Gluon transverse momentum distributions and saturation

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13th International workshop on Multiple Partonic Interactions at the LHC

University of Antwerp
Particle Physics Group

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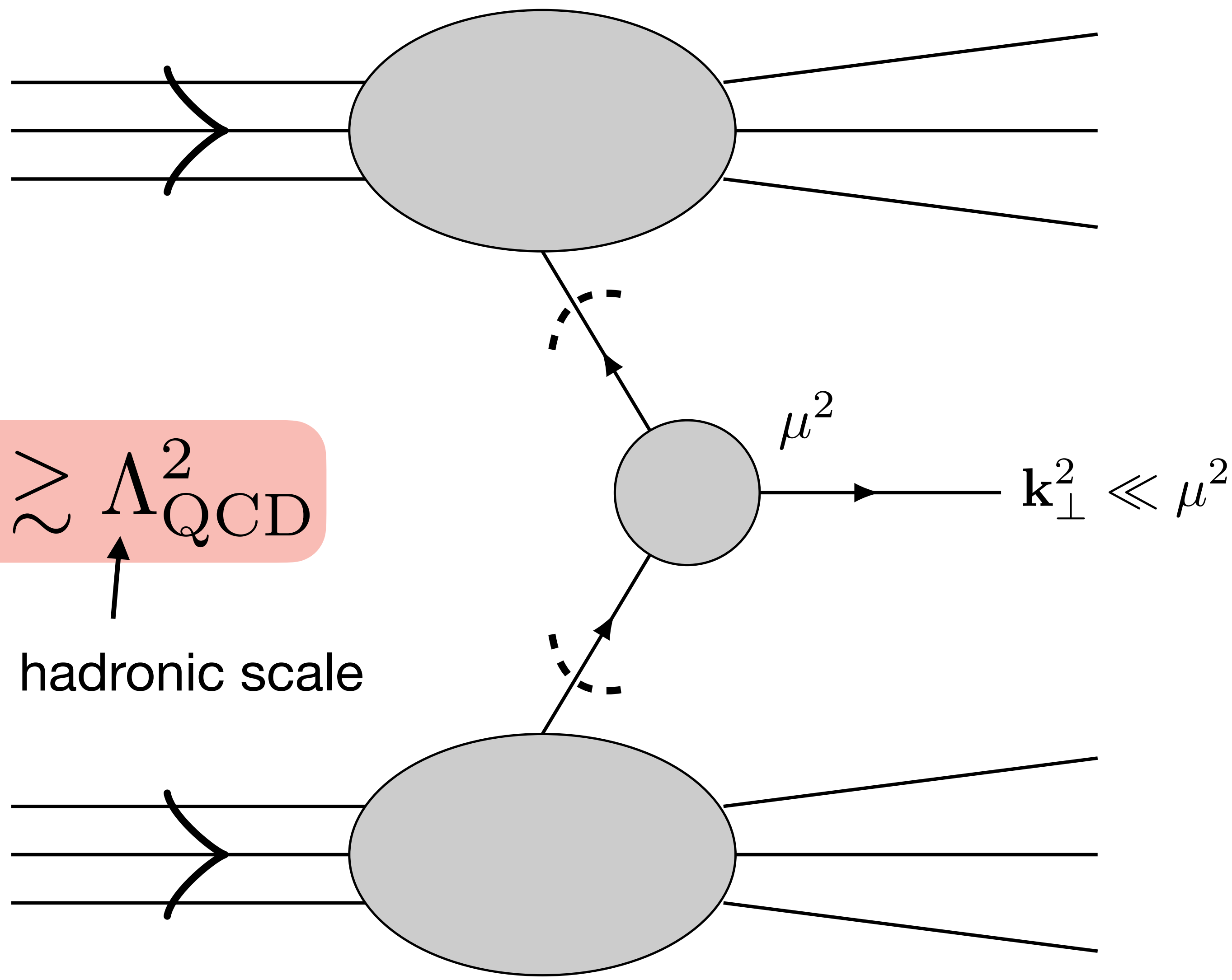
Transverse momentum dependent (TMD) factorisation



$$s \sim \mu^2 \gg \mathbf{k}_\perp^2 \gtrsim \Lambda_{\mathrm{QCD}}^2$$

COM energy² (s) — hadronic scale (Λ_{QCD}^2)

$$\sigma_{\mathrm{TMD}} = \hat{\sigma}(\mu^2) \otimes f(x_a, \mathbf{k}_{a\perp}, \mu^2) \otimes f(x_b, \mathbf{k}_{b\perp}, \mu^2) + \mathcal{O}\left(\frac{\Lambda}{\mu}\right)^n$$

partonic cross section — TMD PDFs — higher twists

Resum both DGLAP logs $\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)$ and Sudakov logs $\ln(\mu^2/\mathbf{k}_\perp^2)$

Collins, Soper, Sterman ('85-'89)
Collins (2011)
Echevarria, Idilbi, Scimemi (2012)

(Gluon) TMDs

Mulders & Rodrigues (2001)
Angelez-Martinez et al. (2015)

PDFs parameterise longitudinal structure of hadron

TMDs parameterise 3D momentum structure + spin correlations

polarisation of gluon

polarisation of proton

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g,\ h_{1T}^{\perp g}$

} → this talk

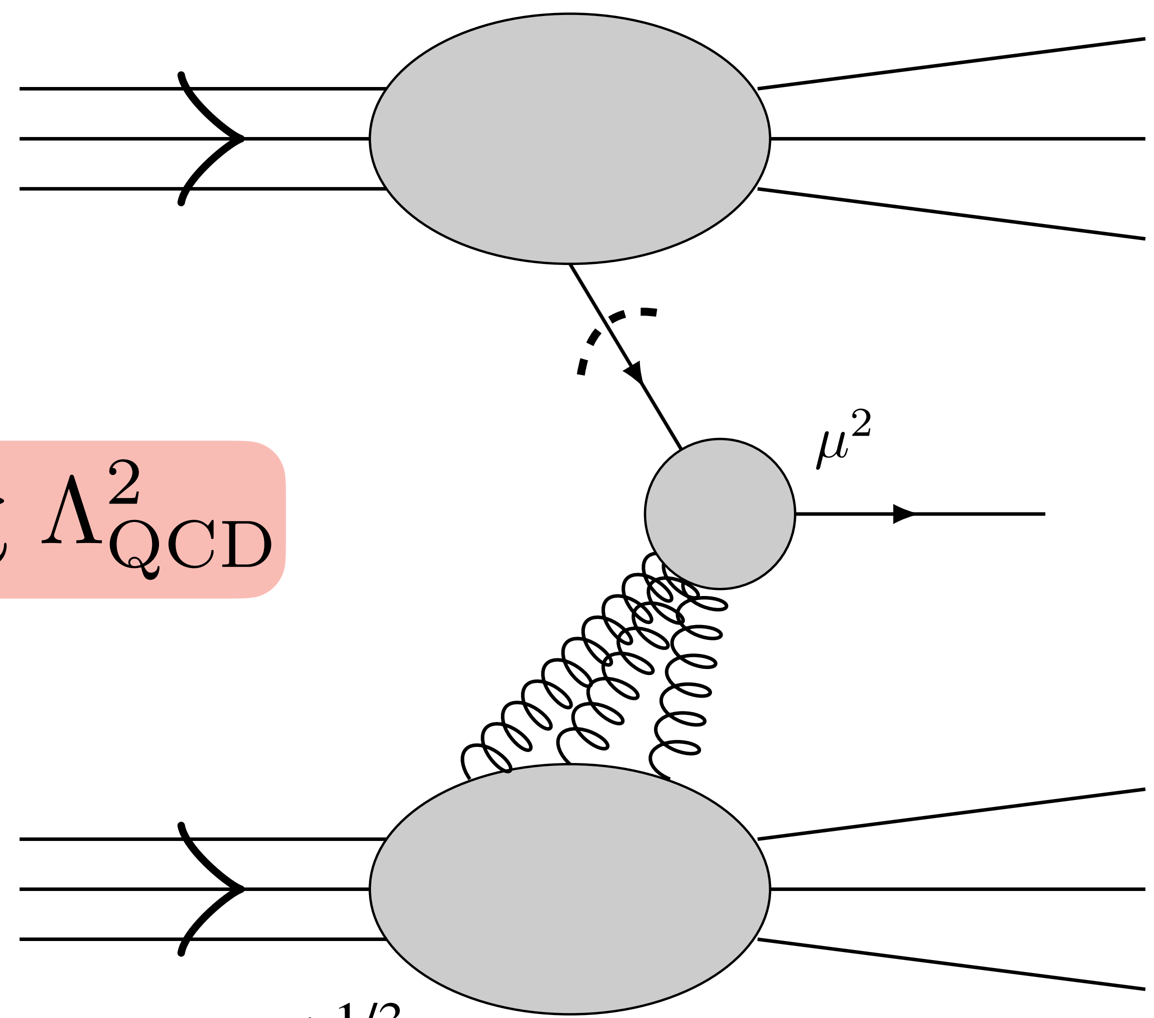
$$\Gamma^{\mu\nu}(x,\mathbf{k}) = \frac{2}{p_A^-}\int \frac{\mathrm{d}\xi^+\mathrm{d}^2\boldsymbol{\xi}}{(2\pi)^3} e^{i\xi^+k^-} e^{-i\boldsymbol{\xi}\mathbf{k}} \langle p_A|\mathrm{Tr}\, F^{-\mu}(0)\mathcal{U}(0,\xi^+,\boldsymbol{\xi})F^{-\nu}(\xi^+,\boldsymbol{\xi})|p_A\rangle$$

(Gluon) TMDs are process-dependent through gauge links / Wilson lines

Only large-k_T tail known = unintegrated gluon distribution (UGD)

Kutak, Sapeta (2012)

Color Glass Condensate (CGC)



$$s \gg \mu^2 \gtrsim Q_s^2 \gtrsim \Lambda^2_{\mathrm{QCD}}$$

Saturation scale $Q_s^2(A,x) \simeq \frac{A^{1/3}}{x^{0.3}}$, e.g. $Q_s^2 \simeq 35\,\mathrm{GeV}^2$ for lead at $x = 10^{-3}$

All-twist framework, all hadronic operators $\sim Q_s^2/\mu^2$ included

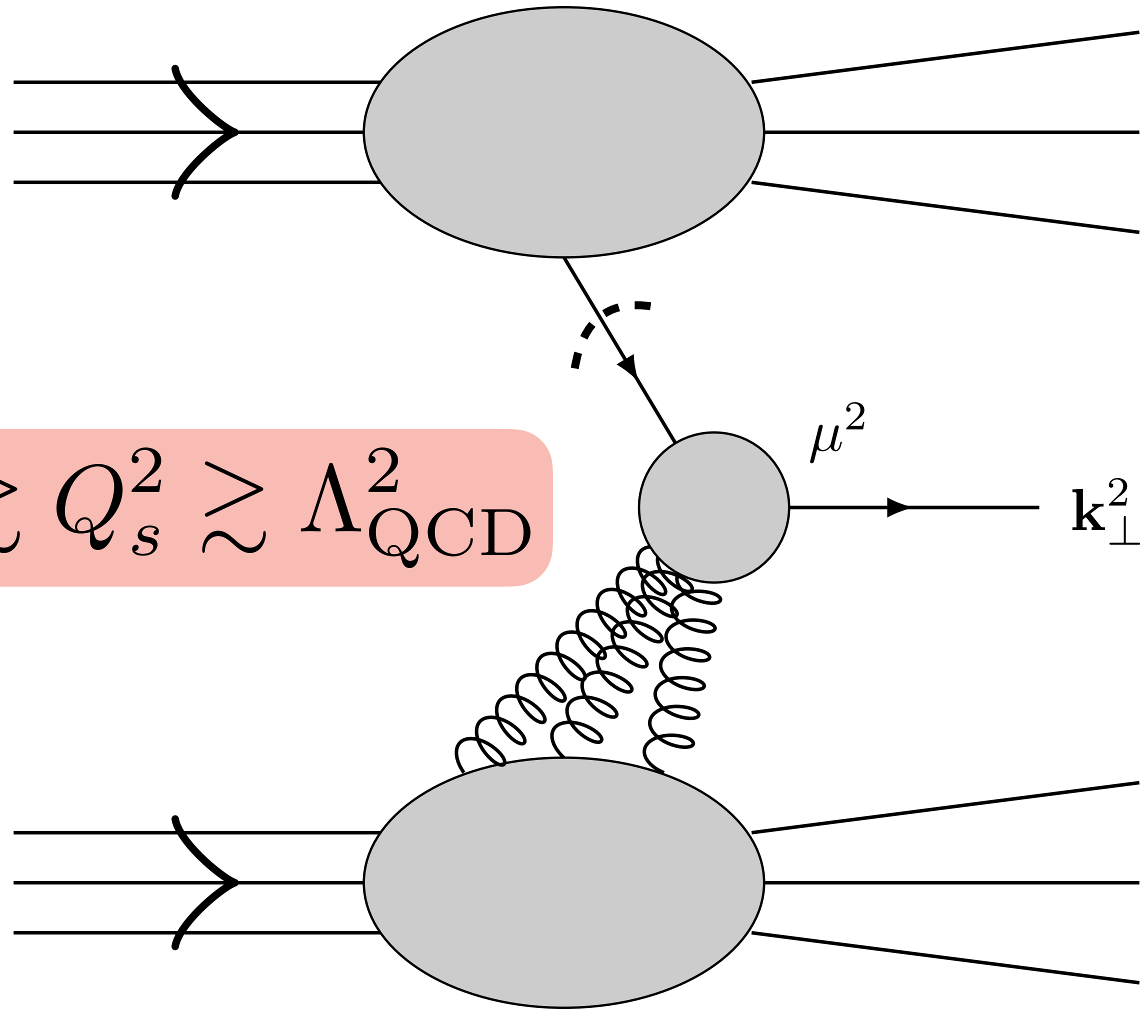
$$\sigma_{\mathrm{CGC}} \neq \hat{\sigma} \otimes \mathcal{G}$$

Resum high-energy logs $\ln(s/\mu^2) \sim \ln(1/x)$

Mueller, McLerran, Venugopalan, Jalilian-Marian, Kovner, Leonidov, Iancu, Weigert (1990-2001)

Gluon TMDs at low x

$s \gg \mu^2 \gg \mathbf{k}_\perp^2 \gtrsim Q_s^2 \gtrsim \Lambda_{\rm QCD}^2$



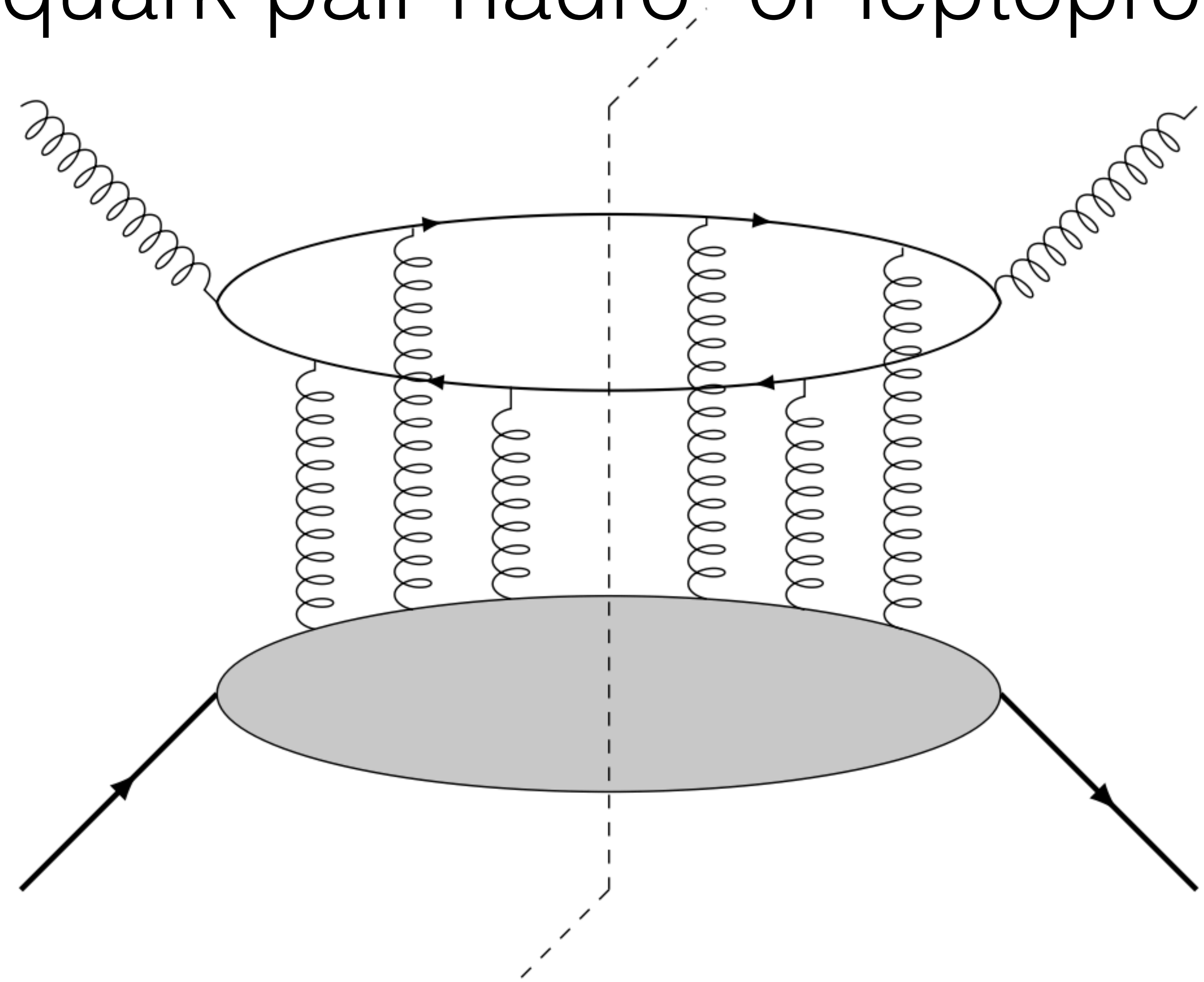
$$\sigma_{\rm CGC,\,TMD} = \hat{\sigma}(\mu^2) \otimes f(x_a, \mu^2) \otimes f(x_b, \mathbf{k}_\perp, \mu^2) + \mathcal{O}(Q_s^2/\mu^2) + \mathcal{O}(\alpha_s^n)$$

TMD PDFs at low x

higher twists

Dominguez, Marquet, Xiao, Yuan (2011)

Heavy-quark pair hadro- or leptoproduction



CGC result:

$$\frac{\mathrm{d}\sigma^{gA\to q\bar{q}X}}{\mathrm{d}^3k_1\mathrm{d}^3k_2} = \frac{\alpha_s}{2}\frac{\delta\left(\left(k_1^+ + k_2^+\right)/p^+ - 1\right)}{p^+}\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{y}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{x}'}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{y}'}{(2\pi)^2} e^{i\mathbf{k}_{1\perp}\cdot\left(\mathbf{y}-\mathbf{y}'\right)} e^{i\left(\mathbf{k}_{2\perp}-\mathbf{p}_\perp\right)\cdot\left(\mathbf{x}-\mathbf{x}'\right)}$$

$$\times \sum_{\lambda s s'} \phi^{\lambda *}_{s's}\left(p^+, k_1^+, \mathbf{x}-\mathbf{y}\right)\phi^{\lambda}_{s's}\left(p^+, k_1^+, \mathbf{x}'-\mathbf{y}'\right)$$

⟶ perturbative $g \to Q\bar{Q}$ splitting

$$\times \Big(C\left(\mathbf{x},\mathbf{y},\mathbf{x}',\mathbf{y}'\right) + D_A\left(z\mathbf{x} + (1-z)\,\mathbf{y}, z\mathbf{x}' + (1-z)\,\mathbf{y}'\right)$$

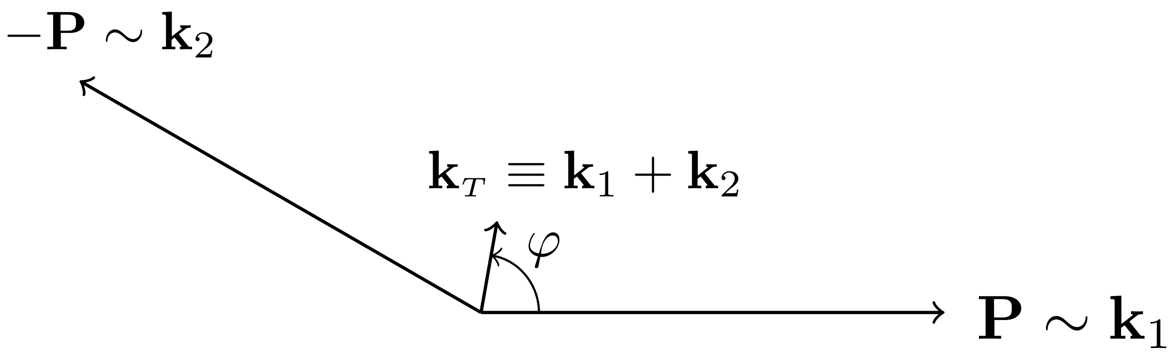
$$- S^{(3)}\left(\mathbf{x}, z\mathbf{x}' + (1-z)\,\mathbf{y}', \mathbf{y}\right) - S^{(3)}\left(\mathbf{y}', z\mathbf{x} + (1-z)\,\mathbf{y}, \mathbf{x}'\right)\Big)$$

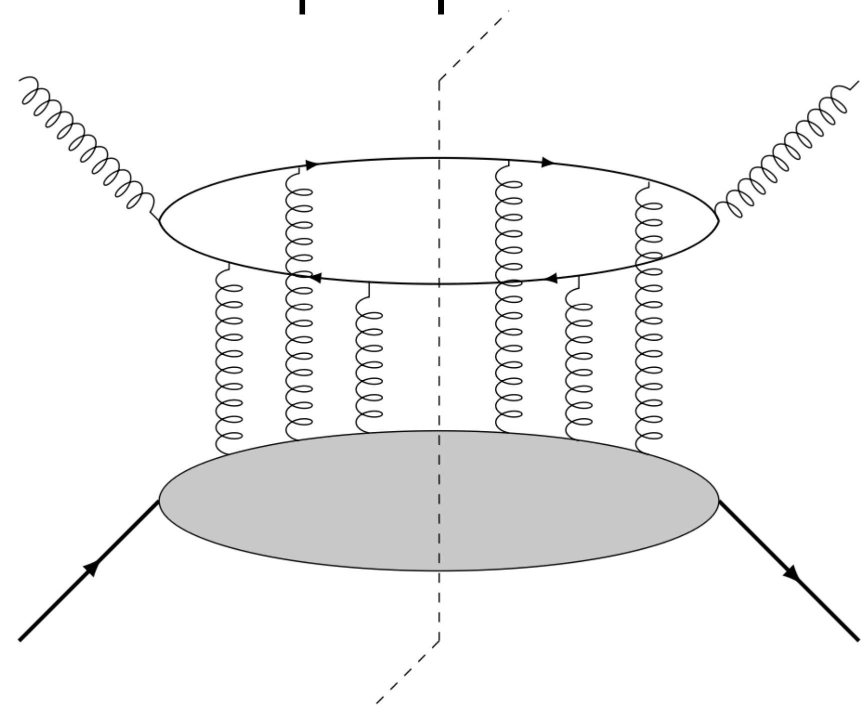
⟶ Wilson lines encode multiple scattering

Marquet, Roiesnel, PT (2018)

Heavy-quark pair hadro- or leptoproduction

Back-to-back limit





Recover low-x TMD factorisation:

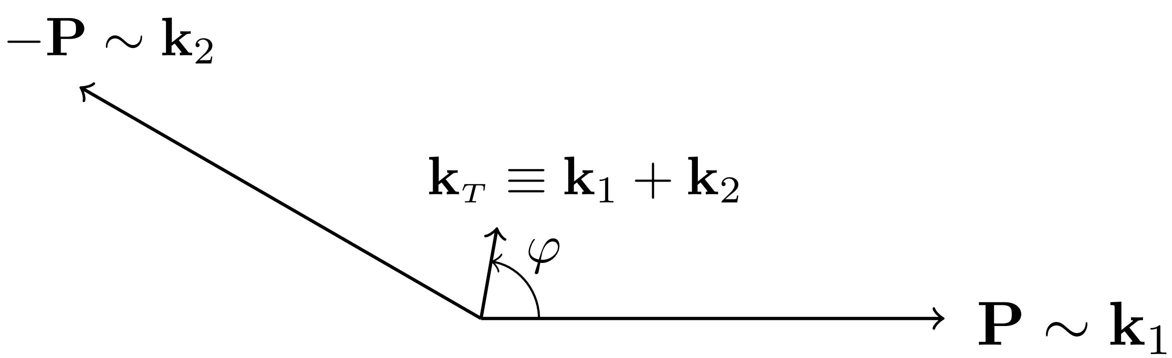
$$s \gg \mathbf{P}^2 \gg \mathbf{k}_T^2 \gtrsim Q_s^2$$

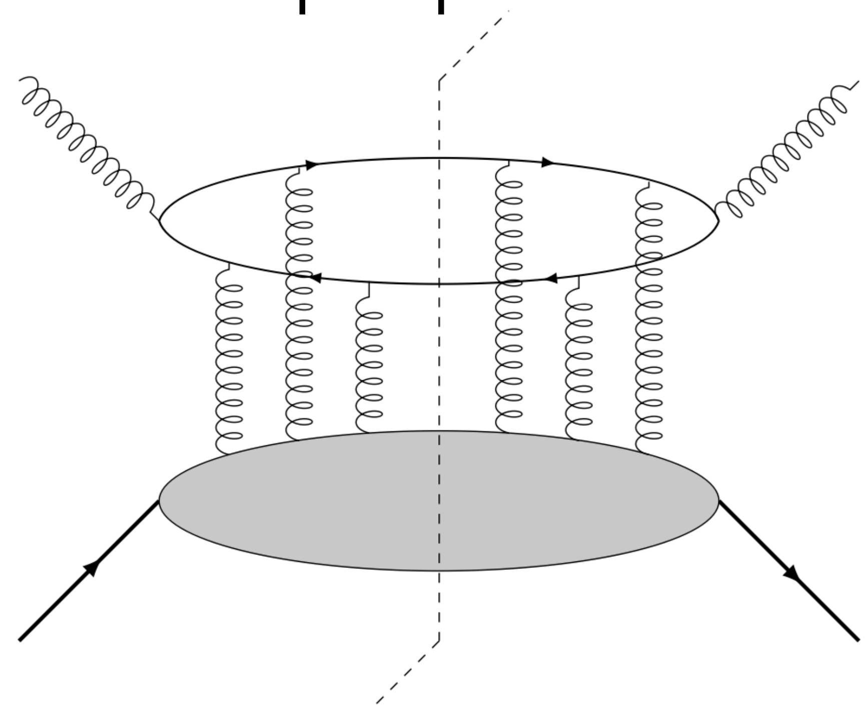
$$\left.\frac{\mathrm{d}\sigma_{gA}}{\mathrm{d}z\mathrm{d}^2\mathbf{k}_1\mathrm{d}^2\mathbf{k}_2}\right|_{\mathrm{TMD}} = \frac{\alpha_s^2}{2C_F}\frac{1}{\left(\mathbf{P}^2+m^2\right)^2}\left\{\left(\frac{z^2+\bar{z}^2}{2}+\frac{2z\bar{z}m^2\mathbf{P}^2}{\left(\mathbf{P}^2+m^2\right)^2}\right)\right.$$
$$\times\left(\mathcal{F}_{gg}^{(1)}\left(x_A,\mathbf{k}\right)-2z\bar{z}\mathcal{F}_{\mathrm{ADP}}\left(x_A,\mathbf{k}\right)-\frac{1}{N_c^2}\mathcal{F}_{\mathrm{WW}}\left(x_A,\mathbf{k}\right)\right)$$
$$+\frac{2z\bar{z}m^2\mathbf{P}^2}{\left(\mathbf{P}^2+m^2\right)^2}\cos\left(2\varphi\right)$$
$$\left.\times\left(\mathcal{H}_{gg}^{(1)}\left(x_A,\mathbf{k}\right)-2z\bar{z}\mathcal{F}_{\mathrm{ADP}}\left(x_A,\mathbf{k}\right)-\frac{1}{N_c^2}\mathcal{H}_{\mathrm{WW}}\left(x_A,\mathbf{k}\right)\right)\right\}$$

Marquet, Roiesnel, PT (2018)

Heavy-quark pair hadro- or leptoproduction

Back-to-back limit





Recover low-x TMD factorisation:

$$s \gg \mathbf{P}^2 \gg k_T \gtrsim Q_s^2$$

$$\left.\frac{d\sigma_{gA}}{dz\,d^2\mathbf{k}_1 d^2\mathbf{k}_2}\right|_{\mathrm{TMD}} = \frac{\alpha_s^2}{2C_F}\frac{1}{\left(\mathbf{P}^2+m^2\right)^2}\left\{\left(\frac{z^2+\bar{z}^2}{2}+\frac{2z\bar{z}m^2\mathbf{P}^2}{\left(\mathbf{P}^2+m^2\right)^2}\right)\right.$$
$$\times\left(\mathcal{F}_{gg}^{(1)}\left(x_A,\mathbf{k}\right) - 2z\bar{z}\mathcal{F}_{\mathrm{ADP}}\left(x_A,\mathbf{k}\right) - \frac{1}{N_c^2}\mathcal{F}_{\mathrm{WW}}\left(x_A,\mathbf{k}\right)\right)$$
$$+\frac{2z\bar{z}m^2\mathbf{P}^2}{\left(\mathbf{P}^2+m^2\right)^2}\cos\left(2\varphi\right)$$
$$\left.\times\left(\mathcal{H}_{gg}^{(1)}\left(x_A,\mathbf{k}\right) - 2z\bar{z}\mathcal{F}_{\mathrm{ADP}}\left(x_A,\mathbf{k}\right) - \frac{1}{N_c^2}\mathcal{H}_{\mathrm{WW}}\left(x_A,\mathbf{k}\right)\right)\right\}$$

Hard parts depend on hard scale $\mathbf{P}$

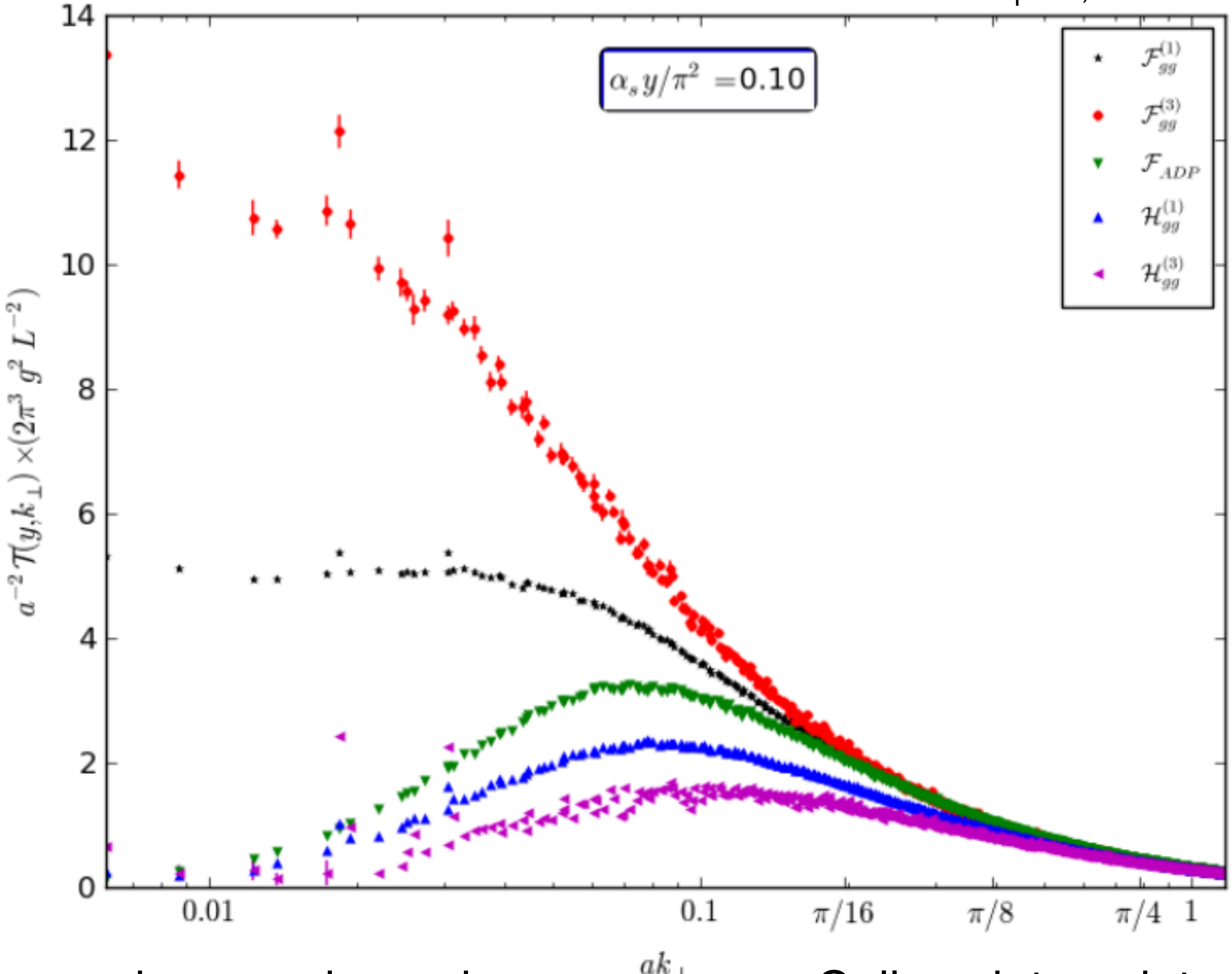
Unpolarised TMDs

Linearly polarised TMDs

Marquet, Roiesnel, PT (2018)

Model + nonlinear high-energy evolution of low x gluon TMDs

Marquet, Roiesnel, PT (2018)



Polarisation and gauge dependence critical when $k_t \lesssim Q_s$

Collaps into unintegrated gluon distribution when $k_t \gg Q_s$

Low-x improved TMD factorisation (ITMD)

Extend applicability of TMD factorisation af low x

$$\sigma_{\rm TMD} \overset{\mu^2 \gg \mathbf{k}_T^2}{=} \hat{\sigma}(\mu^2) \otimes f(x_a, \mathbf{k}_{a\perp}, \mu^2) \otimes f(x_b, \mathbf{k}_{b\perp}, \mu^2) + \mathcal{O}\Big(\frac{\Lambda}{\mu}\Big)$$

Put target gluons off-shell

$$\sigma_{\rm ITMD} = \hat{\sigma}(\mathbf{k}_\perp, \mu^2) \otimes f(x_a, \mu^2) \otimes f(x_b, \mathbf{k}_\perp, \mu^2) + \mathcal{O}(\frac{\Lambda}{\mu}, \frac{Q_s}{\mu})$$

Neglect higher twists

$$\sigma_{\rm CGC} \neq \hat{\sigma} \otimes \mathcal{G}$$

See Piotr Kotko's talk on Friday

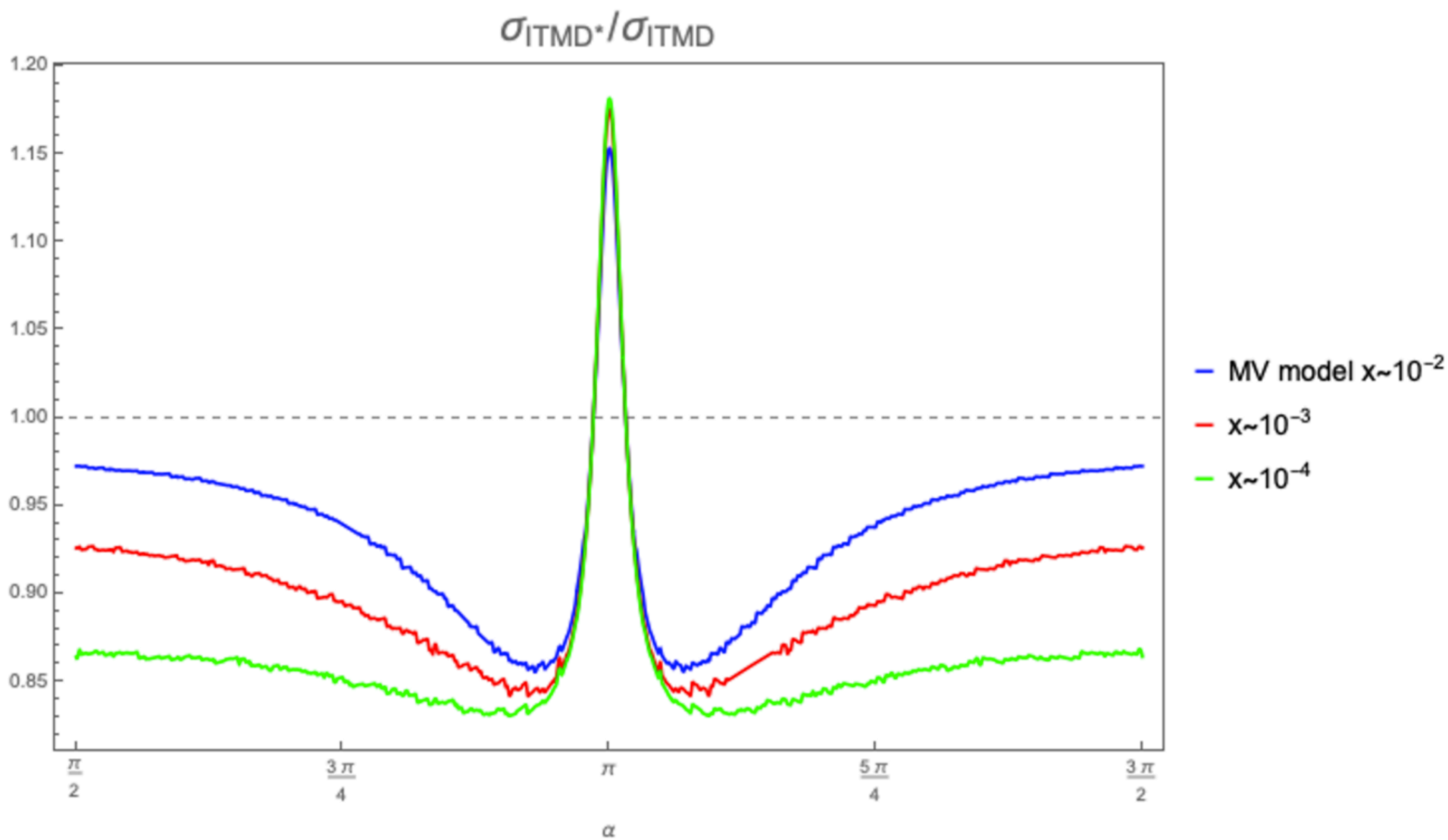
Catani, Hautmann (1994)
Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren (2015)
Altinoluk, Boussarie, Kotko (2019)

Heavy-quark pair hadro- or leptoproduction in ITMD

$$\frac{\mathrm{d}\sigma_{gA}}{\mathrm{d}z\mathrm{d}^2\mathbf{k}_1\mathrm{d}^2\mathbf{k}_2} = \frac{\alpha_s^2}{4C_F}\Bigg[H_{\mathrm{ADP}}^{ns}\mathcal{F}_{\mathrm{ADP}}(x_A,\mathbf{k})+H_1^{ns}\left(\mathcal{F}_{gg}^{(1)}(x_A,\mathbf{k})-\frac{1}{N_c^2}\mathcal{F}_{\mathrm{WW}}(x_A,\mathbf{k})\right)$$

$$+H_1^h\left(\mathcal{H}_{gg}^{(1)}(x_A,\mathbf{k})-\mathcal{F}_{gg}^{(1)}(x_A,\mathbf{k})-\frac{1}{N_c^2}\mathcal{H}_{\mathrm{WW}}(x_A,\mathbf{k})+\frac{1}{N_c^2}\mathcal{F}_{\mathrm{WW}}(x_A,\mathbf{k})\right)\Bigg]$$

Importance of linearly polarised gluons:

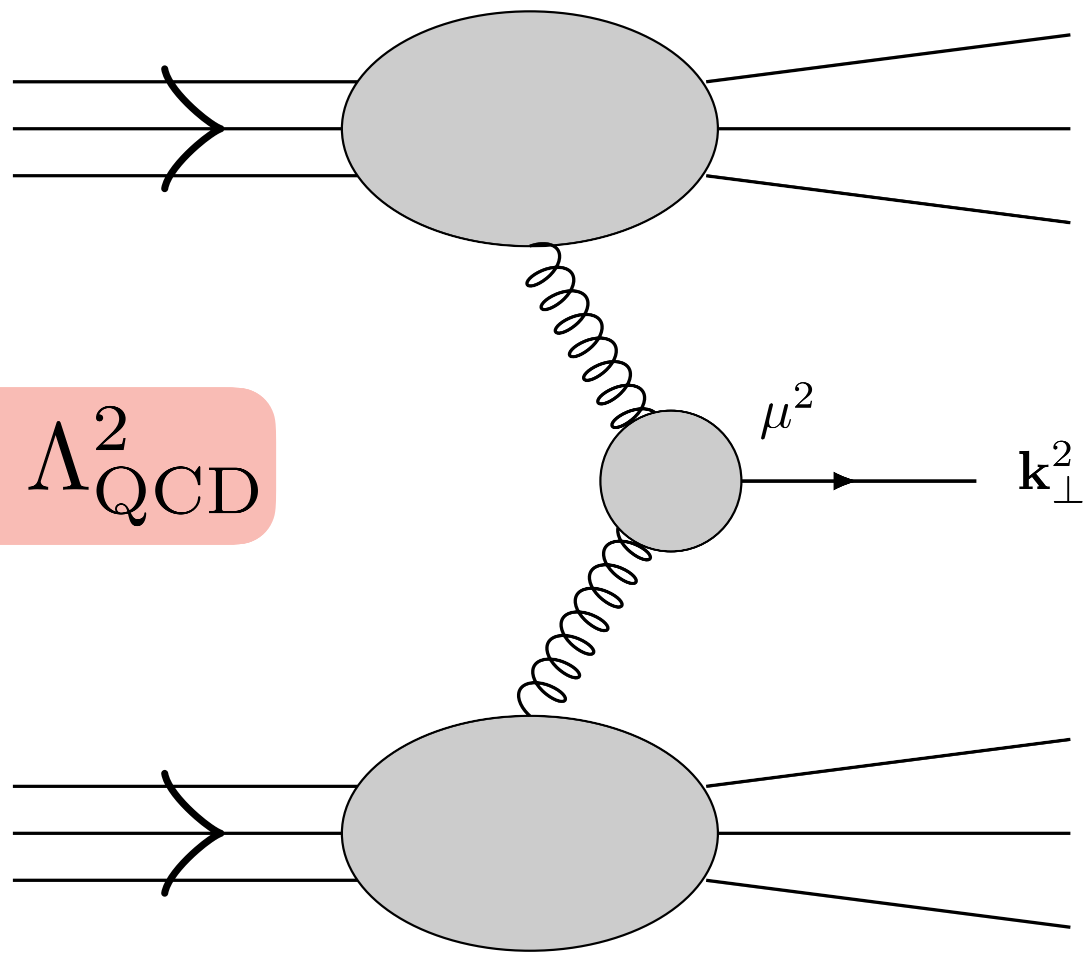


ITMD*: Bury, van Hameren, Kotko, Kutak (2020)

Altinoluk, Marquet, Taels (2021)

Combining low-x and Sudakov resummation

$$s \gg \mu^2 \gg \mathbf{k}_\perp^2 \gtrsim \Lambda_{\rm QCD}^2$$



Simultaneous resummation of $\ln(1/x)$ and $\ln(\mu^2/\mathbf{k}_\perp^2)$?

Many approaches and implementations:

See Maxim Nefedov's talk

SW: Balitsky, Tarasov (2015)

HEF: Deak, Hautmann, Jung, Kutak, van Hameren, Sapeta, Hentschinski (2016-2021)

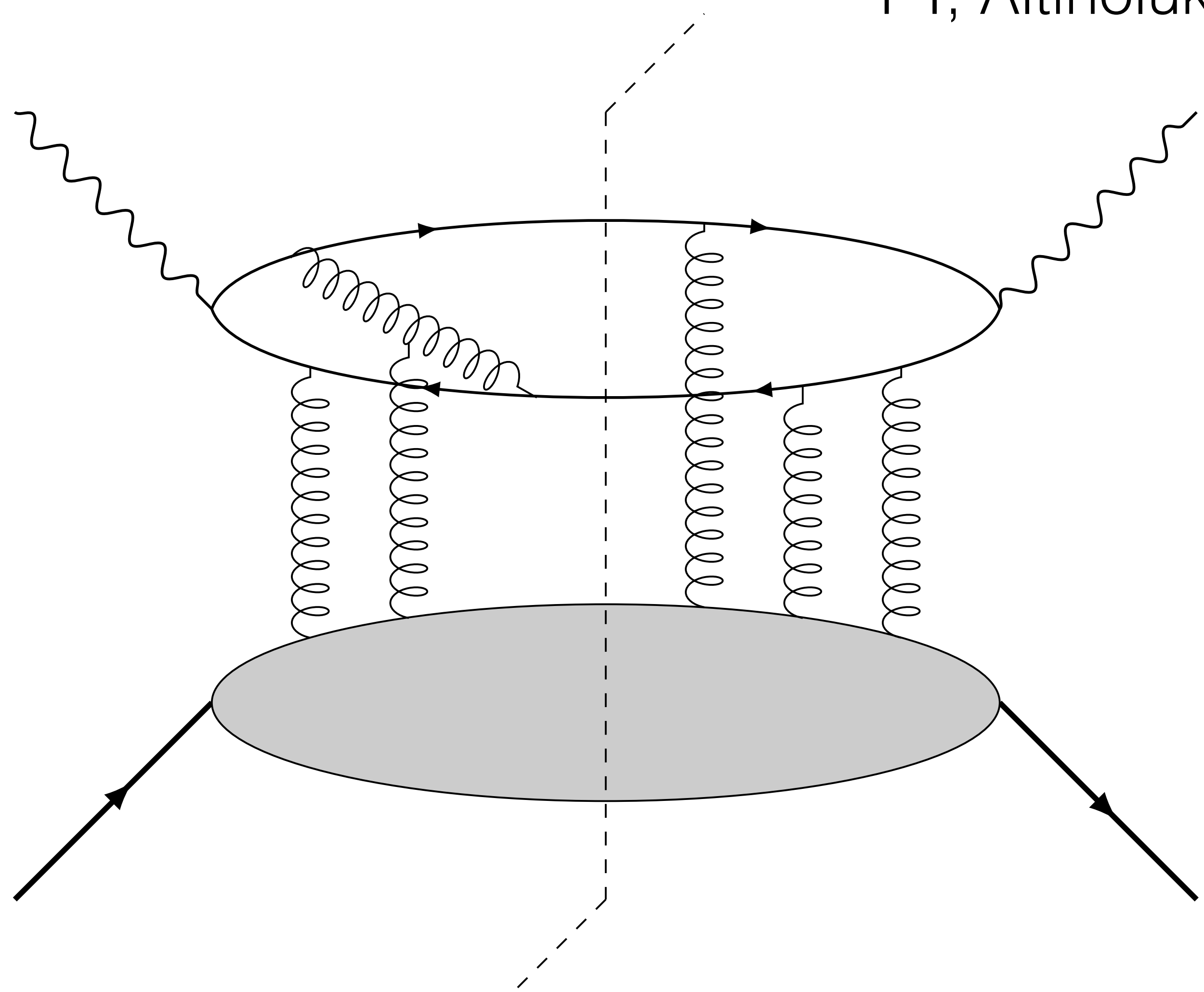
BFKL: Nefedov (2021)

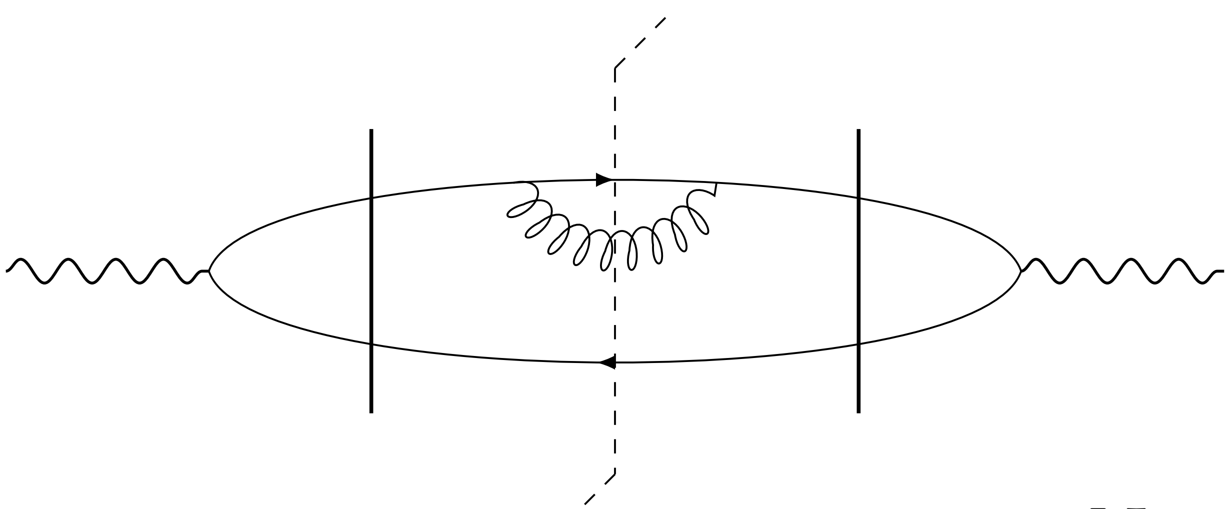
PB: Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek (2022)

CGC: Mueller, Xiao, Yuan (2011); Xiao, Yuan, Zhou (2017); Stasto, Wei, Xiao, Yuan (2018); PT, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022)

Dijet photoproduction at NLO in the CGC

PT, Altinoluk, Beuf, Marquet (2022)





Final-state radiation leads to large Sudakov logarithms which drive TMD evolution:

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{TMD}} = \mathrm{d}\sigma_{\mathrm{LO}}^{\mathrm{TMD}} \times -\frac{\alpha_s N_c}{4\pi} \ln\left(\frac{\mathbf{P}_\perp^2 (\mathbf{b}-\mathbf{b}')^2}{c_0^2}\right)^2 + \text{fact. breaking}$$

See Paul Caucal's talk on Friday

Outlook

Contribution of linearly polarised gluon distribution important depending on process

Rigorous inclusion of Sudakov resummation becomes closer provided kinematically improved high-energy evolution

k_t -factorisation needs extension when $\mu^2 \gg \mathbf{k}_\perp^2 \sim Q_s^2$
… but then TMD factorisation is seemingly broken beyond LO

Thanks for your attention !