Perturbative Color Correlations in Double Parton Scattering

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Aim and Results

- The aim of my talk is to estimate color correlations in Double Parton Scattering (DPS)

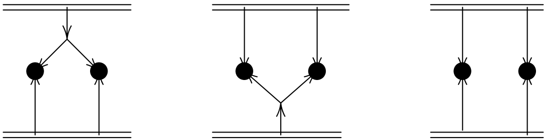

Figure: $1+2$ and $2+2$ processes

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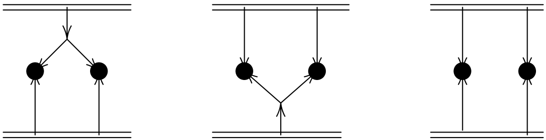

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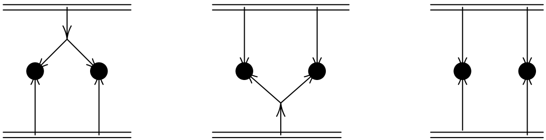
- The total contribution of color correlation account only to 5% of the total cross section. However this contribution is approximately constant in Q (relative to the total DPS cross section)



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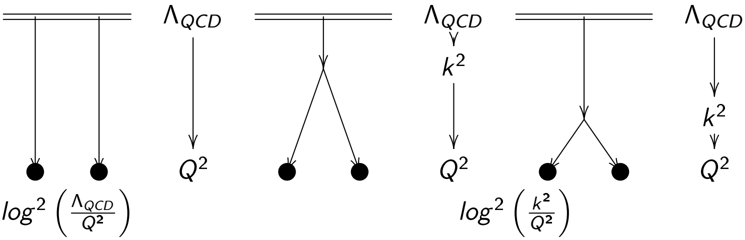
Relaxation of Sudakov Suppression

- The Sudakov-like suppression comes from evolution of non-singlet ladders



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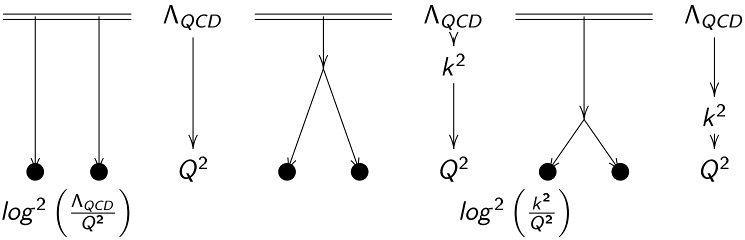

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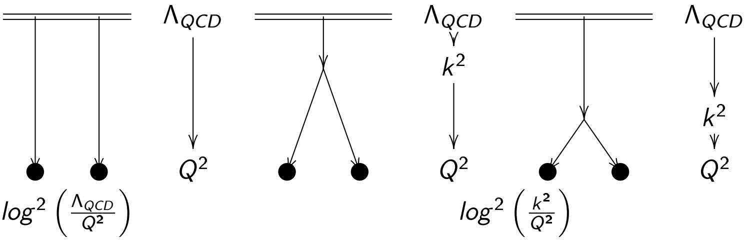
- The effective splitting scale for non-singlet GPD increases with Q^2



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k^2 dependence of GPD

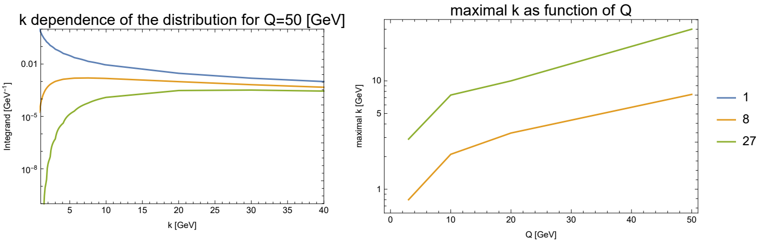

Figure: The integrand of the GPD, after performing the z and y integrals, in the LHC kinematics and $Q = 50$ $[GeV]$

Outline

- Computation of the non-singlet $1+2$ Generalized Parton Distribution (GPD)
  - Non singlet DGLAP
  - Formula for GPD
  - Regularization of divergent integrals
- Contribution to final states: the interference diagram
- Numeric results
- Conclusion

Computation of the GPD

Method and Scope

- In calculation of GPD we calculate $1 \to 2$ contribution to non-singlet GPD in LLA by generalizing the DDT formula for singlet case
- Non-singlet GPD were also considered in [Diehl, Gaunt, Plossl, 2021] and [Diehl, Gaunt, Pichini, Plossl, 2021] using a different method
- We expect the results to agree in 1-loop approximation

Non-Singlet DGLAP

- The DGLAP equation gets contribution from real (ladder) and virtual (self energy) diagrams
- For color non-singlet channels these have different color factors (C_A^A and $\overline{C}_A^A$)
- These color factors are known ([Diehl, Ostermeier, Schafer, 2012,2016])

$$\frac{\partial \overline{D}_A^B\left(x;Q^2,k^2\right)}{\partial ln\left(k^2\right)}$$

$$= -\frac{\alpha_s\left(k_0^2\right)}{4\pi}\sum_C \int_0^1 \frac{dz}{z}\left[\underbrace{\Phi_A^C(z)\,\overline{D}_C^B\left(x;Q^2,k^2\right)}_{virtual\ \ contributions} - \underbrace{\overline{\Phi}_A^C(z)\,z^2\overline{D}_A^B\left(x;Q^2,k^2\right)}_{real\ \ contributions}\right]$$

Non-Singlet DGLAP Solution

- The fundamental solution of the non-singlet DGLAP has the form:

$$\overline{D}_A^B\left(k^2, Q^2 = k^2, x\right) = \delta_A^B \delta\left(1 - x\right)$$

$$\overline{D}_A^B\left(x; Q^2, k^2\right) = \overline{S}_A\left(Q^2, k^2\right) \widetilde{D}_A^B\left(x; Q^2, k^2\right)$$

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- $\overline{S}_A \sim e^{\left(C_A^A - \overline{C}_A^A\right) log^2\left(\frac{Q^2}{k^2}\right)}$ is a Sudakov suppression factor
- $\widetilde{D}$ is a solution of the non-singlet evolution and can be evaluated numerically

$$\frac{\partial \widetilde{D}_A^B\left(x; Q^2, k^2\right)}{\partial ln\left(k^2\right)} = -\frac{\alpha_s\left(k_0^2\right)}{4\pi} \sum_C \int_0^1 \frac{dz}{z} \left[\underbrace{\overline{\Phi}_A^C\left(z\right) \widetilde{D}_C^B\left(x; Q^2, k^2\right)}_{new\ \ kernels} - \underbrace{\overline{\Phi}_A^C\left(z\right) z^2 \widetilde{D}_A^B\left(x; Q^2, k^2\right)}_{new\ \ kernels}\right]$$

GPD of $1 \to 2$ Process From DGLAP

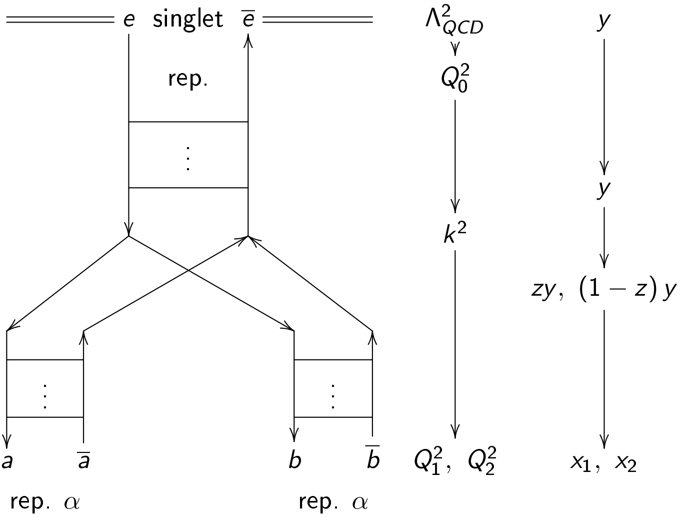

Figure: Different channels in $1 \to 2$ process

GPD of $1 \to 2$ process

- From the picture above we can write explicitly the GPD:

$$
{}^{\alpha}_{[1]}\overline{D}_h^{AB}\left(x_1, x_2, Q_1, Q_2\right) = \underbrace{\sum_{E,A',B'} \int\limits_{Q_0^2}^{min\left(Q_1^2, Q_2^2\right)} \frac{dk^2}{k^2} \frac{\alpha_s\left(k^2\right)}{2\pi} \int \frac{dy}{y} G_h^E\left(y; k^2\right)}_{\text{singlet evolution of E up to } k^2}
$$

$$
\times \underbrace{\int \frac{dz}{z\left(1-z\right)} V_E^{A'}\left(z\right) \frac{n_{A'}}{n_E} {}^{\alpha}\overline{C}_{A'}^{B'}}_{\text{splitting kernel}}
$$

$$
\times \underbrace{{}^{\alpha}\overline{D}_{A'}^{A}\left(\frac{x_1}{zy}; Q_1^2, k^2\right) {}^{\alpha}\overline{D}_{B'}^{B}\left(\frac{x_2}{\left(1-z\right)y}; Q_2^2, k^2\right)}_{\text{non-singlet evolution of A and B from } k^2 \text{ to } Q^2}
$$

- G_h^E are the distribution of single parton E in the hadron
- Equation for singlet case was given in [Blok, Dokshitser, Frankfurt, Strikman, 2011]
- The same quantity was considered in [Diehl, Gaunt, Plossl, 2021] in a different way

Divergent Integrals

- It can be shown that (using the methods of [Dokshitser, Diakonov, Troian, 1980]):

$$\overline{D}_A^A \overset{x \to 1}{\propto} \frac{1}{(1-x)^{1-\xi \overline{C}_A^A}}, \ \xi = \frac{3}{\beta_0} ln \left[\frac{\alpha_s\left(k^2\right)}{\alpha_s\left(Q^2\right)} \right]$$

- For negative color kernels $\overline{C}_A^A$ the GPD integrals *dzdy* diverge
  - at either $zy \to x_1$ or $(1-z)\,y \to x_2$

Regularization of Divergent Integrals

- We use a regularization method due to [Gel'fand, 1958] (and similar method also used by [Manohar and Waalewijn, 2012])

$$\int_0^1 dx \frac{f(x)}{x^\lambda} := \int_0^1 \frac{f(x) - f(0)}{x^\lambda} dx + \frac{f(0)}{1-\lambda}$$

- Which we generalize to a 2d case (straight forward)

The Interference Diagram

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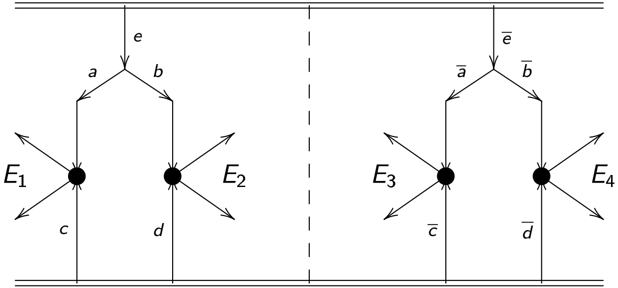


Figure: Double parton scattering diagram and its complex conjugate

- When squaring the amplitude 2 possible contractions
  - E_1 to E_3 is the direct diagram
  - E_1 to E_4 is the **interference** diagram (events must be close in phasespace)

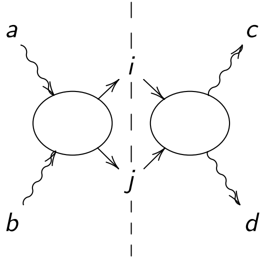
Final States Considered

- We only look at gluon distributions because its color factors should be bigger
- We look at the following final states:
  - $2 \times q + \overline{q}$ (two quark dijets)
  - $2 \times g + g$ (two gluon dijets)
  - $2 \times \chi_1$
  - $2 \times J/\psi$
  - $\chi_0 \; + \; J/\psi$
- We only look at central kinematics ($Q_1 = Q_2$ and $x_1 = x_2 = x_3 = x_4$) where we expect maximal contribution
- We use color projectors to compute the cross section (e.g [Buffing, Diehl, Kasemets, 2018] and others)

Example of Computation: The Hard Process

- We give example of the cross section calculation
- We first write the hard process in terms of color projectors:

$$\frac{1}{N^2-1}\cdot$$



$$= \frac{\mathcal{M}_{ab;ij}\mathcal{M}^{\dagger}{}_{cd;ij}}{N^2-1} = \Sigma^{\alpha} P^{\alpha}_{ab;cd}$$

Figure: The hard process amplitude partially contracted with its complex conjugate.

Example of Computation: The Interference Diagram

- The interference diagram amplitude can then be written in the form:

$$\sigma_I = \left({}^{\alpha}_{[1]}\overline{D}P^{\alpha}_{a\bar{a};b\bar{b}} \right) \left({}_{[2]}\overline{D}P^{1}_{c\bar{c};d\bar{d}} \right) \left(\Sigma^{\beta} P^{\beta}_{ac;\overline{bd}} \right) \left(\widetilde{\Sigma}^{\gamma} P^{\gamma}_{bd;\overline{ac}} \right)$$

- For example in the 2 gluons dijet case we have:

$$\Sigma^{ab;cd}_{gg\to gg} = N^2 C_a P^{8_a}_{ab;cd} + 4N^2 C_s P^{1}_{ab;cd} + N^2 C_s P^{8_s}_{ab;cd} + 4C_s P^{27}_{ab;cd} + 4C_s P^{0}_{ab;cd}$$

$$C_a = \frac{g^4}{(N^2-1)} \left(-\frac{4\hat{t}\hat{u}}{\hat{s}^2} - \frac{3\hat{s}^2}{\hat{t}\hat{u}} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} + 9 \right)$$

$$C_s = \frac{g^4}{(N^2-1)} \left(\frac{\hat{s}^2}{\hat{t}\hat{u}} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} + 1 \right)$$

- So σ_I can be written completely in terms of s, t and u

Numeric Results

Numeric results for the GPD

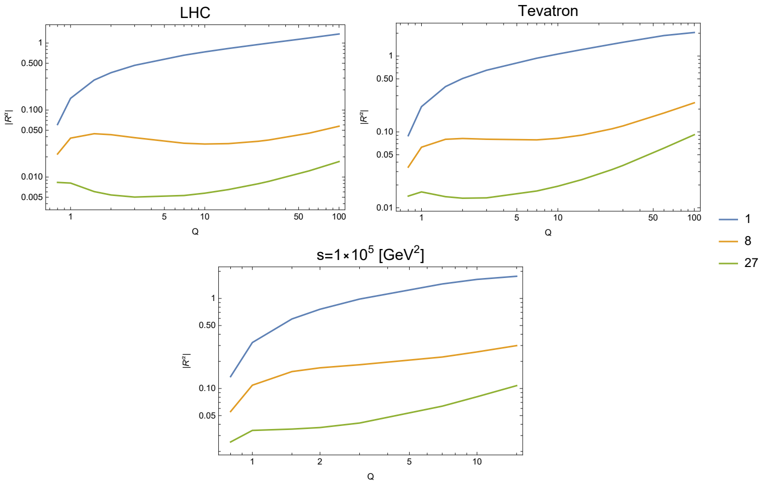


Figure: Ratio (in absolute value) of different channels of $1 \to 2$ distribution to the singlet DPD for gluonic GPD. The fact that these distributions can be negative is not a surprise [Diehl, Gaunt, Pichini, Plossl, 2021].

Numeric results for the Cross Section

The ratio $R_{tot}{}^{\alpha}$ at the LHC kinematics

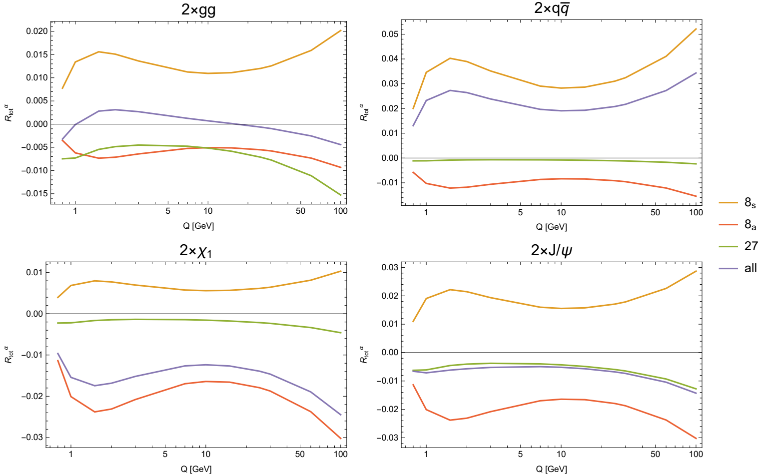


Figure: Relative contribution of non-singlet channels to the DPS cross section

Numeric results for the Cross Section

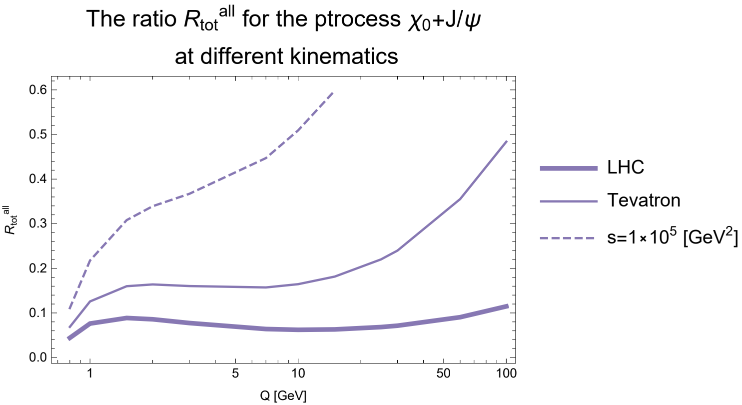


Figure: Relative contribution of non-singlet channels to the $\chi_0 + J/\psi$ cross section

Conclusion

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- We did not account for non-diagonal ladders (i.e soft gluons connecting different partons [Gaunt, 2013]) that can contribute to the direct diagram
- Although the contributions are small they might be observed experimentally when comparing different kinematics (e.g as function of $\frac{x_1}{x_2}$ and $\frac{x_3}{x_4}$), we will leave this for future research

Thank You for Listening