Transverse momentum broadening from NLL BFKL to all orders in pQCD

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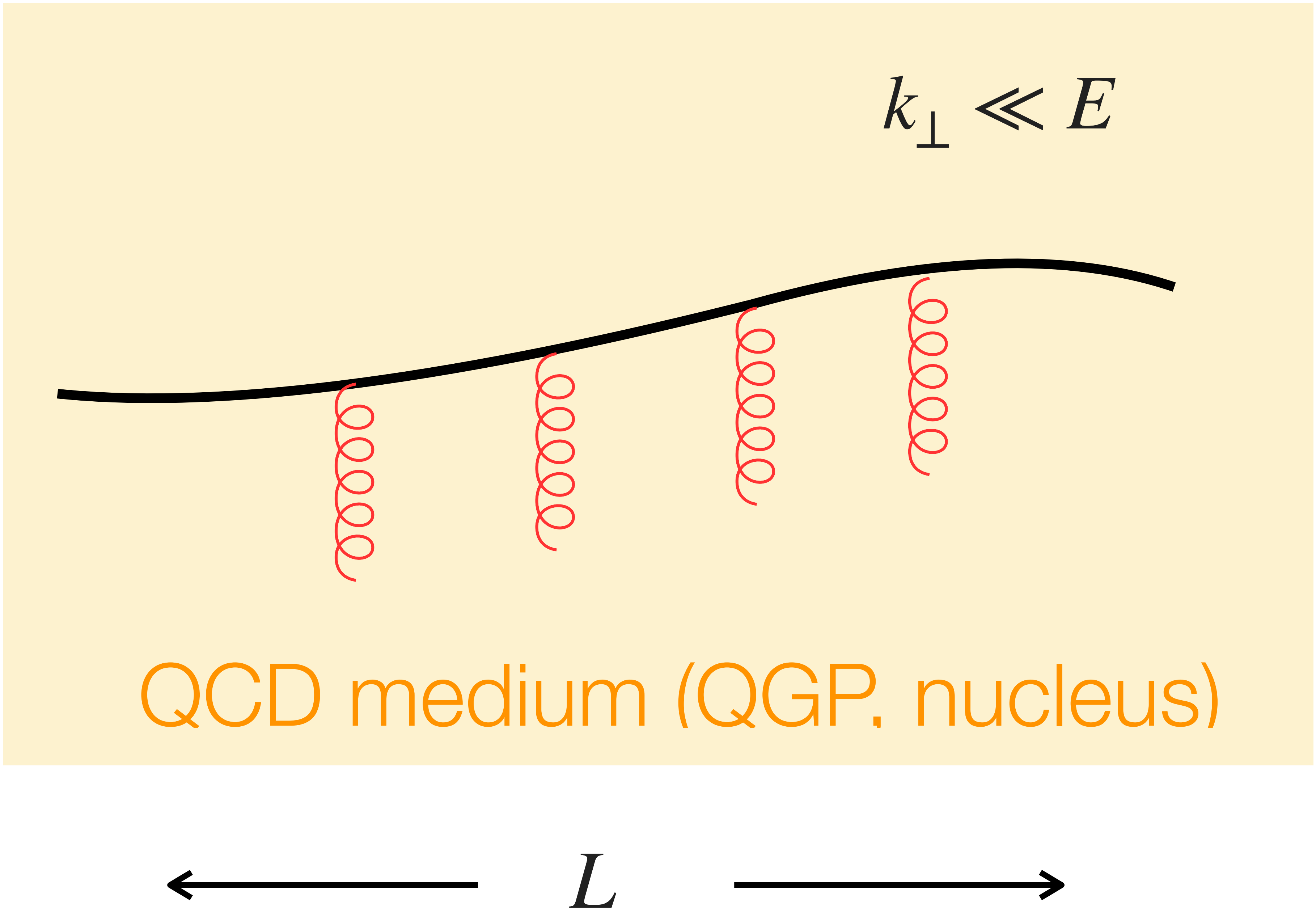
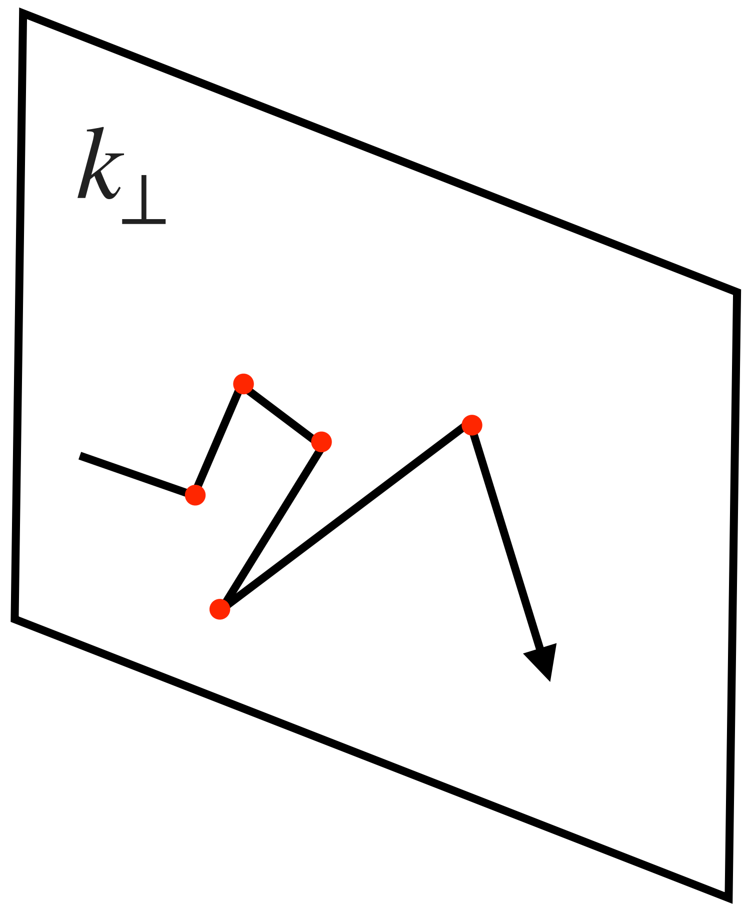
Transverse momentum broadening (TMB)

- High energy partons experience random kicks in hot or cold nuclear matter that cause their transverse momentum to increase over time





Transverse plane

- Normal diffusion scaling at LO (from 2 to 2 matrix element)

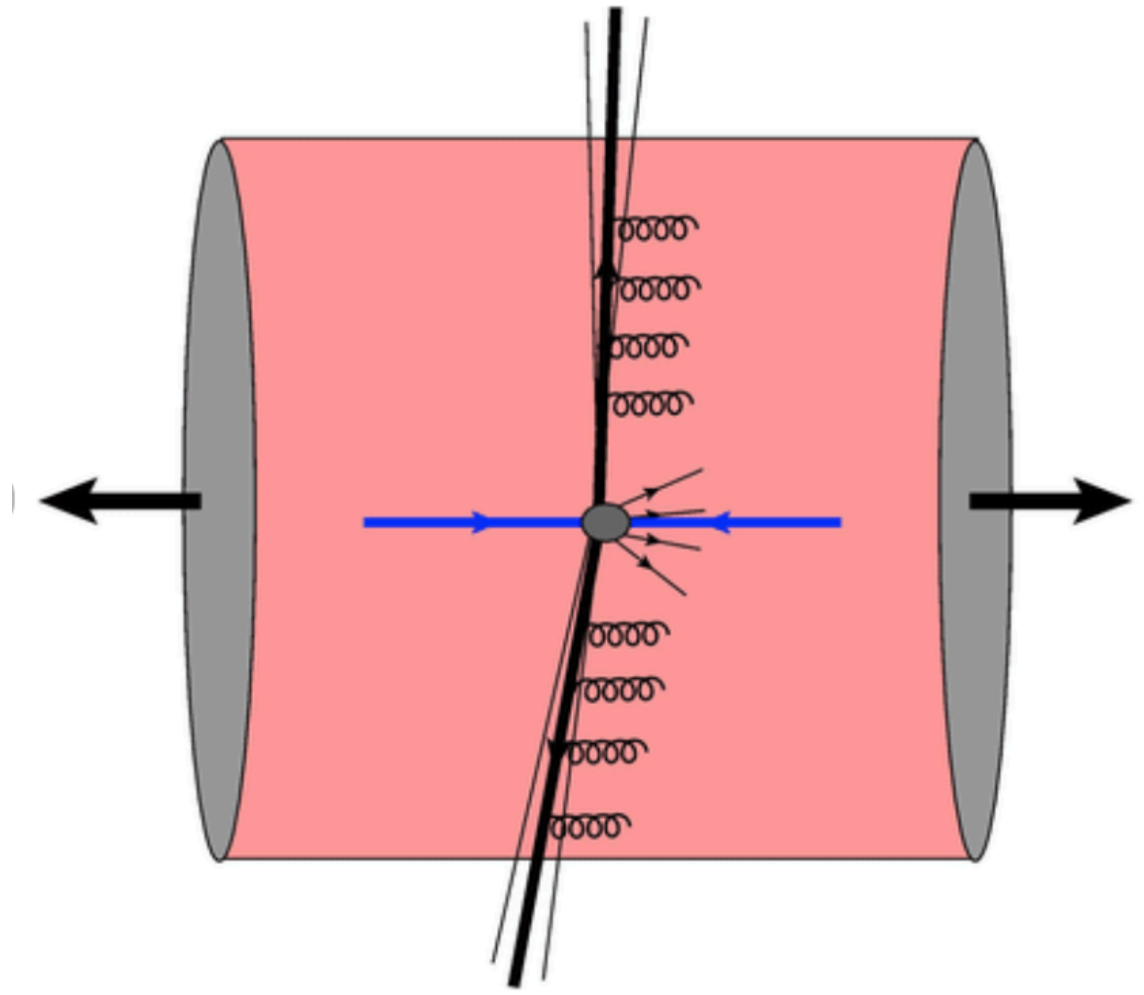
$$\langle k_\perp^2 \rangle_{\rm typ} \propto \hat{q}\, t$$

Q: What are the effects of quantum corrections on transverse momentum broadening?

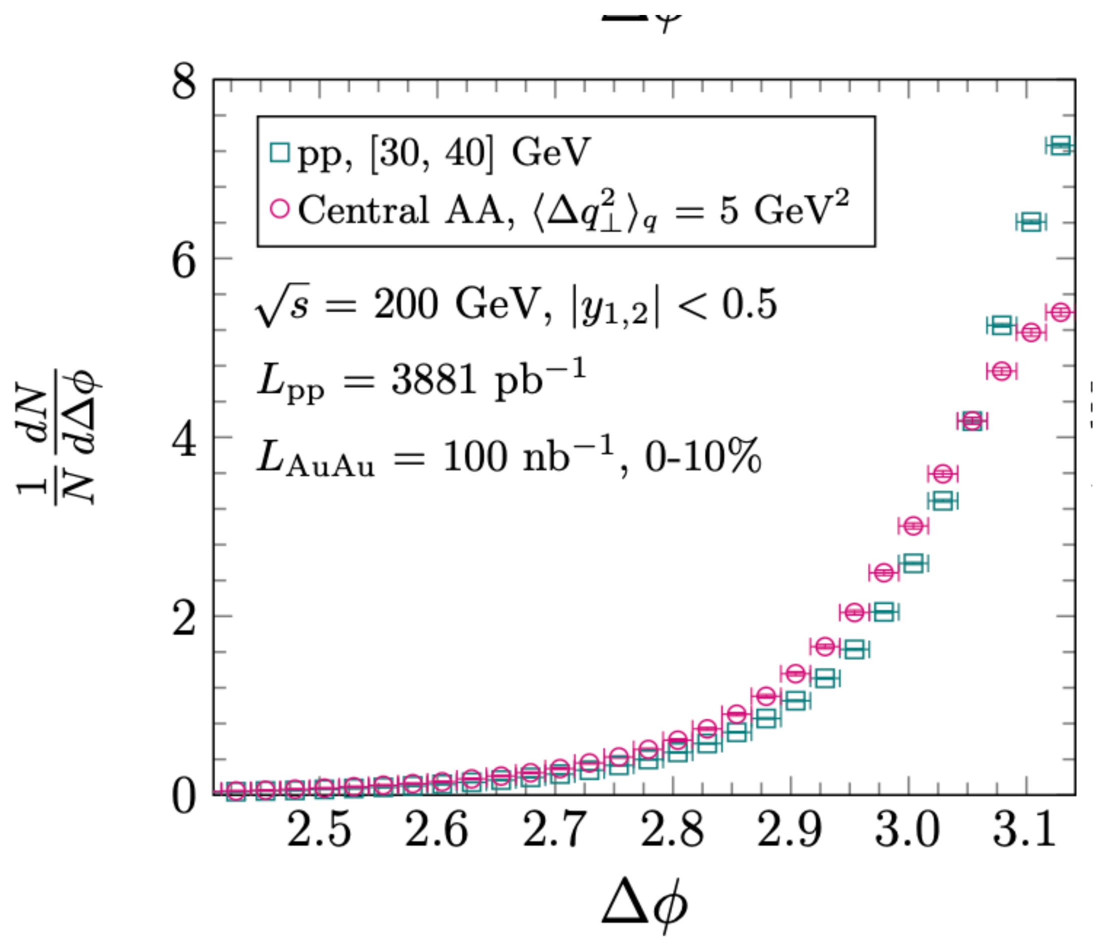
Transverse momentum broadening (TMB)

- Probe the QGP in Heavy Ion Collisions: dijet azimuthal de-correlation, jet quenching

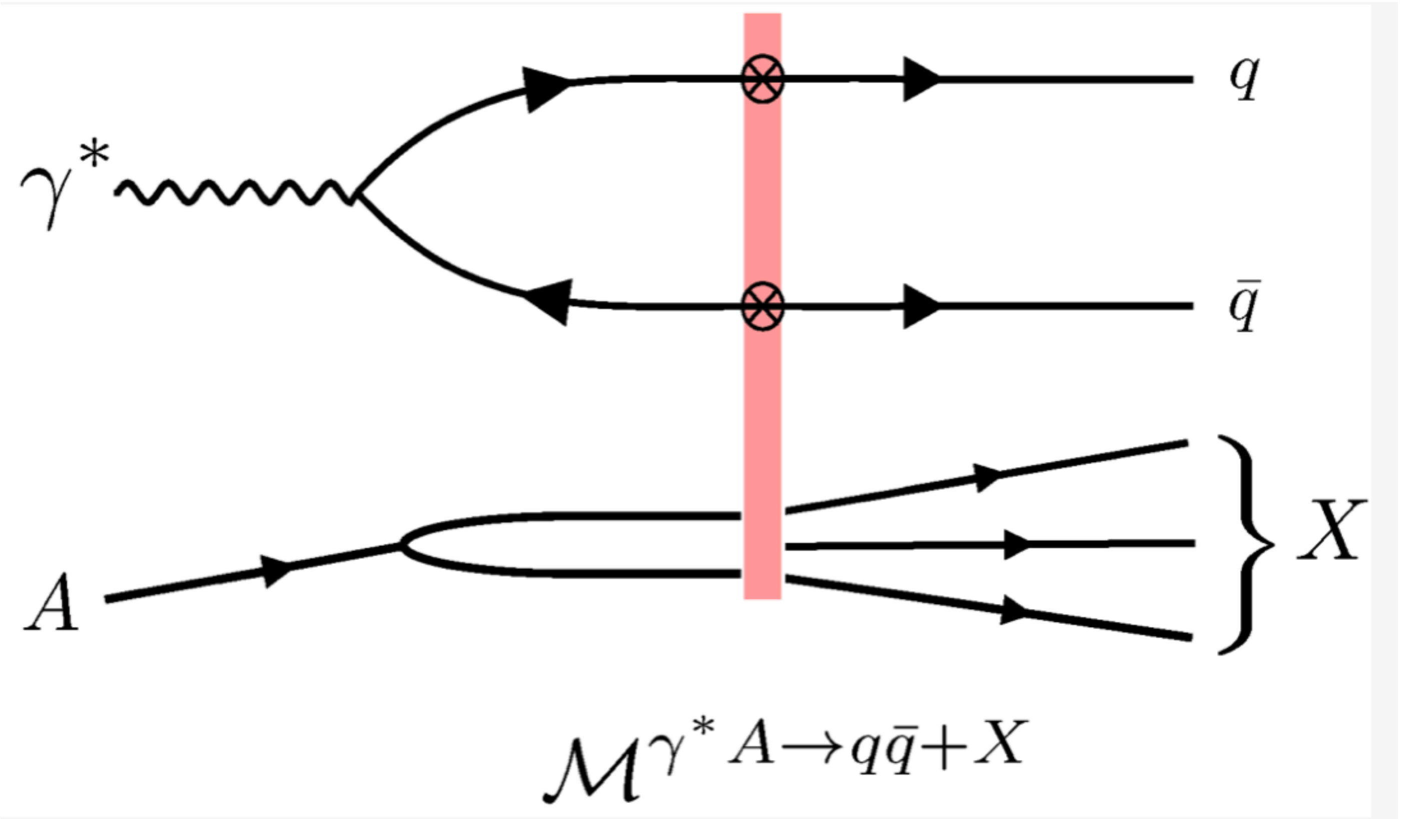
Mueller, Wu, Xiao, Yuan (2016)



Jia, Xiao, Yuan (2019)



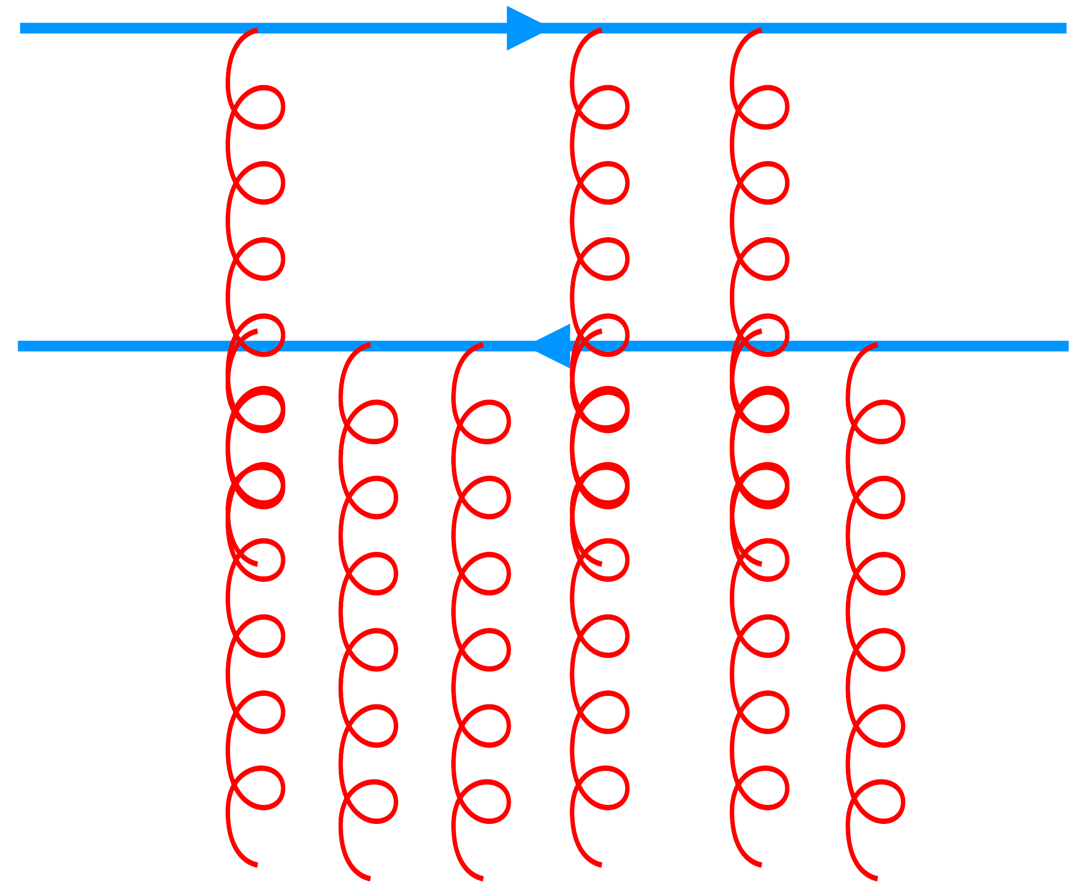
- Probe cold nuclear matter: SIDIS and forward dijet production in eA



TMB and Dipole S-matrix

- TMB is related to the scattering of color dipole off a strong background field A^μ

$$\mathcal{P}(\boldsymbol{k}_\perp) \equiv \frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}_\perp} = \int \mathrm{d}^2\boldsymbol{x}_\perp\, \mathrm{e}^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\, S(\boldsymbol{x}_\perp)\,.$$



- Dipole S-matrix

$$S(\boldsymbol{x}_\perp) \equiv \frac{1}{N_c}\mathrm{Tr}\langle U(\boldsymbol{x}_\perp)U^\dagger(\boldsymbol{0})\rangle\,,$$

Path ordered exponential:

$$U(\boldsymbol{x}_\perp) \equiv \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty}\mathrm{d}x^+ t^a A_a^-(x^+,\boldsymbol{x}_\perp)\right]$$

Relation to $\hat{q}$

$$S(x_\perp) = \mathrm{e}^{-\frac{1}{4}x_\perp^2 L\,\hat{q}(1/x_\perp, L)}$$

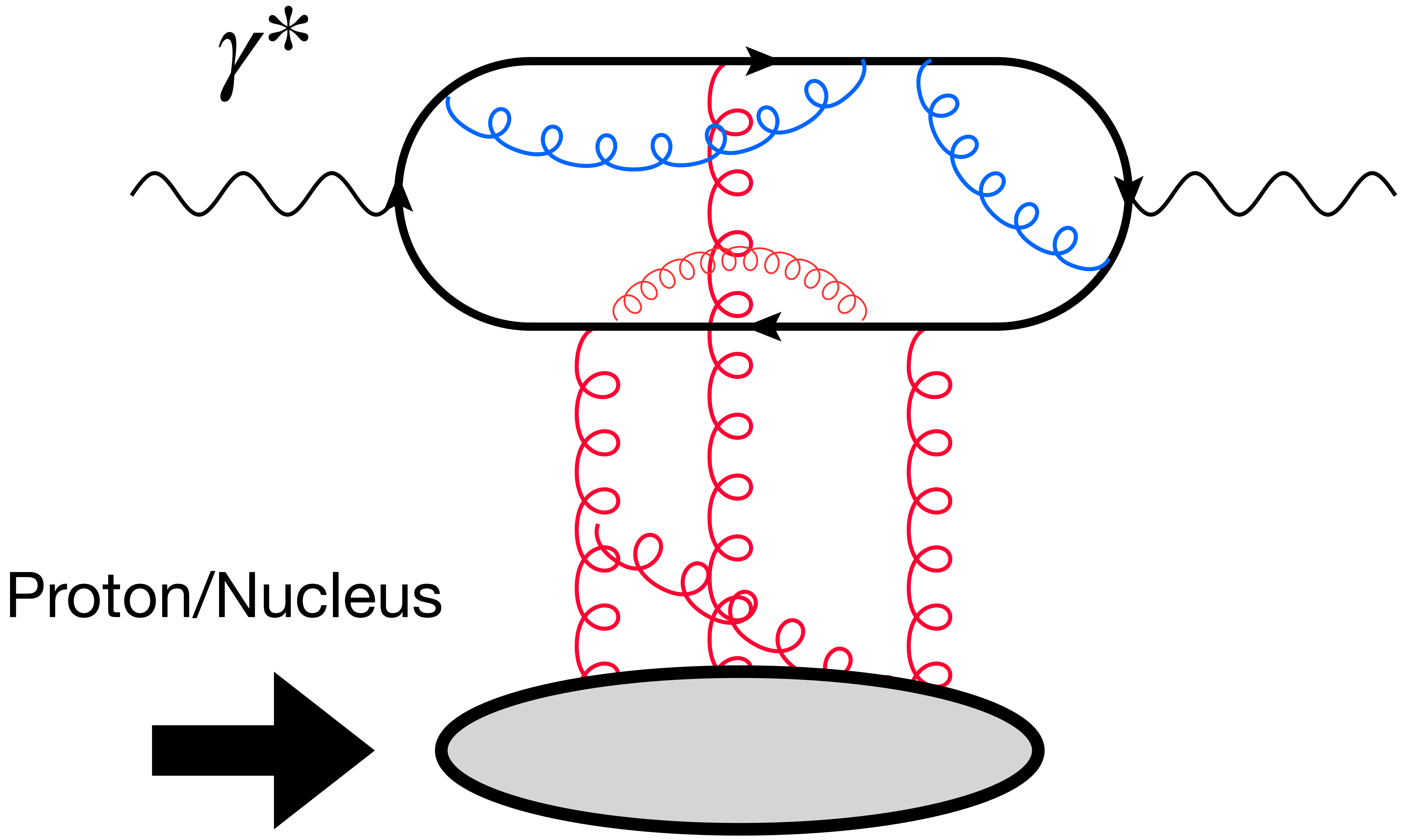
Yet another quantum effect in eA

- In addition to the standard long-lived gluon fluctuations resummed by small x evolution in the shock wave approximation

$$\ln \frac{1}{x}$$

- Short-lived quantum fluctuations inside the nucleus

$$\ln L \sim \ln A^{1/3}$$



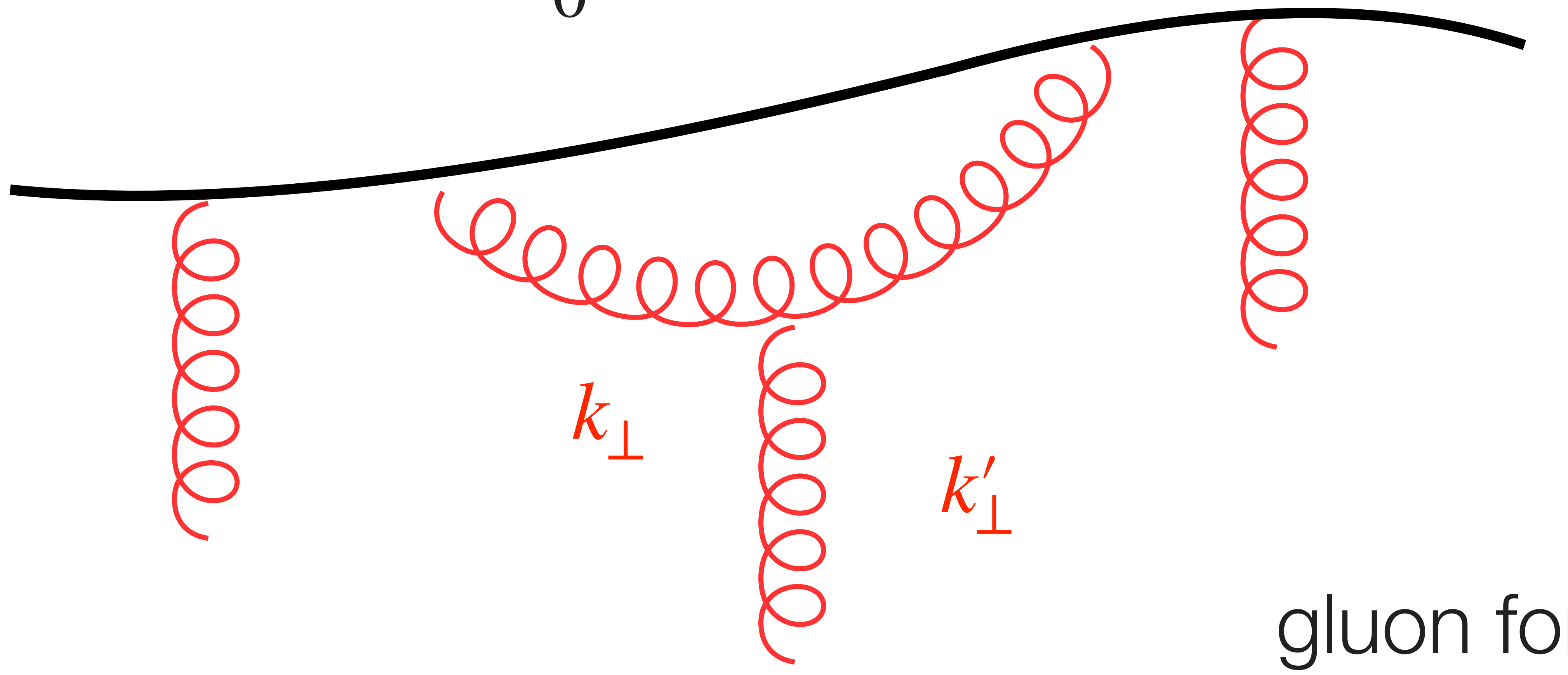
Deep Inelastic Scattering at small $x = Q^2/s$

Quantum corrections to $\hat{q}$ (or Q_s)

- Potentially large double logs (DL) in transverse momentum broadening at NLO

$$\mathrm{NLO} \sim \bar{\alpha} \int \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \int \frac{\mathrm{d}\tau}{\tau}$$

$$\tau_0 \ll \tau \ll L$$



[Liou, Mueller, Wu (2013)
Blaizot, Dominguez, Iancu, MT (2014)]

$$\langle k_\perp^2 \rangle = \hat{q}_0 L \left(1 + \frac{\bar{\alpha}}{2} \log^2 \frac{L}{\tau_0} \right)$$

$$\tau_0 \sim \ell_{\mathrm{mfp}} \ll \tau \ll L$$

gluon formation time

$$\tau \equiv \frac{\omega}{k_\perp^2}$$

- Not the standard DGLAP double log: the factor 1/2 reflects the presence of multiple scattering constraint $k_\perp > \hat{q}\,\tau \equiv Q_s^2$ (saturation boundary)

Geometric scaling and superdiffusion

- **Traveling waves solutions:** Derive sub-asymptotic behavior. We follow Brunet and Derrida's (1988) and U. Ebert and W. van Saarloos (2000) approaches to FKPP equation (Fisher-Kolmogorov-Petrovsky, Piskunov) - (population growth, wave propagation, etc)
- First application to QCD in small x (Balitsky-Kovchebov equation) by Munier and Peschanski (2003)

- The all order results was first derived in the DL approximation

Asymptotically: Lévy distribution

$$S(r_\perp, L) \to \mathrm{e}^{-(r_\perp^2 Q_s^2(L))^{1-2\sqrt{\bar{\alpha}}}}$$

⤳ non-Gaussian

Anomalous scaling: super diffusive process

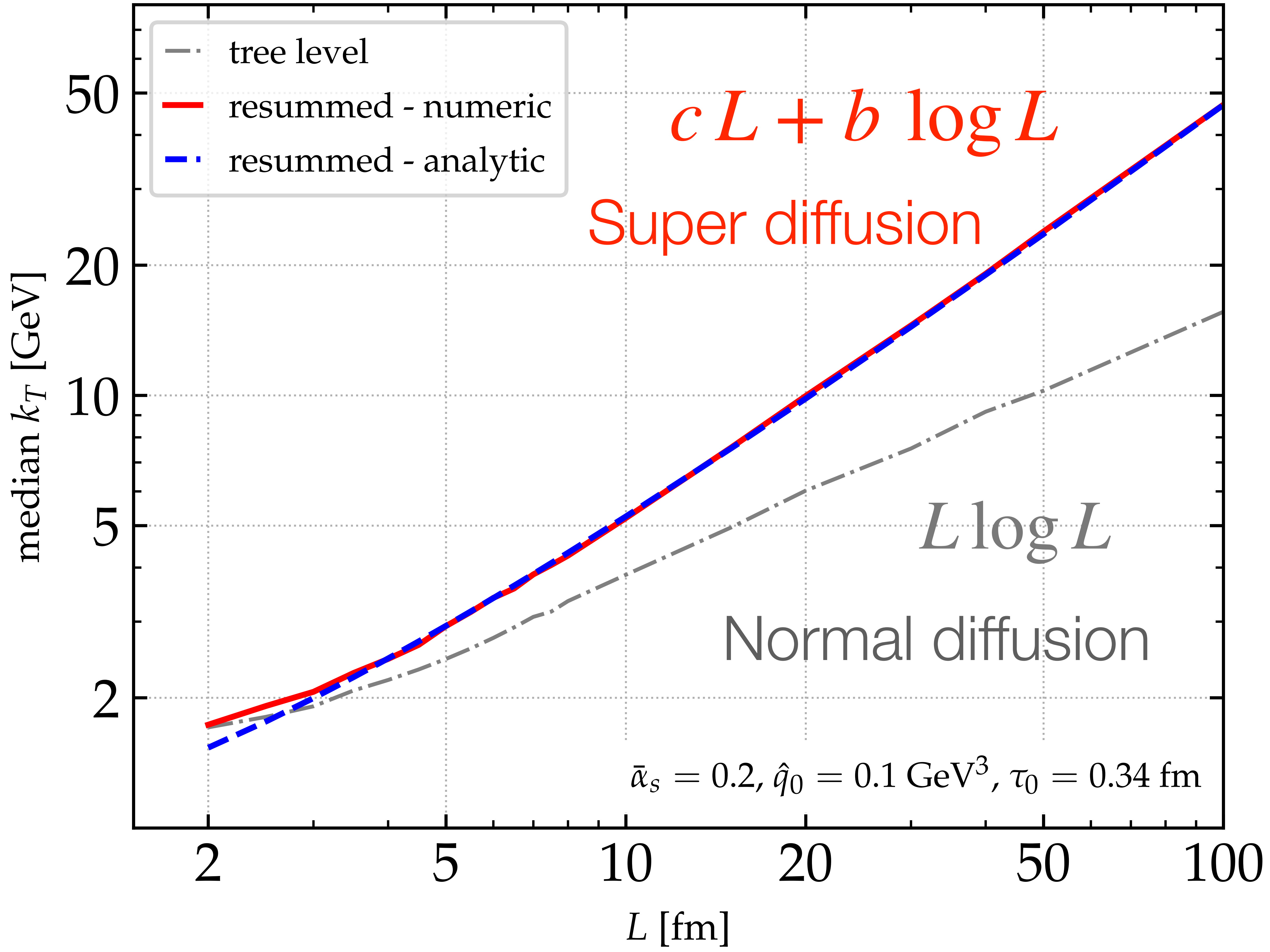
$$Q_s^2(L) \propto L^{1+2\sqrt{\bar{\alpha}}}$$

MT, Caucal 2203.09407 [hep-ph]

Time dependence of the typical transverse momentum

$$\rho_s(Y) = \log Q_s^2(Y) = cY + b \log Y + \text{const}.$$

$$Y \equiv \log \frac{L}{\tau_0} \qquad c \simeq 1 + 2\sqrt{\bar{\alpha}}$$



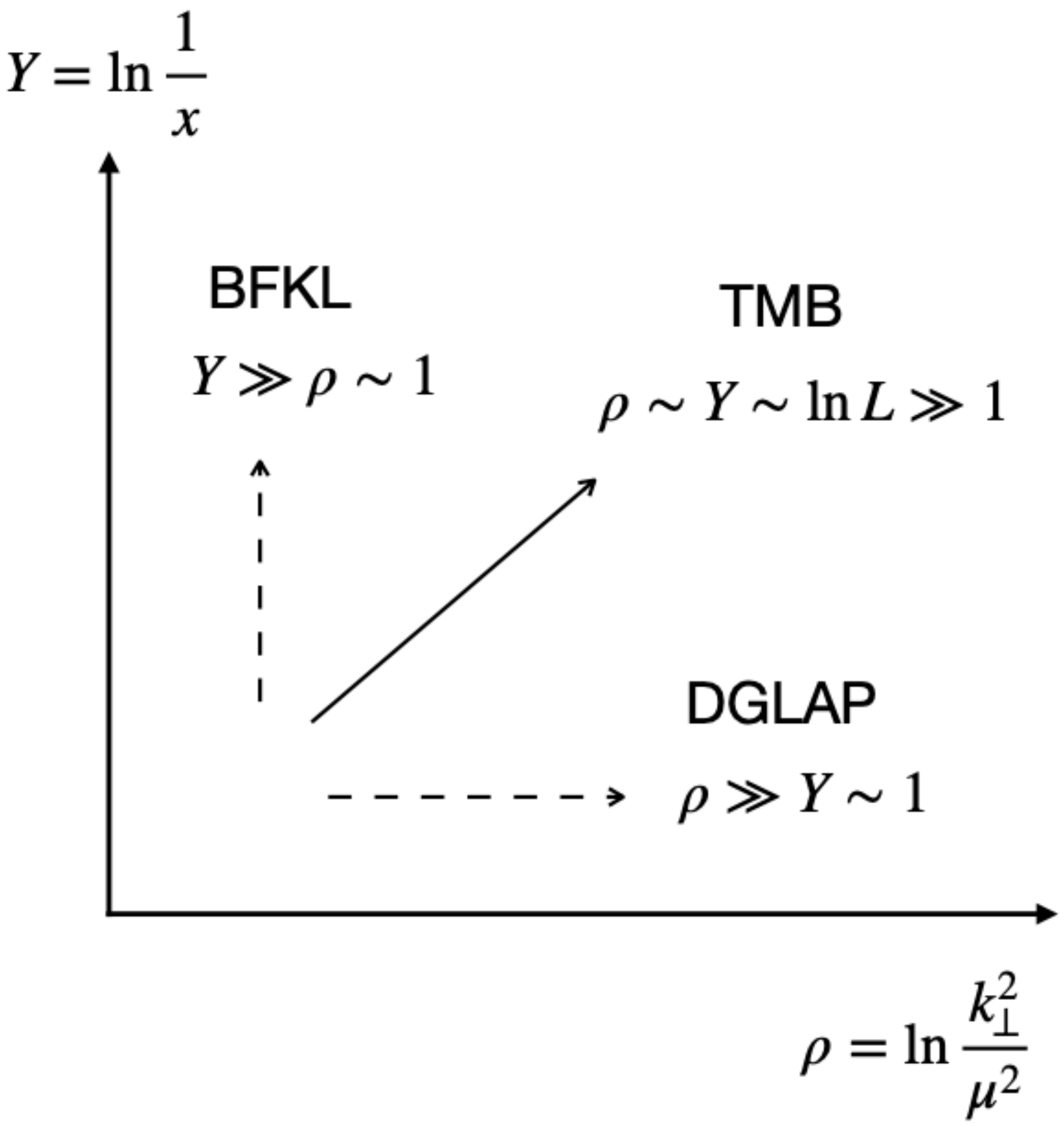
Nonlocal quantum corrections: anomalous system size dependence (super diffusion)

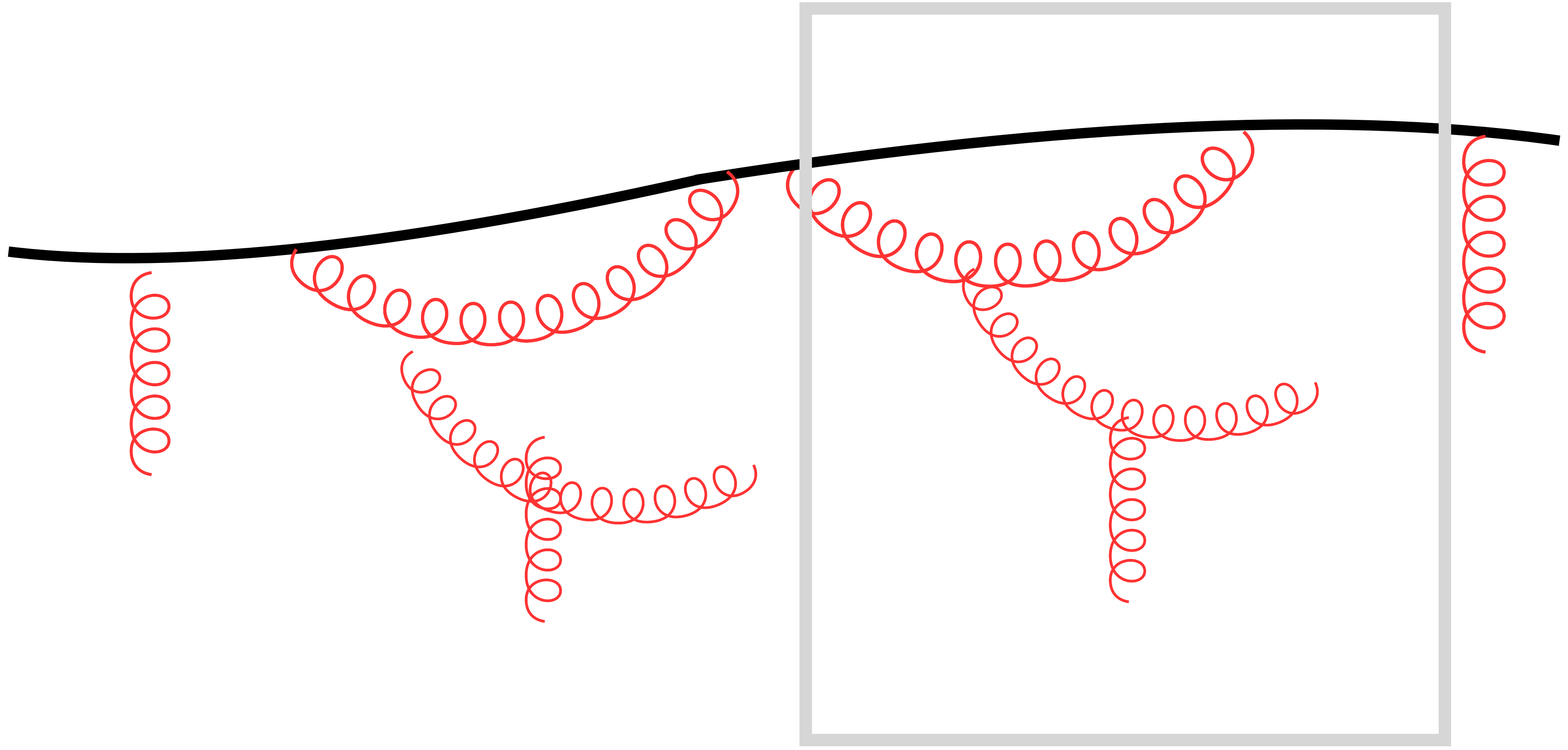
$$Q_s^2 \simeq \langle k_\perp^2 \rangle_{\text{median}} \propto L^{1+2\sqrt{\bar{\alpha}}}$$

Universal behavior observed down to $L \simeq 3$ fm

Next to double logarithmic evolution

- Resummation of NDLA from BFKL or DGLAP + saturation boundary





BFKL or DGLAP evolution

$$\frac{d}{dY}\rho_s(Y) \simeq \simeq 1 + 2\sqrt{\bar{\alpha}} + \mathcal{O}(\bar{\alpha})$$

DLA NDLA

Next to double logarithmic evolution

- Solve linear NLL-BFKL with saturation boundary $\rho > \rho_s(Y)$ (non-linear effects)

$$\frac{\partial \hat{q}}{\partial Y} = \chi_{\rm LL}(\partial_\rho)\,[\bar{\alpha}_s(\rho)\hat{q}(\rho,Y)] + \bar{\alpha}_s^2(\rho)\tilde{\chi}_{\rm NLL}(\partial_\rho)\hat{q}(\rho,Y)$$

$$\rho \equiv \ln\frac{k_\perp^2}{\mu^2} \qquad Y \equiv \ln\frac{L}{\tau_0}$$

Iancu (2014) MT, Blaizot (2014) Vaidya (2021)

- Saturation condition $\rho > \rho_s(Y) \equiv \ln\frac{Q_s^2(L)}{\mu^2}$
- N.B. $Q_s^2(L) \equiv \hat{q}(L)\,L$
- The details of non-linear terms are not relevant to derive the asymptotics of the saturation scale and the behavioor of the TMB distribution at $k_\perp > Q_s(L)$ [Mueller, Triantafyllopoulos (2002) (ancu, Itakura, McLerran (2002)]
- Expansion of BFKL kernel around DLA: $\gamma = 0$

$$\chi_{\rm LL}(\gamma) = \frac{1}{\gamma} + 2\zeta(3)\gamma^2 + \mathcal{O}(\gamma^4)\,,$$

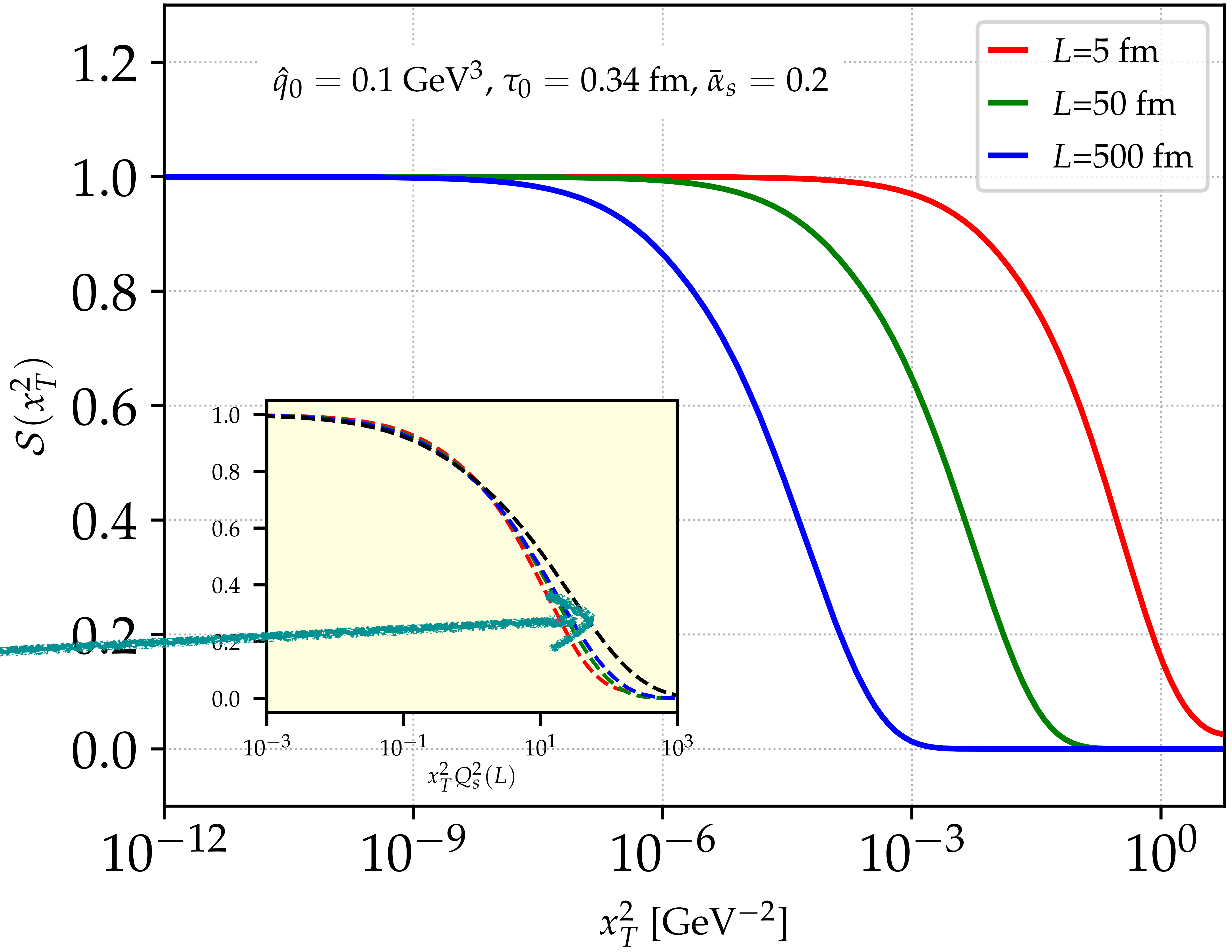
All orders - Geometric scaling

- We look for a solution of the form (solve around the scaling solution for running coupling)

$$\hat{q}(\rho, Y) = e^{\rho_s(Y)-Y} e^{\beta x} f(x, Y)\,, \quad x = \frac{\rho - \rho_s(Y)}{\sqrt{Y}}$$

- To obtain the sub-asymptotic corrections we use the so-called leading-edge expansion

$$f(x, Y) = \sum_{n=-1}^{\infty} Y^{-n/6} G_n \left(\frac{x}{Y^{1/6}}\right)$$



U. Ebert and W. van Saarloos (2000)

Universal terms up to $Y^{-3/2}$ for

$$\dot{\rho}_s = 1 + \frac{4b_0}{\bar{Y}^{1/2}} + \frac{2\xi_1 b_0}{\bar{Y}^{5/6}} + b_0\left(1 - 8b_0 + 4b_0 B_g\right)\frac{1}{\bar{Y}} - \frac{7\xi_1^2 b_0}{270}\frac{1}{\bar{Y}^{7/6}} - (5 + 1944 b_0)\frac{\xi_1 b_0}{81}\frac{1}{\bar{Y}^{4/3}} - 2b_0^2\left(1 - 8b_0 + 4b_0 B_g\right)\frac{\ln(\bar{Y})}{\bar{Y}^{3/2}} + \mathcal{O}(\bar{Y}^{-3/2}),$$

NLL BFKL

$$\bar{Y} = 4b_0 Y$$

$$b_0 = \frac{1}{\beta_0} = -\frac{1}{B_g}$$

$$B_g = -\frac{11}{12} - \frac{N_f}{6N_c^3} \approx -\frac{11}{12}.$$

$$\rho_s(Y) \equiv \ln\frac{Q_s^2(L)}{\mu^2} \equiv \ln\frac{\hat{q}(L)L}{\mu^2}$$

MT, Caucal 2209.08900 [hep-ph]

- Non universal terms start at order $Y^{-3/2}$ as can be seen by the substitution $Y \to Y + Y_0$
- NNLO BFKL and beyond do not contribute to the universal terms

Summary

- TMB in QCD is a super-diffusive process (non-gaussian) due to logarithmically enhanced quantum corrections $\longrightarrow$ anomalous system size dependence
- Exhibits geometric scaling and heavy tails akin to Lévy random walks $\longrightarrow$ Substantial departure from the LO Molière scattering
- Systematic approach for computing universal asymptotic and pre-asymptotic solutions for transverse momentum broadening
- Using DGLAP or NLL BFKL (due to the DL nature of the problem) we have computed all of the universal terms in the asymptotic expansion of the saturation scale
- Application: non-Gaussian initial condition for BK evolution at small x

Backup

Universal terms up to $Y^{-3/2}$

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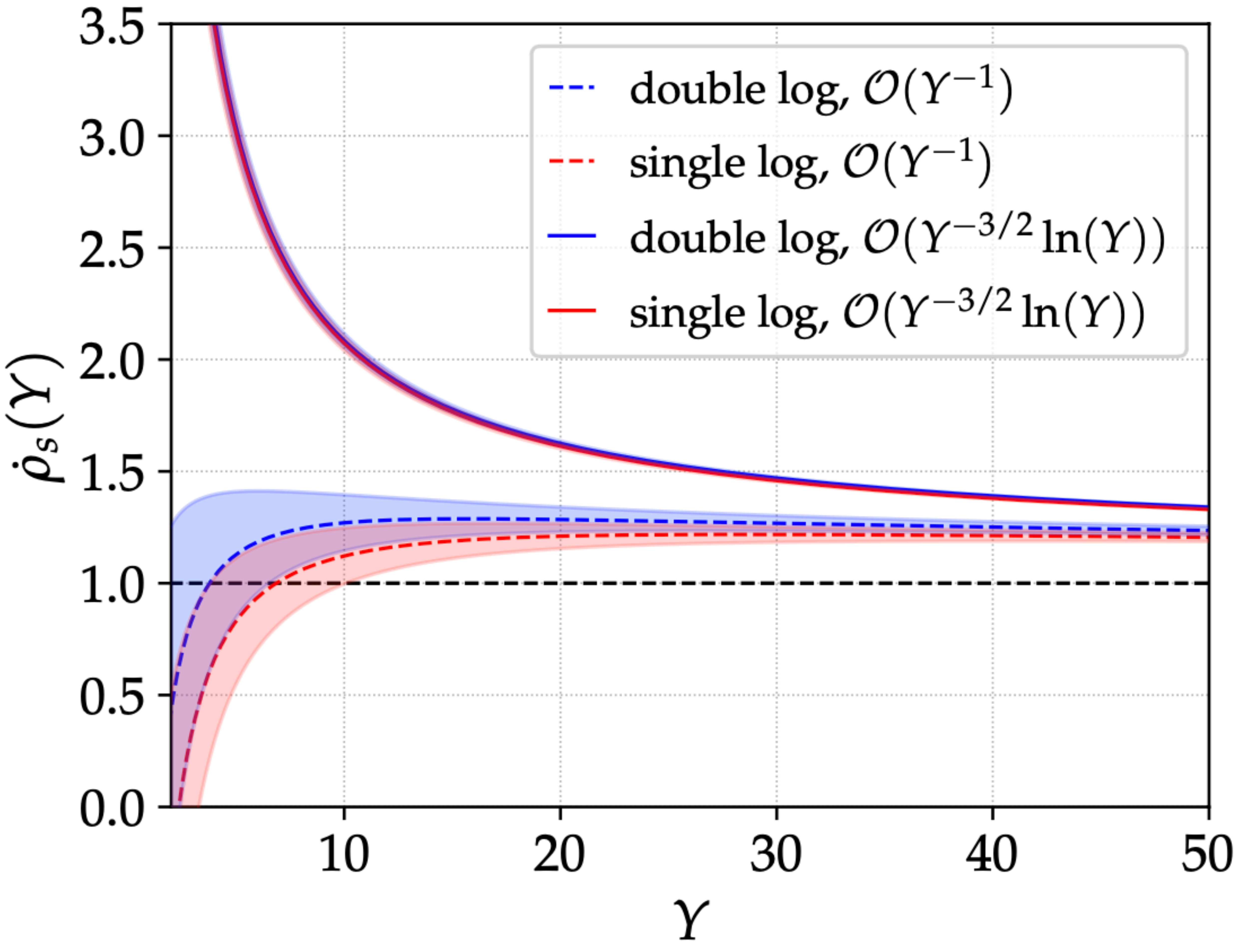
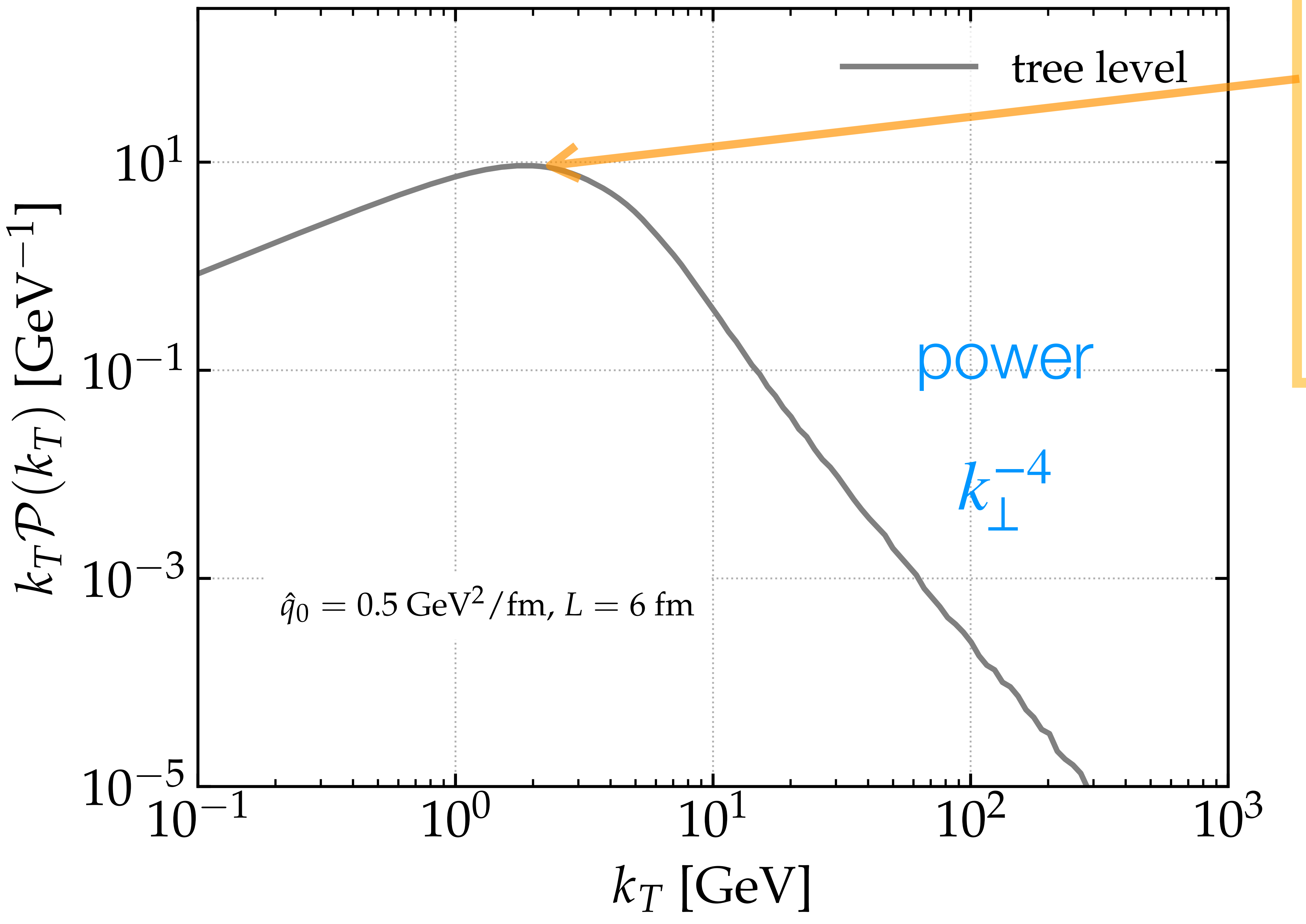


Tree Level and large media

TMB distribution at leading order - resummation of multiple scattering

Gaussian for $k_\perp < Q_s \sim \hat{q}\,L$ and exhibits the power law tail $k_\perp^{-4}$ for $k_\perp > Q_s$



multiple scattering

$$P(k_\perp) = \frac{4\pi}{\hat{q}L} e^{-\frac{k_\perp^2}{\hat{q}L}}$$

Normal diffusion

$$\langle k_\perp^2 \rangle_{\mathrm{typ}} \propto L$$

L: Medium size

Diffusion coefficient at tree level

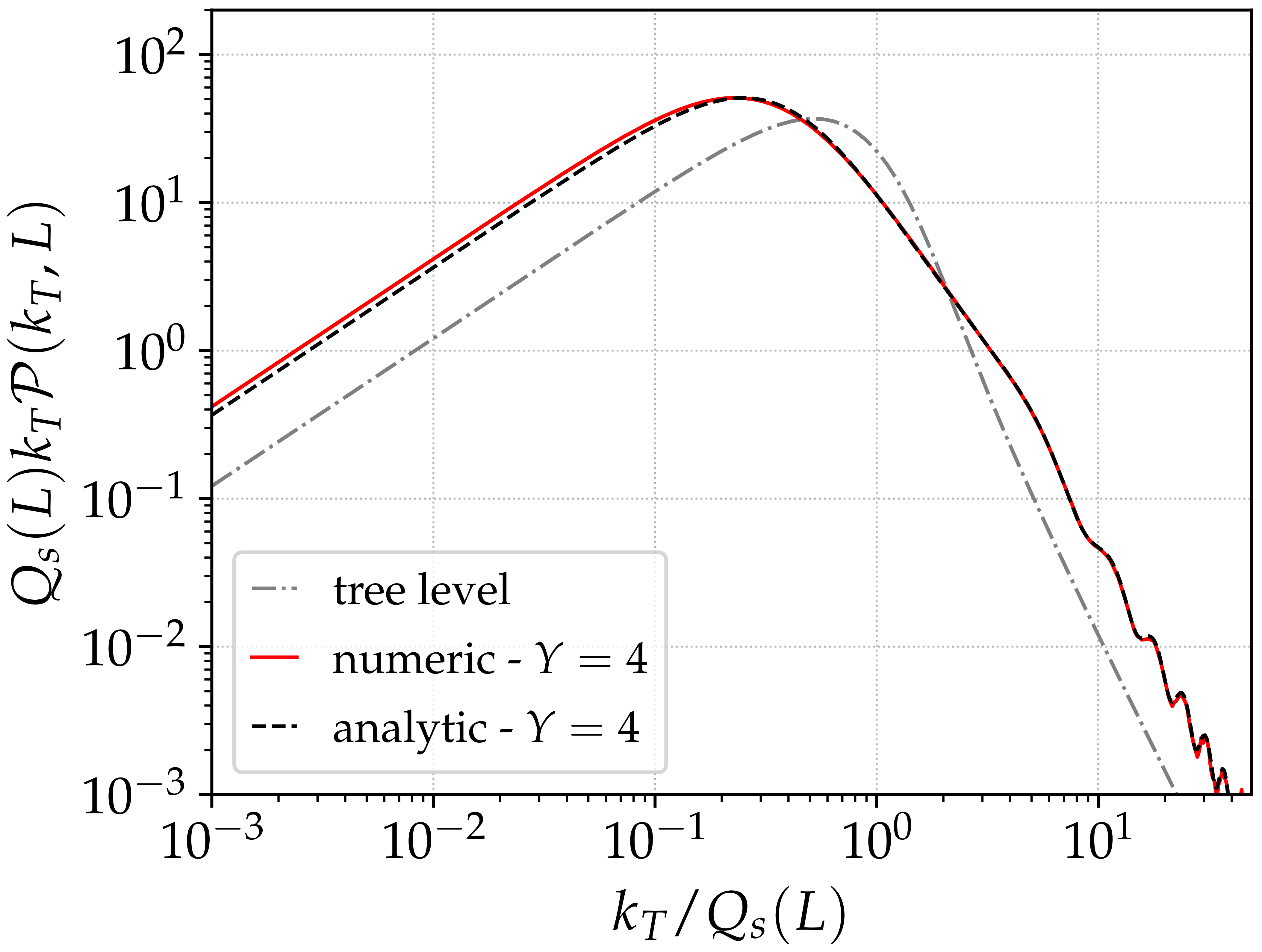
$$\hat{q} = C_R \int_{q_\perp} q_\perp^2 \frac{d^2\sigma}{d^2 q_\perp} \simeq 4\pi\alpha_s^2 n \log \frac{q_{\max}}{\mu^2}$$

Running coupling case (vs numerics)

- At large Y (leading contribution):

$$G(\zeta) = \frac{2^{1/3} b_0^{1/6}}{\mathrm{Ai}'(\xi_1)} \mathrm{Ai}\left[\xi_1 + 2^{-1/3} b_0^{1/3} \zeta\right]$$

- Small Y (leading edge expansion diverges). Instead expand around the saturation line



$$\hat{q}_<(Y, x) = \hat{q}_0\, \mathrm{e}^{\rho_s(Y) - Y} \,\exp\left(\frac{\dot{\rho}_s - 1}{\dot{\rho}_s} x + \frac{1}{2}\frac{\ddot{\rho}_s}{\dot{\rho}_s^3} x^2 + \mathcal{O}(x^3)\right)$$

Systematic approach for universal sub-leading terms

- Fixed coupling

Caucal, MT, 2109.12041 [hep-ph]

$$\rho_s(Y) = cY - \frac{3c}{1+c}\ln(Y) + \kappa - \frac{6c\sqrt{2\pi(c-1)}}{(1+c)^2}\frac{1}{\sqrt{Y}} + \mathcal{O}\left(\frac{1}{Y}\right)$$

- Running coupling

Caucal, MT, 2203.09407 [hep-ph]

$$\rho_s(Y) = Y + 2\sqrt{4b_0Y} + 3\xi_1(4b_0Y)^{1/6} + \left(\frac{1}{4} - 2b_0\right)\ln(Y) + \kappa$$
$$+\frac{7\xi_1^2}{180}\frac{1}{(4b_0Y)^{1/6}} + \xi_1\left(\frac{5}{108} + 18b_0\right)\frac{1}{(4b_0Y)^{1/3}} + b_0\left(1 - 8b_0\right)\frac{\ln(Y)}{\sqrt{4b_0Y}} + \mathcal{O}\left(Y^{-1/2}\right)$$

First four terms conjectured by Iancu and Triantafyllopoulos (2015)

Running coupling case

- One-loop running coupling:

$$\bar{\alpha}(k_\perp) \simeq \frac{b}{\ln k_\perp^2/\Lambda_{QCD}^2} \qquad \text{with} \qquad b = \frac{12N_c}{11N_c - 2N_f}$$

- Slower evolution w.r.t. $Y \equiv \log \frac{L}{\tau_0}$

- Modified scaling variable

$$x \equiv \log \frac{k_\perp^2}{Q_s^2} \quad \rightarrow \quad x \equiv \frac{\log \frac{k_\perp^2}{Q_s^2}}{\sqrt{Y}} \sim \sqrt{\bar{\alpha}(Y)} \log \frac{k_\perp^2}{Q_s^2}$$