COMPARISON OF GS09 + PYTHIA DOUBLE PDFS

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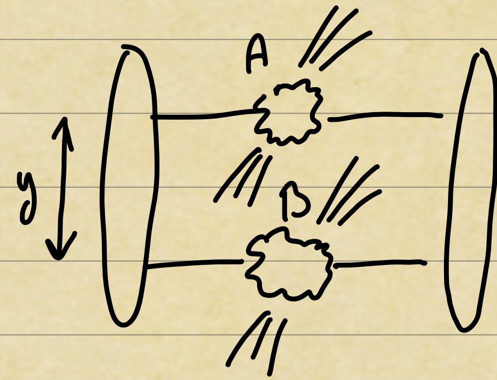
Based on arXiv: 2208.08197 with Oleh Fedkevych

OUTLINE:

- Brief recap of double PDFs (dPDFs), sum rules & Pythia model of dPDFs.
- How well do Pythia dPDFs satisfy sum rules?
- Compare Pythia dPDFs to GS09 dPDFs. How do they differ & why?

Diehl, JG, Schönwald, 1702.06486
See also other work by Paver, Treleani, Mekhfi, Blok, Dokshitzer, Frankfurt, Strikman, Diehl, Ostermeier, Schafer, Plößl, Nagar, Vladimirov, ...



$$\sigma_{DPS} = \frac{1}{1+\delta_{AB}} \sum_{j_i} \int_{|y|>1/Q} d^2\underline{y}\, dx_i\, \Gamma_{j_1 j_2}(x_1, x_2, \underline{y}, Q)$$

$$\times\, \Gamma_{j_3 j_4}(x_3, x_4, \underline{y}, Q)\, \hat{\sigma}_{j_1 j_3 \to A}\, \hat{\sigma}_{j_2 j_4 \to B}$$

↑ Double parton distributions, DPDs

Can define:

$$D_{j_1 j_2}(x_1, x_2, Q) = \int_{|y|>1/Q} d^2\underline{y}\, \Gamma_{j_1 j_2}(x_1, x_2, \underline{y}, Q)$$

↑ Double PDF, dPDF.

MOMENTUM & NUMBER SUM RULES

Important theory constraints on dPDFs.

Available momentum after "taking out" parton j_1 with momentum x_1

Momentum rule: $$\sum_{j_2} \int dx_2 \, x_2 \, D_{j_1 j_2}(x_1, x_2) = (1 - x_1) \, f_{j_1}(x_1)$$

Number of j_2 quarks - number of $\bar{j}_2$ quarks after "taking out" parton j_1.

Number rule: $$\int dx_2 \, D_{j_1 j_{2v}}(x_1, x_2) = (N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}) \, f_{j_1}(x_1)$$

First written down in JG, Stirling 0910.4347 & proved in Diehl, Plößl, Schäfer 1811.00289. See also Blok, Dokshitzer, Frankfurt, Strikman, 1306.3763

[In fact only hold exactly for $\overline{MS}$ dPDFs - for dPDFs as defined on previous slide they hold up to corrections of order α_s and/or Λ^2/Q^2].

If one assumes

$$\Gamma_{j_1 j_2}(x_1, x_2, \underline{y}, Q) \approx D_{j_1 j_2}(x_1, x_2, Q)\, F(\underline{y}) \qquad \text{(A)}$$

↑ smoothly varying function, width ~ proton radius

Then:

$$\sigma_{DPS} = \frac{1}{1+\delta_{AB}} \frac{1}{\sigma_{eff}} \sum_{j_i} \int dx_i \, D_{j_1 j_2}(x_1, x_2, Q)\, D_{j_3 j_4}(x_3, x_4, Q)$$

$$\times\, \sigma_{j_1 j_3 \to A}\, \sigma_{j_2 j_4 \to B} \qquad \text{(B)}$$

Nowadays it is known that (A) cannot hold. See e.g. Blok, Dokshitzer, Frankfurt, Strikman, 1306.3763, 1106.5533, JG 1207.0480, JG & Stirling 1103.1888, JG, Diehl, Schönwald, 1702.06486, Ryskin & Snigirev 1103.3495, 1203.2330, Manohar & Waalewijn 1202.5034...

Nonetheless, (B) is still used in MC DPS (& MPI) models (e.g. Pythia) & in pheno studies.

Ⓢ

PYTHIA DOUBLE PDFS

Sjostrand, Skands
hep-ph/0402078
hep-ph/0408302

$$D_{j_1 j_2}(x_1, x_2, Q) = f^{r}_{j_1}(x_1, Q)\, f^{m \leftarrow j_1, x_1}_{j_2}(x_2, Q)$$

$f^{r}_{j_1}$ ↑ usual "raw" PDF

$f^{m \leftarrow j_1, x_1}_{j_2}$ ↑ PDF that has been modified due to removal of parton 1

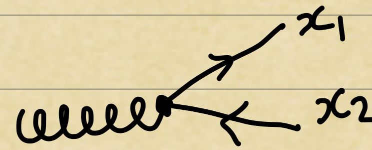
How is $f^{m \leftarrow j_1, x_1}_{j_2}$ obtained?

(I) MOMENTUM "SQUEEZING". To ensure $x_2 < 1 - x_1$: $f_{j_2}^{m \to j_1, x_1}(x_2, Q) = \frac{1}{1-x_1} f_{j_2}^{r}\left(\frac{x_2}{1-x_1}, Q\right)$

If this was the only modification: $\sum_{j_2} \int dx_2\, x_2\, f_{j_2}^{m \to j_1, x_1}(x_2, Q) = 1 - x_1$ (*)

(II) VALENCE NUMBER SUBTRACTION: if j_1 & j_2 are valence quarks of the same flavour: $f_{j_{2v}}^{m \to j_{1v}, x_1}(x_2, Q) = \left(\frac{N_{j_1 v} - 1}{N_{j_1 v}}\right) \frac{1}{1-x_1} f_{j_{2v}}^{r}\left(\frac{x_2}{1-x_1}\right)$

(III) COMPANION QUARK ADDITION: if j_1 is a sea quark & $j_2 = \bar{j}_1$, add an extra contribution to the PDF: $q_c(x_2, x_1) = C(x_1)\, P_{g \to q\bar{q}}\left(\frac{x_1}{x_1+x_2}\right) g(x_1+x_2)/(x_1+x_2)$



(IV) SEA QUARK & GLUON RESCALING: Steps (II) & (III) break (*). To fix this we rescale all sea quark & gluon distributions by a factor "a" → restores (*).

Pythia dPDFs satisfy sum rules when integrating over second "modified" parton "by design" - can show analytically & numerically.

BUT These dPDFs are not symmetric under $j_1 \leftrightarrow j_2$, $x_1 \leftrightarrow x_2$

Simplest proposal: $D^{sym}_{j_1 j_2}(x_1, x_2, Q) = \dfrac{D_{j_1 j_2}(x_1,x_2,Q) + D_{j_2 j_1}(x_1,x_2,Q)}{2}$

Satisfies sum rules reasonably well (~10-25% level). Some bigger deviations in places!

x_1	Momentum sum rule ($j_1 = u$). Should = 1	$u\bar{u}$ number sum rule. Should = -1	$\bar{u}u$ number sum rule. Should = 3
10^{-6}	0.979	-1.227	2.961
10^{-3}	0.980	-0.847	3.351
10^{-1}	1.014	-0.925	3.491
0.2	1.047	-0.928	3.580
0.4	1.133	-0.884	3.858
0.8	1.679	-0.740	7.048

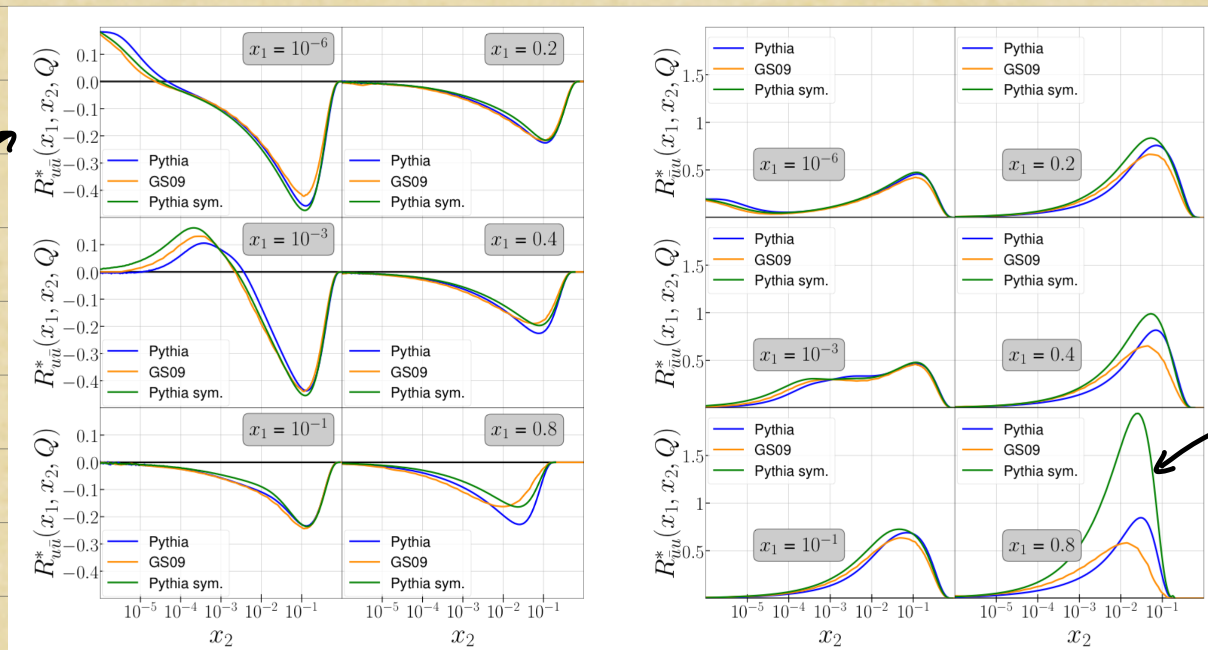
Connected to companion quark mechanism when both quarks have large x

JG, Stirling, 0910.4347.

Let's compare to GS09 dPDFs. Input designed to approximately satisfy sum rules & then evolved to higher scales using inhomogeneous double DGLAP (preserves sum rules).

"Response functions" (~ integrands of sum rules)

$$\frac{x_2 D_{j_1 j_2 v}(x_1,x_2)}{f_{j_1}(x_1)}$$



Large violation of sum rule by Pythia sym. here

Very similar!

Compare dPDFs themselves. Let's do this in the context of a toy pheno study.

Drell-Yan process (to probe quarks & antiquarks). Only cut is $|y_{leptons}| < 5$.
We always set renormalisation & factorisation scales to 91 GeV ($\sim M_Z$).

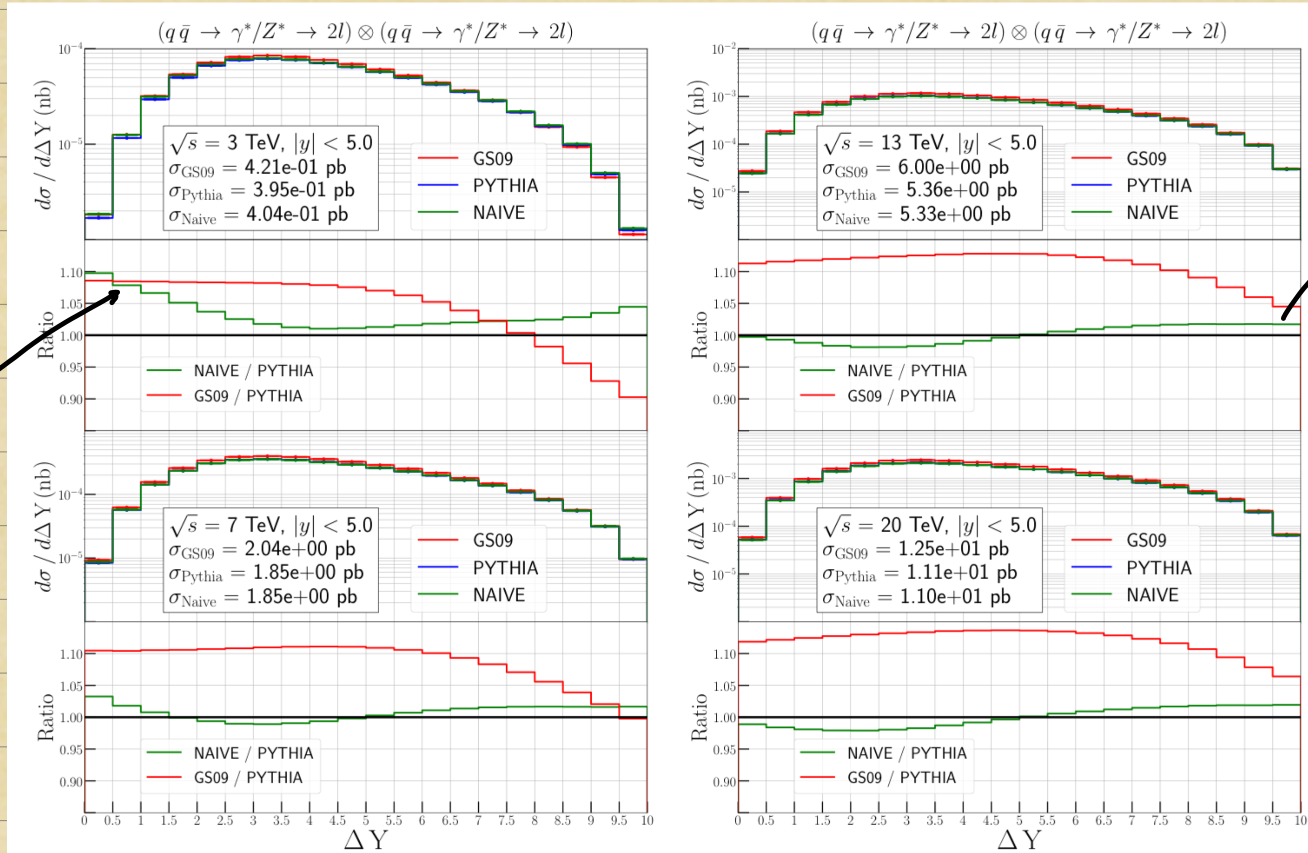
Use naive dPDF formula to compute cross sections:

$$\sigma_{DPS} = \frac{1}{1+\delta_{AB}} \frac{1}{\sigma_{eff}} \sum_{j_i} \int dx_i \, D_{j_1 j_2}(x_1, x_2, Q) \, D_{j_3 j_4}(x_3, x_4, Q) \times \hat{\sigma}_{j_1 j_3 \to A} \, \hat{\sigma}_{j_2 j_4 \to B}$$

Compare to predictions with naive dPDFs:

$$D^{naive}_{j_1 j_2}(x_1, x_2, Q) = f_{j_1}(x_1, Q) \, f_{j_2}(x_2, Q)$$

Plot $\Delta Y = \max |y_i - y_j|$ for different $\sqrt{s}$



Small ΔY: Pythia/naive ~1 except at smallest $\sqrt{s}$. GS09/Pythia ~1.1 at small & intermediate ΔY.

large ΔY: Ratio Pythia/naive decreases as $\sqrt{s}\downarrow$. GS09/Pythia decreases as $\sqrt{s}\downarrow$

Interesting differences in shape, ~10% level. Can we explain these?

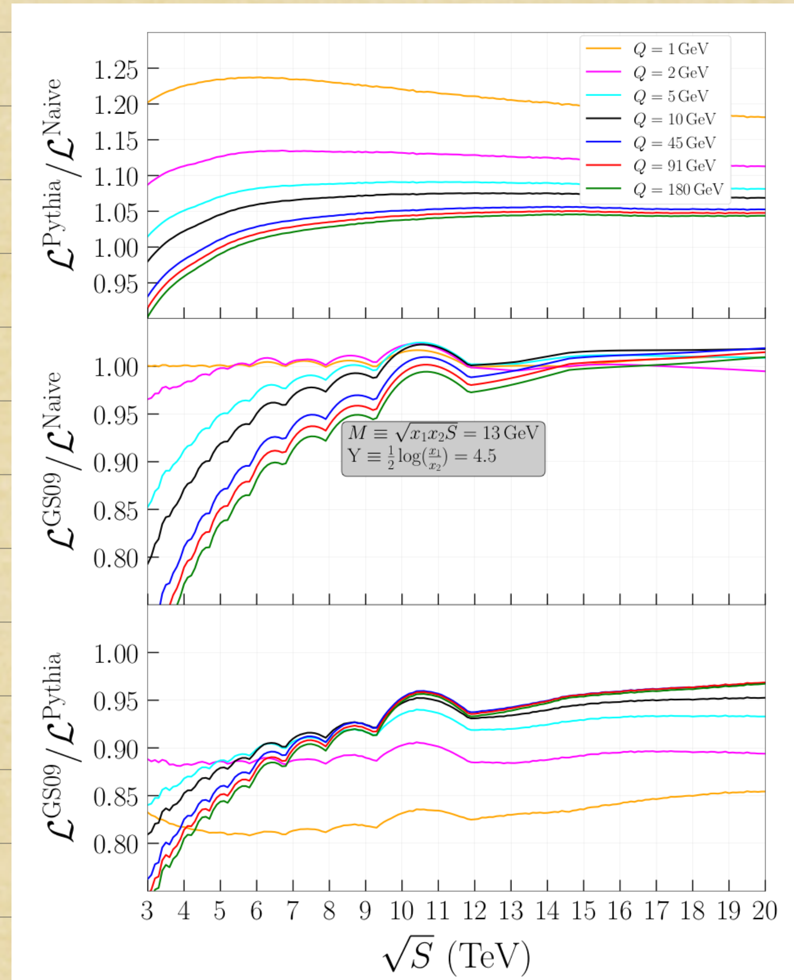
Large $\Delta Y \sim 9-10$.

Dominated by

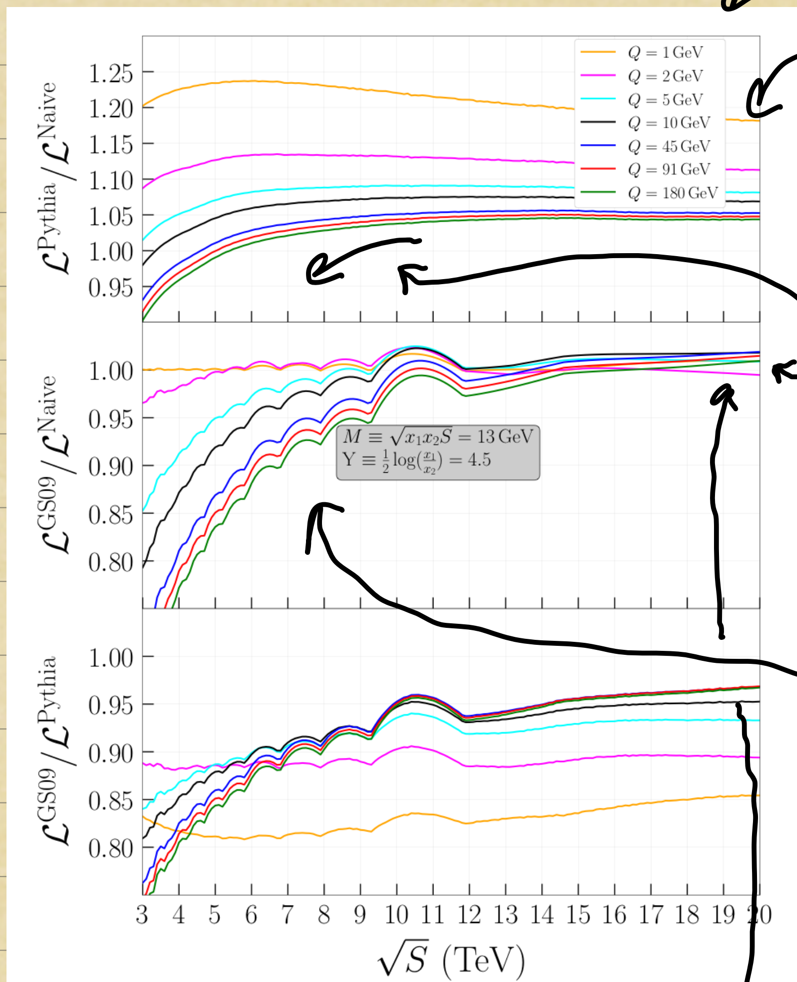
large x u (valence) $\longrightarrow$ $\longleftarrow$ small x $\bar{u}$

small x $\bar{u}$ $\longrightarrow$ $\longleftarrow$ large x u (valence)

Plot luminosity for this config $\longrightarrow$



Plot as a function of factorisation (+ renormalisation) scale Q



Mostly driven by $\bar{u}$ modifications!

a factor initially huge (~1.2-1.3)! Reduces to ~1.1 @ Q~5GeV & then doesn't change much. ~flat in $\sqrt{s}$

Squeezing effect grows as $\sqrt{s}$ ↓ & if Q ↑. Drives this shape.

GS09 / Naive ~ 1 at Q=1GeV (by design!)

At low $\sqrt{s}$ (higher x) GS09 drops below naive as Q↑. Due to suppressions in dPDFs at higher x migrating to lower x.

At high $\sqrt{s}$ (lower x) GS09 rises above naive, due to 1→2 feed.

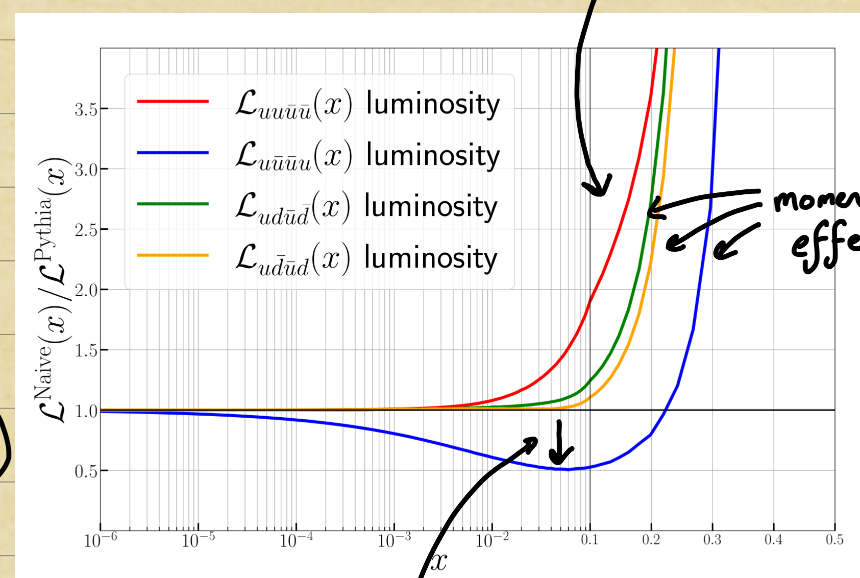
Small $\Delta Y \sim 0 - 0.5$.

Dominated by equal x values in dPDFs, even mix of

$$\begin{matrix} q_i \longrightarrow \longleftarrow \bar{q}_i \\ q_j \longrightarrow \longleftarrow \bar{q}_j \end{matrix} \quad \& \quad \begin{matrix} q_i \longrightarrow \longleftarrow \bar{q}_i \\ \bar{q}_j \longrightarrow \longleftarrow q_j \end{matrix}$$

Plot luminosity for these configs.
Compare first Pythia & naive:

valence number + momentum effects



momentum effect

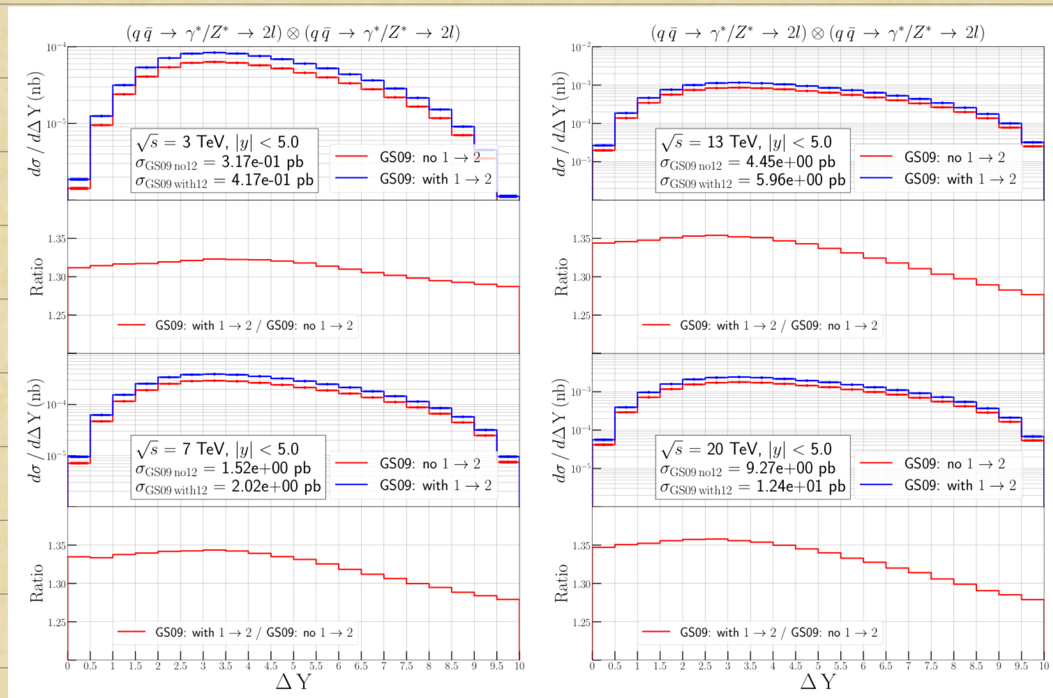
companion quark effect

Momentum & valence number effects suppress Pythia wrt naive at higher x (or smaller $\sqrt{s}$)

Effect actually penetrate down to fairly small x!

Why is GS09 bigger than Pythia at small & intermediate ΔY?

$1 \to 2$ splitting effects during evolution:



SUMMARY

- DPDs $\Gamma_{j_1 j_2}(x_1, x_2, \underline{y}, Q)$ & dPDFs $D_{j_1 j_2}(x_1, x_2, Q)$. DPDs appear in DPS cross section. dPDFs are ~ integral of DPD over $\underline{y}$, satisfy sum rules.

- Pythia has a model of dPDFs. "Asymmetric" dPDFs satisfy sum rules when integrating over 'modified' parton only. Symmetrising in a simple way yields dPDFs that satisfy sum rules to 10-25% level for $x \lesssim 0.4$.

- Comparing Pythia dPDFs to GS09 dPDFs:

  - response functions (~ sum rule integrands) are quite similar!

  - dPDFs themselves / cross section predictions show some differences. Can explain in terms of the different procedures for generating these dPDFs.