Quantum Black Holes and Holographic Complexity

Antonia Micol Frassino

ICCUB

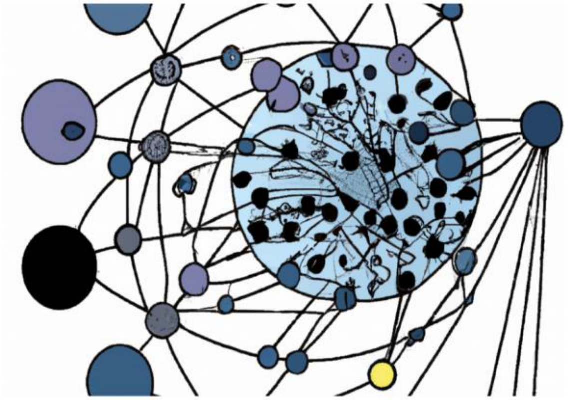
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU 2020-2023

UNIVERSITAT DE BARCELONA

COMPLEXITY:

Between Field Theory and Gravity



IFT, Madrid, 22 May 2023

Motivations

What is it that makes quantum physics so different from classical physics?

- Entanglement; the fact that you can know everything that can be known about a system and know nothing about its parts.
- Feynman pointed out the second unique characteristic of quantum mechanics, which is the remarkable potential complexity of quantum states.

Entanglement

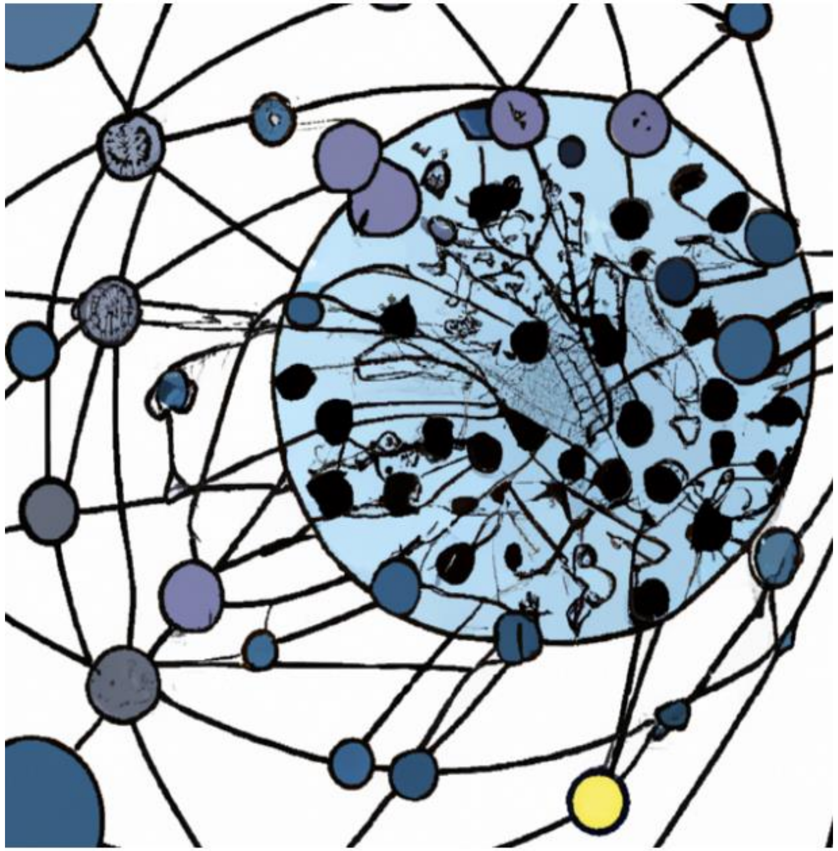
Over time, a considerable amount of progress has been made in understanding **the correlation between gravity and quantum information**

- The concept of entanglement, particularly in the AdS/CFT correspondence framework, exemplifies this idea quite well.
- The Ryu-Takayanagi prescription links entanglement in field theory spacetime regions to minimal surfaces in the bulk.
  This relationship is often summarised as **"entanglement=geometry".**

Holographic Complexity

There's a proposal that entanglement alone may not explain “extreme” physics.
Dual CFT state complexity can offer more insights.

Holographic complexity is currently understood in the large c limit of the CFT ($c \to \infty$) and it is implicitly defined for states $|\psi\rangle$ for some classical bulk observable X_{bulk}

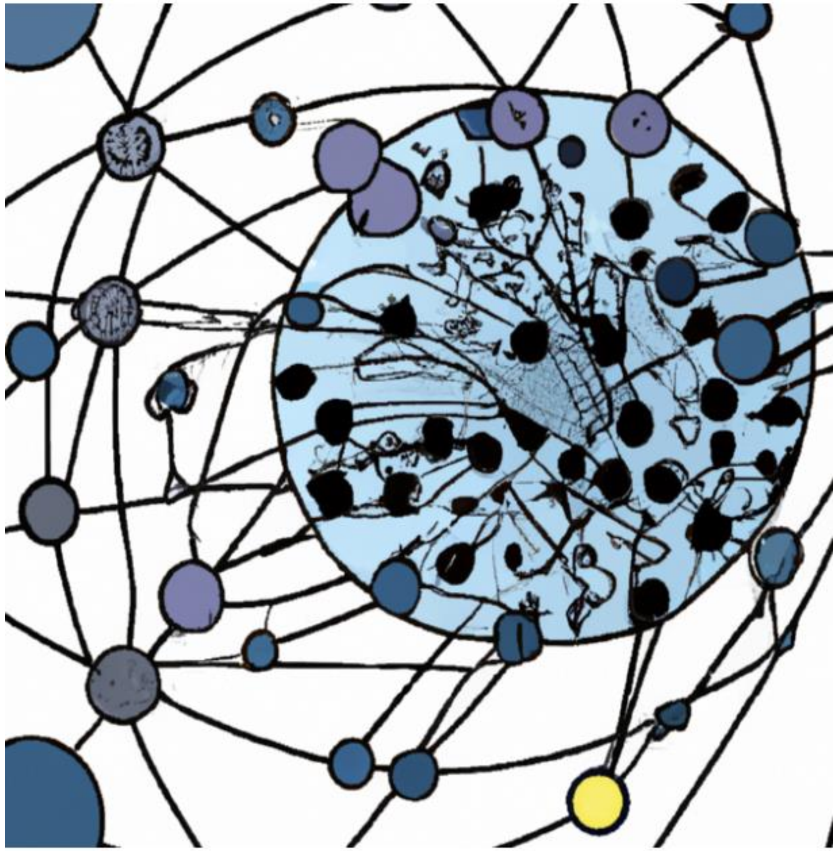


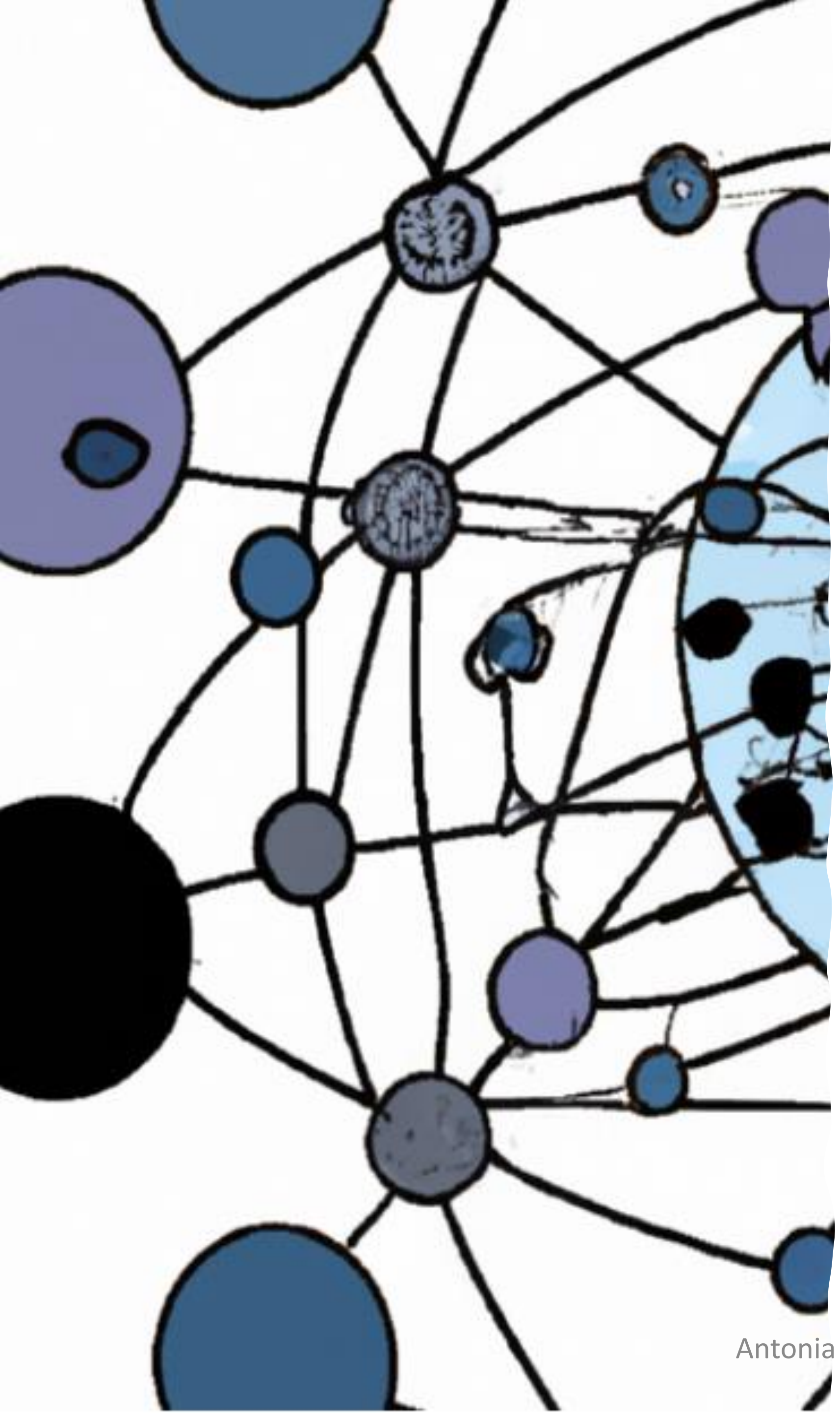
Complexity

And the properties that are usually demanded for this quantity

$$C(|\Psi\rangle) \propto X_{\text{bulk}}$$

1. This classical bulk observable probes the BH interior
2. The late time growth of C (i.e., dC/dt) has a constant flow given approximately by the energy of the state (which is approximately TS)
3. If you have a perturbation in the past, it takes a scrambling time t$\sim\beta \log S$ to scramble and complexity should start growing after this scrambling time





Outline

- We analyze different holographic complexity proposals for **black holes that include corrections from bulk quantum fields.**
- The 1/c corrections correspond to the bulk quantum corrections, and we have the expansion:

$$C\left(|\Psi\rangle\right) \propto X_{\text{bulk}} + C_{\text{q.c.}}\left(|\Psi\rangle\right)$$

- The specific setup is the **quantum BTZ BH**
  - It encompasses the effects of conformal fields with large central charge in the presence of the black hole, including the backreaction corrections to the BTZ metric.

Quantum Backreaction

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

→ Classical Einstein tensor & metric

→ Quantum matter renormalized stress tensor (<u>many fields</u>)

<u>Perturbative backreaction</u>

- Coupled system: metric+ <QFT>
- Very hard to solve simultaneously

<u>Exact backreaction</u>

- 2D models: CGHS, JT+CFT
- **Holographic reformulation**

Holographic Reformulation

Conventional AdS/CFT has *fixed* boundary geometry

→ Move the boundary inwards: **Braneworld holography**

→ And get dynamic on the brane

[H. Verlinde, Gubser (1999)]

Braneworld Black Holes

Braneworld models

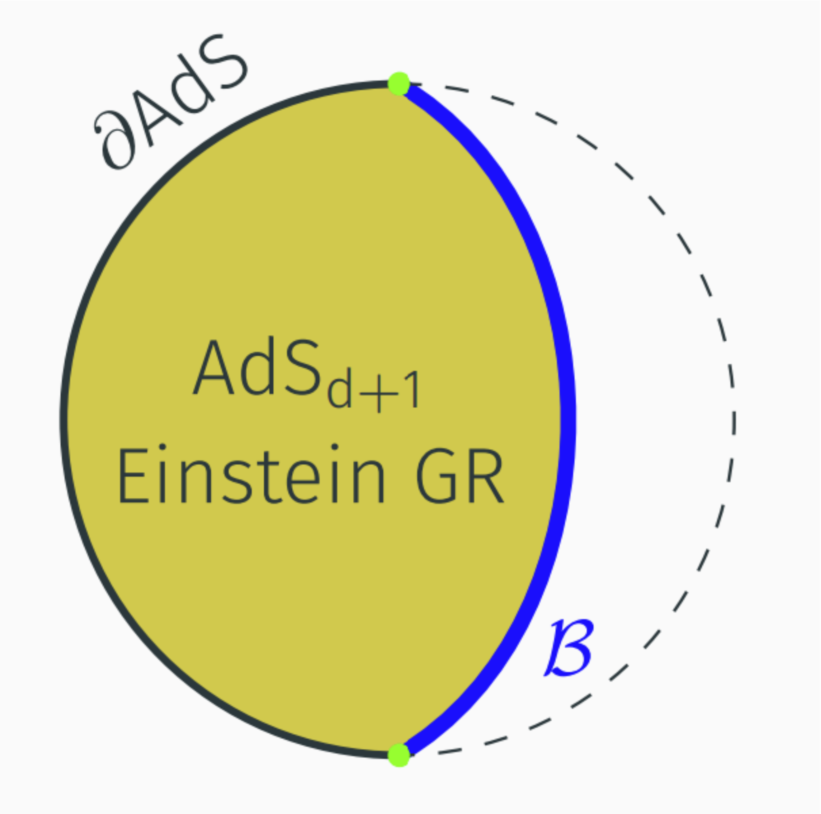
Basic idea of BRANEWORLD gravity:
Recover gravity localized on a lower dimensional surface of a higher dimensional bulk spacetime.

One of the most popular models is that of Randall and Sundrum (RS), which consists of a domain wall universe living in five-dimensional AdS spacetime

Interestingly, although the RS model is an empirical braneworld set-up it can be related to, or motivated by, string theory in several ways.

KR model: it has an effective negative cosmological constant residing on the branr. KR branes are thus AdS slicing of AdS.

Braneworld Holography



Israel equations give us the general formalism for getting our braneworld metric

However, finding actual solutions can be a far trickier matter!

One can resort, as with standard gravity, to two main approaches:

- Local Physics or perturbation theory
- "Big Picture" or geometry, **find exact solutions assuming symmetries**

Black holes and Holography

Linearized gravity results for an isolated mass and the brane cosmology metric suggest a deeper importance to braneworlds and black holes.

- The corrections to the Newtonian potential coincide precisely with the 1-loop corrections to the graviton propagator

M. J. Duff and J. T. Liu, PRL (2000)

- The cosmological dark radiation term in the brane Friedman equation corresponds (up to a factor) to the energy density of a CFT at the Hawking temperature of the black hole

S. S. Gubser (2001)

These clues, and analogies with lower dimensional branes, led to **the black hole holographic conjecture** of Emparan, Fabbri and Kaloper

Black holes and Holography

Linearized gravity results for an isolated mass and the brane cosmology metric suggest a deeper importance to braneworlds and black holes.

- The corrections to the Newtonian potential coincide precisely with the 1-loop corrections to the graviton propagator

M. J. Duff and J. T. Liu, PRL (2000)

- The cosmological dark radiation term in the brane Friedman equation corresponds (up to a factor) to the energy density of a CFT at the Hawking temperature of the black hole

S. S. Gubser (2001)

If we have a classical solution to the RS model then we can interpret the braneworld as a quantum corrected 4D spacetime.
In the case of the black hole, this would mean that **we have a quantum corrected BH!**

Black holes and Holography

This conjecture has implications in two directions:

1. It allows us to view the brane-induced modifications of the metric of a D-dimensional black hole as quantum corrections from a CFT
2. <u>We can use the conjecture to infer, from the known properties of the classical bulk solutions, the properties of the cutoff CFT coupled to gravity.</u>

Holographic interpretation (AdS/CFT):

- 'Bulk' governed by GR + brane $\mathcal{B}$: $I_{\text{bulk}} = I_{\text{EH}} + I_{\text{brane}}$
  with: $\Rightarrow I_{\text{brane}} = -\tau \int_{\mathcal{B}} d^d z \sqrt{-h}$
- Integrate out bulk from ∂AdS to $\mathcal{B}$: $I_{\text{eff}}^{\mathcal{B}} = I_{\text{grav}}[\mathcal{B}] + I_{\text{CFT}}^{\text{cut-off}}[\mathcal{B}]$
- Include the CFT action

Induced gravity: scales

Holographically induced scales:

$$G_d = \frac{d-2}{2L_{d+1}} G_{d+1} \qquad \frac{1}{L_d^2} = \frac{2}{L_{d+1}^2}\left(1 - \frac{4\pi G_{d+1} L_{d+1}}{d-1}\tau\right)$$

Keeping the other bulk parameters fixed (AdS radius and Newton constant), it follows that the tuning of the brane tension controls the effective CC on the brane

$$\delta\tau = \frac{\delta\Lambda_d}{8\pi G_d}$$

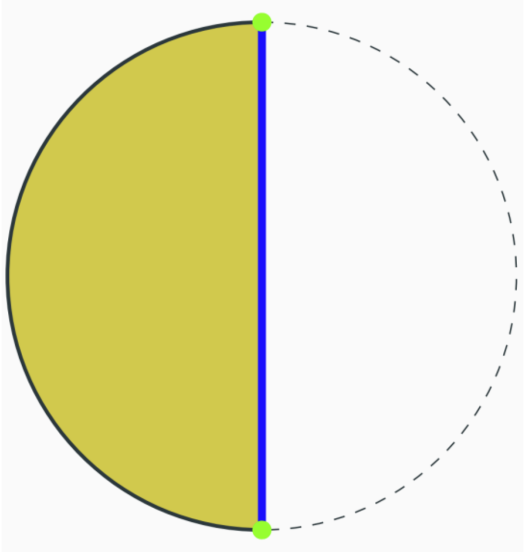
AMF, Pedraza, Svesko, Visser, PRL (2023)

This hypothesis allows us to define a q.c. (thermodynamic) volume for the BH on the brane

Brane limits

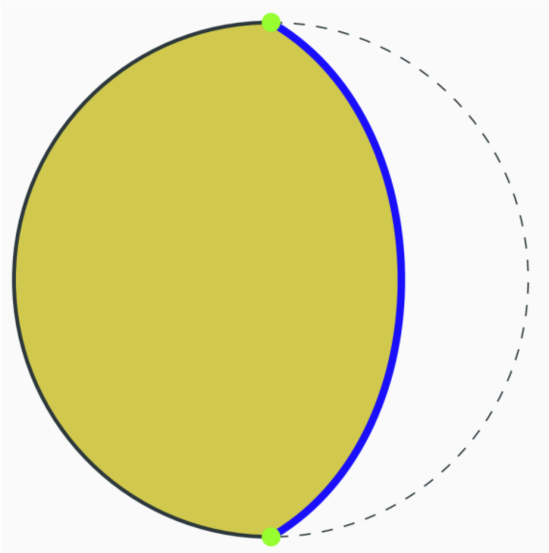
$$\tau \propto \frac{1}{G_4 \ell}$$

$\ell \to \infty$



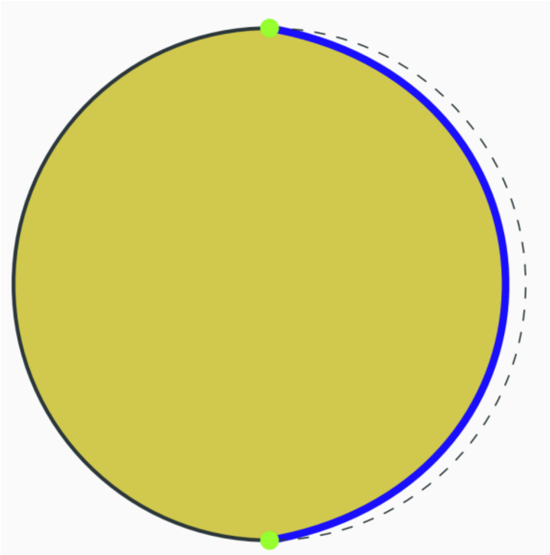
the graviton mass becomes as large as possible (unreasonable limit from the brane perspective)

$0 < \ell < \infty$



massive gravity theory on the brane

$\ell \to 0$



almost massless graviton

Black hole on the brane

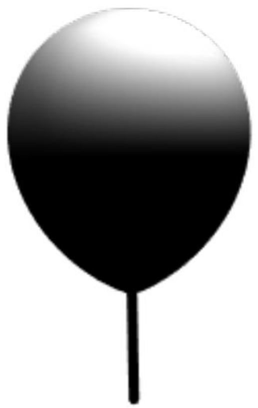
- It is possible to construct exact 4D solutions describing localized black holes bound to a brane
- To obtain these solutions, notice that a black hole on a brane in AdS is accelerating!

There is a solution to Einstein's equation that describes accelerating black holes: **the C-metric**

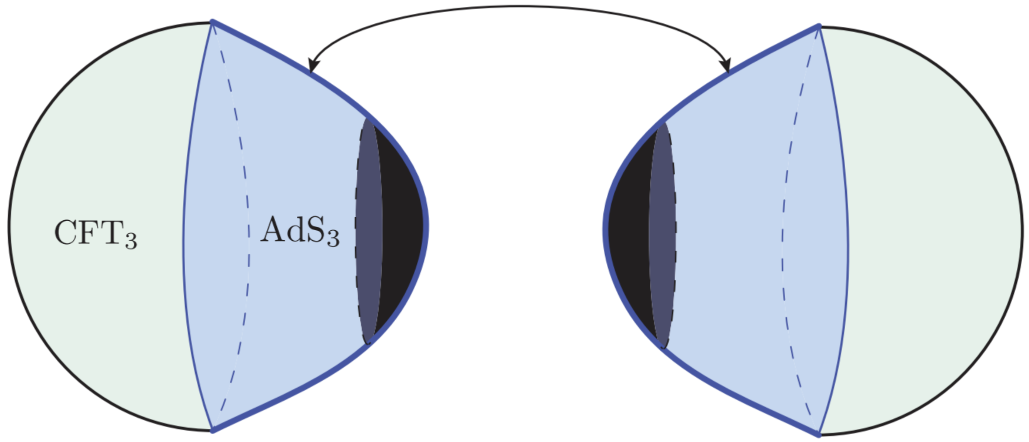
- This solution can be extended to include the CC
- Mechanism to accelerate these black holes: cosmic strings pulling on the black holes
  - usually contains conical singularities along the axis from the BH to ∞

AdS4 C-metric

Accelerated BH



Black hole on the brane



AdS4 C-metric

The classical metric of this state:

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

ℓ_3 Brane curvature radius

ℓ Brane position, tension^(-1)

μ Quantum corrections *on the brane*

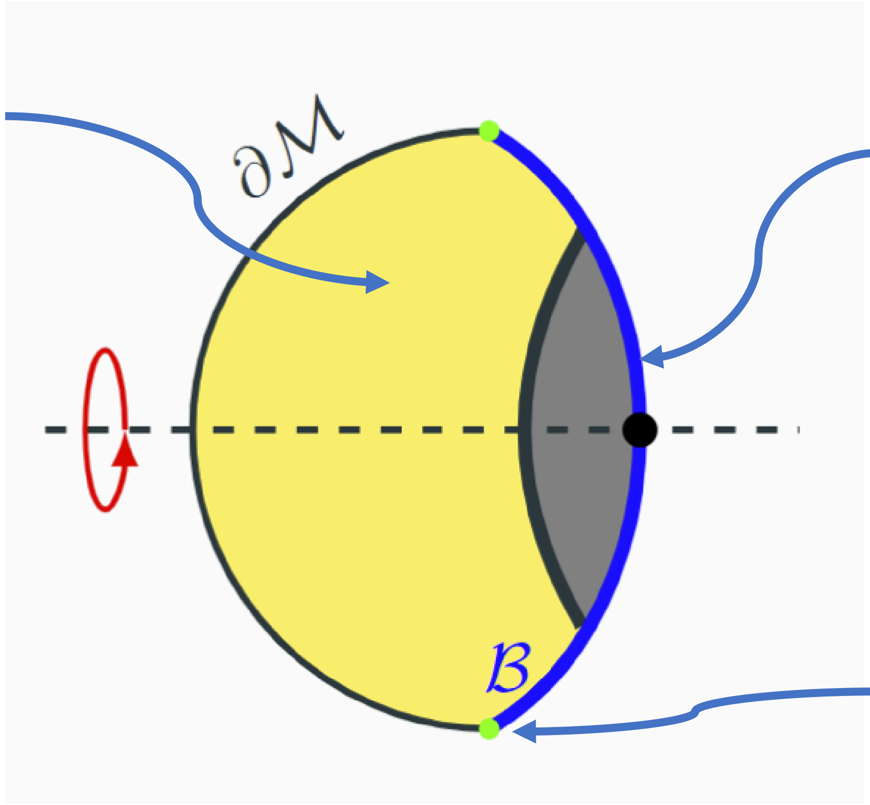
Note: the black hole is NOT in the center of AdS4 – it's accelerating

AdS$_4$ C-metric

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

EE with negative CC in 4D

The bulk spacetime is a solution of the vacuum D



The BH in the bulk is intersecting the brane

3D CC obtained from:
4D CC & brane tension

$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$

Transparent boundary conditions for the CFT
btw the boundary and the brane
(we consider the dCFT)

Metric quBTZ

The 3D metric induced on the brane at x=0

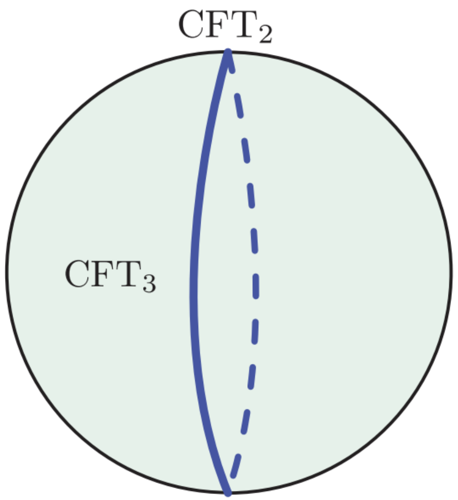
$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right) dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}} dr^2 + r^2 d\phi^2$$

$\mu \neq 0 \quad \kappa = -1$ quBTZ, different properties of the horizon, it has curvature singularity

Interpretation: is as a solution of a theory of 3D gravity, with higher curvature terms, coupled to a large number of quantum conformal fields, namely, the holographic CFT dual to the 4D bulk.

The whole setup: Double holography

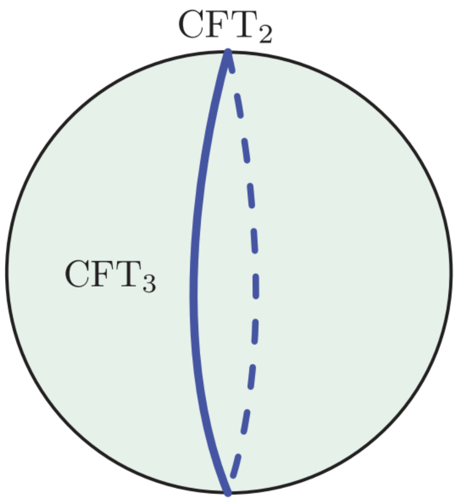
1. Start with a 3d CFT on a space that is topologically a sphere and then put a 2d defect on the equator

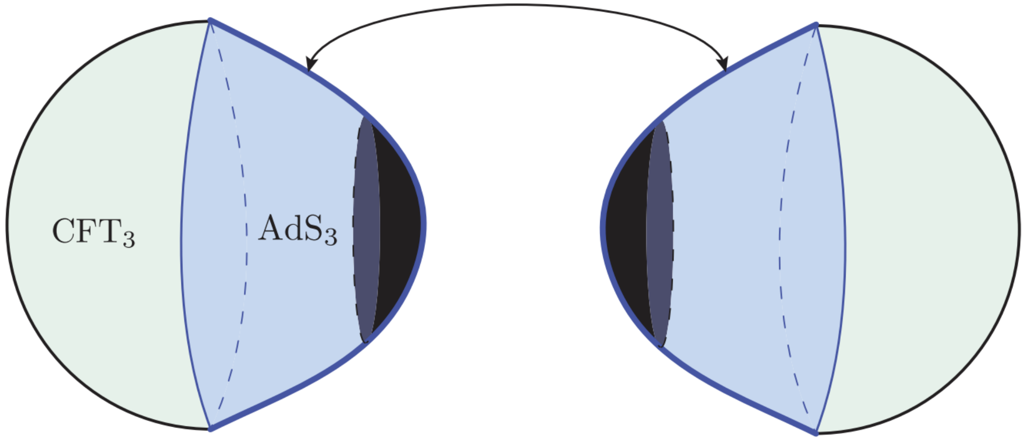


The whole setup: Double holography

1. Start with a 3d CFT on a space that is topologically a sphere and then put a 2d defect on the equator

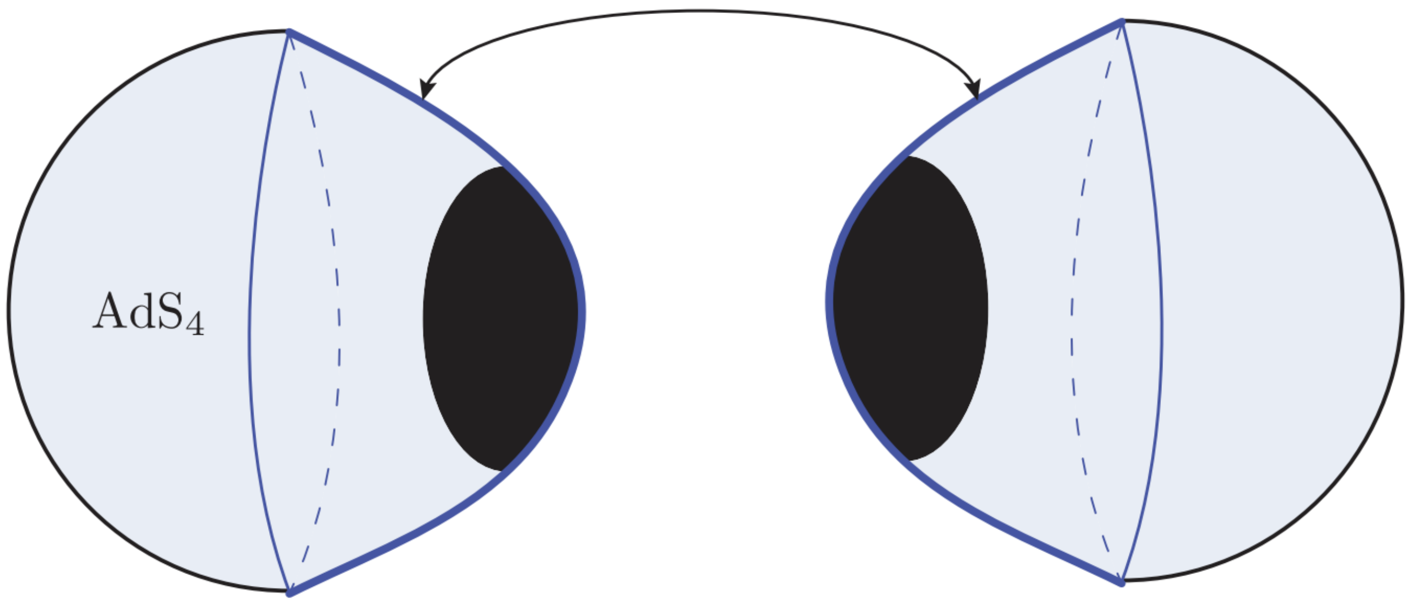
2. Replace the defect by a AdS_3 theory. Since the 2d CFT is in a thermal state, the AdS_3 contains a (semiclassical) BH





The whole setup: Double holography

3. Put the whole system on a 4D bulk



This is the idea of translating quantum effects in the brane perspective to classical quantities into 4D perspective

The precise limits we are taking to have this classical description are:

$$c_3 \to \infty \qquad \text{and} \qquad \frac{c_3}{c_2} \ll 1$$
$$c_2 \to \infty$$

We can see it from the induced effective action on the 3D brane

Generalized Entropy

Emparan, AMF, Way (2020)

A way to see how powerful it is this method of computing quantum corrections using the classical theory is by calculating the generalized entropy of the state.

- Calculate the 4D thermodynamic entropy based on the RT formula

$$\frac{A_{d+1}}{4G_{D+1}} \quad \text{(Bulk entropy)}$$

- This quantity can be interpreted in the 3D theory as $S_{\text{Wald}} + S_{\text{outside}}$

$$\Rightarrow \quad S_{\text{gen}} = \frac{A_{d+1}}{4G_{D+1}} \equiv S_{\text{Wald}} + S_{\text{outside}}$$

Generalized Entropy

Emparan, AMF, Way (2020)

$$S_{\text{gen}} = S_{\text{Wald}} + S_{\text{outside}}$$

Bulk BH entropy

$$S_{gen} = \frac{A_{D+1}}{4G_{D+1}}$$

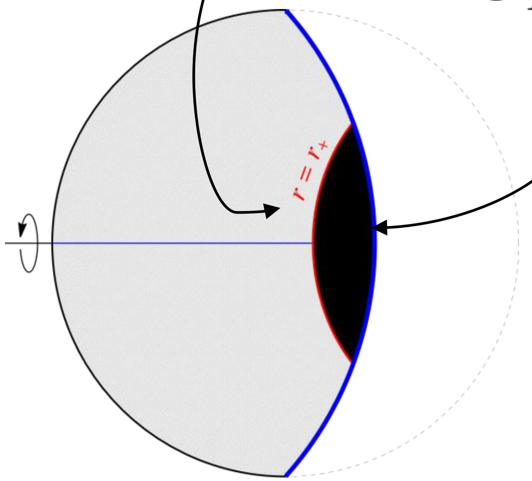
Brane BH entropy

$$S_{\text{Wald}} = \frac{A_D}{4G_D} + \dots$$

Entropy of the CFT that leaves in the presence of the BH

→ Obtained by subtraction:

$$S_{\text{ent}} = S_{\text{gen}} - S_{\text{Wald}}$$



Emparan, AMF, Way (2020)

Quantum entropy: 1^{st} and 2^{nd} law

If the holographic interpretation of braneworld is consistent, then:

$$T dS_{\text{gen}} = dM - \Omega dJ$$

1^{st} law: Not trivial

In D+1 dim bulk

On D dim brane w/ higher curvature gravity

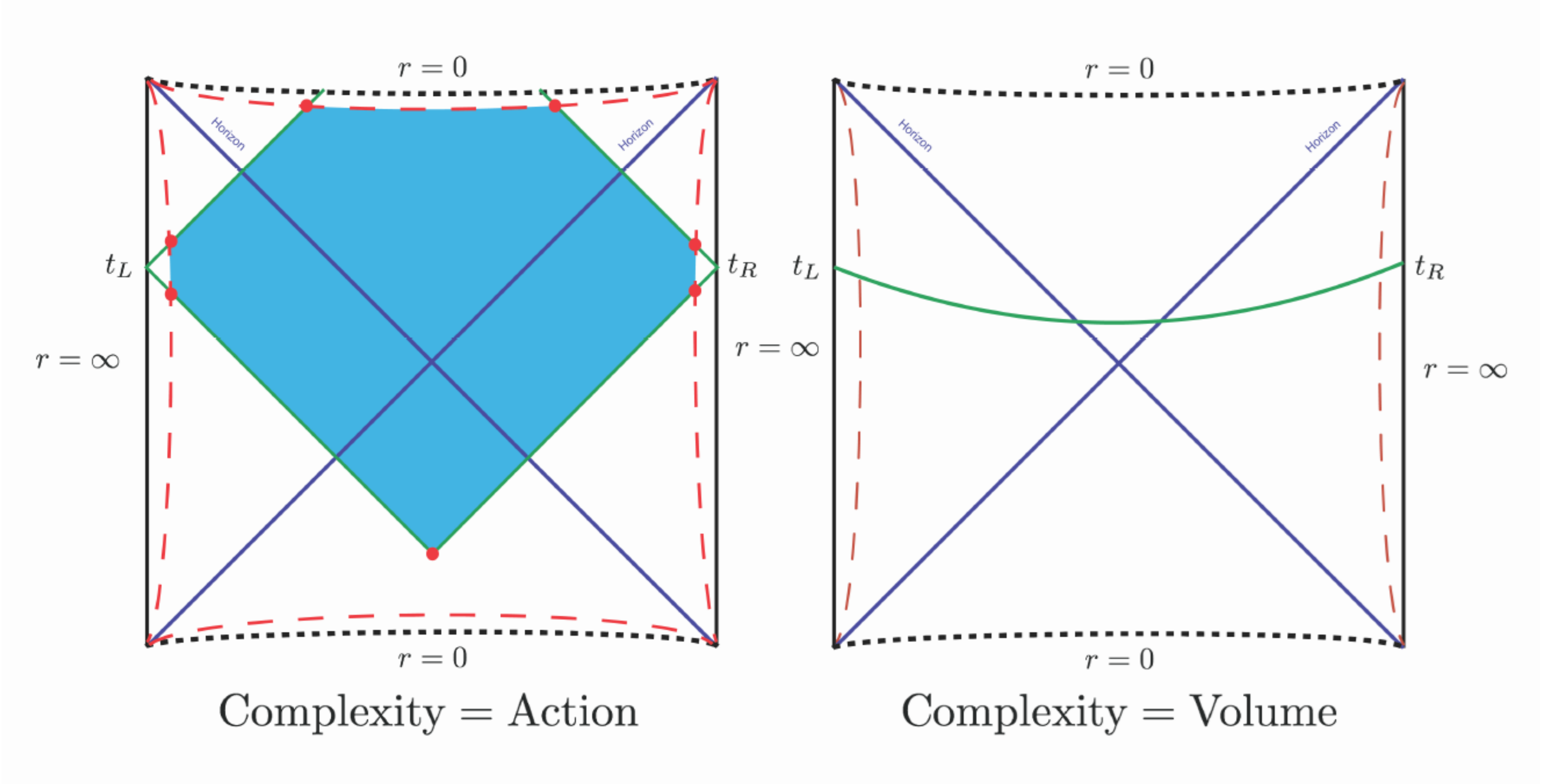
$$\Delta S_{\text{gen}} \geqslant 0$$

2^{nd} law: Trivial

S_{Wald} alone will not satisfy these relations!

Holographic Complexity

- Action and Volume



Holographic Complexity

- Volume and Action – several proposals
- New developments in the definition of Holographic Complexity
- The key property that sustains VC and AC as plausible measures of the complexity for the chaotic time-evolution of the black hole:

$$\left.\frac{d\,\mathcal{C}_{V,A}}{dt}\right|_{t\gg\beta} \sim TS$$

<u>Using the C-metric we can compute quantum corrections using a classical setup</u>

Quantum corrections

- These proposals (action/volume) have passed many tests and they give the same results in almost all cases
- They are similar in spirit to the RT formula, where we also connected a purely geometric notion (area) to a quantum property of the state (von Neumann entropy)

However, in the case of RT, **adding quantum corrections** led to unexpected new results that purely classical bulk could not have given us

Engelhardt, Wall ‘14
Penington `19
Almheiri, Engelhardt, Marolf, Maxfield `19

Holographic complexity

Emparan, AMF, Sasieta, Tomašević (2022)

→ Our system describes a 2-sided BH (given by the 2-copies of the system)
→ We consider a thermal state of a 3D CFT coupled to gravity
→ We compute the CV complexity of this state by the volume in 4D or the CA by the action in 4D

$$C_V(|\mathrm{quBTZ}\rangle) = \frac{\mathrm{Vol}_4(\Sigma)}{G_4 L} \qquad C_A(|\mathrm{quBTZ}\rangle) = \frac{I_4}{\hbar}$$

- These quantities should compute the complexity of the quBTZ, including the bulk complexity of the quantum fields in the large limit of the c_3 central charge.
- Note that there are no extra terms because we have a purely tensional brane
- We are NOT computing the volume/action on the brane - this, in principle, includes the bulk complexity of the quantum fields, the higher curvature terms, etc, but in the large c_3 limit

Quantum corrections

Emparan, AMF, Sasieta, Tomašević (2022)

→ Volume

$$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\, L} + \frac{\delta\mathrm{Vol}(\Sigma) + \mathcal{V}(\Sigma)}{G\hbar\, L} + \mathcal{C}_V^{\mathrm{bulk}}\left(|\phi\rangle\right) + \dots$$

→ Action

$$\mathcal{C}_A\left(|\Psi\rangle\right) = \frac{I(\mathcal{W})}{\hbar} + \delta\mathcal{C}_A\left(|\Psi\rangle\right) + \dots$$

Quantum corrections

Emparan, AMF, Sasieta, Tomašević (2022)

→ Volume

$$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\, L} + \frac{\delta\mathrm{Vol}(\Sigma) + \mathcal{V}(\Sigma)}{G\hbar\, L} + \mathcal{C}_V^{\mathrm{bulk}}\left(|\phi\rangle\right) + \dots$$

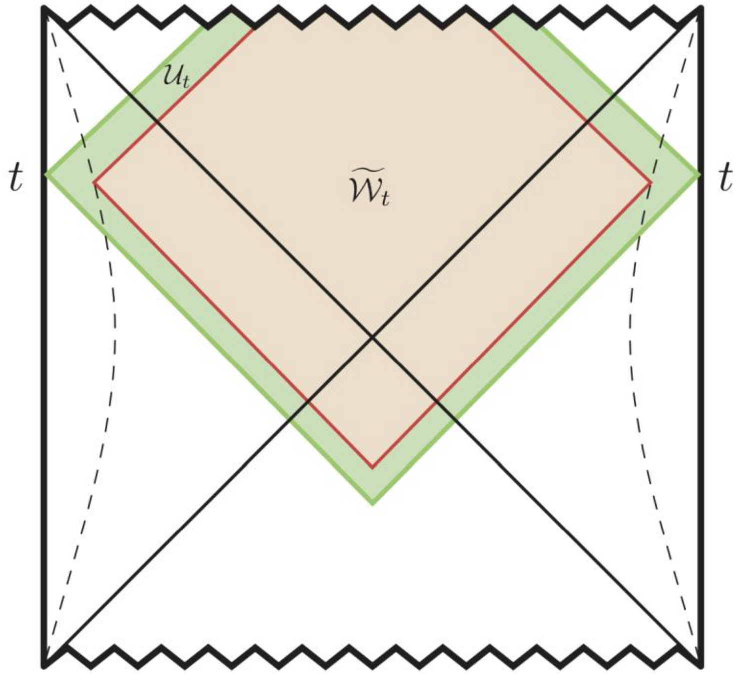
→ Action

$$\mathcal{C}_A\left(|\Psi\rangle\right) = \frac{I(\mathcal{W})}{\hbar} + \delta\mathcal{C}_A\left(|\Psi\rangle\right) + \dots$$

Action complexity

Emparan, AMF, Sasieta, Tomašević (2022)

CA is complicated, because the metric of the spacetime is not spherically symmetric (so the WdW patch will not be a diamond)



- Introduce a bulk IR-cutoff at $r = \infty$ which lies at finite proper distance from the brane.
- Then consider two different spacetime domains
- The WdW patch W lies in between the red causal diamond $\widetilde{W}$, and the green causal diamond U_t
- The $\widetilde{W}$ is anchored to a finite proper distance in the bulk

$$\widetilde{\mathcal{C}}_A(t) = \frac{I(\widetilde{\mathcal{W}}_t)}{\pi}$$

CA late time behavior

Emparan, AMF, Sasieta, Tomašević (2022)

$$C_A(t) = \tilde{C}_A(t) + C_{UV}(t)$$

Since the spacetime is static outside the black hole, it is very natural to expect that C_{UV} is constant at late times, and thence, all the time-dependence of CA is incorporated in the first term

CA late time behavior

Emparan, AMF, Sasieta, Tomašević (2022)

The late time growth of CA is proportional to the mass of the quBTZ but, in the final results, is **independent of the effective coupling** on the brane

$$\frac{dC_A}{dt} = 8M f(G_3 M)$$

All the term in the calculation of the action on the WdW patch depend on the effective coupling (c_3/c_2) but then when we sum, there is a cancellation

P. Braccia, A. L. Cotrone, and E. Tonni (2019)
S. Chapman, D. Ge, and G. Policastro (2018)

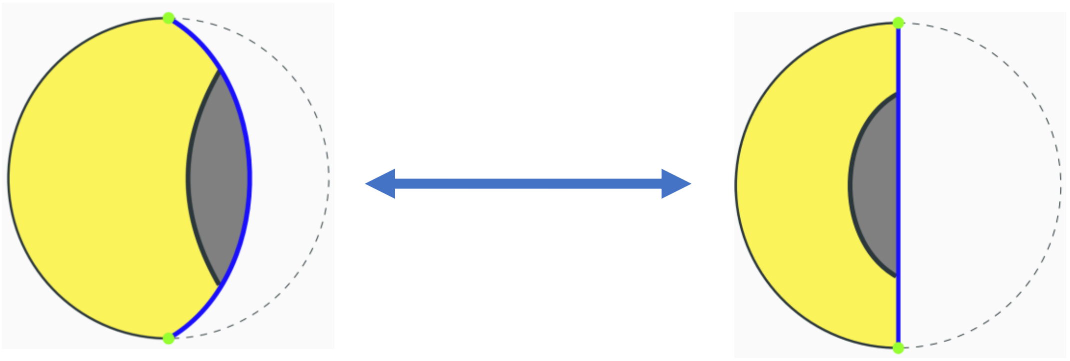
Emparan, AMF, Sasieta, Tomašević (2022)

Results for the Action I

It doesn’t reproduce the complexity rate of the BTZ:

$$\frac{dC_A}{dt} \neq (TS)_{BTZ} \text{ when } \frac{c_3}{c_2} \left(\text{or} \frac{\ell}{\ell_3} \right) \to 0$$

And, since the same parameters control the inverse tension of the brane, it doesn’t distinguish between different bulk geometry:



$\ell \to 0$ $\qquad$ $\ell \to \infty$

$$C_A(|\text{quBTZ}\rangle) \neq C_A^{\text{BTZ}} + \delta C_A$$

$$\mathcal{O}(c_2) \qquad \mathcal{O}(c_3)$$

Results for the Action II

Emparan, AMF, Sasieta, Tomašević (2022)

It doesn't reproduce the complexity rate of the BTZ:

$$\frac{dC_A}{dt} \neq (TS)_{BTZ} \text{ when } \frac{c_3}{c_2}\left(\text{or} \frac{\ell}{\ell_3}\right) \to 0$$

- The discontinuity of AC is (likely) coming from discontinuity in the character of the singularity
- the WdW patch reaches the inner singularity, whose structure changes qualitatively due to quantum backreaction and which gives rise to large quantum contributions to AC.
- Our result point to a more fundamental characteristic of AC: it is a magnitude that allows to probe the black hole interior in a more precise way than VC.

Quantum corrections

Emparan, AMF, Sasieta, Tomašević (2022)

→ Volume

$$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\, L} + \frac{\delta\mathrm{Vol}(\Sigma) + \mathcal{V}(\Sigma)}{G\hbar\, L} + \mathcal{C}_V^{\mathrm{bulk}}\left(|\phi\rangle\right) + \dots$$

→ Action

$$\mathcal{C}_A\left(|\Psi\rangle\right) = \frac{I(\mathcal{W})}{\hbar} + \delta\mathcal{C}_A\left(|\Psi\rangle\right) + \dots$$

These quantities should compute the complexity of the quBTZ including the bulk complexity of the quantum fields in the large limit of the c_3 central charge.

Volume complexity

For CV, the tensor network rationale already hints at a generalization of the schematic form

$$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\, L} + \frac{\delta\mathrm{Vol}(\Sigma) + \mathcal{V}(\Sigma)}{G\hbar\, L} + \mathcal{C}_V^{\mathrm{bulk}}\left(|\phi\rangle\right) + \ldots$$

Semiclassical backreaction of the state of the quantum fields in the bulk

higher derivative corrections

complexity of the state of the quantum fields in the bulk

The expression resembles the quantum-corrected holographic entanglement entropy

Volume complexity

Whit the 4D prescription of the volume, we get

$$C_V(|\mathrm{quBTZ}\rangle) = \frac{\mathrm{Vol}_4(\Sigma)}{4G_4} = \frac{\mathrm{Vol}_3(\sigma)}{4G_3} + C_{q.c.}$$

Leading order: 3D complexity of the (standard) BTZ state.

NOTE: In the UV description this would correspond to the C of the defect dof (it scales as c2) $\mathcal{O}(c_2)$

These quantum corrections admit an **expansion** in the effective coupling on the brane

$$C_{q.c.} = C_{q.c.}^{(0)}\left(1 + \mathcal{O}\left(\frac{c_3}{c_2}\right)\right)$$

Leading term for quantum corrections

The quantum corrections admit a leading term that scales as $O(c_3)$ + some corrections which are subleading in the parameter (c_3/c_2) which is the effective coupling on the brane

The leading quantum correction has 2 terms:

$$C^{(0)}_{q.c.} = C_{\text{UV}} + \frac{\delta \text{Vol}(\sigma)}{G_3 L}$$

This is UV divergent and it happens when the extremal volume slice touches the asymptotic boundary of the CFT3

(We didn't compute it, it requires holographic renormalization with nontrivial asymptotics, but it's constant in time)

not constant in time, depends only on the backreaction (this will change the slope)

Results for the Volume I

Emparan, AMF, Sasieta, Tomašević (2022)

The quantum-corrected **VC formula reproduces the expected computation rate for a semiclassical black hole**!

$$\left.\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}\bar{t}}\right|_{t\gg\beta} = 2M\left(1+\sqrt{2}\,\frac{\mu\ell}{\ell_3}+\ldots\right)$$

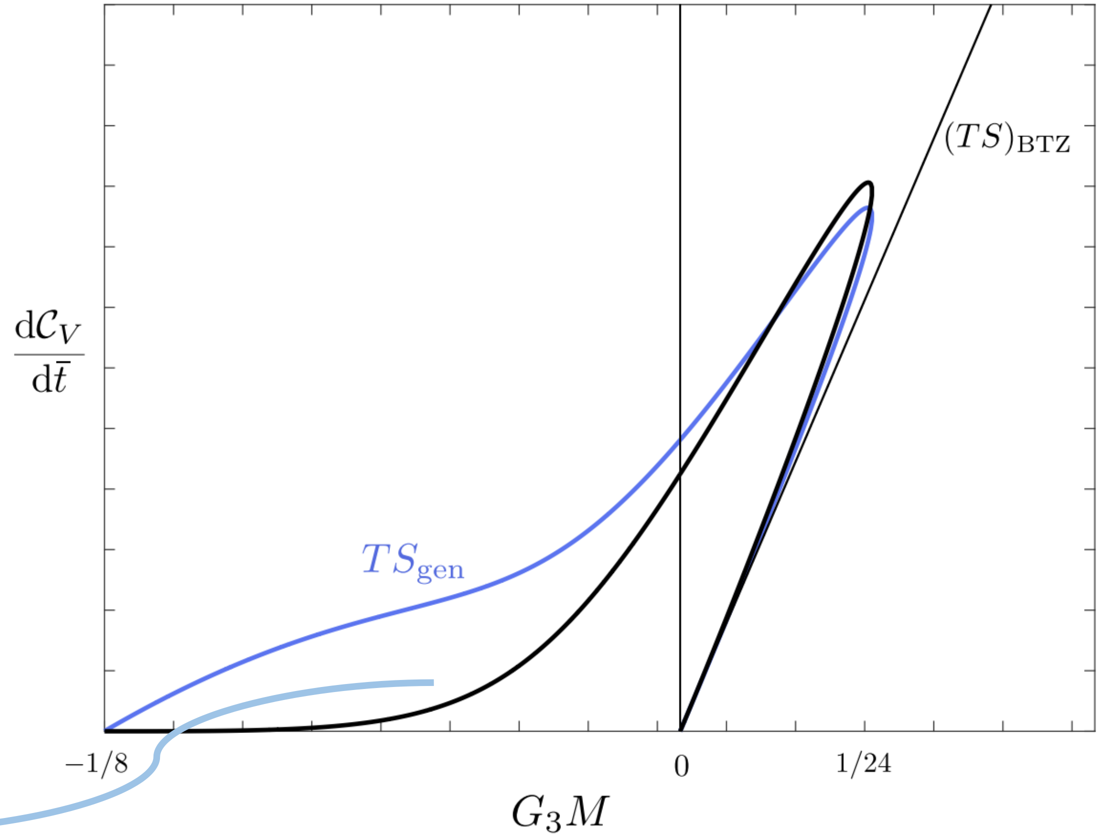
- The quBTZ computes at a faster rate than the respective BTZ

Late-time regime

Emparan, AMF, Sasieta, Tomašević (2022)

$$\left.\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}t}\right|_{t\gg\beta} \sim TS_{\text{gen}} > (TS)_{BTZ}$$

up to an O(1) coefficient that depends on the mass of the black hole.



For the branch of ‘quantum-dressed’ conical defects (M < 0), the rate of computation is a quantum effect suppressed with respect to TSgen.

Thermodynamic Volume

AMF, Pedraza, Svesko, Visser, PRL (2023)

- Treat tension as a variable, like fluid surface tension
- Brane performs work on the bulk BH system

Bulk first law:

$$\boxed{dM = TdS + A_\tau d\tau}$$

$$A_{tau} \equiv \left(\frac{\partial M}{\partial \tau}\right)_S$$ – "regularized brane area"

Treating backreaction small $\nu << 1$:

$$V_3 = V_{BTZ} + 8\pi\ell_3^2\left[\frac{z^3(1+z^2)}{(1+3z^2)^3}\right]\nu + \dots.$$

⋆ Classically $V_{BTZ} \sim S_{BTZ}^2$

⋆ Backreaction modifies volume from geometric volume

Relate the thermodynamic volume to holography, and in particular to the quantum complexity of the boundary state.

Summary and outlook I

- We analyzed different holographic complexity proposals for **black holes that include corrections from bulk quantum fields.**
- The specific setup is the **quantum BTZ black** hole, which encompasses the effects of conformal fields with large central charge in the presence of the black hole, including the backreaction corrections to the BTZ metric

- Exact solution that allowed us to compute the generalized entropy, including the von Neumann entropy of the CFT

Summary and outlook II

- The solution allows computing quantum corrections to Volume-Complexity and Action-Complexity
- The quantum-corrected VC formula correctly reproduces the expected computation rate for a semiclassical black hole
- AC remains still puzzling because it does not give the correct classical limit
- Sensitivity of the AC to the singularity

- Possible relation between CV and the thermodynamic volume?

Summary and outlook III

- Add an explicit EH term to the brane and then study it. However, the problem is that there are no known analytical solutions (one would need to do numeric)
- Discontinuity in CA given by the singularity (Consider the rotation?)
- Try to decouple the CFT_3 bath of this description by adding a second brane in the bulk. This setup is the so-called wedge holography.

Thank you!

Metric quBTZ

The 3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right)dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}}dr^2 + r^2 d\phi^2$$

<u>Classical limits</u> $\mu = 0$

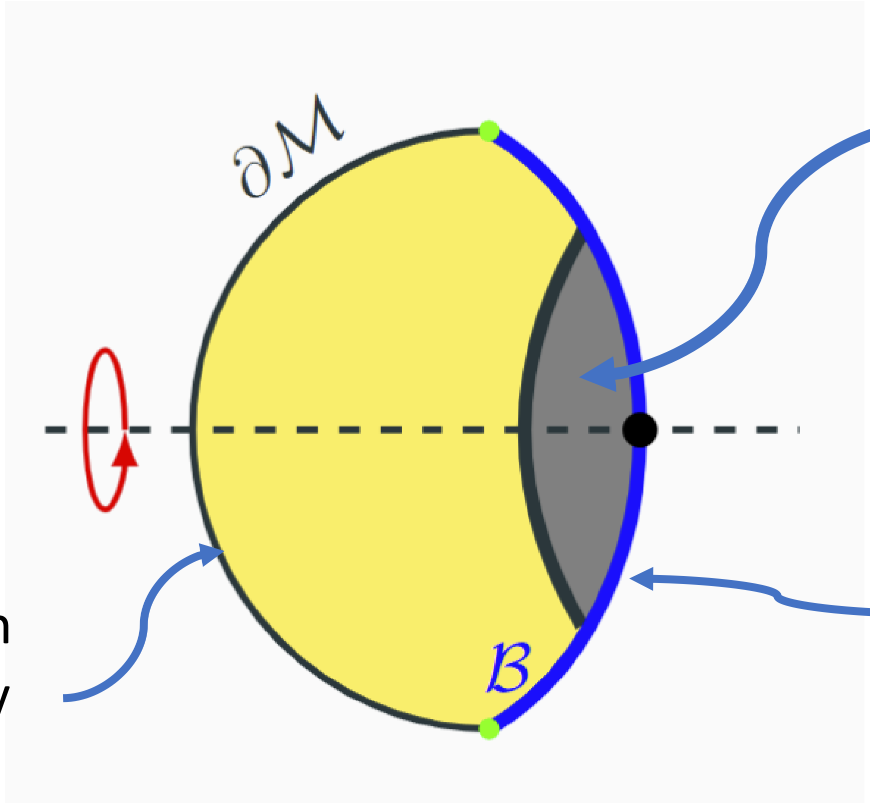
- $\kappa = -1$ BTZ
- $\kappa = +1$ Global or Conical AdS3

$\mu \neq 0$ $\kappa = -1$ quBTZ, different properties of the horizon, has curvature singularity

AdS$_4$ C-metric

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

The BH in the bulk is intersecting the brane



CFT$_3$ on the brane

The 4D theory has an asymptotic boundary where the CFT$_3$ lives

CFT Stress tensor

$$\langle T^a{}_b \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \text{diag}\{1, 1, -2\} + \ldots$$

Comparison with other calculations

Free conformal scalar in BTZ

- Method of images
- Transparent bc's at boundary of AdS_3
- Satisfy KMS, HH at horizon
- Same structure – different $F(M)$

Holographic w/out backreaction:

- Exact BTZ at boundary AdS
- Very different method of construction
- but recovered as limit $\ell \to 0$ of ours