak position:  $C_K(t) \approx 2b_\infty t$

► Misses build-up of the tail of the wave packet.

# K-COMPLEXITY REGIMES: SUMMARY AND NUMERICS



(E. Rabinovici, ASG, R. Shir, J. Sonner; 2023)

# CONTINUUM LIMIT

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Lanczos coefficients:  $b_n = J \sqrt{\frac{1 - e^{-\lambda n}}{\lambda(1 - q)}}$

Recurrence equation (**Schrödinger**) for  $\phi_n(t) = i^n \varphi_n(t)$ :

$$\dot{\varphi}_n(t) = b_n \varphi_{n-1}(t) - b_{n+1} \varphi_{n+1}(t)$$

*(J. Barbón et al., 2019)*

Continuum limit: *(E. Rabinovici, ASG, R. Shir, J. Sonner, 2023)*

$$\lambda \rightarrow 0, \quad n \rightarrow \infty, \quad x := \lambda n \quad \text{fixed}$$

$$\implies b_n \longrightarrow \frac{J}{\lambda} \sqrt{1 - e^{-x}} + O(\lambda^0) \equiv b(x)$$



# CONTINUUM APPROXIMATION TO RECURRENCE EQUATION

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► Can promote  $\varphi_n(t)$  to  $f(t, x)$  such that  $\varphi_n(t) = f(t, n\lambda)$ .

Recurrence equation becomes:

$$\partial_t f(t, x) = -v(x)\partial_x f(t, x) - \frac{v'(x)}{2}f(t, x) + O(\lambda)$$

Where:

$$v(x) = 2\lambda b(x) = 2J\sqrt{1 - e^{-x}} + O(\lambda) \xrightarrow{\lambda \rightarrow 0} 2J\sqrt{1 - e^{-x}}.$$

Redefining:

$$dy = \frac{dx}{v(x)} \quad \text{and} \quad g(t, y) := \sqrt{v(x(y))}f(t, x(y))$$

We get:

$$\left(\partial_t + \partial_y\right) g(t, y) = 0 + O(\lambda).$$

Chiral wave equation.

# CONTINUUM APPROXIMATION – TRAJECTORY

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The solution just **propagates initial condition**:

$$g(0, y) \equiv g_0(y) \quad \implies \quad g(t, y) = g(y - t)$$

Given  $\varphi_n(0) = \delta_{n0} \implies g_0(y) \propto \delta(y) \implies g(t, y) \propto \delta(y - t)$

Position expectation value ( $\sim$  **K-complexity**) given by position of the peak:

$$t = \int_0^{y_p(t)} dy = \int_0^{x_p(t)} \frac{dx}{v(x)} \quad \equiv \quad \int_0^{n_p(t)} \frac{\cancel{\lambda} dn}{2 \cancel{\lambda} b_n} = \int_0^{n_p(t)} \frac{dn}{2 b_n}$$

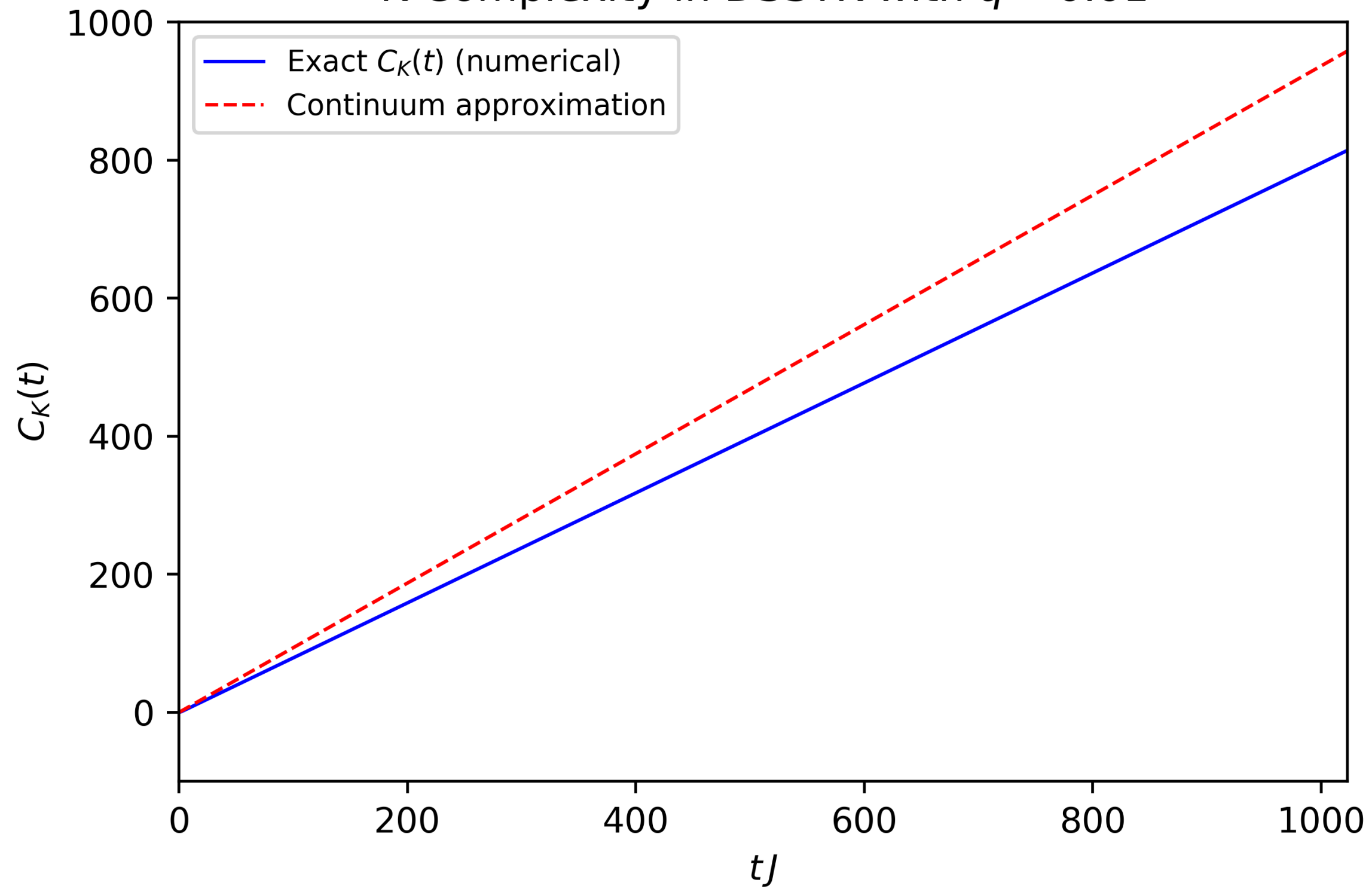
Performing the integral and solving for  $n_p(t)$  we find:

$$C_K(t) \approx n_p(t) = \frac{2}{\lambda} \log \left\{ \cosh \left[ tJ \sqrt{\frac{\lambda}{1-q}} \right] \right\} \quad \textit{Expected to be good at small } \lambda$$

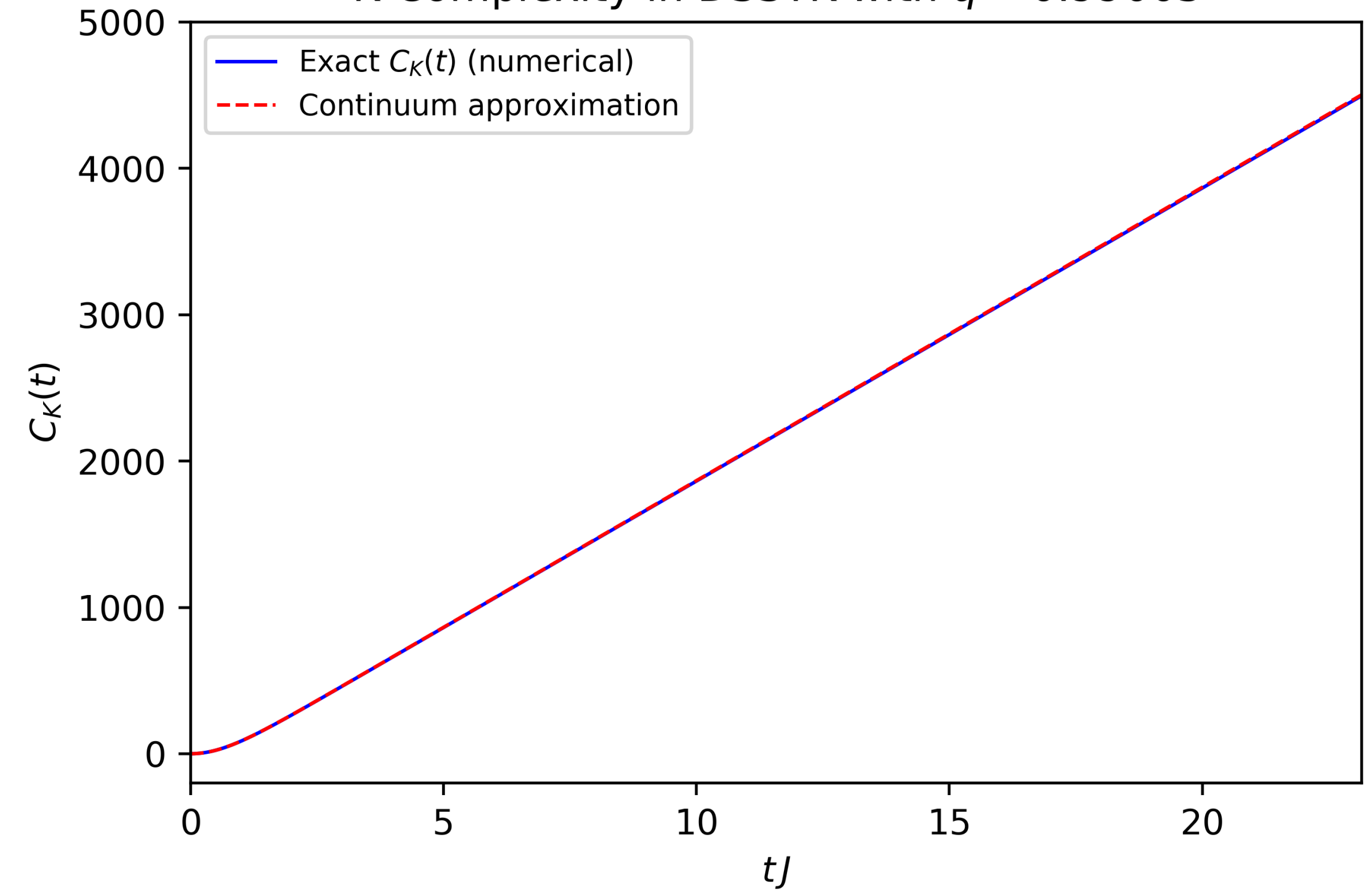
# CONTINUUM APPROXIMATION VS NUMERICS

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K-Complexity in DSSYK with  $q = 0.01$



K-Complexity in DSSYK with  $q = 0.99005$



*(E. Rabinovici, ASG, R. Shir, J. Sonner, 2023)*

# CONNECTION TO LIOUVILLE (CLASSICAL)

---

► Going back to the continuum limit for the variable  $\lambda n = x \equiv \frac{l}{L}$ .

Trajectory satisfying  $\dot{x} = v(x) = 2J\sqrt{1 - e^{-x}}$  is a solution of the EOM of:

$$H' \equiv E_0 + 2\lambda J \left( \frac{L^2 k^2}{2} + \frac{2}{(2\lambda)^2} e^{-\frac{l}{L}} \right) \quad \textit{Liouville!}$$

Why?  $\longrightarrow$  **Classical limit** of effective DSSYK Hamiltonian: *(H. Lin, 2022)*

$$\tilde{T}_{\text{class}} \sim -\frac{2J}{\lambda} \cos(\lambda L k) \sqrt{1 - e^{-\frac{l}{L}}}$$

Both have **same EOM**  $\longrightarrow$  Connection to Liouville is only **classical** so far.

# CONNECTION TO LIOUVILLE (QUANTUM)

---

► Hence the need of the **triple-scaling** limit.

$$\lambda \rightarrow 0, \quad l \rightarrow \infty, \quad \frac{e^{-\frac{l}{L}}}{(2\lambda)^2} =: e^{-\frac{\tilde{l}}{L}} \quad \text{fixed.} \quad (\text{H. Lin, 2022})$$

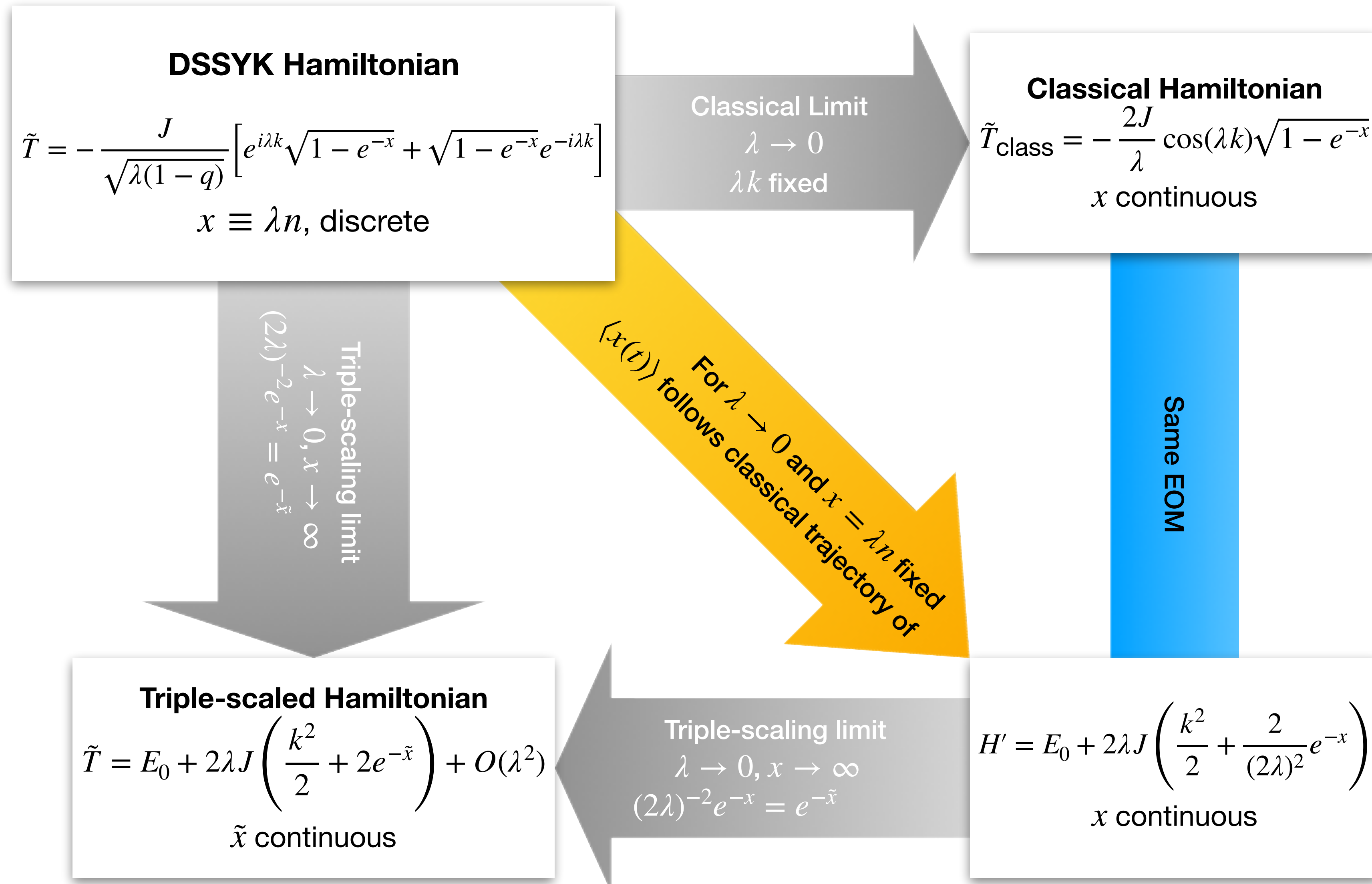
Such that

$$\tilde{T} = E_0 + 2\lambda J \left( \frac{L^2 k^2}{2} + 2e^{-\frac{\tilde{l}}{L}} \right) + O(\lambda^2), \quad E_0 \sim -\frac{2J}{\lambda}$$

i.e. the DSSYK Hamiltonian is Liouville QM near its ground state.

On top of this one can still perform classical approximations.

# HAMILTONIANS: SUMMARY



# REGULARIZED LENGTH FROM K-COMPLEXITY IN TRIPLE-SCALED LATTICE

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► Can envision  $\frac{\tilde{l}}{L} \equiv \tilde{x}$  as the continuum limit of a lattice s.t.  $\tilde{x} = \lambda \tilde{n}$ .

► Triple-scaled Lanczos coefficients:

$$b_{\tilde{n}} = b - 2\lambda J q^{\tilde{n}} + O(\lambda^2)$$

► Cont. approx gives **K-complexity from EOM of triple-scaled Hamiltonian:**

$$\lambda \widetilde{C}_K(t) = \frac{\tilde{l}(t)}{L} = \tilde{x}_0 + 2 \log \left\{ \cosh \left( 2\lambda J e^{-\tilde{x}_0/2} t \right) \right\}$$

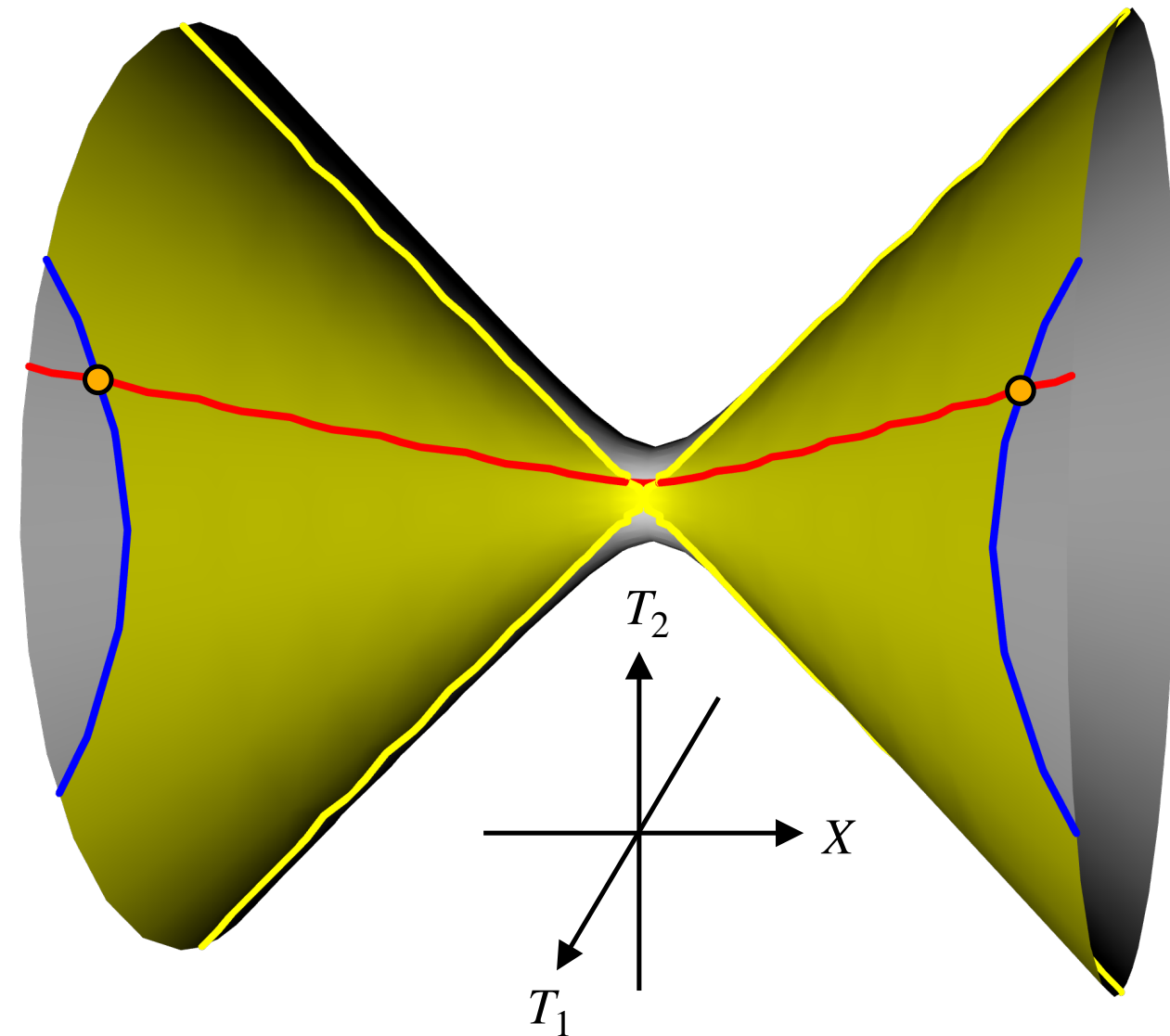
TFD:  $\tilde{x}_0 = 0$

(E. Rabinovici, ASG, R. Shir, J. Sonner, 2023)

# GRAVITY MATCHING: REMINDER OF JT RESULTS

► Two-sided length in  $\text{AdS}_2$ :

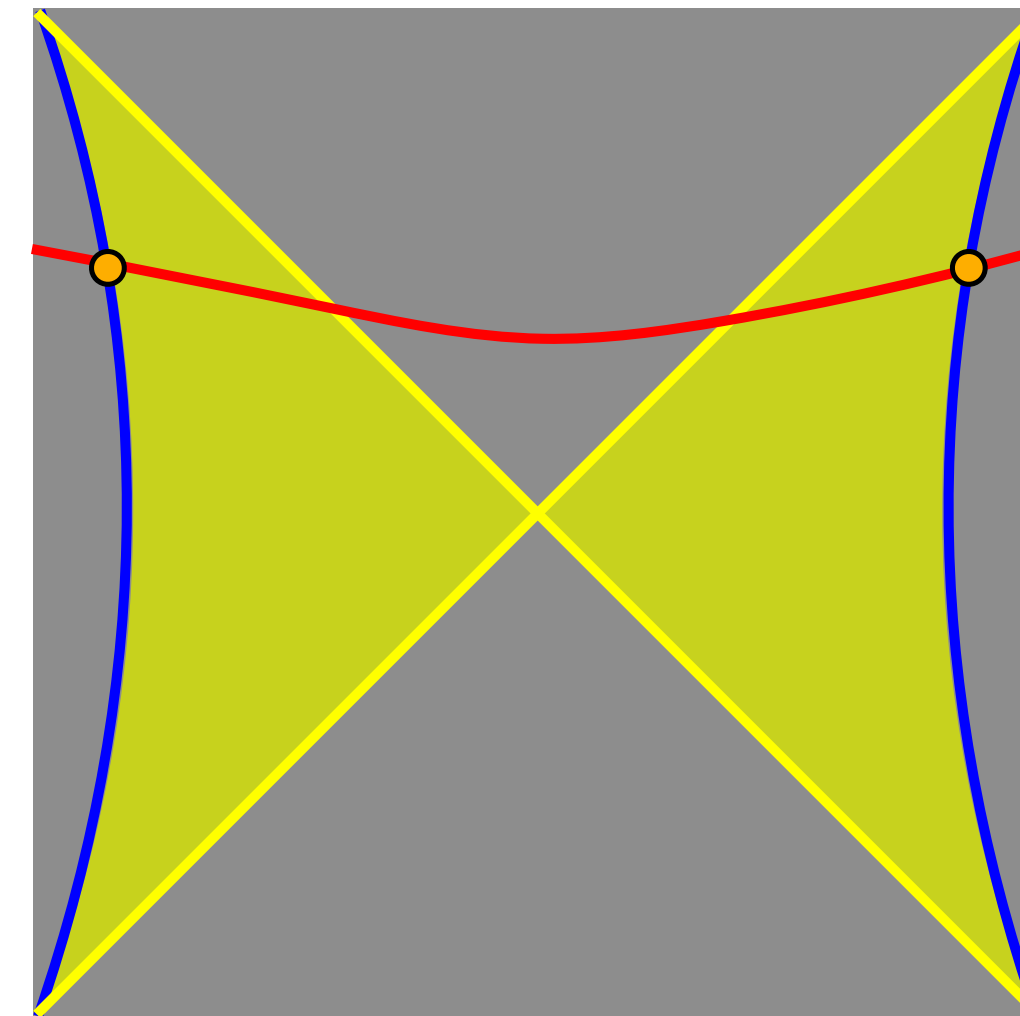
$$\begin{aligned} \frac{\tilde{l}}{l_{\text{AdS}}} &= l - 2 \log \left( \frac{2\phi_b}{\epsilon} \right) \\ &= 2 \log \left[ \cosh \left( \frac{\Phi_h}{l_{\text{AdS}} \phi_b} t_b \right) \right] - 2 \log \Phi_h \end{aligned}$$



► JT Hamiltonian:

$$H = \frac{1}{l_{\text{AdS}} \phi_b} \left( \frac{l_{\text{AdS}}^2 P^2}{2} + 2e^{-\tilde{l}/l_{\text{AdS}}} \right)$$

*(D. Harlow & D. Jafferis, 2018)*





# GRAVITY MATCHING: CORRESPONDENCE AND PARAMETER IDENTIFICATIONS

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➤ **K-complexity eigenstates are bulk length eigenstates** because:

1. Krylov elements are fixed chord number states. (*E. Rabinovici, ASG, R. Shir, J. Sonner, 2023*)
2. Fixed chord number states are bulk length eigenstates. (*H. Lin, 2022*)

➤ Parameter identifications:

1. From **Hamiltonian**:  $L = l_{AdS}$ ,  $2\lambda J = \frac{1}{l_{AdS} \phi_b}$
2. From **classical evaluation**:  $\tilde{x}_0 = -2 \log \Phi_h$

# CONCLUSION

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- Krylov complexity as a candidate for holography.
- DSSYK as a system where the holographic dictionary is well established.
- Under this dictionary, K-complexity is exactly 2-sided bulk length.
  
- Saturation of complexity: finite size on boundary  $\longleftrightarrow$  higher genus corrections in bulk
- Corrections to JT from higher orders in  $\lambda$ ?
- Operators?
- Higher dimensions?

i Thanks!

Photo:

*Los huevos de Lucio*

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