

# Back-to-back dijets production in DIS at small- $x$ : Sudakov suppression and gluon saturation at NLO

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**SUBATECH**

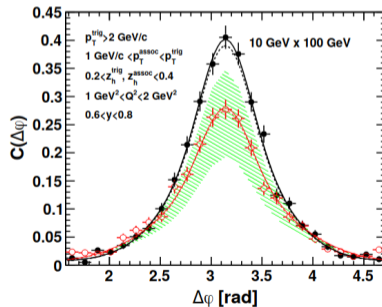
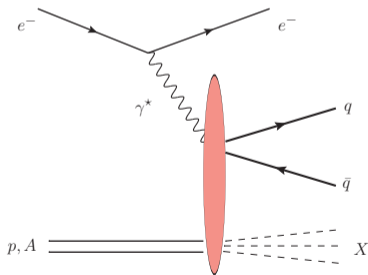
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JHEP 2021 (11), 1-108, [arXiv:2208.13872](https://arxiv.org/abs/2208.13872) and work in progress

# Inclusive dijet production in DIS at small- $x$

- ⇒ probe of the saturated regime of QCD
- ⇒ access to the Weizsäcker-Williams gluon TMD in the back-to-back limit.



Zheng, Aschenauer, Lee, Xiao, 1403.2413

# Dijets in DIS at NLO and small- $x$ : many recent progresses!

- Dihadrons production. [Bergabo, Jalilian-Marian, 2207.03606](#), [Iancu, Mulian, 2211.04837](#)
- Photo-production limit. [Taels, Altinoluk, Beuf, Marquet, 2204.11650](#)
- Related processes: exclusive dijet, [Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371](#),  
Single inclusive hadron production [Bergabo, Jalilian-Marian, 2210.03208](#) ,  
Diffractive dihadron [Fucilla, Grabovsky, Li, Szymanowski, Wallon, 2211.05774](#)
- Results from different approaches: cross-check of a challenging computation!

## In this talk: NLO impact factor for inclusive dijet production in DIS

- Reliable QCD prediction requires to account for NLO corrections.
- Systematic determination of the theoretical uncertainties.
- Analytic expressions in back-to-back kinematics that simplify the numerical calculation.

# LO cross-section at small-x

- Differential cross-section at leading order:

$$\left. \frac{d\sigma^{\gamma^*+A \rightarrow q\bar{q}+X}}{d^2\mathbf{k}_\perp d^2\mathbf{p}_\perp d\eta_q d\eta_{\bar{q}}} \right|_{\text{LO}} = \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6} \int d^8\mathbf{X}_\perp e^{-ik_\perp r_{x'x'}} e^{-ip_\perp r_{yy'}} \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp) \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}'_{xy})$$

- Factorization** between **perturbative factor** describing the  $\gamma^* \rightarrow q\bar{q}$  splitting...

$$\mathcal{R}_{\text{LO}}^L(\mathbf{r}_{xy}, \mathbf{r}'_{xy}) = 8z_q^3 z_{\bar{q}}^3 Q^2 K_0(\bar{Q}r_{xy}) K_0(\bar{Q}r_{xy'})$$

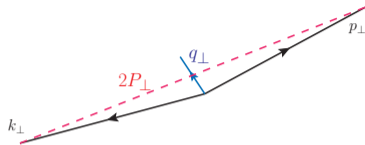
- ... and a **color structure** describing the interaction of  $q\bar{q}$  with the dense target

$$\Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) = \left\langle \underbrace{Q(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp)}_{\text{quadrupole}} - D(\mathbf{x}_\perp, \mathbf{y}_\perp) - \underbrace{D(\mathbf{y}'_\perp, \mathbf{x}'_\perp)}_{\text{dipole}} + 1 \right\rangle_Y$$

$$\text{Dipole: } D(\mathbf{x}_\perp, \mathbf{y}_\perp) = \frac{1}{N_c} \langle \text{Tr}(V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp)) \rangle$$

# The back-to-back limit

- Def:  $|\mathbf{P}_\perp| = |z_{\bar{q}}\mathbf{k}_\perp - z_q\mathbf{p}_\perp| \gg |\mathbf{q}_\perp| = |\mathbf{k}_\perp + \mathbf{p}_\perp|$

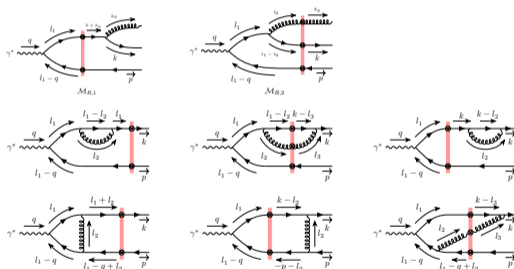


- LO: TMD factorization [Dominguez, Marquet, Xiao, Yuan, 1101.0715](#)

$$\frac{d\sigma^{\gamma^* \rightarrow q\bar{q}+X}}{d^2\mathbf{P}_\perp d^2\mathbf{q}_\perp} \Big|_{\text{LO}} \propto \mathcal{H}(\mathbf{P}_\perp) \int d^2\mathbf{b}_\perp d^2\mathbf{b}'_\perp e^{-i\mathbf{q}_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)} \underbrace{G_{\text{WW}}(\mathbf{b}_\perp, \mathbf{b}'_\perp)}_{\left\langle \frac{1}{N_c} \text{Tr} [\partial_i V^\dagger(\mathbf{b}_\perp) V(\mathbf{b}'_\perp) \partial_j V^\dagger(\mathbf{b}'_\perp) V(\mathbf{b}_\perp)] \right\rangle_Y} + \mathcal{O}\left(\frac{q_\perp}{P_\perp}\right) + \mathcal{O}\left(\frac{Q_s}{P_\perp}\right)$$

# Inclusive dijet at NLO and small- $x$ in the CGC

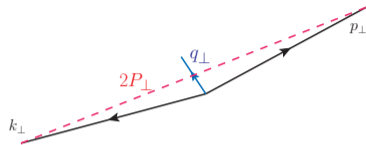
- We have done the full computation for general kinematics in [JHEP 2021 \(11\), 1-108](#)
- The diagrams are



- The cross-section is UV finite, IR divergences cancel provided one uses jets, light cone singularity absorbed into the JIMWLK evolution of the LO color structure.

# Back-to-back limit at NLO: emergence of large Sudakov logarithms

- NLO: large **Sudakov logarithms** vs **small-x logarithm**.



$$\frac{d\sigma^{\gamma^* \rightarrow q\bar{q}+X}}{d^2\mathbf{P}_\perp d^2\mathbf{q}_\perp} \Big|_{\text{NLO}} \propto \mathcal{H}(\mathbf{P}_\perp) \int d^2\mathbf{b}_\perp d^2\mathbf{b}'_\perp e^{-i\mathbf{q}_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)}$$

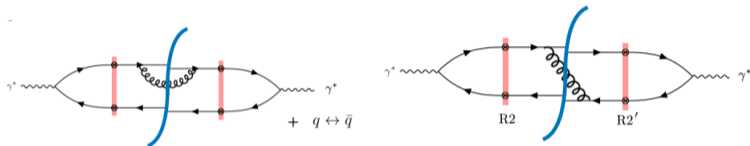
$$\times \left[ 1 - \frac{\alpha_s N_c}{4\pi} \ln^2 \left( \frac{P_\perp^2 (\mathbf{b}_\perp - \mathbf{b}'_\perp)^2}{c_0^2} \right) + \dots + \alpha_s \ln \left( \frac{1}{x_{Bj}} \right) \mathcal{K}_{LL} \otimes \right] G_{\text{WW}}(\mathbf{b}_\perp - \mathbf{b}'_\perp)$$

See Mueller, Xiao, Yuan, 1308.2993 based on Higgs production in pA.

- $\mathbf{b}_\perp - \mathbf{b}'_\perp$  conjugate to  $\mathbf{q}_\perp \Rightarrow$  double log of the form  $\ln^2(P_\perp/q_\perp)$

# Sudakov logarithms in our computation

- Real diagrams with soft divergences.



- However: the integration over the soft gluon gives the Sudakov **with a positive sign!**

$$\begin{aligned}
 d\sigma_{\text{NLO}}^{\gamma^* \rightarrow q\bar{q}+X} &\sim \mathcal{H}(\mathbf{P}_\perp) \int d^2\mathbf{b}_\perp d^2\mathbf{b}'_\perp e^{-iq_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)} \\
 &\times \left[ 1 + \frac{\alpha_s N_c}{4\pi} \ln^2 \left( \frac{\mathbf{P}_\perp^2 (\mathbf{b}_\perp - \mathbf{b}'_\perp)^2}{c_0^2} \right) + \dots + \alpha_s \ln \left( \frac{k_f^-}{\Lambda^-} \right) \mathcal{K}_{\text{LL}} \otimes \right] G_{\text{WW}}(\mathbf{b}_\perp - \mathbf{b}'_\perp)
 \end{aligned}$$

- Problem: overlapping phase space between soft gluons and slow gluons included in  $\mathcal{K}_{\text{LL}}$ .



# Solution: collinearly improved small- $x$ evolution of the WW

- Kinematic improvement: impose both  $k_g^-$  and  $k_g^+$  ordering (lifetime ordering).
  - ⇒ Resum large transverse double logarithms to all orders.
  - ⇒ Solve the instability of NLO B-JIMWLK evolution.

Beuf, 1401.0313, Taels, Altinoluk, Beuf, Marquet, 2204.11650

- In practice, add an additional constraint in the LL evolution kernel

$$k_g^+ \geq k_f^+ \implies k_g^- \leq \frac{\mathbf{k}_{g\perp}^2}{Q_f^2} k_f^-$$

with  $Q_f^2 \sim Q^2 \sim \mathbf{P}_\perp^2$ .

- With this modification  $\mathcal{K}_{LL} \rightarrow \mathcal{K}_{LL,\text{coll}}$ , one recovers the expected double logarithm.

$$\begin{aligned} d\sigma_{\text{NLO}}^{\gamma^* \rightarrow q\bar{q}+X} &\sim \mathcal{H}(\mathbf{P}_\perp) \int d^2\mathbf{b}_\perp d^2\mathbf{b}'_\perp e^{-i\mathbf{q}_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)} \\ &\times \left[ 1 - \frac{\alpha_s N_c}{4\pi} \ln^2 \left( \frac{\mathbf{P}_\perp^2 (\mathbf{b}_\perp - \mathbf{b}'_\perp)^2}{c_0^2} \right) + \dots + \alpha_s \mathcal{K}_{LL,\text{coll}} \otimes \right] G_{\text{WW}}(\mathbf{b}_\perp - \mathbf{b}'_\perp) \end{aligned}$$

# Sudakov resummation at single log accuracy

- Exponentiation of the Sudakov logarithms  $G_{\text{WW}}(\mathbf{r}_{bb'}) \rightarrow G_{\text{WW}}(\mathbf{r}_{bb'}) \mathcal{S}(\mathbf{P}_{\perp}^2, \mathbf{r}_{bb'}^2)$

$$\mathcal{S}(\mathbf{P}_{\perp}^2, \mathbf{r}_{bb'}^2) = \exp \left( - \int_{c_0^2/\mathbf{r}_{bb'}^2}^{\mathbf{P}_{\perp}^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2) N_c}{\pi} \left[ \underbrace{\frac{1}{2} \ln \left( \frac{\mathbf{P}_{\perp}^2}{\mu^2} \right)}_{\text{double log}} + \underbrace{\frac{C_F}{N_c} s_0 - s_f}_{\text{single log}} \right] \right)$$

- Double and **single** Sudakov logarithms with **exact**  $N_c$  dependence:
- Dijet geometry single log  $s_0$

$$s_0 = \ln \left( \frac{2(1 + \cosh(\Delta Y_{12}))}{R^2} \right) + \mathcal{O}(R^2)$$

See also Hatta, Xiao, Yuan, Zhou, 2106.05307

- Single log from the interplay between small- $x$  and Sudakov resummation:

$$s_f = \ln \left( \frac{\mathbf{P}_{\perp}^2 x_{\text{Bj}}}{z_1 z_2 Q^2 c_0^2 x_f} \right) \Rightarrow x_f \text{ factorization scale dependence!}$$

# Finite terms in $\alpha_s$ in the back-to-back limit

## Azimuthal anisotropies from soft gluon radiations

- We can also access pure  $\alpha_s$  (and non power suppressed) corrections.
- Some of them are coming from soft gluon radiations. [Hatta, Xiao, Yuan, Zhou, 2010.10774](#)
- Azimuthally averaged x-section sensitive to the **linearly polarized gluon TMD** at NLO!

$$\langle d\sigma \rangle = \dots + \mathcal{H}(\mathbf{P}_\perp) \times \int \frac{d^2 \mathbf{r}_{bb'}}{(2\pi)^4} e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_{bb'}} \hat{h}(\mathbf{r}_{bb'}) \mathcal{S}(\mathbf{P}_\perp^2, \mathbf{r}_{bb'}^2) \\ \times \frac{\alpha_s}{\pi} \left\{ \frac{N_c}{2} + C_F \ln(R^2) - \frac{1}{2N_c} \ln(z_1 z_2) \right\}$$

- The  $\cos(2\phi)$  anisotropy is also sensitive to the **unpolarized gluon TMD**.

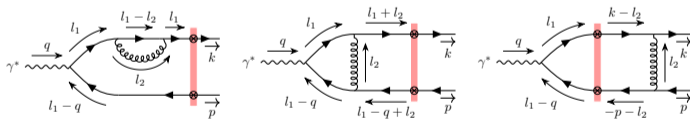
$$\langle \cos(2\phi) d\sigma \rangle = \dots + \mathcal{H}(\mathbf{P}_\perp) \times \int \frac{d^2 \mathbf{r}_{bb'}}{(2\pi)^4} e^{-i\mathbf{q}_\perp \cdot \mathbf{r}_{bb'}} \cos(2\theta) \hat{G}(\mathbf{r}_{bb'}) \mathcal{S}(\mathbf{P}_\perp^2, \mathbf{r}_{bb'}^2) \\ \times \frac{\alpha_s}{\pi} \left\{ N_c + 2C_F \ln(R^2) - \frac{1}{N_c} \ln(z_1 z_2) \right\}$$

# NLO hard factors

- Some other pure  $\alpha_s$  involve new "hard factor" and do not break TMD factorization:

$$\left. \frac{d\sigma^{\gamma^* \rightarrow q\bar{q}+X}}{d^2\mathbf{P}_\perp d^2\mathbf{q}_\perp} \right|_{\text{NLO}} \propto \alpha_s \mathcal{H}_{\text{NLO}}^{ij}(\mathbf{P}_\perp) \int d^2\mathbf{b}_\perp d^2\mathbf{b}'_\perp e^{-i\mathbf{q}_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)} G_{\text{WW}}^{ij}(\mathbf{b}_\perp, \mathbf{b}'_\perp)$$

- They come from virtual graphs in which the gluon does not cross the SW:



- Do they appear in the TMD literature for this process?

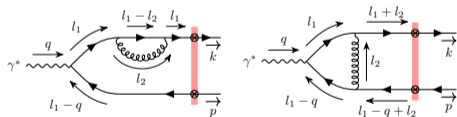
# NLO hard factors

- Remarkably, they can be computed fully analytically!
- They have the form

$$\mathcal{H}_{\text{NLO}}(\mathbf{P}_\perp) = \mathcal{H}_{\text{LO}}(\mathbf{P}_\perp) \times f\left(\frac{Q}{P_\perp}, z_1, z_2\right)$$

with  $f$  linear combination of logarithms and dilogarithms.

- Example:



$$\mathcal{H}_{\text{SE}_1+\text{V}_1}^{\lambda=L,ij}(\mathbf{P}_\perp) = \mathcal{H}_{\text{LO}}^{ij}(\mathbf{P}_\perp) \times \left[ 4 - 4 \ln\left(\frac{u}{c_0}\right) - 4 \ln\left(1 + \frac{1}{u^2}\right) \right]$$

with  $u = \sqrt{z_1 z_2} Q / P_\perp$ .

# TMD factorization at NLO and small- $x$

- Can we put the full result in a TMD factorized form?
- The small- $x$  evolution of the WW is not closed. [Dominguez, Mueller, Munier, Xiao, 1108.1752](#)
- At NLO, non-trivial color correlators, e.g.

$$\frac{N_c}{2} \langle 1 - D_{y'x'} + Q_{zy,y'x'} D_{xz} - D_{xz} D_{zy} \rangle_Y$$

which does not reduce to the WW gluon TMD, unless at least  $Q_s^2 \ll k_{g\perp}^2 \sim$  dilute limit.

- Maybe there is an other argument to neglect these complicated finite terms beyond the dilute limit...

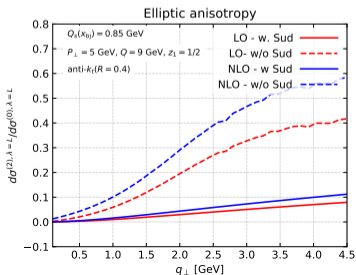
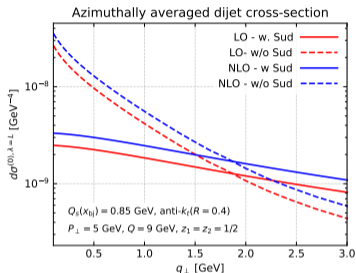
# Preliminary numerical results (with many caveats)

✓ Include:

- Sudakov with double and single log **but** at fixed coupling.
- **All** finite terms that do not manifestly break TMD factorization.

✗ Do not include:

- Proper small- $x$  evolution. The WW is parametrized by  $Q_s(x_{Bj})$  in the Gaussian approx.
- Factorization breaking terms.



# Summary and outlook

- NLO results for inclusive dijet production at small- $x$  in the CGC EFT.
- Back-to-back limit: Sudakov double and single log at exact  $N_c$ , and impact factor.
- Necessity to use a collinearly improved small- $x$  evolution to find the correct Sudakov double log.
- Towards a numerical evaluation of the impact factor with saturation corrections: **very challenging...**  
But recent progresses with the complete evaluation of all non factorization breaking terms!

**THANK YOU!**