Brookhaven National Laboratory

# Parton transport in anisotropic media

**16th November 2022, MPI@LHC 2022**

João Barata, BNL

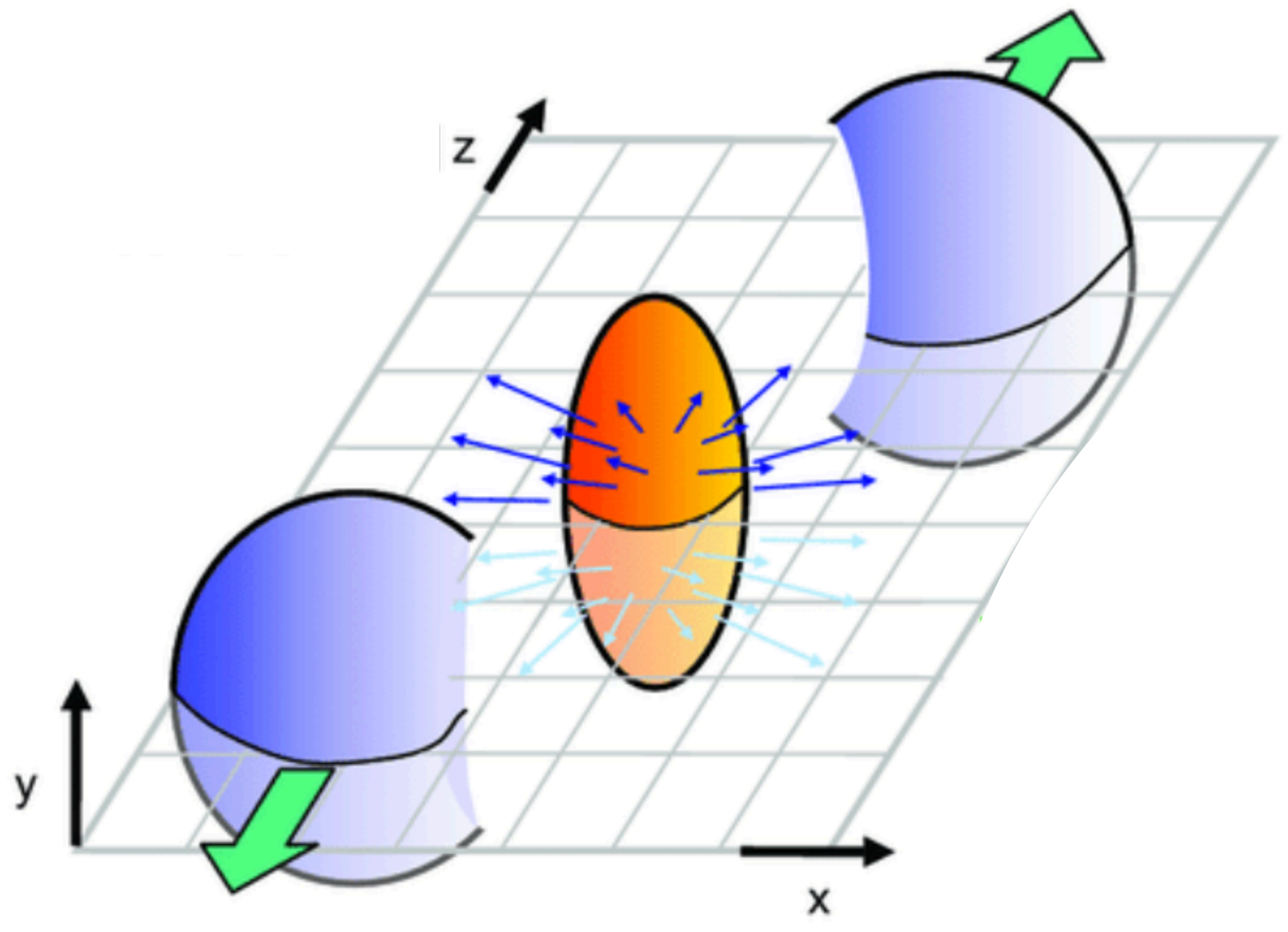
Based on 2210.06519 with A. Sadofyev and X.-N. Wang

# Overview

**Today: parton transport in anisotropic media**



## Why?




## How?

**BDMPS-Z**

Use BDMPS-Z technology

to derive effective kinetic theory

**Kinetic Theory**

See Andrey's talk for details

$$\left\langle \mathcal{G}\left(\boldsymbol{Y}+\frac{\boldsymbol{y}}{2};\boldsymbol{X}+\frac{\boldsymbol{x}}{2}\right)\mathcal{G}^{\dagger}\left(\boldsymbol{Y}-\frac{\boldsymbol{y}}{2};\boldsymbol{X}-\frac{\boldsymbol{x}}{2}\right)\right\rangle$$

Allows to derive particle distribution in pert. theory

Closed form expressions

Painful beyond leading order

$$\left(\partial_L+\frac{\boldsymbol{p}\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}}{E}-\frac{\hat{q}(\boldsymbol{Y})}{4}\partial^2_{\boldsymbol{p}}\right)W(\boldsymbol{Y},\boldsymbol{p})=$$

$$\nabla_i\nabla_j\rho\times\int_{\boldsymbol{q}}\left[\kappa\frac{\partial^2}{\partial p_i\partial p_j}\delta^{(2)}(\boldsymbol{q})-V_{ij}(\boldsymbol{q})\right]W(\boldsymbol{Y},\boldsymbol{p}-\boldsymbol{q})$$

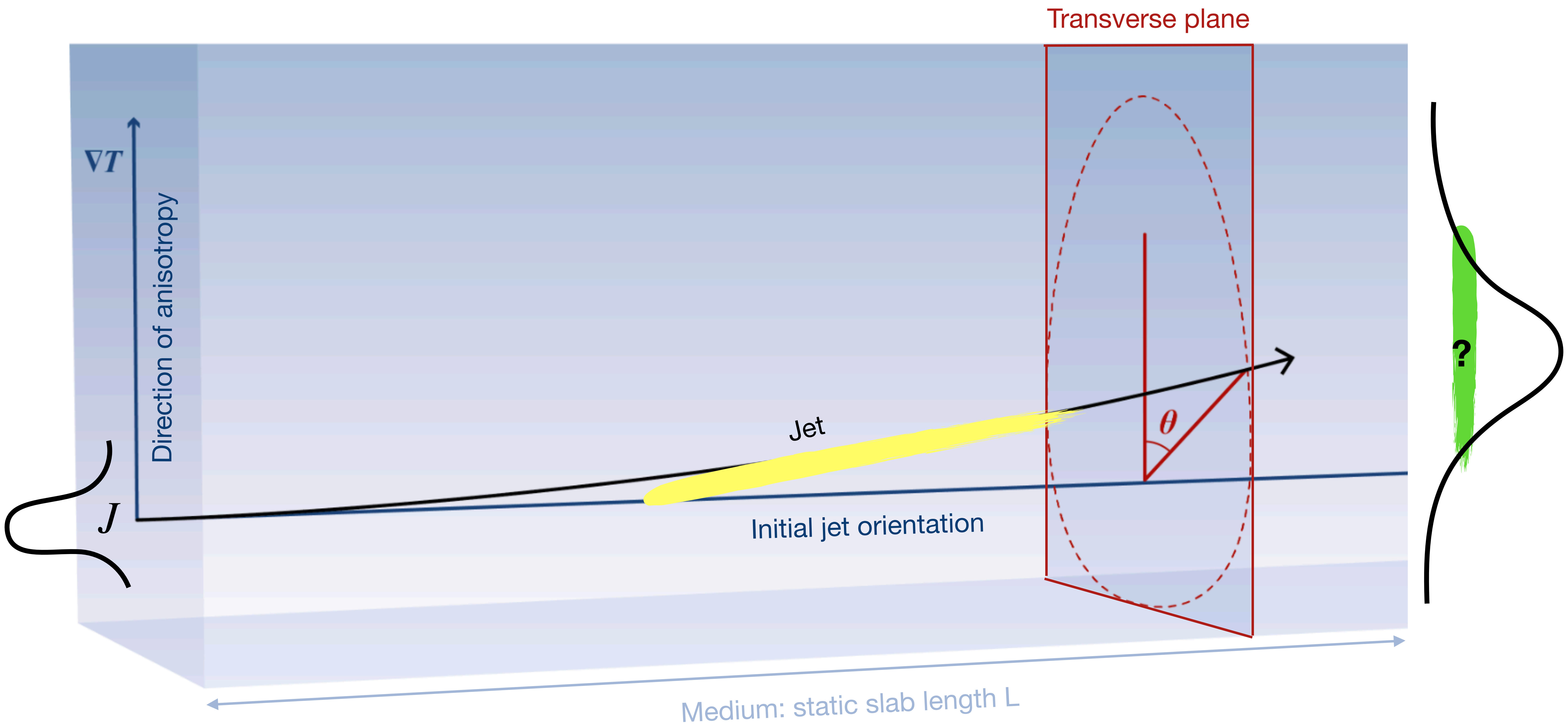
Allows to derive master evolution equations

Valid at all orders in gradients

No closed form solutions

# Recap of momentum broadening

Brookhaven National Laboratory




**Two related questions:** What is the form of the particle distribution at late times? See Andrey's talk

Which effective equation rules the intermediate evolution?

# Transport approach to jet quenching

Parton evolution in the medium can be formulated in terms of an effective kinetic description

Earlier work by Baym, Heinz, ...

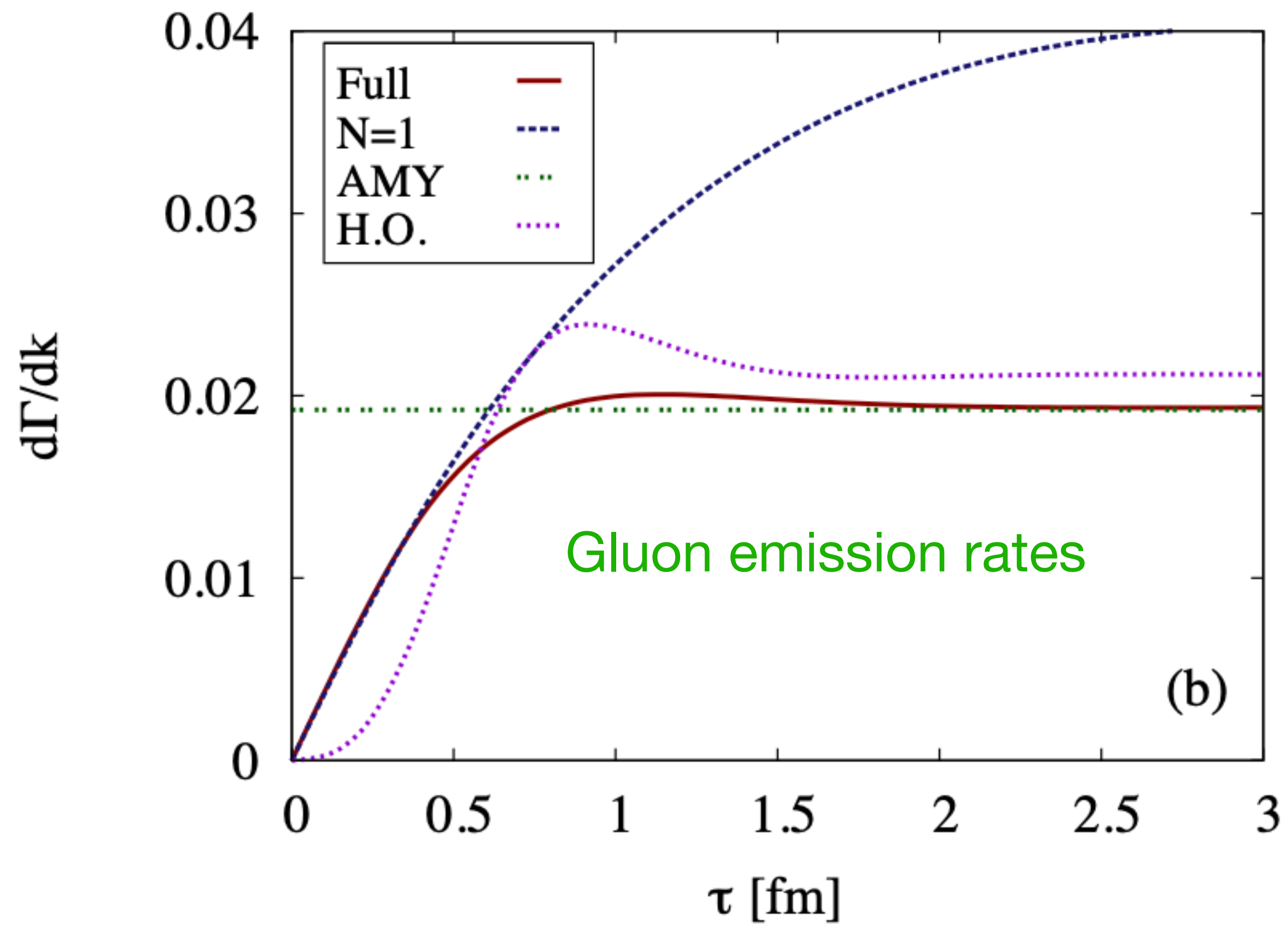
$$\left(\partial_t + \frac{\boldsymbol{p}}{|\boldsymbol{p}|}\cdot\nabla_x\right) f_a(\boldsymbol{p},\mathbf{x},t) = -C_a^{2\leftrightarrow 2}[\{f_i\}] - C_a^{1\leftrightarrow 2}[\{f_i\}]$$

AMY, 2000's

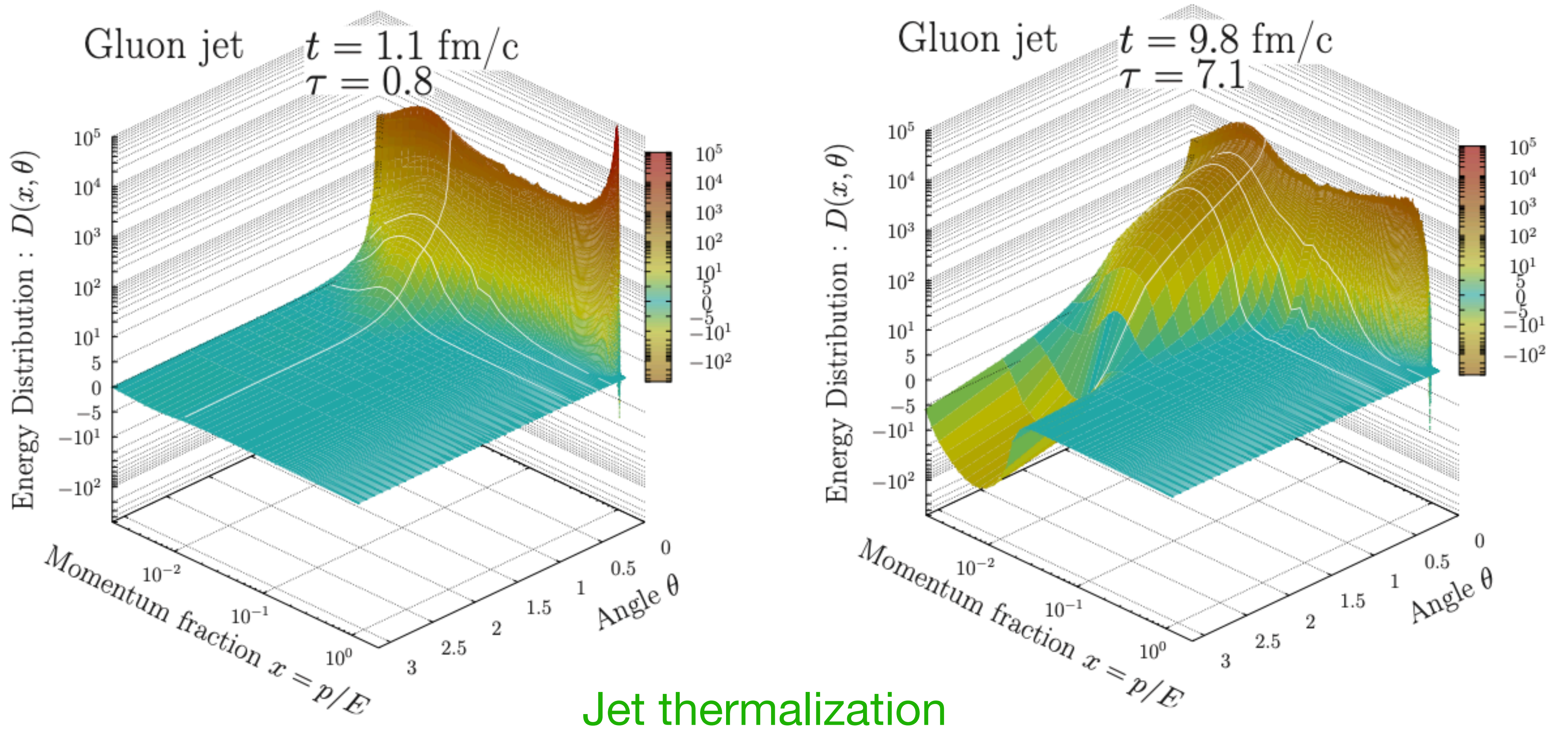
One important comment: K.T. description does **not** emerge from first principle QFT calculation

It has seen ample theory and pheno applications

Caron-Huot, Gale, 1006.2379




Mehtar-Tani, Schlichting, Soudi, 2209.10569



# Transport approach to jet quenching

An effective kinetic picture can also be derived starting from BDMPS-Z formalism

**Simplest example:** Momentum broadening

At leading order we have the classical distribution

$$\mathcal{P}(\boldsymbol{k},t) \equiv \int_{\boldsymbol{x}} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} e^{-\mathcal{V}(\boldsymbol{q})t} = \frac{4\pi}{\hat{q}t} e^{-\frac{\boldsymbol{k}^2}{\hat{q}t}}$$

One can check that taking the time derivative

$$\frac{\partial}{\partial L}\mathcal{P}(\boldsymbol{k},L) = -\int_{\boldsymbol{q}} \mathcal{V}(\boldsymbol{q})\mathcal{P}(\boldsymbol{k}-\boldsymbol{q},L)$$

harmonic approximation →

$$\left(\partial_L - \frac{\hat{q}}{4}\partial_{\boldsymbol{p}}^2\right)\mathcal{P}(\boldsymbol{p},L) = 0$$

Boltzmann + diffusion approx. + spatial isotropy

# Transport approach to jet quenching

2210.06519 J.B., A. Sadofyev, X.-N. Wang

The key approximation taken is the assumption that the medium is isotropic

$$\left(\partial_t + \frac{\boldsymbol{p}}{|\boldsymbol{p}|}\cdot\nabla_x\right) f_a(\boldsymbol{p},\boldsymbol{x},t) = -C_a^{2\leftrightarrow 2}[\{f_i\}]$$

but I promised **Today: parton transport in anisotropic media**

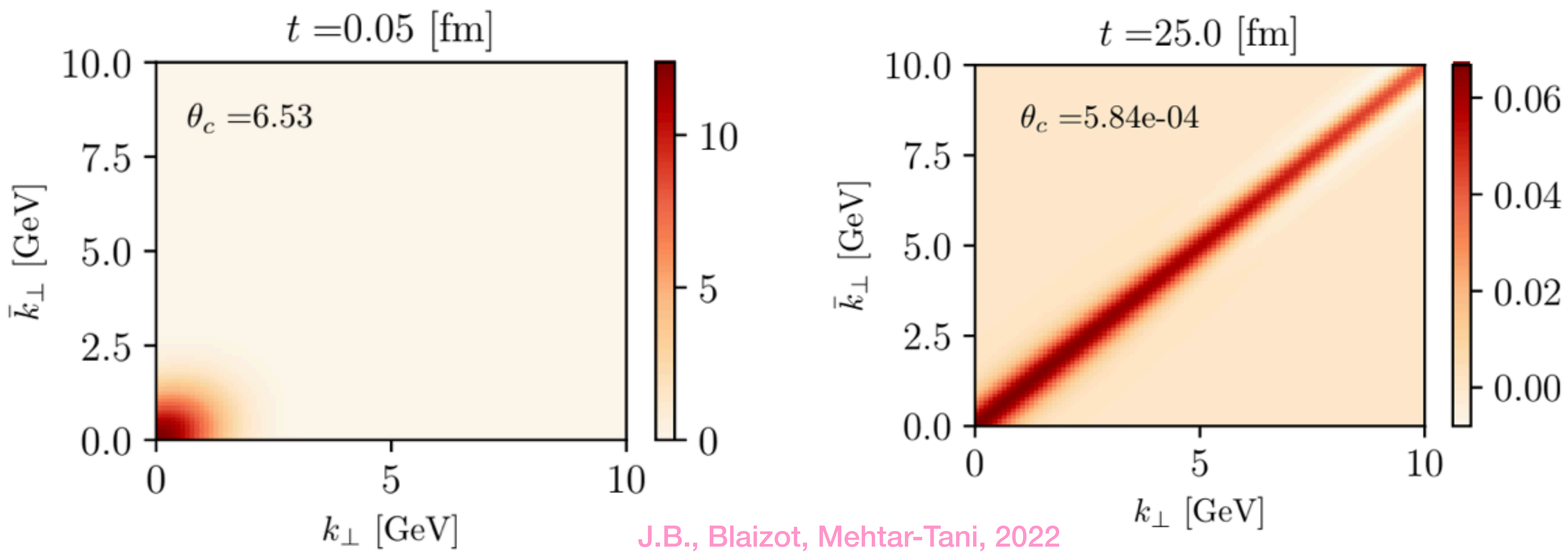
X.-N. Wang et al, 2020, 2022

To go beyond, one needs to introduce the parton Wigner function / density matrix

Generalized correlator from previous talk

$$W_L(\boldsymbol{Y},\boldsymbol{p}) \equiv \int_{\boldsymbol{y},\boldsymbol{x},\boldsymbol{X},\boldsymbol{p}_0} e^{-i(\boldsymbol{p}\cdot\boldsymbol{y}-\boldsymbol{p}_0\cdot\boldsymbol{x})}\, W_0(\boldsymbol{X},\boldsymbol{p}_0)\times\left\langle \mathcal{G}\left(\boldsymbol{Y}+\frac{\boldsymbol{y}}{2};\boldsymbol{X}+\frac{\boldsymbol{x}}{2}\right)\mathcal{G}^\dagger\left(\boldsymbol{Y}-\frac{\boldsymbol{y}}{2};\boldsymbol{X}-\frac{\boldsymbol{x}}{2}\right)\right\rangle$$

Even for homogeneous media allows to further study novel quantum effects




J.B., Blaizot, Mehtar-Tani, 2022

# Transport approach to jet quenching

2210.06519 J.B., A. Sadofyev, X.-N. Wang



We can compute $\left\langle \mathcal{G}\left(\boldsymbol{Y}+\frac{\boldsymbol{y}}{2};\boldsymbol{X}+\frac{\boldsymbol{x}}{2}\right)\mathcal{G}^{\dagger}\left(\boldsymbol{Y}-\frac{\boldsymbol{y}}{2};\boldsymbol{X}-\frac{\boldsymbol{x}}{2}\right)\right\rangle$ order by order in gradients

With the resulting form, one can do the analog computation to:

$$\frac{\partial}{\partial L}\mathcal{P}(\boldsymbol{k},L) = -\int_{\boldsymbol{q}} \mathcal{V}(\boldsymbol{q})\mathcal{P}(\boldsymbol{k}-\boldsymbol{q},L) \quad \xrightarrow{\text{harmonic approximation}} \quad \left(\partial_L - \frac{\hat{q}}{4}\partial_{\boldsymbol{p}}^2\right)\mathcal{P}(\boldsymbol{p},L) = 0$$

To prove that the Wigner function satisfies Boltzmann-Lorentz-diffusion transport

Kinetic theory with local interaction in $\boldsymbol{Y}$

$$\left(\partial_L + \frac{\boldsymbol{p}}{E}\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}} - \frac{\hat{q}(\boldsymbol{Y})}{4}\partial_{\boldsymbol{p}}^2\right)W(\boldsymbol{Y},\boldsymbol{p}) = \mathcal{O}\left(\partial_\perp^2\hat{q}\right) \quad \text{with} \quad \hat{q}(\boldsymbol{Y}) \simeq \hat{q} + \boldsymbol{Y}\cdot\boldsymbol{\nabla}\hat{q} \longrightarrow \hat{q}(\boldsymbol{Y})$$

Note: Integrating over $\boldsymbol{Y}$ gives the usual diffusion equation

# Transport approach to jet quenching

2210.06519 J.B., A. Sadofyev, X.-N. Wang

Brookhaven National Laboratory

## Phenomenologically why is this result relevant?

The minimal replacement $\hat{q}(Y)$ form justifies usual approach to include geometry $\hat{q} \sim T^3 \rightarrow T^3(t, \vec{x})$

**Gradient Tomography of Jet Quenching in Heavy-Ion Collisions**


Yayun He,[1] Long-Gang Pang,[1] and Xin-Nian Wang[1,2,*]

[1] *Key Laboratory of Quark & Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

[2] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*


$$\frac{\partial f_a}{\partial t} + \frac{\vec{k}_\perp}{\omega} \cdot \frac{\partial f_a}{\partial \vec{r}_\perp} = \frac{\hat{q}_a}{4} \vec{\nabla}^2_{k_\perp} f_a(\vec{k}, \vec{r})$$

**Jet-temperature anisotropy revealed through high-$p_\perp$ data**


Stefan Stojku, Jussi Auvinen, Lidija Zivkovic, and Magdalena Djordjevic*

*Institute of Physics Belgrade, University of Belgrade, Serbia*

Pasi Huovinen

*Institute of Physics Belgrade, University of Belgrade, Serbia and Incubator of Scientific Excellence—Centre for Simulations of Superdense Fluids, University of Wrocław, Poland*


(Dated: March 1, 2022)

## When does this stop being correct?

A clue:

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\boldsymbol{r}(\tau),\tau)\right) \mathcal{P}\exp\left(i\int_0^L d\bar{\tau}\, t^b_{\text{proj}} v^b(\bar{\boldsymbol{r}}(\bar{\tau}),\bar{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau\left[1 + \frac{\boldsymbol{r}(\tau)+\bar{\boldsymbol{r}}(\tau)}{2}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right]\mathcal{V}\left(\boldsymbol{r}(\tau)-\bar{\boldsymbol{r}}(\tau)\right)\right\}$$

**Asymmetric transverse momentum broadening in an inhomogeneous medium**


Yu Fu,[1] Jorge Casalderrey-Solana,[2] and Xin-Nian Wang[1,3,*]

[1] *Key Laboratory of Quark and Lepton Physics (MOE) & Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

[2] *Departament de Física Quàntica i Astrofísica & Institut de Ciències del Cosmos (ICC), Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain*

[3] *Nuclear Science Division, Lawrence Berkeley National Laboratory, CA 94720, Berkeley, USA*


which is usually assumed to take the all order form

$$\frac{1}{N_c}\left\langle\!\left\langle tr\{W(\boldsymbol{r}_1)W^\dagger(\boldsymbol{r}_2)\}\right\rangle\!\right\rangle \approx \exp\{-\int_{x_0^+}^{x_f^+} \mathrm{d}x^+ \frac{1}{4\sqrt{2}}\hat{q}(\boldsymbol{R})\,\boldsymbol{r}^2\}$$

A direct calculation shows it fails at the 2nd order

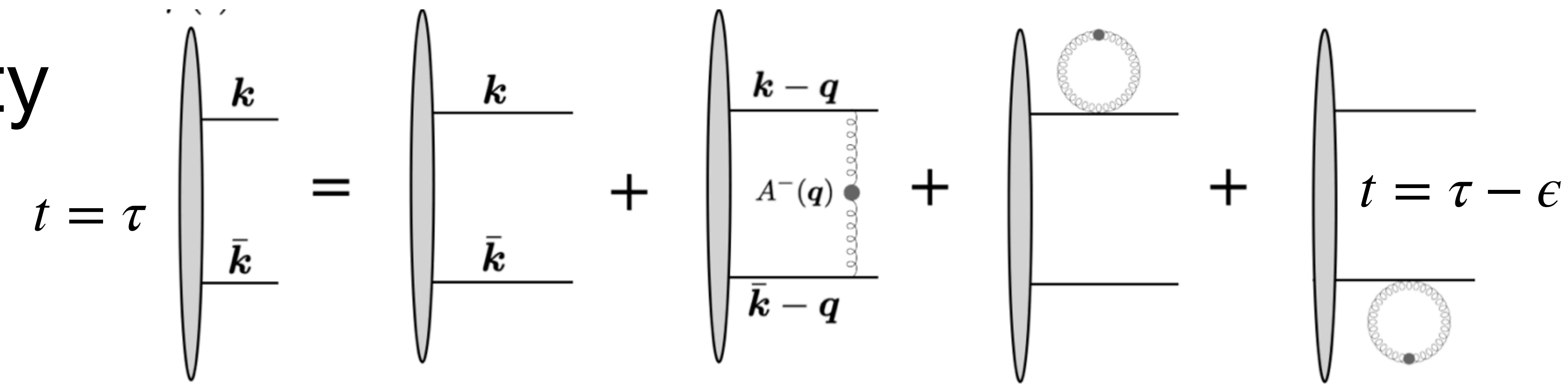
# Transport approach to jet quenching

2210.06519 J.B., A. Sadofyev, X.-N. Wang

Brookhaven National Laboratory

To solve this question, we construct the evolution kernel accounting for density gradients

In general, we have the convolution property




$$\langle \mathcal{G}^\dagger(\bar{\boldsymbol{k}}, L+\epsilon; \bar{\boldsymbol{k}}_0, 0)\, \mathcal{G}(\boldsymbol{k}, L+\epsilon; \boldsymbol{k}_0, 0)\rangle = \int_{\boldsymbol{l},\bar{\boldsymbol{l}}} \langle \mathcal{G}^\dagger(\bar{\boldsymbol{k}}, L+\epsilon; \bar{\boldsymbol{l}}, L)\, \mathcal{G}(\boldsymbol{k}, L+\epsilon; \boldsymbol{l}, L)\rangle \times \langle \mathcal{G}^\dagger(\bar{\boldsymbol{l}}, L; \bar{\boldsymbol{k}}_0, 0)\, \mathcal{G}(\boldsymbol{l}, L; \boldsymbol{k}_0, 0)\rangle$$

Valid for Gaussian background + locality of average

No need to compute $W$, i.e. we can go to all order transport equation:

$$\partial_L W(\boldsymbol{k}, \bar{\boldsymbol{k}}) = -i\,\frac{\boldsymbol{k}^2 - \bar{\boldsymbol{k}}^2}{2E} W(\boldsymbol{k}, \bar{\boldsymbol{k}}) - \int_{\boldsymbol{q},\bar{\boldsymbol{q}},\boldsymbol{l},\bar{\boldsymbol{l}}} \mathcal{K}(\boldsymbol{q}, \bar{\boldsymbol{q}}; \boldsymbol{l}, \bar{\boldsymbol{l}}) W(\boldsymbol{l}, \bar{\boldsymbol{l}})$$

$$\mathcal{K}(\boldsymbol{q}, \bar{\boldsymbol{q}}; \boldsymbol{l}, \bar{\boldsymbol{l}}) = -(2\pi)^4\, C\, v(\boldsymbol{q}) v(\bar{\boldsymbol{q}}) \times \Big\{ \rho(\boldsymbol{q}-\bar{\boldsymbol{q}})\, \delta^{(2)}(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{l})\delta^{(2)}(\bar{\boldsymbol{k}}-\bar{\boldsymbol{q}}-\bar{\boldsymbol{l}}) - \frac{1}{2}\rho(\boldsymbol{q}+\bar{\boldsymbol{q}})\, \delta^{(2)}(\boldsymbol{k}-\boldsymbol{l})\delta^{(2)}(\bar{\boldsymbol{k}}-\boldsymbol{q}-\bar{\boldsymbol{q}}-\bar{\boldsymbol{l}}) - \frac{1}{2}\rho^\dagger(\boldsymbol{q}+\bar{\boldsymbol{q}})\, \delta^{(2)}(\boldsymbol{k}-\boldsymbol{q}-\bar{\boldsymbol{q}}-\boldsymbol{l})\delta^{(2)}(\bar{\boldsymbol{k}}-\bar{\boldsymbol{l}}) \Big\}$$

# Transport approach to jet quenching

2210.06519 J.B., A. Sadofyev, X.-N. Wang

In this form it is hard to digest: expand in gradients + harmonic approximation

0th: Boltzmann + diffusion ✓

1st: Boltzmann + diffusion + $\hat{q}(Y)$ ✓

2nd: Boltzmann + diffusion + $\hat{q}(Y)$ + quantum corrections

$$\kappa = 2\pi^2 C \int_{\boldsymbol{q}} v^2$$

$$\left(\partial_L + \frac{\boldsymbol{p}\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}}{E} - \frac{\hat{q}(\boldsymbol{Y})}{4}\partial^2_{\boldsymbol{p}}\right) W(\boldsymbol{Y},\boldsymbol{p}) = \nabla_i\nabla_j\rho \times \int_{\boldsymbol{q}} \left[\kappa \frac{\partial^2}{\partial p_i \partial p_j}\delta^{(2)}(\boldsymbol{q}) - V_{ij}(\boldsymbol{q})\right] W(\boldsymbol{Y},\boldsymbol{p}-\boldsymbol{q})$$

$$V_{ij}(\boldsymbol{q}) = \frac{C}{2}\left(\left\{2q_iq_j\left[vv'' - v'v'\right] + vv'\delta_{ij}\right\} - (2\pi)^2\delta^{(2)}(\boldsymbol{q})\int_{\boldsymbol{l}}\left\{2l_il_j\left[vv''-v'v'\right] + vv'\delta_{ij}\right\}\right)$$

Now the equation is sensitive scattering rate + corrections inherited from kinetic phases

Comparing to naive K.T., one needs to consider transport with non-local interactions

# Transport approach to jet quenching

2210.06519 J.B., A. Sadofyev, X.-N. Wang

Brookhaven National Laboratory

## How large can such corrections be?

It is natural to look at $\hat{q}L = \int_{\boldsymbol{p},\boldsymbol{Y}} \boldsymbol{p}^2 W(\boldsymbol{p},\boldsymbol{Y},L)$

## A simple calculation gives

Coef. homogenous case

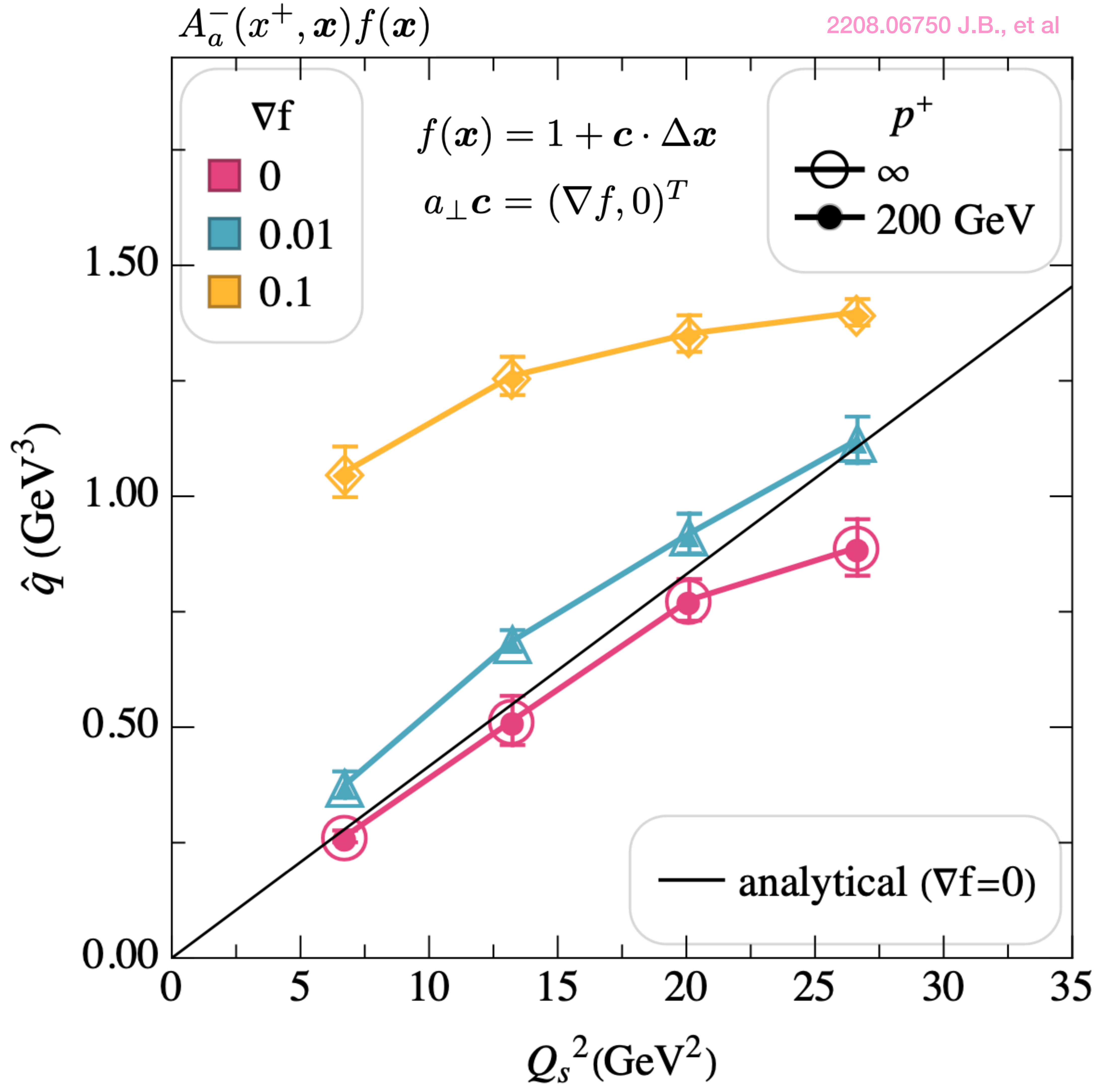
$$\hat{q}_r = \hat{q} + \boldsymbol{\nabla}^2 \hat{q} \left( \frac{\hat{q}L^3}{12E^2} + \eta \right)$$

Full coeff.

Coef. due to anisotropy effects

with $\eta = \rho\,\kappa/(2\pi^2\hat{q}) + \frac{C\rho}{2\hat{q}} \int_{\boldsymbol{q}} \boldsymbol{q}^2 v^2 [\boldsymbol{q}^2 v'/v]'$

2208.06750 J.B., et al

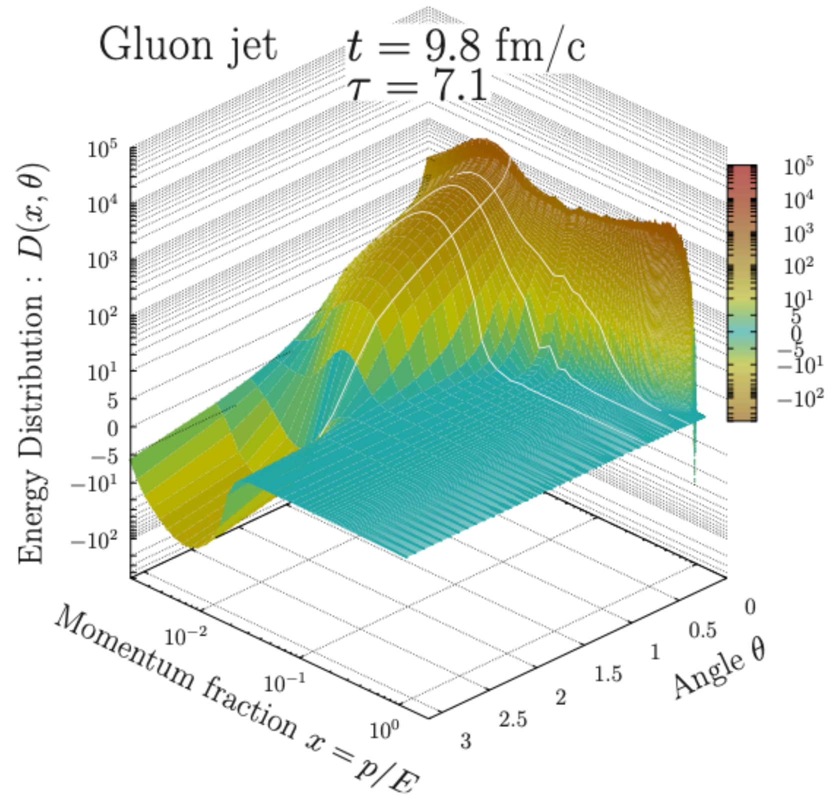


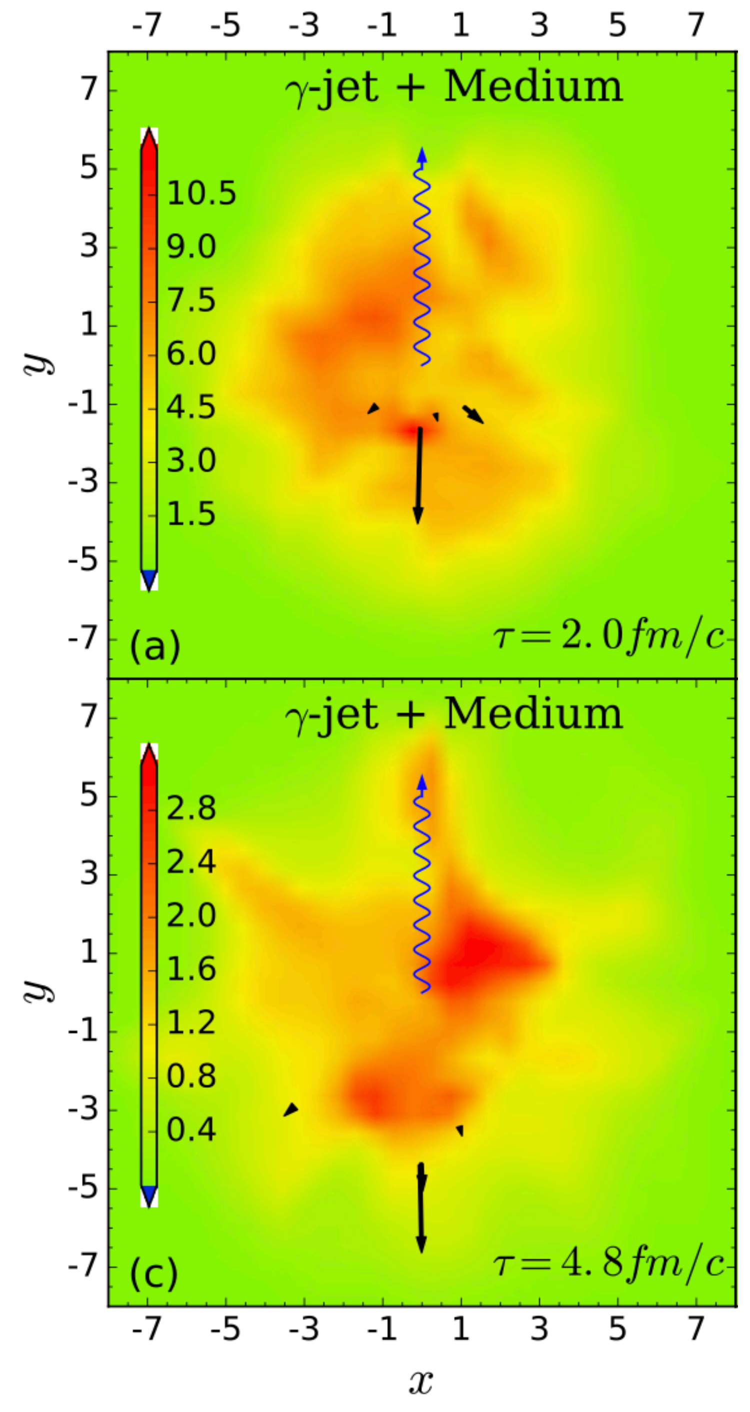
# Conclusion and Outlook

→ We constructed the all order in gradients transport equation for partons in anisotropic media

$$\left(\partial_L + \frac{\boldsymbol{p}\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}}{E} - \frac{\hat{q}(\boldsymbol{Y})}{4}\partial^2_{\boldsymbol{p}}\right) W(\boldsymbol{Y},\boldsymbol{p}) = \nabla_i\nabla_j\rho \times \int_{\boldsymbol{q}} \left[\kappa\frac{\partial^2}{\partial p_i \partial p_j}\delta^{(2)}(\boldsymbol{q}) - V_{ij}(\boldsymbol{q})\right] W(\boldsymbol{Y},\boldsymbol{p}-\boldsymbol{q})$$

→ Can be implemented in current numerical models; it is half the story for medium response




1704.03648, Chen et al